

Gravitation and Phase Transitions  
in the Early Universe

by

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(1977)

SUBMITTED TO THE DEPARTMENT OF  
PHYSICS IN PARTIAL  
FULFILLMENT OF THE REQUIREMENTS  
FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 1982

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Submitted to the Department of Physics on August 6 in Partial  
Fulfillment of the Requirements for the Degree of Doctor of Philosophy

ABSTRACT

An investigation is made of the possibility that gravity is the low energy effective theory resulting from a phase transition at some high energy. I focus on the instabilities of classical gravity in order to build a model for the universe shortly after such a transition which is assumed to be of first order. The dynamics of the evolution of this initial state are investigated in detail, and the implications for such processes as baryosynthesis and monopole production are discussed. Also, the initial state is investigated in detail, with consideration of the validity and possible refinements of the initial approximations, and connection is made to the outstanding horizon, flatness, and cosmological constant problems in cosmology. Finally, I briefly discuss other treatment of gravity which may have implications for phase transitions in the early universe.

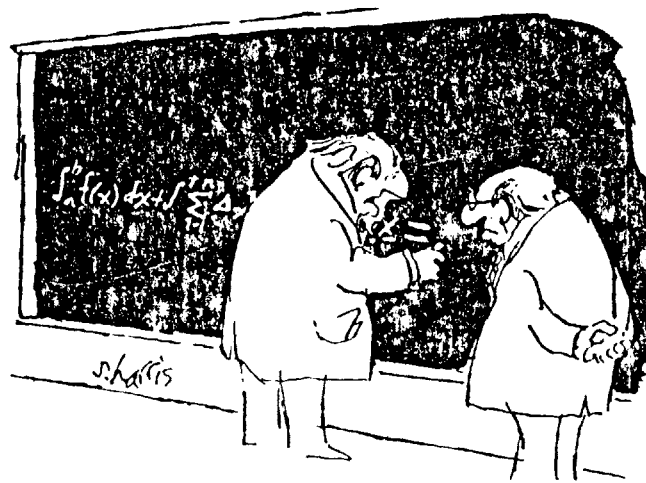
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*"This is the part I always hate."*

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of artist, Sidney Harris*

## Introduction

Science has come closest to theology on a subject of abiding interest to both: the origin and nature of the large scale universe. Indeed it is hard to distinguish at times which field has required greater leaps of faith, or imagination. The last half century has, however, ushered in a remarkable transition in scientific cosmology. Emerging out of an era of primarily unfounded hypotheses, it is now possible to seriously consider constraints which can be imposed on various theoretical models of the first  $10^{-35}$  sec of the big bang expansion. Two factors have made possible these striking developments. On the one hand, vast improvements in the experimental apparatus of observational astronomy have allowed the accumulation of fundamental data, including the discovery and nature of the Hubble expansion and of the  $3^\circ$  black body radiation background.<sup>1</sup> At the same time rapid progress in elementary particle physics has allowed us to extend the regime of validity of the equations of state for matter by many orders of magnitude. If the observed big bang expansion implies the universe was hotter as we look earlier in time, then each new breakthrough in understanding particle interactions at higher energies gives, in principle, a new tool to dig one layer deeper into the cosmological fossil record which the present universe provides.

The situation is complicated, however, by the fact that this fossil record is extremely meager. Evidence indicates that for the period immediately preceding the formation of the objects we observe today (beginning at  $kT_{\text{space}} \sim 1 \text{ MeV}$ ) the universe was

in thermal equilibrium. The equilibrium state at that time was largely insensitive to the detailed dynamics which led to equilibrium. The net result is that many high-energy effects of interest to particle physicists are washed out by equilibrium. The only relevant quantities which survive today are remnants of non-equilibrium processes; such as the baryon to entropy ratio,<sup>2</sup> and inhomogeneities in matter<sup>3</sup> and radiation<sup>4</sup> distributions. That there are very few such quantities limits the applicability of many particle physics calculations for cosmology. However it also provides a challenge to particle physicists to search out developments which may allow a derivation of these fundamental quantities.

Recently the application of the concept of spontaneous symmetry breaking in field theory has provided a framework for models to unify interactions and classify particle types up to energies of the order of  $10^{15}$  GeV.<sup>5</sup> Such energies may have only been achieved in the very early universe. Yet because of the efficiency of modern detection technology, these grand unified symmetries may be measured indirectly in our terrestrial laboratories through such processes as proton decay and neutrino mass measurements.<sup>6</sup> Naturally the development and potential experimental verification of grand unified theories has resulted in a rush of activity to investigate their many implications. One of these is the fact that phase transitions in matter interactions as a function of energy are mirrored by transitions in the dynamics of universe evolution at various critical temperatures. This possibility of non-equilibrium behavior specifically determinable within the framework of

particle physics has, for the reasons described earlier, infused new life into early universe model building by particle physicists to derive the few parameters of observational cosmology.

While the number of such constraints may be insufficient to imply a unique empirical model of the early universe they can be supplemented in order to point out important directions of inquiry. This is because our present, clearly incomplete models of the neo-natal universe contain a number of paradoxes which must be resolved to give an acceptable description of this era. While their "natural resolution" will not be an empirical test, it may further restrict the class of reasonable models. Even if the net result is merely to point out where our ignorance is greatest these combined constraints of cosmology may, as grand unified theories are beginning to demonstrate, provide the only valuable, if not empirical, directions for progress in particle theory.

In this sense one of the most exciting aspects of recent developments is that they motivate consideration of an era where quantum, or semi-classical gravitational effects may become important. The consistent application of quantum mechanics to gravity has presented insurmountable difficulties up to the present time. If we can use the limited developments in this area to probe for significant effects in the cosmological era now under investigation we may gain important new insights for quantum gravity.

For these reasons I consider in this work the relationship between various problems in cosmology and the peculiarities of gravity as a field theory, and focus on the role gravity may



play in early universe phase transitions. While I will briefly discuss potential important effects of gravity in matter phase transitions, I will concentrate on possible phase transitions more intimately tied to the nature of gravity itself. For reasons that will be described in more detail in the following chapter I consider the possibility that classical gravity is low energy effective theory, the remnant of a phase transition. Rather than attempting to find the explicit symmetry breaking which may be responsible for this transition I investigate aspects of the classical theory which may signal the existence of such a transition, which might occur in the early universe. These are then used to build an ansatz for the physics of the transition region, and the implications of this ansatz can then be investigated.

We find that the semi-classical effects on which our transition scenario is based can significantly alter early universe dynamics, leading to novel methods of treating the horizon problems, cosmological constant, and baryosynthesis problems of the standard model, and perhaps avoids the monopole production problems of other phase transition scenarios. While only suggestive, our results indicate the potential importance of the application of early universe studies to our understanding of quantum gravity.

The specific outline of this work is as follows:

Chapter one provides in more detail the cosmological and field theoretic motivations for first order phase transitions in the early universe, and for our hypothesis in particular, and gives an introduction to the standard FRW model, and to the

phenomena of first order phase transitions in field theory.

Chapter two gives an introduction to the instabilities and peculiar thermodynamic behavior of semi-classical gravity which will be used to build a model. Black hole thermodynamics are discussed in some detail.

Chapter three provides a brief overview of the model, its dynamics, and some of its implications.

In Chapter four I give first a detailed review of the model, a derivation of relevant dynamical equations, and a qualitative treatment of their solutions. I then provide a quantitative description of its dynamics based on a numerical evaluation of the equations.

Chapter five then provides a reexamination in some detail of some of the assumptions which went into the description given in the previous chapter. I also describe how several paradoxes of the standard FRW model are treated in the context of this work.

Chapter six is devoted to an investigation of several detailed implications of this scenario. Specifically discussed are baryosynthesis, monopole production, the nature of inhomogeneities, and also implications for supersymmetric scenarios.

Finally, the last chapter briefly outlines other applications of gravity to phase transitions in the early universe, as well as problems and perspectives for future studies of gravity at finite temperatures.

Chapter 1: The Standard Model  
and Phase Transitions in Cosmology and Gravity

1.1 The problem of adiabaticity

The standard cosmological model, a hot big-bang followed by an adiabatic isotropic homogeneous expansion is necessarily incomplete. The model loses predictive power at the inevitable singularity at  $t=0$ , and requires the imposition of ad-hoc physical initial conditions at some time  $t>0$ .<sup>1</sup> Moreover, the initial conditions which must be chosen so that the model agrees with present observations are highly unnatural. To further explain this requires a brief description of the standard Friedman-Robertson-Walker (FRW) model.

An isotropic and homogeneous universe is described in general in terms of comoving coordinates by a metric of the form:<sup>2</sup>

$$d\tau^2 = dt^2 - \frac{1}{c^2} R^2(t) \left\{ \frac{dr^2}{1-\kappa r^2} + r^2 [d\theta^2 + \sin^2\theta d\phi^2] \right\} \quad (1.1-1)$$

where  $\kappa$  is a constant parameterizing closed, open, or flat universes. The particular combination of  $\kappa$  and  $R$  which is physically relevant can be determined by considering Einstein's equations for the scale factor  $R(t)$ , which can be written as

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{\kappa c^2}{R^2} = \frac{8\pi G}{3} \rho \quad (1.1-2a)$$

$$\frac{d(\rho c^2 R^3)}{dt} = -p \frac{dR^3}{dt} \quad (1.1-2b)$$

where  $\rho c^2$  is the total energy density, and  $p$  is the pressure

of matter and radiation. Eq. 2(b) represents the statement of energy conservation. Eq. 2(a) implies that it is only  $\frac{\kappa c^2}{R^2}$  which is physically significant. This is why we are free to rescale  $R$  and  $r$  so that  $\kappa$  takes on the standard values: +1, 0, and -1 for closed, flat, or open universes respectively. It is also assumed that this expansion is adiabatic:

$$\frac{d(sR^3)}{dt} = 0 \quad (1.1-3)$$

where  $s$  is the entropy density.

To solve these we must supplement them by an equation of state  $p(\rho, T)$  for matter. If we assume that the early universe was dominated by radiation in thermal equilibrium ( $\kappa T \gg$  particle masses), then we can use the equations of state for radiation  $\rho \propto T^4$ ,  $s \propto T^3$ , to rewrite Eq. (1)-(3) in terms of temperature. Note first that (3) implies:

$$RT = \text{const} \quad , \Rightarrow \quad \frac{\dot{R}}{R} = -\frac{\dot{T}}{T} \quad (1.1-4)$$

As we shall show shortly, our universe is approximately flat ( $R_{\text{present}}$  is very large), and the approximation  $k=0$  gets continually better as we extrapolate back in time. Hence, for early times we can now solve Eq. (2a) in terms of temperature, neglecting the second term, yielding the relation  $T^2 \sim t^{-1}$ . This implies from (1.1-4) that  $R(t) \sim t^{1/2}$  for a radiation dominated expansion. Using the fact that  $t_{\text{present}} \approx 10^{10}$  years, one finds that the size of the presently observable region of the universe at earlier times is given by  $L(t) \approx 10^{20} t^{1/2}$  cm.

On the other hand, from the metric (1.1-1), the distance light will travel in time  $t$  is given by:

$$l(t) = c R(t) \int_0^t dt' R^{-1}(t') = 2ct \quad (1.1-5)$$

The expressions for  $L(t)$  and  $l(t)$  are graphed on figure 1. From this, we see that the presently observed universe was made up of many ( $\sim 10^{75}$  at  $t \sim 10^{-35}$  sec) causally disconnected volumes at early times. Why these regions should combine to yield an isotropic and homogeneous universe is problematic (the horizon problem).

We now demonstrate why the adiabatic assumption implies that the approximation  $\kappa=0$  becomes better at early times, requiring extremely fine tuning of  $\rho(t)$  at early times. Consider the case  $|\kappa| = 1$  ( $\kappa=0$  is contained in this parameterization when  $R \rightarrow \infty$ ). Eq. (2a) is thus equivalent to:

$$\left| \frac{8\pi G}{3} \frac{(\rho - \rho_{\text{crit}})}{\rho} \right| = \left| \frac{c^2}{R^2 \rho} \right| \quad (1.1-6)$$

where  $\rho_{\text{crit}}(t)$  is the value of  $\rho(t)$  for a flat universe ( $\kappa=0$ ). Now for an adiabatic radiation dominated expansion  $R \sim t^{1/2}$  and  $\rho \sim t^{-2}$ . Hence (6) implies

$$\left| \frac{(\rho - \rho_{\text{crit}})}{\rho} \right| \sim \tilde{C} t \quad (1.1-7)$$

where  $\tilde{C}$  is a positive constant.

Hence by choosing times to that  $\tilde{C}t \ll 1$  the ratio on the left hand side becomes arbitrarily small. When the constant  $\tilde{C}$  is evaluated<sup>3</sup> we find that this ratio becomes vanishingly small even well after the planck time, implying that we must fix an initial

condition  $\rho = \rho_c$  with extreme accuracy (the flatness problem).

Clearly both the horizon and flatness problems stem from extrapolating back the assumption of adiabaticity, requiring very large entropy densities at very early times. In an effort to relax this assumption it is natural to consider ways in which entropy may have been generated in the early universe. The possibility that it arose through dissipative hydrodynamic processes during the expansion seems to have been ruled out.<sup>4</sup> However, during the non-equilibrium period associated with a first order phase transition significant entropy may be generated via latent heat. Such a possibility has been suggested, associated with the breaking of grand unified symmetries.<sup>5</sup> To understand the significance of this suggestion, and because it will prove useful later, I shall briefly review the phenomenology of first order phase transitions in field theory in the next section.

## 1.2 First order phase transitions in the early universe

The well-defined techniques of statistical mechanics for the description of phase transitions<sup>6</sup> can be carried over with few changes to describe phase transitions associated with symmetry breaking in particle physics. Of particular interest here are first order transitions which, in statistical mechanics, are defined as changes of state with discontinuities in various thermodynamic state functions (energy, entropy, etc.) at the transition point, as opposed to second order transitions, where the change of state is continuous. Mathematically, an "order

parameter" is defined which takes on non-zero values in one phase ("non-symmetric" or "ordered" phase) and is zero in the symmetric or disordered phase. First order transitions involve a discontinuous shift in this parameter.

Also relevant is the fact that at the transition point of a first order transition both phases coexist: i.e., both correspond to local minima of their respective free energy function  $\Phi(P,T)$  and thus represent equilibrium (though possibly metastable) states. At the transition point the free energies (thermodynamic potentials) are equal. Since both phases are local minima there exists the possibility for supercooling however. Regardless, when the discontinuous transition occurs, it must take place locally at the interface of two phases. For second order transitions this situation is impossible. Only one phase represents a local minimum at any time. The whole system is either in one state or in the other and the transition describes a singular situation where large-scale fluctuations cause a global transition.

In quantum field theory the role of the free energy is played by an effective potential  $V_{\text{eff}}(\bar{\phi})$  which is a function of a classical field  $\bar{\phi}$  representing the expectation value of a quantum field  $\phi$ .  $\bar{\phi}$  plays the role of an order parameter for the transition. Consider for example the field theory described by the Lagrangian:

$$\mathcal{L} = \frac{1}{2} \partial_\nu \phi \partial^\nu \phi - U(\phi) \quad (1.2-1)$$

To lowest order in  $\hbar$  the absolute minimum of the classical potential

$U(\phi)$  corresponds to the vacuum state of the quantum system and the value of the classical field at this minimum ( $\bar{\phi}$ ) is the expectation value of the quantum field  $\phi$  in this state.  $U(\phi)$  must be altered to take into account both quantum and thermal corrections to (1.2-1), and the quantity which has the above properties to all orders in  $\hbar$  and temperature is called the effective potential  $V_{\text{eff}}(\bar{\phi})$ . These corrections and the calculation of  $V_{\text{eff}}(\bar{\phi})$  are discussed in Appendix I. The symmetries of  $U(\phi)$  determine the symmetries of the Lagrangian (1.2-1), but if  $U(\phi)$  (or  $V_{\text{eff}}(\phi)$ ) has a non-zero absolute minimum, these symmetries may not be possessed by the ground state of the theory (spontaneous symmetry breaking). Moreover, if  $V_{\text{eff}}(\bar{\phi})$  has a local minimum at  $\bar{\phi} = 0$  as well as the absolute minimum at  $\bar{\phi} \neq 0$  (see Fig. 2) then the symmetry breaking may take place via a first-order transition from the metastable symmetric state. Thus  $\bar{\phi}$  acts as an order parameter describing the transition and  $V_{\text{eff}}(\bar{\phi})$  is equivalent to the free energy function in statistical mechanics. Its form as both a function of temperature, and higher order radiative (loop) contributions (see Appendix I) gives the phase structure of the quantum theory.

For example consider the finite temperature one loop effective potential shown in Figure 3(a). There is only one minimum of  $V_{\text{eff}}(\bar{\phi})$  at each temperature, and  $\bar{\phi}_{\text{Min}}$ , the expectation value at this minimum, continuously approaches  $\bar{\phi} = 0$  as a function of temperature. This thus represents a second order transition.

(N.B.: the smooth approach to  $\bar{\phi} = 0$  is due to the fact that

$$M_{\phi} = \left. \frac{d^2 V}{d\phi^2} \right|_{\bar{\phi}=0} \text{ at } T = T_c, \text{ allowing the long range correlations}$$



necessary for such a global transition.)

An alternative possibility, the one on which we shall concentrate, is shown in Figure 3(b). If the system is originally at high temperatures  $T > T_c$  the ground state will be the symmetric minimum at  $\bar{\phi} = 0$ . If the system cools to temperatures  $T < T_c$ ,  $\bar{\phi} = 0$  becomes a metastable local minimum which may decay via quantum tunnelling resulting in a local first order transition. This possibility has recently taken on cosmological significance because the standard big bang model allows precisely these thermal conditions, and because the finite temperature effective potential of a wide variety of realistic grand unified models is of this general form. Moreover, there exists the possibility for substantial entropy generation depending on the decay rate of the metastable state, the decay mechanism, and universe dynamics during the transition.

Both the decay rate and decay mechanism for first order transitions induced by effective potentials of the above form can be determined semi-classically, by extending to field theory methods used to calculate barrier penetration in quantum mechanics,<sup>7</sup> the details of which are presented in Appendix II. The result closely parallels nucleation processes in statistical physics, with quantum fluctuations replacing thermodynamic ones. Bubbles of the phase  $\bar{\phi} = \phi_1$  materialize (via tunnelling) amidst the phase  $\bar{\phi} = 0$ . If it is energetically favorable for them to grow, they will - converting metastable vacuum to true vacuum. The decay rate per unit volume ( $\Gamma$ ) is determined from the re-

lation  $\Gamma = \frac{2}{\hbar} \text{Im } E_0$ , where  $E_0$  is the energy of the metastable state and, using a functional integral formalism, is related to the Euclidean action of the solution to the equations of motion describing the bubble at the instant of its formation. The critical size bubble ( $V_0$ ) is also calculable at least numerically by a variational method.

An example of how this mechanism may lead to substantial entropy production within the framework of an expanding universe is given by the inflationary universe scenario of Guth.<sup>8</sup> If the decay rate of the metastable vacuum is slow compared to universe expansion rate, substantial supercooling may be possible. As the metastable phase supercools its energy density approaches a fixed value related to  $V_{\text{eff}}(0) - V_{\text{eff}}(\bar{\phi})$  (see Fig. 3(b)). Solving Eq. (1.1-2a) for fixed  $\rho_0$  yields the asymptotic relation:

$$R(t) \sim e^{\chi t} ; \quad \chi^2 = \frac{8\pi G}{3} \rho_0 \quad (1.2-2)$$

Thus in this scenario the universe may undergo a non-equilibrium period of exponential expansion during the transition. After the transition is complete, sufficient entropy may have been generated so that the initial pre-transition conditions do not involve the horizon and flatness problems (see Fig. 1). At the moment this scenario has fundamental problems associated with it,<sup>9</sup> but several variants are being considered.<sup>10</sup> It does illustrate however the cosmological appeal of the hypothesis of a first order transition in the early universe. We next consider whether such a transition may be related to the nature of gravity.

### 1.3 Gravity and symmetry breaking

Immediate problems are encountered when attempts are made to quantize classical gravity. Its Lagrangian:

$$\mathcal{L} = \frac{-1}{16\pi G} \sqrt{g} R \quad (1.3-1)$$

contains a coupling constant  $G$  with negative dimensions of mass [ $\sim M_p^{-2}$ ], and thus dimensional arguments imply that it is naively non-renormalizable in perturbation theory. Higher order diagrams become increasingly divergent requiring an infinite number of counterterms in the bare Lagrangian proportional to powers of the curvature tensor and its covariant derivatives. Gravity coupled to matter has other divergences as well (some of which may be of cosmological interest<sup>11</sup>).

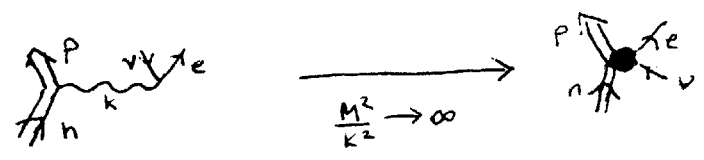
It may be that coupling gravity to matter in specific ways demanded by certain symmetries might allow a cancellation of divergences, as is the hope in supergravity.<sup>12</sup> Alternatively, perhaps these divergences are just a disease of perturbation theory about flat space and non-perturbative or curved space effects may remedy this deficiency. They might, for example, provide some natural physical cutoff at large momenta, thus changing the short distance structure of the theory. This last possibility is particularly plausible in the case of gravitation, which is after all measured purely by its macroscopic effects.

The simplest natural cutoff is provided by new dynamical degrees of freedom which are frozen out at some large mass scale, by symmetry breaking for example, and which leave an effective

non-renormalizable theory at low energies. As can be demonstrated easily using functional integrals, or by considering Feynman graphs, when heavy dynamical degrees of freedom are integrated out of a theory they leave behind at low energies induced effective non-renormalizable terms suppressed by powers of the energy over the mass scale being integrated out. For example the Fermi theory of weak interactions contains a non-renormalizable four fermion coupling (with dimensional coupling  $G_F \sim O(M^{-2})$ ):

$$\frac{G_F}{\sqrt{2}} (\bar{\Psi}_p \gamma_\alpha \Psi_n \cdot \bar{\Psi}_e \gamma^\alpha \Psi_\nu + \text{h.c.}) \quad (1.3-2)$$

This term is induced via heavy intermediate vector boson exchange in a fundamental renormalizable theory with symmetry breaking at high energies:



The diagram shows a transition from a tree-level diagram to a contact term. On the left, a fermion  $\Psi_n$  with momentum  $p$  emits a vector boson with momentum  $k$  and becomes  $\Psi_p$ . The boson is absorbed by a fermion  $\Psi_\nu$  with momentum  $\nu$ , becoming  $\Psi_e$ . An arrow labeled  $\frac{M^2}{k^2} \rightarrow \infty$  points to the right, where the diagram is a contact vertex with four external fermion lines:  $\Psi_n$ ,  $\Psi_\nu$ ,  $\Psi_p$ , and  $\Psi_e$ .

$$\sim \frac{g^2 \bar{\Psi}_p \gamma_\alpha \Psi_n \cdot \bar{\Psi}_e \gamma^\alpha \Psi_\nu}{M_w^2 - k^2} \quad \sim \left[ \frac{g^2}{M_w^2} \right] \bar{\Psi}_p \gamma_\alpha \Psi_n \cdot \bar{\Psi}_e \gamma^\alpha \Psi_\nu$$

$$\sim \frac{G_F}{\sqrt{2}}$$

Another example which will be discussed in greater detail later is the non-linear  $\sigma$  model with  $O(N)$  internal symmetry, and which in many ways resembles the Lagrangian of General Relativity.<sup>13</sup> It can be induced from spontaneously broken linear  $\sigma$  model with the symmetry  $O(N)$  breaking parameter going to infinity. The generating functional (see Appendix I) for the

spontaneously broken linear  $O(N)$  theory is given by:<sup>14</sup>

$$Z_L[\vec{J}] = \int \prod d\vec{\chi}(x) \exp(i \int d^4x [\frac{1}{2}(\partial\vec{\chi})^2 - \frac{1}{4}\alpha(\vec{\chi}^2 - f_\pi^2) + \vec{J}\cdot\vec{\chi}]) \quad (1.3-3)$$

If  $f_\pi^2$  is positive and  $\alpha \rightarrow \infty$  the long wavelength ( $\partial_\mu \vec{\chi} \rightarrow 0$ ) behavior is given by:

$$Z_L(\vec{J}) = \int \prod d\vec{\chi}(x) \prod_x \delta(\vec{\chi}^2 - f_\pi^2) \exp(i \int d^4x [\frac{1}{2}(\partial\vec{\chi})^2 + \vec{J}\cdot\vec{\chi}]) \quad (1.3-4)$$

The delta function fixes one component of  $\vec{\chi}$ , say  $\chi_0$  (i.e.  $m_{\chi_0} \rightarrow \infty$ ), leaving  $(N-1)$  dynamical fields  $\pi(x)$ . Setting  $J_0=0$ , and integrating over  $\chi_0$ , we obtain

$$Z_{NL}(\vec{J}) = \int \prod d\pi(x) \exp\left\{i \int d^4x \left[ \frac{1}{2}(\partial\vec{\pi})^2 + \frac{\lambda_0}{2} \frac{(\vec{\pi} \cdot \partial\vec{\pi})^2}{1 - \lambda_0 \pi^2} + \vec{J} \cdot \vec{\pi} \right] \right\} \quad (1.3-5)$$

(plus other measure-dependent terms which are zero in dimensional regularization).<sup>14</sup> The  $\vec{\pi}$  fields transform non-linearly under  $O(N)$  transformations, and the effective theory (1.3-5) contains the non-renormalizable coupling  $\lambda_0 \sim 1/f_\pi^2$ .

