A “Permanent” High-Temperature Superconducting Magnet Operated in Thermal Communication with a Mass of Solid Nitrogen

by

Benjamin J. Haid

Submitted to the Department of Mechanical Engineering in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Mechanical Engineering at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Author

Department of Mechanical Engineering

May 4, 2001

Certified by.

Yukikazu Iwasa

Research Professor, Francis Bitter Magnet Laboratory, and Senior Lecturer, Department of Mechanical Engineering, MIT

Thesis Supervisor

Accepted by

Ain A. Sonin

Chairman, Committee on Graduate Students
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Abstract

This thesis explores a new design for a portable “permanent” superconducting magnet system. The design is an alternative to permanent low-temperature superconducting (LTS) magnet systems where the magnet is cooled by a bath of liquid helium. The new design involves a high-temperature superconducting (HTS) magnet that is cooled by a solid heat capacitor.

An apparatus was constructed to demonstrate stable operation of a permanent magnet wound with Bi2223/Ag conductor while in thermal communication with a mass of solid nitrogen. The system includes a room-temperature bore and can function while it stands alone, detached from its cooling source, power supply, and vacuum pump. The magnet is operated in the 20–40 K temperature range. This apparatus is the first to demonstrate the operation of a superconducting magnet with a permissible temperature variation exceeding a few degrees kelvin while a magnetic field is maintained for a useful duration. Models are developed to predict the experimental system’s warming trend and magnetic field decay. The models are validated with a good agreement between simulations based on these models and experimental results.

Potential performance advantages of a solid nitrogen cooled permanent HTS (SN2/HTS) magnet system over a liquid helium cooled LTS (LHe/LTS) system are explored for various applications. The SN2/HTS system design includes a second solid heat capacitor that cools a radiation shield. Recooling of the heat capacitors is performed with a detachable cryocooler. The SN2/HTS system offers both improved stability and improved portability over an LHe/LTS system design.

Design codes are constructed to compare the SN2/HTS system design with a LHe/LTS design for two different applications. The first application is a general permanent superconducting magnet employing a room-temperature bore. The second application is a superconducting mine countermeasures system (SCMCM) that is used to sweep passive magnetic influence mines. The codes predict the important system attributes, namely minimum volume and minimum weight, that should be
expected for a given set of design requirements (i.e. field magnitude and bore size, or magnetic dipole moment) and a given set of conductor properties. Their results indicate that present HTS conductor critical current and index are not yet sufficient for producing SN2/HTS systems of a size that is comparable to that expected for a LHe/LTS system. However, the conductor properties of Bi2223/Ag have been consistently improving over the last decade, and new HTS conductors are expected to be developed in the near future. Therefore, the codes are used to determine the minimum HTS properties that are necessary for constructing a cryocooled SN2/HTS system with a size comparable to that expected for a LHe/LTS system.

Thesis Supervisor: Yukikazu Iwasa
Title: Research Professor, Francis Bitter Magnet Laboratory, and Senior Lecturer, Department of Mechanical Engineering, MIT
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Chapter 1

Introduction

1.1 History of Superconductors

The electrical resistivity of some materials becomes zero when cooled to low temperatures, defining a state known as superconductivity. In 1911, a Dutch physicist, Kamerlingh Onnes, discovered this condition when he cooled mercury to 4.2 K (the boiling point of liquid helium at atmospheric pressure) and detected no measurable resistance. He later discovered that several other metals, such as lead and tin, also exhibited the same change of state. However, these early superconductors did not promise any practical advantage because they could not carry significant current densities while maintaining their superconducting state. Superconducting materials with maximum current densities (or critical current densities) high enough to permit design improvements over the use of normal conductive metals, such as copper, were not discovered until several decades later.

1.1.1 Type II Superconductors

In the early 1950s, Nb$_3$Sn and NbTi were found to exhibit superior current-carrying performance even at high magnetic-fields. Their critical current densities at 4.2 K are greater than the maximum current density of copper conductor in well-designed water-cooled copper magnets. These superconductors are classified as Type II. The
Type I classification applies to the monatomic metals with relatively low critical currents that Onnes discovered. Type I and Type II superconductors also differ in their response to external magnetic field. Below a critical field, $H_{c1}$, a body of either type will completely exclude the field from its interior while it is superconducting, a behavior known as the Meissner effect. The magnitude of $H_{c1}$ varies with material and temperature. Above $H_{c1}$, Type I superconductors become non-superconducting; Type II materials allow field penetration throughout the body while remaining superconducting until the field reaches $H_{c2}$, which is several orders of magnitude greater than $H_{c1}$.

The critical current density of Type II superconductors varies with temperature and magnetic field. There is a maximum temperature, $T_c$, as there is a maximum field strength, $H_{c2}$, and within these limits the materials are in the superconducting state. Figure 1-1 shows three-dimensional plots depicting the critical surfaces for NbTi and a high-temperature superconductor (HTS) compound of bismuth (Bi), lead (Pb),
strontium (Sr), calcium (Ca), copper (Cu), and oxygen (O) – BiPbSrCaCuO – often abbreviated as BSCCO-2223 [1], or Bi2223/Ag when referring to silver-sheathed conductor.

1.1.2 High Field Superconducting Magnets

High field magnets may be constructed with Type II superconductor that has been fabricated in the form of a wire. The wire is usually a composite of the superconductor and a conductive metal, such as copper, which improves the thermal stability of the superconductor while it carries current. High field magnets wound with Type II superconductor pose two key advantages over copper magnets. To begin with, superconducting magnets can produce the same field as copper magnets while using a smaller volume of conductor, because the critical current density of superconducting wire at 4.2 K typically exceeds the maximum current density for water-cooled copper magnets. A second advantage of high-field superconducting magnets is they require less power to operate. Most of the required power is used for refrigeration to maintain the magnet at its operating temperature. Little heat dissipation is caused by the operating current because the conductor has negligible resistance. As a result, the refrigeration load only depends on the heat leak from the warm surroundings into the cryostat that houses the magnet.

For copper magnets, high power is required for driving current through the conductor because the resistance across a long length of copper conductor is significant. The difference in power consumption between a superconducting magnet system and an equivalent copper magnet system depends on the field requirements. For a low-field magnet (i.e. < 5 T), the reduction in power consumption for a superconducting magnet as compared to a copper magnet is not large enough to justify the use of a superconducting magnet because of the inherent complexities of operating a superconducting magnet system. On the other hand, the power savings may be quite substantial for high-field systems. For example, a typical 15-T, 5-cm bore DC superconducting magnet system requires a few hundred watts to operate, while an equivalent copper magnet requires several megawatts [2].
The low power dissipation associated with superconducting magnets permits an additional operational mode that cannot be implemented with a copper magnet. This additional mode is commonly referred to as the persistent-mode, where the magnet produces a constant field without an external power supply to drive current through the superconductor. A persistent-mode superconducting coil is constructed by joining the ends of the wire such that the wire forms a closed superconducting loop. Then, once a current is induced in the superconductor, the current will not decay because the electrical resistance of the superconducting loop is tiny, and the stored magnetic energy is dissipated very slowly.

Stand-alone operation of a magnetic system, where the device may be detached from its cooling source, power supply, and vacuum pump, is desirable for portable systems. The greatest advantage offered by superconducting persistent-mode magnets to portable systems is that the power supply may be detached after the magnet has been charged. The "permanent" field that the superconducting magnet provides may exceed what is obtainable with a system that only utilizes ferromagnetic materials as a source of magnetization. For bore magnets, ferromagnetic materials presently cannot produce fields larger than 2 T, while persistent-mode superconducting magnet systems have been operated at fields up to 17 T at 4.2 K [3]. All other types of electromagnets require continuous operation of a power supply to drive current through the conductor. Therefore, superconducting magnets provide the only means of constructing stand-alone high-field systems. However, the refrigeration requirement of superconducting magnets still implies either a steady power consumption for a cryocooled persistent-mode system, or at least periodic refilling of the liquid helium supply for a liquid helium cooled persistent-mode system. Nonetheless, a liquid helium cooled system can still be designed to stand alone for a limited but reasonably long duration.

1.1.3 High-Temperature Superconductors

In 1986, Karl Alex Muller and Johann Georg Bendnorz of the Zurich IBM Research Laboratory reported an alloy of Ba-La-Cu-O as having a critical temperature of 35 K, over 10 K above the highest critical temperature known at that time. The discovery
started a worldwide effort pursuing superconductors with even higher critical temperatures. Within a few years, materials with critical temperatures in excess of 100 K were produced. Many of these "high-temperature" superconductors (HTS) have critical current densities at liquid nitrogen temperatures that are comparable to the current densities of water-cooled copper magnets. The ability to produce high fields at liquid nitrogen temperatures is a key advantage over materials which must be cooled by liquid helium (low-temperature superconductors or LTS), as cryogenics is more efficient at higher operating temperatures. Therefore, HTS magnets offer better cryogenic efficiencies over LTS magnets.

However, the critical current of all superconductors decreases with increasing temperature. The operating temperature cannot be made too high because a lower permissible operating current density implies a larger conductor mass and volume for producing a given field. Increasing the operating temperature too far will result in an HTS magnet and system size that is much larger than an equivalent LTS system. The larger size often leads to performance disadvantages. Also, the critical current density of HTS conductor operating at any temperature in fields less than 15 T is smaller than the critical current density of LTS conductor operating at liquid helium temperature. Therefore, the mass of an HTS magnet always is larger than the mass of an LTS magnet that is designed to produce the same field. Nonetheless, there may be a temperature that is low enough to permit a manageable HTS system size, but high enough to offer superior cryogenic efficiency in comparison to an LTS system.

A second advantage offered by HTS magnets is superior thermal stability in comparison to LTS magnets. The higher critical temperature of HTS conductor permits operation at temperatures that are significantly higher than liquid helium temperature. By comparing a magnetic system employing silver-sheathed BSCCO-2223 (Bi2223/Ag) conductor operating at 20 K or above with an LTS system that is operated at liquid helium temperature (4.2 K), two factors leading to improved stability for the HTS system may be identified. To begin with, the critical temperature of LTS conductor is only a few degrees kelvin above the temperature of liquid helium. The critical temperature of Bi2223/Ag conductor, on the other hand, is close to 110 K,
so the difference between the critical temperature and the operating temperature for HTS conductor operating at 20 K is close to 100 K. Secondly, the heat capacity of the matrix metals used in both conductors increases by over two orders of magnitude as the temperature is increased from 4.2 K to 20 K. As a result, the energy required to drive a given volume of HTS conductor from 20 K into the normal state is 3–4 orders of magnitude larger than the energy required to drive the same volume of LTS conductor normal.

The superior thermal stability of HTS conductor may offer advantages for systems where the magnet is expected to absorb some form of thermal dissipation. Improved thermal stability reduces the likelihood that a quench may occur unexpectedly, which could damage the magnet, or at least be troublesome to those who rely on the magnet’s operation. Unexpected quenches are not unusual for LTS systems where tiny and unpredictable sources of dissipation, like cracking of the coil’s epoxy impregnant, can lead to a quench and subsequent discharge of the magnet. This behavior is not observed for HTS magnets. Therefore, an HTS magnet might offer superior reliability for a variety applications, such as the Superconducting Minesweeper discussed in Section 5.8 where a large mechanical disturbance caused by an exploding mine might lead to thermal dissipation in the magnet winding.

In addition to their lower critical current, there are other disadvantages associated with HTS materials that lie in their manufacturing complexity and mechanical properties. Most are ceramic and very brittle. Their maximum acceptable strain is smaller and more care must be taken in designing high field magnets, where the conductor is inevitably stressed under high Lorentz forces. Strain cycling will also cause the critical current to suffer. In terms of manufacturing, most HTS materials contain at least four constituents, making it difficult to reduce defects and maintain a consistent quality. The critical current often varies along the length of the conductor. The manufacturing difficulties make HTS conductor far more expensive than LTS conductor.
1.2 Advantage of a Solid Heat Capacitor Cooled HTS System

The high critical temperature of HTS superconductor opens up new possibilities for the cryogenic part of stand-alone superconducting magnet systems. LTS magnets can only be operated close to liquid helium temperature and as a result, are typically operated in a liquid helium bath. HTS magnets, on the other hand, may be operated at much higher temperatures which permits the use of cryogens with boiling points that are significantly higher than the boiling point of liquid helium. Liquid nitrogen is commonly used to maintain the operating temperature of an HTS magnet at the boiling point of 77K. Liquid nitrogen has several characteristics that make it an attractive means of maintaining a low temperature: 1) it is easy to handle, 2) is typically available at 1/20th the cost of liquid helium per volume, and 3) has a latent heat of vaporization per volume that is more than 60 times that of liquid helium. However, the critical current density of HTS conductor at 77K is significantly less than the critical current of LTS conductor at 4.2 K, so the conductor mass needs to be substantially larger for a liquid nitrogen cooled HTS system. For a high field system, it might not even be possible to construct a magnet that can produce the desired field. Additionally, the larger magnet size might be a major disadvantage for certain applications.

In an attempt to reduce the magnet mass of stand-alone HTS systems, the first logical step is to consider cryogens that have a lower boiling point than liquid nitrogen. There are only two cryogens that have boiling point temperatures between the boiling point of liquid helium and the boiling point of liquid nitrogen. Hydrogen boils at 20.4 K, and neon boils at 27.1 K, both are attractive temperatures for operating HTS magnets. Unfortunately, both cryogens have significant disadvantages that prevent their use in practice. Neon is very expensive, and hydrogen, when mixed with air, is dangerously explosive. Another possible cooling scheme for an HTS magnet system is to start with a solid cryogen and operate near its melting point. Two possibilities include, solid nitrogen with a melting temperature of 63.2 K, and solid oxygen with a
melting temperature of 54.4 K. But, as was the case with liquid nitrogen, the resulting HTS magnet size for a high field system operating at these temperatures is still significantly larger than for a liquid helium cooled LTS system.

Having eliminated the possibility of relying solely on a latent heat of some substance to provide cooling, there still remains the possibility of using a solid heat capacitor that absorbs heat by permitting the temperature of the heat capacitor (and the magnet) to change. This is not a possibility for LTS systems because the operating temperature is typically only a few degrees kelvin below the conductor critical temperature. However, the critical temperature of HTS conductor is typically near 100 K, thus offering a permissible temperature variation on the order of 10s of degrees kelvin. For example, if it is determined that a practical magnet size can be obtained for an operating temperature of 40 K, the heat capacitor could be cooled to 20 K and then the cooling source would be disconnected. The system could then stand alone until the solid heat capacitor warms back to 40 K [4].

The internal energy change per unit volume associated with a 20–40 K warming is significant for certain substances. Figure 1-2 shows the expected warming trend for various substances assuming a heat leak of 1 W, (a) a volume of 21, and (b) a mass of 2 kg. The curves were calculated from data listed in [5], [6], and [7]. The volume based curves correspond to 21 of substance at room-temperature for the room-temperature solids, or 21 of liquid at the boiling point for the substances with a boiling point that is below room-temperature. The volume based curve for ice corresponds to 21 of solid at the melting point. Both figures indicate the duration required to boil off the same quantity of liquid helium.

The fixed volume curve is more meaningful because the heat capacitor typically accounts for a small fraction of the total system mass, at least for the applications considered in Chapter 5. The mass of the magnet and the structural materials is larger because they are usually much denser, unless a dense metal such as lead is used as the heat capacitor. On the other hand, the volume of the heat capacitor often does account for a significant fraction of the total system volume, while a larger system volume implies a larger mass of structural materials. As a result, the total
Figure 1-2: Predicted warming trends for a heat leak of 1 W, (a) a volume of 21, and (b) a mass of 2 kg. The duration required to boil off the same quantity of liquid helium is also indicated.
system mass and volume are strong functions of the heat capacitor volume but weak functions of the heat capacitor mass.

The use of a solid heat capacitor that is not permitted to boil off offers the possibility of constructing a system where the heat capacitor does not have to be replenished. Instead, it would simply need to be recooled periodically to maintain its temperature below the maximum permissible operating temperature of the magnet. This design may not offer any advantage when recooling is performed with a liquid cryogen. However, if a cryocooler is used to recool the system, then the system may be operated for extended periods of time without the handling of liquid cryogens. If the cryocooler is detachable, the system may stand alone for some useful duration. A detachable cryocooler system has been proposed in [4]. This thesis is the first work to include detailed analyses of systems that are based on this concept. Two solid nitrogen cooled HTS magnet systems that may be recooled with a detachable cryocooler are considered in Chapter 5.

It appears that the ability to operate a permanent high field magnet system without the handling of liquid cryogens is the greatest advantage offered by the solid heat capacitor cooled HTS concept. No other high field permanent magnet system design offers the possibility of a stand-alone system that does not require the handling of liquid cryogens. Systems that depend on the continuous operation of a cryocooler to maintain the magnet operating temperature also do not require liquid cryogens; however, they must remain connected to a power source during operation and therefore cannot stand alone. Additionally, the mass of the cryocooler may increase the total system mass by more than the increase associated with the inclusion of a solid heat capacitor used in constructing a stand-alone system.

The warming curves shown in Figure 1-2 do not actually show a fair comparison of the expected system sizes between a liquid helium system and a solid heat capacitor system. In typical liquid helium systems, heat leak into the liquid helium bath is reduced by enclosing the bath within vapor-cooled radiation shields, as shown in Figure 1-3a. Cold vapor that boils off from the bath is fed into heat exchangers that are thermally anchored to the shields, so that the cold vapor intercepts radiation heat
Figure 1-3: Schematic drawings of superconducting permanent magnet systems for (a) a liquid helium cooled LTS magnet, and (b) an HTS magnet cooled by a solid heat capacitor. The HTS system includes a radiation shield that is thermally anchored to a second solid heat capacitor.
transfer from the warm cryostat wall. Ultimately, the bath is enclosed by a surface at a temperature that is significantly less than the cryostat wall and so the heat leak into the bath by radiation and conduction through the supports is significantly reduced. The radiation heat leak is generally much larger than the conduction heat leak through the supports for systems that are not intended to experience significant mechanical loading.

There is no vapor boil-off that may be channeled to intercept the radiation and conduction heat leaks in a solid heat capacitor system. The absence of radiation shields leads to a heat leak into the cold container that is orders of magnitude larger than what can be achieved with vapor-cooled shields. As a result, even though the heat capacitor can afford to intercept a much larger quantity of heat than an equal volume of liquid helium, the hold time offered by the solid heat capacitor is still significantly less than the hold time that may be achieved with an equal volume of liquid helium. However, if a radiation shield that is thermally anchored to a second heat capacitor is included, as depicted in Figure 1-3b, a system size that is comparable to what is achieved with a liquid helium system may be possible. Chapter 5 includes an analysis for predicting system size as a function of hold time for a liquid helium system with two radiation shields, and for a system with two heat capacitors where a primary heat capacitor is housed within the cold container and a secondary heat capacitor is coupled to a radiation shield surrounding the cold container. It is determined that solid nitrogen and solid ammonia are attractive choices for the primary and secondary heat capacitors, respectively; particularly for a system that uses a detachable cryocooler to recool the heat capacitors.

For both the liquid helium system and the solid heat capacitor system, heat leak into the cold container may be further reduced by the inclusion of additional radiation shields. However, the number of radiation shields has been limited in order to make a comparison of designs that are not extremely difficult to construct.
1.3 Overview

This thesis explores a new design for a portable permanent magnet system that stands alone, without connections to a power source, a refrigeration system, or a vacuum pump, and may be operated without liquid cryogen handling. The design is an alternative to LTS persistent-mode magnet systems where the magnet is cooled by a bath of liquid helium. The LTS magnet and liquid helium bath are replaced by an HTS magnet that is cooled by a mass of solid nitrogen.

In Chapter 2, an apparatus that was constructed to demonstrate the stable operation of a stand-alone system involving a persistent-mode (or permanent) HTS magnet in thermal communication with a mass of solid nitrogen is described. The magnet was tested in the 15–40 K temperature range. The system warming trend was demonstrated over the 20–40 K temperature range. A minimum temperature of 20 K was chosen because the additional enthalpy associated with lower initial temperatures is relatively small for solid nitrogen, while recooling becomes increasingly expensive as the initial temperature is decreased. The maximum temperature of 40 K was chosen because it approximates the optimum maximum temperature for various applications as calculated by a design analysis presented in Chapter 5. This apparatus is the first to demonstrate the operation of a superconducting magnet with a permissible temperature variation exceeding a few degrees kelvin while a magnetic field is maintained. Recooling of the magnet and the solid nitrogen is accomplished by circulating liquid helium through a heat exchanger located within the cold container which houses the nitrogen and the magnet. The system includes a room-temperature (RT) bore. Models are developed in Chapter 3 to predict the experimental system's warming trend and the field decay of the charged magnet. The models are validated in Chapter 4 with a good agreement between simulations based on these models and experimental results.

Potential performance advantages of a (SN2/HTS) permanent magnet system over a liquid helium cooled LTS (LHe/LTS) system are explored for various applications in Chapter 5. Two different system designs are analyzed and compared. Each system
utilizes a layer-wound solenoidal superconducting coil so that the magnet may be charged with a power supply. The first system is an LHe/LTS system which is used for comparison of an SN2/HTS system with present permanent superconducting magnet system design. The second system is an SN2/HTS system that may be recooled with a detachable cryocooler. It employs one radiation shield that is cooled by solid ammonia. This system offers both improved stability and improved portability over an LHe/LTS design. Design codes are developed for both systems to predict the important system attributes, namely minimum volume and minimum weight, that should be expected for a given set of design requirements (i.e. field magnitude and bore size, or magnetic dipole moment) and a given set of conductor properties. The system designs are compared based on the system size that is predicted for a set of design requirements that are specific to certain applications.

Two applications are considered. The first is a general permanent magnet system employing a room-temperature bore. For this application, the dominant mode of heat leak is the radiation heat transfer between the cryostat wall, radiation shields, and the cold container. The second application is a superconducting mine countermeasures system (SCMCM) that is used to sweep passive magnetic influence mines. The magnet is designed to produce a specified magnetic dipole moment. The dominant mode of heat leak is the conduction heat transfer through the internal supports.

The design code calculations indicate that present HTS conductor critical current and index are not yet sufficient for producing SN2/HTS systems of a size that is comparable to that expected for a LHe/LTS system. However, the conductor properties of Bi2223/Ag have been consistently improving over the last decade, and new HTS conductors are expected to be developed in the near future. Therefore, the codes are used to determine the minimum Bi2223/Ag properties that are necessary for constructing a cryocooled SN2/HTS system with a size comparable to that expected for a LHe/LTS system. The calculated properties may be used to determine when SN2/HTS systems should be seriously considered as an alternative to LHe/LTS systems, especially for applications where system stability or cryogen handling is a major concern.
Chapter 2

Experimental Apparatus and Procedures

An apparatus for demonstrating the operation of a persistent-mode (or “permanent”) high-temperature superconducting (HTS) magnet in thermal communication with a mass of solid nitrogen was constructed. It is also intended for demonstrating that a system involving a solid nitrogen cooled persistent-mode HTS magnet may stand alone, where the cooling source, vacuum pump, and power supply are detached from the system while a magnetic field is maintained for a useful duration. The apparatus includes a room-temperature (RT) bore. The system is depicted schematically in Figure 2-1. This apparatus is the first to demonstrate the operation of a superconducting magnet with a permissible temperature variation exceeding a few degrees kelvin.

An HTS coil comprised of 6 double-pancakes, each wound with Bi2223/Ag tape, was prepared by the Korean Electrotechnology Research Institute (KERI). It is housed inside of a copper container (referred to as the cold container) that is suspended within an aluminum cryostat. Three access tubes leading from a room-temperature flange at the top of the cryostat to the cold container permit it to be filled with liquid nitrogen and cooled to \( \approx 15 \text{ K} \) with liquid helium. Filling the container with liquid nitrogen provides substantial savings in liquid helium in comparison to filling the container by condensing room-temperature nitrogen gas using liquid helium.
Figure 2-1: Schematic drawing of the experimental apparatus used to operate the HTS magnet in thermal communication with solid nitrogen. Dimensions are given in mm.
Just 4.0 liters of liquid helium are required to cool 1.0 kg of liquid nitrogen at 77 K to 15 K, while 12.4 liters of liquid helium are required to cool 1.0 kg of nitrogen gas at 293 K to 15 K, based on the assumption of maximum usage of the available cooling after the nitrogen has been condensed. The calculation of these volumes is described in Appendix B.

The useful duration for which the system may remain detached from its cooling source was arbitrarily chosen as one day. Reaching this target required some design effort to reduce the heat leak into the cold container. To reduce the conduction heat leak, disconnectable current leads are used to initially energize the magnet. Radiation heat leak is minimized by shielding the cold container and access tubes from room-temperature surfaces with multiple layers of superinsulation. This type of high vacuum insulation typically performs an order of magnitude better than alternative types. A getter mounted to the cold container maintains a high vacuum (≤10^{-5} torr) after the cryostat is detached from the vacuum pump. The getter absorbs gases inside the cryostat.

Detachment of the power supply requires the magnet to be operated in persistent-mode, where the superconductor forms a continuous superconducting loop. Two methods have been demonstrated for magnetizing these types of systems. The first method requires the following sequence of steps: 1) place the normal-state superconducting magnet into the bore of a background magnet that generates a constant field; 2) cool the magnet so that it becomes superconducting in the presence of the field; 3) force the background field to zero, either by de-energizing the background magnet or physically separating the two magnets. Step 3 induces a current in the superconducting magnet thereby leaving the superconducting magnet energized. The second method involves energizing the superconducting magnet directly, by providing it with a superconducting switch, as depicted schematically in Figure 2-2. The switch is simply a section of superconductor contacted by a heater that warms the switch superconductor into the normal state while the rest of the magnet remains superconducting. If the normal-state switch is sufficiently resistive, most of the current from an external power supply may be injected into the magnet. After the current
reaches a steady value in the magnet, the heater is shut off to permit the switch to cool back into the superconducting state. The current from the supply is then ramped down, causing the current in the magnet to be diverted through the switch. The second method is demonstrated in this experiment because the first method is generally impractical.

The superconducting switch was wound using superconductor fabricated with a silver/1at% gold matrix. At low temperatures, silver/gold alloys have resistivity values, both electrical and thermal, that are much greater than those of pure silver. Both of these properties are advantageous for a superconducting switch. The larger resistance reduces the current leak through the switch while charging the magnet. The smaller thermal conductivity reduces the heat input necessary to drive the superconductor normal. In order to drive and maintain the switch superconductor normal with minimum heat leak to the cold environment, the switch is well-insulated in its own sealed container at the top of the magnet.
2.1 Apparatus Design

Detailed machine drawings for all of the machined parts used in the apparatus are included in Appendix A.

2.1.1 Cold Container

The vertical walls of the cold container consist of two copper tubes, one inside the other to form an annulus as shown in the assembly drawing of Figure 2-3. The inner tube is necessary to permit the RT bore to extend down into the field of the magnet. The tubes are part of two separate pieces that are joined by stainless steel machine screws and sealed with an indium o-ring when the system is assembled for operation. The inner tube is capped at its lower end with a circular copper plate and joined to an annular flange at its upper end. The flange is constructed so that its periphery has the necessary geometry for providing a vacuum tight indium seal and structural coupling to the outer tube. All of the electrical feedthroughs for instrumentation and current as well as stainless steel tubes for the helium circuit and nitrogen inputs pass through this flange and are soldered to it.

The outer tube is also sealed at its lower end by a circular copper plate, while a ring which mates with the copper flange is soldered to the upper end. Capping the ends of the tubes requires the inner tube to be slightly shorter than the outer tube. Although capping the ends prevents the RT bore from extending all the way through the container, which may be useful for some applications, assembly is much simpler as we do not require a second indium seal at the bottom end of the container.

2.1.2 Cryostat and Room-Temperature Flange

The aluminum cryostat and the RT flange are shown in Figure 2-4. The top of the cryostat includes a channel for a rubber o-ring to produce a vacuum tight seal when mated with the RT flange. The RT flange is machined from 12.7mm thick brass plate. Eight holes surrounding the o-ring channel permit the flange and cryostat to be fastened together. A pumpout port extending from the side of the cryostat permits
Figure 2-3: Assembly drawing of the cold container showing the components of the cold container flange. The fasteners have been omitted for clarity.
Figure 2-4: Illustration of the cryostat and the room-temperature flange. The compression fittings are labeled according to the penetrations which pass through them.
attachment of a vacuum pump by a 2 inch Ladish flange. The pumpout port includes a valve to allow detachment of the vacuum pump. A Teflon stop is inserted into a slot inside the wall of the cryostat to prevent the cold container from contacting the cryostat wall in case the system is turned on its side by mishandling. The location of the stop is shown in Figure 2-1.

Holes are drilled into the RT flange to allow various penetrations: a helium input, a helium output, an access tube to the cold container for nitrogen filling and boil-off, a 5-pair thermocouple vacuum feedthrough, a 10-conductor electrical vacuum feedthrough, disconnectable leads, a RT bore, and an ionization vacuum gauge. Compression fittings are soldered into each hole to provide a vacuum tight seal with each of these components. The location of each compression fitting is shown in Figure 2-4. Three eyebolts (not shown) are fastened into threaded holes located near the circumference of the RT flange. The eyebolts permit the system to be hung from a chain-fall.

2.1.3 Supports and Access Tubes

Three thin-wall stainless steel tubes extend from the cold container flange up through the RT flange. The tubes are shown without the RT flange in Figure 2-3. Two of the tubes provide a path for the helium circuit while the third provides a path for filling the container with liquid nitrogen (and the exit of nitrogen gas during warm-up). These tubes also function as the structural supports for hanging the cold container within the cryostat from the RT flange. They are positioned symmetrically about the centerline of the container (120° apart), at a radius of 52 mm. The helium and nitrogen inputs are actually constructed of two different diameter tubes joined together in series. The lower section consists of a thinner tube with an outer diameter (OD) of 3.2 mm and a length of 115 mm between the top of the cold container and the bottom of the coupling that joins it to the upper section. The helium output is a single 3.2 mm OD tube.

The thinner tubes provide enough flexibility in the supports to permit a 13 mm horizontal deflection of the cold container without the structural supports yielding.
This distance is equal to the gap width between the cold container and the stop on
the inside of the cryostat and is the maximum deflection that can occur when the
superinsulation surrounding the cold container is absent. The supports are designed
this way to protect against mishandling of the apparatus. Additionally, the thinner
tube provides a much larger thermal resistance for reducing conduction heat leak into
the cold container.

At the top of the nitrogen input and helium output, the access tubes are coupled
to 100 mm lengths of 12.7 mm OD stainless steel tubes which extend back down
through the compression fittings. These outer tubes act as standoffs to prevent the
compression fitting o-rings from getting cold when cold gas exits the inner tubes. The
o-rings inside the compression fittings cannot supply the necessary load to support
the cold container and its contents, so stops are soldered to the standoffs and the
helium input tube. The stops rest on top of the compression fittings.

The RT bore is constructed from a fourth stainless steel tube with an OD of
12.7 mm. It extends from the RT flange down into the inner copper tube of the
cold container along the container's centerline. The lower end of this tube is capped
and positioned 14 mm below the midpoint of the six double-pancake axis. The gap
between the RT bore and the inner cold container tube is large enough so that if the
container horizontally deflects to the Teflon stop, there is no contact between the RT
bore and the cold container.

2.1.4 Liquid Nitrogen Filling and Cooling Systems

Figure 2-5 shows a liquid nitrogen transfer line that is constructed to fill the cold
container with liquid nitrogen prior to sub-cooling it with liquid helium. When filling
the cold container, the upper end of the transfer line is attached to a pressurized liquid
nitrogen storage dewar using a flare fitting, while the lower end is inserted down into
the cold container through the nitrogen input access tube. Copper tubing leads from
the storage dewar to just above the top of the nitrogen input tube. The tubing is
insulated with an elastomeric foam insulation. As the nitrogen input access tube is
constructed with two tubes of different diameter coupled in series, two stainless steel
Figure 2-5: Section drawing of the transfer line used to fill the cold container with liquid nitrogen from a pressurized storage dewar.

tubes coupled together lead from the insulated copper tubing down into the cold container. The use of a larger OD tube over most of the length substantially reduces the time required to fill the cold container when compared with using a thinner tube over the entire length.

The upper section of the helium input (see Figure 2-3) consists of a 12 mm inner diameter (ID) tube. This larger diameter permits a helium transfer line to be inserted and extend roughly 225 mm below the RT flange. A short length of surgical tubing is used to seal the gap between the helium input and the transfer line at the top of the helium input. The lower section of the helium input consists of a 3.2 mm OD tube as described in the previous section. The 3.2 mm tube extends into the cold container to a coil of 12.7 mm OD copper tubing, having 5 2/3 turns wound over a diameter of 92 mm. These dimensions are chosen so that there is negligible pressure drop across the helium coil and enough surface area to allow the system to be cooled to 15 K in under 2 hours. A simulation of the cooling trend is included in Appendix B.

The opposite end of the helium transfer line is inserted into a helium storage dewar with the end below the liquid helium surface. Compressed helium gas is then used to
pressurize the helium dewar, thereby forcing liquid helium to flow through the helium circuit shown schematically in Figure 2-6. The helium exits the copper tubing into the helium output access tube which extends above the RT flange. The gas then travels through 1.25 m of 6.4 mm ID plastic tubing into a 6 m length of finned 13 mm ID copper tubing that is submerged in room-temperature water. The helium gas exits this coil at approximately room-temperature and travels along a 1.5 m length of 6.4 mm ID plastic tubing to a volumetric flow meter. The gas is vented to the atmosphere as it exits the flow meter. When helium is not being transferred, the helium input and output are sealed with stoppers.

2.1.5 Instrumentation Leads

Voltage Taps

A 10-pin electrical feedthrough attached to the RT flange permits passage of voltage taps and current leads for the superconducting switch's heater from outside of the cryostat into the vacuum space. Each pin is rated for 10 A maximum current. A second 10-pin cryogenic electrical feedthrough with the same current rating permits
passage across the cold container flange. Nickel/chrome wires, No. 40 and 530 mm long, run between the feedthroughs to connect the appropriate pins used for voltage measurements. A fine wire with a low thermal conductivity is used to reduce conductive heat leak. The wire is wrapped 10 times around the helium input as it travels from the RT flange to the cold container in order to maintain a stable position. Inside the cold container, no. 30 Teflon insulated copper wire leads from the feedthrough pins to the magnet. The ends are soldered to the superconductor with indium solder. A solder with a low melting temperature is used to prevent overheating the superconductor. All wires are connected to the feedthrough pins by soldering the wire ends to sockets which mate with the feedthrough pins.

Voltage taps are placed across each double-pancake for critical current measurements. During persistent-mode tests, taps are soldered to the current ports to measure the voltage across the superconducting switch.

**Thermocouples**

Temperature is measured with 5 type-E thermocouple pairs. Measurements are made at the following locations:

1. the middle of the heated section of the superconducting switch,
2. one end of the heated section,
3. one current port at the top of the magnet,
4. the inside radius of the magnet between the fourth and fifth double-pancake (counting from the top),
5. bottom of the cold container.

Feedthroughs designed for type-E thermocouples provide passage across the RT flange and the top flange of the cold container. The feedthrough fixed to the cold container is rated for cryogenic temperatures. 450 mm lengths of no. 40 Teflon insulated thermocouple wire travel from the RT flange to the cold container while completing
10 turns around the helium output. Inside the container, no. 28 thermocouple wire travels from the feedthrough to the desired temperature measurement location where the ends of the pairs are soldered together. At all locations, the thermocouple ends are electrically insulated with 25\( \mu \text{m} \) thick Kapton insulation.

Feedthrough pins are connected to the thermocouple wires using sockets of an appropriate material, to which the ends of the thermocouple wire is soldered. Since the cross-sections of the sockets and solder joints are much larger than the cross-sections of the wires, there should not be a significant temperature gradient across the joints and therefore uncertainties in the temperature measurement should be within 1 degree kelvin. No. 28 wire travels from the feedthrough to a liquid nitrogen bath outside of the cryostat, which is used as a reference temperature. The ends of the thermocouple wires are submerged in the bath and soldered to fine copper wires that exit the bath for voltage measurement. Temperature can be inferred from the voltage measurement. The bath temperature is measured with a silicon diode.

**Heater Current Leads**

Heater current travels between the same electrical feedthroughs that are used for voltage measurements. Each lead traveling between the RT flange to the cold container consists of 4 lengths of 530 mm long no. 30 brass wire twisted together and fed through Teflon tubing which acts as insulation. As with the voltage taps, the leads complete 10 turns around the helium input as they travel between the feedthroughs. Each brass lead has a resistance of approximately 140 m\( \Omega \).

The conductor thickness is chosen so that the maximum temperature in the leads does not exceed 300 K when they carry a current of 1.5 A. This current is estimated to be two times the current necessary to drive the switch superconductor normal (>105 K) when the cold container is at 20 K. The value is based on experiments done at 77 K with the switch submerged in a liquid nitrogen bath. Since the thermal conductivity of the switch materials and the resistivity of the heater material do not vary significantly between 20 K and 77 K [2][8], the heater power is expected to increase by a factor of 3 to account for a factor of 3 increase in temperature gradient.
A numerical code is used to predict the temperature profiles in the leads while they carry 1.5 A, and to predict the steady-state heat leak through the leads when the current is shut off. The lead material is chosen to minimize heat leak into the cold container as it warms from 20–40 K, when no current is being supplied to the switch heater. Brass is selected after a comparison of three additional materials already on hand: copper, stainless steel, and Constantan. The selection process and the code are described in detail in Appendix B.

No. 20 copper wire is used to carry current from the power supply to the room-temperature feedthrough. No. 28 copper wire carries current from the cold container feedthrough to the leads which exit the switch container. Both can comfortably carry 2 A with negligible dissipation. The leads inside of the switch container are described in Section 2.1.10.

### 2.1.6 Disconnectable Current Leads

The current lead construction is depicted in Figure 2-7. Disconnectable leads are used to reduce conduction into the cold container. The leads are designed to carry a current of 125 A based on an optimum vacuum lead criterion. This current rating is significantly greater than the critical current of the test magnet superconductor so that the magnet may be replaced by one wound with high quality superconductor if the opportunity arises in the future. A calculation supporting the selection of the conductor dimensions is included in Appendix B.

The room-temperature current ports are machined from brass rod. Cables from a power supply may be fastened to the current port by machine screws which press the cable connectors against a flat surface at the top of the current ports. Copper conductor of 3.2 mm diameter runs along the axis of the lead into the brass current port at the conductor’s top end and a brass cylinder at its bottom. Lead/tin solder maintains an electrical connection at both ends. The brass cylinder fits inside of a brass tube at the very bottom of the mobile section of the lead. The lower section of the brass tube is threaded to fit a Multilam socket. The lead is made rigid by a 12.7 mm OD tube which runs from inside the current port at the top down into
Figure 2-7: Illustration of the current lead construction including: (a) design details of a single lead, and (b) drawing of the assembled mobile pair. Dimensions are given in mm.
the threaded brass tube at the bottom. Most of the structural tube is constructed from G-10 to electrically isolate the conductor from the RT flange. The bottom 50 mm consists of stainless steel 304 to prevent scorching the G-10 when soldering the bottom pieces. The bottom 15 mm of the G-10 tube is recessed and inserted into the steel tube to provide a rigid connection. The G-10 and steel are bonded together with Stycast epoxy. The stainless steel tube is soldered to the brass tube with antimony/tin solder which has a higher melting temperature than lead/tin solder. This is necessary for maintaining a good joint between the steel and brass tubes when soldering the brass cylinder, brass tube, and Multilam socket together.

Vertical motion is permitted by steel bellows. A brass flange couples the top flange of the bellows to the G-10 tube and the phenolic spacer that is inserted between the brass flange and the current port. Brass flange, tube, phenolic spacer, and current port are bonded together with Stycast epoxy. The bottom flange of the bellows is soldered to a brass piece which extends into a compression fitting. The compression fitting is soldered to the RT flange.

The screws which press the cable connectors against the current port also pass through short brass cylinders inside of the G-10 tube which join the current ports of the two leads. Vertical positioning of the lead pair is controlled by rotation of a threaded rod which passes through a brass cylinder at the middle of the G-10 tube. The bottom of the threaded rod is anchored to a brass plate that rests on top of the compression fittings and is soldered to the brass pieces attached to the bottom flange of the bellows. The three brass cylinders inside of the G-10 tube do not touch so that the leads and the RT flange are electrically isolated from each other.

The leads connect to the magnet circuit as the mobile section is moved downwards. The Multilam socket mates with a plug that is soldered at the top of a Ceramaseal feedthrough. The feedthrough consists of 6.4 mm copper conductor that passes from the vacuum space into the cold container. It provides an electrical connection across the container wall while maintaining a vacuum tight seal and electrical isolation from the cold container. The feedthrough and the container wall are sealed together by soldering the feedthrough's stainless steel lip to the flange at the top of the container.
Figure 2-8: Depiction of how the copper braid is wrapped around the helium coil. Only one turn around the coil is shown for clarity.

A solder lug joins the end of the feedthrough that is inside the container to 1.5 m of 10 mm x 2 mm copper braid which leads to the magnet current port. The copper braid is wrapped in Kapton insulation. The braid is wrapped around the helium coil 5 times, as depicted in Figure 2-8, so that heat conduction through the leads may be intercepted by helium circulating through the coil. The lower end of the copper braid is soldered to the magnet current ports. The number of wraps around the helium coil is justified by a calculation described in Appendix B.
2.1.7 Thermal Insulation

The cold container is suspended inside of an evacuated aluminum cryostat as shown in Figure 2-1. A diffusion pump maintains a vacuum better than $10^{-5}$ torr before the system is cooled below room-temperature. Performance of the superinsulation is observed to improve with time while a good vacuum is maintained within the cryostat. Gas trapped between the inner layers of superinsulation permits significant conduction between the layers and therefore reduces the thermal resistance between the layers. Even after 6 weeks of continuous pumping, a noticeable decrease in heat leak is observed after pumping for an additional 2 weeks.

A charcoal getter is attached to the bottom of the cold container to maintain a good vacuum while the vacuum pump is detached from the cryostat. At low temperature, the getter serves to absorb gases that may leak or outgas into the cryostat. For this system, a vacuum better than $2 \times 10^{-5}$ torr can be maintained for more than one day after closing the pumpout port and detaching the vacuum pump while the temperature of the cold container is below 40 K. The vacuum is measured with a hot-filament ionization gauge that is inserted through the RT flange.

The superinsulation surrounds the cold container and access tubes. Each insulated surface is wrapped with 40 layers. Performance gain per additional layer decreases with the number of layers already applied because evacuation of the inner layers becomes more obstructed. 40 layers were used because increasing the number of layers beyond this point usually leads to a small improvement in performance that is not worth the increase in system size [9][10][11][12]. The superinsulation wrapped around the outer cylindrical surface of the cold container consists of 1.65 m long by 310 mm wide strips. Adjacent strips overlap by roughly 100 mm. There are two different blankets covering this surface (see Figure 2-1). The lower blanket is wrapped at a slight angle downward. After each turn around the circumference a circular end-piece, shown in Figure 2-9a, is aligned and attached at the bottom of the cold container. Tabs extending from the circular end-piece are folded at a right angle to the plane of the circle and taped to the outer vertical layer. As more layers are
Figure 2-9: The superinsulation end-pieces attached to (a) the bottom and (b) the top of the cold container.

wrapped around the container the wrapping diameter increases. To account for the increasing diameter, the diameter of the end-pieces are incremented every 10 layers as illustrated in Figure 2-10.

The 40 layers constituting the top blanket that surrounds the outer cylindrical surface of the cold container overlaps the bottom blanket by 100 mm. The outer diameter of the lower superinsulation blanket and the outer diameter of the cold container flange are closely matched, providing an even surface and dimensional stability for the upper blanket. As with the lower blanket, the upper blanket is wrapped at a slight angle to the horizontal, in this case upwards. After each turn around the circumference a circular end-piece is aligned and attached at the top of the wrapping. The top end-pieces are constructed and fastened similarly to the bottom end-pieces in that the diameter is incremented every 10 layers and tabs are used to fasten an end-piece to each vertical layer. Additionally, the top end-pieces have holes cut in them to permit penetration of the access tubes as shown in Figure 2-9b. Slits extending from the end-piece circumference to the penetration holes are required for installation. Additional details of the penetration holes are described in the following paragraphs.
Figure 2-10: Arrangement of the superinsulation end-pieces that are attached to the bottom of the cold container. The diameter is incremented every 10 layers.

Each access tube is wrapped with 40 layers of superinsulation from 280 mm wide strips. The superinsulation is applied 10 layers at a time, with the lower edge of the wrap vertically realigned every 10 layers by a distance corresponding to the thickness of 10 top end-pieces, as shown in Figure 2-11. The diameter of the holes in the top end-pieces are increased every 10 layers. The holes are cut so that their diameter matches the diameter of the outermost access tube layer that penetrates it. The 10 innermost layers wrapped around the access tubes terminate on top of the 10 innermost end-pieces.

The lower portion of the RT bore which extends into the superinsulation covering the cold container is wrapped with 330 mm wide strips at a slight downward angle. The length of each layer extending below the end of the RT bore is folded over to completely cover the layers inside. Holes are cut in the center of the end-pieces on top of the cold container to permit penetration of the RT bore. The diameter of these holes are decremented every 10 layers to account for the smaller wrapping diameter of the inner layers, as illustrated in Figure 2-12. The top 5 end-piece layers have the smallest hole diameter which closely matches the diameter of the stainless steel tube.
Figure 2-11: Arrangement of the superinsulation at the access tube penetrations.
that the RT bore is constructed from. The diameter of the hole in the 10 innermost end-pieces closely matches the diameter of the outermost layer wrapped around the RT bore.

The disconnectable leads reduce the conduction heat leak but are likely to cause a significant additional radiative heat flow because the tips of the current feedthroughs must remain bare to permit them to mate with the Multilam socket. Each end-piece on top of the cold container has a hole slightly larger than the diameter of the Multilam plug to permit its penetration. An effort is made to shield the plug's view from as much room-temperature surface as possible. The brass pieces at the ends of the room-temperature portion of the disconnectable leads are covered with two layers of superinsulation with the aluminized surface facing outward, as shown in Figure 2-13. Holes are cut in the appropriate places to permit the plugs to pass into the Multilam socket. Two sheets of superinsulation are wound into truncated cones with the top diameter matching the diameter of the brass pieces. The aluminized surface of the superinsulation faces inward. Each cone is installed with the top edge fastened to the
brass piece and the bottom edge fastened to the outermost superinsulation end-piece covering the cold container. With this configuration, the cold tips can only see a room-temperature, high-emissivity surface through the hole leading to the Multilam socket. The leads are connected and disconnected several times at room-temperature to verify that the cones stay together.

2.1.8 Magnet Design

An 840-turn, 6 double-pancake magnet was wound by KERI using Bi-2223/Ag composite tape conductor. Conductor specifications and magnet parameters are given in Table 2.1. The winding of each double-pancake is illustrated in Figure 2-14. Each double-pancake is wound around a G-10 coil form with a continuous length of 25 μm thick Kapton insulation separating the turns. A recess with the same width as the conductor is machined into the coil form to permit the conductor to pass between the two pancakes while maintaining a flat surface for winding the first turn of each pancake on. This recess is commonly referred to as a “crossover-guide”. 

Figure 2-13: Illustration of the superinsulation surrounding the connector surfaces of the disconnectable leads.
Figure 2-14: Illustration of how the double-pancake coils were wound. Dimensions are given in mm.
Table 2.1: Magnet Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winding i.d.</td>
<td>mm</td>
<td>75</td>
</tr>
<tr>
<td>Winding o.d.</td>
<td>mm</td>
<td>107</td>
</tr>
<tr>
<td>#turns/double-pancake (DP)</td>
<td></td>
<td>140</td>
</tr>
<tr>
<td>As-received conductor $I_c$ at 77 K*</td>
<td>A</td>
<td>32</td>
</tr>
<tr>
<td>Bare conductor width</td>
<td>mm</td>
<td>3.2</td>
</tr>
<tr>
<td>Bare conductor thickness</td>
<td>mm</td>
<td>0.23</td>
</tr>
<tr>
<td>Conductor length/DP</td>
<td>m</td>
<td>42</td>
</tr>
<tr>
<td># of DP</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>Total # of turns</td>
<td></td>
<td>840</td>
</tr>
<tr>
<td>Overall magnet (6 DP) height</td>
<td>mm</td>
<td>47.1</td>
</tr>
<tr>
<td>Self Inductance</td>
<td>mH</td>
<td>50.4</td>
</tr>
<tr>
<td>Approximate $I_{op}$ at 40 K</td>
<td>A</td>
<td>20</td>
</tr>
<tr>
<td>Center Field at $I_{op}$</td>
<td>T</td>
<td>0.21</td>
</tr>
<tr>
<td>Maximum axial field</td>
<td>T</td>
<td>0.29</td>
</tr>
<tr>
<td>Maximum radial field</td>
<td>T</td>
<td>0.17</td>
</tr>
</tbody>
</table>

*Criterion of $E_c = 1.0 \mu\text{V/cm}$.

The 6 double-pancakes are stacked on top of each other. The pancakes are separated by 8 rectangular G-10 spacers, 0.8 mm thick, oriented radially. The ends of the spacers are rounded to match the inner and outer curvature of the pancakes. This same spacer configuration is used to separate the stack of pancakes from the switch container located above the top pancake as in Figure 2-15. The dimensions of the coil forms are such that when stacked on top of each other, each single-pancake is one spacer thickness apart. The bottom pancake rests on a 3.2 mm thick G-10 plate. Brass threaded rods are used to fasten the coil forms, bottom plate, and switch container together. To account for the larger contraction of G-10 than brass when cooled to cryogenic temperatures, 4 Belleville washers are placed inside of the nuts at each end of the threaded rods.

The pancakes are lap-spliced together using indium solder over 440 mm of conductor (one full turn around the outer circumference). 880 mm of conductor extends from the top pancake and 1.5 m extends from the bottom pancake to wind around the outside of the magnet up to the switch container. The extensions are lap-spliced to the switch conductor.
Figure 2-15: Illustration of the magnet assembly (a) prior to attachment of the superconducting switch and (b) completed.
2.1.9 Superconducting Switch Design

The switch is constructed by winding two turns of BSCCO-2223 superconductor with a 1 at% gold/silver matrix around an 88 mm diameter cylinder. The switch conductor has the same critical current rating as the magnet conductor. Before winding, one layer of 25 μm thick Kapton is adhered to the outer surface of the conductor. Apiezon-N grease is applied over the Kapton along a length of conductor that constitutes the two turns wound around the cylinder. A heater made of 25 μm thick stainless steel shim with the same width as the conductor is placed over this region after current leads are soldered to its ends. The heater has a resistance of 3.8 Ω at 77 K. Each lead is a 290 mm length of no. 28 copper wire.

The cylinder is covered with Mylar tape. A polystyrene coating is applied to the surface of the tape. After winding the two turns of conductor, Kapton, and steel, thermocouples are placed at the middle and one end of the heater. The turns are then coated with Stycast epoxy. After the epoxy cures, the polystyrene coating on the cylinder is dissolved with acetone and the epoxy coated turns are carefully slid off. Each end of the heated turns has a 650 mm long conductor extension.

The switch is housed inside its own container that sits on top of the magnet. The container is depicted in Figure 2-16. The epoxy coated turns are centered around a cylindrical G-10 piece that has a flange at the bottom. The turns are separated from the inner cylindrical wall and the bottom flange using polyethylene foam spacers, 1.6 mm thick. The outer wall of the container is fabricated from phenolic tube. The top wall is a G-10 circular plate with a slot machined into it to allow passage of the conductor, thermocouples, and heater leads. The container pieces are bonded together and sealed with Stycast epoxy. The slot in the cover is also sealed with Stycast epoxy. A strip of polyethylene foam (not shown) is fastened underneath the slot to prevent the epoxy from dripping through. The length of conductor between the end of the heated turns to its exit from the container is 210 mm. A guide machined from phenolic plate provides a flat surface for the conductor to travel from the slot to the container’s outer cylindrical surface. The guide is bonded to the cover with epoxy.
Figure 2-16: Assembly drawings of the superconducting switch. Starting with the components fabricated as in (a), the phenolic ring is adhered to the G-10 form with epoxy and the phenolic guide is adhered to the cover as shown in (b). After the epoxy cures, the conductor extensions are fed through the slot in the cover and the two pieces are adhered together as shown in (c), sealing the superconducting switch.
Two current ports are cut from 0.8 mm thick copper plate. The plates are curved to match the curvature of the phenolic guide at their locations. Slots are machined into the guide to recess the current ports so that the outer surface of the current ports align with the outer surface of the guide. The current ports are held in place with machine screws which fasten the current ports to brackets that are adhered to the top of the guide with epoxy. The switch conductor is lap-spliced to the ends of the magnet conductor on the outside of the container wall over a length of 340 mm using indium solder. The splices begin at the copper current ports and extend down towards the magnet.

2.1.10 Field Measurement

The axial field produced by the magnet is measured by a Hall probe positioned on the magnet axis and placed in the RT bore at the midpoint of the 6 double-pancake coil. The Hall probe is adhered to the bottom end of a 6.4 mm diameter phenolic rod. A stop which rests on the top of the RT bore is attached to the rod 740 mm above the Hall probe, holding the probe at its desired position.

2.1.11 Instrumentation

All voltage outputs are monitored with Keithley voltmeters and recorded with a digital computer using LabView data acquisition software. A DC power supply operates in current mode to provide current for charging the magnet. A smaller power supply operates in voltage mode to provide power to the switch heater. A small calibrated current source powers the Hall probe.
2.2 Experimental Procedures and Results

2.2.1 Cooling the Cold Container Below 20 K

Figure 2-6 depicts the method by which helium is used to cool the system from 77 K to below 20 K. The cold container and 1.6 kg of nitrogen are cooled using the following steps. Experiments are conducted with a cryostat pressure of less than $2 \times 10^{-5}$ torr.

1. The liquid nitrogen transfer line is inserted through the nitrogen input tube and then connected to a pressurized nitrogen storage dewar.

2. With the cold container initially at room-temperature, the storage dewar valve is opened and liquid nitrogen (and cold gas) is injected into the cold container until liquid begins to overflow out of the nitrogen input. Then, the storage dewar valve is closed and the transfer line is removed. This step requires about 8 hours to complete.

3. The stoppers are removed from the helium inputs and outputs. A plastic tube leading to the RT heat exchanger is connected to the helium output. A liquid helium transfer line is inserted into a liquid helium storage dewar. The opposite end of the transfer line is then inserted into the helium input and sealed at the top of the helium input with surgical tubing. The end of the transfer line is positioned approximately 13 mm above the coupling that connects the 12.7 mm and 3.2 mm OD tubes.

4. A helium gas cylinder is connected to the liquid helium storage dewar. Helium transfer begins after pressurizing the dewar to 3 psi. Initially, warm gas exits the helium transfer line and travels through the helium coil causing liquid nitrogen to boil off. Roughly 300 ml of liquid is lost before the helium transfer begins to provide cooling to the nitrogen.

5. To replenish the nitrogen supply, a plastic tube is connected between the nitrogen input and a nitrogen gas cylinder. A nitrogen pressure of roughly 10 psi is applied. The helium pressure is constantly adjusted between 0 and 1 psi to
maintain the cold container wall close to 65 K (above 63.1 K, the freezing point of nitrogen). When the container is full of liquid, the nitrogen gas flow rate drops, at which time the nitrogen gas cylinder is disconnected and the nitrogen input tube is sealed with a stopper.

6. The helium pressure is increased to 3 psi and the system is cooled until the cold container wall reaches 12 K.

7. The helium transfer line is removed, the helium output is disconnected from the RT heat exchanger, and both the helium input and outputs are sealed with stoppers.

8. The pumpout port valve is closed and the vacuum pump is disconnected.

Figure 2-17 shows temperature traces recorded while cooling the system according to the steps above. The temperature of the magnet current port, as measured by thermocouple#3 (TC#3), drops the fastest because the current leads are wrapped around the helium coil. The temperature of the cold container (TC#5) also drops more rapidly than the magnet and superconducting switch temperatures because a significant portion of its surface is separated from the cold container by only a few mm of nitrogen. Even though the temperature of the cold container is measured at its bottom, a greater distance from the helium coil than the magnet, the high thermal conductance of the cold container (copper) causes a thermal short. A significant thermal resistance exists between TC#4 and the container wall because the thermocouple is located at the inner radius of the magnet, away from the highly conductive cold container wall. Similarly, TC#1 and TC#2 lag behind because they are sealed within the switch container.

The temperature traces begin to level at 0.5 hr because the helium flow rate is slowed to maintain the temperatures above the freezing point of nitrogen. This permits the nitrogen to condense into a liquid, which creates a denser solid than a solid condensed directly from a gas. Additionally, condensing the gas as a liquid prevents overfilling the container with subcooled solid, which could lead to damaging stresses.
Figure 2-17: Temperature traces obtained when cooling the cold container and its contents from 77 K to below 20 K.
Figure 2-18: Warming trend observed over the temperature range of 20–40 K.

as the solid tries to expand when the cold container is warmed. After 0.9 hr, the nitrogen gas is disconnected and the helium flow rate is increased.

The horizontal region seen at roughly 63 K is due to the latent heat of the liquid-solid transition. A second phase transition is observed at roughly 36 K; at 35.6 K solid nitrogen undergoes a solid-solid phase transition that absorbs heat (see Figure 1-2). At 2.4 hr, the cold container wall is at 12 K when the helium flow is stopped and the temperatures begin to rise. The heat leak and container size is small enough that the difference in temperatures measured by each of the thermocouples is negligible at 5.5 hours.

Figure 2-18 shows the 20–40 K warming trend that followed this cooling trial. With the pumpout port valve closed the cryostat pressure slowly rises from $2.0 \times 10^{-6}$ torr when the cold container temperature is at 20 K, to $2.0 \times 10^{-5}$ torr at 40 K. The 20–40 K rise time corresponds to an average heat leak of 550 mW (see Section 3.1).
The horizontal region at 35.6 K is due to the latent heat of the solid-solid phase transition of solid nitrogen.

After the experiments are completed and the system is permitted to warm to 77 K, the amount of nitrogen in the cold container is measured by connecting the nitrogen input to the room-temperature heat exchanger with a plastic tube. The volumetric flow meter measures how much nitrogen boils off based on the volume of gas at room-temperature. The cold container held 1.6 kg of nitrogen in this trial.

### 2.2.2 Charging the Magnet

The magnet was charged at three different temperatures: 15 K, 25 K, and 35 K. Preliminary measurements were also made at 77 K in a liquid nitrogen bath before installing the magnet. The following steps describe how the magnet was charged at temperatures below 40 K while in thermal communication with 1.5 kg of solid nitrogen. A schematic of the charge circuit is shown in Figure 2-2.

1. The cold container is cooled below the desired temperature for charging the magnet by the same method described in the previous section for cooling the cold container from 77 K to below 20 K. The helium storage dewar pressure is then reduced to zero, but the helium transfer line is not removed.

2. When the system has warmed to its desired initial operating temperature, voltage is applied across the heater leads by a power supply. The helium storage dewar pressure is adjusted to maintain the system at this initial operating temperature.

3. After the temperature of the switch conductor exceeds 105 K, as measured by the thermocouple at the middle of the heated region, the disconnectable leads are attached. The helium storage dewar pressure is readjusted again to maintain the initial operating temperature.

4. Current through the disconnectable leads is slowly ramped up to 22 A from another power supply. This maximum current is chosen because the transport
current approximately equals the smallest critical current (1 μV/cm criterion) in the magnet for the field profile produced when the magnet is carrying 22 A at 20 K. At higher temperatures, the critical current is less, but not so much that 22 A causes significant dissipation in the magnet.

5. The superconducting switch heater supply is shut off, and the switch conductor cools back to the temperature of the cold container.

6. After the voltage across the switch diminishes, the current through the disconnectable leads is ramped down to zero, diverting the current through the superconducting switch.

7. The leads are disconnected and the helium flow is stopped. The transfer line is removed and the helium input and output tubes are sealed with stoppers.

For each trial the following are recorded over 12.5 hours:

- the heater power supply voltage at the room-temperature terminals,
- the voltage across the superconducting switch,
- the disconnectable lead current,
- the axial magnetic field at the center of the 6 double-pancake coil,
- and the temperatures measured by all of the thermocouples.

The charging of the magnet at ≈25 K is recorded and shown in Figure 2-19. The temperature of the current port (TC#3) and the cold container (TC#5) drop somewhat due to the helium circulation that is employed to counter the heat leak from the disconnectable leads and the dissipation from the superconducting switch’s heater. The magnet temperature (TC#4) rises due to the heater dissipation. At roughly 12 min, the current leads are disconnected, the helium pressure is released, and the transfer line is removed. All of the temperatures converge at 21 min and ≈25 K. Figure 2-20 shows the recorded data of the subsequent field decay over a period of 12 hours.
Figure 2-19: Traces obtained when charging the magnet at 25 K.
Figure 2-20: Hall probe and temperature traces depicting the magnetic field decay after the magnet is charged with an initial temperature of 25 K.
Chapter 3

Theoretical Modeling

The system attributes, such as weight and volume, of any superconducting system depend on the magnetic and cooling requirements. In order to demonstrate the advantages of a solid nitrogen cooled permanent HTS magnet system, theoretical models are required to estimate system attributes for a given set of system requirements. These models may be used to compare the attributes of this design with the attributes of other designs for a particular application. In this chapter, models are developed to predict the experimental system’s warming trend and magnetic field decay. In chapter 4, the models are validated with a good agreement between simulations based on these models and experimental results. In chapter 5, similar models are used to identify selected superconducting magnet applications where solid nitrogen cooled HTS magnet systems are best suited.

Prediction of the cold container’s warming trend involves solving the differential form of the first law of thermodynamics applied to a control volume that encloses the cold container, as shown schematically in Figure 3-1:

\[
\frac{dE_{cc}}{dt} = \dot{Q}_t
\]  

(3.1)

where \( E_{cc} \) is the internal energy of the cold container and its contents, and \( \dot{Q}_t \) is the total heat flow into the cold container from outside the control volume. Dissipation within the magnet does not make a significant contribution to the warming of the cold
container because the stored magnetic energy of the charged magnet is on the order of 10 J, while the energy change associated with a 20–40 K warming of the cold container and its contents is on the order of 50 kJ. The control volume used in this analysis is illustrated by the dashed lines in Figure 3-2. Calculations described in Appendix C support the assumption that the temperature inside the cold container is uniform. Therefore, a one-to-one correspondence between $E_{cc}$ and $T_{cc}$ may be derived.

3.1 20–40 K Rise Time Prediction

With the exception of nitrogen, thermal contraction of the cold container and the materials contained within it is insignificant for temperatures below 80 K with respect to the densities at 80 K. Therefore, the cold container and its contents may be treated as a constant volume system so that a relation between energy and temperature may be derived from thermodynamic data for each material by following a path of constant specific volume. In the absence of thermal gradients, the nitrogen resides in a two-phase (solid and vapor) equilibrium after it is cooled below its triple point of 63.1 K. The vapor pressure of nitrogen in the 20–40 K temperature range is very small [13]. As a result, the specific energy of the nitrogen is very close to the specific enthalpy of the solid phase at its saturation pressure because the warming of the nitrogen approximates a constant pressure process. The same arguments apply to the other materials enclosed within the control volume.
Figure 3-2: Depiction of the control volume (dashed line) used for predicting the cold container's 20–40 K warming trend. Most details have been removed for clarity. The boundary of the control volume is considered to lie just beneath the innermost layer of superinsulation.
Ec is calculated as a function of $T_{ec}$ using data listed in [2], [5], and [6], and the mass, $m$, of each material inside of the control volume:

$$E_{cc} = m_{N_2}e_{N_2} + m_{Cu}e_{Cu} + m_{G10}e_{G10} + m_{Bi2223/Ag}e_{Bi2223/Ag}$$

(3.2)

where the subscript $N_2$ denotes solid nitrogen, $Cu$ is copper, $G10$ is G-10 fiberglass reinforced epoxy, and $Bi2223/Ag$ is the conductor. The control volume encloses 6.8 kg of copper, 1.6 kg of solid nitrogen, 0.16 kg of G-10, and 1.93 kg of conductor. The internal energy of the conductor per unit mass is assumed equal to the internal energy of silver per unit mass. $e_{G10}$ is approximated using values for epoxy [2].

$\dot{Q}_t$ is equal to the sum of three components, each representing a mode of heat transfer:

$$\dot{Q}_t = \dot{Q}_{cnd} + \dot{Q}_{rad} + \dot{Q}_{cnv}$$

(3.3)

where $\dot{Q}_{cnd}$ is the heat transfer by solid conduction, $\dot{Q}_{rad}$ is the heat transfer by radiation between surfaces that see each other, and $\dot{Q}_{cnv}$ is the heat transfer by convection through a fluid medium. The system was designed with the intent of reducing the total heat leak by considering each of the heat transfer modes separately. Convection heat transfer from the cryostat wall to the cold parts is made negligible by maintaining a vacuum better than $10^{-5}$ torr inside the cryostat. Convection heat transfer is expected to be negligible. Calculation supporting this claim is illustrated in Appendix C. Radiation heat transfer is reduced by enclosing the cold container within 40 layers of NRC-2 superinsulation, as described in Chapter 2. The combination of various nonuniform conditions make analytical prediction of the heat leak across the superinsulation difficult, so empirical relations obtained from the literature are used to estimate the heat flux through the superinsulation in Section 3.1.1.

Conduction into the cold container is reduced by the design of the components that span from the cold container to room-temperature. The access tubes and supports were constructed using thin-wall stainless steel tubes which have a high strength to thermal conduction ratio. Instrumentation leads consist of thin wire of a material with low thermal conductivity. The superconducting switch current leads are designed for
the lowest possible heat leak when power is not being delivered to the heater, as described in Chapter 2 and Appendix B.

The access tubes and instrumentation leads, which penetrate the superinsulation enclosure, cause a significant contribution to the heat leak into the cold container. The heat leak through the penetrations result from a combination of radiation traveling down the tube, and conduction through the walls of the tube and the instrumentation leads that are wrapped around them. The heat leak through each penetration may be estimated based on theory as described in Section 3.1.2.

Summing the estimates of the penetration heat leaks with an estimate for the heat leak through the superinsulation based on empirical relations should yield a reasonable prediction of the total heat leak, $\dot{Q}_t$:

$$\dot{Q}_t = \dot{Q}_{\text{ins}} + \dot{Q}_{\text{pen}}$$  \hspace{1cm} (3.4)

where $\dot{Q}_{\text{ins}}$ is the heat leak through the superinsulation, and $\dot{Q}_{\text{pen}}$ is the heat leak through the penetrations. Estimates for both of these terms are described separately below. $\dot{Q}_t$ is predicted to be in the range of 270–700 mW.

### 3.1.1 Heat Leak through the Superinsulation

NRC-2 superinsulation consists of a 6.4 $\mu$m thick polyester film that is metalized on one side with aluminum 25 nm thick. The insulation is often mounted without a separate spacer material between the layers. Gaps are maintained between the layers by crinkling the superinsulation and applying it loosely. Heat is transfered between layers of superinsulation by radiation that is emitted from the entire surface of each layer and conduction across regions where the adjacent layers are in contact. The superinsulation was applied to the cylindrical wall of the cold container by wrapping sheets around it, so conduction along the sheets is also expected to contribute to the heat transfer between layers. Convective heat transfer between the inner layers may also be significant because pumping is severely obstructed and outgasing degrades the vacuum.
Table 3.1: Reported Performance of NRC-2 Superinsulation

<table>
<thead>
<tr>
<th>Reference</th>
<th>Configuration</th>
<th>$Q''_{rep}$ [W/m²]</th>
<th>$Q''_{adj}$</th>
<th>$k_\perp$ [μW/mK]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[10]</td>
<td>11 layers/cm</td>
<td>0.64</td>
<td>0.61</td>
<td>102</td>
</tr>
<tr>
<td></td>
<td>$T_H=277$ K, $N_{rep}=30$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[14]</td>
<td>35 layers/cm</td>
<td>-</td>
<td>-</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>$T_H=300$ K</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[15]</td>
<td>20 layers/cm</td>
<td>0.786</td>
<td>0.454</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>$T_H=300$ K, $N_{rep}=25$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

There is little recent performance data reported for NRC-2 superinsulation mounted without a spacer material between the layers. Most works focus on double-aluminized mylar film (DAM) (aluminized on the front and back surface) or aluminum foil applied with a spacer material between each layer, such as silk or polyester net, or glass paper. Researchers have focused on this type of configuration because it provides superior performance. However, the materials are much more expensive, and the proper application of DAM and spacer material requires much experience. Therefore, since this project is a concept demonstration that does not focus on improving insulation techniques, the simpler method involving NRC-2 without a spacer is used.

Some experimentally measured values of the performance of crinkled NRC-2 superinsulation without a spacer material have been reported, and are listed in Table 3.1. $Q''_{rep}$, $Q''_{adj}$, and $k_\perp$ are discussed below. The data are for heat flux in the direction perpendicular to the superinsulation layers. However, for most superinsulation configurations, predictions for real systems based on reported values should be expected to be off by at least a factor of 2 [16][17], for several reasons. To begin with, most of the reported values are for insulating simple geometries, such as a flat plate which is much simpler to enclose with a flat sheet than a cylindrical tank. When applying sheets of superinsulation to a complicated geometry, gaps inevitably exist at the joints between different surfaces, such as between the layers wound around the curved surface of a cylinder and the layers applied to the top flat surface of the cylinder. Secondly, application over corners and tight curves compresses the layers together, increasing conduction heat flux across the layers. Thirdly, penetrations through the
superinsulation also increase thermal communication between layers. The high thermal conductivity parallel to the superinsulation layers (due to the aluminum coating) tends to increase the area of superinsulation that behaves poorly when the layers are thermally shorted.

There are other differences between the techniques used in obtaining the literature data and the techniques used for the experiment of this project that will further increase the discrepancy between predicted and observed heat leak. To reduce the time required to mount the superinsulation, it was applied to the cylindrical wall of the cold container using a rolled configuration, where the layers are built up by wrapping a continuous sheet around the cylinder. Most reported works are based on a laminated configuration where each layer of superinsulation is applied as a separate sheet. The rolled configuration does not perform as well because heat leaks by conduction and radiation interreflection along the superinsulation in the direction it was rolled. Also, some researchers use different boundary temperatures and a different number of layers.

For large temperature differences, the heat flux through the superinsulation, \( \dot{Q}_\perp'' \) is usually dominated by radiation heat transfer, which is proportional to the difference in the fourth power of the bounding temperatures:

\[
\dot{Q}_\perp'' \propto T_H^4 - T_C^4
\]

where \( T_H \) is the temperature of the outermost layer and \( T_C \) is the temperature of the innermost layer.

The data listed in Table 3.1 are all for \( T_H \geq 277 \text{K} \), and \( T_C = 77 \text{K} \). The value of \( T_C \) makes less than a 2% difference in the heat flux for these conditions. This claim is supported by experimental observations listed in [18] for other superinsulation configurations, and was observed using the experimental system of this project by comparing the boil-off rate when the cold container was filled with liquid nitrogen at the boiling point with the heat leak calculated from the 20–40 K warming time. However, variation in \( T_H \) leads to a very significant difference. The warm wall of our experimental system was at room-temperature (\( \approx 293 \text{K} \)). The difference in temper-
ature between the outermost layer and the warm wall has been reported by various researchers to be negligible (< 5 K) \cite{9,10,19}, so $T_H$ is assumed to be equal to the temperature of the warm wall. According to Equation 3.5, increasing $T_H$ from 277 K to 293 K should lead to a 25% increase in heat flux.

Finally, 40 layers of superinsulation are used in our experiment, yet most researchers do not report data for exactly 40 layers. For a superinsulation system dominated by radiation, the heat flux is expected to be inversely proportional to 1 plus the number of layers, $N$:

$$\dot{Q}_{\perp}'' \propto \frac{1}{N + 1}. \quad (3.6)$$

In applying the literature data, the expected heat flux values are adjusted depending on the configuration used. The adjusted values are listed in the second to last column of Table 3.1. Where a different $T_H$ was used, the adjusted heat flux value, $\dot{Q}_{adj}''$, was obtained based on the following relation in accordance with Equation 3.5:

$$\dot{Q}_{adj}'' = \left(\frac{293 \text{K}}{T_H}\right)^4 \dot{Q}_{rep}'' \quad (3.7)$$

where $\dot{Q}_{rep}''$ is the value reported. Where a heat flux could not be discerned for 40 layers based on the data reported for $N_{rep}$ layers:

$$\dot{Q}_{adj}'' = \frac{N_{rep} + 1}{40 + 1} \dot{Q}_{rep}'' \quad (3.8)$$

The last column of Table 3.1 lists an equivalent thermal conductivity in the direction perpendicular to the layers, $k_{\perp}$, calculated using a warm wall temperature of 293 K, a cold wall temperature of 77 K, and a blanket thickness based on 40 layers with the reported layer density.

Figure 3-3 depicts the model used for predicting heat leak through the superinsulation without considering the heat leak through the penetrations. The superinsulation enclosure is broken down into 6 blankets, each individually analyzed: the upper outer cylinder, the lower outer cylinder, the inner cylinder, the outside bottom end, the inside bottom end, and the top end. The heat flux through each blanket is based on
Figure 3-3: Cross-section of the cold container identifying the breakup of the superinsulation into blankets that are analyzed individually for predicting the total heat leak crossing the layers of superinsulation. The system dimensions used in the calculations for each blanket are identified.
an innermost layer temperature of 77 K because all of the values listed in Table 3.1 were determined for a cold wall at 77 K, and the heat flux through the superinsulation is actually weakly dependent on the inner wall temperature if it is below 100 K and the warm wall temperature is up near room-temperature, as described above. The outermost superinsulation layer is assumed to be equal to the temperature of the cryostat wall, approximately 293 K. Modeling the superinsulation with an equivalent thermal conductivity, a heat leak is calculated for each blanket using an appropriate relation. For the cylindrical surfaces:

\[
\dot{Q}_\perp = \frac{2\pi k_\perp l \Delta T}{\ln(D_o/D_i)}
\]  

(3.9)

where \( l \) is the length of the cylindrical surface, \( \Delta T \) is the temperature difference between the inner and outer layers (216 degrees kelvin), \( D_i \) is the diameter of the innermost layer, and \( D_o \) is the diameter of the outermost layer. The diameters dimensioned in Figure 3-3 are the inner diameters used in the calculations. The top and bottom ends are modeled as flat plates so that:

\[
\dot{Q}_\perp = \frac{A_{end} k_\perp \Delta T}{t_{ins}}
\]  

(3.10)

where \( A_{end} \) is the surface area of the innermost layer covering the end, and \( t_{ins} \) is the total superinsulation thickness between the innermost and outermost layers. Using the value from Table 3.1 that is based on a layer density close to the layer density used in the experiment of 36 layers/cm, i.e. 42 \( \mu \)W/mK [14], \( \dot{Q}_\perp \) is predicted for each blanket and listed in Table 3.2. It should be noted that if the layers of [10] were compressed to the same layer density as [14], the effective thermal conductivity of [10] would be between the values for [14] and [15] if the heat flux remained the same. The total heat flux through the superinsulation is expected to be 160 mW when assuming that the heat only flows perpendicular to the layers.

The heat leak values predicted by Equations 3.9 and 3.10 only consider conduction perpendicular to the superinsulation layers. However, the effective thermal conductivity in the direction parallel to the layers can be quite high due to combined conduction
Table 3.2: Predicted Heat Leak Through Each Superinsulation Blanket

<table>
<thead>
<tr>
<th>Blanket</th>
<th>Heat Leak [mW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>upper outer cylinder</td>
<td>36</td>
</tr>
<tr>
<td>lower outer cylinder</td>
<td>82</td>
</tr>
<tr>
<td>inner cylinder</td>
<td>17</td>
</tr>
<tr>
<td>outside bottom end</td>
<td>10</td>
</tr>
<tr>
<td>inside bottom end</td>
<td>&lt;1</td>
</tr>
<tr>
<td>top end</td>
<td>12</td>
</tr>
<tr>
<td>Total</td>
<td>≈160</td>
</tr>
</tbody>
</table>

through the highly conductive aluminum coating and radiation interreflection between adjacent superinsulation layers. Therefore, the heat flow along the superinsulation layers may be quite significant at certain locations, particularly where the end of a superinsulation blanket is exposed. Four superinsulation blankets have an exposed end where the edges of the superinsulation layers view the room-temperature cryostat wall. These include the blankets covering the access tubes (nitrogen input, helium input, and helium output) and the upper outer cylinder blanket which surrounds the top portion of the cold container's cylindrical surface.