All such non-renormalizable interactions are suppressed by inverse powers of the mass-scale of the fundamental dynamics. (In the above model, even though  $\alpha \rightarrow \infty$ , symmetry restoration is associated with masses  $\sim \lambda_0^{-1}$ ).<sup>15</sup> Thus it is expected that at low energies observable interactions will be renormalizable. Gravitation, however, although it is incredibly weak, involves coherent long-range macroscopic interactions. Since it is only these macroscopic interactions we detect, it is natural to suppose that the non-renormalizable Lagrangian (1.3-1) is an effective one. Indeed, in order to have long-range coherence, any macroscopically detectible interaction must involve exchange

of massless bosons. Moreover, Lorentz invariance alone constrains the interactions of massless spin two particles to satisfy the Principle of Equivalence.<sup>16</sup> The unique such theory at long distances is General Relativity.<sup>17</sup>

If there is strong field theoretic motivation for considering gravity to be an effective interaction, there remains the problem of explicitly determining the fundamental interaction from which gravity is induced. Whether such a fundamental theory can be explicitly deduced merely by knowledge of an effective theory is not clear.<sup>18</sup> The effort most likely must be supplemented by certain guiding principles, such as renormalizability in the case of the weak interactions. I will briefly discuss in the final chapter of this thesis an extension of this idea related to the treatment of the non-linear  $\sigma$  model described earlier which has been suggested for the case of gravity along with renormalization group techniques used to implement the idea. There I will also mention various models which have been proposed as fundamental theories from which general relativity might be explicitly induced.

The body of this work relies on a different approach, however. Presumably at the scale at which a cross-over occurs between the dynamics of general relativity and those of a fundamental theory there is a phase transition. As has been discussed, such transitions at large energy scales have important consequences. I investigate here what physical properties of the classical theory might signal the possible onset of a transitional region, and characterize its physical properties. This heuristic approach has the advantage that we need not know the structure of a high-

energy phase (indeed we may gain as a result some insight into its structure). Moreover we can use the constraints of cosmology to test our hypotheses. Presumably these physical constraints are more reliable than the current prejudices of field theorists.

## 2 Gravitational Instabilities and Thermodynamics

Due to its universally attractive nature classical gravity is beset by instabilities. On the most dramatic scale these instabilities manifest themselves in gravitational collapse, and the formation of singularities in space-time. Indeed, it has been shown that such a singularity in the past is inevitable in space-time under very general conditions.<sup>1</sup> In the semi-classical treatment of gravity, the event horizons associated with gravitational collapse result in interesting and peculiar thermodynamics. It is this thermodynamic structure associated with gravitational instabilities that I believe most likely points in the direction of a possible gravitational phase transition, in ways in which I will describe below.

The peculiar relationship between the instabilities of universally attractive gravity and thermodynamics manifests itself even at the Newtonian level through the Jeans instability.<sup>2</sup> Consider for example a spherically symmetric distribution of gas in equilibrium under its own gravitational field.<sup>3</sup> The outward pressure gradient must equal the inward gravitational force:

$$\frac{dP}{dr} = -n \frac{GM(r)}{r^2} \quad (2.1)$$

where  $n$  is the density and  $M(r)$  is the mass enclosed within a radius  $r$ . Multiplying both sides by  $4\pi r^3$  and integrating by parts we get:

$$\int_0^R 3p \, 4\pi r^2 dr = \int_0^R n \frac{GM(r)}{r} \, 4\pi r^2 dr \quad (2.2)$$



The L.H.S. is  $3\bar{P}V$  ( $\bar{P}$  is average pressure) and the R.H.S. is the negative gravitational energy, yielding the virial condition

$$3\bar{P}V = -\phi_G \quad (2.3)$$

Assuming the average gas pressure is given by an ideal gas at temperature T:

$$P \approx \left(\frac{N}{V}\right)KT \quad (2.4)$$

(2.3) yields (for a uniform density):

$$3NKT \approx \frac{3}{5} \frac{GM^2N^2}{R} \quad (2.5)$$

where N is the number of gas particles (of uniform mass M). Now, for an ideal gas the kinetic energy is  $3/2 PV = 3/2 NkT$ . Thus (2.5) can be rewritten as:

$$2E_k = -\phi_G$$

Hence the total energy is:

$$E_T = E_k + \phi_G = -E_k = -\frac{3}{2}kTN = -\frac{3}{10} \frac{GM^2N^2}{R} \quad (2.6)$$

This expresses the virial theorem identity that for a self-gravitating system the kinetic energy is half the absolute gravitational energy. Hence for such a system (2.6) shows that the lower its total energy the higher its kinetic energy. If the system radiates it gets hotter, and thus has negative specific heat. The total energy of the system decreases as it collapses because the potential

energy decreases twice as fast as the kinetic energy.

This thermodynamic instability is not special to Newtonian gravity. Negative specific heats arise as a feature of general relativity as well - when it is self-consistently coupled to quantized matter systems. This is no accident. In a fundamental way it is tied to the sickness of gravity as a field theory. The Einstein Action is not positive semi-definite, even when continued to Euclidean space (a remnant of the virial condition (2.6)). This instability is reflected in the functional integral formalism of quantum gravity, where in principle we are instructed to integrate over all field configurations. However  $R$  (the curvature) can be arbitrarily large with either sign, causing the integral to be ill-defined. Moreover at finite temperature, the functional integral is defined in terms of a canonical ensemble of states in a heat bath (see Appendix 1). The presence of gravitational states with negative specific heats, even in the classical theory, implies a breakdown of the ergodicity postulate at the basis of this ensemble.<sup>4</sup> At fixed temperatures some classical trajectories (associated with negative specific heats) run into the boundaries of the allowed regions of phase space. Another way of demonstrating this is to recall that in the canonical ensemble the number of subsystems which, in a loosely coupled large system, are in a given energy state  $E_i$ , is proportional to  $\exp(-E_i/T)$ . If the number of energy levels of one of the subsystems between  $E$  and  $E+dE$  is  $\rho(E)dE$  then the probability of the subsystem having energy in this range is  $\rho(E)\exp(-E/T)dE$ . However for certain subsystems with negative specific heat (such

as the black holes we will shortly describe) the entropy as a function energy is such that  $\rho(E) \sim \exp(E^2)$ .<sup>5</sup> At fixed T this grows faster than the thermal factor, so the total probability diverges indicating again a breakdown of the ensemble.

It seems reasonable when investigating the possibility of a phase transition to concentrate on this thermodynamic instability, which is of course related to the dynamic instability of gravitational collapse and the subsequent formation of singularities.

The peculiar thermodynamic behavior associated with formation of event horizons when gravity is semi-classically coupled to quantized matter fields was first investigated by Hawking.<sup>6</sup> Since his discovery, his results have been confirmed and rederived via a vast number of independent and equivalent means. For the purpose of simplicity I will give here the most heuristic derivation.<sup>7</sup> Other methods of obtaining the same result are given in Appendix 3.

Associated with gravitational collapse, and before the formation of a singularity occurs the formation of an event horizon, inside of which the gravitational field is so strong that classically not even radiation can escape to infinity. This surface forms the boundary of a classical black hole, shielding the singularity inside. (Indeed, it is currently hypothesized that all singularities in space-time must be cloaked behind an event horizon<sup>8</sup>). For a non-rotating uncharged hole this surface occurs at the Schwarzschild radius given by  $r \sim \frac{2GM}{c^2}$ . The area of the event horizon is thus given by:

$$A = 16\pi \frac{G^2 M^2}{c^4} \quad (2-7)$$

The "no hair theorem"<sup>9</sup> asserts that the only properties one can ascertain about a black hole a macroscopic ones associated with the existence of long-range fields (i.e., its mass, charge, etc.). Hence no details about the specific particles states inside the black hole are available, even though geodesic completeness implies this region must be considered as part of space-time, implying an inherent entropy associated with the black hole event horizon.<sup>10</sup> Bekenstein<sup>11</sup> was the first to point out that the relation:

$$d(Mc^2) = \frac{c^6}{16\pi G^2 M} dA + B dJ + C dQ \quad (2-8)$$

relating the change in equilibrium energy of black holes to changes in the area of the event horizon  $A$ , and changes in its angular momentum  $J$ , and charge  $Q$ , ( $B$ ,  $C$  are constants which do not interest us here), is similar to the first law of thermodynamics:

$$dU = Tds - PdV \quad (2-9)$$

This relation between event horizon and entropy becomes especially suggestive when one recognizes that classically the area of the event horizon of a black hole can only increase.

This, combined with the fact that two black holes which collide will merge and thus the entropy of the final hole should be greater than or equal to that of the original separate holes suggests the relation  $S = \gamma A$  (where  $\gamma$  is a constant to be deter-

mined). Classically, the number of possible configurations inside a black hole of mass  $M$  (horizon area  $A$ ) would be infinite due to the possible presence of an indefinitely large number of massless particles. However, for the same reason that quantum mechanics cures the infinities in the distribution of radiation in a box, the fact that the Compton wavelengths of particles might be restricted to be less than the radius of the black hole reduces the number of possible internal configurations to be large but finite. An explicit computation<sup>12</sup> yields  $\gamma = \frac{kC^2}{4GM}$ , giving:

$$S = \frac{kC^3 A}{4GM} = \frac{4\pi k G M^2}{kC^3} \quad (2-10)$$

Given an entropy in terms of energy we use the thermodynamic relation  $T^{-1} = \left(\frac{\partial S}{\partial E}\right)_{J,Q}$  to yield

$$kT_{\text{hole}} = \frac{kC^3}{8\pi GM} \quad (2-11)$$

This result implies that black holes have associated with them a finite temperature, and they thus radiate particles with a thermal spectrum. Moreover (2-11) indicates that black holes too have negative specific heat - as they radiate they become hotter.

The fact that black holes radiate, implies that under certain conditions they can exist in equilibrium with radiation. Clearly for the reasons described earlier, stable isothermal equilibrium is impossible. However in a box of fixed volume with fixed total energy  $E$ , equilibrium is possible if:

$$(a) \quad T_{\text{Hole}} = T_{\text{Rad}} \quad (b) \quad \frac{\partial T_{\text{Rad}}}{\partial E_{\text{Rad}}} > - \frac{\partial T_{\text{Hole}}}{\partial E_{\text{Hole}}}$$

Condition (a) implies that  $S_{\text{TOT}}$  is maximized subject to  $E_{\text{Hole}} + E_{\text{Rad}} = E$ , while (b) implies that if the black hole momentarily emits more radiation than it absorbs, the temperature of radiation must increase more than that of the black hole. Otherwise, due to its negative specific heat, the black hole would continue to get hotter and radiate faster.

Constraints (a) and (b) together with the relations  $T_{\text{Rad}} \propto \alpha_1 E_{\text{Rad}}^{1/4}$  and  $T_{\text{Hole}} \propto \alpha_2 E_{\text{Hole}}^{-1}$  imply that for equilibrium:

$$\begin{aligned} \frac{1}{T_{\text{Rad}}} \frac{\partial T_{\text{Rad}}}{\partial E} &> - \frac{1}{T_{\text{BH}}} \frac{\partial T_{\text{BH}}}{\partial E} \\ \Rightarrow \frac{1}{T_{\text{Rad}}} \frac{k}{4} E_{\text{Rad}}^{-3/4} &> \frac{k'}{T_{\text{BH}}} E_{\text{BH}}^{-2} \\ \Rightarrow \frac{1}{4E_{\text{Rad}}} &\geq \frac{1}{E_{\text{Hole}}} \Rightarrow E_{\text{Hole}} \geq 4E_{\text{Rad}} \end{aligned} \quad (2-12)$$

This result is independent of the constants  $\alpha_1$ ,  $\alpha_2$  and hence is independent of the number of helicity states which make up the radiation. Note also that for a given mass black hole this provides a constraint on the volume in which it can remain in equilibrium. This volume must be small enough so that the total energy of radiation is less than 1/4 the energy of the hole. As long as  $V$  is less than this critical volume  $V_c$

(and greater than the Schwarzschild volume) the most probable state of a system with total energy  $E$  will be that of a single black hole surrounded by radiation at the same temperature. Alternatively, if one raises the energy density in a fixed volume the equilibrium state will eventually be that of a black hole and radiation, coexisting at a temperature less than that of pure radiation with the same total energy. This is illustrated in Figure 4.

This instability under the formation of black holes can also be demonstrated in a fixed temperature canonical ensemble. Among the total ensemble of states the system can occupy at fixed temperature will be states involving black holes at that temperature. Thus at least part of the time we would expect any finite temperature system to evolve such a state. Of course once formed this state will be unstable against evaporation or accretion of matter, unless the heat bath keeping the system at fixed temperature is removed. What is particularly interesting is that such a state can form not only by thermal density fluctuations, but by semi-classical quantum tunnelling effects.<sup>13</sup> The Schwarzschild black hole represents a solution to the Euclidean equations of motion with period  $B = 1/T$  in imaginary time, and hence gives an instanton contribution to the finite temperature Euclidean functional integral which gives the ground state free energy of the theory (see Appendices I and II). Moreover, the Gaussian fluctuations about this saddlepoint involve negative modes,<sup>14</sup> giving an imaginary part to the free energy. As described in Appendix II, this yields a decay rate for nucleation of black holes from hot "flat"

space via quantum tunnelling effects.<sup>14</sup>

As we have discussed above these semiclassical thermodynamic instabilities are inherent to the description of gravity coupled to matter. In addition, the above arguments suggest that they should play an increasingly important role as energy densities and temperatures are increased, and are thus especially relevant to considerations of the early universe. If then, as we suspect, gravity is a low energy effective interaction these instabilities are likely a reflection of that fact and we might hypothesize that they play a dominant role in the region near a first order phase transition involving gravity. Based on the properties described above, black hole thermodynamics is ideally suited to the formulation of a model for such a transition region. Using such a model we will investigate the justification and implications of this hypothesis in the subsequent chapters.



### 3. A Model for Dynamics Near a Gravitational Phase Transition

#### 3(a) Initial State

If black hole configurations become more probable as energy densities and temperatures are increased, we may imagine that such configurations were more probable in the early universe. There has already been speculation that density fluctuations during the early periods of the Robertson-Walker expansion resulted in an abundance of primordial black holes.<sup>1</sup> We consider here however the possibility that such black holes were already present in the initial state from which the Robertson-Walker expansion resulted.

In the standard cosmological model there is a natural justification for assuming black hole configurations to be present in the initial state, based on the behavior of particle horizons at early times. In the FRW radiation dominated model the horizon volume  $\ell^3 \sim t^3$  and the local energy density  $\rho \sim t^{-2}$ . Hence the total energy contained in a horizon volume goes as  $\ell^3 \rho \sim t$ . We also know the radiation temperature goes as  $T \sim t^{-1/2}$ . The mass of a black hole which radiates at the temperature is  $M \sim T^{-1}$ . Hence the mass of black holes which could be in equilibrium with the radiation goes as  $M \sim t^{1/2}$ . Thus the ratio of black hole energy at this temperature to the energy per horizon volume goes as  $t^{-1/2}$ . Once this ratio (with the proper numerical factor in place) gets larger than 4/5 then, based on earlier arguments about black hole equilibrium<sup>2</sup>, a state in each horizon volume involving one black hole surrounded by radiation becomes the favored

equilibrium configuration.

Hence the "natural" condition at early times in the Robertson-Walker expansion involves a black hole radiation mixture. Indeed as the above argument suggests, if this initial state arose as the result of a local physical process (e.g., tunnelling), black hole configurations would likely be favored.

Any first order transition involving gravitation is such a process. While we have concentrated here on the possibility of a transition to semi-classical gravity as a low energy effective theory, one should note that these remarks will be relevant to any transition for which gravity plays a dynamical role (i.e. the effective potential is a function of the metric as a dynamical field).

There are several reasons one might associate the presence of black holes to a first order transition. First, we remark that as a black hole radiation "gas" evolves into a state involving pure radiation alone, its specific heat goes from a negative value (dominated by the black holes) to a positive value. This is reminiscent of canonical behavior near a first order transition. Such a turnaround in the thermodynamic behavior in a quantum system, has been linked before to the possible onset of a transition.<sup>3</sup>

In addition there is a natural mechanism for the production of black holes in first order transitions. Based on the field theory description of such transitions<sup>4</sup>, they may be described

by nucleation of bubbles of one phase amidst the surrounding the metastable phase. This is the mechanism by which quantum tunneling allows the expectation value of a quantum field to relax to the absolute minimum of the effective potential. After such bubbles form they evolve classically with the fields inside to relaxing to their equilibrium expectation values. If, during bubble formation, the energy density and size of the bubble are such that a metric configuration which involves a black hole surrounded by radiation at a temperature  $T$  is classically favored, we would expect such a field configuration to arise inside the "bubble" as a result of the tunneling process. After the bubbles have occupied all of space, completing the transition we are left with the remnant black hole "gas" in local equilibrium with radiation which we may take as the initial state for the Robertson-Walker expansion.

The actually decay mechanism and decay rate cannot be calculated until we have a fundamental quantum theory whose effective potential describes the transition. A calculation using the classical gravitational Lagrangian (supplemented by surface terms) of the instability of hot flat space to nucleate black holes done by Gross et al.<sup>5</sup>, gives us some confidence that the time reversed picture of this process supports our assumptions. Of course, calculations using the low energy (classical) theory are only suggestive, and strong quantum effects which presumably govern any gravitational phase transition will, drastically affect decay rates, etc. We thus take the above initial state as an ansatz, and investigate its consequences (i.e. we consider time as

measured from the moment this state is formed--see Section 4.3 for a more detailed discussion of this point). My recent letter (see next section) gives a preliminary view of this investigation. The quantitative results are described here in the sections which follow.

GRAVITATION, PHASE TRANSITIONS AND THE BIG BANG

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We introduce a model of the early universe based on the possibility of a first order phase transition involving gravity, and arrived at by a consideration of instabilities in the semi-classical theory. The evolution of the system is very different from the standard FRW big band scenario, indicating the potential importance of semi-classical finite temperature gravitational effects. Baryosynthesis and monopole production in this scenario are also outlined.

\*This work is supported in part through funds provided by the U.S. DEPARTMENT OF ENERGY (DOE) under contract DE-AC02-76ER03069

Submitted to Physical Review Letters

May, 1982  
CTP #994

The rapprochement between particle physics and cosmology can not be complete until quantum gravity is fully understood, when it will be possible to quantitatively trace the big bang to times  $t_{\text{planck}} (=5.4 \times 10^{-44} \text{ s})$ . Developments in particle theory however have motivated a consideration of periods shortly thereafter. Not only might one explain such fundamental quantities as the observed baryon to photon ratio,<sup>1</sup> but the early universe may have undergone phase transitions during which its dynamics may have differed greatly from that of the adiabatic Robertson-Walker model.<sup>2</sup> Thus the early universe can serve as a laboratory in which to test our models of particle interactions at high energies. In particular, the resolution of various problems of cosmology may be tied to understanding the peculiarities of gravity as a field theory.

The model we present, based on treating classical gravity as a remnant of a phase transition, is somewhat speculative and preliminary, but illustrates several important aspects of such an approach: (1) The attempt to couple quantum mechanics and general relativity is strongly tied to thermodynamics. Resulting effects will be important in the early universe, and need further investigation. (2) Quantum, or semi-classical, gravitational effects may be relevant at temperatures below the planck temperature.

Specifically our model indicates that space may never have been hotter than the critical temperature for restoration of Grand Unified gauge symmetries. At the same time it may be possible to generate the observed baryon excess while suppressing monopole production. We here briefly outline these results, leaving more detailed discussions to a future paper.

Although they present some problems, first order transitions

may play a crucial role in early universe dynamics, perhaps resolving several paradoxes of the standard FRW adiabatic model. Indeed, given the possibility that baryon number may not be conserved all the observed matter and entropy of the present universe may have been generated in such a transition.<sup>2</sup> Thus the big bang explosion itself may have been the result of a first order phase transition. In an earlier article I<sup>3</sup> suggested that it may be feasible to connect such a possibility to the nature of classical gravity. The gravitational Lagrangian with its dimensional coupling  $K = \frac{1}{16\pi G} \sim 0 (m_{\text{planck}}^{-2})$  has the form of a non-renormalizable low energy effective interaction in an expansion in inverse powers of a large mass scale at which some heavy degree of freedom is frozen out. In this sense it resembles the Fermi weak effective Lagrangian. Also, Weinberg demonstrated on general grounds that any such effective interaction, in order to have detectable macroscopic effects at large distances might reasonably have long range dynamics governed by a Lagrangian like that of gravity.<sup>4</sup>

Whether it is possible to explicitly deduce from an effective theory the existence of a transition and the nature of a fundamental high energy theory is not clear, although renormalization group techniques may offer some possibilities.<sup>5</sup> A more intuitive approach involves investigating the classical theory for instabilities which may signal the onset of a transition and may characterize the relevant physics of the transition region. This is the approach of the present work. I thus produce an ansatz for the physics of a state immediately following a transition to a vacuum effectively describable by a semi-classical coupling of gravity to quantized matter fields. It is then possible to evolve this state using the equations of general

relativity, in order to investigate alternative early universe behavior and relevant semi-classical gravitational effects therein.

Classical gravity is beset by instabilities. Even Newtonian gravity involves the Jeans instability.<sup>6</sup> In General Relativity instabilities lead to gravitational collapse and the formation of singularities in space time,<sup>7</sup> which are particularly relevant for studies of the early universe as they indicate points where the predictive power of the classical theory breaks down. If such naked singularities are cloaked behind an event horizon<sup>8</sup> this results in the formation of black holes (BH's). Since such singularities imply the incompleteness of the classical theory the formation of associated BH's may be important in the region of a gravitational phase transition. Indeed, if classical gravity is self-consistently coupled to quantized matter fields, BH's exhibit thermodynamic behavior relevant to the description of a transition. Associated with their finite event horizon, BH's have finite entropy:<sup>9</sup>

$$S = kc^3(4Gk)^{-1}A_H = 4\pi kGM^2/kc \quad (1)$$

where  $A_H$  is the area of the event horizon. Thus BH's radiate at a temperature:

$$T_H^{-1} \approx \partial S / \partial E_H = (\pi c^3 / 8\pi k G M)^{-1} \quad (2)$$

and thus have negative specific heat.

It can easily be shown<sup>10</sup> that this implies that BH's can exist in equilibrium with radiation in a box with fixed total energy if:

$$T_H = T_{space}; \quad \partial T_{space} / \partial E_{space} > \partial T_{hole} / \partial E_{hole}$$

For radiation,  $T_{space} \propto E_{space}^{1/4}$  and using (2) this then implies

$$4 E_{space} \leq E_{hole} \quad (3)$$

Thus if one raises the energy density in a fixed volume the equilibrium state will eventually be that of a black hole and radiation at a temperature which is less than the equilibrium temperature of pure radiation with the same energy density. [For a similar result, using a fixed temperature ensemble see Ref. 6].

This suggests that BH configurations should become more important in the early universe, where energy density and temperature are increased. Moreover, a "black hole gas" would have non-standard thermodynamic properties reminiscent of a system near a first order transition, being dominated by the negative specific heat of the BH's [Note: The possibility of an abundance of primordial BH's has been considered elsewhere for other reasons.<sup>11</sup>]

Let us next consider how such a state may arise out of a first order transition. Based on semi-classical calculations in model field theories,<sup>12</sup> such a transition occurs locally at random sites via the nucleation of "bubbles" of fixed size and energy density which then evolve classically until the phase transition to a new equilibrium state is completed via percolation. If the transition is to a state described by semi-classical gravity coupled to quantized matter, and if the bubble size and energy density are within the proper range, then the state which is tunnelled to inside the bubbles will involve a BH surrounded by radiation.

We will assume here that such a situation describes to some approximation the universe shortly after a transition. After it

is completed we are left with a remnant "gas" of BH's with a mean mass and volume per hole (with which each hole is in thermal contact). While a fundamental theory is needed to calculate the parameters of such a transition, we can take this ansatz as an initial state condition and investigate its consistency and the consequences of its evolution in time.

The pre-tunnelling state may have had an arbitrary long time to relax into a metastable equilibrium (as we have dispensed with the big bang as the origin of time). Then, if the nucleation rate is sufficiently fast we may imagine that on a scale large compared to the volume per hole that the universe is sufficiently isotropic and homogenous to describe its evolution by the Einstein equations for the Robertson Walker metric scale factor  $R(t)$ :

$$(\dot{R}/R)^2 + \kappa c^2/R^2 = 8\pi G\rho/3 \quad (4)$$

$$d(\rho c^2 R^3)/dt = -p dR^3/dt \quad (5)$$

where  $\rho c^2$  is the total energy density, and  $p$  is the pressure and Eq. (5) represents the statement of energy conservation. To solve these we must supplement them by an equation of state  $p(\rho, T)$  for a BH-radiation mixture. Assuming the standard equation of state for radiation ( $p=\rho/3$ ), and that BH's act like massive dust particles ( $p=0$ ) (5) implies:

$$\dot{\rho}_{\text{TOT}}/\rho_{\text{TOT}} = -\left(3 + \rho_{\text{space}}/\rho_{\text{TOT}}\right) \dot{R}/R \quad (6)$$

where we will hence refer to the quantity in brackets as  $K(t)$ , which smoothly goes from the matter value  $K(t)=3$  to that of radiation,  $K(t)=4$  as the BH's decay.

One can also show that the universe expansion, combined with Eq. (2) implies that BH's lose mass at a rate:

$$\dot{M}/M = -3.8 \times 10^{15} N/M^3 + 2.0 \times 10^{-15} M \rho_{\text{space}} \quad (7)$$

(in MKS units) where  $N$  is the standard helicity factor dependent on the number of massless fermionic and bosonic degrees of freedom;  $N = \frac{1}{2} (N_B + \frac{7}{8} N_F)$ .

Then, using Eqs. (4) (choosing  $\kappa=0$ ) and (6), we have

$$\dot{\rho}_{\text{TOT}} = -K(t) \sqrt{\kappa} \rho_{\text{TOT}}^{3/2}; \quad (\sqrt{\kappa} = 2.3 \times 10^{-5}) \quad (8)$$

Finally, the time behavior of  $\rho_{\text{BH}}$  and  $\rho_{\text{space}}$  (using the fact that  $\rho_{\text{space}} = \rho_{\text{TOT}} - \rho_{\text{BH}}$ ) is given by:

$$\dot{\rho}_{\text{BH}}/\rho_{\text{BH}} = \dot{M}/M - 3 \dot{R}/R \quad (9)$$

$$\dot{\rho}_{\text{space}}/\rho_{\text{space}} = -4\sqrt{\kappa} \rho_{\text{TOT}}^{1/2} - \rho_{\text{BH}}/\rho_{\text{space}} (\dot{M}/M) \quad (10)$$

Equations (7), (8), (9), and (10) allow one in principle to evolve an initial state with BH's of mean mass  $M_0$  and mass density  $\rho_{\text{BH}}^0$ . In practice, they must be solved numerically and we shall describe our quantitative results in a future paper. However, the general qualitative features are easily described. Depending on the initial parameters there may be an adiabatic period where  $T_{\text{space}} = T_{\text{BH}}$  and both are increasing. However, it is easy to show that once



$4\rho_{\text{space}} > \rho_{\text{BH}}$  black holes must go out of equilibrium. (In fact they will often go out of equilibrium before this, depending on the relative magnitudes of  $\dot{M}$  and  $\dot{R}$ ). After this point the BH's, at a mass  $M_C$ , increase in temperature and evaporate in a time scale of order:<sup>13</sup>  $\tau \approx 10^{-18} M_C^3$  sec, while the temperature of space reaches a maximum and then decreases.

The initial values  $M_0$  and  $\rho_{\text{BH}}^0$  are constrained by a variety of requirements. First, for a given  $M^0$ ,  $\rho_{\text{BH}}^0$  must be less than the value given by dense packing of BH's,  $\rho_{\text{crit}}$  (in practice  $\rho_{\text{BH}}^0 \ll \rho_{\text{crit}}$  in order for our approximations to be valid) and greater than a minimum value below which BH states would no longer be favored in the initial tunnelling bubble formation. This dual requirement then can be shown to imply:  $M_0 \geq 10-100 M_{\text{planck}}$  ( $kT_0^{\text{BH}} \leq 10^{17}$  GeV).

There are also limits on primordial BH density for  $M_0 \geq 10^9$  kg in order not to affect big bang nucleosynthesis,<sup>14</sup> so we will take our initial mass constraint as:  $10^{-6}$  kg  $< M_0 < 10^9$  kg.

This range can be restricted further by considering baryon and monopole production by black holes. It has been shown that unless CP is not microscopically conserved, black holes may produce a net baryon number only via superheavy X-boson production.<sup>15</sup> The advantage of such production in our scenario is that if  $kT_0^{\text{BH}} < 10^{14}$  GeV, all X-bosons produced subsequently by black hole evaporation will be out of equilibrium (inverse decays are suppressed) and will decay producing net baryon number. Hence mass limits on the X-boson needed in the standard model in order to get departure from thermal equilibrium are unnecessary. Noting that X-bosons will only be radiated after  $kT_{\text{BH}} > M_X$  ( $M_{\text{hole}} > M_i$ ) one can estimate the number of such particles produced per black hole:

$$N_x \approx M_i / 3k\langle T \rangle N^* = 2.5 \times 10^8 / N^* \quad (11)$$

where  $\langle T \rangle$  is the average temperature at which the BH radiates after reaching mass  $M_i$ , and  $N^*$  is the number of species of particles being radiated.

If the expansion of the universe is adiabatic after the X-particle decay products thermalize then

$$(n_B/n_T)_{\text{present}} = 7\epsilon (N_x \rho_{\text{BH}}^0 / n_0) [s_0 + \{n_x N_x \rho_{\text{BH}}^0 / 3kT_i M_i\}]^{-1}$$

where  $\rho_{\text{BH}}^0$ ,  $M_0$ ,  $s_0$  are the initial values of BH mass density and mass and the total entropy density respectively,  $t_i$  and  $\rho_{\text{BH}}^i$  are the values of the temperature of space and BH mass density at the time the X-bosons are emitted ( $M_{\text{BH}} = M_i$ ), and  $\epsilon$  is the net baryon number per X,  $\bar{X}$  pair decay. Note that the term in curly brackets arises because X-bosons are out of equilibrium and their decay produces significant entropy. (This term has been neglected by other authors<sup>14</sup> but it need not be small). Without this term  $(n_B/n_T)$  would be comparable to that of the standard scenario. Hence, although we avoid problems with tuning the X-boson mass, the potential entropy production by the X-boson constrains the size of the temperature difference between the BH's and radiation at the time X-bosons are radiated, and can in turn further constrain  $M_0$  and  $\rho_{\text{BH}}^0$ .

On the other hand, monopole production via phase transitions provides a severe constraint on grand unified theories.<sup>15</sup> One might

naively imagine that in our model if  $T_{\text{space}}$  is always less than  $T_{\text{crit}}$  for such a transition that, as the transition never occurs, no monopoles are produced and we avoid these constraints. However in fact one may expect production of monopoles via a thermal background and via BH evaporation. The latter effect may be exponentially suppressed by semi-classical effects,<sup>16</sup> and by the finite size of the monopole (model calculations to provide estimates are underway), and background thermal production may be suppressed by Boltzman factors if  $T_{\text{space}} \ll T_{\text{crit}}$  at all times during which equilibrium may be achieved.

Thus the early universe may have been much cooler than naive extrapolations would imply. In this scheme, the temperature of space need never have exceeded the critical temperature for the restoration of grand unified symmetry. If so, (modulo various numerical computations now underway) it seems possible in principle to allow baryosynthesis, while suppressing monopole production. Also being considered are such questions as: the possibility of producing remnant inhomogeneities on the scale of galaxies; refinements to include an initial mass distribution of BH's; and a discussion of the horizon, flatness, and cosmological constant problems in the context of our model. While such investigations, in the absence of a fundamental theory, provide only circumstantial evidence for the existence of a gravitational transition, they illustrate the possibility that finite-temperature gravitational effects may significantly alter our models of the early universe, as well as our understanding of quantum gravity.

I would like to thank Roscoe Giles for immeasurable aid, A. Guth for insightful discussions, and S. Glashow, D. Gross, N. Isgur, J. Preskill, and J. Primack for useful comments.

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#### 4. Detailed Dynamics

As noted previously,<sup>1</sup> a complete description of the dynamics of the black hole-radiation mixture is described by a set of coupled differential equations involving the total mass density  $\rho(t)$ , the black hole mass  $M(t)$ , the radiation mass density  $\rho_{\text{Rad}}(t)$ , and the black hole mass density  $\rho_{\text{BH}}(t)$ . These equations can be derived from the Einstein equations coupled with an equation of state for the mixture, plus the relations between black hole temperature as a function of mass, and radiation temperature as a function of energy density as the number of massless degrees of freedom change. We describe first in detail the derivation of these equations. Before a numerical analysis of the resulting dynamics is presented in Section 4.3, we describe in Section 4.2 the qualitative dynamical behavior of the system in various regions of interest, where we can give approximate analytical solution of the evolution equations.

##### 4.1 Derivation of evolution equations

Under the assumptions described earlier (we shall review in detail all our approximations in the next chapter) dynamics are governed by the Einstein equations for an isotropic homogeneous expansion:

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{\kappa c^2}{R^2} = \frac{8\pi G}{3} \rho \quad (4.1-1)$$

$$\frac{d(\rho c^2 R^3)}{dt} = -p \frac{dR^3}{dt} \quad (4.1-2)$$

The solution of these equations depended on an ansatz for an equation of state for the black hole mixture. This allows us to use (4.1-2) to express  $\dot{\rho}$  in terms of  $\dot{R}$ . For a

pure radiation state we know  $p = \rho c^2/3$ . Then (4.1-2) yields the relation

$$\dot{\rho}_{\text{rad}}/\rho_{\text{rad}} = -4 \dot{R}/R \quad (4.1-3)$$

The equation of state for a black hole gas is however not obvious. We assume that the black holes, if dilute enough, act like noninteracting dust particles in which case  $p=0$ . We shall justify this assumption in the next chapter, based on our numerical results. Eq. (4.1-2) then yields

$$\dot{\rho}_{\text{BH}}/\rho_{\text{BH}} = -3 \dot{R}/R \quad (4.1-4)$$

Combining (4.1-3) with (4.1-4) yields for the black hole-radiation mixture:

$$\dot{\rho}_{\text{TOT}}/\rho_{\text{TOT}} = -\left(3 + \rho_{\text{rad}}/\rho_{\text{TOT}}\right) \dot{R}/R = B(t) \dot{R}/R \quad (4.1-5)$$

This immediately yields, when combined with (1) the time development for the total mass density: (using  $\kappa=0$ )

$$\dot{\rho}_{\text{TOT}} = B(t) \sqrt{c} \rho_{\text{TOT}}^{3/2}; \quad \left(c = \frac{8\pi G}{3}\right) \quad (4.1-6)$$

Next, we need to determine the time development of  $M(t)$ , the black hole mass. Now for a black-body radiating at temperature  $T$  the intensity of emission is given by:

$$J = \frac{N}{2} \sigma_B T^4 ; \quad \left( \sigma_B = \frac{\pi^2 k^4}{60 \hbar^3 c^2} \right) \quad (4.1-7)$$

where  $N$  is related to the number of helicity degrees of freedom in the radiation:  $N_e = (N_B + 7/8 N_F)$  where  $N_B$ ,  $N_F$  are the number of boson and fermion degrees of freedom, respectively. In  $SU(5)$   $N \approx 80$  ( $T > 10^{14}$ ),  $N \approx 25$  ( $T > 100$  GeV).

Assuming for the moment that the black body approximation is reasonable, the black hole will radiate at a rate (we will use MKS units throughout):

$$\dot{M} = \frac{J}{c^2} A = \frac{N}{2} \left( \frac{\hbar c^4}{60(256) \pi G^2 M^2} \right) = \frac{3.8 \times 10^{15}}{M^2} \left( \frac{N}{2} \right) \text{ kg/sec} \quad (4-1-8)$$

where  $A =$  surface area  $=$  area of event horizon  $= 16\pi G^2 M^2/c^4$ . Note that while the black hole temperature is independent of the number of massless particles, its emission rate is not. Also, Eq. (4.1-8) gives an upper bound on the emission rate. Suppression factors due to nonzero spin, etc., will stop some

particles which are emitted by the black hole from reaching infinity for black holes with mass  $> 10^{14}$ . Page<sup>2</sup> has estimated these suppression effects, and shows that they reduce the emissive power by a factor of 2.6 from the naive frequency independent estimates based on a geometric cross section  $27\pi M^2$ , thus making the effective emission area  $\sim 10\pi M^2$ . This factor is 1.6 less than the area factor used here, and hence our assumption of black body emission is, to a first approximation quite reasonable.

The effect of radiation with an energy density  $\rho_{\text{rad}}$  being absorbed by the black hole is given, using the relation  $\rho_{\text{rad}} = 4/c J$ , when  $J$  now equals the mass intensity incident on the surface. Using this we get

$$\dot{M}_{\text{incident}} = JA = \frac{4\pi G^2 M^2 \rho_{\text{rad}}}{c^3} = 2.0 \times 10^{-45} \rho_{\text{rad}} M^2 \quad (4.1-9)$$

(Note again that the helicity factor for the incident radiation is included in  $\rho_{\text{rad}}$ .) Hence the total rate of change of  $M$  is given by:

$$\dot{M} = \frac{-3.8 \times 10^{15}}{M^2} \left(\frac{M}{2}\right) + 2.0 \times 10^{-45} \rho_{\text{rad}} M^2 \quad (4.1-10)$$

Using the relation  $\rho_{\text{BH}} \sim \alpha M/R^3$  ( $\rho_{\text{BH}} \sim$  mass density of black holes) we get

$$\frac{\dot{\rho}_{\text{BH}}}{\rho_{\text{BH}}} = \frac{\dot{M}}{M} - 3 \frac{\dot{R}}{R} \quad (4.1-11)$$

where the right hand side is given by (4.1-10) and (4.1-1). Finally, we can derive the time development of  $\rho_{\text{rad}}$  as follows since  $\dot{\rho}_{\text{rad}} = \dot{\rho}_{\text{rot}} - \dot{\rho}_{\text{BH}}$ . Then (4.1-6), (4.1-11) and (4.1-1) imply:

$$\begin{aligned}\dot{\rho}_{\text{rad}} &= -\left(3 + \frac{\rho_{\text{rad}}}{\rho_{\text{rot}}}\right) \sqrt{c} \rho_{\text{rot}}^{3/2} - \frac{\dot{M}}{M} \rho_{\text{BH}} + 3 \sqrt{c} \rho_{\text{rot}}^{1/2} \rho_{\text{BH}} \\ &= -3 \sqrt{c} \rho_{\text{rot}}^{3/2} - \rho_{\text{rad}} \sqrt{c} \rho_{\text{rot}}^{-1/2} - \frac{\dot{M}}{M} \rho_{\text{BH}} + 3 \sqrt{c} \rho_{\text{rot}}^{1/2} (\rho_{\text{rot}} - \rho_{\text{rad}})\end{aligned}$$

$$\therefore \frac{\dot{\rho}_{\text{rad}}}{\rho_{\text{rad}}} = -4 \sqrt{c} \rho_{\text{rot}}^{1/2} - \frac{\rho_{\text{BH}}}{\rho_{\text{rad}}} \frac{\dot{M}}{M} \quad (4.1-12)$$

Thus, in principle (4.1-1), (4.1-6), (4.1-10), (4.1-11), and (4.1-12) allow us to derive the complete time evolution from the initial state, giving  $R(t)$ ,  $M(t)$ ,  $T_{\text{black hole}}(t)$ ,  $T_{\text{rad}}(t)$ . Of course, this set of coupled differential equations is difficult to solve analytically in any useful fashion, and we will discuss its solution via numerical methods in Section 4.3. It is possible, however, to get a great deal of qualitative information without the explicit solution of these equations, and we now discuss these results.

## 4.2 Qualitative Dynamics

The evolution of the black hole radiation state is governed by two competing processes. The expansion of the universe will tend to reduce the temperature of radiation, thus driving the initial state configuration away from equilibrium. On the other hand, as the black holes get hotter than the ambient radiation, they will pump energy into the background via particle emissions. Which process wins out for different initial conditions can be determined numerically. However, we can analytically describe the behavior of the system in a variety of regimes.

First, consider the possibility of a period of adiabatic expansion. The total entropy of the system is given by

$$S = C_1 M^2 + V C_2 T_{rad}^3 \quad (4.2-1)$$

where  $C_1$  and  $C_2$  are constants and  $V \sim R^3$ . Imposing the condition  $dS/dt = 0$  implies:

$$\frac{\dot{T}_{rad}}{T_{rad}} = -\frac{\dot{R}}{R} - \left[ \frac{2 S_{BH}}{3 S_{rad}} \frac{\dot{M}}{M} \right] \quad (4.2-2)$$

If the term in brackets were zero (4.2-2) would describe the standard adiabatic relation in the radiation dominated FRW universe. As in that case we can determine the temperature evolution of the system by using the energy conservation condition (4.1-2). Since the total energy density is given by:

$$\rho_T \sim C_3 T_{rad}^4 + \frac{C_4 M}{R^3} \quad (4.2-3)$$



Eq. (4.1-2) then implies the relation

$$\frac{\dot{T}_{\text{rad}}}{T_{\text{rad}}} = -\frac{\dot{R}}{R} - \frac{P_{\text{BH}}}{4P_{\text{rad}}} \frac{\dot{M}}{M} \quad (4.2-4)$$

Demanding that (4.2-2) and (4.2-4) both be satisfied implies the condition:

$$\frac{P_{\text{BH}}}{4P_{\text{rad}}} = \frac{2S_{\text{BH}}}{3S_{\text{rad}}} \quad (4.2-5)$$

Using the explicit form for these quantities one can show that (4.2-5) then implies the condition for adiabatic expansion:

$$T_{\text{rad}}(t) = \frac{hc^3}{8\pi GM(t)} \quad (4.2-6)$$

This is exactly the condition that the temperature of radiation equal the temperature of black holes, which agrees with our intuition that adiabatic expansion implies the system always remains near equilibrium.

This process cannot go on indefinitely, and it is easy to derive limits on when it must cease. Even if the black holes could radiate at an arbitrarily large rate, (4.2-6) cannot always be satisfied. To see this, we note that (4.2-6) implies:

$$\frac{D}{M(t)} = D' \left( \frac{E_{\text{rad}}(t)}{R^3(t)} \right)^{1/4} \quad (4.2-6A)$$

Now, neglecting the energy loss incurred by radiation in doing work in the expansion (this will only extend the possible period of adiabaticity) we have:

$$E_{\text{rad}} = N_{\text{BH}} (M_0 - M(t)) c^2 + E_{\text{rad}}^0 \quad (4.2-7)$$

where  $N_{\text{BH}}$  gives the total number of black holes in the initial state. Eq. (4.2-7) and (4.2-6A) then imply:

$$\begin{aligned} V(t) \sim R^3(t) &= \left(\frac{D'}{D}\right)^4 M^4(t) E_{\text{rad}}(t) \\ &= \left(\frac{D'}{D}\right)^4 M^4(t) (N_{\text{BH}} [M_0 - M(t)] c^2 + E_{\text{rad}}^0) \\ &= \left(\frac{D'}{D}\right)^4 [ (N_{\text{BH}} M_0 c^2 + E_{\text{rad}}^0) M^4(t) - N_{\text{BH}} M^5(t) c^2 ] \\ &= \left(\frac{D'}{D}\right)^4 [ E_{\text{tot}}^0 M^4(t) - N_{\text{BH}} M^5(t) c^2 ] \end{aligned} \quad (4.2-8)$$

Although we can give an explicit form for  $R^3(t)$  as we shall show shortly, all we need at this point is that the expression of the universe implies:

$$\frac{dV(t)}{dt} > 0 \quad (4.2-9)$$

Now plugging (4.2-9) into (4.2-8) and differentiating with respect to time yields,

$$\frac{dV}{dt} = \left(\frac{D'}{D}\right)^4 M^3(t) [ 4E_{\text{tot}}^0 - 5N_{\text{BH}} M(t) c^2 ] \dot{M}(t) \quad (4.2-10)$$

The left-hand side of (4.2-10) is always positive

Since  $m < 0$ , the right-hand side becomes negative when

$$N_{\text{BH}} M(t) c^2 \leq \frac{4}{5} E_{\text{tot}}^0 \quad (4.2-11)$$

Since the left-hand side of (4.2-11) gives the total energy in the form of black holes, this inequality expresses the fact that adiabatic expansion is impossible once the total energy in the form of black holes is less than 4/5 the total energy of the system. However, this is merely a reflection of the well known fact, which we have previously derived<sup>3</sup>, that a black hole radiation state cannot be in equilibrium in a box if radiation accounts for more than 1/5 of the total energy. Thus, our derivation implies the reasonable constraint (which we could have guessed) that adiabatic expansion is impossible when equilibrium is impossible.

On top of this, however, there exists a dynamic constraint on adiabatic expansion, if the black hole decay rate is small compared to the expansion rate. Using the relation  $T_{\text{rad}} = T_{\text{BH}}$  during the adiabatic phase, (4.2-4) then implies during this period

$$\frac{\dot{T}_{\text{BH}}}{T_{\text{BH}}} = \frac{\dot{T}_{\text{rad}}}{T_{\text{rad}}} \left( = \frac{-\dot{R}}{R} - \frac{\rho_{\text{BH}}}{4\rho_{\text{rad}}} \frac{\dot{M}}{M} \right) \quad (4.2-12)$$

Then, using the fact that  $\dot{T}_{\text{BH}}/T_{\text{BH}} = -\dot{M}/M$ , we see that in order for adiabatic expansion:

$$\left[ \frac{\rho_{\text{BH}}}{4\rho_{\text{rad}}} - 1 \right] \frac{\dot{M}}{M} = -\frac{\dot{R}}{R} \quad (4.2-13)$$

Again, we see that if  $\rho_{\text{BH}} < 4\rho_{\text{rad}}$  this relation cannot be satisfied. However, (4.2-13) also gives a constraint on the value of  $\dot{M}/M$  which is needed to maintain adiabatic expansion. Also, setting  $\dot{T}/T = 0$  in (4.2-4) gives the minimum value

of  $\dot{M}/M$  which will stop the temperature of space from decreasing:

$$\frac{\rho_{BH}}{4\rho_{rad}} \frac{\dot{M}}{M} = -\frac{\dot{R}}{R} \quad (4.2-14)$$

Regardless of whether the expansion is adiabatic or not, we can analytically derive the approximate behavior of  $\rho_{tot}(t)$  and  $R(t)$  over large time intervals. This is because the factor  $B(t)$  in (4-6) changes only from 3-4 as black hole density varies over its entire range. Hence we can integrate (4.1-6) and (4.1-1) (with  $\kappa=0$ ) using the approximation  $B(t) = B = \text{const}$ . With the boundary conditions  $\rho_{tot}(t=0) = \rho_{tot}^0$ ,  $R(t=0) = R_0$  we get:

$$\rho_{tot}(t) = \rho_{tot}^0 / \left( \frac{B}{2} \sqrt{c} \rho_{tot}^0{}^{1/2} t + 1 \right)^2 \quad (4.2-15)$$

$$R(t) = R_0 \left[ \frac{B}{2} \sqrt{c} t \rho_{tot}^0{}^{1/2} + 1 \right]^{2/B} \quad (4.2-16)$$

(the result (4.2-15) was used in the derivation of (4.2-16)). Thus the value of  $\rho(t)$  is largely insensitive to the variation of  $B$ , and asymptotically has the same  $t^{-2}$  behavior of the standard model. On the other hand, the scale factor  $R(t)$  has a factor of  $B$  in the exponent. When  $B=3$  and black holes dominate the mass density  $R(t) \sim t^{2/3}$ . This value will then shift to the standard  $R(t) \sim t^{1/2}$  behavior as the universe becomes radiation dominated.

If  $\dot{M}/M$  begins much smaller than  $\dot{R}/R$ , we may expect this shift to occur quite rapidly in the final stages of black hole

evaporation. For in this case we can analytically solve (4.1-11) and (4.1-12) (for constant B), yielding

$$\rho_{\text{rad}}(t) \approx \rho_{\text{rad}}(0) \left[ 1 + \beta_2 \sqrt{c} \rho_{\text{tot}}^{-1/2} t \right]^{-3/B} \quad (4.2-17)$$

$$\rho_{\text{BH}}(t) \approx \rho_{\text{BH}}(0) \left[ 1 + \beta_2 \sqrt{c} \rho_{\text{tot}}^{-1/2} t \right]^{-6/3} \quad (4.2-18)$$

Hence, in this case  $B(t)$  will approach the value 3, as long as radiation loses more energy doing work on the expanding universe than the black holes do due to evaporation. Also, from our previous discussion, in this case we expect that the temperature of space will drop considerably before the black hole begins to evaporate. At this point the first term of (4.1-10) dominates indicating that the black hole will evaporate freely with  $M(t)$  given by:

$$M(t) = (M_0^3 - 3\sigma t)^{1/3} ; \sigma = \frac{N}{2} (3.8 \times 10^{15}) \quad (4.2-19)$$

Thus, to summarize: From the initial state, the expansion will go at a rate  $R(t) \sim t^{2/3}$ , and  $\rho_{\text{tot}}(t)$  will drop proportionally to  $t^{-2}$ . Depending on the initial values,  $\rho_{\text{tot}}^0$  and  $M^0$ , then may be a short period of adiabatic expansion with the temperature of space increasing. The temperature of space will then fall while the black hole temperature continues to increase. Finally, in the last moments of their evaporation, when  $\dot{M}/M$  becomes very large black holes will rapidly pump energy into space, heating it up.