It is difficult to estimate how much heat leaks from the warm exposed ends of these blankets to the cold container because of the nonuniform conditions across the superinsulation blankets, coupled effects between the conduction parallel and perpendicular to the layers, and the uncertainty of the quality of joints between adjacent blankets. It is likely that heat flowing into the exposed ends reaches the cold container by traveling through gaps at the joints. But despite the difficulties in making an accurate estimate of the heat leak contribution by radiation incident on the superinsulation layer edges, an upper bound on the heat flow parallel to the layers from the exposed edges can be determined with a simple calculation.

The model used to calculate upper bounds on the heat leak contributions from conduction and radiation interreflection parallel to the superinsulation layers is illustrated in Figure 3-4. The exposed end is assumed to be at the ambient temperature, $T_H$. At some distance into the blanket from the end, $z_{pc}$, the heat flow parallel to the
Figure 3-4: Illustration of the model used to calculate the upper bound on the heat leak contribution from conduction and radiation interreflection parallel to the superinsulation layers.
layers becomes small. As a result, the temperature profile across the thickness of the blanket at this location is completely determined by the steady heat flux perpendicular to the superinsulation layers.

For a cylindrical medium, the temperature profile resulting from a steady heat flux in the radial direction is determined as:

\[ T(r) = T_i - \frac{(T_i - T_o)}{\ln(r_o/r_i)} \ln(r/r_i) \]  \hspace{1cm} (3.11)

where \( r \) is the radial distance from the cylinder axis, and the subscripts \( i \) and \( o \) indicate the inner and outer surfaces, respectively. Based on this temperature profile, heat flow along the superinsulation layers, \( \dot{Q}_\parallel \), from the warm edge to the location assumed to have the temperature profile of Equation 3.11 may be calculated. The following integral relation assumes heat flow only in the parallel direction:

\[ \dot{Q}_\parallel = \int_{r_i}^{r_o} \frac{2\pi \tilde{k}_\parallel}{z_{pc}} [T_H - T(r)] r dr \]  \hspace{1cm} (3.12)

where \( \tilde{k}_\parallel \) is the average effective thermal conductivity of the superinsulation in the direction parallel to the layers (\( z \)-direction) over the temperature range of \( T(r) - T_H \). Combining Equations 3.11 and 3.12 while assuming \( T_i \) equal to \( T_C \) and \( T_o \) equal to \( T_H \), and treating \( \tilde{k}_\parallel \) as a constant leads to a relation for calculating \( \dot{Q}_\parallel \):

\[ \dot{Q}_\parallel = \frac{\pi \tilde{k}_\parallel (T_H - T_C)}{z_{pc}} \left[ \frac{r_o^2 - r_i^2}{2 \ln(r_o/r_i)} - r_i^2 \right]. \]  \hspace{1cm} (3.13)

[20] lists experimentally measured values of \( k_\parallel \) for NRC-2 superinsulation as a function of temperature. An approximate value for \( \tilde{k}_\parallel \) is obtained from the data by finding the average of \( k_\parallel \) over the temperature range \( \frac{T_H + T_C}{2} - T_H \). The data needs to be extrapolated over the 240–293 K temperature range. A value for \( \tilde{k}_\parallel \) of 0.060 W/mK is used in calculating the upper bounds on \( \dot{Q}_\parallel \).

The selection of appropriate values for \( z_{pc} \) is the last step in determining the upper bound on \( \dot{Q}_\parallel \) for each of the superinsulation blankets with exposed ends. For the access tube penetrations, \( z_{pc} \) is taken as the distance from the top edge of the
superinsulation blanket enclosing the penetration to the joint with the top end blanket of the superinsulation enclosing the cold container, which is a length of 230 mm. For the upper outer cylinder blanket enclosing the cold container, \( z_{pc} \) is taken as the axial length which the upper outer cylinder overlaps the lower outer cylinder, equal to 100 mm. This length is chosen because the inner surface of the upper outer cylinder blanket sees the cold container at locations past this distance from the exposed end. Heat flow parallel to the superinsulation layers will have the greatest influence on the net heat flux from the inner surface of the blanket to the cold container when the inner layer of the superinsulation is in direct thermal communication with the cold container. \( \dot{Q}_i \) as calculated using Equation 3.13 for this value of \( z_{pc} \) is not negligible, but is still an upper bound on the additional heat leak resulting from the parallel heat flow. When \( \dot{Q}_i \) is not negligible, the average temperature across the blanket thickness at \( z_{pc} \) will be higher than the average temperature determined by the temperature profile described by Equation 3.11, so Equation 3.13 overestimates \( \dot{Q}_i \) and an upper bound is still obtained.

Table 3.3 lists the calculated upper bounds on the additional heat leak by parallel conduction for each of the superinsulation blankets with exposed ends. The total heat leak through the superinsulation is expected to be greater than the value predicted by assuming only perpendicular heat flux across the superinsulation thickness (see Table 3.2), but less than the sum of this value and the total upper bound calculated for heat flow parallel to the superinsulation layers from exposed ends of 430 mW. Therefore, \( \dot{Q}_{ins} \) is expected to be between 160 mW and 590 mW.

### Table 3.3: Upper Bounds on Heat Leak from Exposed Blanket Ends

<table>
<thead>
<tr>
<th>Blanket</th>
<th>Maximum ( \dot{Q}_i ) [mW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helium Input</td>
<td>18</td>
</tr>
<tr>
<td>Helium Output</td>
<td>7</td>
</tr>
<tr>
<td>Nitrogen Input</td>
<td>11</td>
</tr>
<tr>
<td>Upper Outer Cylinder (Surrounds Cold Container)</td>
<td>395</td>
</tr>
<tr>
<td>Total</td>
<td>( \approx 430 )</td>
</tr>
</tbody>
</table>
Table 3.4: Predicted Heat Leak Through Penetrations and Instrumentation Leads

<table>
<thead>
<tr>
<th>Penetration</th>
<th>Heat Leak [mW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helium Input</td>
<td>37</td>
</tr>
<tr>
<td>Helium Output</td>
<td>12</td>
</tr>
<tr>
<td>Nitrogen Input</td>
<td>20</td>
</tr>
<tr>
<td>Current Feedthroughs</td>
<td>26</td>
</tr>
<tr>
<td>Voltage Taps</td>
<td>&lt;1</td>
</tr>
<tr>
<td>Thermocouples</td>
<td>&lt;1</td>
</tr>
<tr>
<td>Heater Leads (both)</td>
<td>16</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>≈110</td>
</tr>
</tbody>
</table>

3.1.2 Heat Leak Through The Penetrations

There are five components which penetrate the superinsulation: the three stainless steel access tubes (the helium input, the helium output, and the nitrogen input) and the two current feedthroughs that pass through the cold container to mate with the mobile part of the disconnectable leads. The instrumentation leads are considered to be thermally isolated from the access tubes that they are wrapped around. With this assumption, the heat leak through the instrumentation leads is small.

Two models are developed: one for the access tubes, and one for the current feedthroughs. In all cases, we are interested in the steady-state conduction into the cold container since the transient effects become negligible in less than an hour, while the 20–40 K warming time of the system exceeds one day. The predicted heat leak through each penetration is listed in Table 3.4 along with the predictions for the instrumentation leads. $\dot{Q}_{pen}$ is expected to be approximately 110 mW.

Access Tubes

Heat leak associated with the access tubes results from a combination of conduction through the wall of the tube, radiation transmitted down the inside, and radiation emitted from the outer surface of the section near the cold container that is not covered in superinsulation. Estimates of the heat transfer by combined conduction...
and radiation interreflection parallel to the superinsulation layers surrounding the access tubes was included in Section 3.1.1. The net heat leak down each access tube is estimated by considering the radiation transmission along the inside of the tube to be independent of the conduction through the tube wall and radiation emission from the tube outer surface. The conduction through the tube wall is calculated while assuming no radiation heat transfer between points on the inner surface of the tube, such that the inner surface is adiabatic. Then, the total heat leak through the access tube, $\dot{Q}_t$, is estimated as the sum of two individual components:

$$\dot{Q}_t = \dot{Q}_c + \dot{Q}_r \quad (3.14)$$

where $\dot{Q}_c$ is the heat conduction along the wall of the tube, and $\dot{Q}_r$ is the radiation transmitted through the inside of the tube. In the actual system, the inner surface of the tube is not adiabatic. There is a net heat transfer by radiation from one location on the inner surface to all other locations on the inner surface that are at a different temperature, including the the end caps. However, assuming the radiation and conduction modes to be decoupled will yield a conservative overestimate of the total heat leak through the access tube.

A numerical simulation was developed to estimate the heat conduction through the walls of a single length of tube using a one-dimensional finite difference scheme employing forward-Euler differencing in time. The model is depicted in Figure 3-5. The top and bottom ends are capped with surfaces of fixed temperature, $T_{top}$ and $T_{bot}$. The outer wall of the tube is assumed to be adiabatic over the length that is wrapped in superinsulation. The inner wall is assumed to be adiabatic as described in the previous paragraph. The simulation solves for the steady-state temperature profile along the length of the tube, and the associated conduction heat transfer rate entering the top of the tube, $\dot{Q}_c$.

Rotational symmetry about the tube axis permits the temperature profiles and heat transfer rates within the tube to be described by a modified form of the 1-D heat
Figure 3-5: Model for calculating conduction along a tube with the ends at fixed temperatures. The tube is broken up into node segments so that a finite difference scheme may be applied.
conduction equation. The modified form is obtained by starting with the traditional form of the heat conduction equation:

\[
c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + g(x, t)
\]  

(3.15)

where \( T \) is temperature, \( t \) is time, \( x \) is the distance from the bottom of the tube, \( c \) is the volumetric heat capacity, \( k \) is the thermal conductivity, and \( g \) is the heat dissipation term. There is no source of heat dissipation in the walls of the tube, so \( g \) is equal to zero. However, some means of accounting for radiation heat transfer from the uninsulated section of the outer surface is required, so \( g(x, t) \) is replaced with a term that represents the radiation heat transfer, \(-r(x, t)\), yielding:

\[
c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) - r(x, t)
\]  

(3.16)

To apply this model to a numerical simulation, the tube is broken up into ring elements of equal length referred to as nodes (see Figure 3-5). Each node transfers heat with its adjacent nodes by conduction. Additionally, nodes that are not wrapped in superinsulation transfer heat to the surroundings by radiation. The partial differential equation, Equation 3.16, is converted into a numerically usable form by substituting Taylor expansions for the partial differentials:

\[
c \frac{\partial T}{\partial t} = \frac{T^{i+1} - T^i}{\Delta t} - r(x, t)
\]  

(3.17)

where \( \Delta t \) is the time step size, and \( i \) denotes which time step. The first term on the right-hand side of Equation 3.16 represents the divergence of conduction along the tube and can be written simply as \( \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) \) for a one-dimensional system. This term is transformed into the following discrete form:

\[
\nabla \cdot (k \nabla T) = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) = \frac{1}{\Delta x} \left[ k_{m+1/2} \frac{T_{m+1} - T_m}{\Delta x} - k_{m-1/2} \frac{T_m - T_{m-1}}{\Delta x} \right]
\]  

(3.18)

where \( \Delta x \) is the node length and \( m \) denotes which node. Equation 3.18 includes a
subscript for $k$ denoted as $m \pm 1/2$ which indicates the value of the thermal conductivity on the half-step. There are several acceptable means for calculating $k_{m+1/2}$, such as simply taking the arithmetic mean of $k_m$ and $k_{m+1}$. In this heat transfer problem $k_{m \pm 1/2}$ is determined according to:

$$k_{m \pm 1/2} = 2 \frac{k_m k_{m \pm 1}}{k_m + k_{m \pm 1}}. \quad (3.19)$$

This relation permits a more intuitive form of the discrete heat equation by defining another value, $R_j$, which represents the thermal resistance between the midplane of node $m$ and the midplane of an adjacent node $j$ ($j = m \pm 1$) and is given by:

$$R_j = \frac{1}{A_w} \left( \frac{\Delta x}{2k_m} + \frac{\Delta x}{2k_j} \right) = \frac{\Delta x}{2A_w} \left( \frac{1}{k_m} + \frac{1}{k_j} \right) \quad (3.20)$$

where $A_w$ is the cross-sectional area of the wall. Combining Equations 3.16–3.20 and rearranging terms leads to the following:

$$T_{m+1}^i = \frac{\Delta t}{c_{m}^i} \left( \frac{1}{A_w \Delta x} \sum_j \frac{T_j^i - T_m^i}{R_j} - r_m^i \right) + T_m^i. \quad (3.21)$$

Now all that remains in obtaining a numerical relationship for the simulation is to establish a means for calculating $r_m^i$.

The lower 65 mm of each access is not wrapped in superinsulation and therefore, its outer surface should not be considered adiabatic. Instead, radiation is emitted from the outer surface of the tube into the gap between the cold container and the superinsulation. The rate of radiation heat transfer between two surfaces with uniform temperature is determined by the following relation:

$$\dot{Q}_{a-b} = \frac{\sigma (T_a^4 - T_b^4)}{1 - \epsilon_a} \frac{1}{\epsilon_a A_a} + \frac{1}{A_a F_{ab}} + \frac{1 - \epsilon_b}{\epsilon_b A_b} \quad (3.22)$$

where $a$ and $b$ denote which surface, $A$ is the surface area, $\sigma$ is the Stefan-Boltzmann constant, $\epsilon$ is the emissivity of the surface, and $F_{ab}$ is the view factor which depends
on the geometry and relative orientation of both surfaces. The subscript \(a-b\) indicates a positive direction of heat transfer from surface \(a\) to surface \(b\).

The outer surface of the uninsulated tube section is assumed to only face surfaces that are at the cold container temperature. Therefore, at any time step, the rate of radiation heat transfer leaving one of the nodes in the uninsulated section is given by:

\[
\dot{Q}_{m-cc} = \frac{\sigma(T_m^4 - T_{cc}^4)}{1 - \epsilon_m} \left( \frac{1}{\epsilon_mA_m} + \frac{1}{\epsilon_cA_c} \right)
\]

where the subscript, \(cc\), refers to the cold container, and \(A_m\) is the node surface area on the outside of the tube. The area of the cold container surface seen by the tube is much larger than the surface area of the tube, and the emissivity of the cold container surface is significant because it is heavily oxidized, so the last term in the denominator on the right-hand side of Equation 3.23 may be neglected. Additionally, the surface on the outside of the tube only sees surfaces at the cold container temperature so the view factor, \(F_{m-cc}\), may be set equal to 1. With these modifications, Equation 3.23 becomes:

\[
\dot{Q}_{m-cc} = \sigma\epsilon_m A_m(T_m^4 - T_{cc}^4)
\]

Therefore, \(r_m^i\) is calculated as:

\[
r_m^i = \begin{cases} 
0 & \text{for } x \geq x_{bare} \\
\sigma\epsilon_{ss} A_m(T_m^4 - T_{cc}^4) & \text{for } x < x_{bare}
\end{cases}
\]

where \(\epsilon_{ss}\) is the emissivity of stainless steel 304 (SS304), and \(x_{bare}\) is the length of the uninsulated section. \(\epsilon_{ss}\) can vary significantly depending on how the surface is processed. [21] and [22] list a range of values for \(\epsilon_{ss}\) of 0.26–0.40. A value of 0.40 is used because it leads to a conservative overestimate of \(\dot{Q}_{m-cc}\), \(\dot{Q}_c\), and \(\dot{Q}_t\).

Having defined all terms in Equation 3.21, \(\dot{Q}_c\) may be calculated by a numerical simulation. The simulation is initiated with a linear temperature profile connecting \(T_{bot}\) at the bottom end with \(T_{top}\) at the top end. The temperature of each node is recalculated during each time step of the simulation according to Equation 3.21. As the time steps progress, the simulated temperature profile approaches a steady-state
solution. Stable solutions are obtained only for small enough $\Delta t$. $\Delta x$ must be reduced until the solution converges such that further reduction of $\Delta x$ does not significantly change the calculated solution. The simulated steady-state temperature profile and the associated conduction heat transfer rate into the top end should then be a close match with the actual.

Accurate simulation of the access tubes requires relations for the thermal properties of the associated materials. Two thermal properties of the tube material (SS304), thermal conductivity $k$ and specific heat $c$, are strongly temperature dependent and must be recalculated for each node at every time step. Polynomial correlations for $c$ and $k$ are based on data reported in [23]. It should be noted that any value for $c$ can be used in this simulation since the steady-state solution does not depend on it. However, actual values are used to verify that a steady-state is reached in less than 1 hour.

The radiation transmitted through the tube, $Q_{ro}$, is estimated while assuming zero conduction along the walls of the tube. This problem was solved in [24] in terms of the tube length to diameter ratio, $l/d$, the rate of radiation entering the warm end of the tube, $\dot{Q}_{ri}$, the emissivity of the inner tube surface, and whether the surface is a diffuse or specular reflector. The cold end temperature is assumed to be zero, which models the situation of negligible radiation flux entering the cold end. The solutions are illustrated in Figure 3-6 for (a) a black or diffuse reflecting gray wall surface, and for (b) a specularly reflecting gray wall surface. The solution for a diffuse reflecting gray surface is identical to the black surface solution, regardless of the emissivity of the gray surface.

The access tubes are constructed with SS304, which probably behaves as a specular reflector. Calculation of $\dot{Q}_{ro}$ begins by calculating $\dot{Q}_{ri}$. For a warm end open to ambient surroundings at a temperature, $T_H$ (taken as 293 K for the experimental system), $\dot{Q}_{ri}$ is calculated as:

$$\dot{Q}_{ri} = \sigma T_H^4 A_{tube} = \sigma T_H^4 \pi d^2 / 4 \tag{3.26}$$

where $A_{tube}$ is the cross-sectional area of the tube. Then the radiation exiting the
Figure 3-6: Solutions for calculating radiation transmission through a tube with zero conduction for (a) a black or diffuse reflecting gray wall surface, and for (b) a specularly reflecting gray wall surface [24]. The cold end temperature is assumed to be zero.
bottom of a diffuse reflecting tube with the same dimensions as the access tube, \( \dot{Q}_{rod} \), is calculated from \( l/d \) according to Figure 3-6a. Finally \( Q_{ro} \) is calculated from the emissivity and \( l/d \) according to Figure 3-6b. The lower bound of typical emissivity values for SS304 (0.26–0.40) is used in the calculation. This value produces a conservative overestimate because the radiation transmission increases with decreasing emissivity for specularly reflecting surfaces.

The helium output is the only access leading from the cold container to room-temperature that is constructed from a single tube. The methods described above are used to predict the heat leak through the helium output without modification. The helium input and the nitrogen input, on the other hand, consist of two tubes of different diameter coupled together in series (see Chapter 2) as illustrated in Figure 3-7. In this case the simulation is applied to both tubes in an iterative manner, adjusting the temperature where the tubes meet, \( T_{cpl} \), until the heat flow exiting Tube 1, \( \dot{Q}_{1o} \), is equal to the heat flow entering Tube 2, \( \dot{Q}_{2i} \).

Figure 3-8 illustrates an energy balance on a control surface that surrounds the coupling. The radiation entering the top of Tube 2, \( \dot{Q}_{2ro} \), is determined as the radiation flux exiting Tube 1 multiplied by the cross-sectional area of Tube 2, such that:

\[
\dot{Q}_{2ri} = A_2 \frac{\dot{Q}_{1ro}}{A_1}
\]  

where \( A_1 \) and \( A_2 \) are the cross-sectional areas of the tubes. The radiation exiting Tube 2, \( \dot{Q}_{2ro} \), is calculated from \( \dot{Q}_{2ri} \) using Figure 3-6. The conduction into Tube 2 is balanced with the conduction from the top tube and the radiation incident on the coupling which is assumed to be a black surface that emits an insignificant amount of radiation in comparison to what it absorbs, such that:

\[
\dot{Q}_{2c} = \dot{Q}_{1c} + (1 - A_2/A_1) \dot{Q}_{1ro}.
\]

The coupling probably emits a significant amount of radiation. However, most of it is reabsorbed by the tube wall near the coupling so that the effective heat flow into the control volume from above is not significantly overestimated by Equation 3.28.
Figure 3-7: Model for predicting the heat leak through an access tube that is composed of two tubes coupled together with different diameters.
Figure 3-8: Energy balance on a control surface that surrounds the coupling.

When the value of $T_{cpl}$ that satisfies Equation 3.28 is found, the heat leak associated with the access tube penetration, $\dot{Q}_t$, is determined as:

$$\dot{Q}_t = \dot{Q}_{2c} + \dot{Q}_{2ro}. \quad (3.29)$$

The magnitudes of the radiation transmission through the helium output, Tube 2 of the nitrogen input, and Tube 2 of the helium input are all expected to be less than 1 mW. Radiation transmission through Tube 1 of the helium input and Tube 1 of the nitrogen input are predicted to be, respectively, 17 mW and 3 mW.

**Cold Container Current Feedthroughs**

The plugs extending from the current feedthroughs that are attached to the cold container flange pass through the superinsulation, as shown in Figure 2-13. While the disconnectable current leads are detached, the lower end of the mobile portion rests just 1–2 mm above the plug. An effort was made to reduce the radiation heat transfer to the plugs, which should be at roughly the same temperature as the cold
container. The brass pieces at the ends of the mobile portion of the disconnectable leads are covered with two layers of superinsulation and the connector surfaces are enclosed within a superinsulation cone, as described in Section 2.1.7. With this configuration, the cold tips can only see a room-temperature, high-emissivity surface through the hole leading to the Multilam socket.

A simple but approximate method is used to estimate the heat leak into the plugs because the contribution is not expected to be too significant. Since the spacing between the plug and the socket is small, the plug is assumed to have a view factor of 1 with the hole. The hole is assumed to be a black surface. The tip of the plug, which is a copper surface that is somewhat oxidized, is also assumed to have a black surface. The remaining surfaces seen by the plug consist of the low emissivity superinsulation. These are assumed not to transfer significant amounts of heat with the plug. With these assumptions, the following relation is used to estimate the heat leak through one feedthrough:

$$\dot{Q}_{h-p} = \frac{\sigma(T_h^4 - T_p^4)}{A_h} \left( \frac{1}{1 - \epsilon_p} + \frac{1}{\epsilon_p A_p} \right)$$  \hspace{1cm} (3.30)

where the subscript \(h\) denotes the hole and \(p\) denotes the plug. If the area of the plug surface that faces the hole, \(A_p\), is approximately equal to the cross-sectional area of the hole, \(A_h\), and \(\epsilon_p\) is assumed equal to 1, then Equation 3.30 reduces to:

$$\dot{Q}_{h-p} = \sigma A_h (T_h^4 - T_p^4).$$  \hspace{1cm} (3.31)

Assuming \(T_h\) equal to 293 K, the radiation heat leak to both plugs combined is calculated to be 26 mW, as listed in Table 3.4.

### 3.2 Simulation of the Magnetic Field Decay

After the magnet is charged, the magnetic field and transport current decay due to resistive dissipation in the winding. A numerical simulation is developed here to predict the field variation with time. Figure 3-9 is an electrical circuit schematic.
modeling the magnet while it operates in persistent-mode. The circuit model is a simple \( RL \) circuit containing two resistances in series, where each resistor represents one of two mechanisms expected to cause dissipation. The circuit equation is:

\[
L \frac{dI_t}{dt} = -(V_s + V_n)
\]  

(3.32)

where \( I_t \) is the transport current, \( V_s \) is the voltage due to dissipation in the splices, and \( V_n \) is the voltage due to index dissipation. Both sources of dissipation are explained in the following sections.

### 3.2.1 Splice Resistance

The first source of dissipation is the splice resistance which is the resistance arising from the lap-splices that create the electrical joint between adjacent double-pancakes, and the lap-splices that join the superconducting switch and the stack of 6 double-pancakes. The lap-splices are made by soldering overlapping lengths of conductor from each coil together with indium (see Chapter 2). With the coil charged, the current must exit the superconducting fibers and travel through the conductor’s silver matrix and the indium in order to cross the splice as it travels from double-pancake
to double-pancake. The indium and the silver are not superconducting and therefore contribute some resistance. These resistances are expected to behave as ohmic resistors, where the total resistive voltage from splicing, \( V_s \), is equal to the transport current passing through them, \( I_t \), multiplied by the total splice resistance, \( R_s \):

\[
V_s = I_t R_s. \tag{3.33}
\]

\( R_s \) is calculated by summing together the expected resistance of each individual splice, \( R_{1s} \).

There are at least two resistive mechanisms which contribute to the splice resistance; the resistance of the solder, and the resistance of the matrix metal. The effect of the matrix resistance is neglected because it is expected to be small in comparison with the solder resistance in low fields. This claim is supported by theory described in [25] that is related to resistance measurements conducted at 4K for copper-sheathed Nb-Ti superconductor wire with a rectangular cross-section of 0.53 \( \times \) 0.68 mm. Although there are differences between the Nb-Ti conductor and the Bi2223/Ag conductor used in this experiment, which consists of a silver matrix and a rectangular cross-section of 0.23 \( \times \) 3.2 mm, it is still a fair comparison because the resistivity of silver at 40 K is comparable to the resistivity of copper at 4 K [2]. Additionally, the current crosses the splices through a smaller thickness of matrix metal for the Bi2223/Ag conductor.

\( R_{1s} \) is calculated based on a contact resistance, \( R_{ct} \), of \( 5 \times 10^{-9} \Omega \text{cm}^2 \) for a joint between two copper contacts soldered together with lead/tin solder, as reported in [25] and [26]. Although indium is used as the solder for making the splices in this experiment, no measurement of \( R_{ct} \) for contacts soldered together with indium could be located in the literature. The lead/tin joint data is used to make a conservative estimate of the splice resistance. \( R_{ct} \) resulting from a lead/tin soldered splice should be larger than an indium soldered splice since the resistivity of alloys are larger. \( R_{ct} \) is assumed constant since the resistivity of alloys are usually weakly dependent on temperature.
Each splice extends over one full turn around the outer diameter of the magnet and has a contact area, $A_{ct}$, of approximately $10.75 \text{ cm}^2$. The resistance of each individual splice, $R_{1s}$, calculated according to:

$$R_{1s} = \frac{R_{ct}}{A_{ct}} \quad (3.34)$$

leads to $R_{1s}$ equal to $4.65 \times 10^{-10} \Omega$, and a total splice resistance for all 7 splices, $R_s$, of $3.26 \times 10^{-9} \Omega$.

### 3.2.2 Index Resistance

The second resistive mechanism is the index resistance, which results from the conductor not being a perfect superconductor. The electric field in the conductor, $E$, is often expressed as a function of $I_t$ as follows:

$$E = E_c \left(\frac{I_t}{I_c}\right)^n \quad (3.35)$$

where $I_c$ is the critical current defined at an arbitrarily chosen electric field $E_c$ (usually 0.1 or 1.0 $\mu$V/cm), and $n$ is the index number. For a perfect superconductor, $n$ is equal to infinity; implying that $E$ is zero for $I_t < I_c$. Typical LTS conductor used to wind persistent-mode magnets has a value of $n$ greater than 60, leading to decay time constants on the order of many years. Currently, $n$ for long lengths of Bi2223/Ag conductor is reported to be in the range of 8–16, leading to a much faster magnetic field decay when $I_t$ approaches $I_c$, and a much lower sustainable field than for LTS conductor. The lower field is the result of $I_t$ needing to be a smaller fraction of $I_c$ and of Bi2223/Ag having a smaller $I_c$.

Critical current, $I_c$, is temperature and field dependent. The conductor manufacturer (NORDIC) reports an $I_c$ for the as-received tape conductor of 32 A (1.0 $\mu$V/cm criterion) at 77 K and in zero background field. This value can be applied to data reported by IGC [27] which gives the ratio between $I_c$ and its value at 77 K and zero background field for a range of temperature and field. The data is based on
tests of their own Bi2223/Ag multi-filamentary conductor, in its straight as-received condition. The expected $I_c$ of the as-received NORDIC conductor for a range of temperature and field is shown graphically in Figure 3-10: (a) for field applied parallel to the wide tape conductor surface, and (b) for field applied perpendicular to the wide tape conductor surface. Zero-field values are predicted based on data reported in [28]. When field components exist both perpendicular and parallel to the wide conductor surface, $I_c$ is calculated by the following relation:

$$I_c(T, B_{\perp}, B_{\parallel}) = I_c(77, 0, 0) \times \left[ \frac{I_c(T, B_{\perp}, 0)}{I_c(77, 0, 0)} \right] \times \left[ \frac{I_c(T, 0, B_{\parallel})}{I_c(T, 0, 0)} \right]$$

(3.36)

where $T$ is the conductor temperature, $B_{\perp}$ is the field perpendicular to the wide conductor surface, and $B_{\parallel}$ is the field parallel to the wide conductor surface.

The critical current of the conductor is also affected by the bending strain that results from winding the as-received conductor into a coil. The expected $I_c$ value for each conductor turn is adjusted in the simulation according to data reported in [29], which gives $I_c/I_{co}$ for Bi2223/Ag tape operating at 77K in self-field over a range of bending radius, where $I_{co}$ is the critical current of the as-received straight conductor. The data is reported for various samples, among which there is some deviation in the results. The $I_c/I_{co}$ predictions used in the simulation are based on a line fitted to this data. Also, the measurements were performed on a conductor with a larger thickness than what was used to wind the 6 double-pancake coil. This difference is accounted for in the simulation by applying to each turn in the coil the value of $I_c/I_{co}$ reported in the source for the same ratio of conductor thickness to bending diameter. This ratio gives a close approximation of the maximum bending strain in the conductor.

The effects of mechanical strain are believed to be extrinsic [30] such that the dependency of $I_c$ on temperature and field follow the same characteristic for bent conductor as the as-received straight conductor. The degradation of $I_c$ resulting from bending strain is accounted for in the simulation after using Equation 3.36 to predict $I_c$ of straight conductor for the temperature and field conditions. The degradation in $I_c$ caused by the bending strain in the innermost turn of the coil is expected to
Figure 3-10: Critical current density of Bi2223/Ag versus background field at various temperatures, for (a) field oriented parallel to the tape conductor’s wide surface and (b) field perpendicular to the tape conductor’s wide surface. The plots are based on IGC data [27]. The zero-field values are based on [28].
be less than 10% of the as-received value. Therefore, the degradation is expected to be less than 10% everywhere in the coil because the largest conductor bending strain exists in the innermost turns because they have the smallest bending radius.

The conductor index, $n$, also varies with magnetic field. According to [31], which examines the variation of $I_c$ and $n$ with magnitude and direction of an applied magnetic field at 77 K, the field dependency of $n$ seems to follow a trend similar to that of $I_c$. However, the reduction in $n$ is not quite as large as it is for $I_c$. Therefore, $n$ is calculated as a function of temperature and field, both perpendicular and parallel to the wide conductor surface, according to the following relation:

$$ n(T, B_{\perp}, B_{||}) = n_0 \times \left[ \frac{I_c(T, B_{\perp}, B_{||})}{I_c(T, 0, 0)} + X \left( 1 - \frac{I_c(T, B_{\perp}, B_{||})}{I_c(T, 0, 0)} \right) \right] $$

(3.37)

where $X$ is a dimensionless number less than 1 used to account for $n$ not being as strongly dependent on field as $I_c$, and $n_0$ is the index in zero field. The data reported in [31] indicates a degradation in $n$ that is approximately 10% less than the degradation in $I_c$, so $X$ is set equal to 0.1 in the simulation. A value for $n_0$ of 11.0 is used. $n_0$ is considered to be independent of temperature.

Temperature gradients within the magnet are expected to be negligible based on calculations described in Appendix C. Experimental temperature traces obtained while the magnet is charged support this claim. The magnetic field decay simulation calculates index resistance while assuming the magnet has a uniform temperature that varies according to the temperature-time traces that were recorded during the experiment.

### 3.2.3 Calculation of the Total Index Dissipation

The magnetic field within the winding, on the other hand, depends strongly on location. Figure 3-11 illustrates how $V_n$ may be calculated for a given temperature and transport current. Each turn is assigned a nodal coordinate, $(m, p)$, where $m$ denotes which turn counting from the inside of the winding, and $p$ denotes which of
the twelve pancakes (each double-pancake has two pancakes) counting up from the bottom. The electric field within each turn, \( E_{m,p} \), is calculated based on the field components within the turn, the magnet temperature, and the transport current. The dissipative index voltage contribution from each turn is determined by multiplying the turn length, \( l_{m,p} \), by \( E_{m,p} \) so that \( V_n \) is calculated as:

\[
V_n = \sum_{m,p} l_{m,p} E_{m,p}.
\]  

The magnetic field components inside any turn are proportional to \( I_t \). To reduce computation time, proportionality matrices \( R \) and \( Z \) are calculated with a solenoid design program [32] prior to starting the simulation. Only two components are necessary because the winding is rotationally symmetric and we expect the azimuthal component to be zero. The program calculates the field components at every node \((m, p)\) based on the location of the node, \((r_{m,p}, z_{m,p})\), the dimensions of the magnet, the
dimensions of the conductor, and unit current. In the simulation, these values are
simply scaled according to the value of $I_t$, such that:

$$B_{r,m,p} = I_t \times R_{m,p}$$  \hspace{1cm} (3.39)

$$B_{z,m,p} = I_t \times Z_{m,p}$$  \hspace{1cm} (3.40)

where $B_{r,m,p}$ and $B_{z,m,p}$ are the field components in the radial and axial directions, respectively.

The field components and the temperature trace recorded from the experimental trial are used to calculate the critical current, $I_{c,m,p}$, and index, $n_{m,p}$, for each turn according to Equations 3.36 and 3.37, respectively. $E_{m,p}$, is then determined as a function of $I_t$ and $T$ according to Equations 3.35–3.37. Equation 3.35 is rewritten here to include the nodal coordinates of each spatially varying quantity:

$$E_{m,p}(I_t, T) = E_c \left[ \frac{I_t}{I_{c,m,p}(T, B_{⊥ m,p}, B_{∥ m,p})} \right]^{n_{m,p}(T, B_{⊥ m,p}, B_{∥ m,p})}$$  \hspace{1cm} (3.41)

where the field components $B_{r,m,p}$ and $B_{z,m,p}$ have been replaced with $B_{⊥ m,p}$ and $B_{∥ m,p}$ as dictated by the orientation of the tape conductor in the magnet. The calculation of $V_n$ is completed by substituting Equation 3.41 into Equation 3.38:

$$V_n = E_c \sum_{m,p} l_{m,p} \left[ \frac{I_t}{I_{c,m,p}(T, B_{⊥ m,p}, B_{∥ m,p})} \right]^{n_{m,p}(T, B_{⊥ m,p}, B_{∥ m,p})}.$$  \hspace{1cm} (3.42)

### 3.2.4 Simulation

Substituting the expressions for splice voltage, Equation 3.33, and index voltage, Equation 3.41, into Equation 3.32 yields:

$$L \frac{dI_t}{dt} + R_s I_t + E_c \sum_{m,p} l_{m,p} \left[ \frac{I_t}{I_{c,m,p}(T, B_{⊥ m,p}, B_{∥ m,p})} \right]^{n_{m,p}(T, B_{⊥ m,p}, B_{∥ m,p})} = 0.$$  \hspace{1cm} (3.43)
Table 3.5: Conductor and Splice Properties Applied in the Simulation

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total splice resistance, $R_s$</td>
<td>$(\Omega)$ 3.26x$10^{-9}$</td>
</tr>
<tr>
<td>As-received $I_c$ at 77 K*, $I_c(77 K)$</td>
<td>(A) 32</td>
</tr>
<tr>
<td>$I_c$ degradation at inner wind diameter, $I_c/I_{co}$</td>
<td>0.93</td>
</tr>
<tr>
<td>$I_c$ degradation at outer wind diameter, $I_c/I_{co}$</td>
<td>0.96</td>
</tr>
<tr>
<td>Zero-Field Index, $n(0T,0T)$</td>
<td>11.0</td>
</tr>
<tr>
<td>Index field dependency adjustment, $X$</td>
<td>0.10</td>
</tr>
</tbody>
</table>

*Criterion of $E_c = 1.0 \, \mu V/cm.$

This equation can be transformed into a numerically usable form by substituting a Taylor expansion for the derivative:

$$\frac{dI_t}{dt} = \frac{I_t^{j+1} - I_t^j}{\Delta t}$$

(3.44)

where $\Delta t$ is the time step size, and the exponent $j$ denotes which time step. Substitution of Equation 3.44 into Equation 3.43 with some rearrangement of the terms and discretization of the time-dependent quantities yields a forward-Euler finite difference equation for simulating the current decay of the charged magnet:

$$I_t^{j+1} = I_t^j - \frac{\Delta t}{L} \left[ R_s I_t^j + E_c \sum_{m,p} l_{m,p} \left( \frac{I_t^j}{I_{c,m,p}(T^j, B_{\perp m,p}, B_{\parallel m,p})} \right) n_{m,p}^{j}(T, B_{\perp m,p}, B_{\parallel m,p}) \right]$$

(3.45)

where, as described previously, the temperature at each time step, $T^j$, is taken from the experimental temperature versus time trace of the trial being simulated. The simulation will be unstable if $\Delta t$ is not chosen small enough.

Each simulation is initiated with a current that corresponds to a point in time after the thermal gradients that arise during the charging procedure (see Chapter 2) have subsided. The initial current, $I_t^0$, is set equal to the current calculated by a solenoid design code that corresponds to the initial field of the portion of the experimental field trace to be simulated. Table 3.5 lists the conductor and splice properties applied in the simulation. These values were justified in the previous sections.
Chapter 4

Results and Discussion

In this chapter, the experimental results and theoretical predictions are presented and compared. The cold container warming trend shows reasonable agreement with the predictions made according to the methods described in Chapter 3. The model of the field decay, on the other hand, underestimates the rate of decay due to the lack of data required for accurately calculating the dissipation caused by the conductor index. Specifically, the critical current was measured for the as-received conductor at 77 K and self-field before it was wound into a coil. The critical current at other temperatures and applied field is predicted based on data reported by a different manufacturer from the one that manufactured the conductor used in the experimental system. The critical current dependency on temperature and field may vary among manufacturers and possibly among batches. Secondly, the critical current is known to be degraded when the conductor is wound into a coil and strained. Reduction in critical current due to bending strain is predicted based on data reported for a sample of conductor that might show a different response to bending strain, as the effects of bending radius on the critical current may also vary among conductor manufacturers and among batches. Finally, it has been shown that conductor index varies strongly with the magnetic field applied to the conductor [31], but experimental measurements of the index dependency on magnetic field have not been reported for the temperature range used in the experiment of this project.
4.1 20–40 K Warming Trend of the Cold Container

Figure 4-1 shows the experimentally observed warming trend (solid lines) for the cold container in the temperature range 20–40 K. The temperature measurements for each thermocouple are included. There is no significant temperature difference between any of the thermocouple measurements, indicating that the warming is slow enough such that temperature gradients within the cold container are negligible. This observation is consistent with prediction described in Appendix C.

Based on the internal energy data for each of the materials contained within the cold volume, the experimental 20–40 K warming trend indicates a heat leak of 550 mW. This value is within the range predicted in Chapter 3 (dashed lines in Figure 4-1), although it is significantly greater than the optimistic theoretical pre-
diction of 270 mW that assumes the heat flux perpendicular to the superinsulation layers to dominate the net heat flow through the blankets. It can still be shown that this discrepancy is within what should be expected when basing heat leak predictions on heat flux values measured for a flat plate in a carefully constructed laboratory experiment. If it is assumed that the penetration heat leak value predicted in Chapter 3 (110 mW) is correct, then the total heat leak across the superinsulation by itself is equal to 440 mW. This is a factor of 2.75 greater than the predicted heat leak across the NRC-2 superinsulation of 160 mW (when neglecting conduction and radiation interreflection parallel to the layers), but well within the 2–3 difference past researchers have observed [16][17] when comparing flat-plate measurements with those obtained when the superinsulation is applied to a cylindrical tank. Various factors may account for the difference between the reported flat-plate heat flux values obtained from carefully controlled experiments and the values observed in this experiment: 1) gaps inevitably exist at the joints between different surfaces, such as between the layers wound around the outer cylindrical surface of the cold-container and the layers placed on the top flat surface of the container; 2) penetrations through the superinsulation increase thermal communication between layers and may degrade the performance over significant distances from the penetrations due to the high thermal conductivity in the direction parallel to the layers; 3) the reported flat-plate performances were obtained for laminar configurations but the superinsulation was applied to the cylindrical surfaces of the cold container using a rolled configuration; and 4) the bottom edge of the upper outer cylindrical blanket was exposed such that it was in direct thermal communication with the room-temperature cryostat wall.

The predicted warming trend based on the experimentally measured heat leak and the internal energy versus temperature data for the materials contained within the cold container is also shown in Figure 4-1 by the dash-dot curve. The close agreement between this curve and the observed warming trend indicates that the internal energy versus temperature relation that was calculated in Chapter 3 is reasonably accurate.
4.2 Magnetic Field Decay

The magnet was charged to approximately 22 A three times, each time at a different temperature: 15 K, 25 K, and 35 K. The recorded field traces for each trial are shown as the solid lines in Figures 4-2–4-4. Each reported trace begins shortly after the thermal gradients that arise during the charging procedure (see Chapter 2) have subsided. The absence of thermal gradients is expected according to calculation illustrated in Appendix C, and is revealed by a negligible difference among the thermocouple temperature measurements.

Each field trace was simulated according to the model described in Section 3.2. The magnetic field (and current) decays due to two sources of dissipation. The first is the splice resistance which arises from the joints between adjacent double-pancakes and the joints between the superconducting switch and the top and bottom pancakes. These resistances are expected to behave as ohmic resistors, where the total resistive voltage from splicing, $V_s$, is equal to the transport current, $I_t$, passing through them multiplied by a total splice resistance, $R_s$:

$$V_s = I_t R_s.$$  \hspace{1cm} (3.33)

$R_s$ is expected to be approximately $3.3 \times 10^{-9} \Omega$.

The second source of dissipation is the index resistance in the superconductor which is a result of the conductor not being a perfect superconductor. The electric field in the conductor arising from this form of dissipation is calculated according to the following equation:

$$E = E_c \left( \frac{I}{I_c} \right)^n$$ \hspace{1cm} (3.35)

where $I_c$ is the critical current defined at an arbitrarily chosen electric field $E_c$ (1.0 $\mu$V/cm), and $n$ is the index. $I_c$ depends strongly on temperature and the field components both parallel and perpendicular to the tape conductor’s wide surface. It decreases with increasing temperature and with increasing field, with a stronger dependency on the perpendicular field component. $n$ follows the same general char-
Figure 4-2: Experimental (solid) and simulated (dashed) field decay for an initial temperature of 16 K. The ideal case with a decay time constant of $10^8$ hr is also shown (dotted).

The characteristic as $I_c$ as a function of field, but the index in zero field is considered to be weakly dependent on temperature. A total index voltage, $V_n$, is obtained by multiplying the electric field in each turn by the turn length and summing these contributions together. The current (and field) decay is then determined by the following relation:

$$L \frac{dI_t}{dt} = -(V_n + V_s). \quad (3.32)$$

The simulations started with an initial current corresponding to a solenoid design code’s prediction of the current required to produce the initial field of the portion of the experimental trace that was simulated. The magnet was assumed to have a uniform temperature that varies with time as recorded during each trial. $I_c$ was adjusted for each turn in the magnet at every time step based on temperature and
Figure 4-3: Experimental (solid) and simulated (dashed) field decay for an initial temperature of 25K. The ideal case with a decay time constant of $10^8$ hr is also shown (dotted).
Figure 4-4: Experimental (solid) and simulated (dashed) field decay for an initial temperature of 36 K. The ideal case with a decay time constant of $10^8$ hr is also shown (dotted).
field (as calculated by a solenoid design code) according to data reported by IGC [27]. Additionally, $I_c$ was adjusted for each turn to account for bending strain degradation according to data reported in [29].

The same splice resistance, $R_s$, and property relations for $I_c$ as a function of temperature and field, and $n$ as a function of field were assigned in the simulation of each trace. The results from the simulation are shown as dashed lines in the figures. The initial decay rate matches the experimental traces, but significant discrepancy is observed after 0.5 hr. Ideal performance is shown as the dotted line, where an acceptable field decay time constant is chosen as $10^8$ hr. Simulated traces based on the expected splice resistance while neglecting index dissipation have also been produced; however they are not included in the figures because they are indistinguishable from the ideal traces for the axes range of the plots.

It is obvious that the experimentally observed field decay is mainly due to index dissipation and that the discrepancy between the experimental and simulated traces is due to underestimation of the index dissipation. There are various reasons to suspect inaccuracies in the relations used to calculate electric field in the conductor as a function of current, temperature, and field. To begin with, the $I_c$ variation with temperature and field was based on data supplied by a different conductor manufacturer (IGC) from the one that fabricated the tape used in this experiment (NORDIC). A 10% variation in the calculated $I_c$ from the actual will lead to an electric field calculation that is off by a factor greater than 2.5 assuming $n$ equal to 11. Not only was the field and temperature dependency based on measurements performed by a different manufacturer, but the measured samples were straight. The conductor used in this experiment was bent as it was wound into double-pancakes. Bending the conductor may alter the $I_c$ dependency on field and temperature. The predicted $I_c$ degradation due to bending strain was also based on measurements conducted on samples produced by a different manufacturer.

Secondly, the index dependency on magnetic field was assumed to follow the same general characteristic as $I_c$. This assumption is supported by measurements performed at 77 K that are reported in [31]. However, index measurements against applied field
have not been performed at other temperatures. The index dependency on magnetic field might not follow the $I_c$ dependency on field in the temperature range of the field decay experiments in quite the same way as it does at 77 K.

Finally, the relation used for calculating the electric field in the conductor, Equation 3.35, may be an unrefined generalization. It is usually used to predict electric field for values of current in the neighborhood of $I_c$, which is based on some electric field criterion, typically $E_c = 0.1$ or 1.0 $\mu$V/cm. A simple calculation can reveal that the average electric field in the conductor was much smaller than this during most of the duration of each field trace. If the dissipation is assumed to be dominated by the index, the average electric field in the conductor may be derived from the decay rate using Equation 3.32. The average electric field is estimated by setting $V_s$ equal to zero, calculating $LdI_t/dt$ from the experimental trace, and dividing the resulting $V_n$ by the total conductor length. For the first experimental trace, the average electric field in the conductor at 2 hr was on the order of 1.0 nV/cm, which is 2 or 3 orders of magnitude less than $E_c$.

Clearly, poor conductor index is a much greater obstacle than splice resistance to fabricating a persistent-mode HTS magnet with a sufficiently large decay time constant to be useful. Unfortunately, there has been little discussion concerning improvements in the conductor index; most discussion focuses on improvements in $I_c$. Reduction in the index dissipation will require research efforts by the tape manufacturer. The quality of HTS conductor has improved steadily over the last 8 years; but as of yet, we do not suspect any manufacturer to be capable of producing conductor that can offer practical persistent-mode performance.
Chapter 5

Design Analysis of Potential Applications

In this chapter, a design study is conducted for a solid nitrogen cooled “permanent”
high-temperature superconducting (SN2/HTS) magnet system. The system offers
three improvements to the performance of current stand-alone permanent magnet
system design which involves a persistent-mode low-temperature superconducting
magnet cooled by a bath of liquid helium (LHe/LTS). The improvements include,
1) significantly better thermal stability that eliminates the possibility of a quench
during operation, 2) simpler system dynamics, and 3) improved portability by per-
mitting the system to be recooled with a detachable cryocooler, thereby eliminating
the need to transport liquid cryogens. A detachable cryocooler scheme is described
in this chapter. Improvement in the system dynamics results from the solid nitrogen
being rigidly fixed within the system, as opposed to a liquid cryogen system where
the free surface permits cryogen motion that is independent of the system motion.
The superior thermal stability of HTS conductor was explained in Section 1.1.3.

Two different system designs are analyzed and compared. Each system utilizes
a layer-wound solenoidal superconducting coil so that the magnet may be charged
with a power supply. The first system is an LHe/LTS system which is used for
comparison of an SN2/HTS system with present permanent superconducting magnet
system design. The second system is an SN2/HTS system that may be recooled
with a detachable cryocooler. It employs one radiation shield that is cooled by solid ammonia. This system offers both improved stability and improved portability over an LHe/LTS system design. Design codes are developed for each type of system to predict the important system attributes, namely minimum volume and minimum weight, that should be expected for a given set of design requirements (i.e., field magnitude and bore size, or magnetic dipole moment) and a given set of conductor properties.

The system designs are compared by predicting the system weight and volume that should be expected for a set of design requirements that are specific to a given application. Two applications are considered. The first is a general permanent magnet system employing a room-temperature (RT) bore. The mass and volumes to be expected from each of the system designs are compared over a range of bore size and field strength. Details of the system designs for the permanent RT bore magnet application are described in Section 5.1. The second application is a superconducting mine countermeasures system (SCMCM) that is used to sweep passive magnetic influence mines. The structural and magnetic requirements for this system are described in Section 5.8.

The expected weight and volume of the system designs are predicted based on various design parameters, including: magnetic field requirements, heat capacitor internal energy data, expected mechanical loading, operating temperature ranges, and conductor properties. The codes are used to determine the minimum HTS properties that are necessary to construct a SN2/HTS system with a size comparable to that expected for a LHe/LTS system.
5.1 Design Details of the Permanent Superconducting Magnet Systems with a RT Bore

5.1.1 Design of a LHe/LTS System

The LHe/LTS system design is illustrated in Figure 5-1. The magnet and liquid helium supply are housed within a cylindrical cold container that is suspended inside the cryostat. The cryogenic design is similar to standard liquid helium dewars that have been used for several decades. These dewars typically include vapor-cooled shields to substantially reduce the heat leak into the liquid helium bath, where the cold helium vapor exiting the bath exchanges heat with the shield in order to intercept the heat leak associated with the radiation emitted from the room-temperature cryostat walls.

Two cylindrical radiation shields, an inner shield and an outer shield, surround the cold container. The shields consist of a thin sheet of a highly conductive material, typically copper, formed into a shell that completely encloses the contents within it.

Figure 5-1: Design of a permanent LTS superconducting magnet system where the magnet is cooled by liquid helium.
A highly conductive material is used to eliminate temperature gradients along the shield's surface. The cold container and the shields are covered on their outer surface by superinsulation blankets to further reduce the radiation heat leak. Vapor exiting the cold container is fed into a heat exchanger that is attached to the inner shield (see Figure 1-3a). The vapor exiting the heat exchanger is fed to a second heat exchanger that is attached to the outer shield. Finally, the vapor is vented from the cryostat after it exits the outer shield heat exchanger. In theory, more than two radiation shields could be included to further reduce the heat leak into the cold container, but construction of the system becomes increasingly difficult as the number of radiation shields is increased. Few systems utilize more than two vapor-cooled radiation shields.

The shields and the cold container are suspended within the cryostat by fiberglass support straps. The straps are based on a design used in the High Temperature Superconducting Space Experiment II (HTSSE II) [33] involving a six strap configuration. The straps are thermally anchored at each end to the shield or container they attach to so that the helium vapor used to intercept the radiation heat leak to the shields is also used to intercept heat leak through the support straps. Several thin-wall stainless steel access tubes extend from outside of the cryostat to the cold container, penetrating the superinsulation blankets. The tubes provide access for filling the cold container with liquid helium and for inserting copper conductors that are part of detachable vapor-cooled current leads that deliver current to the cold container for charging the magnet. A superconducting switch is located inside the cold container.

5.1.2 Design of a Cryocooled SN2/HTS System

The second system, depicted in Figure 5-2, is based on the permanent magnet system demonstrated in this thesis where solid nitrogen is used as the primary heat capacitor to maintain the magnet temperature below 40 K. In this case, there is no helium boil-off that may be channeled to intercept radiation heat leak, so a secondary heat capacitor is used to cool a radiation shield. Without the radiation shield coupled to
Figure 5-2: Design of a permanent HTS superconducting magnet system where solid heat capacitors are used to cool the magnet and a radiation shield. The heat capacitors may be recooled by a detachable cryocooler.
the secondary heat capacitor, the total volume of the solid nitrogen system would need to be orders of magnitude greater than the volume of the liquid helium system if both systems are designed to offer the same hold time. Selection of the secondary heat capacitor is explained in Section 5.4.2. Additional radiation shields and heat capacitors could be included to further reduce the heat leak into the cold container, as was stated for the liquid helium system. However, only one radiation shield is included in an attempt to show that a system size that is comparable to the LHe/LTS system may be achieved without an extremely complicated design.

The HTS magnet and the solid nitrogen are housed within the same cold container. The cold container is surrounded by a superinsulation blanket and suspended within the radiation shield by fiberglass support straps which penetrate the superinsulation blanket. The support straps are based on the same design used for the LHe/LTS system described in the previous section. The radiation shield is thermally shorted to the secondary heat capacitor container that is located coaxially with the cold container. The shield is also covered on its outer surface by a superinsulation blanket. Again, fiberglass straps penetrate the blanket so that the radiation shield, secondary heat capacitor container, and the cold container may be suspended within the cryostat. The straps are thermally anchored to the radiation shield so that the secondary heat capacitor absorbs heat conduction through the support straps as well as radiation emitted from the cryostat wall.

Four access tubes extend from outside of the cryostat to the cold container, penetrating both superinsulation blankets. The tubes provide access for filling the cold container with nitrogen (not shown), for inserting current leads to charge the magnet, and for inserting a copper cold bus that extends from the second stage of a cryocooler head for recooling the cold container. Two more access tubes extend from outside of the cryostat to the secondary heat capacitor container. These permit filling of the container with the secondary heat capacitor (not shown) and insertion of a cold bus that extends to the first stage of a cryocooler head for recooling the secondary heat capacitor. The cryocooler head is housed within its own shell from which the cold buses extend. The shell can be sealed against the outer surface of the cryostat wall.
after the cold buses are inserted into their access tubes. The head is mounted on a bellows so that the cold buses may be cooled before connecting them to the cold container and the secondary heat capacitor container. A pumpout port is included so that the shell may be evacuated.

5.2 Thermal Isolation Components

A means of estimating the rate of heat leak into the insulated containers of both system designs is needed in order to predict the system sizes. Relations for estimating the rate of heat leak through each component that provides a path for heat flow into the internal containers are derived in the following sections. These relations are used in the design codes described in Sections 5.4, 5.6, 5.9 and 5.10. Heat leak predictions for the superinsulation blankets and the supports straps may be optimistic. However, their dependence on the relevant temperatures should scale properly, and so a fair comparison of the LHe/LTS and SN2/HTS systems is still made.

5.2.1 Superinsulation Blankets

The radiation shields and cryogen containers of both the LHe/LTS and SN2/HTS systems are considered to be covered with a high quality superinsulation blanket design that is described and modeled in [34]. The blankets consist of layers of double-aluminized mylar film (DAM) that are spaced apart with one or more layers of silk net. The DAM layer density may be varied by changing the number of silk net layers between them and by adjusting the tension in the DAM layers. The optimum layer density mainly depends on the temperature of the warmer wall faced by the superinsulation. Two different layer densities are used in the analysis of the RT bore application. A DAM layer density of 12.50 layers/cm is used for the blankets in the SN2/HTS system and for the blankets covering the radiation shields in the LHe/LTS system. The DAM layers are spaced apart with two silk net layers.
The expected heat flux in W/m², \( q \), through a blanket with this configuration may be predicted by the following relation [34]:

\[
q = \frac{C_s}{N + 1} (\overline{N})^{2.56} T_m (T_H - T_C) + \frac{C_r \epsilon_{RT}}{N} (T_H^{4.67} - T_C^{4.67}) \tag{5.1}
\]

where \( \overline{N} \) is the DAM layer density in layers/cm, \( T_m \) is the mean temperature of the cold wall and the warm wall faced by the blanket in degrees kelvin, \( N \) is the total number of DAM layers, \( T_H \) is the warm wall temperature, \( T_C \) is the cold wall temperature, \( \epsilon_{RT} \) is the room-temperature emissivity of the DAM surface, and \( C_s \) and \( C_r \) are coefficients equal to 8.95x10⁻⁸ and 5.39x10⁻¹⁰, respectively. This equation was originally intended for predicting the performance of blankets with layer densities between 26 layers/cm and 47 layers/cm. However, experimental results presented in [34] for layer densities less than 15 layers/cm still show reasonable agreement with Equation 5.1. The relation should be sufficient for the purpose of comparing the LHe/LTS and SN2/HTS systems.

The blanket enclosing the cold container of the LHe/LTS system is considered to have a DAM layer density of 11.0 layers/cm where the DAM layers are spaced apart with three silk net layers. This blanket configuration offers better performance for lower warm wall temperatures that are within the range expected for the inner radiation shield. Again, Equation 5.1 was not originally intended for this configuration, but experimental results [34] indicate reasonable agreement. The experimental performance is actually somewhat better than what is predicted.

In the design analysis that follow, no more than 40 layers are used in any superinsulation blanket because effective evacuation of the inner layers becomes more difficult as the number of layers is increased. Increasing the number of layers beyond 40 typically leads to a negligible improvement in performance [9][10][11][12].

### 5.2.2 Fiberglass Support Straps

Characterization of the support strap heat leak is based on a design used in the High Temperature Superconducting Space Experiment II (HTSSE II) [33]. In the
HTSSE II, an 8.3 kg cold mass maintained at 77 K is supported by 6 fiberglass support straps. Three straps are attached at axial-symmetric points around each face of the cylindrical cold mass. The opposite ends of the straps connect to a warm support structure. Each strap is approximately 100 mm long.

The cold mass of HTSSE II is enclosed in a superinsulation blanket that is similar to the type described in Section 5.2.1. Careful attention was paid in isolating the supports from the main superinsulation blanket by wrapping various layers of silk net spacer around the straps. The silk net prevents the strap from thermally shorting the layers of the blanket, and absorbs radiation that might travel through the gap between the strap and the blanket. If mounted properly, it should be expected that most of the heat leak associated with the support straps is due to conduction along the fiberglass straps. Therefore, the expected heat leak through the support straps should be proportional to the load they are designed for because both tensile strength and heat conduction (for a strap of fixed length and end temperatures) are proportional to the cross-sectional area of the strap. Additionally, the heat leak should be proportional to the difference in the end temperatures, and inversely proportional to the strap length. For HTSSE II, the fiberglass straps were designed to support a load of 8.3 kg under accelerations of 14.8 G r.m.s. Each strap was approximately 100 mm long. The total heat leak through the support straps was predicted by the HTSSE II designers to be 56 mW with the warm end and cold end temperatures fixed at 216 K and 77 K, respectively. Based on this data, the combined heat leak through a set of six supports, \( \dot{Q}_{fss} \), each of length \( l_{fss} \), spanning a temperature difference, \( T_C - T_H \), are predicted according to the relation:

\[
\dot{Q}_{fss} = (3.28 \times 10^{-6} \text{ Wm/kgKG}) \frac{ma}{l_{fss}} (T_H - T_C)
\]  

(5.2)

where \( m \) is the mass supported by the straps and \( a \) is the maximum acceleration in gravitational units (G) the system is designed to withstand. The supports are assumed to be thermally anchored to the radiation shields, cold container, and the cryostat wall for both the LHe/LTS and the SN2/HTS system designs. Equation 5.2
is used to predict heat conduction between successive components. The effective length of the supports is generally somewhat larger than the length of the fiberglass straps themselves due to additional parts for coupling the straps to the structures and mechanisms that permit easy adjustment of the strap tension. However, for this analysis it will be assumed that the hardware for attaching the ends of the straps make an insignificant contribution to the support length. The length of the straps is determined by the gap width they cross and the assumption that the straps extend at a $45^\circ$ angle to the axis of the cold mass they support.

### 5.2.3 Current Lead Access Tubes

In order to charge the magnet, an electrical path traveling from the magnet to the outside of the cryostat must be provided so that current may be delivered to the magnet from an external power supply. A copper rod thick enough to carry the magnet current while spanning from the cold container to the cryostat wall would cause a heat leak of a magnitude that is larger than all other sources of heat leak combined. Therefore, this analysis considers the detachable current lead design depicted in Figure 5-3. This design permits the electrical conductor to be removed so that only a thin-wall tube with a slightly larger diameter remains to provide a thermal conduction path from the cryostat wall to the cold container. Another possible design calls for the thin-wall tube to also detach and become separated from the cold container by a short distance, thereby reducing the conduction through the tube wall. However, the radiation heat transfer from the bottom of the tube to the cold container is difficult to predict, so the simpler design that does not permit detachment of the tube from the cold container will be analyzed to make a conservative estimate of the heat leak through the current lead access tubes.

The design codes require a relation for estimating the heat leak through the access tube during normal operation of the system when the conductor is removed. The heat leak through the tube depends on its material, end temperatures, and dimensions. The end temperatures have already been specified as room-temperature at the warm end and the cold container temperature at the cold end. Stainless steel and epoxy
Figure 5-3: Design of a detachable current lead. The configuration shown pertains to the SN2/HTS system. (a) During normal operation the copper conductor is removed. (b) The current lead is inserted through the stainless steel tube to deliver current to the cold container when charging the magnet.
reinforced fiberglass are the two most common materials used for cryogenic access tubes because of their high strength to thermal conduction ratio. Stainless steel will be considered here. The tube length, $l_{lat}$, for the LHe/LTS system is determined by the width of the radiation gaps as shown in Figure 5-1. $l_{lat}$ for the SN2/HTS system is determined by the length of the secondary heat capacitor container and the widths of the radiation gaps as shown in Figure 5-2. The entire electrical joint is considered to reside within the cold container for both systems. The diameter of the tube depends on the diameter required for the copper conductor that is inserted during the magnet charge procedure.

The diameter of the copper conductor for the SN2/HTS system is determined based on the optimum vacuum lead criterion [2] where the conductor surface, excluding the ends, is assumed to be adiabatic. The criterion is used to determine the dimensions of the conductor that will minimize the heat conduction out of the cold end of the lead while it carries current. The minimum heat leak is achieved for a given transport current, $I_t$, when the ratio of the conductor length, $l_{cnd}$, to its cross-sectional area, $A_{cnd}$, is such that the resistive dissipation within the lead is equal to the conduction into the warm end of the lead. $l_{cnd}/A_{cnd}$ may be calculated according to the following relation:

$$\left( \frac{I_t l_{cnd}}{A_{cnd}} \right) = \sqrt{\frac{2\tilde{k}(T_H - T_C)}{\tilde{\rho}}}$$

(5.3)

where $T_H$ is the temperature of the warm end, $T_C$ is the temperature of the cold end, $\tilde{k}$ is the average thermal conductivity of the conductor material in the temperature range $T_C - T_H$, and $\tilde{\rho}$ is the average electrical resistivity in this temperature range. For a round conductor, the diameter, $d_{cnd}$, is solved for in terms of $l_{cnd}$, $I_t$, the end temperatures, and the material properties by substituting $\pi d_{cnd}^2 / 4$ for $A_{cnd}$ and rearranging terms:

$$d_{cnd} = 2 \left[ \frac{I_t l_{cnd}}{\pi} \left( \frac{\tilde{\rho}}{2\tilde{k}(T_H - T_C)} \right)^{1/2} \right]^{1/2}.$$ 

(5.4)
The diameter of the copper conductor for the LHe/LTS system is based on the optimum vapor-cooled lead criterion [2], where the cold helium vapor expelled from the cold container due to heat leak through the lead is used to cool the conductor. The vapor intercepts heat conduction from the warm end and resistive dissipation within the conductor. The optimum ratio of the length to the cross-sectional area of the conductor, $l_{\text{cnd}}/A_{\text{cnd}}$, that minimizes the heat conduction out of the cold end of the lead when it carries a current, $I_t$, is described by the following relation:

$$
\left( \frac{I_t l_{\text{cnd}}}{A_{\text{cnd}}} \right) = \sqrt{\frac{c_p k_0 \rho_0}{h_L}} \int_{T_C}^{T_h} \frac{dT}{\rho(T)}
$$

where $\rho$ and $k$ are the electrical resistivity and thermal conductivity, respectively, of the conductor, $c_p$ is the specific heat of helium vapor, $h_L$ is the latent heat of vaporization for liquid helium, and the subscript 0 indicates evaluation at the lead cold-end temperature, $T_C$. Solving for the conductor diameter, $d_{\text{cnd}}$, yields the relation:

$$
\frac{d_{\text{cnd}}}{l_{\text{cnd}}} = 2 \left[ \frac{I_t l_{\text{cnd}}}{\pi} \int_{T_C}^{T_h} \frac{dT}{\rho(T)} \left( \frac{h_L}{c_p k_0 \rho_0} \right) \right]^{1/2}.
$$

In practice, the inner diameter of the access tube, $d_{\text{lat}}$, should be slightly larger than $d_{\text{cnd}}$ to permit easy insertion of the conductor. However, the difference should be small enough such that $d_{\text{lat}}$ is reasonably estimated by $d_{\text{cnd}}$ as calculated from Equation 5.4 or Equation 5.6. The difference between $l_{\text{cnd}}$ and $l_{\text{lat}}$ should be small, so they are also considered to be equal. The tube wall thickness should be chosen as small as possible to minimize conduction along the tube. The minimum wall thickness is limited by the manufacturing process and not by the structural requirements involved in supporting the pressure difference between the inside and outside of the tube. Manufacturers of stainless steel tube typically specify a maximum diameter to wall thickness ratio of 50:1 for diameters less than 15 mm. The thickness, $t_{\text{lat}}$, will be calculated by applying this ratio to $d_{\text{lat}}$.

With the tube dimensions determined, the total heat leak through the access tube penetration may now be estimated. There are three modes of heat transfer
Figure 5-4: Thermal model for estimating heat leak associated with the current lead access tubes.

that should be considered as possible contributors to the heat leak as indicated in Figure 5-4: 1) conduction through the tube wall, $Q_c$, 2) radiation transmission through the inside of the tube, $Q_{ro}$, and 3) gaseous conduction through the tube, $Q_g$. Radiation interreflection along the outside of the access tube is assumed to be made small by wrapping or coating the access tube with a high emissivity material (i.e. glass paper). It is shown below that just one mode of heat transfer, conduction through the tube wall, is expected to dominate. Nonetheless, estimates of the other two modes are carried out to support the claim that they are negligible. Estimates of each mode are conducted while assuming each mode to be thermally independent. This assumption leads to conservatively high estimates.

**Conduction Through the Tube Wall**

Conduction, $\dot{Q}_c$, through a medium of length, $l_{lat}$, with constant cross-sectional area, $A_{lat}$, is estimated as:

$$\dot{Q}_c = \frac{\bar{k}A_{lat}}{l_{lat}}(T_H - T_C)$$  \hspace{1cm} (5.7)

where $T_H$ is the temperature of the warm end, $T_C$ is the temperature of the cold end, and $\bar{k}$ is the average thermal conductivity of the medium in the temperature range $T_C$-$T_H$. Equation 5.7 applies to the SN2/HTS system where there is no cold boil-off.
to intercept conduction down the tube. For a thin-wall tube with a diameter to wall thickness ratio of 50:1, $A_{lat}$ is estimated as:

$$A = \pi d_{lat} l_{lat} = \frac{\pi d_{lat}^2}{50}. \quad (5.8)$$

Substituting this expression into Equation 5.7, and replacing $T_C$ with the temperature of the cold container $T_{CC}$, and $T_H$ with room-temperature, 293 K, an expression for estimating the heat conduction for a given $l_{lat}$ and $d_{lat}$ results:

$$\dot{Q}_c = \frac{\tilde{k}}{l_{lat}} \frac{\pi d_{lat}^2}{50} (293 \text{ K} - T_{CC}). \quad (5.9)$$

The value for $\tilde{k}$ used in this analysis is calculated as 12 W/mK for the temperature range $T_{CC}$-293 K from data given in [2], and does not vary significantly with $T_{CC}$ for $T_{CC}$ below 80 K.

**Radiation Transmission Through the Tube**

The radiation heat transfer through the inside of the tube depends on the temperatures of the environments at each end of the tube, and the optical properties of the tube’s inner surface, including the emissivity of the surface and whether radiation is reflected from the surface in a specular or diffuse manner. The radiation heat transfer for a specularly reflecting surface is larger than that for a diffuse reflecting surface, so the inner surface of the tube is considered to have a thin coating that behaves as a diffuse reflector. In this case, the heat leak by radiation exiting the cold end of the tube is equal to what would be calculated if the inner surface of the tube is black (emissivity of 1). A solution to the problem of calculating the heat leak by radiation through a diffuse reflecting or black tube is given in [24]. The solution was derived while assuming conduction through the tube walls to be zero, and the cold end absolute temperature to be zero.

When conduction is considered the radiation component is reduced. Therefore, the reported solution provides a conservative overestimate of the actual radiation
transmission. The assumption of zero absolute temperature at the cold end applies well to the current lead access tube because the radiation heat flux emitted from a surface varies by $T^4$. The radiation influx at the cold end of the tube should be insignificant in comparison to the radiation influx at the warm end of the tube because the warm end absolute temperature is more than 7 times that of the cold end (which corresponds to a radiation influx at the warm end that is over 2400 times greater).

The net radiation heat flow exiting the cold end of the tube, $Q_{ro}$, depends on the heat radiated in from the warm environment, $Q_{ri}$, and the ratio of the length of the tube to its diameter, $l/d$. $Q_{ro}/Q_{ri}$ is plotted against $l/d$ in [24]. The results are reproduced in Figure 3-6a. $Q_{ri}$ is calculated as:

$$Q_{ri} = \sigma T_{H}^4 \frac{\pi d_{lat}^2}{4}$$

(5.10)

where $\sigma$ is the Stefan-Boltzmann constant and $\pi d_{lat}^2/4$ is the cross-sectional area within the tube wall.

The relative magnitudes of $Q_c$ and $Q_{ro}$ are estimated by using reasonable dimensions for a current lead access tube as calculated by the design code for the SN2/HTS system described in Section 5.6.2. The code predicts lead access tube dimensions of $d_{lat}$ equal to 1.6 mm, and $l_{lat}$ equal to 58 mm for a 4.0 T magnet with a 50 mm diameter RT bore. $Q_c$ is calculated to be approximately 9 mW using Equation 5.9. This is almost 200 times larger than the expected value for $Q_{ro}$ of 50 $\mu$W as calculated using Equation 5.10 and Figure 3-6a. Therefore, $Q_{ro}$ is neglected.

**Gaseous Convection Through the Tube**

To prevent condensation from building up at the cold end of the access tube, the tube should be purged with helium which does not condense at the minimum cold container temperature of 20 K. Gaseous convection from the warm end to the cold end of the tube, $Q_g$, is estimated assuming the gas is stagnant, leading to the relation:

$$Q_g = \frac{k}{l_{lat}} \frac{\pi d_{lat}^2}{4} (T_H - T_C)$$

(5.11)
where \( \bar{k} \) is the average thermal conductivity of the gas over the temperature range \( T_C - T_H \). Thermal conductivity data for helium [35], which is larger than the thermal conductivity of air, is used in the calculation of \( \bar{k} \). For \( T_C \) equal to the minimum cold container temperature, 20 K, and \( T_H \) equal to room-temperature, 293 K, \( \bar{k} \) is approximately 97 mW/mK based on thermal conductivity data for helium vapor listed in [35]. For a tube with \( d_{lat} \) equal to 1.6 mm and \( l_{lat} \) equal to 58 mm (dimensions for the 4.0 T, 50 mm bore system), \( \dot{Q}_g \) is calculated to be approximately 900 \( \mu \)W, which is an order of magnitude smaller than \( \dot{Q}_c \) as calculated from Equation 5.9 for the same tube dimensions. Comparison of Equation 5.9 and Equation 5.11 reveals that \( \dot{Q}_c \) and \( \dot{Q}_g \) have the same relative magnitudes irrespective of the tube dimensions. Therefore, \( \dot{Q}_g \) is also neglected in this analysis.

**Total Current Lead Access Tube Penetration Heat Leak**

For the SN2/HTS system, the total heat leak associated with the current lead access tube, \( \dot{Q}_{lat} \) is estimated by summing each of the four contributions described above:

\[
\dot{Q}_{lat} = \dot{Q}_c + \dot{Q}_{ro} + \dot{Q}_g.
\]  

(5.12)

However, it was found that for an access tube of reasonable dimensions, \( \dot{Q}_{ro} \) and \( \dot{Q}_g \) will probably be insignificant. Therefore, \( \dot{Q}_{lat} \) is estimated according to:

\[
\dot{Q}_{lat} = \dot{Q}_c = \frac{\bar{k} \pi d_{lat}^2}{l_{lat}} \frac{293 \text{K} - T_{CC}}{50}
\]  

(5.13)

where \( \dot{Q}_c \) is calculated according to Equation 5.9.

The lead access tubes used in the LHe/LTS system are considered to be cooled by the helium boil-off resulting from the heat leak through the tube. [2] includes an analysis for estimating the conduction heat leak through a vapor-cooled stainless steel support rod. An identical analysis applies to the stainless steel lead access tubes. The results reveal that the heat leak through an access tube without vapor-cooling
should be 80 times larger than the heat leak through an identical access tube that is vapor-cooled. Therefore, \( \dot{Q}_{\text{lat}} \) for the vapor-cooled access tubes is estimated by dividing Equation 5.13 by 80:

\[
\dot{Q}_{\text{lat}} = \frac{k}{l_{\text{lat}}} \frac{\pi d_{\text{lat}}^2}{4000} (293 \text{ K} - T_{CC}).
\]  

(5.14)

5.2.4 Cryocooler Cold Buses

The design considered for the cryocooler detachable cold buses is almost identical to the detachable current lead design of Figure 5-3, except that the disconnectable electrical joint is replaced by a disconnectable thermal joint. The penetration heat leak associated with each cold bus access tube are estimated in a manner similar to the method used for the current lead access tubes. It is assumed that the heat leak associated with these penetrations is dominated by conduction through the access tube wall, so that Equation 5.13 applies. The main difference in estimating the heat leak for the cold bus access tubes and the current lead access tubes is the calculation of the required tube diameter, \( d_{\text{bat}} \). Another difference is that the two cold buses have different lengths and end temperatures. The first cold bus extends from the cryostat wall to the cold container as was the case for the current lead access tubes. The second extends from the cryostat wall to the bottom of the secondary heat capacitor container, and its length is determined by the length of the secondary heat capacitor container and the width of the radiation gap outside of the radiation shield, as shown in Figure 5-2. It is assumed that an actual system can be constructed with disconnectable thermal joints that do not require a reduction in the effective tube length.