### 4.3 Quantitative Dynamics

The coupled set of equations for  $\dot{M}/M$ ,  $\dot{\rho}_{\text{tot}}/\rho_{\text{tot}}$ ,  $\rho_{\text{rad}}/\rho_{\text{rad}}$ ,  $\dot{\rho}_{\text{B}}/\rho_{\text{BH}}$  were numerically integrated using a fourth order Runge-Kutta approximation. (see Appendix IV for computer program). The evolution was determined by the initial value conditions on  $\rho_{\text{BH}}^0$  and  $M^0$ . (In this scenario with  $k=0$  in equation (4.1-1),  $R^0$  is a free variable whose value can be determined by requiring the final state to agree with the FRW model at a given temperature). The constraint that  $T_{\text{BH}}^0 = T_{\text{rad}}^0$  was also used, in order that the initial state be an equilibrium configuration. For a given  $M^0$ , the range of acceptable  $\rho_{\text{BH}}^0$  was determined by requiring that  $\rho_{\text{BH}}^0$  satisfy the dual requirement that it be less than the value given by dense packing of black holes (by a minimum factor of  $10^{-2}$ ) and greater than the value below which black hole configurations would no longer be favored in the initial state (i.e.,  $\rho_{\text{BH}} > 4\rho_{\text{rad}}$ ). This can be shown to imply:

$$10^{-2} \frac{3c^6}{32\pi G^3 M^2} \geq \rho_{\text{BH}}^0 \geq \frac{N}{2} \frac{\pi^2 (16)^4}{15 h^3 c^5 M^4} \quad (4.3-1)$$

or in MKS units:

$$\frac{1}{1.3 \times 10^{-82} \text{ M}^2} \geq \rho_{\text{BH}}^0 \geq \frac{N/2}{1.4 \times 10^{-61} \text{ M}^4} \quad (4.3-2)$$

This restriction implies that  $\rho_{\text{BH}}^0$  has a nonzero range only when  $M^0 > 10^{-6} \text{ kg} = 10^2 M_{\text{p}}$  and the acceptable range for  $\rho_{\text{BH}}^0$  increases with increasing  $M_{\text{BH}}^0$ . The constraint  $M^0 > 10^2 M_{\text{p}}$  is

also self-consistent because it implies that the distance scales and densities under consideration will be safely within the "classical regime", where we expect our evolution equations to be valid. On the other hand, this constraint implies that if the initial state results from effects near the Planck scale, then the parameters of the state will be tightly constrained. We shall discuss this point in more detail when we discuss the stability of this initial state in the next section.

Results for two initial value of  $M^0$  ( $=10^{-2}, 10^{-4}$  kg) are plotted on Figures 5(a,b). We note immediately that there is a very short period of adiabatic expansion, during which the temperature remains nearly constant. This is due primarily to the fact that the "expansion" rate has not yet become significant enough to reduce the temperature of radiation. As can be seen, black hole decay is not significant factor during this period and once the expansion rate reaches its asymptotic  $\sim t^{2/3}$  value, the temperature of space drops more or less monotonically. As can be seen, in Figure 4(b), there may be short periods during which ratio of the mass density of black holes to that of radiation is sufficiently large so that even small levels of black hole decay stop the monotonic decrease in the temperature of space. This is not a general feature, however, and occurred for only one black hole mass value. It is true, however, that the decrease in temperature of space is significantly less than that predicted by (4.2-17), indicating black hole evaporation has a significant effect. On the other hand, the time evolution of black hole mass  $M(t)$  differs negligibly

from the formula (4.2-19) indicating that the back reaction due to the nonzero temperature of space never significantly affects the black hole decay rate. Also, we note that for black holes of mass  $< 10^{-4}$  kg, the decay rate time is decreased by a factor of 3 due to effect the increase in the number of massless degrees of freedom in Eq. (4.1-8). The initial values of  $M_0, \rho_{\text{BH}}^0$  used in the computation, plus the value of the temperature of space after the black holes have evaporated are shown in Table 1.

Shown on Figure 5(a) is the behavior of the scale factor  $R(t)$  with time. As expected, it rather quickly approaches the expected asymptotic  $t^{2/3}$  behavior, with a shift to  $t^{1/2}$  only after the last moments of the black hole evaporation. The speed with which it approaches its asymptotic behavior is a function of the initial mass density, also as expected from Eq. (4.2-16). Shown on Figure 6(a) is the behavior of  $R(t)$  as a function of temperature. The adiabatic Robertson-Walker value is  $R(t) \sim T^{-1}$ . During the black hole decay phase the behavior of  $R(t)$  is quite different from this value - an example of the nonadiabatically in this model. Also, the temperature of space falls faster as a function of  $R(t)$  for smaller initial values of  $\rho_{\text{tot}}^0$ , which is again a reflection of the fact that black hole decay is slowing the decrease of the temperature of space.

Another point which will prove useful later is that although  $R(t_{\text{decay}})/R_0$  decreases with decreasing initial mass density for fixed  $M_0$ , the distance between black holes increases with decreasing mass density. Hence the final physical



distance between points where the black holes were initially located will be constant for fixed  $M_0$ .

As a final note we point out that the initial boundary value  $t=0$  in this scenario has a different connotation from that in the standard FRW model. There it represents the time when the total energy density becomes singular. Here  $t$  measures time of evolution from an initial local equilibrium state of black hole and radiation. Hence it may appear that  $t=0$  is somewhat arbitrary, and as long as the initial state is in local equilibrium, re-definition of  $t_0$  can be compensated by possible changes in the values of  $\rho_{\text{BH}}^0$  and  $M^0$ . While this is true in principle, the degree of flexibility of choice in  $t_0$  is restricted by the fact that we have seen that black holes and radiation go out of equilibrium on a timescale associated with the expansion timescale. This timescale is tied to real physical scale which enters the problem in order that the initial volume per black hole describes a possible equilibrium configuration, and it is this timescale which is not arbitrary. The time at which this expansion sets in connects time in our scenario with the time variable in the standard model. We see from Tables 1, 2, 3 and Appendix IV that this timescale becomes closer to the Planck timescale as the initial black hole masses get smaller, and the distances between them approach the Planck scale. The fact that we are always limited to time and distance scales larger than the Planck scale is associated with the equilibrium condition (4.3-1) and this timescale can be thought of roughly as the time required to form the initial state configuration.

## 5. The Initial State Reexamined

The preceding discussions relied on an initial state configuration which involved on large scales an isotropic and homogeneous pressureless gas of black holes of mass  $M_0$ . In such an initial state self-consistent? The quantitative results we have obtained now allow an examination of the stability of this state, and the validity of the approximation used to derive its behavior. As it invariably must, the rapid expansion of the universe plays a vital role in resolving these questions. Also of importance is the fact that black holes have a finite lifetime. Our discussions also shed light on how the horizon, flatness, and cosmological constant puzzles, or their resolution, fit into our initial state assumptions.

### 5.1 Noninteracting Black Holes?

Assuming that  $p \approx 0$  for the black hole configurations in Section 4 was equivalent to the assumptions that black holes behave somehow like noninteracting dust particles. This seems nonintuitive. Unless the black holes are extremely "dilute" we expect the gravitational potential between neighboring black holes to be large. By recalling the definition of pressure in relativistic fluid mechanics, we can see how this strong interaction might manifest itself in nonzero pressure.

The pressure of a moving fluid measures the deviations from the average motion. Pressure is defined by the flux of momentum out of a comoving volume element.<sup>1</sup> Clearly if all particles are motionless with respect to the average flow

the pressure is zero. Likewise if particles are noninteracting the average momentum flow out of a volume is zero. Interactions however change the momentum of particles, and this may lead to a net momentum flux. If deviations from the average state are associated with a random velocity  $\hat{v}$ , then the relation between pressure and mass density in a relativistic fluid is given by<sup>2</sup>

$$\frac{P/c^2}{\rho} = \frac{1}{3} \frac{\hat{v}^2}{c^2} \quad (5.1-1)$$

Thus, if interactions induce velocities of order  $c$ , then the pressure of the system is significant. The  $p=0$  approximation for massive bodies is based on the assumption that their random motion is thermal in character in which case  $\hat{v}^2/c^2 \ll 1$  in general. For example, if initial black hole motion is thermal, using the fact that  $kT \approx (1.6/M)$  Joules, we can estimate  $\hat{v}^2$  from the relation  $1/2 m\hat{v}^2 \approx 3/2 kT$ . For the black hole masses considered previously this yields  $\hat{v}^2/c^2 < 10^{-4}$ . The argument that black hole motion always remains predominately thermal is clearly false in general, as we know at some critical density interactions must become important. It also ignores possible relevant curved space effects.

We can immediately remedy this last point. For a Schwarzschild metric  $g_{00} \approx (1 - 2GM/c^2r)$ . This metric is time independent, and with the expectation (which we here hope to justify) that particle velocities in a comoving frame will be small we recognize that if the second term is small we can use the weak field approximation<sup>3</sup>  $g_{00} = 1 + 2\phi/c^2$ , where  $\phi$  is the Newtonian gravitational potential. If  $r \gg 0(10GM/c^2)$  then this

approximation will be justified. Thus if black holes are separated by this order or greater we may do our calculations in the Newtonian approximation.

The effect of interactions on particle motion is dependent not only on the characteristic strength associated with encounters, but also on a characteristic time scale over which they become important. If, considering the combined effects, we can show that the effects at interactions are negligible on the time scales with which we are concerned, then the  $p=0$  approximation is justified. This approach is standard in astrophysics, in stellar dynamics for example<sup>4</sup>, where interaction effects are parameterized in a time variable, the relaxation time. This is defined as the time it takes on average for encounters to produce deviations in the velocities of stars which are of the order of magnitude of the original velocity. Because in the case of stars each encounter results in a small deviation  $\Delta v$ , the relaxation time is usually many orders of magnitude greater than the time it takes for a star to traverse the general dimensions of the system.

In the black hole system, however, each encounter is significant. To get a quantitative idea of this, consider the simplified two body gravitational encounter with impact parameter  $\ell$  shown in Figure 8. Each path can be approximated by straight lines if the resultant change in velocity  $\Delta v$ , perpendicular to the original direction of motion, is small compared to initial velocity  $v$ . This change  $\Delta v$  is of order:<sup>5</sup>

$$\Delta v \approx \frac{2GM}{\ell v} \quad (5.1-2)$$

where  $m$  is the mass of each particle. For thermal velocities ( $v \approx \sqrt{3kt/m}$ ) considered here the impact parameter needed so that  $\Delta v \lesssim v$  is of order  $10^8$ - $10^{12}$  times the average initial distance between black holes in our model. Indeed, by the virial theorem, for particles reaching equilibrium under their mutual gravitational interactions,  $v_{av} \sim \sqrt{Gm/2R}$ , where  $R$  is the distance between particles. For masses and initial densities we consider this implies  $v \sim .1 - .3c$ . This average random velocity is large enough to result in nonzero pressure for the relativistic black hole gas.

Hence, the only way in which the  $p=0$  approximation can be justified is if no encounter between black holes can take place before they evaporate. We show below that this is indeed the case, for certain initial conditions. Our argument is based on three properties of the initial state. First, the black holes are not in global gravitational equilibrium at small times. The particle horizon is initially smaller than the distance between holes, although it grows almost immediately to include nearest neighbors. This implies that we can treat the initial random motion as thermal in origin. Next, the assumption of an approximately flat ( $\kappa=0$ ) configuration to agree with observation, implies that each black hole at rest in a comoving frame will asymptotically escape to infinity in spite of the gravitational pull of its neighbors. Small initial fluctuations counteract the effects of this expansion in order to become significant. However, the time scale for this to occur is longer than the characteristic expansion time for the period under consideration.

(Such suppression of interactions due to expansion is standard, in the case of monopole-antimonopole annihilation.<sup>6</sup>) Finally, the isotropic nature of the initial configuration cancels out much of the force between any two neighboring black holes.

Consider, for example, the nearest neighbor configuration of Figure 9. (Interactions with black holes at larger distances will be suppressed). Initial thermal fluctuations will cause the black hole at 0 to move a distance  $\delta r = v_{\text{thermal}} \delta t$ . Once the particle moves from the center point it begins to feel a differential force acting in the direction of the nearest black hole. If  $\delta r$  is small, then in Figure 9 we can neglect the forces from C, D. The resulting force is then given by:

$$\begin{aligned} \delta F_x &= \frac{GM^2}{(r' + \delta r)^2} - \frac{GM^2}{(r' - \delta r)^2} \\ &= -4 \frac{GM^2}{r'^2} \left( \frac{\delta r}{r'} \right) \end{aligned} \quad (5.1-3)$$

where  $r' = 2r$  where  $r$  is given approximately by  $r \sim (M/\rho_{\text{BH}})^{1/3}$ .

Thus the instantaneous accelerations of the particle towards A is given by

$$a(\delta r) = \frac{4MG}{r'^2} \left( \frac{\delta r}{r'} \right) \quad (5.1-4)$$

Now, from Chapter 4 we know that distance intervals are expanding at a rate

$$R(t) = R_0 \left( 1 + \frac{3}{2} (p_{\text{ror}}^0)^{1/2} \sqrt{c} t \right)^{2/3} \quad (5.1-5)$$

Also if  $\Delta v = \int_0^{t_{\text{decay}}} a(r,t) dt$  is small enough so that  $\Delta v < v$ , then throughout this time interval  $v_{\text{av}} \approx v_{\text{initial}}$ . The distance travelled by a particle with this velocity during the expansion (5.15) is  $\ell \approx 3v t_{\text{decay}}$  (if  $t_{\text{decay}} \gg 3/2(\rho_{\text{tot}}^0)^{1/2}/\sqrt{C}$ ). Thus if we set  $\delta r \approx 3v_0 t$ , then we can show that  $(\delta r(t)/r(t))_{\text{av}} \approx (\delta r(t))/r(t)_{\text{final}}$ . Thus the average acceleration during this period is

$$a_{\text{av}} \approx \frac{3GM}{(2r)_{\text{av}}^2} \left( \frac{\delta r}{2r} \right)_{\text{av}} \quad (5.1-6)$$

where, from (5.15)  $r_{\text{av}} \approx 3/5 r_{\text{final}}$ .

The self-consistency of this procedure can then be verified by checking that  $\Delta v = a_{\text{av}} t_{\text{decay}} \lesssim v_0$ , and that  $t_{\text{decay}} \gg 3/2(\rho_{\text{tot}}^0)^{1/2}/\sqrt{C}$  and finally that  $(\delta r(t)/2r(t))_{\text{final}} \ll 1$ . If, all these conditions are met, then during the time interval over which the black holes decay their mutual gravitational interaction induced by initial thermal fluctuations never significantly increases their initial velocity. Since  $v_0^2/c^2 \ll 1$  this then implies that  $p \approx 0$  during the period under consideration. Also, since  $(\delta r/2r)_{\text{av}} < 1$  for this period, actual black hole collisions do not occur. Table 2 lists the calculation of these variables for various initial masses  $M_0$  and mass densities  $p_0$ . Note that the results do not depend on initial mass density for each initial black hole mass, because as we pointed out earlier, the expansion rate and the initial volume per hole very inversely so that  $r_{\text{final}}$  is constant.

Based on the results in Table 2, the above approximation is self-consistent in the case of initial masses of  $10^{-4}$  and  $10^{-2}$  kg. Here  $\Delta v$  is less than  $v$  so that the actual average velocity does not differ significantly from  $V_0$  (in fact  $V_{av} \lesssim 2V_0$ ). Also,  $(\delta r/r)_{av} \lesssim 10^{-2}$  so that the approximation used to derive (5-3) and the equations which follow are valid. For other masses we cannot conclude from this argument that the  $p=0$  approximation is invalid--just that the simple approximations used here to check its validity break down. It is interesting to note why. For masses smaller than  $10^{-4}$  kg, the short lifetime of the black hole keeps velocity increase  $\Delta v$  down, but the expansion rate does not have time to significantly increase  $r$ , so that  $\delta r/r$  can become large. On the other hand, for masses larger than  $10^{-2}$  the expansion rate is sufficient to keep  $\delta r/r$  small, but the lifetime of the black holes is long enough to allow  $\Delta v$  to be of order  $v$ .



## 5.2 Stability--Initial Mass Distribution

The expansion rate of the universe is also crucial in stabilizing the density fluctuations arising from an initial distribution of black hole masses, as opposed to a fixed uniform initial mass. There are many ways such distributions might arise in a transition, due to fluctuations in initial bubble size, and the amount of growth after formation, for example. Such a distribution of black holes, if static, would be unstable. Holes smaller than the mean would evaporate, and holes larger than the mean would continue to grow through accretion of radiation. However, the data in Table 1 clearly indicates that the expansion rate of the universe is much larger than the evaporation rate for black holes for all reasonable initial mass densities. This implies that each black hole evaporates for the most part independently of the evaporation of other black holes. The dynamics of mean temperature, mean mass density of space, and mean decay time will approximate those of the uniform initial mass model. The temperature of space will initially fall at a slower rate, but then the rate will increase so that at the time of evaporation the temperature will not differ significantly from the naive model.

Consider, for example, a mean initial mass of  $10^{-2}$  kg, with a mid-range mean initial mass density of  $10^{77}$  kg/m<sup>3</sup>. Say 10% of the initial density is in the form of  $10^{-3}$  and  $10^{-1}$  kg holes, respectively, and 1% in the form of  $10^{-2}$  and  $10^0$  kg holes. Using the data from Table 2 and Table A-1(Appendix 4)

we can estimate the following behavior. The  $10^{-4}$  kg black holes (with initial mass density  $10^{75}$  kg/m<sup>3</sup>) will evaporate in a time  $\sim 10^{-30}$  sec. At the time of their evaporation, their mass density will be  $\sim 10^{66}$  kg/m<sup>3</sup>. This will increase the temperature of space from  $\sim 10^{10}$  GeV to  $\sim 10^{11}$  GeV. However, it will also raise the ratio of radiation density to black hole mass density by four orders of magnitude, thus decreasing the effect of black hole evaporation on suppressing the decrease in the temperature of space. Thus this temperature will begin to fall at a faster rate. The evaporation of the black holes of mass  $10^{-3}$  kg will occur at  $\sim 3 \times 10^{-27}$  sec, when their mass density is  $\sim 10^{61}$  kg/m<sup>3</sup>. This will raise the temperature of space to  $6 \times 10^9$  GeV if it has already fallen below that value. This evaporation will once again increase the ratio of radiative energy to black hole energy to the value  $\sim .5$ . This will mean that from this time until the black holes of mass  $10^{-2}$  kg evaporate at time  $\sim 10^{-24}$  sec the temperature of radiation will drop as  $\sim t^{-8/3.5} = \sim t^{2.2}$  (see Eq. (4.2-17)). Thus as  $t$  increases by a factor  $10^3$ ,  $\rho_{\text{rad}}$  will fall by a factor  $\sim 4 \times 10^6$ . Thus at time  $t \sim 3 \times 10^{-24}$  sec, just before the black holes of mass  $10^{-2}$  evaporate the density of radiation will be  $\sim 10^{55}$  kg/m<sup>3</sup>, almost exactly the same values as in the uniform mass model of Table A-1 (Appendix 4). Similarly one can show that the black holes of original mass  $10^0$  kg will not accrete enough matter to change their mass significantly by the time they evaporate. Thus an initial state with a range of initial masses

results in similar dynamics to those of the uniform mass initial state.

### 5.3 Horizon, Flatness and Cosmological Constant Problems

As has been described before the standard adiabatic Robertson-Walker model suffers from what has become known as the horizon and flatness problems.<sup>7</sup> In addition, observations restrict a possible cosmological constant to be very nearly zero<sup>8</sup>, while theory gives no such constraint. We now consider how these paradoxes are reflected in our initial state conditions. Each of them puts a strong constraint on the initial state, which in turn implies that their resolutions in this scenario is closely linked with the parameters of the hypothesized initial phase transition.

The finite horizon in the initial state black hole configuration proved to be an important feature in our stability arguments--keeping the different black hole regions from being in initial thermal contact and in gravitational equilibrium. On the other hand, there is natural mechanism which keeps causally disconnected regions at the same temperature. The temperature of each region is governed by the temperature of the black holes within. These black holes will have the same mass, thus each region has the same temperature.

While on the surface this appears to be a natural resolution of the horizon problem as it is traditionally framed, the problem arises in a different form. Why are all black holes of the same mass? This is presumably a reflection of the fact that the parameters of a local phase transition via bubble nucleation (i.e.,  $\rho_0$  and  $v_0$ ) are fixed by the initial

parameters of the effective potential. On the other hand, this implies that the pre-transition phase was globally uniform in order for the conditions for a local transition to be the same everywhere (i.e., in order for the parameters of the effective action to be the same everywhere.) Hence the resolution of the standard horizon problem in this scenario puts the strong constraint that there exist no horizon problem in the pre-transition phase. It can be claimed, justifiably, that this sweeps the problem under the rug, pushing it backwards in time to a region about which we claim to have little or no information. On the other hand, the standard horizon problem stems from the demand that the Robertson-Walker metric be good arbitrarily close to  $t=0$ . Once the assumption of a phase transition is made, the initial state may have had an arbitrarily long time to relax to metastable equilibrium. The assumption that this phase was uniform, while entirely ad hoc, is more plausible physically, than the assumption of uniformity in the initial Robertson-Walker state.

Similarly, in order to describe the evolution of our initial state, the flatness ( $\kappa=0$ ) condition was put in by hand. Hence unlike the inflationary inverse scenario<sup>9</sup> there can be no explicit self-consistent resolution of this problem. On the other hand, the flatness problem in the Robertson-Walker model is a reflection of the fact that the initial state has arbitrarily large total entropy. The real point is that there is no explanation within the model of why this entropy should be

so large. In our scenario, however, the final Robertson-Walker state which results out of the black hole-phase transition scenario has a radiation entropy as large as  $10^{14}$  times the initial radiation entropy. The entropy of the final Robertson-Walker state can be viewed as a remnant of that generated in the first order transition which produced the highly entropic initial black hole configuration. This is again a reflection of the motivation behind the original inflationary universe model--that the only known way to generate large entropies is during the non-equilibrium period of a first order transition. The specific way in which large entropies in the final state are tied to the flatness constraint in post-transition black hole state is interesting. As we pointed out earlier, the fact that the initial relative velocity of the black holes was sufficient so they could asymptotically escape to infinity ( $K=0$  flatness condition) allowed the initial state to evolve in the manner we have described. Thus in order to generate the required entropy in the observed Robertson-Walker state via a physical mechanism like the one proposed here requires that the initial black hole configuration be created with at least zero "gravitational" (potential plus kinetic) energy. From the point of view of a possible gravitational phase transition this is not an unreasonable constraint. On the other hand we will see that the fine tuning aspect of the flatness problem crops up in another form in our scenario when we discuss inhomogeneities.

Finally, there is an intriguing anthropic connection between the observed small value of the cosmological constant and a phase transition as envisioned above. We might imagine

that the initial state contains in addition to black holes, a nonzero cosmological constant. Locally then the configuration would have a Schwarzschild-de Sitter metric, given by

$$ds^2 = -\left(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}\right) dt^2 + dr^2 \left(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}\right)^{-1} + r^2 d\Omega^2 \quad (5.3-1)$$

where  $\Lambda$  is the cosmological constant.

As can be seen from (5.3-1), if  $\Lambda > 0$ , and  $9\Lambda m^2 < 1$  the factor  $g_{00}$  is zero at two positive values of  $r$ . This implies there are two event horizons, one associated with the black hole, and a larger cosmological event horizon. Gibbons and Hawking<sup>10</sup> have demonstrated that it is possible to associate a temperature,  $T_C$ , with the surface gravity associated with this second event horizon. In this case  $T_C < T_{BH}$  where  $T_{BH}$  is the black hole temperature. Thus no equilibrium is possible for such a state, as the black holes will evaporate. Hence the initial state we have described is only in local equilibrium if  $\Lambda=0$ . Put in another way; if there was a phase transition resulting in the initial local equilibrium state described here, we expect  $\Lambda=0$ .

## Chapter Six: Model Predictions

Having analyzed the initial state configuration in detail we now proceed to consider the implications of the dynamics described in Chapter 4. Specifically we investigate baryosynthesis, monopole production, and remnant inhomogeneities in this model.

### 6.1. Baryosynthesis

Grand unified theories provide automatically two of the three ingredients necessary for baryon production in the early universe: C- and CP-non-invariance, and baryon non-conserving interactions.<sup>1</sup> The third ingredient, departure from equilibrium, is more difficult to achieve in general. Particles, such as superheavy X-bosons, whose decays are baryon asymmetric must go out of equilibrium if the baryon number their decay produces is not to be washed out by the effect of inverse decays (see Appendix V). On the other hand, black holes will radiate superheavy particles once their temperature is greater than the mass of these particles. As demonstrated earlier, the temperature of space may be substantially below these black hole temperatures. Thus the particles radiated by the black holes at this time will automatically be out of equilibrium, with inverse decays suppressed, and the subsequent decay of these particles can result in a net baryon-antibaryon asymmetry.

As discussed previously,<sup>2</sup> this removes the constraints on masses of superheavy particles which are normally needed to ensure that their distribution functions move away from their equilibrium values during the cosmological expansion. However it introduces the additional problem that significant entropy



is also generated by out of equilibrium decays. With the data from Section 4 we can estimate this effect to determine the net baryon to photon ratio that results.

We consider here only baryon number generation via super-heavy particle production and subsequent decay. It has also been suggested that black holes may emit a net flux of baryons directly if baryon number and CP are microscopically violated.<sup>3,4</sup> However since baryon number violating interactions do not become significant below energies of order of the superheavy mass scale we expect such a flux will not significantly alter the order of magnitude estimate of the net baryon flux produced via superheavy particle decays.