\( d_{\text{bat}} \) is calculated based on the maximum thermal resistance across the length of the cold bus permitted to allow a typical two-stage cryocooler to extract heat at a rate that would recool the containers within a reasonable amount of time, \( \tau_{rc} \). \( \tau_{rc} \) does not include the time required to precool the cryocooler and the cold buses. If the container being recooled has an associated internal energy change, \( \Delta E \), between
its minimum and maximum temperatures, $T_{min}$ and $T_{max}$, then the average heat capacity, $\tilde{C}_{cnt}$, is determined as:

$$\tilde{C}_{cnt} = \frac{\Delta E}{T_{max} - T_{min}}. \tag{5.15}$$

During recooling, the container temperature, $T_{cnt}$, will vary with time roughly according to the relation:

$$\tilde{C}_{cnt} \frac{dT_{cnt}}{dt} = \dot{Q}_{bus} \tag{5.16}$$

where $\dot{Q}_{bus}$ is the heat conduction through the cold bus, and $t$ is time. The rate of heat transfer through a cold bus of length, $l_{bus}$, and cross-sectional area, $A_{bus}$, is determined as:

$$\dot{Q}_{bus} = \frac{\bar{k}_{bus} A_{bus}}{l_{bus}} (T_{cnt} - T_{rfg}) \tag{5.17}$$

where $T_{rfg}$ is the temperature at the cryocooler end of the cold bus, and $\bar{k}_{bus}$ is the average thermal conductivity of the cold bus material in the temperature range $T_{rfg}$–$T_{cnt}$. If it is assumed that $T_{rfg}$, $\bar{k}_{bus}$, and the heat capacity of the container materials are approximately constant with temperature, and that the cold bus has a circular cross-section such that $A_{bus} = \pi d_{bus}^2 / 4$, the required bus diameter is determined using Equations 5.15–5.17:

$$d_{bus} = \frac{2 \sqrt{\Delta E l_{bus}}}{\bar{k}_{bus} r_{rc} (T_{max} - T_{min}) \ln \left( \frac{T_{max} - T_{rfg}}{T_{min} - T_{rfg}} \right)} \tag{5.18}$$

The cold bus that extends to the cold container is assumed to be attached to the second stage of a cryocooler. A typical cryocooler might have a first stage which operates close to 80 K and a second stage which operates close to 20 K. Therefore, $T_{rfg}$ is set equal to 20 K in determining $d_{bus}$ for the cold container cold bus. $\bar{k}_{bus}$ is assumed to be equal to 1500 W/mK which approximates the thermal conductivity of copper in the temperature range 20 K–40 K. The cold bus which extends to the secondary heat capacitor container is assumed to be attached to the cryocooler first stage because a larger $\Delta E$ is associated with this container, and it therefore requires the larger
cooling rate that can be afforded by the first stage in order to recool the secondary heat capacitor within the specified $\tau_{rc}$. $T_{rfg}$ is set equal to 80 K in determining $d_{bus}$ for the secondary heat capacitor bus, as described above. $\bar{k}_{bus}$ is assigned the value of 400 W/mK which is the value of the thermal conductivity of copper in the temperature range above 80 K where it is essentially constant with temperature. It is assumed that the primary heat capacitor is cooled to 1 degree kelvin above the cryocooler second stage temperature, and the secondary heat capacitor is cooled to 5 degrees kelvin above the cryocooler first stage temperature. In symbolic form, $T_{min} - T_{rfg} = 1$ K for the primary heat capacitor, and $T_{min} - T_{rfg} = 5$ K for the secondary heat capacitor.

In practice, there would be a gap between the bus and the inner diameter of the access tube, $d_{bat}$, to permit easy insertion and to thermally isolate the bus from the access tube. However, it is assumed that the gap width is small enough to be considered negligible so that $d_{bus}$ calculated according to Equation 5.18 is a fair approximation of $d_{bat}$.

### 5.2.5 Fill Lines

The access tubes for the current leads and the cold buses are required to be straight to permit insertion and detachment of the associated copper rods, so their lengths are determined by the other system dimensions. This is not a requirement for the access tubes used in the helium and heat capacitor fill lines. These tubes may bend so that considerably longer lengths of tube can be used to travel from the containers to the cryostat wall. It should then be possible to use a large enough length to diameter ratio to yield a negligible heat leak associated with these penetrations. Therefore, the heat leak for each of the fill line penetrations are assumed to be negligible.

### 5.3 Container Wall Thicknesses and Masses

The cryostat and internal containers are constructed using stainless steel 304 (SS304), which is a common choice of materials for constructing cryostats. In addition to its high strength to weight ratio and tendency to remain ductile at cryogenic tempera-
tures, series 300 stainless steels have the added advantage that they are non-magnetic. Therefore, components constructed with SS304 will not disturb the magnetic fields that the system is designed to produce.

The thicknesses of the cylindrical walls are calculated using two different relations. The first relation is for cylindrical vessels under internal pressures and applies to the outer wall of the cold container for both the LHe and SN2 systems, and the outer wall of the secondary heat capacitor container. These wall thicknesses are calculated in accordance with ASME Code, Section VIII, Division 1 [36] which specifies:

\[
t_{cip} = \max \left\{ \frac{P_i r_{cip}}{2SE + 0.4P_i}, \frac{P_i r_{cip}}{SE - 0.6P_i} \right\}
\]  

(5.19)

where \( t_{cip} \) is the required thickness of a cylindrical wall for a container under internal pressure, \( P_i \) is the magnitude of the internal pressure, \( r_{cip} \) is the inner radius of the cylindrical wall, \( S \) is the allowable stress for the container material, and \( E \leq 1 \) is the joint efficiency and depends on the properties of the weld joints used at the seams. The wall of the RT bore in the permanent magnet application has negligible thickness.

The thickness of the circular end plates that cap the ends of the cylindrical walls are also calculated according to the ASME code, regardless of whether they support an internal or external pressure since the loading on the plates is the same for both cases. The following relation is specified for a circular flat head:

\[
t_{cfh} = d_{cfh} \sqrt{\frac{CP_i}{SE}}
\]  

(5.20)

where \( t_{cfh} \) is the required thickness of the circular flat head, \( d_{cfh} \) is the inner diameter of the cylindrical shell that the head is attached to, and \( C \) is the attachment factor which depends on the method used in attaching the head to the cylindrical shell.

The maximum pressure expected to be seen by the internal containers would occur when the containers are being filled. During this time, the container may be left open to the atmosphere, at least briefly. During normal operation, the pressure within the
containers should be equal to the vapor pressure of the substance it contains, which is less than atmospheric pressure. Therefore, the wall thicknesses are calculated for $P_i$ equal to atmospheric pressure. The allowable stress, $S$, is chosen as the lower limit specified for SS304 at room-temperature of 200 MPa (the strength of SS304 actually increases with decreasing temperature). The attachment factor for the circular flat heads, $C$, is chosen as 0.33 which is a conservative value for most types of attachments. Finally, the joint efficiency, $E$, is chosen equal to 1 which corresponds to the use of high quality weld joints.

The thickness of the cylindrical walls under external pressure are calculated according to a criterion given in [37]. The prevention of buckling is the main concern in choosing the wall thickness for a cylinder under external pressure. [37] lists the maximum ratio of container diameter to wall thickness required to prevent buckling under an external pressure equal to atmospheric pressure, for various materials. A value of 105 is given for stainless steel at room-temperature. Therefore, the wall thicknesses, for the cylindrical cryostat wall and the inner cylindrical walls of the secondary heat capacitor and cold containers are calculated as:

$$t_{cep} = \frac{d_{cep}}{105}$$

where $t_{cep}$ is the required wall thickness for a cylindrical shell under external pressure, and $d_{cep}$ is the shell’s diameter. [37] also lists maximum acceptable shell length to diameter ratios. However, these ratios are not approached in this design analysis.

Once the container diameter, length, and wall thicknesses have been established, the mass of the empty container, $m_{wall}$, is calculated simply by multiplying the volume of the wall material, $V_{wall}$, by the density of the wall material, $\rho_{wall}$:

$$m_{wall} = \rho_{wall} V_{wall}.$$  \hspace{1cm} (5.22)

The value of $\rho_{wall}$ used in the analysis for all of the containers (cold container, secondary heat capacitor container, and cryostat) is the density of SS304 at room-temperature, 7900 kg/m$^3$ [21].
5.4 Codes for Predicting System Mass and Volume of the Cryogenic Systems Without the Magnet

Codes are developed here to generate plots of predicted system size versus hold time for the cryogenic systems without considering the inclusion of a magnet or the access tube penetrations. These codes will be used to select a secondary heat capacitor for the SN2/HTS system and to demonstrate that a solid nitrogen system may be constructed with a system size that is comparable with what would be expected for the liquid helium system. While still considering cylindrical geometry, the inner diameter and the inner length of the cold containers are fixed equal to each other in order to minimize the ratio of surface area to volume. The number of superinsulation layers in each blanket is optimized to yield the smallest system size. The cryostat tends to make the largest contribution to the total system mass. Reduction of the superinsulation blanket thickness may reduce the size of the cryostat, depending on the associated increase in the size of the internal containers required for providing the same hold time. The optimum number of layers varies with hold time for both systems.

The LHe system is analyzed for a cold container temperature equal to the boiling point of liquid helium, while the SN2 system is analyzed for operation over the temperature range 20–40 K. The selection of this temperature range is explained in Section 5.4.2.

5.4.1 LHe System

In the analysis of the liquid helium system the shields are assumed to be isothermal with a uniform temperature. Under this condition, optimum performance is achieved by permitting the liquid helium vapor that is circulated through a shield to warm to the shield temperature before it exits the shield. For a given set of the system dimensions defined in Figure 5-5a, the shield temperatures and the helium boil-off rate, \( \dot{m}_{\text{He}} \), may be solved for by applying the first law of thermodynamics for an open
Figure 5-5: Illustration of the thermal model used to calculate the system size as a function of hold time for the liquid helium system without the magnet. The system dimensions and location of the control volumes are defined in (a). The superinsulation has been omitted for clarity. The heat leak components applied to each control volume are shown in (b).
system separately to the three control volumes illustrated in Figure 5-5a. Figure 5-5b is a schematic describing the heat leak components that apply to each control volume.

For the control volume encompassing the cold container:

$$\dot{m}_{He} = \frac{\dot{Q}_{CC}(T_{CC}, T_{I})}{h_{vap}}$$  \hspace{1cm} (5.23)

where $h_{vap}$ is the latent heat of vaporization for liquid helium, and $\dot{Q}_{CC}$ is the heat leak into the cold container from the inner shield and is a function of the cold container and inner shield temperatures, $T_{CC}$ and $T_{I}$. $T_{CC}$ is equal to 4.2 K, the boiling point of liquid helium at atmospheric pressure. $\dot{Q}_{CC}$ is determined by summing the individual heat leak components shown in Figure 5-5b:

$$\dot{Q}_{CC}(T_{CC}, T_{I}) = \dot{Q}_{CC_{ins}}(T_{CC}, T_{I}) + \dot{Q}_{CC_{fss}}(T_{CC}, T_{I})$$  \hspace{1cm} (5.24)

where $\dot{Q}_{CC_{ins}}$ is the heat leak through the superinsulation blanket, and $\dot{Q}_{CC_{fss}}$ is the heat leak through the fiberglass supports. $\dot{Q}_{CC_{ins}}$ is calculated according to Equation 5.1 by substituting $T_{CC}$ for $T_{C}$, $T_{I}$ for $T_{H}$, and multiplying the result by the surface area of the inner shield. An $\bar{N}$ of 11.0 layers/cm is used in determining $\dot{Q}_{CC_{ins}}$. $\dot{Q}_{CC_{fss}}$ is calculated according to Equation 5.2 based on the cold container mass when it is full of liquid and substituting $T_{I}$ for $T_{H}$ and $T_{CC}$ for $T_{C}$.

The first law relation for a control volume encompassing the inner shield is determined as:

$$\dot{Q}_{I}(T_{I}, T_{II}) = \dot{Q}_{CC}(T_{CC}, T_{I}) + \dot{m}_{He}(h_{II} - h_{v,4.2K})$$  \hspace{1cm} (5.25)

where $\dot{Q}_{I}$ is the heat flow from the outer shield to the inner shield, $h_{II}$ is the enthalpy of helium vapor at the temperature of the inner shield, and $h_{v,4.2K}$ is the enthalpy of helium vapor at the boiling point. It is assumed that the enthalpy of the vapor remains constant as it travels from the cold container to the inner shield. $\dot{Q}_{I}$ is equal to the sum of the heat leak components between the outer shield and the inner shield:

$$\dot{Q}_{I}(T_{I}, T_{II}) = \dot{Q}_{I_{ins}}(T_{I}, T_{II}) + \dot{Q}_{I_{fss}}(T_{I}, T_{II}).$$  \hspace{1cm} (5.26)

$\dot{Q}_{I_{ins}}$ and $\dot{Q}_{I_{fss}}$ are calculated in the same way as described in the previous paragraph,
except the bounding temperatures are now $T_I$ and $T_{II}$, where $T_{II}$ is the temperature of the outer radiation shield. An $\bar{N}$ of 12.50 layers/cm and the surface area of the outer shield are used in the calculation of $\hat{Q}_{\text{ins}}$.

Finally, for a control volume encompassing the outer shield:

$$\hat{Q}_{II}(T_{II}, 293 \text{ K}) = \hat{Q}_I(T_{II}, 293 \text{ K}) + \bar{m}_{He}(h_{T_{II}} - h_{T_I})$$  \hspace{1cm} (5.27)

where $\hat{Q}_{II}$ is the heat flux from the room-temperature (293 K) wall to the outer shield, and $h_{T_{II}}$ is the enthalpy of helium vapor at the temperature of the outer shield, $T_{II}$. $\hat{Q}_{II}$ is equal to the sum of the heat leak components from the cryostat wall to the outer radiation shield:

$$\hat{Q}_{II}(T_{II}, 293 \text{ K}) = \hat{Q}_{\text{ins}}(T_{II}, 293 \text{ K}) + \hat{Q}_{\text{fss}}(T_{II}, 293 \text{ K}).$$  \hspace{1cm} (5.28)

An $\bar{N}$ of 12.50 layers/cm and the surface area of the cryostat wall are used in the calculation of $\hat{Q}_{\text{ins}}$.

A computer code was constructed to calculate the shield temperatures and $\bar{m}_{He}$ by solving Equations 5.23–5.28. The code starts with some arbitrary values for the cold container diameter, $d_{CC}$, and the number of layers in each superinsulation blanket, $N_I$, $N_{II}$, and $N_{III}$. As stated previously, the length of the cold container, $l_{CC}$, is set equal to $d_{CC}$. Based on these values it calculates all of the dimensions (see Figure 5-5a) and mass quantities required to solve the equations, including the wall thicknesses for the cold container and the cryostat using Equations 5.19–5.21. The gap widths, $g_I$, $g_{II}$, and $g_{III}$, are determined as:

$$g = \frac{N}{\bar{N}} + s$$  \hspace{1cm} (5.29)

where $s$ is an additional spacing between the blankets that accounts for the thickness of the copper radiation shield and spacing required for the support attachments. $s$ is set equal to 3 mm.
After determining all of the dimensions, the code calculates the total system volume and mass. Finally, Equations 5.23-5.28 are solved and the hold time, \( \tau_h \), is calculated based on \( \dot{m}_{He} \) and the volume of the cold container, such that:

\[
\tau_h = \frac{\rho_{LHe} V_{CC}}{\dot{m}_{He}} = \frac{\rho_{LHe} \pi d_{CC}^2 V_{CC}}{4 \dot{m}_{He}} = \frac{\rho_{LHe} \pi d_{CC}^2}{4 \dot{m}_{He}}
\]

(5.30)

where \( \rho_{LHe} \) is the density of liquid helium, and \( V_{CC} \) is the volume of the cold container. These calculations are repeated over a range of \( d_{CC} \) that lead to an interesting range of hold times. For each value of \( d_{CC} \), the values of \( N_I, N_{II}, \) and \( N_{III} \) are varied and the calculations are repeated until the optimum combination that leads to the smallest system size is found. Helium enthalpy data was taken from [35]. Total system mass and total system volume are plotted against \( \tau_h \) in Figures 5-7 and 5-8, respectively. \( T_I \) and \( T_{II} \) depend on the system size, but are in the ranges \( 40 \text{K} < T_I < 53 \text{K} \) and \( 161 \text{K} < T_{II} < 174 \text{K} \) for \( 6 \text{days} < \tau_h < 58 \text{days} \). A smaller \( \tau_h \) leads to higher \( T_I \) and \( T_{II} \). \( N_I, N_{II}, \) and \( N_{III} \) vary between 1-5, 3-12, and 9-31, respectively for the same range of \( \tau_h \). A smaller \( \tau_h \) leads to fewer superinsulation layers.

### 5.4.2 Simulation of the Solid Heat Capacitor System and Selection of the Secondary Heat Capacitor

A slightly more complicated numerical scheme needs to be developed for the solid nitrogen system because the shield and cold container temperatures are not held constant. The liquid helium system, on the other hand, operates in a steady-state because the cold container temperature is maintained at the boiling point of liquid helium. It would be impractical to consider a stand-alone system involving subcooled liquid helium for several reasons: 1) liquid helium is not easily obtained as a subcooled liquid, 2) even if the liquid is subcooled close to the lambda point, the latent heat of vaporization would still account for the majority of internal energy that is available for cooling, and 3) the liquid will remain subcooled only for a very short duration because if the liquid is subcooled then there will not be any boil-off to keep the
shields cold, leading to a heat leak into the helium bath that is orders of magnitude larger than what is expected when the liquid is boiling.

In contrast, the cold container temperature cannot be constant for the solid nitrogen system because heat absorption by nitrogen below 60 K is only realized by an accompanying rise in temperature, except at 35.6 K where there is a solid-solid phase transition. But, unlike the liquid-gas transition in liquid helium, the latent heat associated with the nitrogen solid-solid transition is smaller than the change in internal energy realized when the nitrogen is warmed from close to 0 K to the transition temperature. Therefore, in order to make full use of the available cooling from solid nitrogen, the temperature is permitted to rise from a specified initial minimum temperature to a specified maximum temperature that is reached after a duration equal to the desired hold time.

The minimum and maximum solid nitrogen temperatures, $T_{l_{\text{min}}}$ and $T_{l_{\text{max}}}$, must be selected and applied in the analysis in order to compare the two systems. It will be revealed in Sections 5.7 and 5.10 that the ideal maximum temperature varies among applications as it depends on the various design parameters such as magnetic field, hold time, mechanical loading, and the HTS conductor properties. For this analysis, $T_{l_{\text{max}}}$ is set equal to 40 K because it approximates the optimum $T_{l_{\text{max}}}$ that is determined for the applications discussed in later sections. A minimum temperature of 20 K is chosen because the internal energy change of nitrogen between 0 K and 20 K is less than 10% of the internal energy change associated with a warming from 0 K to 40 K. Since recooling the system becomes increasingly costly as the system temperature decreases, it does not make much sense to design a system with $T_{l_{\text{min}}}$ below 20 K since the increase in hold time would be insignificant. This same reasoning is used to justify demonstration of the experimental system in the 20–40 K temperature range.

Having specified the operating temperature range of the primary heat capacitor (solid nitrogen), it remains for the substance and operating temperature range of the secondary heat capacitor to be chosen. The ideal substance to be used in the secondary heat capacitor is one which has a high heat capacity to intercept radiation
from the room-temperature wall, at a low enough temperature to significantly reduce
the radiation flux to the primary heat capacitor. As the desired temperature is
increased, higher heat capacities are observed for the various candidate substances
that may be used as the secondary heat capacitor. As a result, there is a trade-off
between increasing the operating temperature range of the secondary heat capacitor
which reduces the required volume of the secondary heat capacitor, and reducing
the operating temperature range of the secondary heat capacitor which reduces the
radiation flux to the primary heat capacitor and therefore reduces the required volume
of the primary heat capacitor. However, once an initial temperature for the secondary
heat capacitor, $T_{Imin}$, has been justified and a substance has been chosen, all that
remains is the straight-forward task of identifying what heat-capacitor volumes are
required to provide a specified hold time. There may be a maximum acceptable
temperature for the secondary heat capacitor (i.e. the boiling point of the substance
if a system that does not require regular refilling of the heat capacitors is desired)
but this temperature does not necessarily need to be reached. A lower $T_{Imin}$ may
be ideal.

The analysis is performed for two different values of $T_{Imin}$, 20 K and 80 K. Al-
though a $T_{Imin}$ of 20 K may be more difficult and costly to reach, results for an
analysis based on $T_{Imin}$ equal to 20 K is included below because better performance
is expected for a different set of substances than the ones predicted to offer the best
performance for the 80 K case. 80 K may be more practical because a two-stage cry-
ocooler can extract heat faster through its first stage which is typically close to 80 K,
while a larger quantity of heat must be extracted from the secondary heat capacitor
than from the cold container during recooling.

**Numerical Calculation**

A computer code is constructed to calculate the minimum required heat capacitor
volumes for various substances to be considered as the secondary heat capacitor. The
code is split into two functions. The first function simulates the temperature variation
with time for both heat capacitors given the initial temperatures, system dimensions,
and the superinsulation blanket parameters. The heat capacitor volumes and the radiation shield are assumed to have a uniform temperature. The temperature traces are simulated by applying the first law of thermodynamics for a closed system to two control volumes. The first control volume surrounds the cold container, and the second surrounds the radiation shield and the secondary heat capacitor container, as shown in Figure 5-6a. Figure 5-6b is a schematic describing the heat leak components that apply to each control volume. For the primary heat capacitor:

\[ \rho_{I,\text{sat}} V_I \frac{d e_I}{d t} = \dot{Q}_I(T_I, T_{II}) \]  

(5.31)

where \( \rho_{I,\text{sat}} \) is the density of the saturated liquid state for the primary heat capacitor, \( V_I \) is the volume of the cold container, \( e_I \) is the specific internal energy of the primary heat capacitor, and \( \dot{Q}_I \) is the heat leak from the radiation shield to the cold container. \( \dot{Q}_I \) is determined by summing the individual heat leak components shown in Figure 5-6b:

\[ \dot{Q}_I(T_I, T_{II}) = \dot{Q}_{I,\text{ins}}(T_I, T_{II}) + \dot{Q}_{I,\text{fss}}(T_I, T_{II}) \]  

(5.32)

where \( \dot{Q}_{I,\text{ins}} \) is the heat leak through the superinsulation blanket, and \( \dot{Q}_{I,\text{fss}} \) is the heat leak through the fiberglass supports. \( \dot{Q}_{I,\text{ins}} \) is calculated according to Equation 5.1 by substituting \( T_I \) for \( T_C \), \( T_{II} \) for \( T_H \), and multiplying the result by the inner surface area of the radiation shield. A layer density of 12.50 layers/cm is used in determining \( \dot{Q}_{I,\text{ins}} \). \( \dot{Q}_{I,\text{fss}} \) is calculated according to Equation 5.2 based on the cold container mass and substituting \( T_{II} \) for \( T_H \) and \( T_I \) for \( T_C \).

For the secondary heat capacitor:

\[ \rho_{II,\text{sat}} V_{II} \frac{d e_{II}}{d t} = \dot{Q}_{II}(T_{II}, 293K) - \dot{Q}_I(T_I, T_{II}) \]  

(5.33)

where \( \rho_{II,\text{sat}} \) is the density of the saturated liquid state for the secondary heat capacitor, \( V_{II} \) is the volume of the secondary heat capacitor container, \( e_{II} \) is the specific internal energy of the secondary heat capacitor, and \( \dot{Q}_{II} \) is the heat leak from the room-temperature cryostat wall to the radiation shield. \( \dot{Q}_I \) is calculated according to
Figure 5-6: Illustration of the thermal model used to calculate the system size as a function of hold time for the solid heat capacitor system without the magnet. The system dimensions and the locations of the control volumes are defined in (a). The superinsulation has been omitted for clarity. The heat leak components applied to each control volume are shown in (b).
Equation 5.32 and \( \dot{Q}_{II} \) is determined by summing the heat leak components shown in Figure 5-6b:

\[
\dot{Q}_{II}(T_{II}, 293 \text{ K}) = \dot{Q}_{IIins}(T_{II}, 293 \text{ K}) + \dot{Q}_{IIfss}(T_{II}, 293 \text{ K})
\]  

(5.34)

where \( \dot{Q}_{IIins} \) is the heat leak through the superinsulation blanket, and \( \dot{Q}_{IIfss} \) is the heat leak through the fiberglass supports. \( \dot{Q}_{IIins} \) is calculated according to Equation 5.1 by substituting \( T_{II} \) for \( T_C \), 293 K for \( T_H \), and multiplying the result by the surface area of the cryostat wall. A layer density of 12.50 layers/cm is used in determining \( \dot{Q}_{IIins} \). \( \dot{Q}_{IIfss} \) is calculated according to Equation 5.2 based on the sum of the masses of the cold container, secondary heat capacitor container, and their contents, and substituting 293 K for \( T_H \) and \( T_{II} \) for \( T_C \).

A forward-Euler finite difference method is used to simulate the temperature rise of the heat capacitors from their minimum temperature. The method involves calculation of the specific enthalpy of both heat capacitors at discrete time steps. The first law equation describing the primary heat capacitor, Equation 5.31, is discretized to yield the following form:

\[
e^i_{j+1} = e^i_j + \Delta t \left( \frac{\dot{Q}_{I}(T^i_j, T^i_{II})}{\rho_{sat} V_I} \right)
\]  

(5.35)

where \( \Delta t \) is the time step size, and the superscript \( i \) denotes which time step. For the secondary heat capacitor:

\[
e^i_{II+1} = e^i_{II} + \Delta t \left( \frac{\dot{Q}_{II}(T^i_{II}, 293 \text{ K}) - \dot{Q}_{I}(T^i_j, T^i_{II})}{\rho_{II,sat} V_{II}} \right).
\]  

(5.36)

At each time step, the temperatures \( T^i_j \) and \( T^i_{II} \) are recalculated based on the corresponding internal energy values by interpolation of the internal energy versus temperature data. In order to obtain an accurate solution, \( \Delta t \) must be kept small enough such that the solution converges. The simulation of the temperature rise is initiated with the heat capacitor temperatures, \( T^0_j \) and \( T^0_{II} \), set equal to their initial values \( T_{I_{min}} \) and \( T_{I_{II min}} \). The simulation terminates when either \( T^j_j \) reaches \( T_{I_{max}} \) (40 K), or
Table 5.1: Candidate Substances for the Secondary Heat Capacitor

<table>
<thead>
<tr>
<th>Substance</th>
<th>$T_{IImin}$ [K]</th>
<th>$T_{IImax}$ [K]</th>
<th>Phase</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ammonia</td>
<td>20</td>
<td>195</td>
<td>s</td>
<td>[21][38]</td>
</tr>
<tr>
<td>Ammonia</td>
<td>80</td>
<td>195</td>
<td>s</td>
<td>[21][38]</td>
</tr>
<tr>
<td>Argon</td>
<td>20</td>
<td>87</td>
<td>s&amp;l</td>
<td>[7]</td>
</tr>
<tr>
<td>Methane</td>
<td>20</td>
<td>111</td>
<td>s&amp;l</td>
<td>[39][40]</td>
</tr>
<tr>
<td>Methane</td>
<td>80</td>
<td>111</td>
<td>s&amp;l</td>
<td>[39][40]</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>20</td>
<td>77</td>
<td>s&amp;l</td>
<td>[5]</td>
</tr>
</tbody>
</table>

$T_{II}$ reaches some maximum specified temperature such as the boiling point or melting point of the secondary heat capacitor. The simulated time at the termination is then returned to the calling function.

The second function is the main function which calls the simulation function just described. When it calls the simulation function, it specifies the system dimensions. It begins with arbitrarily small values for the number of layers in each superinsulation blanket, $N_I$ and $N_{II}$, and the cold container diameter, $d_I$. $N_I$, $N_{II}$, and $d_I$ determine the length of the cold container ($l_I = d_I$) and all other diameters as illustrated in Figure 5-6a. It then adjusts the length of the secondary heat capacitor container, $l_{II}$, until the desired hold time is returned by the simulation function, or some arbitrarily large $l_{II}$ value is reached. This process is repeated for various $N_I$, $N_{II}$, and $d_I$ values until the optimum combination that leads to a minimum total system volume or mass is found.

The code is used to generate minimum system volume and mass versus hold time plots for each of the secondary heat capacitor substances listed in Table 5.1. Different permissible temperature ranges, $T_{IImin}$-$T_{IImax}$, are specified for each substance. The value of $T_{IImax}$ corresponds to either the melting or boiling point of the substance at atmospheric pressure, and determines whether or not the heat capacitor is permitted to melt. The phases considered are indicated in the table (s denotes solid, and l denotes liquid). $T_{II}$ does not actually reach $T_{IImax}$ before $T_I$ reaches 40 K for the solid ammonia system. Sources for the secondary heat capacitor internal energy data and the saturated liquid density are also given.
Figure 5-7 shows predicted system mass as a function of $\tau_h$ for the various candidate substances used as the secondary heat capacitor. Figure 5-8 shows predicted system volume as a function of $\tau_h$. The resulting system dimensions when using the criterion of minimum system mass are different from the case of minimum system volume. Two slightly different codes must be used in the generation of each graph. Also shown in the figures for comparison is the predicted system mass and volume for the liquid helium system (dashed line). The analysis of this section reveals that a cryogenic system utilizing solid nitrogen in the 20–40 K temperature range as a primary heat capacitor, and solid ammonia as a secondary heat capacitor with $T_{I, I_{min}}$ equal to 80 K, may offer a total system size that is somewhat smaller than what is expected for a liquid helium system with the same hold time.

5.5 Magnet Performance Characteristics

The coil dimensions and operating current used in the permanent RT bore magnet application are chosen to provide a specified axial field strength at the magnet mid-plane, $B_z$, and decay time constant, $\tau_B$, for a specified wind radius, $a_i$, and typical conductor dimensions. The calculation of $\tau_B$ is not usually a concern for LTS magnets because the conductor has a high index leading to a large $\tau_B$ when the operating current is less than the critical current. However, this is not the case for HTS magnets, as was revealed in Chapter 4, and so a method for estimating $\tau_B$ is required when designing a permanent HTS magnet.

In calculating the magnet dimensions the geometry that yields the minimum conductor volume that can fulfill the design requirements is assumed. This minimum conductor criterion also implies the minimum magnet mass. The combination of this criterion with a specified field strength and bore size allows the magnet dimensions to be specified as a function of current density. However, the decay time constant, which depends on the operating temperature, current density, as well as the magnet dimensions will not necessarily fulfill its requirement. For a given operating temperature, a current density that is small enough to yield the specified decay time constant must
Figure 5-7: Predicted mass versus hold time for the solid heat capacitor system without the magnet. Predictions are shown for each of the secondary heat capacitor substances listed in Table 5.1. The predicted mass for the liquid helium system is shown as the dashed line for comparison.
Figure 5-8: Predicted volume versus hold time for the solid heat capacitor system without the magnet. Predictions are shown for each of the secondary heat capacitor substances listed in Table 5.1. The predicted volume for the liquid helium system is shown as the dashed line for comparison.
Figure 5-9: Coil dimensions required for calculating the field strength and the decay time constant as a function of current density.

be chosen. The next two sections describe calculation of the magnet dimensions as a function of current density and inner wind diameter, and calculation of the decay time constant.

5.5.1 Determination of the Magnet Geometry

Figure 5-9 illustrates the dimensions that are required for calculating the field strength at the midpoint of the axis, $B_z(0,0)$, of a coil with a rectangular cross-section. [41] gives the following solution:

$$B_z(0,0) = \mu_0 \lambda j a_i F(\alpha, \beta)$$

(5.37)

where $\mu_0$ is the magnetic permeability of free space, $\lambda j$ is the average current density across the cross-section of the wind, $a_i$ is the inner wind radius, and $F(\alpha, \beta)$ is the following function of the geometric parameters $\alpha$ and $\beta$:

$$F(\alpha, \beta) = \beta \ln \left[ \frac{\alpha + (\alpha^2 + \beta^2)^{1/2}}{1 + (1 + \beta^2)^{1/2}} \right]$$

(5.38)

where $\alpha$ is the ratio of the outer wind radius to the inner wind radius, $a_o/a_i$, and $\beta$ is the ratio of half the wind length to the inner wind radius, $b/a_i$. It can be inferred
from Equation 5.37 that for a set of field requirements, \([B_z(0, 0), a_i]\), and a fixed value for \(\lambda j\), the value of \(F\) is determined as:

\[
F(\alpha, \beta) = \frac{B_z(0, 0)}{\mu_0 \lambda j a_i}. \tag{5.39}
\]

Any set of the geometrical parameters \((\alpha, \beta)\) that yields a value for \(F\) equal to the right hand side of Equation 5.39 will produce the desired field. The \(\alpha\) and \(\beta\) used in the analysis are the values that minimize the conductor volume. The set of geometrical parameters that will yield the minimum conductor volume is determined by realizing that the conductor volume, \(V_{cnd}\) may be calculated as:

\[
V_{cnd} = 2\pi a_i^3 v(\alpha, \beta) \tag{5.40}
\]

where \(v(\alpha, \beta)\) is calculated based on the geometric parameters:

\[
v(\alpha, \beta) = \beta(\alpha^2 - 1). \tag{5.41}
\]

Figure 5-10 is a contour plot indicating the value of \(F\) over a range of values for \(\alpha\) and \(\beta\). Also shown in the plot is a curve that identifies the \(\alpha\) and \(\beta\) that minimize \(v(\alpha, \beta)\) on the contours of constant \(F\). Therefore, once \(F\) has been chosen, the geometric parameters that yield the minimum conductor volume are found by locating the intersection of the minimum volume curve with the contour that corresponds to the desired \(F\). The magnet mass, \(m_{mag}\), is determined by multiplying the conductor density, \(\rho_{cnd}\), by \(V_{cnd}\) such that:

\[
m_{mag} = \rho_{cnd} V_{cnd}. \tag{5.42}
\]
Figure 5-10: Contour plot of $F$ as a function of $\alpha$ and $\beta$. The solid curve indicates the values of the geometric parameters that yield the minimum conductor volume.
5.5.2 Calculation of the Field Decay Time Constant

A circuit schematic modeling a layer-wound superconducting magnet operating in persistent-mode was shown in Figure 3-9. Assuming the splice resistance, $V_s$, is negligible, the magnetic field decay time constant, $\tau_B$, may be determined based on the magnet inductance, $L_{mag}$, and the index resistance, $R_n$:

$$\tau_B = \frac{L_{mag}}{R_n} = \frac{L_{mag}I_t}{V_n} \quad \text{(5.43)}$$

where $I_t$ is the transport current, and $V_n$ is the index voltage which is equal to $I_tR_n$. $L_{mag}$ is determined by the conductor and magnet geometry. [42] outlines a method for estimating the inductance of a coil of any proportions. It lists the following relation:

$$L_{mag} = (1.9739 \times 10^{-6} \text{ H/m turn}^2)N^2\alpha_i \left[\frac{(\alpha + 1)^2}{4\beta}\right] k'(\alpha, \beta) \quad \text{(5.44)}$$

where $N$ is the total number of turns, and $k'$ is a parameter that is a function of the geometric parameters and is calculated as the difference of two other parameters that are tabulated in [42], $K$ and $k$, such that:

$$k'(\alpha, \beta) = K(\alpha, \beta) - k(\alpha, \beta) \quad \text{(5.45)}$$

$I_t$ is related to the overall current density, $\lambda j$, and the conductor cross-section, $A_{cond}$. In the analysis that follow, the electrical insulation cross-section is assumed negligible, such that:

$$I_t = A_{cond}\lambda j \quad \text{(5.46)}$$

$V_n$ may be calculated by summing the index voltage contributions from each turn of the coil as described in Section 3.2:

$$V_n = \sum_{m,p} l_{m,p}E_{m,p} \quad \text{(5.47)}$$

where the subscript $(m,p)$ is a coordinate specifying the location of the turn within
the wind such that \( m \) indicates which layer and \( p \) indicates which turn in the layer, \( E_{m,p} \) is the electric field in a particular turn caused by the index, and \( l_{m,p} \) is the length of the turn. \( E_{m,p} \) is determined by the conductor properties, operating temperature, transport current, and the magnetic field components both parallel and perpendicular to the tape conductor's wide surface. In this chapter, \( E_{m,p} \) is calculated for the HTS conductor using a modified form of Equation 3.41:

\[
E_{m,p}(I_t, T) = E_c \left( \frac{I_t}{I_{c,m,p}(T, B_{\perp m,p}, B_{\parallel m,p})} \right)^n
\]

(5.48)

where \( E_c \) is the electric field produced when the transport current, \( I_t \), is equal to the critical current, \( I_c \), \( T \) is the operating temperature, \( B_{\perp m,p} \) is the field component perpendicular to the conductor's wide surface, \( B_{\parallel m,p} \) is the field component parallel to the conductor's wide surface, and \( n \) is the conductor index. It was assumed that \( n \) is also a function of temperature and the magnetic field components in the modeling of the experimental system of Section 3.2. However, since the variation of \( n \) with temperature and field for HTS is still uncertain, a constant value of \( n \) representing an overall effective index is assumed. \( I_{c,m,p}(T, B_{\perp m,p}, B_{\parallel m,p}) \) is calculated in the same manner as in Section 3.2, based on the conductor \( I_c \) at 77 K in the absence of a background field, \( I_c(77\, \text{K}, 0, 0) \), and data reported by IGC [27], such that:

\[
I_c(T, B_{\perp}, B_{\parallel}) = I_c(77\, \text{K}, 0, 0) \times \left[ \frac{I_c(T, B_{\perp}, 0)}{I_c(77\, \text{K}, 0, 0)} \right] \times \left[ \frac{I_c(T, 0, B_{\parallel})}{I_c(T, 0, 0)} \right]
\]

(3.36)

where the terms in brackets are found by interpolation of the IGC data. Satisfaction of the decay time constant criterion is checked by calculation of \( \tau_B \) for \( T \) equal to the maximum permitted cold container temperature, \( T_{max} \).

For LTS conductor, even of rectangular cross section, \( I_c \) is not dependent on the individual field components, but on the overall magnitude, \( |B_{m,p}| \). Therefore, \( E_{m,p} \) is calculated as:

\[
E_{m,p}(I_t, T) = E_c \left( \frac{I_t}{I_{c,m,p}(T, |B_{m,p}|)} \right)^n.
\]

(5.49)

Also, since the magnet operates at the boiling point of liquid helium for the LTS
system, satisfaction of the decay time constant is checked while setting $T$ equal to 4.2 K. The variation of $I_c$ with $|B|$ is reported in [2] for both NbTi and Nb$_3$Sn superconductors at 4.2 K, so that $I_c(4.2 \text{ K}, |B|)$ is calculated as:

$$I_c(4.2 \text{ K}, |B|) = I_c(4.2 \text{ K}, 0) \times \left[ \frac{I_c(4.2 \text{ K}, |B|)}{I_c(4.2 \text{ K}, 0)} \right]$$

(5.50)

where the term in brackets is derived from the reported data.

### 5.6 Design Codes for the Permanent Superconducting RT Bore Magnet System

Numerical codes are constructed to predict the size of a system with a specified bore size, $d_{bs}$, field, $B_{zs}$, hold time, $\tau_{ha}$, and field decay time constant, $\tau_{Bs}$. The subscript $s$ is added to the symbols when referring to the specified value. The mass of the RT bore magnet system is dominated by the magnet mass and the cryostat mass. Reduction of the gap widths reduces the inner wind radius and therefore reduces the mass of the magnet, but increases the liquid helium or heat capacitor volumes. For all system specifications considered, the number of layers in each superinsulation blanket should be optimized to make a fair comparison of the LHe/LTS and SN2/HTS systems. However, based on a more detailed examination than what is presented below of the smallest (2.0 T, 10 mm bore) and largest (6.0 T, 50 mm bore) systems considered, the optimum number of layers in each blanket does not vary significantly with the system specifications. Therefore, a fixed number of layers is chosen for each blanket in each system to significantly reduce the computation time of the codes. The fixed blanket thicknesses lead to calculated system sizes that are less than 5% larger than what the optimum blanket thicknesses would yield. For the LHe/LTS system, $N_I$, $N_{II}$, and $N_{III}$ are set equal to, respectively, 1, 2, and 5. For the SN2/HTS system, $N_I$ and $N_{II}$ are both set equal to 5. The radiation gap widths for both the LHe/LTS and SN2/HTS designs are calculated according to Equation 5.29 with $s$ set equal to 3 mm.

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Figure 5-11: Dimensions used in describing the LHe/LTS system. The superinsulation and access tubes are omitted for clarity.

5.6.1 LHe/LTS System Design Code

This section describes the numerical processes followed by the code used to predict the mass and volume of the LHe/LTS system. Dimensional symbols are defined in Figure 5-11. A detailed description of the code is included in Appendix D.

The code begins by calculating the magnet dimensions based on \( d_{Bs} \) and \( B_{zs} \). The inner wind radius, \( a_i \), is determined by \( d_{Bs} \), the radiation gap widths, and \( t_{CCci} \) which is calculated according to Equation 5.21. Then, an arbitrary number of conductor turns per layer, \( N_t \), is chosen. The geometrical parameter, \( \beta \), is determined by \( N_t \), the conductor width, and \( a_i \). The number of layers, \( N_t \), is calculated to yield a value for \( \alpha \) such that \( [\alpha, \beta] \) lies on the minimum volume curve of Figure 5-10. A value for \( \lambda J \) is determined based on \( a_i \), \( B_{zs} \), \( \alpha \), and \( \beta \) according to Equations 5.37 and 5.38. Finally, \( \tau_B \) is calculated for this geometry and current according to the method of Section 5.5.2. \( N_t \) is adjusted and the process is repeated until \( \tau_B \approx \tau_{Bs} \). Once the magnet geometry is determined the magnet mass is calculated using Equations 5.40–5.42.
With the magnet geometry determined, the code can calculate the remaining system dimensions that will yield the desired hold time, \(\tau_{hs}\). It begins by choosing a value for \(d_I\) that is slightly larger than \(2a_o\). \(l_I\) is then calculated as:

\[
l_I = \frac{d_I \beta}{\alpha}
\]

which yields cold container dimensions that are geometrically similar to the magnet dimensions. The remaining system dimensions are calculated where the cold container and cryostat wall thicknesses are determined according to the relations in Section 5.3. The lead access tube diameter is calculated based on the magnet transport current and Equation 5.6. Finally, the heat leak into the cold container, \(\dot{Q}_{CC}\) is calculated. The first law control volume model for predicting the heat leak into the cold container is depicted in Figure 5-12. The method of determining the shield temperatures and heat leak into the cold container, \(\dot{Q}_{CC}\), is identical to the method described in Section 5.4.1, except an additional component corresponding to the heat leak through the current lead access tubes, \(2\dot{Q}_{lat}\), is calculated using Equation 5.14 and added to \(\dot{Q}_{CC}\). The initial volume of liquid helium, \(V_{LHe}\), is taken as the difference between the empty cold container volume and the magnet volume. \(\tau_h\) is then calculated as:

\[
\tau_h = \frac{V_{LHe}}{\dot{Q}_{CC}}.
\]

\(d_I\) is adjusted and the process is reiterated until \(\tau_h \approx \tau_{hs}\), at which point the system dimensions have been determined and its mass is calculated as the sum of the magnet mass, helium mass, cold container mass, and the cryostat mass.

### 5.6.2 SN2/HTS System Design Code

This section describes the numerical processes followed by the code used to predict the mass and volume of the SN2/HTS system. Dimensional symbols are defined in Figure 5-13.
Figure 5-12: Thermal model for predicting the heat leak into the cold container of the LHe/LTS system.
Figure 5-13: Dimensions used in describing the SN2/HTS system. The superinsulation and access tubes are omitted for clarity.
The code begins by choosing an arbitrary value for the maximum primary heat capacitor, $T_{I_{max}}$. Then, the magnet dimensions and operating current are calculated in the same manner as for the LHe/LTS system while calculating the conductor critical current using the relation for HTS magnets, Equation 3.36. In this case, the temperature of the magnet must be considered (assumed equal to 4.2 K for the LTS case). The magnet mass and dimensions are determined for a temperature equal to $T_{I_{max}}$ because the highest magnet temperature leads to the smallest decay time constant.

With the magnet geometry determined, the code can then calculate the remaining system dimensions that will yield the desired hold time, $\tau_{hs}$. It begins by choosing a value for $d_I$ that is slightly larger than $2a_o$ and calculates $l_I$ such that the cold container dimensions are geometrically similar to the magnet dimensions. The remaining system diameters are calculated where the thicknesses of the cylindrical shells are determined according to the relations of Section 5.3. Then, an arbitrary value for the length of the secondary heat capacitor container, $l_{II}$, is chosen and the remaining system dimensions are calculated.

The system hold time is calculated while assuming the mass of the primary heat capacitor is equal to the density of the saturated liquid multiplied by the difference between the empty cold container volume and the magnet volume. The mass of the secondary heat capacitor is equal to the density of the saturated liquid multiplied by the volume of the secondary heat capacitor container. Reduction in the secondary heat capacitor container volume caused by the cold container access tubes passing through it, as shown in Figure 5-2, is neglected.

The first law control volume model for predicting the warming trend of the cold container and the secondary heat capacitor container is depicted in Figure 5-14. The initial temperatures, $T_{I_{min}}$ and $T_{I_{11min}}$, are fixed and must be specified for the heat capacitors. A maximum permissible temperature, $T_{I_{11max}}$, must also be assigned for the secondary heat capacitor. $T_{I_{11max}}$ corresponds to either the melting point or boiling point for the substance used as the secondary heat capacitor. The variation of $T_I$ and $T_{II}$ with time are simulated using the same method as the one described
Figure 5-14: Model used to predict the warming trends of the cold container and the secondary heat capacitor container.
in Section 5.4.2, with a few adjustments. To begin with, the internal energy of the cold container contents is calculated while taking into account the internal energy associated with the magnet mass. And secondly, the method of determining the heat leak into the cold container, \( \dot{Q}_I \), is identical to the method described in Section 5.4.2, except two additional components are included. The first is the heat leak through the current lead access tubes, \( 2\dot{Q}_{Ilat} \), which is calculated as a function of \( T_I \) in accordance with Equations 5.4 and 5.13. The second is the heat leak through the cold bus access tube, \( \dot{Q}_{Ibat} \), which is calculated as a function of \( T_I \) in accordance with Equations 5.13 and 5.18. \( \dot{Q}_I \) is then determined as \( \dot{Q}_{Iins} + \dot{Q}_{Ifs} + 2\dot{Q}_{Ilat} + \dot{Q}_{Ibat} \). Another heat leak component that corresponds to the conduction through the cold bus access tube that extends into the secondary heat capacitor container, \( \dot{Q}_{Ibat} \), is added to the heat leak into the the secondary heat capacitor, \( \dot{Q}_{II} \). An approximate value for \( T_{II\text{max}} \) is used in the calculation of \( \dot{Q}_{II\text{bat}} \). \( \dot{Q}_{II} \) is determined as the sum \( \dot{Q}_{II\text{ins}} + \dot{Q}_{II\text{fs}} + \dot{Q}_{II\text{bat}} \).

\( \tau_h \) is equal to the simulated time when \( T_I = T_{I\text{max}} \) or \( T_{II} = T_{II\text{max}} \). \( l_{II} \) is increased until \( \tau_h \approx \tau_{hs} \) or until \( l_{II} \) exceeds some arbitrarily large value. The system dimensions have then been determined and the system mass and volume may be calculated. \( d_I \) and \( T_{I\text{max}} \) are varied and the process is reiterated until the set of values, \([d_I, T_{I\text{max}}]\), that lead to the smallest system size have been identified.

### 5.7 Minimum HTS Conductor Properties

The design codes were used to identify the minimum HTS conductor critical current density (average across the entire conductor cross-section) at 77 K and 0 field, \( \lambda J_c(77\text{ K}, 0, 0) \), required to permit a system mass that is comparable with the system mass expected for the LHe/LTS system. The cross-section of the insulation is assumed to be negligible.

The minimum \( \lambda J_c(77\text{ K}, 0, 0) \) was determined for a range of axial field, \( 2\text{ T} \leq B_{zs} \leq 6\text{ T} \), RT bore diameter, \( 10\text{ mm} \leq d_{Bs} \leq 50\text{ mm} \), and conductor index, \( n \). The remaining design specifications were fixed at the following values: \( \tau_{hs} = 5\text{ days} \), \( \tau_{ Bs} = 1 \times 10^{11} \text{ s} \), and a maximum system acceleration of 1.0 G. For the cryocooled
SN2/HTS system, $\tau_{res}$ is specified as 10 hr (which does not include the time required to precool the cryocooler and the cold buses).

Conductor dimensions were selected to be consistent with what is commonly available. For the LTS magnet, Nb$_3$Sn conductor is considered with a ratio of matrix to superconductor of 2.5:1. Values for the critical current density of the superconductor (not including the matrix metal) and the dependency on field is taken from [2]. The cross-sectional dimensions were selected to be 2 mm $\times$ 1 mm. The density of the conductor, $\rho_{cond}$, is set equal to the density of copper. For the HTS magnet, Bi2223/Ag conductor is considered with cross-sectional dimensions of 3.2 mm $\times$ 0.23 mm. A sample of conductor was weighed to determine a value for $\rho_{cond}$ of 8750 kg/m$^3$.

$\lambda J_c(77 K, 0, 0)$ of presently available Bi2223/Ag conductor now approaches $2 \times 10^8$ A/m$^2$, while $n$ in zero field at 77 K approaches 20. HTS conductor properties clearly must be improved in order to make permanent HTS magnets practical. However, the performance of Bi2223/Ag conductor has steadily improved over the last decade. Also, other high-temperature superconductors are expected to be fabricated into long lengths of wire over the next few years. Short lengths of YBCO and Bi2212 conductor have already been demonstrated. A 1 m long sample of YBCO conductor consisting of a thin film of YBCO deposited on normal metal had a $J_c(77 K, 0, 0)$ in the film of $1 \times 10^{10}$ A/m$^2$ [43]. Therefore, although permanent HTS magnets presently are not practical, it is likely that they will become practical.

Table 5.2 identifies Bi2223/Ag property targets that will permit the construction of a practical cryocooled SN2/HTS system. It lists the values of $\lambda J_c(77 K, 0, 0)$ required for constructing a cryocooled SN2/HTS system with a mass that is twice the mass expected for an equivalent LHe/LTS system for various field strength, RT bore diameter, and index. Also listed is the expected system mass (twice the LHe/LTS system mass) and the maximum cold container temperature, $T_{max}$. The maximum solid ammonia temperature varies between 158 K for the largest system (6.0 T, 50 mm bore) and 178 K for the smallest system (2.0 T, 10 mm bore).
Table 5.2: Required Bi2223/Ag Critical Current Properties for the SN2/HTS System

<table>
<thead>
<tr>
<th>$B_z$ [T]</th>
<th>$d_B$ [mm]</th>
<th>$n$</th>
<th>$\lambda J_c(77K, 0, 0)$ [A/m$^2$]</th>
<th>Mass [kg] (twice LHe/LTS)</th>
<th>$T_{max}$ [K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>10</td>
<td>20</td>
<td>$1.6 \times 10^9$</td>
<td>3.1</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td></td>
<td>$1.2 \times 10^9$</td>
<td>3.1</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td></td>
<td>$1.1 \times 10^8$</td>
<td>3.1</td>
<td>50</td>
</tr>
<tr>
<td>25</td>
<td>20</td>
<td></td>
<td>$1.6 \times 10^9$</td>
<td>4.4</td>
<td>47</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td></td>
<td>$1.2 \times 10^9$</td>
<td>4.4</td>
<td>47</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td></td>
<td>$1.1 \times 10^9$</td>
<td>4.4</td>
<td>47</td>
</tr>
<tr>
<td>50</td>
<td>20</td>
<td></td>
<td>$1.8 \times 10^9$</td>
<td>6.9</td>
<td>46</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td></td>
<td>$1.4 \times 10^9$</td>
<td>6.9</td>
<td>46</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td></td>
<td>$1.2 \times 10^9$</td>
<td>6.9</td>
<td>46</td>
</tr>
<tr>
<td>4.0</td>
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<td>4.6</td>
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<td>45</td>
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<td>40</td>
<td></td>
<td>$2.1 \times 10^9$</td>
<td>6.5</td>
<td>42</td>
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<td>10.7</td>
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<td>40</td>
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<td>10.7</td>
<td>37</td>
</tr>
<tr>
<td>6.0</td>
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<td>20</td>
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<td>7.4</td>
<td>37</td>
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<td></td>
<td>30</td>
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<td>$2.3 \times 10^9$</td>
<td>7.4</td>
<td>37</td>
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<td>$2.0 \times 10^9$</td>
<td>10.8</td>
<td>37</td>
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<td>20</td>
<td></td>
<td>$2.8 \times 10^9$</td>
<td>17.9</td>
<td>36</td>
</tr>
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<td></td>
<td>30</td>
<td></td>
<td>$2.2 \times 10^9$</td>
<td>17.9</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td></td>
<td>$1.9 \times 10^9$</td>
<td>17.9</td>
<td>36</td>
</tr>
</tbody>
</table>
5.8 Superconducting Mine Countermeasures

The second application discussed is a superconducting mine countermeasures (SCMCM) system, currently being considered by the US Navy. It involves a large solenoidal magnet that imitates the magnetic signature of a ship. The system is intended to trigger submerged magnetic influence mines that detect the magnetic field while the system is towed across the surface.

A design for a SCMCM system was proposed by Golda et al. [44]. Figure 5-15 illustrates their design. A large solenoidal superconducting magnet, wound with LTS conductor, is submerged in a liquid helium bath. The magnet and the liquid helium are contained within a cylindrical SS304 vessel. This system is designed to withstand high mechanical shock caused by exploding mines so it involves a much more rugged support structure for suspending the magnet vessel within its SS304 cryostat than was considered for the RT bore magnet application considered previously. The cylindrical SCMCM magnet vessel is supported by two G-10 support cones that are fastened to each end. G-10 is used because it has a high strength to thermal conduction ratio. The larger ends of the cones connect to the ends of the cylindrical cryostat. Two vapor-cooled radiation shields are included to reduce the radiation component of heat leak into the magnet vessel. Superinsulation blankets cover each shield.
Table 5.3: SCMCM Magnet Specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner Radius, $a_i$</td>
<td>250 mm</td>
</tr>
<tr>
<td>Outer Radius, $a_o$</td>
<td>268 mm</td>
</tr>
<tr>
<td>Wind Height, $2b$</td>
<td>325 mm</td>
</tr>
<tr>
<td>Total Turns, $N$</td>
<td>11,375</td>
</tr>
<tr>
<td>Transport Current, $I_t$</td>
<td>75 A</td>
</tr>
<tr>
<td>Current Density, $\lambda J$</td>
<td>$1.5 \times 10^6$ A/m$^2$</td>
</tr>
<tr>
<td>Magnetic Dipole Moment, $</td>
<td>M</td>
</tr>
<tr>
<td>Peak Field</td>
<td>2.44 T</td>
</tr>
</tbody>
</table>

The magnet is operated in persistent-mode. It is charged using a superconducting switch and an external power supply. Current is delivered to the magnet vessel by a pair of vapor-cooled detachable current leads. The magnet operates at a low current density of 15,000 A/cm$^2$, which is probably less than 20% of the conductor critical current density in a field equal to the expected peak field in the wind of 2.44 T. The low current density and peak field reduces the possibility of quenching, and the possibility that the magnet will be damaged if a quench does occur. It is especially important to consider the quench behavior of this system because quenching is commonly caused by mechanical sources of dissipation in LTS systems, and this system is intended to withstand high mechanical shock. The magnet dimensions and operating parameters are listed in Table 5.3. For this application, the main field parameter is the magnitude of the magnetic dipole moment, $|M|$. $|M|$ may be estimated for a solenoid as:

$$|M| = I_t N \bar{A}$$

(5.53)

where $I_t$ is the transport current, $N$ is the number of turns, and $\bar{A}$ is the mean cross-sectional area, $\pi(a_o^2 + a_i^2)/2$.

The design code described below estimates the total system mass, $m_{sys}$, for a specified hold time, $\tau_{hs}$. The code calculated $m_{sys}$ for a $\tau_{hs}$ of 17 hours with a safety factor of 2 so that only 50% of the initial volume of liquid helium boils off. $m_{sys}$ is predicted to be 360 kg. The optimum gap widths, $g_I$, $g_{II}$, and $g_{III}$, are, respectively, 33 mm, 48 mm, and 93 mm. The inner and outer shield temperatures are, respec-
tively, 20 K and 80 K. The code does not consider the mass associated with support attachments, or the mass associated with support structures that would be required to prevent buckling of the cryostat, the magnet vessel, and the radiation shields under high loads. The mass of the support cones and the radiation shields are also neglected because they are expected to be small.

5.9 Prediction of the SCMCM System Mass

The use of a rigid support structure for the SCMCM system leads to a relatively high rate of conduction heat leak in comparison to the permanent superconducting RT bore magnet discussed previously. For the RT bore magnet system, heat conduction through the support straps was small in comparison to the heat leak through the superinsulation blanket and the access tubes. As a result, the system mass was minimized by adjusting the thickness of the superinsulation blankets, because the cryostat constitutes a large portion of the total system mass, and the mass of the cryostat is strongly dependent on its diameter which increases with increasing gap widths. But for the SCMCM system, conduction is expected to dominate. Conduction can be reduced by increasing the gap width between the shields which corresponds to an increase in the length of the supports between the shields. Therefore, optimization of this LHe/LTS system also requires the optimization of the gap widths, $g_I$, $g_{II}$, and $g_{III}$.

When the number of layers in a superinsulation blanket is increased, the performance of the inner layers is degraded because evacuation is inhibited. Therefore, there is a maximum practical superinsulation blanket thickness. Increasing the thickness beyond this point offers negligible improvement in the overall performance of the blanket. The optimum gap widths for the minesweeper system are larger than these maximum thicknesses, and so the thickness of the superinsulation blankets are fixed. Variation of the magnet vessel diameter, $d_{MV}$, reveals that the optimum $d_{MV}$ is equal to its minimum value which is the outer wind diameter of the magnet, $2a_o$. Therefore, $d_{MV}$ is also fixed.
Figure 5-16: Dimensions used to describe the SCMCM system. The superinsulation and access tubes have been omitted for clarity.

The heat flux through the superinsulation blanket covering the outer radiation shield is calculated for a configuration of 37 DAM layers each separated by a single silk net layer with $\bar{N}$ equal to 14.6 layers/cm. This blanket configuration has been tested and its performance is predicted reasonably well by Equation 5.1 [34]. The heat flux through the superinsulation blankets covering the inner radiation shield and the magnet vessel is calculated for a configuration of 25 DAM layers each separated by three silk net layers with $\bar{N}$ equal to 11.0 layers/cm. This lighter blanket offers better performance than the heavier blanket for lower warm boundary temperatures.

5.9.1 Conduction Heat Leak

The dimensions used to describe the support cone geometry are indicated along with the other system dimensions in Figure 5-16. The surface of the cone slopes in at an angle to the base, $\theta$, of 45°. The outer diameter of the base that attaches to the cryostat wall is equal to the inner diameter of the cryostat wall, $d_{CR}$. The outer diameter of the base that attaches to the magnet vessel is equal to the outer diameter
of the magnet vessel, $d_{MV} + 2t_{MVC}$. The thickness of the cones are chosen to prevent buckling under axial compressive loads equal to the maximum design load, which is a more restrictive criterion than considering just the static stresses. [45] contains a criterion for calculating the maximum acceptable axial compressive load, $F$, as a function of the support cone wall thickness, $t_{sc}$, and the material properties:

$$F = 0.3(2\pi Et_{sc}^2 \cos^2 \theta)$$  \hspace{1cm} (5.54)$$

where $E$ is the Young's modulus of the cone material (G-10). G-10 is a fiberglass-epoxy composite. [2] lists a value for $E$ in the direction crosswise to the fibers (which is smaller than for the lengthwise direction) of 14 GPa.

Mechanical loads are assumed to be opposed by both cones acting equally, one in compression and one in tension. The compressive load magnitude that will cause a cone to fail is smaller than the tensile load magnitude, so the required wall thickness is calculated with Equation 5.54. Designing for accelerations of 100 G, a $\theta$ of 45°, and a safety factor of 10, $t_{sc}$ is calculated in m as:

$$t_{sc} = (6.1 \times 10^{-4})\sqrt{m}$$  \hspace{1cm} (5.55)$$

where $m$ is the mass in kg supported by the cones. $m$ is calculated as the sum of the magnet vessel mass, the magnet mass, and the mass of the liquid helium when the vessel is full.

In estimating the heat leak by conduction to the radiation shields and the magnet vessel, it is assumed that the radiation shields are thermally anchored to the support cones so that the temperature of the cone at the circles where the radiation shields connect is equal to the temperature of the radiation shield. Conduction heat leak is then intercepted by the cold helium vapor circulated through the shields just as was assumed for the support straps of the RT bore magnet application. The conduction through the cones can be determined by separately considering three sections of cones with fixed end temperatures: 1) spanning from the magnet vessel to the inner radiation shield, 2) from the inner radiation shield to the outer radiation shield, and
3) from the outer radiation shield to the cryostat wall. A conservative estimate of the conduction through a section of support cone, \( \dot{Q}_{sc} \), is made by modeling the cones as cylinders with diameters equal to the diameter at the larger end of the section, and lengths equal to the length measured along the cone surface, resulting in:

\[
\dot{Q}_{sc} = \frac{\bar{k}_{G10} A}{\sqrt{2g}} (T_H - T_C) = \frac{\bar{k}_{G10} (\pi d_{base} t_{sc})}{\sqrt{2g}} (T_H - T_C) \tag{5.56}
\]

where \( T_H \) is the temperature of the warmer end of the cone section, \( T_C \) is the temperature of the colder end, \( \bar{k}_{G10} \) is the average thermal conductivity of the cone material (G-10) in the temperature range \( T_C - T_H \), and \( g \) is the gap width for the section being considered which is equal to the height of the cone section. \( \bar{k}_{G10} \) is calculated based on data reported in [2].

### 5.9.2 Design Optimization Code for the LHe/LTS System

The system mass is predicted while assuming the magnet geometry given in Table 5.3, so functions that calculate the magnetic field and field decay time constants do not need to be included in this design code. What is required is a code comprised of three functions. The first function calculates the system dimensions (including the cone thicknesses), masses, volumes, and surface areas for a given set of radiation gap widths, \( [g_I, g_{II}, g_{III}] \). The second function solves a thermal model like the models presented in Sections 5.4.1 and 5.6.2 to determine the inner shield temperature, \( T_I \), the outer shield temperature, \( T_{II} \), and the heat leak into the magnet vessel, \( \dot{Q}_{MV} \). \( T_I, T_{II}, \) and \( \dot{Q}_{MV} \) are solved for using the method of Section 5.6.2, with one minor adjustment to the heat balance equations. The conduction heat leak through the support straps, \( \dot{Q}_{fss} \), is replaced by the conduction heat leak through the support cones, \( \dot{Q}_{sc} \). \( \dot{Q}_{sc} \) may be calculated for each shield and the magnet vessel using Equation 5.55, after calculating \( t_{sc} \) according to Equation 5.54. An additional component of heat leak is estimated for the current lead access tubes, \( \dot{Q}_{lat} \), according to Equations 5.6 and 5.14 after calculating \( \dot{Q}_{MV} \) according to the method of Section 5.4.1. \( \dot{Q}_{lat} \) is added to \( \dot{Q}_{MV} \) and \( \tau_h \) is determined according to Equation 5.30 while assuming the initial
volume of liquid helium is equal to the difference between the empty magnet vessel volume and the magnet volume.

The final function is the main function which calls the first two functions for various values of $g_I$ and $g_{III}$. $g_{III}$ is increased from some arbitrarily small value until $\tau_h \approx \tau_{hs}$. $g_I$ and $g_{III}$ are varied and the functions are reiterated until the optimum dimensions which lead to the minimum system size are identified. The functions are described in detail in Appendix D.

5.10 A SN2/HTS SCMCM System

An alternative SCMCM system design is illustrated in Figure 5-17. This design is based on the SN2/HTS system that employs a solid ammonia cooled radiation shield. The support cone geometry is based on the same criterion as for the LHe/LTS system where Equation 5.54 is used to calculate their thicknesses. However, the thickness of the cone section that spans from the radiation shield to the magnet vessel is calculated based on the mass of the magnet vessel and its contents, while the thickness of the cone section that spans from the cryostat wall to the radiation shield is based on the sum of the mass of the magnet vessel and its contents and the mass of the ammonia container and its contents. Current is delivered to the magnet vessel for charging the magnet through detachable vacuum leads (not shown in the figure) of the design described in Section 5.2.3. The heat capacitors are cooled (and recooled) using a detachable cryocooler system (not shown) like the one described in Section 5.1.2.

Since the conductor properties are different from the LTS conductor used for the LHe/LTS system, we cannot assume that the HTS magnet dimensions are the same as the dimensions of the LTS magnet. Instead the outer diameter, $a_o$, and the magnet length, $2b$, are scaled up proportionally until the magnetic field decay time constant, $\tau_B$, is above some minimum acceptable value while the conductor carries a large enough current density to produce the desired magnetic dipole moment, $|M|$. $\tau_B$ is chosen as $1 \times 10^{11}$ s. The outer magnet dimensions remain geometrically
Figure 5-17: Design of a SN2/HTS SCMCMS system with a solid ammonia cooled radiation shield. The heat capacitors are recooled with a detachable cryocooler (not shown).

similar to the outer dimensions of the LTS magnet. The inner wind diameter, \( a_i \), remains unchanged.

The dimensions used to describe this system are indicated in Figure 5-18. There are three independent design variables that must be optimized in order to describe the system with the dimensions that yield the smallest system size for the desired performance: the radiation gap widths, \( g_I \) and \( g_{II} \), and the maximum magnet vessel temperature, \( T_{imax} \). A design code was compiled to optimize the design variables assuming an initial magnet vessel temperature, \( T_{imin} \), of 20 K and an initial solid ammonia temperature, \( T_{imin} \), of 80 K. As with the LHe/LTS SCMCMS system, the code does not consider the mass associated with support attachments, or the mass associated with support structures that would be required to prevent buckling of the cryostat, the magnet vessel, and the radiation shields under high loads. The mass of the support cones and the radiation shield are also neglected because they are expected to be small.

Five functions are involved in the design code. The functions are described in detail in Appendix D. The first function calculates the required magnet dimensions when given \( T_{imax} \) using the criterion described above. The second function calculates the mass, volume, and surface area of the magnet vessel which has inner dimensions...
equal to the outer dimensions of the magnet. It also calculates the inner surface area of the radiation shield. The third function determines the required length of the solid ammonia container, \( l_{II} \), required to yield the specified hold time, \( \tau_{hs} \). The fourth function calculates the cryostat dimensions, system volume, and system weight. The last function is the main function which calls the other functions for various values of \( g_I \), \( g_{II} \), and \( T_{\text{Imax}} \), and identifies the optimum values which lead to the minimum system size for a specified hold time, \( \tau_{hs} \).

All of these functions have been described in Section 5.6.2, except for the function that calculates the magnet dimensions (see Appendix D). The other functions of Section 5.6.2 are easily adapted to this system by replacing the conduction heat leak associated with the fiberglass support straps, \( \dot{Q}_{fs} \), with the conduction heat leak calculated for the support cones, \( \dot{Q}_{sc} \), determined by Equations 5.54 and 5.55. The heat flux through the superinsulation blanket covering the radiation shield is calculated based on the parameters for the heavier superinsulation blanket described in Section 5.9 with \( \bar{N} \) equal to 14.7 layers/cm. The parameters for the lighter superinsulation blanket with \( \bar{N} \) equal to 11.0 layers/cm is used for the blanket covering the magnet vessel.
5.10.1 Minimum Conductor Properties

The design code is used to determine the minimum Bi2223/Ag critical current density (average across the entire conductor cross-section) at 77 K and 0 field, $\lambda J_c(77 \text{ K}, 0, 0)$, that will permit a system mass of 450 kg, which is just 25% larger than the system mass predicted for the LHe/HTS SCMCM system. The same HTS conductor dimensions that were used for the RT bore magnet application are applied. A value of $1.2 \times 10^9 \text{ A/m}^2$ is calculated when an index of 20 is assumed. The optimum $T_{I,max}$ is 45 K, and the maximum solid ammonia temperature is 175 K. The optimum radiation gap widths, $g_I$ and $g_{II}$, are, respectively, 63 mm and 49 mm.

The criterion used for the mass of the SN2/HTS RT bore magnet system was double the LHe/LTS system mass. A closer system mass is obtainable for this application because the current density in the SCMCM LTS magnet is reduced for protection purposes. However, protection should not be as important for the HTS system because HTS conductor has superior stability, as described in Section 1.1.3.
Chapter 6

Conclusions and Recommendations

In this study, the operation of a high-temperature superconducting permanent magnet in thermal communication with a solid heat capacitor was considered. An experimental system was constructed to demonstrate that a permanent HTS magnet could operate stably over the temperature range of 20–40 K when cooled by a mass of solid nitrogen. Additionally, the experiment demonstrated operation of a stand-alone solid nitrogen cooled permanent HTS magnet (SN2/HTS) system. Models were developed to predict the magnetic field decay of the energized magnet and to predict the warming trend of the cold mass. The experimental and predicted warming trends indicate a discrepancy in the heat leak prediction that is within a factor of 2–3, which is typical for the superinsulation system used. The experimental and predicted magnetic field decay, due mostly to index dissipation, also shows discrepancy. However, the best agreement was obtained by considering the index to be dependent on magnetic field. Little work has been published concerning the dependence of Bi2223/Ag index on magnetic field and temperature. The discrepancy in the field decay simulation is almost certainly a result of inaccuracies in the relations used to predict the conductor properties as a function of temperature and field.
6.1 Design Study Conclusions

A design study was conducted to identify the advantages that may be offered by a SN2/HTS system over presently used permanent superconducting magnet systems involving an LTS magnet that is cooled by a liquid helium bath (LHe/LTS). The study reached the following conclusions:

1. A cryogenic system involving two solid heat capacitors, one to provide a temperature less than 40 K, and a second to absorb heat leak to a radiation shield that surrounds the first heat capacitor, may be constructed with a total system size that is comparable to that for a liquid helium system that offers the same hold time. A good choice of heat capacitors is solid nitrogen as the primary, and solid ammonia as the secondary.

2. The greatest advantage to portable high-field permanent magnet systems offered by the SN2/HTS design is the ability to recool the heat capacitors with a detachable cryocooler, thus eliminating the need to handle liquid cryogens.

3. Two more advantages of the solid heat capacitor cooled HTS system include: a) significantly improved thermal stability such that the risk of unexpected quenching is virtually nonexistent, and b) improved dynamic stability as a result of the solid heat capacitor being rigidly fixed within the system (as opposed to a liquid cryogen system where the free surface permits motion of the cryogen that is independent of the system).

4. The critical current and index of currently available HTS conductor is insufficient to permit a system size that is comparable to what may be achieved with a LHe/LTS system. However, HTS conductors are relatively new and their performance has been improving consistently over the past decade. The Bi2223/Ag conductor property values that are required to permit a reasonable SN2/HTS system size are given in Table 5.2 for a range of magnetic field requirements. The table serves as a guideline for determining when the SN2/HTS system should be considered as an alternative to LHe/LTS systems.
6.2 Recommendations for Future Work

This work was an initial assessment of the potential advantages offered by a solid heat capacitor cooled HTS magnet. The conclusions of this assessment indicate questions that should be addressed in order to determine with greater certainty whether portable magnetic systems will benefit from the solid heat capacitor cooled HTS magnet concept. Recommendations for future work that may help answer these questions are summarized below:

1. Present HTS conductor properties are not sufficient for constructing high-field magnets of a size comparable to what may be obtained using LTS conductor. A theoretical assessment of the fundamental physics should be conducted to determine if there is a theoretical maximum for conductor index and critical current for HTS conductors, and how the properties vary with temperature and field.

2. The original magnet supplied for the experiment was wound with a conductor of a lesser quality than what is presently available. The magnet should be replaced to test the persistent-mode performance of the best available HTS conductor.

3. A two-stage solid heat capacitor system should be constructed to experimentally demonstrate that the cryogenic system size may be comparable to that of a liquid helium system that is designed for the same hold time.