As described in our previous letter,<sup>5</sup> it is easy to estimate the net baryon number produced in black hole decays. X bosons (X generically may stand here for vector or higgs bosons) will only be radiated significantly after  $kT_{BH} > M_X$ , provided of course that at this time the mass of the hole is greater than the mass of the X particle. This implies (in units  $\hbar=c=1$  and  $M_{pl} = \text{planck mass} = 10^{-5} \text{ gm}$ )

$$kT_{BH}^0 = \frac{M_{pl}^2}{8\pi M_{BH}^0} \geq M_X$$

$$\Rightarrow M_{BH}^0 \leq .1 M_{pl} \left( \frac{M_{pl}}{M_X} \right) \quad (6.1-1)$$

We can determine the average temperature of black holes after their mass drops below this value, using the relation (eq. 4.2-19)

$$M(t) \sim (M_{BH}^0 - 3\sigma t)^{1/3}; \quad \sigma \approx 80(3.8 \times 10^{15}):$$

$$\langle M \rangle = \frac{1}{\tau} \int_0^{\tau} M(t) dt \quad \text{where} \quad \tau = \frac{(M_{BH}^0)^3}{3\sigma}$$

$$\Rightarrow \langle M \rangle = \frac{3}{4} M_{BH}^0 \quad \therefore \langle kT \rangle = \frac{4}{3} kT_{BH}^0 = \frac{4}{3} M_X \quad (6.1-2)$$

Assuming that in a thermal distribution of particles each particle carries energy  $\sim \frac{3}{2}kT$ , and letting  $N$  be the ratio of total number of helicity states of particles being emitted to the number of  $\bar{X} + X$  particle helicity states at these temperatures then the average number of  $X$  particles emitted per black hole is given by: (using 6.1-1, 6.1-2)

$$N_x = \frac{M_{BH}^0}{\frac{3}{2} \langle kT \rangle N} = \frac{0.1 M_{Pl}}{4 M_x N} \left( \frac{M_{Pl}}{M_x} \right) \approx \frac{5 \times 10^3}{N} \quad (6.1-3)$$

where we have assumed  $M_x \approx 10^{14}$  GeV.

The total number density of  $X, \bar{X}$  pairs emitted is then related to the number density of black holes at the time of emission ( $\approx$  decay time):

$$n_x = N_x \left( \frac{\rho_{BH}}{M_{BH}} \right) \Big|_{t=t_{decay}} = \frac{5 \times 10^3}{N} \left( \frac{\rho_{BH}}{M_{BH}} \right)_{t=t_{decay}} \quad (6.1-4)$$

The net number density of baryons produced will then be given by  $n_B = \Delta B n_x$ , where  $\Delta B$  is related to the CP violating parameter of the theory<sup>6</sup> (see Appendix V).

We can derive the ratio of baryon density to photon density at present by considering the baryon to entropy density ratio, which is constant for the adiabatic expansion presumed to follow the black hole decay. Since  $n_\gamma \approx \frac{s}{7}$  ( $s$  = entropy density) we have:

$$\left( \frac{n_B}{n_\gamma} \right)_{\text{present}} = \frac{7 \Delta B n_x}{s^c/k} = 2.6 \times 10^9 \frac{\Delta B}{N} \left( \frac{\rho_{BH}}{M_{BH}} \right)_{t=t_{decay}} \times \frac{kT^f}{\rho_{BH}^+ c^2} \quad (6.1-5)$$

where  $S^f$ ,  $T^f$ ,  $\rho_T^f$  are the entropy density, temperature, and total mass density just after the black holes have evaporated, and  $S^f$  is given by:

$$\frac{S^f}{K} = \frac{4 P_{\text{TOT}}^f c^2}{3 K T^f} \quad (6.1-6)$$

where we assume the final state is radiation dominated.

Several features of this result are notable. First;  $\left(\frac{n_B}{n_Y}\right) < \frac{\Delta B}{N}$ . This is due to the fact that the R.H.S. is the baryon to radiation ratio produced by the black hole. This ratio can only be decreased by the subsequent out of equilibrium decays and annihilation of particles to produce photons. Thus, the lower the temperature of space when the black holes decay, the greater the entropy produced, and the smaller the ratio (6.1-5).

Next, as we have noted earlier, due to the relationship of expansion rate to black hole density the final mass densities, temperatures, and thus entropy densities are the same for all initial mass densities for a given black hole initial mass. Hence, for initial mass  $M_0$  holes, baryosynthesis is independent of the original mass density.

From the data in Table 3 (Appendix IV), and using  $N \approx 40$  we get:

$$M^0 = 10^{-4} \text{ kg} \Rightarrow \left(\frac{n_B}{n_Y}\right)_{\text{present}} \approx 1.4 \times 10^{-3} \Delta B \quad (6.1-7)$$

$$M^0 = 10^{-2} \text{ kg} \Rightarrow \left(\frac{n_B}{n_Y}\right)_{\text{present}} = 1.6 \times 10^{-7} \Delta B \quad (6.1-8)$$

Thus, the larger the initial black hole mass, the smaller the baryon to photon ratio. For  $M^0 \approx 10^{-4}$  kg that the ratio

produced is somewhat lower than that produced via non-equilibrium decay in the standard FRW cosmological expansion.<sup>7</sup> This means that SU(5) in its simplest form, with  $\Delta B \lesssim 10^{-17}$ , is not compatible with the observed  $\frac{n_B}{n_\gamma} \approx 10^{-10 \pm 1}$  if baryon production is via the above mechanism, just as it is not compatible with the standard baryosynthesis mechanism.<sup>8</sup> However, schemes with more complicated higgs structure, and more complicated gauge groups which have been developed for the standard baryosynthesis mechanism, with  $\Delta B \lesssim 10^{-5}$ , are phenomenologically acceptable in this model. Thus we conclude that if  $M^0 \lesssim 10^{-4}$  kg, acceptable baryosynthesis is possible in this scenario. Moreover since  $T_{\text{space}} < 10^{14}$  GeV at all times, this scenario provides the significant advantage of removing the constraints on particle masses needed in the standard model to produce a non-equilibrium situation. Thus, in this scenario, smaller higgs (and/or vector) masses result in larger net baryon to photon ratios, a result very different from the standard model.<sup>9</sup> If the higgs mass  $\approx 10^{13}$  GeV then this may increase  $\frac{n_B}{n_\gamma}$  in (6.1-8) by a factor of  $10^2$ . We also note, that if there exists an initial distribution of black holes, or if the temperature of space were at some early point rather hotter than the temperature of black holes, then larger mass holes would tend to reduce any initial symmetry - first by asymmetric absorption, then by increasing the total entropy as they evaporate. These effects depend on factors such as initial asymmetry, black hole densities and masses as discussed in Section 5.2.

## 6.2 Monopole Production

The existence of superheavy magnetic monopoles, which may have recently been observed<sup>10</sup>, is one of the most exciting predictions of grand unified theories. On the other hand, monopoles provide one of the strongest constraints restricting grand unified cosmology. It is difficult to devise a transition from the symmetric to broken phase that does not produce too many monopoles to agree with present mass densities<sup>11</sup> and galactic magnetic fields.<sup>12</sup> However, estimates of monopole production and remnant densities are problematic. Monopoles are classical solutions in a quantum field theory. As such they must not be considered as elementary quantum excitations out of the vacuum, but rather as a coherent state of quantum excitations. As a result, it is possible that their production is suppressed.<sup>13</sup> At the very least their production is intimately tied to the complicated phase transition dynamics of the elementary Higgs fields on which they are built.

Due to the novel thermal behavior in our model scenario it is interesting to consider the possibility that within it monopole production is at acceptable phenomenological levels. We will consider here a number of possibilities, related first to production in the initial transition, and then to production via subsequent black hole evaporation. The latter subject is interesting in its own right. Estimates are presented which indicate that such production is suppressed. We will consider this question in more detail in a subsequent publication.

As far as production resulting out of transition to a black hole "gas" is concerned, it was demonstrated in Section 4 that the temperature of space may always have been lower than the critical temperature associated with GUT transitions. This will happen if  $M^0 \geq 10^{-4}$  kg. Monopole formation is associated with local zero of the Higgs expectation value (symmetric regions) surrounded by regions where the Higgs takes on its low temperature nonzero expectation value.<sup>14</sup> The question then becomes one of whether or not any such locally symmetric regions are produced during the transition. The initial expectation is that if the transition occurs at low temperatures, the Higgs will have a nonzero expectation value in both phases (the universe will always be in the nonsymmetric state) and no monopoles will be produced. However, one is somewhat uncomfortable with prescribing certain Higgs behavior in a transition whose parameters we do not know. It is not clear, for example, that the Higgs expectation value is well defined in a "pre-gravitational transition" phase. Alternatively, it may be possible that before the state settled down to the equilibrium black hole configuration that high temperatures occurred. Thus we investigate the conservative alternative that Higgs expectation values in different regions are uncorrelated, with the possibility of local zero values, and use standard topological-horizon arguments to estimate maximum densities which may result.

Following the arguments of Kibble<sup>15</sup> and Einhorn<sup>15</sup> we assume that if this situation exists, there will be on the order of  $\sim .1$  monopole per horizon volume by the time the transition is completed. The results from topological frustrations when one

tries to extend certain uncorrelated regions with nonzero Higgs expectation continuously into a central region. In the black hole configuration resulting from such transition, the initial horizon volume is approximately the inverse black hole density. Thus, we can estimate the initial monopole density as a function of black hole density, and from the initial temperature, the initial monopole to photon ratios. However, during the subsequent evolution significant entropy is generated. If we assume for the moment that no significant monopole production occurs during this period then the initial monopole to photon ratio is severely diluted. Moreover, if initial density is that  $n_m/n_\gamma \gg 10^{-10}$ , the annihilation processes may reduce this ratio<sup>17</sup> to  $\sim 10^{-10}$  before the black holes evaporate. Table 3 gives the result of calculations for  $M_0$  initial mass  $10^{-4}$  kg and  $10^{-2}$  kg for a range of initial mass densities. We estimate the effect of annihilation processes on reducing initial mass densities by calculating the two terms in the relation:

$$\frac{dn_m}{dt} = -Dn_m^2 - 3\frac{\dot{R}}{R}n_m \quad (6.2-1)$$

where we have used Preskill's<sup>18</sup> estimate for  $D$  for temperature  $< 10^{-2} M_{\text{monopole}}$  (in units  $\hbar = c = 1$ ,  $D \sim 10^2 T^{-2}$ ). When  $Dn^2 \gg 3\dot{R}/R$ , we expect that the initial ratio ( $n_m/n_\gamma$ ) will quickly be reduced to  $\sim 10^{-10}$ . Final estimates of ( $n_m/s$ ) are presented with and without this assumption.

We see first that final monopole density is suppressed more in initial transitions resulting in larger black holes,

due to the entropy production by these holes. With the assumption that annihilation to a level of  $(n_m/n_\gamma) \sim 10^{-10}$  occurs when  $Dn_0^2 > \dot{R}/R n_0$  we see that it is possible (even if monopoles are produced in abundance in the initial transition), that for  $M^0 \geq 10^{-2}$  kg the ratio  $(n_m/s)$  can be made small enough not to violate present mass density limits  $(n_m/s)_{\text{present}} \leq 10^{-24}$ , and definitely small enough not to affect nucleosynthesis constraints  $(n_m/s) < 10^{-19}$ . For  $M_0 \times 10^{-4}$  kg, monopole densities that are produced in the transition are approximately small enough not to violate nucleosynthesis limits, but must be reduced by subsequent annihilation (perhaps in galaxies) in order to agree with present mass density limits. We recall that these limits only exist if the Higgs field does not have a uniform well defined nonzero expectation value in the pre-transition phase. If it does, then we expect little or no production in the transition.

We now investigate the possible production of monopoles during black hole evaporation. It may first be expected<sup>19)</sup> that black holes may produce monopoles at a thermal rate once their temperature exceeds the monopole mass, which is  $\sim M/\alpha \sim 10^{16}$  GeV in simple grand unified theories.<sup>20)</sup> This results in prohibitively large production rates in our scenario. (i.e.  $n_{\text{monopole}}/n_{\text{baryon}} \sim 10^{-2}$ ) However, this naive expectation must be refined to take into account the peculiar dynamics and structure of monopoles. Indeed, as we demonstrate, there



are a variety of reasons to expect that this production will be exponentially suppressed, at the very least. Due to uncertainties in the production mechanisms, we consider a number of different alternatives. All are suppressed, but for different reasons.

The first problem we encounter when trying to make sense out of monopole production, is that black hole radiation has a thermal spectrum, i.e., a black hole in thermal equilibrium at temperature  $T$  will emit as many particles in each mode as it absorbs in that mode from the ambient thermal radiation. Monopoles, however, are complicated objects, and thus have a unique thermal behavior. Below  $T_c$ , when the Higgs field  $\phi$  has a well defined nonzero expectation value, monopoles exist, with a structure determined by this expectation value. Since their mass is greater than  $T_c$ , however, population in thermal equilibrium is suppressed. Above  $T_c$ , it is not clear that monopoles have any meaning. The Higgs expectation value is zero, and while local fluctuations may produce local nonzero values, these are rapidly changing, and no asymptotic structure can be defined. Thus, it is a nontrivial question to consider whether monopoles can exist in thermal equilibrium at temperatures  $T > T_c$ . Nevertheless we will investigate the possibility that monopole-antimonopole pairs are produced near the event horizon of black holes which radiate at a temperature  $T > M_{\text{monopole}}$ .

The production cross-section for monopole-antimonopole pairs may also be expected to depend on the production mechanism, i.e., are monopole-antimonopole pairs produced directly by the gravitational field near the horizon, or do they result from thermal fluctuations of elementary Higgs configurations produced near the horizon? Also, if one member of a monopole-antimonopole pair is absorbed by the hole while the other escapes to infinity, the hole will become magnetically charged. Not only do we expect significant corrections in production due to this large fluctuating magnetic charge, but if the energy of the black hole changes significantly, we might expect the semiclassical approximation of fixed background metric to break down. Nevertheless we can proceed undaunted to estimate maximum production rates, making simplifying assumptions whenever possible which can only enhance them, and by attempting to determine rates which are independent of the specific mechanism of production-limited only by energetics.

Particle production by black holes is determined by the condition that at thermal equilibrium, production rates and absorption rates are equal. Here departures from black-body thermal spectrum can be obtained by estimating suppression in absorption cross-sections.<sup>21</sup> Thus, we examine absorption of a monopole of mass  $\sim M_x/\alpha \sim 10^{16}$  GeV by a neutral black hole of mass  $M_{BH} (\approx (8\pi G KT)^{-1}, \text{ where } KT = M_{\text{monopole}})$ . The classical size of an equilibrium monopole configuration is  $\sim M_x^{-1} \sim 10^{-14} \text{ GeV}^{-1} \sim (10^{-30}$  in MKS units). A black hole of the required mass ( $< 10^{-6}$  kg) has Schwarchild radius  $\sim 10^{-33} M$ .

In order to be absorbed by the black hole, tidal forces must crush the monopole down to the size of the hole. If this occurs and the monopole is captured, then the black hole will become magnetically charged, with long range  $\vec{B}$  fields. We can roughly estimate the energy gain by the black hole (we will give a better estimate based on monopole energetics later) by considering its magnetic energy. Magnetic energy density is  $\sim 1/2 B^2$  and a spherical object of size  $R_s$  with magnetic charge  $1/g$  ( $g \sim$  GUT coupling) has a field  $B \sim 1/gr$  outside  $R_s$ . Thus its total energy is

$$\frac{1}{2} \int_{r=R_s} B^2 d^3r \sim \int_{r=R_s} \frac{d^3r}{g^2 r^4} \sim \frac{1}{g^2 R_s} = 10^{19} \text{ GeV} \quad (6.2-2)$$

Hence the mass of the black hole increases by more than the rest mass of the monopole. We then expect suppression of the monopole absorption cross-section in much the same way that absorption of massless particles with nonzero spin is suppressed for long wavelengths.<sup>22</sup> There, the suppression is essentially due to the fact that angular momentum transferred to the hole when they are absorbed will be large compared to the initial energies of those particles.

That energetics inhibit monopole absorption by black holes, when considered in light of the implied suppression in production via quantum process, is a manifestation of the fundamental fact that monopoles are classically extended structures with size larger than their Compton wavelength. We might expect that to produce a monopole as if

it were normal quantum excitation, that the initial region of support for the wavefunction of the monopole must be at most of the order of its Compton wavelength, or of the order of the size of the quantum state from which it is being produced. This can be shown to lead to a soliton-like production suppression factor  $\exp(-A/g^2)$ , usually derived using perturbative arguments.<sup>23</sup> We will discuss this point in more generality in a subsequent publication. Here we discuss how this factor may explicitly be derived by direct consideration of monopole production by black holes, via an energetics argument similar to the one which lead to (6.2-2).

Consider a black hole of temperature  $T \gtrsim M_{\text{monopole}}$ . Assume monopoles are produced freely near the event horizon (neglecting complications due to magnetic charge fluctuations of the hole.) Their characteristic scale must initially be of order of the scale of the black hole radius. After production they may relax to their equilibrium scale. We may, however, estimate the energy of the initial configuration. Assuming, for simplicity, a 't Hooft-Polyakov<sup>24</sup> hedgehog type monopole with the Higgs field responsible for symmetry breaking in the adjoint representation of the gauge group, we can write the energy density of the monopole schematically (suppressing group indices, etc.) as:

$$E \sim \frac{1}{2} (D_\mu \phi)^2 + \frac{1}{2} B^2 + V(\phi) \tag{6.2-3}$$

where  $V(\phi) = \lambda(\phi^2 - v^2)^2$ ,  $D_\mu = \partial_\mu - igA_\mu$

The monopole is made up of two asymptotic regions. In a central region of scale  $R_1$ ,  $\phi$  will have its symmetric expectation value  $\phi=0$ . At large distances  $\phi \rightarrow v$  and  $D_\mu \phi \rightarrow 0$ . There will be a transition region of size  $R_2$  between these regions. We note that in the broken phase  $\phi=v$ , gauge particles get a mass  $m_V \sim gv$  and the remaining massive Higgs get a mass  $m_H \sim \sqrt{\lambda} v$ .

In region one, the energy density is given purely by the potential term  $V \sim \lambda v^4$ . Hence the total energy in this region is of order  $\sim R_1^3 \lambda v^4$ . In the outer region only the second term contributes. Since the field in this region is  $B \sim 1/gr^2$  we have  $E_3 \sim \int B^2 d^3r \sim 1/g^2 R_2$ . Outside the central region the gauge field aligns itself outside this region so that the covariant derivative is zero, so that we expect that in this region the contribution from the covariant derivative will be at most of the order of the pure derivative part. In the hedgehog solution the component of  $\partial_\mu$  which is important is the angular piece  $\sim 1/r \partial/\partial\theta$ , which acts on the  $\phi$  field (which has constant value  $v$  and rotates around the sphere) to give a contribution  $\sim v/r$ . Hence in this region  $E_2 \sim \int_{R_1}^{R_2} d^3r v^2/r^2 \sim v^2(R_2 - R_1)$ . Hence the total energy can be approximated by:

$$E_T \sim R_1^3 \lambda v^4 + v^2(R_2 - R_1) + \frac{1}{g^2 R_2} \quad (6.2-4)$$

Minimizing this separately with respect to  $R_1, R_2$  gives:

$$R_1^2 \sim \frac{1}{\lambda v^2} = \frac{1}{m_H^2} \quad ; \quad R_2^2 \sim \frac{1}{g^2 v^2} \approx \frac{1}{m_V^2} \quad (6.2-5)$$

which is why we associate a scale  $R_2 \sim M_V^{-1}$  with the monopole.

This variational calculation implies that at the minimum configuration each term in (6.2-4) contributes equally. This implies  $E_T \sim M_{\text{monopole}} \sim 1/g^2 R_2 \sim 1/g^2 M_V$ , which is where the mass estimate of the monopole equilibrium configuration comes from. Now if we constrain the monopole so that  $R_1, R_2 \ll m_H^{-1}$ , then the energy (6.2-4) is minimized if  $R_2 \approx R_1$  (from the second term). Since in this case the first term is much less than its equilibrium value, this implies that the energy of the new configurations will be dominated by the third term.

Hence  $E'_T \sim M'_{\text{monopole}} \sim 1/g^2 R_2$ .

If  $R_2$  is constrained to be  $\approx R_{\text{BH}}$ , then from the relations (in units  $\hbar = c = 1$ )  $R_{\text{BH}} \sim 2 M_{\text{BH}}/M_p^2$ , we get

$$M' \sim \frac{1}{2g^2} \frac{M_p^2}{M_{\text{BH}}} \quad (6.2-6)$$

On the other hand, for the black hole we have  $T = 1/8\pi GM = M_p^2/8\pi M_{\text{BH}}$ . Hence, we have (modulo factors of  $2\pi$ , etc., from volume factors, etc.)

$$\frac{M'}{T} \sim \frac{4\pi}{g^2} \sim 50 \quad (6.2-7)$$

In a thermal state at this temperature this state will occur with probability  $\exp -M'/T \sim \exp(-4\pi/g^2)$ . Hence, whether or not the monopoles are produced with thermal probabilities directly via the gravitational field, or whether they are produced via fluctuations of Higgs fields produced by the hole, we expect they will be suppressed. The factor is of the form of the standard soliton production suppression

factor. The situation is not helped by going to higher temperatures. Not only will the size of the black hole emitting at higher temperatures be decreased, thus increasing the mass of the monopole state, but the total mass of a black hole at this temperature becomes less than the mass of the monopole.

On the other hand, one might expect that, if the Higgs field expectation value (and hence the Higgs field mass) are reduced near the black hole surface that this suppression might be less severe. After all, the monopole equilibrium mass goes as  $M \sim m_x/g^2$ , which goes to zero as  $m_x (m_H) \rightarrow 0$ . However, if this occurs, then the equilibrium size of the monopole increases (so that the B field contribution to the energy is reduced.) Hence a greater size suppression is required, and the net energy of the monopole configuration produced near the black hole horizon will be the same. (This reflects another difference between classical solutions, and true quantum excitations, which can remain point-like even as their mass changes).

One might, however, carry this argument further and expect that symmetry will be restored in large regions around a black hole, in which the temperature is effectively greater than  $T_c$ . On the borders of this region one might expect thermal monopole production. However, Candelas<sup>25</sup>, has shown that outside a black hole ( $T \gg T_c$ ) radiating into cold space that  $T_{\text{EFF}}^2 \sim r^{-2}$ . This implies that the region over which symmetry is restored has radius at most of order  $m_H^{-1}$ <sup>26</sup>. If monopoles are thermally produced with initial size at most the order of this radius their mass will be

approximately equal to their equilibrium mass  $\sim T_c/g^2$ . Hence thermal production in this region will be suppressed by a factor  $\sim (\exp-1/g^2)$ . Even if monopoles produced at the border of the symmetric and non-symmetric regions had much smaller masses one could estimate that their thermal production, over the lifetime of a black hole hot enough to produce a large symmetric region, would be negligible.<sup>21</sup>

Thus, we conclude from this section that monopole production via black hole evaporation is strongly suppressed, and even if no monopoles are produced during the initial transition, very few are produced afterwards. Based on our earlier considerations, if monopoles are produced in abundance during the transition, this number is not increased afterwards, and thus may still be small enough to agree with constraints of astrophysical phenomenology.



### 6.3 Inhomogeneities

It is interesting to consider whether the density fluctuations in the initial black hole-radiation state result in remnant inhomogeneities which may be related to the formation of galaxies. It is currently felt that galaxies formed on the scales they have because shortly after the recombination time ( $t \sim 10^{12}$  sec) the Jean's length drops sharply.<sup>27</sup> This allows any surviving fluctuations on scales longer than this length to become unstable under their own gravitational interactions, fermion clumps which subsequently evolve into galaxies and clusters of galaxies. On the other hand, these relic inhomogeneities must not be too large if they are not to collapse into black holes, as we shall describe shortly.

Whether an initial fluctuation will survive until this era depends on its initial form: adiabatic or isothermal. Adiabatic fluctuations are ones in which matter fluctuations (characterized by the local density of baryons) are coupled to those of radiation, so that the net ratio of baryon to photon density remains constant throughout space. This implies  $\delta\rho/\rho = 3\delta T/T$  ( $T =$  temperature of radiation.) During universe expansion in the radiation dominated phase, adiabatic perturbations larger than the Jean's scale will grow as fast as  $\sim t$ , while those smaller will exhibit damped acoustic oscillations.<sup>28</sup> It is estimated that only those that result in fluctuations on mass scales greater than  $\sim 10^{11} M_{\text{solar}}$  at the time of recombination will survive this damping.<sup>29</sup> While this is in the range of the mass of galaxies today, adiabatic fluctuations at recombination time should also be reflected in the black-body radiation spectrum. Such fluctuations have not yet been conclusively

observed.<sup>30</sup> Hence there is some interest in whether the relevant primordial fluctuations might have been isothermal in character.

Isothermal fluctuations are those in which matter fluctuations are independent of radiation so that the net radiation temperature is constant throughout space. Such fluctuations remain constant during the one-recombination expansion period, when matter and radiation are coupled via thermal processes. Since the relevant Jean's mass at recombination is  $\sim 10^5 M_{\text{solar}}$ , about the size of a globular cluster, any initial isothermal fluctuations on this scale will, after recombination, grow.