4. The detachable cryocooler concept should be demonstrated with the two-stage heat capacitor system.

5. The applications considered in Chapter 5 represent a small fraction of the total number of potential applications for a SN2/HTS permanent magnet system. Design codes should be compiled to analyze other applications, such as: a MAGLEV system without an on-board compressor and a portable superconducting gradiometer for underwater target detection.
Appendix A

Apparatus Part Descriptions

In this appendix, the components of the experimental apparatus are described in detail. Dimensions are given in inches unless otherwise noted. Figure A-1 is an assembly drawing of the experimental apparatus. The system is divided into 6 groups, labeled A–F. The parts in each group are described in the tables. The tables include part identification numbers, part descriptions, and a figure number that refers to machine drawings for some of the more complicated parts. Detailed descriptions of the superinsulation and the instrumentation leads that extend from the room-temperature flange to the cold container were included in Chapter 2.
Figure A-1: Assembly drawing of the experimental apparatus. The system is divided into 6 groups, labeled A–F.
Group A: Room-Temperature (RT) Flange

Table A.1: Group A Part Descriptions

<table>
<thead>
<tr>
<th>Part</th>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>A-2,A-3</td>
<td>machined from 1/2” thick brass plate</td>
</tr>
<tr>
<td>A2</td>
<td>A-2</td>
<td>Ceramaseal™ 10-pin electrical feedthrough, part no. 10094-01-W</td>
</tr>
<tr>
<td>A3</td>
<td>A-2</td>
<td>Ceramaseal™ 5-pair type-E thermocouple feedthrough, part no. 12029-01-W</td>
</tr>
<tr>
<td>A4</td>
<td>A-2</td>
<td>Glass Ionization Gauge Tube, 3/4” Nominal Size</td>
</tr>
<tr>
<td>A5</td>
<td>A-2</td>
<td>1/2” OD SS304 tube, 0.010” wall thickness, 291/2” length</td>
</tr>
<tr>
<td>A6</td>
<td>A-2</td>
<td>1/2” ID copper end cap, 1/32” thick</td>
</tr>
</tbody>
</table>

Figure A-2: Illustration of the room-temperature flange and selected components. The compression fittings that are soldered to the plate are labeled according to the penetration with the diameter given in the square brackets.
Figure A-3: Machine drawing of the room-temperature flange (Part A1) without the compression fittings. The plate is machined from 1/2" thick brass plate. All holes pass through plate.
### Group B: Parts Connected to the Cold Container Flange

**Table A.2: Group B Part Descriptions**

<table>
<thead>
<tr>
<th>Part</th>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
</table>
| B1   | A-4    | upper tube: 1/2" OD SS304 tube, 0.010" wall thickness, 15" length  
      |        | lower tube: 1/8" OD SS304 tube, 0.005" wall thickness, 7 1/4" length, bottom of the tube extends 2" below the bottom of the flange |
| B2   | A-4    | 1/8" OD SS304 tube, 0.005" wall thickness, 20 1/2" length, bottom of the tube extends 2" below the bottom of the flange |
| B3   | A-4    | upper tube: 1/4" OD SS304 tube, 0.005" wall thickness, 13 1/4" length  
      |        | lower tube: 1/8" OD SS304 tube, 0.005" wall thickness, 5 1/4" length, bottom of the tube is aligned with the bottom of the flange |
| B4   | A-4,A-19a | machined from 3/4" brass rod, A=0.500", B=0.125", C=0.625", D=0.375", E=0.188" |
| B5   | A-4,A-19a | machined from 3/4" brass rod, A=0.250", B=0.125", C=0.375", D=0.375", E=0.188" |
| B6   | A-4,A-19a | machined from 3/4" brass rod, A=0.125", B=0.500", C=0.625", D=0.375", E=0.188" |
| B7   | A-4,A-19a | machined from 3/4" brass rod, A=0.250", B=0.500", C=0.625", D=0.375", E=0.188" |
| B8   | -      | 1/2" to 1/4" copper reducer soldered to a 5" length of 1/4" OD copper tubing, opposite end of the tubing is soldered to a 1/4" to 1/8" copper reducer |
| B9   | -      | 1/2" to 1/4" copper reducer soldered to a 1 1/2" length of 1/4" OD copper tubing, opposite end of the tubing is soldered to a 1/4" to 1/8" copper reducer |
| B10  | A-4    | 1/2" OD SS304 tube, 0.010" wall thickness, 4" length, bottom of the tube is aligned with the bottom of the flange |
Table A.2: Group B Part Descriptions (Continued)

<table>
<thead>
<tr>
<th>Part</th>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>B11</td>
<td>A-4,A-5</td>
<td>machined from 1/32&quot; thick copper sheet</td>
</tr>
<tr>
<td>B12</td>
<td>A-4,A-5</td>
<td>machined from 1/2&quot; thick copper plate</td>
</tr>
<tr>
<td>B13</td>
<td>-</td>
<td>2 1/8&quot; OD, 2&quot; ID copper tube, 10&quot; length</td>
</tr>
<tr>
<td>B14</td>
<td>A-4,A-19b</td>
<td>machined from 3/16&quot; copper plate, $A=2.125''$, $B=0.188''$, $C=4.875''$, $D=0.063''$</td>
</tr>
<tr>
<td>B15</td>
<td>A-4</td>
<td>5 2/3 turns of 1/2&quot; OD copper tubing wound over a 35/8&quot; diameter with a turn spacing of 1/2&quot;, 69&quot; total length</td>
</tr>
<tr>
<td>B16</td>
<td>A-4,A-5</td>
<td>Ceramaseal™ single conductor electrical feedthrough, rated 150 A, part no. 2779-01-W</td>
</tr>
<tr>
<td>B17</td>
<td>A-4,A-5</td>
<td>Ceramaseal™ 10-pin electrical feedthrough, part no. 10236-01-W</td>
</tr>
<tr>
<td>B18</td>
<td>A-4,A-5</td>
<td>Ceramaseal™ 5-pair type-E thermocouple feedthrough, part no. 10236-10-W</td>
</tr>
</tbody>
</table>

Figure A-4: Identification of the cold container flange and the parts that are connected to the cold container flange (Group B). The helium coil is coupled to the helium input and the helium output with, respectively, Part B8 and Part B9.
Figure A-5: Machine drawings of selected Group B parts.
Group C: Cold Container

Table A.3: Group C Part Descriptions

<table>
<thead>
<tr>
<th>Part</th>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>A-6</td>
<td>1/16&quot; diameter indium wire, 99.9% purity, 15.1&quot; length</td>
</tr>
<tr>
<td>C2</td>
<td>A-6,A-7</td>
<td>machined from 1/2&quot; thick copper plate</td>
</tr>
<tr>
<td>C3</td>
<td>A-6</td>
<td>5 1/8&quot; OD 47/8 ID copper tube, 10&quot; length</td>
</tr>
<tr>
<td>C4</td>
<td>A-6,A-19b</td>
<td>machined from 3/16&quot; thick copper plate</td>
</tr>
<tr>
<td>C5</td>
<td>A-7</td>
<td>machined from 1/8&quot; thick copper plate</td>
</tr>
<tr>
<td>C6</td>
<td>A-7</td>
<td>activated charcoal inside a stainless steel container</td>
</tr>
</tbody>
</table>

Figure A-6: Identification of the cold container parts and parts that are fastened to the cold container (Group C). The getter (Part C6) is fastened to the plate for mounting the getter (Part C5) which is adhered to the bottom of the cap with epoxy.
Figure A-7: Machine drawings of selected Group C parts.
Figure A-8: Illustration of the lead construction (Group D). The lead parts are described for two different subsets: an individual lead (Figure A-9), and the lead positioner (Figure A-10).
Table A.4: Group D Part Descriptions

<table>
<thead>
<tr>
<th>Part</th>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>A-9,A-11</td>
<td>machined from 1&quot; diameter brass rod</td>
</tr>
<tr>
<td>D2</td>
<td>A-9,A-11</td>
<td>machined from 1&quot; diameter phenolic rod</td>
</tr>
<tr>
<td>D3</td>
<td>A-9,A-11</td>
<td>machined from 2&quot; diameter brass rod</td>
</tr>
<tr>
<td>D4</td>
<td>A-9</td>
<td>welded bellows, 0.54&quot; ID, 1.5&quot; OD, 1.86&quot; free length, 2.25&quot; extended length, 0.50&quot; compressed length</td>
</tr>
<tr>
<td>D5</td>
<td>A-9,A-11</td>
<td>machined from 2&quot; diameter brass rod</td>
</tr>
<tr>
<td>D6</td>
<td>A-9</td>
<td>1/8&quot; diameter copper rod, 15&quot; length</td>
</tr>
<tr>
<td>D7</td>
<td>A-9</td>
<td>1/2&quot; OD, 3/8&quot; ID G-10 tube, 12.5/8&quot; length</td>
</tr>
<tr>
<td>D8</td>
<td>A-9</td>
<td>1/2&quot; OD SS304 tube, 0.010&quot; wall thickness, 21/4&quot; length</td>
</tr>
<tr>
<td>D9</td>
<td>A-9,A-11</td>
<td>machined from 3/4&quot; diameter brass rod</td>
</tr>
<tr>
<td>D10</td>
<td>A-9,A-11</td>
<td>machined from 3/4&quot; diameter brass rod</td>
</tr>
<tr>
<td>D11</td>
<td>A-9</td>
<td>LAIII Multilam™ Socket, 100 A rated current, Multi-Contact™ part no. 02.0005</td>
</tr>
<tr>
<td>D12</td>
<td>A-9,A-11</td>
<td>plug designed to mate with part D11, Multi-Contact™ part no. 05.0010</td>
</tr>
<tr>
<td>D13</td>
<td>A-10,A-12</td>
<td>machined from 3/4&quot; diameter brass rod</td>
</tr>
<tr>
<td>D14</td>
<td>A-10</td>
<td>1/4&quot; diameter brass rod, 23/4&quot; length</td>
</tr>
<tr>
<td>D15</td>
<td>A-10,A-12</td>
<td>machined from 1/2&quot; diameter brass rod</td>
</tr>
<tr>
<td>D16</td>
<td>A-10,A-12</td>
<td>machined from 1/2&quot; diameter brass rod</td>
</tr>
<tr>
<td>D17</td>
<td>A-10,A-12</td>
<td>machined from 1/2&quot; OD, 3/8&quot; ID G-10 tube</td>
</tr>
<tr>
<td>D18</td>
<td>A-10,A-12</td>
<td>machined from 3/8&quot; diameter brass rod</td>
</tr>
<tr>
<td>D19</td>
<td>A-10,A-12</td>
<td>machined from 1/2&quot; diameter brass rod</td>
</tr>
<tr>
<td>D20</td>
<td>A-10,A-12</td>
<td>machined from 1/2&quot; diameter brass rod</td>
</tr>
<tr>
<td>D21</td>
<td>A-10,A-12</td>
<td>machined from 1/4&quot; thick brass plate</td>
</tr>
</tbody>
</table>
Figure A-9: Identification of the parts in an individual lead.
Figure A-10: Assembly drawing identifying the parts in the lead positioner.
Figure A-11: Machine drawings of selected parts in each lead (Group D).
Figure A-12: Machine drawings of selected lead positioner parts (Group D).
Figure A-13: Machine drawing of the aluminum cryostat (Group E).
Group F: Magnet and Superconducting Switch

Figure A-14: Illustration of the magnet construction (Group F). The magnet parts are described for two different subsets: the double pancakes (Figure A-15), and the superconducting switch (Figure A-16).
<table>
<thead>
<tr>
<th>Part</th>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>A-15,A-17a</td>
<td>machined from 3/8&quot; thick G-10 plate</td>
</tr>
<tr>
<td>F2</td>
<td>A-15</td>
<td>3.2 mm (0.126&quot;) wide, 0.23 mm (0.0091&quot;) thick Bi2223/Ag conductor, 1655&quot; length per double-pancake</td>
</tr>
<tr>
<td>F3</td>
<td>A-15</td>
<td>0.125&quot; wide, 0.031&quot; thick, 1.25&quot; long rectangular G-10 spacer</td>
</tr>
<tr>
<td>F4</td>
<td>A-15</td>
<td>0.126&quot; wide, 0.001&quot; thick Kapton™ insulation with acrylic adhesive</td>
</tr>
<tr>
<td>F5</td>
<td>A-14</td>
<td>10-32 threaded brass rod, 4&quot; length</td>
</tr>
<tr>
<td>F6</td>
<td>A-16,A-18</td>
<td>machined from 3/4&quot; thick G-10 plate</td>
</tr>
<tr>
<td>F7</td>
<td>A-16,A-18</td>
<td>machined from 5&quot; OD, 3 3/4&quot; ID phenolic tube</td>
</tr>
<tr>
<td>F8</td>
<td>A-16,A-18</td>
<td>machined from 1/4&quot; thick phenolic plate</td>
</tr>
<tr>
<td>F9</td>
<td>A-16,A-18</td>
<td>machined from 1/8&quot; thick G-10 plate</td>
</tr>
<tr>
<td>F10</td>
<td>A-16,A-18</td>
<td>machined from 1/8&quot; thick G-10 plate</td>
</tr>
<tr>
<td>F11</td>
<td>A-16,A-18</td>
<td>machined from 1/4&quot; thick phenolic plate</td>
</tr>
<tr>
<td>F12</td>
<td>A-16,A-18</td>
<td>machined from 1/32&quot; thick copper plate</td>
</tr>
<tr>
<td>F13</td>
<td>A-16</td>
<td>no. 28 copper wire, 20&quot; long</td>
</tr>
<tr>
<td>F14</td>
<td>A-16</td>
<td>3.2 mm (0.126&quot;) wide, 0.23 mm (0.0091&quot;) thick Bi2223 conductor with a silver/1 at% gold matrix, 2.3 m (90&quot;) long</td>
</tr>
<tr>
<td>F15</td>
<td>A-16</td>
<td>no. 28 type E thermocouple wire, 20&quot; long</td>
</tr>
<tr>
<td>F16</td>
<td>A-16</td>
<td>0.063&quot; wide, 0.063&quot; thick, 0.65&quot; long rectangular polyethylene foam spacer</td>
</tr>
<tr>
<td>F17</td>
<td>A-16</td>
<td>0.063&quot; wide, 0.063&quot; thick, 0.35&quot; long rectangular polyethylene foam spacer</td>
</tr>
<tr>
<td>F18</td>
<td>A-14,A-17b</td>
<td>machined from 1/8&quot; thick G-10 plate</td>
</tr>
</tbody>
</table>
Figure A-15: Identification of the parts in each double-pancake. The 6 double-pancake stack is held together by three threaded brass support rods (Part F5).
Figure A-16: Identification of the parts in the superconducting switch.
Figure A-17: Machine drawings of (a) the coil forms (Part F1) and (b) the bottom flange of the magnet (Part F18).
Figure A-18: Machine drawings of selected superconducting switch parts (Group F).
Figure A-19: Machine drawings of (a) end caps and (b) tube couplings. The dimensions are specified in the tables.
Instrumentation Lead Connectors

The no. 40 nickel chrome wire leads that extend between the room-temperature flange electrical feedthrough (Part A2) and the cold container electrical feedthrough (Part B17) are connected to the feedthrough using nickel alloy connectors, Ceramaseal™ part no. 0821-01-A. The no. 40 type-E thermocouple leads that extend between the room-temperature flange feedthrough (Part A3) and the cold container feedthrough (Part B18) are connected to the thermocouple feedthroughs using Chromel™ connectors, Ceramaseal™ part no. 11259-05-X, and Constantan™ connectors, Ceramaseal™ part no. 11998-01-X.
Appendix B

Apparatus Design Calculations

This appendix includes various thermal and electrical analyses which justify the dimensions used in the design of the experimental apparatus. The system is designed to demonstrate stand-alone operation while the cold container temperature is in the range of 20–40 K. Therefore, the calculations used to predict the performance of the system during stand-alone operation are based on a minimum temperature of 20 K. However, to properly demonstrate and analyze operation at 20 K and above, the system is cooled to 15 K during the recooling and charging procedures to allow time for thermal gradients that arise during these procedures to subside before the system warms to 20 K. Therefore, calculations used to justify the design of the cooling system are based on a minimum temperature of 15 K.

B.1 Calculation of the Minimum Liquid Helium Required to Cool 1.0 kg of Nitrogen to 15 K

In order to demonstrate the advantage of filling the cold container with liquid nitrogen prior to cooling it to 15 K as compared to producing solid nitrogen at 15 K from a room-temperature source of nitrogen gas, an analysis is presented here for predicting the minimum quantity of liquid helium required for accomplishing both tasks. We intend to show that producing solid nitrogen at 15 K from liquid nitrogen requires
significantly less liquid helium consumption. Figure B-1 shows schematic models representing subprocesses of the two processes to be compared.

In subprocess (a), a liquid helium stream flowing through a control volume condenses nitrogen gas from a room-temperature source at atmospheric pressure. The nitrogen gas enters the control volume while the gas still has the ambient properties. The liquid helium stream enters the control volume as saturated liquid and exits at the temperature of saturated liquid nitrogen. The nitrogen bath and the helium stream are considered to be at atmospheric pressure. A more efficient process would use the helium stream to precool the nitrogen gas such that the helium exits the control volume at room-temperature. However, the former model is a more realistic representation of the experimental system used in this project because the thermal resistance between the helium coil and the nitrogen bath is small.

The quantity of helium required to condense the nitrogen gas is calculated using the first law of thermodynamics for an open system, which yields the following relation:

$$m_{He} = m_{N2} \left[ h_{N2}(293\,K) - e_{N2}(77\,K, l) \right] / \Delta h_{He}^{77\,K, l}$$  \hspace{1cm} (B.1)

where $m_{He}$ is the total mass of helium required for condensing a mass of nitrogen, $m_{N2}$, $e_{N2}(77\,K, l)$ is the specific energy of saturated liquid nitrogen, $h_{N2}(293\,K)$ is the specific enthalpy of nitrogen gas at room-temperature, and $\Delta h_{He}^{T_{N2}}$ is the
enthalpy difference of helium between the saturated liquid state and the vapor state at the saturation temperature of liquid nitrogen (77K).

Figure B-1b is a schematic model illustrating the process of cooling a mass of nitrogen from the saturated liquid state to 15K, which is identical to the second subprocess involved in producing 15K nitrogen from room-temperature gas. A control volume surrounds the nitrogen body of mass $m_{N2}$. A steady flow of liquid helium enters the control volume at its saturation temperature. To make full use of the cooling capacity of the liquid that enters the control volume, each differential helium mass element, $dm_{He}$, must exit the control volume at the current temperature of the nitrogen mass, $T_{N2}$. The first law of thermodynamics for an open system in differential form is applied to the control volume:

$$m_{N2}de_{N2} + \Delta h_{He|_{4.2K,l}} dm_{He} = 0 \quad (B.2)$$

where $e_{N2}$ is the specific energy of nitrogen at $T_{N2}$ and $\Delta h_{He|_{4.2K,l}}$ is the difference in specific enthalpy for helium between the saturated liquid phase and the vapor phase at temperature $T_{N2}$. Equation B.2 is rearranged in order to solve for $m_{He}$:

$$m_{He} = m_{N2} \int_{e_{N2}(T_i)}^{e_{N2}(15K)} \frac{de_{N2}(T_{N2})}{\Delta h_{He|_{4.2K,l}}^{T_{N2}}} \quad (B.3)$$

where $T_i$ is the initial temperature of the nitrogen mass.

Equation B.3 is best solved numerically because the energy function for nitrogen is nonlinear. When implementing a code, the temperature range from $T_i$ to 15K is broken up into small $dT$ increments. $de_{N2}$ and $\Delta h_{He|_{4.2K,l}}^{T_{N2}}$ are mapped according to these temperature increments. The integral of Equation B.3 is then easily solved by summing together the quotients of $de_{N2}$ and the corresponding $\Delta h_{He|_{4.2K,l}}^{T_{N2}}$.

The control volume is sealed from the atmosphere while the liquid nitrogen is subcooled, which causes the nitrogen to reside in a two-phase equilibrium with the condensed phase at saturation. The vapor pressure of the condensed phase is small, such that the cooling approximates a constant pressure process and the specific energy
of the nitrogen is very close to the specific enthalpy of the condensed phase. Enthalpy data as a function of temperature for helium and nitrogen are taken from, respectively, [35] and [46] for temperatures at and below 77 K. The specific enthalpy of room-temperature nitrogen gas is taken from [47]. The calculations reveal that 4.0 liters of liquid helium is required to cool 1.0 kg of liquid nitrogen at 77 K to 15 K, while 8.4 liters of liquid helium is required to condense 1.0 kg of nitrogen gas into the saturated liquid state. Therefore, a total of 12.4 liters of liquid helium is required to produce 1.0 kg of solid nitrogen at 15 K from room-temperature gas.

B.2 Expected Cold Container Cooling Trend

The helium circuit used for cooling the nitrogen from 77 K to 15 K in less than 2 hr is shown schematically in Figure B-2. Liquid helium forced through the transfer line thermally communicates with the nitrogen from inside of the helium coil. Helium vapor exits the helium coil into the helium output access tube which leads to a heat exchanger outside of the cryostat where the vapor is warmed to room-temperature.

In determining the approximate rate of cooling that can be expected from this system, some conservative assumptions are made to simplify the calculation. 1) The helium is assumed to enter the helium coil as saturated vapor because the vapor quality of the helium exiting the transfer line cannot be predicted with accuracy. This assumption leads to a conservative estimate as the available cooling of the helium entering the helium coil is underestimated. Additionally, it permits a simpler single-phase analysis. It will be shown in Section B.2.2 that the cooling provided by the latent heat of the helium is small in comparison with the cooling provided by the warming of the vapor phase. 2) The wall temperature of the helium coil is assumed to be uniform and equal in value to the temperature of the nitrogen. The thermal resistance between the copper wall of the helium coil and the solid nitrogen that is frozen to it should be much smaller than the thermal resistance between the helium vapor and the copper wall of the helium coil. 3) The density and viscosity
Figure B-2: Schematic of the helium circuit.
of helium in the room-temperature heat exchanger is assumed to quickly reach the room-temperature values, which leads to an average vapor density inside of the heat exchanger that is lower than the actual. For a fixed pressure drop across a pipe, decreasing the vapor density causes a decrease in mass flow rate through the pipe. Therefore, it is conservative to use the room-temperature density value since it causes the mass flow rate to be underestimated, which in turn will cause the heat transfer rate in the helium coil to be underestimated. 4) The wall of the helium exit tube is assumed to be adiabatic because the wall is thin, has low thermal conductivity, and is insulated on the outer surface. As a consequence, the helium vapor has the same bulk temperature and physical properties when it exits the tube as it does when it enters. 5) Pressure gradients due to gravity and momentum changes are assumed to be negligible because the resistance to flow through narrow passages is typically dominated by wall friction.

The flow in each section of pipe is expected to be turbulent based on calculation of the Reynolds number, \( Re_D \), for each section:

\[
Re_D = \frac{\rho V D}{\mu} = \frac{4\dot{m}}{\pi \mu D}
\]

where \( D \) is the inner diameter of the pipe, \( V \) is the average fluid velocity, \( \rho \) is the density of the fluid, \( \mu \) is the viscosity, and \( \dot{m} \) is the mass flow rate. The pressure drop across a section of pipe, \( \Delta P \), may be related to \( \dot{m} \) by first calculating a friction factor, \( f \), using Petukhov’s formula for turbulent flow [21]:

\[
f = (0.790 \ln Re_D - 1.64)^{-2}
\]

which is valid in the range \( 10^4 < Re_D < 5 \times 10^6 \). \( \Delta P \) is given in terms of \( \dot{m} \), \( f \), and the tube dimensions by:

\[
\Delta P = \frac{8fL\dot{m}}{\rho \pi^2 D^5}
\]

where \( L \) is the length of the section. Pressure drops across expansions and con-
traction at the connections between adjacent pipe sections may also contribute a significant pressure drop. For a compression \[48\]:

\[
\Delta P = K_c \left( \frac{\dot{m}^2}{2 \rho A_s^2} \right) = 0.4 \left( 1 - \frac{A_s}{A_l} \right) \left( \frac{\dot{m}^2}{2 \rho A_s^2} \right)
\] (B.7)

where \(A_s\) and \(A_l\) are the cross-sections of, respectively, the smaller pipe and the larger pipe. For an expansion \[48\]:

\[
\Delta P = K_e \left( \frac{\dot{m}^2}{2 \rho A_s^2} \right) = \left( 1 - \frac{A_s}{A_l} \right)^2 \left( \frac{\dot{m}^2}{2 \rho A_s^2} \right).
\] (B.8)

The mass flow rate, \(\dot{m}\), may be solved for using Equations B.4–B.8 while realizing that the pressure drops across each section of pipe and area change should add to the gauge pressure inside the helium dewar, \(P_d\). It turns out that the flow restrictions caused by the helium output access tube and the room-temperature heat exchanger are large enough that the pressure drops across the remaining sections (the helium transfer line, the helium input tube, and the helium coil) are negligible; regardless of whether the pressure drops in the sections before the helium output access tube are estimated using properties for saturated vapor or saturated liquid. The lengths of tubing which connect the room-temperature heat exchanger to its adjacent components do not contribute significant flow resistances because they are relatively short. Therefore, a reasonable prediction of \(\dot{m}\) may be obtained by assuming the pressure drops across the pipe sections and area changes which come after the helium coil add to \(P_d\):

\[
P_d = \sum_k \Delta P_k + \sum_l \Delta P_l
\] (B.9)

where each section of pipe after the helium coil is denoted by a value of \(k\), and each area change after the helium coil is denoted by a value of \(l\). \(\Delta P_k\) is calculated according to Equations B.4–B.6 and \(\Delta P_l\) is calculated according to Equation B.7 or Equation B.8, depending on whether it is a contraction or an expansion. When solving Equation B.9, appropriate dimensions and fluid property values must be assigned for each section and area change. The fluid properties are strongly related to temperature.
This does not cause a problem when calculating the pressure drop across the room-temperature heat exchanger because room-temperature values are assumed for that section. However, the fluid properties within the helium exit tube depend on the bulk temperature of the vapor exiting the helium coil, $T_{b,\text{out}}$. Therefore, in order to calculate $\dot{m}$, a means of calculating $T_{b,\text{out}}$ is also required.

**B.2.1 Helium Exit Temperature and Cooling Rate**

A heat transfer coefficient, $h_c$, describing the heat flux entering the helium vapor from the wall of the helium coil, $q_w$, is defined such that:

$$q_w = h_c(T_w - T_b) \quad \text{(B.10)}$$

where $T_w$ is the wall temperature and $T_b$ is the bulk temperature of the helium vapor. It is assumed that $T_w$ is constant over the length of the helium coil. On the other hand, $T_b$ will vary with position, $x$, along the length of the helium coil as the heat influx causes the vapor to warm. An energy balance on a fluid element $dx$ long leads to the following differential equation describing the temperature variation of the helium vapor with $x$:

$$h_c(T_w - T_b)\pi D = \dot{m}c_p \frac{dT_b}{dx} \quad \text{(B.11)}$$

where $c_p$ is the specific heat of the helium vapor. There are two acceptable means of solving this equation. The first is to use average values for the fluid properties and solve the equation analytically. However, this may lead to significant error in the solutions because the fluid properties, specifically density, are strong functions of temperature. A better method involves solving the equation numerically, where the helium coil is broken down into $J$ short elements, or nodes, each of length $\Delta x$ as shown in Figure B-3. A Taylor expansion may be substituted for the derivative in Equation B.11:

$$\frac{dT_b}{dx} = \frac{T_{b,j+1}^j - T_{b,j}^j}{\Delta x} \quad \text{(B.12)}$$
Figure B-3: Model for calculating the temperature profile of the helium vapor along the helium coil.

where the superscript \( j \) denotes which node. The following numerical equation results:

\[
T_{b}^{j+1} = T_{b}^{j} + \frac{\pi D h_{V}^{j}(T_{w} - T_{b}^{j})}{\dot{m} c_{p}^{j}} \Delta x
\]  

(B.13)

where \( h_{V}^{j} \) and \( c_{p}^{j} \) are temperature dependent and must be recalculated at each node. An end condition is required to specify the temperature at the input of the helium coil, \( T_{b}^{0} = T_{b,\text{in}} \), which is assumed to be vapor at the saturation temperature. \( \Delta x \) must be chosen small enough such that further reduction of \( \Delta x \) does not change the calculated solution of the vapor temperature profile along the helium coil.

\( h_{c} \) can be estimated based on calculation of a Nusselt number, \( Nu_{D} \), using Gnielinski’s formula:

\[
Nu_{D} = \frac{h_{c} D}{k} = \frac{(f/8)(Re_{D} - 1000) Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}
\]  

(B.14)

where \( k \) is the thermal conductivity of the helium vapor and \( Pr \) is the Prandtl number. Equation B.14 is valid in the range \( 3000 < Re_{D} < 10^6 \). The values of \( Re_{D} \) and \( f \), which are calculated according to Equations B.4 and B.5 respectively, are determined by the temperature dependent fluid properties, and must be recalculated for each node in order to determine \( h_{c} \). With the temperature profile calculated and the vapor temperature at the output of the helium coil, \( T_{b,\text{out}} = T_{b}^{J} \), known, the total cooling by the helium circulation, \( Q_{c} \), is easily determined by considering a control volume.
enclosing the helium coil:

\[ \dot{Q}_c = \dot{m} [h(T_{b,\text{out}}) - h(T_{b,\text{in}})] \]  

(B.15)

where \( h(T_b) \) is the specific enthalpy of helium at temperature \( T_b \). Alternatively, \( \dot{Q}_c \) could be determined by summing the products of \( q^j_w \) and the surface area of each node, \( A_n \):

\[ \dot{Q}_c = A_n \sum_{j=0}^J q^j_w. \]  

(B.16)

**B.2.2 Numerical Simulation**

Calculation of \( T_{b,\text{out}} \) for a given mass flow rate and wall temperature may now be accomplished using Equations B.4, B.5, B.13, and B.14. Additionally, if \( P_d \) is assigned a value, the system of Equations B.4–B.9 may be solved to determine the mass flow rate as a function of \( T_{b,\text{out}} \). Both the thermal and pressure relations depend on \( \dot{m} \) and \( T_{b,\text{out}} \). Therefore, this system must be solved iteratively as described by the following process:

1. An arbitrary value for the helium exit temperature, \( T_{b,\text{out}1} \), is chosen.

2. Viscosity and density of helium vapor is found for a temperature of \( T_{b,\text{out}1} \) and used to determine a value for \( \dot{m} \) from Equations B.4–B.9;

3. \( \dot{m} \) is applied to the thermal analysis of Section B.2.1 to calculate a second value for the helium exit temperature, \( T_{b,\text{out}2} \). Most likely, \( T_{b,\text{out}2} \) will be significantly different from \( T_{b,\text{out}1} \);

4. \( T_{b,\text{out}1} \) is adjusted based on a comparison between \( T_{b,\text{out}1} \) and \( T_{b,\text{out}2} \);

5. Steps 2–4 are repeated until the difference between \( T_{b,\text{out}1} \) and \( T_{b,\text{out}2} \) is negligible.

Using the values determined for \( \dot{m} \) and \( T_{b,\text{out}} \), the total cooling rate may be calculated using Equation B.15. Figure B-4 shows the expected cooling rate, \( \dot{Q}_c \), as
Figure B-4: Predicted cooling rate as a function of the helium coil’s wall temperature for a dewar pressure of 20.7 kPa.

a function of the helium coil wall temperature, $T_w$, for $P_d$ equal to 20.7 kPa (3 psi). The thermophysical properties of helium vapor (including density, viscosity, heat capacity, and thermal conductivity) were taken from [35] where they are tabulated as a function of temperature and pressure. The dimensions used in the simulation for the helium coil, helium output, and room-temperature heat exchanger are identical to those listed in Section 2.1.4.

With the assumption that the temperature of the contents of the cold container is essentially equal to $T_w$, the cooling curve of the system may be simulated using the enthalpy versus temperature relation for the cold container that is described in Section 3.1 and the cooling characteristic shown in Figure B-4. The resulting prediction is shown in Figure B-5. As intended, the expected 77–15 K cooling time is well within the 2 hour target. The simulation predicts that 9.5 l of liquid helium will be consumed.
Figure B-5: Predicted cooling trend of the cold container
The latent heat of vaporization of helium becomes increasingly significant as the helium coil wall temperature decreases. At 77 K, the rate of heat absorption associated with boiling liquid helium at a rate equal to the mass flow rate calculated by the simulation has a magnitude that is just 6% of the rate offered by warming the vapor phase. This increases to 20% when the wall temperature has reached 28 K, so the rate of cooling is significantly underestimated at the lower temperatures. However, the duration predicted for cooling from 30 K to 15 K accounts for a small fraction of the total time required for cooling from 77 K to 15 K; so the assumption that the helium enters the helium coil as saturated vapor should lead to an insignificant overestimation of the total time required to cool the cold container and its contents to 15 K.

Figure B-5 also includes an experimental temperature trace obtained when cooling the system from 77 K using a dewar pressure of 20.7 kPa. The trace is for the thermocouple that is attached to the inner diameter of the magnet (see Section 2.1.5). In this experiment all of the nitrogen present in the cold container is cooled from the liquid state. Nitrogen that might have boiled off during the initial stage of the cooling procedure was not replenished by condensing nitrogen gas, as was described in the procedures of Section 2.2.1. Instead, the cold container was refilled from a liquid nitrogen dewar after the helium transfer line was precooled. Approximately 14.51 of liquid helium was consumed after cooling the nitrogen to 15 K.

The experimental trial required twice as much time and approximately 50% more liquid helium than predicted. Figure 2-16 shows a cooling curve that includes traces for all 5 thermocouples present in the cold container. It is obvious that significant temperature gradients exist in the cold container based on the temperature difference between the thermocouple attached at the inner radius of the magnet, and the thermocouples attached to the copper container and the magnet current port. If significant temperature gradients are present, the assumption that the helium coil wall temperature is equal to the temperature of the rest of the cold container contents overestimates the temperature difference between the helium vapor and the helium coil wall. Therefore, the heat flux into the helium stream is overestimated as may
be inferred from Equation B.10, and the cooling rate is overestimated such that the cooling duration and the helium consumption are underestimated. Additionally, the helium flow rate may have been overestimated due to the assumption that pressure gradients caused by momentum changes are negligible.

B.3 Disconnectable Lead Conductor Thickness

The disconnectable current leads were designed to carry 125 A. This current rating is significantly greater than the critical current of the test magnet conductor to allow the magnet to be replaced by one wound with higher quality conductor if the opportunity arises in the future.

The lead conductor consists of copper rod that spans from the room-temperature current ports to Multilam sockets over a distance of 320 mm. The surface of the copper rod is surrounded by the vacuum environment. The leads are attached only during the magnet charging procedure, and carry current during most of the duration that they are attached. Reduction of the total heat input when the leads are attached is advantageous in reducing the thermal gradients present within the cold container during the charge procedure. Therefore, the conductor diameter is chosen based on the optimum vacuum lead criterion, described in [2], where the heat leak through the leads when they carry current is minimized by selecting an optimum ratio of conductor length, $l$, to conductor cross-sectional area, $A$. The correct dimensions are calculated according to the following relation:

$$\left( \frac{I_t l}{A} \right)_{ov} = \sqrt{\frac{2\tilde{k}(T_l - T_0)}{\tilde{\rho}}}$$

(B.17)

where $I_t$ is the current the lead is designed to carry, $T_l$ and $T_0$ are the temperatures of the ends, $\tilde{k}$ is the average thermal conductivity over the temperature range $T_0$-$T_l$, and $\tilde{\rho}$ is the average resistivity. The resulting heat leak into the cold end, $\dot{Q}_{ce}$ while
the lead carries its rated current, $I_t$, is given by:

$$\dot{Q}_{ce} = I_t \sqrt{2k\tilde{\rho}(T_1 - T_0)}.$$ 

(B.18)

The end temperatures are assumed fixed at 293 K at the warm end, and 15 K at the cold end. For this range of temperature, $\tilde{k}$ is estimated to be 500 W/mK and $\tilde{\rho}$ is estimated to be $1.0 \times 10^{-8} \Omega$m based on data reported in [2]. With $l$ equal to 0.32 m, and $I_t$ equal to 125 A, $A$ is calculated to be $7.66 \times 10^{-6} \text{m}^2$, which corresponds to a round wire with a diameter of 3.1 mm. Copper rod with a diameter of 3.2 mm ($1/8''$) was used because it was the closest readily available size. The heat leak per lead is expected to be 6.5 W when they are connected and carry a transport current of 125 A.

### B.4 Thermal Anchoring of the Current Leads

It was shown in the previous section that a significant heat leak through the disconnectable leads should be expected when the leads are connected during the charging procedure. This heat leak could cause a temperature rise in the magnet due to heat traveling along the highly conductive copper braid that connects the cold container's current feedthroughs to the magnet's current ports. Therefore, the copper braid was thermally anchored to the helium coil by wrapping the braid several times around it as depicted in Figure B-6, so that a liquid helium stream may intercept the heat leak.

The number of times the copper braid is wrapped around the helium coil is justified using the model illustrated in the figure. A control surface is drawn around the cross-section of the helium coil to enclose a single turn of the copper braid. The control surface has a depth into the figure equal to the width of the copper braid. The helium stream is excluded from the control volume. It is first assumed that there is no heat flux in the azimuthal direction (out of the figure). This is a conservative assumption because any heat flux exiting the control volume would only increase the effective cooling of the mass within it. Secondly, it is assumed that the solids
Figure B-6: Illustration of how the copper braid is wrapped around the helium coil and the model used to calculate the heat leak that may be intercepted by the helium vapor.
separating the copper braid from the helium vapor (which includes the copper wall of the helium coil, solid nitrogen, and a layer of Kapton insulation wrapped around the copper braid) contribute a small thermal resistance in comparison to the convective resistance within the helium stream. A significant portion of the braid is separated from the stream only by thin solid regions. As a result, there should be a negligible temperature difference between the helium coil wall and the copper braid at the point they contact; and the rate of heat transfer from the copper braid to the helium stream is determined by the helium coil wall temperature. If the heat flux is not cooling the mass surrounding the control volume, which would only be the case if the copper braid was excessively cooled and the temperature of the copper braid was less than the temperature of the cold container contents, then it must be intercepting the heat traveling along the braid from the disconnectable current leads. Therefore, the amount of heat leak that may be intercepted by the helium stream, \( Q_{\text{int}} \), is estimated to be the product of the inner surface area of the helium coil that is enclosed within the control surface, \( A_{\text{enc}} \), and the average heat flux into the helium vapor from the helium coil surface, \( \dot{q}_w \):

\[
Q_{\text{int}} = A_{\text{enc}} \dot{q}_w. \tag{B.19}
\]

Based on the analysis of Section B.2, the average heat flux from the surface of the helium coil is 670 W/m\(^2\) when the wall of the helium coil is at 20 K, which corresponds to the minimum practical initial operating temperature for the system. This value is a conservatively low estimate of \( \dot{q}_w \) because an increase in the wall temperature caused by the heat leak through the copper braid should only increase the heat flux into the helium stream. It is acceptable to use the average over the entire helium coil because the portions of the helium coil enclosed within the control surface are equally spaced along the entire length of the helium coil.

The control surface for a single turn of the copper braid encloses a total inner surface area of the helium coil of \( 2.09 \times 10^{-3} \, \text{m}^2 \). Setting \( A_{\text{enc}} \) equal to this value and using a value of 670 W/m\(^2\) for \( q_w \) as discussed in the previous paragraph, the total heat leak that may be intercepted from a single wrap, \( Q_{\text{int}} \), is calculated to be 1.4 W using
Equation B.19. The heat leak per disconnectable lead was calculated in the previous section to be 6.5 W. Therefore, the copper braid is wrapped around the helium coil 5 times to ensure that all of this heat may be intercepted. The temperature traces obtained while charging the coil show that the magnet’s current port temperature actually decreased below the initial temperature, indicating that the copper braid is sufficiently heat sinked. However, the magnet was only charged to 22 A, which leads to a smaller heat leak per lead of approximately 3.5 W.

One final calculation is necessary to demonstrate that the heat dissipation caused by the electrical resistance of the copper braids is negligible in comparison to the heat leak through the disconnectable leads. This dissipation is easily estimated once the resistance across a copper braid is calculated. The resistance is estimated using the dimensions of the braid given in Chapter 2 and a value for the resistivity of copper at 40 K [2], which is warmer than the maximum temperature the magnet is intended to be charged at and yields a conservatively high value. The resistance is estimated to be 40 $\mu\Omega$. For a current of 125 A, a resistive dissipation of 600 mW is expected for each copper braid, which is small in comparison to the 6.5 W of heat leak calculated for a single disconnectable lead.

B.5 Design Basis for the Heater Leads

The system was designed to supply a magnetic field while detached from the vacuum pump and all sources of power and refrigeration. The total heat leak into the cold container when the system stands alone with the magnet charged should be minimized in order to maximize the duration of operation from a thermal standpoint. Therefore, the brass heater leads that carry current from the room-temperature flange to the cold container were designed with the intent of minimizing the steady-state heat leak down the leads while the system stands alone.

The conductor thickness was chosen so that the maximum temperature in the leads would not exceed 373 K when they carry a current of 1.5 A for 12 minutes. This current is an estimate of the current necessary to drive the switch superconductor
normal (>105 K) when the cold container is at 20 K, multiplied by a safety factor of 2. The value is based on experiments done at 77 K with the switch container submerged in a liquid nitrogen bath. Since the thermal conductivity of the switch materials and the resistivity of the heater material do not vary significantly between 77 K and 20 K \cite{2}\cite{8}, the heater power is expected to increase by a factor of 3 to account for a factor of 3 increase in temperature gradient. A duration of 12 minutes was chosen because this was the heating duration required in the preliminary experiments to charge the magnet at a relaxed pace, multiplied by a safety factor of 2.

The lead conductor metal was chosen based on the expected steady-state heat leak down the leads as the cold container warms from 20–40 K, when no current is supplied to the switch heater. The heat leak was calculated for a fixed lead length and a conductor thickness dictated by the 373 K maximum temperature criterion. A numerical code was used to simulate the temperature profiles in the leads while they carry 1.5 A, and to calculate the steady-state heat leak when the leads carry no current. Brass was selected after a comparison of the steady-state heat leak with those expected for three other materials already on hand: copper, stainless steel, and Constantan.