The initial black hole configuration in our model clearly involves initial small scale isothermal density fluctuations. Thus, it is in principle possible that baryosynthesis in this model may be tied to isothermal perturbations-- exactly opposite behavior from that of the standard scenario. This has evoked interest in possibly tying baryosynthesis constraints and galaxy formation.<sup>31</sup> Unfortunately, however, the remnant inhomogeneities which are important are those surviving after black hole evaporation. As demonstrated clearly in section 4, entropy generation by evaporating black holes is at least on the order of the ambient background radiation entropy at the time of decay. Hence any remnant baryon density fluctuations will also involve fluctuations in radiation and will thus be primarily adiabatic.

We first demonstrate easily that such remnant fluctuations on the scale of the black hole inverse density ( $r \sim \{M/\rho\}^{1/3}$ )

will be on scales smaller than the Jeans scale. This scale,  $\lambda_J$ , represents the scale at which pressure gradients can balance gravitational forces. For larger scales gravity dominates, whereas for smaller scales, pressure dominates and perturbations behave like acoustic waves. We can estimate the Jean's scale immediately. Let the gravitational free fall time (ignoring pressure) in a uniform region, be  $t_f$ . If a soundwave can cross the region on a time-scale less than that associated with this gravitational collapse than pressure stabilizes the region. Thus, the Jean's scale is approximately  $v_s t_f$ , where  $v_s$  is the speed of sound in the region. This argument also turns out to be valid in an expanding curved universe, as at the Jean's scale these factors are negligible.<sup>32</sup> Since,  $v_s^2 = (\partial\rho/\partial p)$ , and for radiation  $p \sim \rho c^2/3$ , we have that for a radiation dominated phase  $v_s \sim c/\sqrt{3}$ . Also, since in the early universe  $t_f \approx t$ , we see that the Jean's scale is approximately the horizon distance at time  $t$ .

Now we expect that the maximum initial fluctuations will be on a scale of the order of the distance between black holes. It is clear from the data on Table 1 in section 4, that these scales are much smaller than the horizon scale at the decay time for black holes. Hence, any remnant adiabatic fluctuations will be damped. It is also clear that for black holes of primary interest ( $\sim 10^{-4}, 10^{-2}$  kg) the mass contained within this fluctuation  $\ll 10^{11} M_{\text{solar}}$ , and thus we expect that such adiabatic fluctuations will be negligible at the recombination time. We also note that any accompanying isothermal fluctuation, which will survive until the recombination time will have a

characteristic mass  $< 10^5 M_{\text{solar}}$ , and thus will also not be important after recombination. Even in an initial state with a distribution of black holes of different masses, as described in section 5 would result in inhomogeneities which are negligible, unless the initial holes were so exceedingly large as to be virtually impossible in the initial state.

However, any black holes with masses  $\geq 10^{10}$  would survive until the recombination era. These are too small to contribute to galaxy formation, and so large as to be very improbable in the initial state. Still, if they exist, and survive until galaxies begin to form they may begin to grow by the accretion of matter, and survive until the present era. As such, they may be linked to present quasar densities. Since significant accretion may only begin in the late stages of galaxy development this implies a minimal size for black holes which survive. It may also explain why Quasars are only observed with redshifts less than a certain value. Such assertions are however highly speculative especially in the absence of an initial distribution function.

Unfortunately however the effect of these initial small scale isothermal fluctuations are secondary, when compared to larger scale possible initial adiabatic fluctuations.<sup>33</sup> Consider, for example, the possibility that there are statistical Poisson deviations in the initial spatial distribution of black holes. On the scale of  $N$  black holes we then expect that  $\delta\rho/\rho \sim N^{-1/2}$ . Also, on scales larger than the horizon the initial fluctuation is adiabatic because its wavelength is so large that the density

is uniform over distances where radiation drag might be important.<sup>34</sup> We then expect, during the matter dominated black hole era that these fluctuations can grow as fast as  $\sim t^{2/3}$ . After the black holes evaporate, the adiabatic fluctuations will have modes which grow as fast as  $\sim t$ .

We can now show that such initial fluctuations on the scale of galaxies will become intolerably large. The initial number of radiation quanta which, in the matter dominated era, eventually form galaxies is on the order of  $10^{74}$ .<sup>35</sup> Assuming say, for  $10^{-4}$  kg. black holes, that each hole emits  $10^8$  radiation quanta before it disappears (see section 6.1), then the initial number of black holes which could evolve into a galaxy would be on the order of  $10^{66}$ . Poisson deviations on this scale would lead to initial density fluctuations  $\delta\rho/\rho \sim 10^{-33}$ . From the time when expansion becomes significant ( $t_0 \sim 10^{-38}$  sec) until the black holes decay ( $t_f \sim 10^{-30}$  sec) these large scale fluctuations grow by a factor  $(t_f/t_0)^{2/3} \sim 10^5$ . If they then grow at a rate  $t$  then at a time  $t = 10^{-2}$  sec the density fluctuations  $\delta\rho/\rho \sim 1$ . This is still within the radiation dominated era and at this time the horizon scale is still much smaller than the scale of the fluctuations. This will then probably result in gravitational collapse into a singularity,<sup>36</sup> although the details of this collapse require further study.

While one might imagine mechanisms which slow somewhat the growth of inhomogeneities, the above estimates indicate the need to strongly suppress possible statistical fluctuations (of the Poisson type) in the initial state of our model. This may represent the most prominent "un-natural" fine tuning in this

scenario. Actually the fact that even extremely small fluctuations in the initial state are unacceptable is a remnant of the flatness problem which, as we have observed, is unresolved in both this and the standard scenarios. Thus small density fluctuations on large size scales in the initial density near the critical density will result in collapse on time scales smaller than the present Hubble time. We intend to investigate this phenomena further in a subsequent work.

#### 6.4 Implications for Supersymmetry

We note here, for completeness, that the fact that space might have always existed at temperatures  $< T_{\text{crit}}$  for grand unified theories, as is possible in our scenario, has potentially interesting ramifications for supersymmetry theories. As Weinberg<sup>37</sup> has pointed out, gravity splits the normally degenerate supersymmetric vacuum so that the broken symmetry vacuum we presumably live in is the state of highest energy density, and thus if the universe began at high temperatures in the symmetric phase it is difficult to imagine how symmetry breaking could occur. However, if the universe was always at a temperature less than the temperature where thermal effect favor the symmetric state, it may be possible in principle that this problem can be avoided. If the universe did begin in the nonsymmetric phase, then, as Weinberg has also shown<sup>38</sup> this phase is stable against decay.

## 6.5 Summary and conclusions

We have demonstrated, using the assumptions of an isotropic homogenous initial state involving a mixture of black holes and radiation which may have resulted from a gravitational phase transition, that the dynamics of the early universe could have departed in several important ways from the standard model. These departures are especially relevant to certain problems of interest to particle physicists, and indicate the importance of semi-classical gravitational effects in the early universe.

Specifically, the early universe in this model may have had a radiation temperature which never exceeded the critical temperature for restoration of symmetry in Grand Unified Theories. The early expansion in this model is matter dominated so that  $R(t) \sim t^{2/3}$ . During this period the black hole temperature increases slowly until the final moments of evaporation, while the temperature of space drops almost immediately and continues decreasing monotonically. As the black holes evaporate we have shown that they can generate a baryon to entropy ratio which may be as large as  $10^{-3} \Delta B$ , where  $\Delta B$  is related to the CP violating parameter in grand unified theories. This ratio occurs in spite of the fact that the temperature of radiation never exceeds  $\sim 10^{14}$  Gev because the black holes radiate superheavy particles which subsequently decay out of equilibrium. The generation of entropy by black holes is sufficiently large however, that monopole densities in the initial state may be diluted to phenomenologically

acceptable levels. We have also demonstrated that monopole production during black hole evaporation is strongly suppressed. It thus appears possible to avoid the monopole problem in this scenario.

However the fact that the flatness problem occurs implicitly in our initial state assumptions requires that the initial black hole distribution must be fine tuned. This seems to be a problem common to all models whose scale factor evolution resembles that of the standard model (i.e.  $R(t)$  goes as a power of  $t$ ). Whether this requires an "inflationary" period of growth, or whether the necessary fine tuning can be incorporated naturally in an initial state resulting from a physical process such as a phase transition is an exciting question for the future.



## Chapter 7: Gravity and Phase Transitions: Problems & Perspectives

As I have tried to demonstrate in this work, the possibility of a gravitational phase transition may allow for significantly different dynamics in the early universe, resulting in novel ways of dealing with a certain number of problems at the interface of particle physics and cosmology. Further could be done within the context of the specific model described here, including a systematic treatment of corrections to a number of the initial approximation we have used. More generally, investigations of black hole dynamics are important aside from the context in which they have been described above. There are many reasons to expect that early fluctuations might have resulted in black hole formation even in the standard model.<sup>1</sup> Moreover, as we hinted in section 6, an understanding of such cosmological objects as quasars may also depend on assumptions about primordial black holes. Also, of more direct interest to particle physicists, is the possibility, demonstrated in chapter six, of using the constraints of particle physics to probe the semi-classical gravitational effects responsible for black hole evaporation, and vice-versa.

As far as the general problem of understanding the ultimate high energy behavior of gravity, and of its role in the dynamics of the early universe, the scenario I have presented is only a suggestive, and clearly incomplete, first step. Definite dynamics are required if one hopes to derive a natural initial state configuration. In this final section I will briefly describe other related areas of investigation, and some perspectives on future developments. I will take advantage of the concluding nature of this section to make my remarks schematic

and suggestive. They are intended to present future directions rather than results.

As I outlined in chapter one there is direct field theoretic motivation for considering possible significant changes in the dynamics of gravity at high energies. The most useful method presently available in field theory to describe the high energy evolution of theories is the renormalization group,<sup>2</sup> which also has its origins in statistical mechanics (as does much of modern field theory).

In quantum field theory the renormalization group stems from the fact that in all quantum theories a dimensional scale naturally arises - the scale at which renormalized parameters are defined. The fact that the physical observables of the theory must not depend on the scale at which renormalization is performed implies relations - the so-called renormalization group equations - between the physical parameters of the theory; coupling constants, masses, fields, as a function of changes in the renormalization scale. By equating this scale to the characteristic energy of the interactions being probed, it is possible to determine, to all orders in perturbation theory, the behavior of the above physical parameters with energy.<sup>3</sup>

Weinberg<sup>4</sup> has suggested that certain appropriate behavior of the coupling constants of gravity might imply simplifications in the high energy structure of the theory which would allow an extension of the concept of renormalizability to make the low energy theory "sensible". As discussed earlier the non-renormalizability of the classical

Lagrangian (1.3-1) implies the necessity for an infinite number of divergent counterterms in the bare quantum Lagrangian. Each of these has associated with it a free coupling parameter. The presence of this infinite set of free parameters implies that the theory loses predictive power. Imagine, however, that in the infinite dimensional coupling constant space there exist trajectories of the renormalization group which approach an ultraviolet fixed point. Explicitly this implies the set of coupling constants has behavior governed by a generalized Gell-Mann Low equation:<sup>5</sup>

$$\mu \frac{d}{d\mu} g_i(\mu) = \beta_i(g(\mu)) \quad (7-1)$$

and that as  $\mu \rightarrow \infty$ ,  $\beta_i(g^*) = 0$ , where  $g^*$  is the ultraviolet fixed point. If we constrain the initial couplings to lie on an "ultraviolet critical surface" (i.e. all trajectories on this surface approach  $g^*$ ) and if this surface is finite-dimensional, then all but a finite number of couplings must be fixed, leaving a finite number of free parameters which describe the theory. Weinberg has called such a criterion "asymptotic safety."

What makes this criterion particularly interesting from the point of view of symmetry breaking and effective theories, is the similarity between this behavior, and the observed behavior in effectively non-renormalizable theories which exhibit phase transitions. Models such as the Gross-Neveu model,<sup>6</sup> and the non-linear  $\sigma$  model,<sup>7</sup> described previously, exhibit in certain limits non-trivial ultra-

violet fixed points in their coupling constants. Moreover these points are associated with boundaries between different phase behaviors of the theory.<sup>8</sup> Of particular interest is the non-linear  $\sigma$  model (see section 1.3). In  $2+\epsilon$  dimensions where the theory is effectively non-renormalizable, there exists in the large  $N$  limit a fixed point of order  $\epsilon$ . The value of the coupling constant,  $\lambda_c$ , at this ultraviolet fixed point determines the point at which the  $O(N)$  symmetry, realized non-linearly at low "temperatures" borders on the high "temperature" limit where a new dynamical bound state appears, degenerate with the  $N-1$  fields. These fields transform linearly under  $O(N)$ . Thus the high temperature theory which is deduced is simply the original theory we described in eq. 1.3-3, which was broken to yield the effective non-linear theory. Furthermore the mass of the dynamical bound state is related to the original dimensional coupling parameter.

As far as gravity is concerned, recent studies based on renormalization group arguments also point out the possibility that gravity possesses a non-trivial ultraviolet fixed point, at least in the large  $N$  limit,<sup>9</sup> or in  $2+\epsilon$  dimensions.<sup>10</sup> It would be extremely interesting if this type behavior could be convincingly demonstrated, both from the point of view of asymptotic safety, and from the point of view of the possibility of a gravitational phase transition.

The theory at the critical point is scale invariant. The scale invariance is broken at low energies only by the choice of the critical renormalization group trajectory. This could

relate the value of  $G$  to a symmetry breaking parameter. Such a possibility connects this approach with several others which attempt to derive gravity from a scale invariant theory. One such theory<sup>11</sup> is based on the observation that in the absence of the coupling  $G$ , Eq. (1.3-1) is scale invariant. Thus the assumption is that the scale  $G$  arises as a result of the particular choice of asymptotically flat metric at large distances, so that:

$$\langle 0 | g_{\mu\nu} | 0 \rangle \approx \frac{1}{4\pi G} \eta_{\mu\nu} \quad (7-2)$$

Remarkably, spontaneous dynamical scale symmetry breaking arises in yet another model in which gravity is induced as an effective interaction - this time from a curved space pure matter Lagrangian.<sup>12</sup> Since a scale invariant scalar field theory in curved space has an  $R\phi^2$  coupling term, if symmetry is broken so that  $\langle \phi \rangle \neq 0$ , this induces an effective interaction term proportional to  $R$ , resembling the Lagrangian of pure gravity.

These field theoretic approaches to the small distance structure of gravity have not yet produced conclusive results. However the possibilities of connecting asymptotic safety with symmetry breaking are worth further investigation. From the viewpoint we have stressed in this work, it may then be possible to relate this to explicit phase behavior in the early universe.

However, apart from the possibility of phase transition behavior involving gravity as a low energy effective theory,

the dynamics of a semi-classical gravity coupled to matter may be extremely important. For example, semi-classically coupling gravity to matter fields fundamentally affects vacuum decay phenomena in matter theories.<sup>13</sup> Moreover it has recently been shown that the same semi-classical gravitationally induced particle production effects responsible for black hole radiation (in this case, Hawking radiation in a de Sitter space) may crucially alter the new inflationary universe scenario.<sup>14</sup> Gravitational effects may also determine the phase structure of supersymmetric theories.<sup>15</sup>

In addition to these effects, instanton tunnelling transitions with semi-classical gravity itself, such as the black hole nucleation described earlier, may prove to be important.<sup>16</sup> Of particular interest, and a subject I will investigate in a subsequent work, is the interpretation and implication of de Sitter tunnelling events, and the possibility of decay from de Sitter space, which could have important ramifications for the cosmological constant problem.

Finally, there has yet to be a proper understanding of finite temperature effects in the quantization of gravity. Temperature is not a covariant quantity, yet it is clear that finite temperature renormalization will affect such quantities as the induced renormalized cosmological term that appears when gravity is coupled to matter.<sup>17</sup> Indeed, there exists the more general problem of treating finite temperature effects in the early universe when it is not clear that an isothermal ensemble truly represents the physical situation.

It is thus an exciting time in cosmology. The close connection between problems of cosmology and particle physics is attracting a whole new spectrum of physicists to investigate these problems. A probable concomitant of these investigations will be a better understanding of particle interactions including gravity at high energies. The close interplay between advances in particle theory on the one hand, and semi-classical effects of gravity on the other yields the exciting possibility of real breakthroughs in our understanding of the fundamental forces of nature at its smallest, and largest scales.

Appendix I: Effective Potential and Finite Temperature Field Theory

We describe here, using a scalar field theory for simplicity, how the classical potential function  $V(\phi)$  in the Lagrangian:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - U(\phi) \quad (\text{A-1})$$

must be modified to take into account higher order quantum and thermal corrections.  $U(\phi)$  becomes the first term in an expansion of the effective potential  $V_{\text{eff}}(\phi)$  which gives the symmetry structure of the full quantum theory.

(a) Quantum corrections

We begin by recalling that the full Green's functions, which give the physical content of the quantum theory, can be generated from the functional  $Z(J)$ :

$$Z(J) = \langle 0 | T \exp(i \int J(x) \phi(x) d^4x) | 0 \rangle \quad (\text{A-2})$$

Since

$$G_n(x_1, \dots, x_n) = \frac{\delta^n Z(J)}{\delta J(x_1) \delta J(x_2) \dots \delta J(x_n)} \Big|_{J=0} \quad (\text{A-3})$$

Using the well-known combinatoric fact that the exponential of all connected graphs gives the sum of all graphs;<sup>1</sup> we can define the generating functional for connected Green's functions by:

$$iW(J) = \ln Z(J) \quad (\text{A-4})$$

$Z(J)$  may be represented as a functional integral:<sup>2</sup>

$$Z(J) = N \int [d\phi] \exp(iS(\phi, T)) \quad (\text{A-5})$$



where  $S$  is the action of the theory given in (A-1) in the presence of a source  $J(x)$ . We also can use the well-known fact that the generating functional of connected graphs is related to the generating functional of one particle irreducible graphs. The above-mentioned connection between connected graphs and 1PI graphs can be written (to lowest order in  $a$ ):

$$\exp \left\{ \frac{i}{a} W(J) \right\} = N \int [d\bar{\phi}] \exp \left\{ \frac{i}{a} \left( \Gamma(\bar{\phi}) + \int d^4x J(x) \phi(x) \right) \right\} \quad (\text{A-6})$$

where  $\Gamma(\bar{\phi})$  is the generating function for 1PI graphs, in terms of some field function  $\bar{\phi}$ .

Now the R.H.S. of (A-6) can be evaluated for small  $a$  in the saddle point approximation, yielding (setting  $\int d^4x J(x) \phi(x) = J\phi$ )

$$W(J) = \left[ \Gamma(\bar{\phi}) + J\bar{\phi} \right] \bigg|_{\frac{\delta \Gamma}{\delta \bar{\phi}} = -J} \quad (\text{A-7})$$

Hence we have

$$\frac{\delta W}{\delta J} = \left[ \frac{\delta \Gamma}{\delta \bar{\phi}} \frac{\delta \bar{\phi}}{\delta J} + \bar{\phi} + J \frac{\delta \bar{\phi}}{\delta J} \right] \bigg|_{\frac{\delta \Gamma}{\delta \bar{\phi}} = -J} = \bar{\phi} \quad (\text{A-8})$$

Thus  $\Gamma(\bar{\phi})$  is the Legendre transform of  $W(J)$ , and is given

by the inverse of (A-7):

$$\Gamma(\bar{\phi}) = (W(\tau) - J\phi) \Big|_{\frac{\delta W}{\delta \tau} = \bar{\phi}} \quad (\text{A-9})$$

Hence  $\Gamma(\bar{\phi})$  is the functional such that  $\frac{\delta \Gamma}{\delta \bar{\phi}} = 0$  when  $\bar{\phi} = \left. \frac{\delta W}{\delta J} \right|_{J=0} = \langle \phi \rangle$ , and thus the extrema of  $\Gamma(\bar{\phi})$  determines the ground states of the full quantum theory.

In a transitionally invariant theory  $\bar{\phi}(x)$  is a constant. In this case we can write  $\Gamma(\bar{\phi}) = - \int d^4x [V_{\text{eff}}(\bar{\phi})]$ , defining the effective potential.  $V_{\text{eff}}(\bar{\phi})$  can thus be easily calculated in loop expansion terms of the generating functional for 1PI graphs. To lowest order in  $\hbar$  we have:

$$V_{\text{eff}}(\bar{\phi}) = V(\bar{\phi}) \quad (\text{A-10})$$

(b) Finite temperatures

Finite temperatures imply scattering events take place in a thermal bath at temperature  $T$ , and thus averaging should not be taken with respect to the vacuum state but with respect to the thermal bath. Hence Green's functions change:

$$\langle 0 | T(\phi(x_1) \dots \phi(x_n)) | 0 \rangle \rightarrow N \sum e^{-\beta E(\psi)} \langle \psi | T(\phi(x_1) \dots \phi(x_n)) | \psi \rangle \quad (\text{A-11})$$

( $|\psi\rangle$  represent a complete set of states with energies  $E(\psi)$ ).

One can thus define a finite temperature generating functional:

$$Z^{\beta}(J) = \frac{\text{Tr} \left\{ e^{-\beta H} \exp \left( i \int d^4x J(x) \phi(x) \right) \right\}}{\text{Tr} e^{-\beta H}}$$

This may in turn be written in terms of a functional integral as in (A-5). The important point here is the fact that the only change in the functional integral at finite temperature is in the boundary conditions on the set of paths on which the measure  $[d\phi]$  has support, since the state wave functionals  $\psi(\phi)$  merely determine the boundary condition on the functional integral.<sup>3</sup> This change in boundary conditions becomes especially simple in Euclidean space. Finite temperature greens functions are periodic (anti-periodic for Fermi fields) in Euclidean time with period  $\beta$ ,<sup>4</sup> (as can be shown from A-11 using cyclicity of the trace, and the Poincare transformation properties of  $\psi(\phi)$ ). Hence paths (field configurations) contributing to the functional integral must be likewise periodic (antiperiodic). This implies that the Feynman rules derived via the functional integral form of  $Z^B(J)$  will be unchanged, except for discretization of the  $ik_0$  variable ( $i\beta\omega_n = 2\pi n$  for bose fields). Hence 4-dimensional loop integrals become three-dimensional integrals times a one-dimensional infinite sum. Similarly vertex 4-dimensional delta functions become products of Kronecker delta functions and standard three-dimensional delta functions.

Thus the calculation of  $V_{\text{eff}}(\bar{\phi}, \beta)$  is in principle the same at finite temperatures, except with the new Feynman rules described above.

Appendix II: Semiclassical Metastable State Decay in Field Theory

From quantum mechanics<sup>1</sup> we know the decay probability of an unstable state is given by the imaginary part of its energy (free energy at finite temperature):

$$\Gamma = -\frac{2}{\hbar} \text{Im} E_0 \quad (\text{A2-1})$$

This ground state energy  $E_0$  can be computed by analogy to quantum mechanics, using the relation

$$\begin{aligned} \lim_{T \rightarrow \infty} \langle x | e^{-HT/\hbar} | x \rangle &= \lim_{T \rightarrow \infty} \sum_n e^{-E_n T/\hbar} |\langle x | n \rangle|^2 \\ &= e^{-E_0 T/\hbar} |\langle x | n_0 \rangle|^2 \end{aligned} \quad (\text{A2-2})$$

where  $\langle x |$  is a position eigenstate, and  $n$  are energy eigenstates, and  $n_0$  is the lowest energy eigenstate not orthogonal to  $\langle x |$ ; with energy  $E_0$ . If coordinates are chosen such that  $\langle x=0 |$  is the metastable ground state then we have the relation:

$$E_0 = \hbar \lim_{T \rightarrow \infty} \frac{1}{T} \log \langle 0 | e^{-HT/\hbar} | 0 \rangle \quad (\text{A2-3(a)})$$

Now the RHS of (A2-3) can be written in functional integral form using the Euclidean version of the Feynman path integral:<sup>2</sup>

$$\langle 0 | e^{-HT/\hbar} | 0 \rangle = N \int [dx] e^{-S_E(x)/\hbar} \quad (\text{A2-3(b)})$$

with boundary conditions

$$\begin{aligned} x(T \rightarrow +\infty) &= 0 \\ x(T \rightarrow -\infty) &= 0 \end{aligned} \quad (\text{A2-4})$$

and  $S_E$  is the Euclidean action:

$$S(x) = \int_{-T/2}^{T/2} dt \left\{ \frac{1}{2} \dot{x}^2 + U(x(t)) \right\} \quad (\text{A2-5})$$

The functional integral can then be evaluated in the semiclassical ( $\hbar=0$ ) approximation by performing a Gaussian approximation about each stationary part of  $S_E$  subject to the boundary conditions (A2-4). The calculation in field theory proceeds identically replacing  $x$  by  $\phi$  and  $S_E(x)$  by (see Chapter 2):

$$S_E(\phi) = \int d^4x \left[ \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + U(\phi) \right] \quad (\text{A2-6})$$

The boundary conditions are that  $\phi(\tau \rightarrow \pm\infty, \mathbf{x}) = 0$  (the unstable state in Figure 2), and also the condition  $\phi(\tau, |\vec{\mathbf{x}}| \rightarrow \infty) = 0$ , which ensures that only solutions with finite action give a non-vanishing contribution to the semiclassical evaluation of the functional integral.

Stationary points of (A2-6) are solutions to the Euclidean equation of motion:

$$\partial_\mu \partial_\mu \phi = U'(\phi) \quad (\text{A2-7})$$

(which is the classical equation of motion in an inverted potential  $-U(\phi)$ ), subject to the proper boundary conditions. Assuming these extrema of  $S_E(\phi)$  give the dominant contributions a Gaussian approximation to the integral is performed about each such stationary point. This can be shown to give the relation:<sup>3</sup>

$$N \int [d\phi] e^{-S_E(\phi)/\hbar} \approx N e^{-S_E(\bar{\phi})/\hbar} \left\{ \det[-\partial_\mu \partial_\mu + U''(\bar{\phi})] \right\}^{-1/2} \quad (\text{A2-8})$$

The important point is that while there may be several extrema of  $S_E(\phi)$ , only those with negative eigenvalues of the operator in brackets will contribute to the imaginary part of (A2-3) and hence to  $\Gamma$ . Such solutions are called instantons.

If Figure 2 represented a one-dimensional quantum mechanics potential  $U(x)$  the standard instanton solution which contributes to  $\Gamma$  is called the bounce and consists of a particle rolling off the hill at  $x=0$  in  $-U(x)$  at  $T \rightarrow -\infty$  bouncing off the "wall" at  $\phi=\phi^*$  and returning to  $x=0$  at  $T \rightarrow \infty$ .

In the scalar field theory described here, the minimum action instanton solutions to (A2-7) are  $O(4)$  invariant solutions and the decay rate per unit volume of the metastable state (false vacuum) is proportional to  $\exp(-S_4)$  where  $S_4$  is the Euclidean instanton action.<sup>4</sup>

Quantum corrections to this approximation can be determined by substituting  $V_{\text{eff}}(\phi)$  for  $U(\phi)$  (see Appendix I). Similarly metastable decays at finite temperatures can be calculated by using  $V_{\text{eff}}(\phi, \tau)$ , and by imposing periodic boundary conditions on Euclidean time of period  $\beta = 1/T$  on the functional integral, and its solutions.<sup>5</sup> One then deals with  $O(3)$  invariant "static" solutions to 3-dimensional Euclidean equations of motion, and the decay rate becomes proportional to  $\exp(-S_3/T)$  where  $S_3$  is the Euclidean action of the  $O(3)$  invariant solution.<sup>6</sup>

Under certain simplifying assumptions, these solutions look like spherical bubbles in Euclidean space inside of

which  $\phi=\phi_-$  and outside of which  $\phi=0$ . At the instant of formation ( $t=0$ ) in Minkowski space they are of the same form.

If the bubble once formed, is larger than a critical radius  $R_0$  for it to be energetically favorable to grow, it will. The critical radius  $R_0$  can be calculated from variational calculations.<sup>7</sup> As the energy difference between the two equilibrium states gets smaller, this radius gets larger.

Appendix 3: Black Hole Thermodynamics

The fact that black holes radiate thermally with temperature  $T = \frac{1}{8\pi M}$  (in units  $\hbar=c=G=k=1$ ), has been confirmed in a wide variety of ways, perhaps because it was too remarkable to be accepted on the basis of one derivation. We here describe a number of independent means of demonstrating the result, each of which illuminates some of the intricacies of gravity as a quantum field theory.

First, we note the immediate difficulty that in a curved space background the notion of particle states and vacua becomes ambiguous. In flat space one decomposes field operators into positive and negative frequency components, which are then interpreted as annihilation and creation operators respectively. I.E., if  $\phi$  is a massless scalar field satisfying  $\phi_{;ab}\eta^{ab} = 0$ , we express  $\phi$  as

$$\phi = \sum_i \{ f_i a_i + \bar{f}_i a_i^\dagger \} \quad (\text{A3-1})$$

where  $\{f_i\}$  are a complete basis of solutions of the covariant wave equation  $f_{i;ab}\eta^{ab}=0$ , with positive frequencies with respect to the time coordinate  $x^0=t$ . The vacuum is defined as the unique state such that  $a_i |0\rangle = 0$  for all  $i$  (i.e. one cannot annihilate any particles). However, in curved space, positive frequencies have no invariant meaning, because the "time" coordinate has no invariant meaning. Loosely speaking, it can be "rotated" from one point to another. In a space with local regions of non-zero curvature, the solutions to  $f_{i;ab}g^{ab}=0$  can be defined as having positive frequencies with respect to some asymptotically flat Minkowski time coordinate.



However the basis  $\{f_{1j}\}$  containing only positive frequencies with respect to one asymptotic region will not necessarily be the same as the basis  $\{f_{2i}\}$  defined on some other asymptotic region. Hence the initial vacuum state need not be the same as the final vacuum state, i.e.

$$a'_i |0_1\rangle \neq 0 \text{ even if } a_i |0_1\rangle = 0$$

Hence it will appear that a gravitational field can cause the creation of particles, or at least the definition of a particle state is observer dependent. The problem becomes more acute when one attempts to uniquely define  $\{f_i\}$  in a region of curved space. Here, unlike the asymptotic flat region, there is no unique definition of the subspace of solutions spanned by the  $\{f_i\}$ . Depending on the curvature, one may set up an inertial coordinate system in a region  $U$  of a point  $P$  with a coordinate radius  $R$ , of the order of the radius of curvature.<sup>2</sup> One can then choose a family of  $\{f_i\}$  which are approximately positive frequency with respect to the time coordinate of  $U$ . Clearly as the frequency  $\omega$  gets larger compared with the  $R^{-1}$  the approximation gets better. However for those modes  $\omega$  for which  $\omega < R^{-1}$ , the distinction between positive and negative modes virtually vanishes. This indeterminacy in defining modes of wavelength greater than  $R$  is reflected in an uncertainty in the local energy density of order  $R^{-4}$  (in units  $\hbar=c=G=1$ ).<sup>3</sup> This uncertainty can be thought of as corresponding to the energy density of particles created by the gravitational field. (The back reaction of this on the curvature, via Einsteins equations

is small provided the initial radius of curvature is small compared to the planck length - which is a statement that the semi-classical background metric approximation remains valid).

This particle creation via gravitational field is what is responsible for black hole radiation. The negligible back reaction described above (in order to maintain the validity of the semi-classical approximation), represents the statement that the black hole metric does not change significantly on a time scale given by the inverse rate of particle creation. This will be true for black holes larger than the planck mass.

The fact that the particle creation and emission rate from a black hole are proportional to the surface gravity  $\kappa$  ( $=\frac{1}{4M}$  in dimensionless units  $\hbar=c=G=1$ ) can be seen as follows. Imagine a virtual pair created just outside the event horizon, one particle having negative energy and the other positive. The particle with negative energy is classically forbidden, but if it tunnels through the event horizon, it can exist as a real particle, because negative energies with respect to infinity are allowed inside the black hole. The other member of the pair can escape to infinity where it is detected as thermal radiation. The tunnelling probability is related to the surface gravity  $K$  since this quantity governs the rate of change of the time translation killing vector (i.e. how fast this time-like killing vector becomes space-like - allowing the negative energy particle with respect to infinity to exist classically). We can actually derive an order of magnitude estimate of this process, using a slightly different heuristic description,<sup>4</sup> similar to the mechanism of pair creation by strong

background electric field.<sup>4</sup> If each member of the virtual pair has energy  $\omega$ , the maximum lifetime of the pair is given by a time of order  $\omega^{-1}$  (setting  $\hbar=c=1$ , and neglecting numerical factors). However if a force can do work on these particles, giving them an energy of  $\omega$  in a time of order  $\omega^{-1}$  then they can exist classically as measurable particles. Imagine now the creation in the field of a black hole of mass  $M$ , at a distance ( $r > 2GM$ ). If the particles are separated radially by a distance  $l$  then the tidal force separating them (which can be derived from Newtonian mechanics for the purposes of this argument) is of order

$$F \sim \frac{\omega GM}{r^3} l \quad (\text{Ap 3-2})$$

Hence for real pair creation we need:

$$\int_0^{\omega^{-1}} F dl \geq \omega \quad (\text{Ap 3-3})$$

Using (Ap 3-2) we get

$$GM r^{-3} \omega^{-1} \geq \omega$$

For the outgoing photon to reach infinity,  $r > 2GM$ . Hence:

$$\omega \leq (GM)^{-1} \quad (\text{Ap 3-4})$$

for significant pair creation. If all frequencies up to this value are created uniformly (to this approximation) then the differential luminosity  $\frac{dL}{d\omega} \sim \omega$  up to  $\omega_{\text{max}} \sim (GM)^{-1}$ . This gives

$L \sim (GM)^{-2}$ . If this radiation were actually thermal then we could use the blackbody formula  $L \sim AT^4$  for a radiator of area  $A$  to associate a temperature with this radiation, given by

$$T \sim (GM)^{-1} \quad (\text{using } A \sim (GM)^2) \quad (\text{Ap 3-5})$$

which is qualitatively correct. (The temperature increases as mass decreases because  $r$  decreases at the same rate as  $M$  and thus the tidal forces increase, increasing  $\omega_{\max}$ .)

To describe the exact field theoretic calculation we must however recall our first description of the ambiguity of determining positive energy states in curved space-time. For the case of interest, this ambiguity is explicitly determined for the case of a background Schwarzschild metric in asymptotically flat Minkowski space. Massless fields can then be described using (Ap 3-1) with the  $\{f_i\}$  uniquely defined at past null infinity  $\mathcal{I}^-$  to contain only positive frequencies with respect to the canonical affine parameter (proper time) there. Since Cauchy data on  $\mathcal{I}^-$  is sufficient to determine a massless field everywhere, the form (Ap 3-1) is applicable elsewhere. Another complete boundary value surface consists of future null infinity  $\mathcal{I}^+$  plus the event horizon surface outside the black hole (see the penrose diagram for this case - Figure 7).

Here  $\phi$  can be expressed as:

$$\phi = \sum_i \{ p_i b_i + \bar{p}_i b_i^+ + q_i c_i + \bar{q}_i c_i^+ \} \quad (\text{Ap 3-6})$$

where  $\{p_i\}$  have zero Cauchy data on the event horizon and  $q_i$  have

zero data on  $\mathcal{J}^+$ . Also  $\{p_i\}$  can be unambiguously defined to have only positive frequencies with respect to the affine parameter  $\mathcal{J}^+$ . Since both (Ap 3-1) and (Ap 3-6) describe  $\phi$  everywhere we must have:

$$p_i = \sum_j \{ \alpha_{ij} f_j + \beta_{ij} \bar{f}_j \} \quad (\text{Ap 3-7})$$

(with similar conditions connecting  $b, b^+$  with  $a, a^+$ ).

The critical vacuum state  $|0_- \rangle$  (containing no incoming particles) at  $\mathcal{J}^-$  is defined by  $a_i |0 \rangle = 0$ . However, because  $\beta_{ij}$  may not be zero in general an observer at  $\mathcal{J}^+$  will not measure this to be the vacuum state. Indeed he will find the expectation value of the number of particles in mode  $i$  to be

$$\langle 0_- | b_i^\dagger b_i | 0_- \rangle = \sum_j |\beta_{ij}|^2 \quad (\text{Ap 3-8})$$

Hence the calculation of the number of particles created by the hole and emitted to future null infinity reduces to calculating  $\beta_{ij}$ . The form of this calculation goes as follows. (The details can be found in Ref. (1).) Using continuous normalization, the solutions  $\{p_\omega\}, \{f_{\omega'}\}$  ( $\omega, \omega'$  refers to the continuous frequency variable) are expanded into their Fourier components, in terms of advanced and retarded time. Thus the sums in (Ap 3-7), (Ap 3-8) can be written as integrals. In order to find the phase relationship between  $\{p_\omega\}$  and  $\{f_{\omega'}\}$ , the backward propagation of the part of  $\{p_\omega\}$  which goes through the collapsing body and eventually emerges at  $\mathcal{J}^-$  is studied. This becomes tractable because near the event horizon, the retarded time coordinate

goes to infinity. Thus the effective frequency of the  $p_\omega$  gets very large near the event horizon, and it propagates through the body via the geometrical optics to  $\mathcal{I}^-$  where its form can be estimated. The phase that  $p_\omega$  acquires is determined by comparing the relationship between the retarded time  $u$  and a vector which connects the event horizon to a nearby null surface of constant  $u$  as one translates the vector along the null geodesic generating the future event horizon, and past the end point of this event horizon back to  $\mathcal{I}^-$ .

This phase is determined by the surface gravity of the black hole  $\kappa$ , since  $\kappa$  determines the scale change of the time translation killing vector on the horizon. Once the form of  $p_\omega$  on  $\mathcal{I}^-$  is found,  $\beta_{\omega\omega}$ , in the integral form of (Ap 3-7) can in principle be determined. Of course since continuum normalization is used, the integral form of (Ap 3-8) diverges, as the number of particles created over an infinite time is infinite. However by introducing finite wave packets one can establish the relationship:

$$\begin{aligned} & \text{(probability for black hole to emit particle to } \mathcal{I}^+ \\ & \text{with energy } E) \\ & = e^{-2\pi E/\kappa} \times \text{(probability for black hole to absorb particle} \\ & \text{from } \mathcal{I}^- \text{ with energy } E) \end{aligned} \quad (\text{Ap 3-9})$$

This condition can then be shown to imply that at equilibrium black holes radiate thermally with a temperature  $T = \frac{\kappa}{2\pi}$  (with  $\hbar=G=c=\kappa=1$ ).

Alternatively, a similar calculation can be done using Feynman path integral techniques for calculating particle propagation amplitudes.<sup>5</sup> Here, the amplitude that a particle is

produced by a black hole and detected by an observer outside the hole in a given mode at a point A, is given by an appropriately weighted sum over paths connecting points on the future singularity of the black hole and A. These paths can be analytically continued to paths connected points on the past singularity of an analytically continued complexified Schwarzschild space to A. Hence the above amplitude can be related to the amplitude to propagate to A from a point on this past singularity. This amplitude is just the time reversed amplitude for particle absorption by a black hole. Thus absorption and emission are related and (Ap 3-9) is again derived.

Perhaps the simplest field theoretic argument that black holes have associated with them a temperature is given by examining the functional integral formulation of gravity. Consider the Schwarzschild metric:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad (\text{Ap 3-10})$$

Putting  $t = i\tau$  converts this to a positive definite metric for  $r > 2M$ . The apparent singularity can be shown to be merely a coordinate artifact by defining a radial coordinate  $x = 4M(1 - 2Mr^{-1})^{1/2}$ , giving

$$ds^2 = x^2 \left(\frac{d\tau}{4M}\right)^2 + \left(\frac{r}{2M}\right)^4 dx^2 + r^2 d\Omega^2$$

There is again an apparent singularity at  $r = 2M, x = 0$ , but this can also be shown to be a coordinate artifact by taking  $\tau/4M$

to be an angular coordinate with period  $2\pi$ . This apparent singularity then becomes just like the origin of polar coordinates. Now if  $\tau/4M$  has period  $2\pi$ ,  $\tau$  has period  $8\pi M$ . Thus the Schwarzschild solution is a solution to the Euclidean equations with period  $8\pi M$  in imaginary time. Euclidean Greens functions defined on this background will then automatically also have period  $8\pi M$  in  $\tau$ . However, we know that the Greens functions of a theory at finite temperature  $T$  are periodic in Euclidean time with period  $\beta' = 1/T$ . Hence fields propagating in a black hole background will be propagating in a thermal background with temperature  $T = 1/8\pi M$ .



Appendix IV: Computer Program and Output

```

10 INPUT "Name of output file? ";outfiles
20 OPEN#2,outfiles
100 INPUT "Name of input file? ";infile
110 OPEN#1,infiles
120 ON EOF#1 CLOSE:PRINT"DONE":END
130 INPUT#1,m,number
131 IF m<1e-4 THEN item=80:ELSE item=25
132 IF m>1e7 THEN item=6
133 LET limit=1.0/(1.3e-20*m^2)
134 LET pb=limit*1e-2
135 IF pb<item/(1.4e-1*m^4) THEN GOTO 137
136 GOTO 130
137 PRINT PRINT"pb now set =";pb," and is below minimum allowed density"
138 PRINT#2,PRINT#2;"pb now set =";pb," and is below minimum allowed density"
139 PRINT"we now go to the next m value":PRINT
140 PRINT#2;"we now go to the next m value":PRINT#2;
141 GOTO 130
150 GOSUB 500
155 GOTO 130
200 IF (ERR)=255 THEN END:GOTO 130
210 PRINT PRINT"ERROR NUMBER ";(ERR)," IN LINE ";(LIRLIN)," HAS HALTED EXECUTION" PRINT
220 PRINT#2;PRINT#2;"ERROR NUMBER ";(ERR)," IN LINE ";(LIRLIN)," HAS HALTED EXECUTION" PR
INT#2;
230 GOTO 3050
500 LET FN mtmt(t)=fin*(D*M*ps-t*kappa/M^4)
501 LET FN ptpt(t)=-fin*k*t*c*SQR(pt)
502 LET FN ppsp(t)=-fin*(w*c*SQR(pt)+pb/ps*FN mtmt(t))
503 LET FN pbbD(t)=fin*(FN mtmt(t)-b*c*SQR(pt))
520 iteration=0:factor=0
550 REM ON ERR GOTO 200
600 PRINT"m=";m,"          pb=";pb,"          number=";number
610 PRINT#2;"m=";m,"          pb=";pb,"          number=";number
650 IF m>1e6 THEN fin=1e-10:ELSE fin=1
660 IF fin=1 THEN GOTO 700
670 PRINT PRINT"Time scale is now 1E10 times what it was":PRINT
680 PRINT#2;PRINT#2;"Time scale is now 1E10 times what it was":PRINT#2;
700 LET k=0.8e-15:D=2.0e-15:b=3.MIN=M
703 cns=1
704 vcrp=0
705 LET k=0
710 IF MIN<1e-4 THEN kappa=80:ELSE kappa=25
714 IF m>1e7 THEN kappa=6
715 krite=kappa
717 LET s1=6:low=1e2:twf=25
720 test=0:time=0
725 LET r1=1
730 rscoc=0
750 LET c=2.3e-5
760 LET r=1e-20:y=1e20:j=1e-10:puss=1e10
770 LET yellow=1e5:red=1e-5
780 w=4
790 LET same=1
800 RATE=6.6E9:BATE=1E10:TEMP=1E14
850 v=1
860 twen=50
890 ihou=100
900 sz=7/6:sz=2
950 counter=0
1000 su=rt=25:EIGHTY=80
1050 L=1:J=5:Q=3
1100 LET ps=k*KAPPA/M^4/D
1110 LET pt=pb*ps
1115 PRINT USING 9010;"ps","M","t","TS","TBH","R/RD","F"
1116 PRINT#2 USING 9010;"ps","M","t","TS","TBH","R/RD","T"
1125 IF ps>y THEN GOTO 1185
1175 IF ps>y THEN GOTO 1185
1180 GOTO 1192
1185 LET D=D*c*c*c*puss*pt*pb*r:ps=ps*r:pt=pt*r
1186 LET rate=rate*yellow
1187 LET factor=factor*1
1190 GOTO 1000
1192 IF ps>r THEN GOTO 1195
1193 IF ps>r THEN GOTO 1195
1194 GOTO 1000
1195 LET D=D*r:c=c*j:pb=pb*j:ps=ps*j:pt=pt*j
1196 LET rate=rate*red
1197 LET factor=factor-1
1300 LET kt=b*ps/dt
1700 LET J=1+iteration:iteration=iteration+1

```

```

1750 IF rescoe=1 THEN GOTO 1900
1760 LET h=ABS(same/ FN psp(t))
1770 IF (min-M)/min > .85 THEN rescoe=1
1900 LET ni=h* FN pbb(t)
2000 LET li=h* FN psp(t)
2050 mi=h* FN ptpt(t)
2100 LET ki=h* FN mmt(t):qz=M:v:ps:ap:ln=pb
2110 IF perp=10 THEN GOTO 3845
2120 IF li>twen THEN h=le-4*t:perp=perp+1:GOTO 1900
2150 t=t+fsh:M=q*EXP(f*ki):ps=v*EXP(f*li):pt=a*EXP(f*mi):pb=n*EXP(f*ni)
2200 IF M<(le-7*min GOTO 3700
2250 k2=h* FN mmt(t):l2=h* FN psp(t):m2=h* FN ptpt(t):n2=h* FN pbb(t)
2270 IF l2>twen THEN t=t-fsh:h=le-4*t:M=q:ps=v:pt=a:pb=n:perp=perp+1:GOTO 1900
2300 M=q*EXP(f*k2):ps=v*EXP(f*l2):pt=a*EXP(f*m2):pb=n*EXP(f*n2)
2350 IF M<(le-7*min GOTO 3700
2400 k3=h* FN mmt(t):l3=h* FN psp(t):m3=h* FN ptpt(t):n3=h* FN pbb(t)
2410 IF l3>twen THEN t=t-fsh:h=le-4*t:M=q:ps=v:pt=a:pb=n:perp=perp+1:GOTO 1900
2450 t=t+fsh:M=q*EXP(f*k3):ps=v*EXP(f*l3):pt=a*EXP(f*m3):pb=n*EXP(f*n3)
2500 IF M<(le-7*min GOTO 3700
2550 k4=h* FN mmt(t):l4=h* FN psp(t):m4=h* FN ptpt(t):n4=h* FN pbb(t)
2570 IF l4>twen THEN t=t-fsh:h=le-4*t:M=q:ps=v:pt=a:pb=n:perp=perp+1:GOTO 1900
2600 M=q*EXP(f*k4):ps=v*EXP(f*l4):pt=a*EXP(f*m4):pb=n*EXP(f*n4)
2650 ps=v*EXP(s*(li+x*l2+x*k3+k4))
2700 pt=a*EXP(s*(mi+x*m2+x*m3+m4))
2710 pb=n*EXP(s*(ni+x*n2+x*n3+n4))
2720 LET rt=rt*(kt/g*c*SQR(pt)*h+1)^(s/kt)
2750 IF counter=1 THEN GOTO 2900
2800 IF ps>(pt/5) THEN PRINT"t critical is ":t:PRINT#2;"t critical is ":t:ELSE GOTO 2900
2850 counter=1
2900 TS=RATE*SGR(SQR(ps/kr)) :TBH=BATE/M
2920 LET stand=kr
2940 IF ts>low THEN kr=twf
2950 IF TS>TEMP THEN kr=EIGHTY
2958 IF tbb>low THEN kppa=twf
2960 IF TBH>TEMP THEN kppa=EIGHTY
2970 LET ps=(ps*kr)/stand
3000 LET tes=time
3050 IF ps>v THEN time=1:ELSE time=0
3100 IF time>test THEN GOTO 3450
3150 IF time<test THEN GOTO 3250
3200 GOTO 3500
3250 PRINT:PRINT"ps has started to decrease at t=":t-h:"ps=":ps:"M=":M:"TB=":TBH:"TS=":TS:"R/R0=":rt:"factor=":factor:PRINT
3255 PRINT#2:PRINT#2;"ps has started to decrease at t=":t-h:"ps=":ps:"M=":M:"TB=":TBH:"TS=":TS:"R/R0=":rt:"factor=":factor:PRINT#2:
GOTO 3500
3400 PRINT:PRINT"ps has started to increase at t=":t-h:"ps=":ps:"M=":M:"TB=":TBH:"TS=":TS:"R/R0=":rt:"factor=":factor:PRINT
3450 PRINT#2:PRINT#2;"ps has started to increase at t=":t-h:"ps=":ps:"M=":M:"TB=":TBH:"TS=":TS:"R/R0=":rt:"factor=":factor:PRINT#2:
IF d*ps<(0.001*(k*kppa)/(m*ab)) THEN GOTO 3680
3500 IF i=number GOTO 3600
3550 GOTO 1125
3600 PRINT USING 9000:ps,M,t,TS,TBH,rt,factor:PRINT#2 USING 9000:ps,m,t,ts,tbh,rt,factor:PRINT
3610 IF iteration<1001 GOTO 1125
3650 PRINT"EXECUTION TERMINATING AFTER 1000 ITERATIONS"
3660 PRINT#2:"EXECUTION TERMINATING AFTER 1000 ITERATIONS"
3670 RETURN
3680 LET tau=(m*ab)/(b*k*kppa)
3681 PRINT:PRINT"Black hole now evaporating unimpeded in time t=":tau
3682 PRINT#2:PRINT#2;"Black hole now evaporating unimpeded in time t=":tau
3684 LET t=t+tau
3685 PRINT"total elapsed time is ":t
3686 PRINT#2:"total elapsed time is ":t
3687 LET k1b=0.5*(1-ps/pt)
3688 PRINT"kt is now ":kt
3689 PRINT#2:"kt is now ":kt
3690 LET pt=pt/(1+0.5*kt*c*SQR(pt)*tau)
3691 PRINT"pt is now ":pt
3692 PRINT#2:"pt is now ":pt
3695 LET rt=rt*(kt/2*c*SQR(pt)*tau+1)^(2/kt)
3697 PRINT"rt/R0 is now ":rt:PRINT
3698 PRINT#2:"rt/R0 is now ":rt:PRINT#2:
3699 GOTO 3842
3700 PRINT:PRINT" the smallest value of M is ":M
3705 PRINT" at this time the value of ps is ":ps
3800 PRINT" decay time (tp=t*10E-30) is ":t:PRINT
3810 PRINT#2:PRINT#2:" the smallest value of M is ":M
3820 PRINT#2:" at this time the value of ps is ":ps
3830 PRINT#2:" decay time (tp=t*10E-30) is ":t
3840 PRINT#2:PRINT#2:PRINT#2:PRINT#2:
3842 LET ts=(rate*SGR(SQR(pt/kr)))
3843 PRINT:PRINT"Tsace is now not more than ":ts:"Gev":PRINT
3844 PRINT#2:PRINT#2:"Tsace is now not more than ":ts:"Gev":PRINT#2:
3845 LET limit=limit*le-2:m=min
3847 POP GOTO 134
3848 PRINT:PRINT"we now go to the next m value":PRINT
3849 PRINT#2:PRINT#2:"we now go to the next m value":PRINT#2:
3850 RETURN
9000 IMAGE -6.404E,4X,-3.404E,4X,-8.404E,4X,-6.404E,4X,-8.404E,4X,-6.404E,4X,39
9010 IMAGE 130,150,150,150,150,150,5R