\section*{B.5.1 Heat Leak and Temperature Profile Prediction}

Figure B-7 illustrates the model used to predict the heat leak through the heater leads which span from the room-temperature flange to the cold container, $Q_{hi}$, and the temperature profiles along them. The heater lead conductor is modeled as a circular wire of uniform thickness. It is assumed that there is no significant thermal resistance between the metal conductor and the Teflon insulation; and that there are no temperature gradients within the lead in the radial direction. For this condition, the temperature profiles may be solved for using the one-dimensional form of the heat conduction equation:

$$c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + g(x, t) \quad (B.20)$$
where \( c \) is the volumetric heat capacity, \( k \) is the thermal conductivity, \( g \) is the volumetric heat dissipation, \( x \) is the position along the conductor, and \( t \) is time. This equation could be solved analytically using average values for the thermal properties, which are strongly dependent on temperature. A more accurate solution would be obtained if polynomial correlations were used, but the analytical solution would be much more difficult. Instead, a simple numerical code was written to simulate the profiles using a forward-Euler finite difference method.

The numerical model involves breaking up the lead into \( M \) individual nodes of length, \( \Delta x \), as illustrated in Figure B-7. Equation B.20 can be applied to this model by transforming it into a discrete form using Taylor expansions for the partial differentials:

\[
\frac{\partial T}{\partial t} = \frac{T^{i+1} - T^i}{\Delta t}
\]

where \( \Delta t \) is the time step size, and \( i \) denotes which time step. The first term on the right-hand side of Equation B.20 represents the divergence of conduction along the

Figure B-7: Illustration of the model used to predict the heat leak and temperature profiles along the heater leads.
wire and transforms into the following discrete form:

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) = \frac{1}{\Delta x} \left[ k_{m+1/2}^i \frac{T_{m+1}^i - T_m^i}{\Delta x} - k_{m-1/2}^i \frac{T_m^i - T_{m-1}^i}{\Delta x} \right] \quad (B.22)$$

where $m$ denotes which node. Equation B.22 includes a subscript for $k$ denoted as $m \pm 1/2$ which indicates the value of the thermal conductivity on the half-step. If $k_{m\pm 1/2}$ is calculated as:

$$k_{m\pm 1/2} = \frac{2k_m k_{m\pm 1}}{k_m + k_{m\pm 1}} \quad (B.23)$$

then combining Equations B.20–B.23 leads to the following numerical relation for calculating the temperature profiles:

$$T_{m+1}^i = \frac{\Delta t}{c_m^i} \left( g_m^i + \frac{1}{A \Delta x} \sum_j T_j^i - \frac{T_m^i}{R_j} \right) + T_m^i \quad (B.24)$$

where the subscript $j$ denotes the coordinates of adjacent nodes ($j = m \pm 1$), and $R_j$ is given by:

$$R_j = \frac{\Delta x}{2} \left( \frac{1}{A_w} + \frac{1}{A_{ins}} \right) \left( \left( \frac{1}{1/k_{w,m}} + \frac{1}{1/k_{w,j}} \right)^{-1} + \frac{1}{1/k_{ins,m} + 1/k_{ins,j}} \right) \quad (B.25)$$

where $A_w$ is the cross-sectional area of the wire, $A_{ins}$ is the cross-sectional area of the Teflon insulation, and the subscripts of $k$ indicate which material (either wire or insulation) of which node ($m$ or its adjacent node $j$). Equation B.25 was derived while assuming negligible temperature gradients in the radial direction so that the two mediums may be modeled as parallel resistances in the longitudinal direction. $c_m^i$ is calculated based on the volume fractions of conductor and insulation, such that:

$$c_m^i = \frac{1}{A_w + A_{ins}} \left( A_w c_m^i + A_{ins} c_{m,ins} \right) \quad (B.26)$$

The remaining term in Equation B.24 is the heat dissipation term, $g_m^i$, which is
calculated according to the relation for an ohmic resistor:

\[ P_{pi} = \frac{I^2 \rho_m}{A_w^2} \]  \hspace{1cm} (B.27)

where \( I \) is the current passing through the lead and \( \rho \) is the resistivity which varies with temperature.

Two boundary conditions are required to obtain a solution for the temperature profile. In this case, the temperature of the ends are fixed. The upper end is held at room-temperature (293 K), while the lower end which is attached to the cold container is fixed at 20 K. Also, an initial temperature profile is also required. This profile is calculated by simulating the lead while it carries zero current using a linear initial profile. After a sufficient number of time steps, the simulation will reach a steady-state solution where additional time steps produce no noticeable change in the profile. This profile corresponds to the expected profile after the leads have carried zero current for a long time, and should be an accurate representation of the actual temperature profile just before the charge procedure is initiated. When implementing the code, \( \Delta x \) must be made small enough such that the solution converges and further reduction in the size of \( \Delta x \) produces no significant improvement in accuracy. Additionally, \( \Delta t \) must be kept small enough such that the simulation is stable.

The code was used to compare the various candidate metals to be used as the heater lead conductor. Simulations were performed for the various metals assuming the same lead length, heater current, and end conditions. The analysis of each metal began by selecting a diameter that was larger than necessary to maintain the temperature below 373 K at every point along the conductor while it carried the maximum heater current of 1.5 A for 12 minutes. The diameter of the wire was then decreased and the simulation was repeated several times until the simulated steady-state temperature profile showed a temperature that just barely exceeded 373 K at some location along the conductor. The simulation was repeated one final time with the current set to zero to compute the steady-state conduction through the lead while the heater current was off. The heat leak calculated in this final simulation was the
Table B.1: Expected Heat Leak per Lead for Each Candidate Metal

<table>
<thead>
<tr>
<th>Metal</th>
<th>Diameter (mm)</th>
<th>Zero Current Heat Leak per Lead (mW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>0.265</td>
<td>19.3</td>
</tr>
<tr>
<td>Brass</td>
<td>0.515</td>
<td>8.1</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>1.23</td>
<td>8.6</td>
</tr>
<tr>
<td>Constantan</td>
<td>1.12</td>
<td>13.1</td>
</tr>
</tbody>
</table>

basis for comparing the candidate metals. Thermal and electrical properties for all four metals and the Teflon insulation were taken from [2], [6], and [23]. The same insulation cross-section of $1.48\text{ mm}^2$ was assumed for each metal.

Table B.1 lists the diameters and zero current heat leaks predicted for four candidate metals: copper, brass, stainless steel, and Constantan. Brass and stainless steel seem to be almost equal in offering the best performance. The actual leads used in the experiment consist of 4 lengths of 0.254 mm diameter brass wire twisted together, yielding the same cross-sectional area as a single 0.508 mm wire. Using this diameter, the simulation predicts a heat leak of 8.0 mW per lead.
Appendix C

Calculations Supporting the Theoretical Models

C.1 Thermal Time Constant Estimates

The experimental system is intended for demonstrating operation without thermal gradients within the cold container. The condition of nearly uniform temperature is desirable for a practical system as it maximizes the use of the heat capacitor's enthalpy in reducing the warming rate caused by heat leak from the ambient surroundings. The absence of significant thermal gradients is also a necessary condition for comparing the theoretical models with the experimental system’s performance because the temperature measurements made during the experimental trials must accurately represent the temperature of the entire contents of the cold container. In this section thermal diffusion time constants are calculated to support the claim that the temperature gradients are negligible as the cold container warms through the 20–40 K temperature range.

The vertical copper walls of the cold container are analyzed to show that any thermal gradients that arise along their length due to a sudden thermal disturbance (which may accompany the cooling or charging procedures) will vanish within a short duration. A short duration is considered to be less than both the duration between the completion of the cooling procedure and the time when the cold container reaches...
Figure C-1: An infinite slab model is used to calculate the thermal time constant of the cold container walls.

20 K, and the duration required for the temperature of the cold container to rise a significant amount (arbitrarily chosen as 1 K) during its 20–40 K warming. Both of these time lengths are roughly 1 hour.

Figure C-1 illustrates the calculation of a characteristic time that corresponds to the duration required for thermal gradients within an infinite slab to vanish. Initially, the temperature within the slab is uniform and equal to \( T_o \). Then, at some point in time, \( t=0 \), the temperature of the surface is raised to a new value, \( T_s \), and held there. The temperature at every point within the slab gradually rises due to thermal conduction from the surfaces until the temperature within the slab is nearly uniform again at the new temperature, \( T_s \). The evolution of the temperature profiles may be characterized by the Fourier number, \( Fo \), defined as:

\[
Fo = \frac{\alpha t}{L^2}
\]  

where \( \alpha \) is the thermal diffusivity of the slab material and \( L \) is the semi-width of the
slab. The temperature at the midplane of the slab, $T_c$, which is the location with the slowest response, will complete 90% of the transition from $T_o$ to $T_s$ when $Fo$ is equal to 1 [21]. This value yields the characteristic time, $t_c$, that will be used to support the assumption of negligible thermal gradients, such that:

$$t_c = \frac{L^2}{\alpha}. \tag{C.2}$$

Although the walls of the cold container are not infinite slabs, the model still applies if it is assumed that the walls are adiabatic except at their top end. This is a conservative assumption when calculating how long it will take for a thermal gradient to vanish because now the heat can only diffuse by traveling along the wall and not out of the wall as would be the case in the actual system. The slab model is applied to the cold container walls by assuming an initial uniform temperature, $T_o$, and raising the temperature of the top end to $T_s$ at $t=0$. With these conditions, the wall is equivalent to a section of the slab that extends from the surface to the midplane as shown by the dashed line Figure C-1, because the same boundary conditions apply to both the slab section and the cold container wall: 1) there is no heat flow in the directions parallel to the outer surface of the infinite slab just as there is assumed to be no heat flow in the directions perpendicular to the length of the wall; and 2) the surface at the midplane of the infinite slab is adiabatic due to a symmetric temperature profile about the midplane just as the bottom end of the cold container wall is assumed to be adiabatic. Therefore, the characteristic time, $t_{c,w}$, for the bottom end of the wall to complete 90% of the transition from $T_o$ to $T_s$ is calculated as in Equation C.2, but with $L$ replaced by the length of the cold container wall, $L_w$:

$$t_{c,w} = \frac{L_w^2}{\alpha_{Cu}} \tag{C.3}$$

where $\alpha_{Cu}$ is the thermal diffusivity of copper. The value of $\alpha_{Cu}$ at 40 K is used to yield a conservatively high estimate of $t_{c,w}$ because $\alpha_{Cu}$ decreases with increasing temperature and 40 K is the maximum operating temperature. Setting $L_w$ equal to
the length of the outer wall, 0.267 m, and $\alpha_{cw}$ equal to 0.0028 m$^2$/s [2], $t_{c,w}$ for the outer wall is calculated to be approximately 25 s. This is two orders of magnitudes smaller than the 1 hour criterion.

Having determined that the wall temperature should be uniform throughout each of the experimental trials, it remains to be shown that the contents within the walls of the cold container will not support significant thermal gradients for longer than the 1 hour criterion. Solid nitrogen has the smallest thermal diffusivity of all of the materials housed within the cold container. Therefore, a conservative estimate of the thermal time constant for the contents will be made by assuming that the entire contents has a thermal diffusivity equal to that of solid nitrogen.

It is unclear where the solid nitrogen makes a good thermal contact with the cold container walls. When the cold container is filled with liquid nitrogen, the nitrogen mass assumes the shape of an annulus as dictated by the shape of the cold container. During the cooling procedure, the solid nitrogen shrinks by an amount that is considerably larger than all other materials inside the cold container. It seems likely that the nitrogen annulus would tend to compress the inner container wall, leading to a good thermal contact at the inner surface of the nitrogen annulus. Since it was previously shown that the temperature of the cold container walls should be uniform as the cold container warms through the 20–40 K temperature range, it can be demonstrated that all thermal gradients within the cold container should also vanish by calculating the time required for radially directed thermal gradients in the solid nitrogen annulus to vanish.

Figure C-2 illustrates the model with a cross-section of the nitrogen annulus. A thermal time constant for the solid nitrogen annulus will be estimated using the following assumptions: 1) there is excellent thermal communication with the inner cold container wall, 2) there is poor thermal communication with the outer surface of the nitrogen annulus and the outer cold container wall, so the outer surface is modeled as adiabatic, and 3) the annulus is long and rotationally symmetric so that a one-dimensional model may be used. A relation taken from [49] is used to calculate a characteristic time, $t_{c,a}$, for thermal gradients to vanish across an annulus with these
The temperature is described as a function of time and location by the following group of equations:

\[
\frac{T - T_0}{T_i - T_0} = 1 + 2 \sum_{n=1}^{\infty} e^{-\lambda_n^2 F_0} \frac{Y_1(\lambda_n) J_0(\lambda_n R) - J_1(\lambda_n) Y_0(\lambda_n R)}{\lambda_n \Delta_0(R_i) - \lambda_n R_i \Delta_1(R_i)} \tag{C.4}
\]

\[
\Delta_0(R_i) = Y_0(\lambda_n) J_0(\lambda_n R_i) - J_0(\lambda_n) Y_0(\lambda_n R_i) \tag{C.5}
\]

\[
\Delta_1(R_i) = Y_1(\lambda_n) J_1(\lambda_n R_i) - J_1(\lambda_n) Y_1(\lambda_n R_i) \tag{C.6}
\]

\[
Y_1(\lambda_n) J_0(\lambda_n R_i) - J_1(\lambda_n) Y_0(\lambda_n R_i) = 0 \tag{C.7}
\]

\[
F_0 = \frac{\alpha t}{r_i^2} , R = \frac{r}{r_o} \tag{C.8}
\]

where \( r \) is the radial distance from the axis of the annulus, \( T_0 \) is the initial uniform temperature of the annulus, \( T_i \) is a temperature the inner surface is suddenly raised to at \( t=0 \) s, \( r_i \) is the radius of the inner surface, \( r_o \) is the radius of the outer surface, \( \alpha \) is the thermal diffusivity, \( J \) is the Bessel function of the first kind, \( Y \) is the Bessel function of the second kind, and \( \lambda_n \) is determined by Equation C.7. The time required for the outer surface to complete 90% of the transition from \( T_0 \) to \( T_i \), \( t_{c,o} \), is estimated with the first four terms on the right-hand side of Equation C.4 while setting \( r \) equal to \( r_o \), setting the left-hand side to 0.90, and finding the value of \( t \) that satisfies the equation. \( t_{c,o} \) is solved for using a math program.
\( \alpha_{N_2} \) varies significantly over the temperature range 20–40 K. At 20 K, \( t_{c,a} \) is calculated as 2300 s for \( r_i \) equal to 26 mm, \( r_o \) equal to 62 mm, and \( \alpha_{N_2} \) equal to \( 7.4 \times 10^{-7} \text{ m}^2/\text{s} \) [5]. This is within the 1 hour criterion, and close to the time required for all of the thermocouple temperatures to converge after the helium flow is stopped during the cooling procedure, as may be inferred from Figure 2-16. At 40 K, \( t_{c,a} \) is calculated as \( 1.1 \times 10^4 \text{ s} \) for \( \alpha_{N_2} \) equal to \( 1.6 \times 10^{-7} \text{ m}^2/\text{s} \) [5]. This is approximately 3 times the 1 hour criterion, indicating that the temperature on the outer surface of the nitrogen annulus might lag behind by 10% of the temperature change that occurs over a period of 3 hours. Therefore, a maximum temperature variation across the cold container of 1/4 degree kelvin or so might be expected as the cold container warms past 40 K.

C.2 Convective Heat Leak Estimate

The residual gases inside the cryostat give rise to convective heat transfer between the cold container and the room-temperature cryostat wall. [2] lists values for the expected heat flux between two parallel plates at different temperatures and separated by helium gas at a pressure of \( 10^{-5} \text{ torr} \). This data will be used to make a conservative estimate of the heat leak into the cold container by convection. It will be a conservative estimate for several reasons. 1) The residual gases are unlikely to consist mainly of helium, yet of all the gases expected to have remained inside the cryostat, helium leads to the largest estimate of the convective heat transfer. 2) The cryostat pressure was usually less than \( 10^{-5} \text{ torr} \) (closer to \( 10^{-6} \text{ torr} \)), while conduction through gases in free molecular flow decreases linearly with decreasing pressure. 3) The estimate is based on data for the warm plate at 300 K and the cold plate at 4 K, which is less than the minimum cold container temperature of 20 K. A temperature difference that is larger than the actual leads to an overestimate of the gas conduction. 4) The estimate is based on the area of the larger surface, in this case the inner surface area of the room-temperature cryostat wall. 5) The cold container
was wrapped in superinsulation, so there was no gas conduction occurring directly between the cold container and the cryostat wall.

[2] lists a heat flux due to gas conduction, \( \eta \), between a 300 K surface and a 4 K surface of 47 mW/m\(^2\). Multiplying this by the inner surface area of the room-temperature cryostat wall of 0.45 m\(^2\) leads to a heat leak by gas conduction of 20 mW. This value alone is small compared to the total expected heat leak from conduction and radiation of 260–700 mW. Considering the assumptions that were used in calculating this value, the actual heat leak by convection should be significantly less than 20 mW and certainly should be negligible.
Appendix D

Detailed Descriptions of the Design Codes

D.1 RT Bore Magnet Application

Detailed descriptions for selected numerical processes implemented by the design codes are given in this appendix.

D.1.1 Code for Predicting the LHe/LTS System Size

Dimensional symbols used in describing the LHe/LTS system are defined in Figure 5-11.

Process A: Calculation of the Magnet Dimensions

For the LHe/LTS system, the magnet dimensions may be calculated directly from $d_{Rs}$, $B_{zs}$, and the radiation gap widths. For the SN2/HTS system, the maximum magnet operating temperature, $T_{Imax}$, (which is fixed for the LHe/LTS system) is also required.

A1. An arbitrarily small number of turns per layer, $N_t$, is chosen.

A2. $\beta$ is calculated based on $a_i$, $N_t$, and the conductor width, $w_{cnd}$, such that $\beta = b/a_i = (N_t w_{cnd})/(2a_i)$.
A3. $\alpha$ is determined from $\beta$ and the minimum volume criterion curve of Figure 5-10.

A4. The number of layers, $N_l$, is calculated based on $\alpha$, $a_i$, and the conductor thickness, $t_{cnd}$, such that $N_l = (\alpha a_i)/t_{cnd}$. $N_l$ is rounded up to the closest integer and $\alpha$ is recalculated.

A5. $L_{mag}$ is calculated using the method described in Section 5.5.2.

A6. $F$ is calculated from Equation 5.38.

A7. $\lambda j$ is calculated from Equation 5.37. $I_t$ is calculated as $\lambda j w_{cnd} t_{cnd}$.

A8. The field decay time constant, $\tau_B$ is calculated according to the method of Section 5.5.2.

A9. If $\tau_B < \tau_{Bs}$, $N_l$ is incremented by 1 and Steps A2–A7 are repeated. If $\tau_B > \tau_{Bs}$ the magnet dimensions have been determined.

A10. The magnet volume, $V_{mag}$ is calculated according to Equations 5.40 and 5.41. The magnet mass, $m_{mag}$, is obtained using Equation 5.42.

Increasing $N_l$ leads to a larger magnet with a larger $L_{mag}$, a smaller $\lambda j$ that is calculated in Step A7, and ultimately a larger $\tau_B$. Process A will eventually calculate the appropriate magnet dimensions if the specifications are attainable. However, the initial $N_l$ must be small enough to yield a $\tau_B < \tau_{Bs}$ so that the process is repeated at least once.

Process B: Determination of the Remaining System Dimensions.

In this process, $d_{CC}$ cannot be smaller than the outer wind diameter, $2a_o$, calculated in Process A. Arbitrarily small values are chosen for $N_I$, $N_{II}$, and $N_{III}$, which will be denoted as, respectively, $N_{I\text{min}}$, $N_{II\text{min}}$, and $N_{III\text{min}}$.

B1. $N_I$, $N_{II}$, and $N_{III}$ are set equal to, respectively, $N_{I\text{min}}$, $N_{II\text{min}}$, and $N_{III\text{min}}$.

B2. $g_I$, $g_{II}$, and $g_{III}$ are calculated according to Equation 5.29. $t_{CCci}$ is calculated according to Equation 5.21.
B3. \( a_i \) is calculated as \( d_{Bs}/2 + t_{CCci} + g_I + g_{II} + g_{III} \). (The wall of the RT bore has negligible thickness.)

B4. The magnet dimensions are calculated according to Process A.

B5. \( d_{CC} \) is set equal to a value slightly larger the \( 2a_o \).

B6. The hold time, \( \tau_h \), system mass, \( m_{sys} \), and system volume, \( V_{sys} \) are calculated according to Process C described below.

B7. \( d_{CC} \) is adjusted and Step B6 is repeated until \( \tau_h \) is equal to \( \tau_{hs} \).

B8. \( N_{III} \) is adjusted and Steps B2–B7 are repeated until the \( N_{III} \) value that leads to the smallest \( V_{sys} \) or \( m_{sys} \) for the current values of \( N_I \) and \( N_{II} \) is determined.

B9. \( N_{II} \) is adjusted and Steps B2–B8 are repeated until the \( N_{II} \) value that leads to the smallest \( V_{sys} \) or \( m_{sys} \) for the current value of \( N_I \) is determined.

B10. \( N_I \) is adjusted and Steps B2–B9 are repeated until the \( N_I \) value that leads to the smallest \( V_{sys} \) or \( m_{sys} \) is determined. The system dimensions have now been optimized.

**Process C: Calculation of the Hold Time and System Size**

The first law control volume analysis for predicting the heat leak into the cold container is depicted in Figure 5-12. The method of determining the shield temperatures and heat leak into the cold container, \( \dot{Q}_{CC} \), is identical to the method described in Section 5.4.1, except an additional component corresponding to the heat leak through the current lead access tubes, \( 2\dot{Q}_{lat} \), is calculated using Equation 5.14 and added to \( \dot{Q}_{CC} \).

C1. \( l_{CC} \) is calculated using the same geometric parameters that were determined in Process A for the magnet, such that \( l_{CC} = d_{CC}\beta/\alpha \). The internal volume of the cold container, \( V_{CC} \), is calculated.

C2. \( t_{CCwo} \) and \( t_{CCh} \) are calculated according to, respectively, Equations 5.19 and 5.20.
C3. $d_{CR}$, $l_{CR}$, and the shield dimensions are calculated based on $d_{CC}$, $l_{CC}$, $t_{CCc}$, $t_{CCh}$, $g_I$, $g_{II}$, and $g_{III}$, using the dimensional compatibility defined in Figure 5-11.

C4. $t_{CRc}$ and $t_{CRh}$ are calculated according to Equations 5.21 and 5.20, respectively.

C5. The cold container volume, $V_{CC}$, and the inner surface areas of the inner shield, $A_I$, outer shield, $A_{II}$, and the cryostat wall, $A_{CR}$, are calculated.

C6. The volume of the liquid helium, $V_{LHe}$, is determined as $V_{CC} - V_{mag}$. The mass of liquid helium, $m_{LHe}$, is calculated as $\rho_{LHe} V_{LHe}$ where $\rho_{LHe}$ is the density of liquid helium. The volume and mass of the cold container wall materials, $m_{CC}$, is calculated according to Equation 5.22. The mass of the cryostat, $m_{CR}$ is calculated according to Equation 5.22. $m_{sys}$ is determined as $m_{mag} + m_{LHe} + m_{CC} + m_{CR}$.

C7. The shield temperatures and the heat leak into the cold container through the superinsulation blanket and the fiberglass support straps, $Q_{CCins} + Q_{CCfss}$, is calculated by solving Equations 5.23-5.28. Conduction through the support straps is based on a supported mass of $m_{mag} + m_{LHe} + m_{CC}$.

C8. The diameter of the current lead access tubes, $d_{lat}$, is calculated for the value of $I_t$ determined in Process A using Equation 5.6.

C9. The heat leak through the current lead access tubes, $Q_{lat}$, is calculated according to Equation 5.14. The total heat leak into the cold container, $Q_{CC}$, is determined as the sum $Q_{CCins} + Q_{CCfss} + 2Q_{CClat}$ (see Figure 5-12).

C10. $\tau_h$ is calculated as $\rho_{LHe} V_{LHe} h_{vap}/\dot{Q}_{CC}$. 

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D.1.2 Code for Predicting the SN2/HTS System Size

The following steps describe the numerical processes followed by the code used to predict the mass and volume of the SN2/HTS system. Dimensional symbols are defined in Figure 5-13.

Process D: Optimization of the Dimensions and Operating Temperature

This process is the main function for predicting the SN2/HTS system size. Arbitrarily low values must be chosen for $T_{\text{max}}$, $N_I$, and $N_{II}$, and will be referred to as, respectively, $T_{\text{mm}}$, $N_{\text{imin}}$, and $N_{II\text{imin}}$. $d_I$ cannot be smaller than the outer wind diameter, $2a_o$.

D1. $T_{\text{max}}$, $N_I$, and $N_{II}$ are set equal to, respectively, $T_{\text{mm}}$, $N_{\text{imin}}$, and $N_{II\text{imin}}$.

D2. $g_I$ are $g_{II}$ are calculated according to Equation 5.29. $t_{Ici}$ and $t_{IIci}$ are calculated according to Equation 5.21.

D3. $a_I$ is calculated as $d_{Bz}/2 + t_{Ici} + g_I + g_{II}$. (The wall of the RT bore has negligible thickness.)

D4. The magnet dimensions and $I_t$ are calculated based on $T_{\text{max}}$ using a method that is identical to Process A for the LHe/LTS system, except that Equation 3.36 is used to calculate the conductor critical current.

D5. The diameter of the current lead access tubes, $d_{lat}$ is calculated based on $I_t$ and Equation 5.4.

D6. $d_I$ is set equal to a value slightly larger the $2a_o$.

D7. The system mass, $m_{\text{sys}}$, and system volume, $V_{\text{sys}}$, required to provide the specified hold time, $\tau_{\text{hs}}$, is determined by Process E described below. If the value of $l_{II}$ calculated in Process E exceeds some arbitrarily large value, $d_I$ is increased and this step is repeated.
D8. $d_I$ is adjusted and Step D7 is repeated until the $d_I$ value that leads to the smallest $V_{sys}$ or $m_{sys}$ for the current value of $T_{I\text{max}}$ is determined.

D9. $T_{I\text{max}}$ is adjusted and Steps D4–D8 are repeated until the $T_{I\text{max}}$ value that leads to the smallest $V_{sys}$ or $m_{sys}$ is determined.

D10. $N_{II}$ is adjusted and Steps D2–D9 are repeated until the $N_{II}$ value that leads to the smallest $V_{sys}$ or $m_{sys}$ is determined.

D11. $N_I$ is adjusted and Steps D2–D10 are repeated until the $N_I$ value that leads to the smallest $V_{sys}$ or $m_{sys}$ is determined. $T_{I\text{max}}$ and the system dimensions have now been optimized.

**Process E: Calculation of the System Size**

For this process, an arbitrarily small value must be chosen for $l_{II}$ and will be referred to as $l_{II\text{min}}$. An approximate value for $T_{II\text{max}}$ is chosen in order to calculate $d_{II\text{bat}}$.

E1. $l_I$ is calculated using the same geometric parameters that were determined in Process A for the magnet, such that $l_I = d_I \beta / \alpha$.

E2. $t_{Ico}$ and $t_{Ih}$ are calculated according to, respectively, Equations 5.19 and 5.20.

E3. $d_{sh}$, $d_{CR}$ are calculated based on the outer dimensions of the cold container and the radiation gap widths specified in Process D.

E4. $l_{II}$ is set equal to $l_{II\text{min}}$.

E5. $t_{IIco}$ and $t_{IIh}$ are calculated according to, respectively, Equations 5.19 and 5.20. $d_{II}$ is determined from $d_{sh}$ and $t_{IIc}$.

E6. The mass and volume of the cold container and its contents, $m_I$, and of the secondary heat capacitor container and its contents, $m_{II}$, are calculated.

E7. The inner surface areas of the radiation shield and the cryostat wall, $A_{II}$ and $A_{CR}$, are calculated.
E8. The temperature rise over time is simulated to determine $\tau_h$.

E9. $l_{II}$ is adjusted and Step E8 is repeated until the $l_{II}$ value that makes $\tau_h$ equal to $\tau_{hs}$ is determined. If $l_{II}$ exceeds some arbitrarily long length before $\tau_h > \tau_{hs}$, or if $\tau_h > \tau_{hs}$ for any value of $l_{II}$, then the program returns to Step D4.

E10. $l_{sh}$ and $l_{CR}$ are calculated based on the dimensions of the secondary heat capacitor container.

E11. $t_{CRc}$ and $t_{CRh}$ are calculated according to Equations 5.21 and 5.20, respectively.

E12. All of the system dimensions have been determined for the current values of $d_I$, $T_{Imax}$, $N_I$, and $N_{II}$. $V_{sys}$ and $m_{CR}$ are calculated. Finally, $m_{sys}$ is determined as $m_I + m_{II} + m_{CR}$. 
D.2 Mine Countermeasures Application

D.2.1 Code for Predicting the LHe/LTS System Size

Dimensional symbols used in describing the LHe/LTS SCMCM system are defined in Figure 5-16.

Process F: Optimization of the Dimensions

In this process minimum values for $g_I$, $g_{II}$, and $g_{III}$ are chosen as the thickness of the superinsulation blanket contained within the respective gap. The minimum values are denoted as, respectively, $g_{I\text{min}}$, $g_{II\text{min}}$, and $g_{III\text{min}}$. The diameter of the magnet vessel, $d_{MV}$, is fixed equal to the outer wind diameter of the magnet, $2a_v$. $l_{MV}$ is fixed equal to the melt height, $2b$.

F1. $g_I$, $g_{II}$, and $g_{III}$ are set equal to, respectively, $g_{I\text{min}}$, $g_{II\text{min}}$, and $g_{III\text{min}}$.

F2. The system mass, $m_{sys}$, system volume, $V_{sys}$, and the surface areas of the shields, magnet vessel, and the cryostat are calculated using Process G (described below).

F3. The system hold time, $\tau_h$ is calculated by Process H (described below).

F4. $g_{III}$ is adjusted and Step F3 is repeated until $\tau_h = \tau_{hs}$.

F5. $g_{II}$ is adjusted and Steps F2–F4 are repeated until the $g_{II}$ value that leads to the smallest $V_{sys}$ or $m_{sys}$ for the current value of $g_I$ is determined.

F6. $g_I$ is adjusted and Steps F2–F5 are repeated until the $g_I$ value that leads to the smallest $V_{sys}$ or $m_{sys}$ is determined. The system dimensions have now been optimized.

Process G: Calculation of the Masses, Volumes, and Surface Areas

G1. The internal volume of the magnet vessel, $V_{MV}$, is calculated. The volume of liquid helium when the container is full, $V_{LHe}$, is calculated as $V_{LHe} = V_{MV} - V_{mag}$.
G2. $t_{MVc}$ and $t_{MVh}$ are calculated according to Equations 5.19 and 5.20, respectively.

G3. The mass of the magnet vessel, $m_{MV}$, and the mass of the liquid helium contained in the vessel when it is full, $m_{LHe}$, are calculated.

G4. The dimensions of the shields, the inner dimensions of the cryostat, the surface areas of the shields, $A_I$ and $A_{II}$, and the internal surface area of the cryostat, $A_{CR}$, are calculated.

G5. $t_{CRc}$ and $t_{CRh}$ are calculated according to Equations 5.21 and 5.20, respectively.

G6. The mass of the cryostat walls, $m_{CR}$, and the total system volume, $V_{sys}$, are calculated. The total system mass, $m_{sys}$ is calculated as $m_{CR} + m_{MV} + m_{mag} + m_{LHe}$.

**Process H: Calculation of the Shield Temperatures and Heat Leak**

$T_I$, $T_{II}$, and $\dot{Q}_{MV}$ are solved for using the method of Section 5.4.1, with one minor adjustment to the heat balance equations. The conduction heat leak through the support straps, $\dot{Q}_{fss}$, is replaced by the conduction heat leak through the support cones, $\dot{Q}_{sc}$. $\dot{Q}_{sc}$ may be calculated for each shield and the magnet vessel using Equation 5.56, after calculating $t_{sc}$ according to Equation 5.55. An additional component of heat leak is estimated for the current lead access tubes, $\dot{Q}_{lat}$, according to Equations 5.6 and 5.14 after calculating $\dot{Q}_{MV}$ according to the method of Section 5.4.1. $\dot{Q}_{lat}$ is added to $\dot{Q}_{MV}$ and $\tau_h$ is determined as $\rho_{LHe}V_{LHe}h_{vap}/\dot{Q}_{MV}$. The first law control volume model is similar to the one depicted in Figure 5-12.
D.2.2 Code for Predicting the SN2/HTS System Size

Dimensional symbols used in describing the LHe/LTS SCMCM system are defined in Figure 5-18.

Process I: Calculation of the HTS Magnet Dimensions

I1. An arbitrarily small number of turns per layer, $N_t$, is chosen.

I2. $2b$ is calculated based on $a_i$, $N_t$, and the conductor width, $w_{cnd}$.

I3. $a_o$ is determined from $2b$ such that the outer dimensions of the magnet are geometrically similar to the magnet used in the LTS system.

I4. The number of layers, $N_l$, is calculated based on $a_o$, $a_i$, and the conductor thickness, $t_{cnd}$, such that $N_l = (a_o - a_i)/t_{cnd}$.

I5. $L_{mag}$ is calculated using the method described in Section 5.5.2.

I6. The required transport current, $I_t$, is calculated using Equation 5.53 to yield the desired $|M|$.

I7. The field decay time constant, $\tau_B$ is calculated according to the method of Section 5.5.2.

I8. If $\tau_B < \tau_{Bs}$, $N_t$ is incremented by one and Steps I2–17 are repeated. If $\tau_B > \tau_{Bs}$ the magnet dimensions have been determined.

I9. The magnet volume, $V_{mag}$ is calculated according to Equations 5.40 and 5.41. The magnet mass, $m_{mag}$ is determined using Equation 5.42.

Process J: Optimization of the System Dimensions

In this process minimum values for $g_I$ and $g_{III}$ are chosen as the thickness of the superinsulation blanket contained within the respective gap. The minimum values are denoted as, respectively, $g_{I\text{min}}$ and $g_{III\text{min}}$. Arbitrarily low values for $T_{I\text{max}}$ and $l_{III\text{min}}$, denoted as, respectively, $T_{I\text{min}}$ and $l_{III\text{min}}$, are also chosen. The diameter of the
magnet vessel, \(d_I\), is always equal to the outer wind diameter of the magnet, \(2a_o\). \(l_I\) is fixed equal to the magnet height, \(2b\).

J1. \(g_I, g_{II}, T_{I_{\text{max}}},\) and \(l_{II}\) are set equal to, respectively, \(g_{I_{\text{min}}}, g_{II_{\text{min}}}, T_{I_{\text{mm}}},\) and \(l_{II_{\text{min}}}\).

J2. The magnet dimensions are calculated according to Process I (described above).

J3. The system mass, \(m_{sys}\), system volume, \(V_{sys}\), and the surface areas of the shields, magnet vessel, and the cryostat are calculated using Process K (described below).

J4. The system hold time, \(\tau_h\) is calculated by Process L (described below).

J5. \(l_{II}\) is adjusted and Steps J3–J4 are repeated until \(\tau_h = \tau_{hs}\). If \(l_{II}\) exceeds some arbitrarily chosen large value, \(g_I\) and \(g_{II}\) are increased and Steps J3–J4 are repeated.

J6. \(g_{II}\) is adjusted and Steps J3–J5 are repeated until the \(g_{II}\) value that leads to the smallest \(V_{sys}\) or \(m_{sys}\) is determined.

J7. \(g_I\) is adjusted and Steps J3–J6 are repeated until the \(g_I\) value that leads to the smallest \(V_{sys}\) or \(m_{sys}\) is determined.

J8. \(T_{I_{\text{max}}}\) is adjusted and Steps J2–J7 are repeated until the \(T_{I_{\text{max}}}\) value that leads to the smallest \(V_{sys}\) or \(m_{sys}\) is determined. \(T_{I_{\text{max}}}\) and the system dimensions have now been optimized.

**Process K: Calculation of the Masses, Volumes, and Surface Areas**

K1. The internal volume of the magnet vessel, \(V_I\), is calculated. The mass of solid nitrogen, \(m_{SN2}\), is calculated based on the density of the saturated liquid and the volume difference, \(V_I - V_{mag}\).

K2. \(t_{Ic}\) and \(t_{Ih}\) are calculated according to Equations 5.19 and 5.20, respectively. The mass of the magnet vessel, \(m_I\), is calculated.
K3. The internal dimensions of the solid ammonia container and its internal volume, \( V_{NH3} \), are calculated. The mass of solid ammonia, \( m_{NH3} \), is calculated based on the density of the saturated liquid and \( V_{NH3} \). \( t_{IIc} \) and \( t_{IIh} \) are calculated according to Equations 5.19 and 5.20, respectively. The mass of the solid ammonia container, \( m_{II} \), is calculated.

K4. The dimension of the radiation shield, the inner dimensions of the cryostat, the inner surface area of the shields and the cryostat are calculated.

K5. \( t_{CRc} \) and \( t_{CRh} \) are calculated according to Equations 5.21 and 5.20, respectively.

K6. The mass of the cryostat walls, \( m_{CR} \), and the total system volume, \( V_{sys} \), are calculated. The total system mass, \( m_{sys} \) is calculated as \( m_{CR} + m_{I} + m_{II} + m_{mag} + m_{SN2} + m_{NH3} \).

**Process L: Calculation of the Hold Time**

\( \tau_h \) is calculated using the method of Section 5.4.2, with one minor adjustment to the heat balance equations. The conduction heat leak through the support straps, \( \dot{Q}_{fss} \), is replaced by the conduction heat leak through the support cones, \( \dot{Q}_{sc} \). \( \dot{Q}_{sc} \) may be calculated for each shield and the magnet vessel using Equation 5.56, after calculating \( t_{sc} \) according to Equation 5.55 and the appropriate mass quantities. An additional component of heat leak into the magnet vessel is estimated for the current lead access tubes, \( 2\dot{Q}_{Hat} \), according to Equations 5.4 and 5.13, and for the cold bus access tube, \( \dot{Q}_{Ibat} \), according to Equations 5.13 and 5.18. An additional component of heat leak into the solid ammonia container is calculated for the cold bus access tube, \( \dot{Q}_{IIbat} \). The first law control volume model is similar to the one depicted in Figure 5-14.
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