```

Appendix IV: Table A-1

ps	M	number=5	TS	TEH	R/R0	F
2.9032E+09	1.0000E-02	6.6809E-04	8.9275E+11	1.0000E+12	1.1256E+00	0
1.7744E+09	1.0000E-02	1.4718E-05	7.8936E+11	1.0000E+12	1.2471E+00	0
1.0845E+09	1.0000E-02	2.4386E-05	6.9795E+11	1.0000E+12	1.4265E+00	0
6.6285E+08	1.0000E-02	3.6016E-05	6.1712E+11	1.0000E+12	1.6056E+00	0
4.0513E+08	1.0000E-02	5.0010E-05	5.4535E+11	1.0000E+12	1.8074E+00	0
2.4762E+08	1.0000E-02	6.6848E-05	4.8248E+11	1.0000E+12	2.0347E+00	0
1.5135E+08	1.0000E-02	8.7113E-05	4.2659E+11	1.0000E+12	2.2906E+00	0
9.2506E+07	1.0000E-02	1.1151E-04	3.7719E+11	1.0000E+12	2.5789E+00	0
5.6542E+07	1.0000E-02	1.4088E-04	3.3351E+11	1.0000E+12	2.9037E+00	0
3.4586E+07	1.0000E-02	1.7625E-04	2.9489E+11	1.0000E+12	3.2698E+00	0
2.1125E+07	1.0000E-02	2.1888E-04	2.6074E+11	1.0000E+12	3.6825E+00	0
1.2913E+07	1.0000E-02	2.7030E-04	2.3055E+11	1.0000E+12	4.1481E+00	0
7.8937E+06	1.0000E-02	3.3237E-04	2.0386E+11	1.0000E+12	4.6738E+00	0
4.8257E+06	1.0000E-02	4.0739E-04	1.8026E+11	1.0000E+12	5.2679E+00	0
2.9304E+06	1.0000E-02	4.9823E-04	1.5940E+11	1.0000E+12	5.9455E+00	0
1.8041E+06	1.0000E-02	6.0850E-04	1.4095E+11	1.0000E+12	6.7033E+00	0
1.1035E+06	1.0000E-02	7.4279E-04	1.2465E+11	1.0000E+12	7.5708E+00	0
6.7486E+05	1.0000E-02	9.0706E-04	1.1024E+11	1.0000E+12	8.5612E+00	0
4.1294E+05	1.0000E-02	1.1092E-03	9.7496E+10	1.0000E+12	9.6973E+00	0
2.5277E+05	1.0000E-02	1.3601E-03	8.6237E+10	1.0000E+12	1.1010E+01	0
1.5481E+05	1.0000E-02	1.6749E-03	7.6289E+10	1.0000E+12	1.2539E+01	0
1.4878E+04	1.0000E-02	2.0759E-03	6.7500E+10	1.0000E+12	1.4344E+01	0
5.6205E+04	1.0000E-02	2.5971E-03	5.9738E+10	1.0000E+12	1.6509E+01	0
3.5752E+04	1.0000E-02	3.2925E-03	5.2886E+10	1.0000E+12	1.9159E+01	0
2.1994E+04	1.0000E-02	4.2527E-03	4.6837E+10	1.0000E+12	2.2492E+01	0
1.3556E+04	1.0000E-02	5.6349E-03	4.1499E+10	1.0000E+12	2.6815E+01	0
6.3763E+03	1.0000E-02	7.7197E-03	3.6768E+10	1.0000E+12	3.2612E+01	0
5.1769E+03	1.0000E-02	1.1007E-02	3.2624E+10	1.0000E+12	4.0000E+01	0
3.2046E+03	1.0000E-02	1.6351E-02	2.8937E+10	1.0000E+12	5.1754E+01	0
1.9839E+03	1.0000E-02	2.5148E-02	2.5668E+10	1.0000E+12	6.7278E+01	0
1.2279E+03	1.0000E-02	3.9600E-02	2.2767E+10	1.0000E+12	8.8618E+01	0
7.5785E+02	1.0000E-02	6.3198E-02	2.0193E+10	1.0000E+12	1.1760E+02	0
4.7013E+02	1.0000E-02	1.0155E-01	1.7909E+10	1.0000E+12	1.5663E+02	0
2.7686E+02	1.0000E-02	1.6370E-01	1.5883E+10	1.0000E+12	2.0899E+02	0
1.7974E+02	1.0000E-02	2.6428E-01	1.4086E+10	1.0000E+12	2.7907E+02	0
1.1182E+02	1.0000E-02	4.2695E-01	1.2493E+10	1.0000E+12	3.7278E+02	0
6.8664E+01	1.0000E-02	6.8997E-01	1.1079E+10	1.0000E+12	4.9805E+02	0
4.2601E+01	1.0000E-02	1.1152E+00	9.8258E+09	1.0000E+12	6.6545E+02	0
2.6354E+01	1.0000E-02	1.8026E+00	8.7142E+09	1.0000E+12	8.8916E+02	0
1.6304E+01	1.0000E-02	2.9137E+00	7.7283E+09	1.0000E+12	1.1801E+03	0
1.0086E+01	1.0000E-02	4.7099E+00	6.8540E+09	1.0000E+12	1.5875E+03	0
6.2394E+00	1.0000E-02	7.6133E+00	6.0785E+09	1.0000E+12	2.1212E+03	0
3.0579E+00	1.0000E-02	1.2307E+01	5.3908E+09	1.0000E+12	2.8344E+03	0
2.3678E+00	1.0000E-02	1.9894E+01	4.7810E+09	1.0000E+12	3.7873E+03	0
1.4772E+00	1.0000E-02	3.2158E+01	4.2401E+09	1.0000E+12	5.0667E+03	0
9.1383E-01	1.0000E-02	5.1902E+01	3.7664E+09	1.0000E+12	6.7611E+03	0
5.6532E-01	9.9999E-03	8.4028E+01	3.3369E+09	1.0000E+12	9.0356E+03	0
3.4571E-01	9.9999E-03	1.3503E+02	2.9576E+09	1.0000E+12	1.2073E+04	0
2.1635E-01	9.9999E-03	2.1957E+02	2.6230E+09	1.0000E+12	1.6133E+04	0
1.3384E-01	9.9997E-03	3.5494E+02	2.3261E+09	1.0000E+12	2.1557E+04	0
8.2797E-02	9.9995E-03	5.7376E+02	2.0661E+09	1.0000E+12	2.8804E+04	0
5.2211E-02	9.9991E-03	9.2752E+02	1.8297E+09	1.0000E+12	3.8489E+04	0
3.1687E-02	9.9986E-03	1.4994E+03	1.6227E+09	1.0000E+12	5.1432E+04	0
1.9603E-02	9.9977E-03	2.4241E+03	1.4391E+09	1.0000E+12	6.8727E+04	0
1.2127E-02	9.9963E-03	3.9192E+03	1.2763E+09	1.0000E+12	9.1842E+04	0
7.5023E-03	9.9940E-03	6.3375E+03	1.1319E+09	1.0000E+12	1.2274E+05	0
4.6413E-03	9.9903E-03	1.0249E+04	1.0039E+09	1.0000E+12	1.6445E+05	0
2.8714E-03	9.9842E-03	1.6581E+04	8.9031E+08	1.0000E+12	2.1929E+05	0
1.7766E-03	9.9744E-03	2.6838E+04	7.8960E+08	1.0000E+12	2.9021E+05	0
1.0992E-03	9.9585E-03	4.3474E+04	7.0030E+08	1.0000E+12	3.9220E+05	0
6.8022E-04	9.9326E-03	7.0524E+04	6.2112E+08	1.0000E+12	5.2498E+05	0
4.2101E-04	9.8899E-03	1.1464E+05	5.5092E+08	1.0111E+12	7.0349E+05	0
2.6066E-04	9.8190E-03	1.8711E+05	4.8869E+08	1.0184E+12	9.4450E+05	0

1.6148E-04	9.6992E-03	5.0737E+05	4.3356E+08	1.0310E+12	1.2720E+06	0
1.0016E-04	9.4892E-03	5.1106E+05	3.8472E+08	1.0538E+12	1.7241E+06	0
6.7902E-05	9.1845E-03	7.9116E+05	3.4887E+08	1.0886E+12	2.2443E+06	0
3.2525E-05	8.8581E-03	1.0713E+06	3.2742E+08	1.1287E+12	2.7062E+06	0
4.3921E-05	8.5059E-03	1.3514E+06	3.1310E+08	1.1757E+12	3.2285E+06	0
3.8567E-05	8.1218E-03	1.6315E+06	3.0309E+08	1.2313E+12	3.8212E+06	0
3.5114E-05	7.6978E-03	1.9116E+06	2.9406E+08	1.2991E+12	4.4903E+06	0
t critical is	2.19167E+06					
3.2942E-05	7.2212E-03	2.1917E+06	2.9138E+08	1.3848E+12	4.2399E+06	0
3.1778E-05	6.6723E-03	2.4718E+06	2.8877E+08	1.4987E+12	4.5726E+06	0

ps has started to increase at t=2.63983E+06 ps=3.15036E-05 M=6.15781E-03 TE=1.61395E+11  
 TS=2.88219E+08 R/R0=4.82852E+06 factor=0

Black hole now evaporating unimpeded in time t=819284  
 total elapsed time is 3.51513E+06  
 kt is now 3.35195  
 dt is now 9.8331E-05  
 Rt/R0 is now 5.66274E+06

Tstacc is now not more than 9.6411E+08 Gev

Appendix V: Baryon Production - an outline

In order for a net baryon number to be generated dynamically it is clear that baryon number violating interactions are necessary. Also, in order for more baryons than anti-baryons to be produced these interactions must also violate C and CP invariance. Also, since in equilibrium equal numbers of particles and antiparticles must exist, particle distribution functions must depart from their equilibrium values. Since, in a free expansion massless particle distributions keep their equilibrium values even in the absence of interactions<sup>1</sup>, this implies that relevant baryon violating interactions must involve heavy particles.

Consider a superheavy particle  $X$  with baryon violating decays  $X \rightarrow \ell q$ ,  $\bar{q}\bar{q}$  with branching ratios  $r$ , and  $1-r$  respectively. The mean net baryon number produced when an  $X, \bar{X}$  pair decay independently is then  $B = \frac{1}{3}r - \frac{2}{3}(1-r) + (\frac{-1}{3})\bar{r} + \frac{2}{3}(1-\bar{r}) = r-\bar{r}$ , where  $\bar{r}$  is the branching ratio of  $\bar{X} \rightarrow \bar{q}\bar{\ell}$ . While total rates for particles and anti-particles must be equal by CP, a CP violating phase induced via interference in higher order interactions allows  $r \neq \bar{r}$ .

The order of the interaction at which CP violation first appears thus determines the amount of baryon production (i.e.  $r-\bar{r}$ ), which will be proportional to  $\varepsilon \left(\frac{\alpha_X}{2\pi}\right)^\ell$  where  $\varepsilon$  is the phase angle characterizing CP violation and  $\ell$  is the total number of loops in the graphs whose interference induces the violation. In minimal SU(5) the lowest order occurs in Higgs decay at four loops. Thus  $\Delta B \approx \varepsilon \alpha_X^4 \lesssim 10^{-17}$ . Since at best,  $\frac{n_B}{n_\gamma} = \frac{\Delta B n_X}{n_\gamma} = \frac{\Delta B}{N}$  (where  $N$  is the total number of helicity states), this level of baryon generation is too small. Thus more complicated GUTs must

be used to generate baryon number. The addition of Higgs multiplets, and larger gauge groups, allow one in principle to increase  $\Delta B$  to phenomenologically acceptable levels.<sup>2</sup>

An additional problem occurs in the context of the standard scenario, because in order for X-decay to produce net baryon number, their number density distribution must be out of equilibrium, and inverse decays must be suppressed. This is a highly restrictive problem in general, and further increases the probability that any baryon number generation comes from the Higgs sector of the theory. This particular problem is removed in the black hole scenario because black hole evaporation takes place when the temperature of space is significantly below the mass of particles which generate baryon number. Thus for a further discussion of decay rates and equilibrium problems and their connection with heavy particle masses we refer to the literature.<sup>3,4</sup>

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Table 1. Evolution of the Initial State for Varying  $M_0$ ,  $\rho_{\text{BH}}^0$ 

$M_0$ (kg)	$\rho_{\text{BH}}^0$ (kg/M <sup>3</sup> )	$kT_0$ (GeV)	$kT_{\text{Rad}}^f$ (GeV)	$R_f/R_0$	$t_{\text{decay}}$ (sec)
$10^{-6}$	$7.7 \times 10^{89}$	$10^{16}$	$4.8 \times 10^{14}$	598	$1.09 \times 10^{-36}$
$10^{-6}$	$7.7 \times 10^{87}$	$10^{16}$	$4.8 \times 10^{14}$	157	$1.09 \times 10^{-36}$
$10^{-4}$	$7.7 \times 10^{85}$	$10^{14}$	$8 \times 10^{11}$	$8.9 \times 10^4$	$1.11 \times 10^{-30}$
$10^{-4}$	$7.7 \times 10^{83}$	$10^{14}$	$6.5 \times 10^{11}$	$2.8 \times 10^4$	$1.11 \times 10^{-30}$
$10^{-4}$	$7.7 \times 10^{81}$	$10^{14}$	$6.49 \times 10^{11}$	10374	$1.09 \times 10^{-30}$
$10^{-4}$	$7.7 \times 10^{79}$	$10^{14}$	$6.5 \times 10^{11}$	2743	$1.09 \times 10^{-30}$
$10^{-2}$	$7.7 \times 10^{81}$	$10^{12}$	$3.6 \times 10^8$	$8.1 \times 10^7$	$3.5 \times 10^{-24}$
$10^{-2}$	$7.7 \times 10^{79}$	$10^{12}$	$3.63 \times 10^8$	$2.1 \times 10^7$	$3.5 \times 10^{-24}$
$10^{-2}$	$7.7 \times 10^{77}$	$10^{12}$	$3.64 \times 10^8$	$5.6 \times 10^6$	$3.5 \times 10^{-24}$
$10^{-2}$	$7.7 \times 10^{75}$	$10^{12}$	$3.62 \times 10^8$	$1.4 \times 10^6$	$3.5 \times 10^{-24}$
$10^{-2}$	$7.7 \times 10^{73}$	$10^{12}$	$3.63 \times 10^8$	$3.8 \times 10^5$	$3.5 \times 10^{-24}$
$10^{-2}$	$7.7 \times 10^{71}$	$10^{12}$	$3.64 \times 10^8$	$1.0 \times 10^5$	$3.5 \times 10^{-24}$
$10^0$	$7.7 \times 10^{77}$	$10^{10}$	$3.62 \times 10^5$	$2.1 \times 10^{10}$	$3.5 \times 10^{-18}$
$10^0$	$7.7 \times 10^{75}$	$10^{10}$	$3.62 \times 10^5$	$5.5 \times 10^9$	$3.5 \times 10^{-18}$
$10^0$	$7.7 \times 10^{73}$	$10^{10}$	$3.62 \times 10^5$	$1.45 \times 10^9$	$3.5 \times 10^{-18}$
$10^0$	$7.7 \times 10^{71}$	$10^{10}$	$3.63 \times 10^5$	$3.8 \times 10^8$	$3.5 \times 10^{-18}$
$10^0$	$7.7 \times 10^{69}$	$10^{10}$	$3.62 \times 10^5$	$1.0 \times 10^8$	$3.5 \times 10^{-18}$
$10^0$	$7.7 \times 10^{67}$	$10^{10}$	$3.62 \times 10^5$	$2.6 \times 10^7$	$3.5 \times 10^{-18}$
$10^0$	$7.7 \times 10^{65}$	$10^{10}$	$3.63 \times 10^5$	$6.9 \times 10^6$	$3.5 \times 10^{-18}$
$10^0$	$7.7 \times 10^{63}$	$10^{10}$	$3.64 \times 10^5$	$1.8 \times 10^6$	$3.5 \times 10^{-18}$
$10^2$	$7.7 \times 10^{73}$	$10^8$	$3.62 \times 10^2$	$5.4 \times 10^{12}$	$3.5 \times 10^{-12}$
$10^2$	$7.7 \times 10^{71}$	$10^8$	$3.63 \times 10^2$	$1.4 \times 10^{12}$	$3.5 \times 10^{-12}$
$10^2$	$7.7 \times 10^{69}$	$10^8$	$3.63 \times 10^2$	$3.7 \times 10^{11}$	$3.5 \times 10^{-12}$
$10^2$	$7.7 \times 10^{67}$	$10^8$	$3.62 \times 10^2$	$9.9 \times 10^9$	$3.5 \times 10^{-12}$
$10^2$	$7.7 \times 10^{65}$	$10^8$	$3.63 \times 10^2$	$2.6 \times 10^9$	$3.5 \times 10^{-12}$
$10^2$	$7.7 \times 10^{63}$	$10^8$	$3.63 \times 10^2$	$6.8 \times 10^9$	$3.5 \times 10^{-12}$
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
$10^4$	$7.7 \times 10^{69}$	$10^6$	$3.63 \times 10^{-1}$	$1.4 \times 10^{15}$	$3.5 \times 10^{-6}$
$10^4$	$7.7 \times 10^{67}$	$10^6$	$3.64 \times 10^{-1}$	$3.7 \times 10^{14}$	$3.5 \times 10^{-6}$
$10^4$	$7.7 \times 10^{65}$	$10^6$	$3.62 \times 10^{-1}$	$9.7 \times 10^{13}$	$3.5 \times 10^{-6}$
$10^4$	$7.7 \times 10^{63}$	$10^6$	$3.63 \times 10^{-1}$	$2.5 \times 10^{13}$	$3.5 \times 10^{-6}$
$10^4$	$7.7 \times 10^{61}$	$10^6$	$3.63 \times 10^{-1}$	$6.7 \times 10^{12}$	$3.5 \times 10^{-6}$
$10^4$	$7.7 \times 10^{59}$	$10^6$	$3.62 \times 10^{-1}$	$1.7 \times 10^{12}$	$3.5 \times 10^{-6}$
$10^4$	$7.7 \times 10^{57}$	$10^6$	$3.63 \times 10^{-1}$	$4.6 \times 10^{11}$	$3.5 \times 10^{-6}$
$10^4$	$7.7 \times 10^{55}$	$10^6$	$3.63 \times 10^{-1}$	$1.2 \times 10^{11}$	$3.5 \times 10^{-6}$
$10^4$	$7.7 \times 10^{53}$	$10^6$	$3.62 \times 10^{-1}$	$3.2 \times 10^{10}$	$3.5 \times 10^{-6}$

Table 2: Black Hole Acceleration Resulting From Initial Thermal Fluctuations

$M_0^A$	$\rho_0^A$	$P^B$	$t_{\text{final}}^C$	$k_0^B$	$r_s^B$	$2r_{\text{final}}$	$V_0^A$	$\ell^D$	$(\frac{\ell}{2r_f})^E$	$a_{\text{av}}^F$	$\Delta v$
$10^{-6}$	$10^{89}$	$3 \times 10^{40}$	$3 \times 10^{-37}$	4.64	$1.5 \times 10^{-33}$	$6 \times 10^{-30}$	$10^6$	$9 \times 10^{-31}$	.15	$2.8 \times 10^{42}$	$8 \times 10^5$
...	$10^{87}$	$3 \times 10^{39}$	...	21.8	...	...	...	...	...	...	...
$10^{-4}$	$10^{85}$	$3 \times 10^{38}$	$3 \times 10^{-31}$	4.64	$1.5 \times 10^{-31}$	$2.8 \times 10^{-25}$	$10^4$	$9 \times 10^{-27}$	$3 \times 10^{-2}$	$1 \times 10^{34}$	$3 \times 10^3$
	$10^{83}$	$3 \times 10^{37}$	...	21.8	...	...	...	...	...	...	...
	$10^{81}$	$3 \times 10^{36}$	...	100	...	...	...	...	...	...	...
	$10^{79}$	$3 \times 10^{35}$	...	$4.64 \times 10^2$	...	...	...	...	...	...	...
$10^{-2}$	$10^{81}$	$3 \times 10^{36}$	$10^{-24}$	4.64	$1.5 \times 10^{-29}$	$2.9 \times 10^{-20}$	$10^2$	$3 \times 10^{-22}$	$1 \times 10^{-2}$	$75 \times 10^{25}$	$6 \times 10^1$
	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.
	$10^{71}$	$3 \times 10^{31}$	...	$10^4$	...	...	...	...	...	...	...
$10^0$	$10^{77}$	$3 \times 10^{34}$	$10^{-18}$	4.64	$1.5 \times 10^{-27}$	$1.4 \times 10^{-15}$	$10^0$	$3 \times 10^{-18}$	$2 \times 10^{-3}$	$1.2 \times 10^{18}$	$1 \times 10^0$
	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.
	$10^{63}$	$3 \times 10^{27}$	...	$2.1 \times 10^5$	...	...	...	...	...	...	...
$10^2$	$10^{73}$	$3 \times 10^{32}$	$10^{-12}$	4.64	$1.5 \times 10^{-25}$	$6.2 \times 10^{-11}$	$10^{-2}$	$3 \times 10^{-14}$	$5 \times 10^{-4}$	$1.2 \times 10^{10}$	$1.2 \times 10^{-2}$

Key: A. all quantities in MKS units

B.  $r(t) = k_0 r_{\text{Schwarzschild}} k(t) = k_0 r_s (1 + \frac{3}{2} (\rho_{\text{TOT}}^0)^{1/2} (2.3 \times 10^{-5}) t)^{2/3} = k_0 r_s (1 + Pt)^{2/3}$

C.  $t_f = 3 \times 10^{-1} t_{\text{final}}$

D.  $\ell = 3 V_0 t_f$

E.  $r_{\text{av}} = 3/5 r_{\text{final}}$

F.  $a_{\text{av}} = \frac{3GM}{(2r)_{\text{av}}^2} \times (\frac{\ell}{r_f})$

Table 3: Monopole to Entropy Ratio Arising from Initial State Production

$T^0$ (GeV)	$M_0$ (kg)	$\rho_0^{\text{BH}}$ (kg/M <sup>3</sup> )	$n_H^0 \sim V_H^{-1}$	$\frac{s^0}{k} = \frac{4\pi^2}{.45} \frac{N}{2} \frac{(kT)^3}{(\hbar c)^3}$	$(\frac{\ln H}{s^0/k})$	$D\hat{n}^{2*}$	$\frac{\dot{R}}{R}\hat{n}^*$	Dilution factor	$(\frac{n_M}{s_F})_{\text{Max}}$	$(\frac{n_M}{s_F})$
$10^{14}$	$10^{-4}$	$10^{85}$	$10^{89}$	$10^{91}$	$10^{-3}$	$4.6 \times 10^{57}$	$1 \times 10^{55}$	$7.2 \times 10^{-9}$	$7 \times 10^{-12}$	$7 \times 10^{-19}$
...	...	$10^{83}$	$10^{87}$	...	$10^{-5}$	$4.6 \times 10^{53}$	$10^{53}$	$3.4 \times 10^{-7}$	$4 \times 10^{-12}$	$4 \times 10^{-17}$
...	...	$10^{81}$	$10^{85}$	...	$10^{-7}$	$4.6 \times 10^{49}$	$10^{49}$	$5.2 \times 10^{-6}$	$5 \times 10^{-13}$	$5 \times 10^{-16}$
...	...	$10^{79}$	$10^{83}$	...	$10^{-9}$	$4.6 \times 10^{45}$	$10^{46}$	$2.3 \times 10^{-4}$	$2 \times 10^{-13}$	$2 \times 10^{-14}$
$10^{12}$	$10^{-2}$	$10^{81}$	$10^{83}$	$10^{85}$	$10^{-3}$	$4.6 \times 10^{49}$	$10^{47}$	$7 \times 10^{-14}$	$7 \times 10^{-17}$	$7 \times 10^{-24}$
...	...	$10^{79}$	$10^{81}$	...	$10^{-5}$	$4.6 \times 10^{45}$	$10^{44}$	$4.6 \times 10^{-16}$	$4.6 \times 10^{-17}$	$4 \times 10^{-22}$
...	...	$10^{77}$	$10^{79}$	...	$10^{-7}$	$4.6 \times 10^{41}$	$10^{41}$	$2.9 \times 10^{-10}$	$3 \times 10^{-17}$	$3 \times 10^{-20}$
...	...	$10^{75}$	$10^{77}$	...	$10^{-9}$	$4.6 \times 10^{37}$	$10^{38}$	$3 \times 10^{-8}$	$3 \times 10^{-12}$	$3 \times 10^{-18}$
...	...	$10^{73}$	$10^{75}$	...	$10^{-11}$	...	...	$1.3 \times 10^{-6}$	$1.3 \times 10^{-17}$	$1 \times 10^{-17}$
...	...	$10^{71}$	$10^{73}$	...	$10^{-13}$	...	...	$7 \times 10^{-5}$	$7 \times 10^{-18}$	$7 \times 10^{-18}$

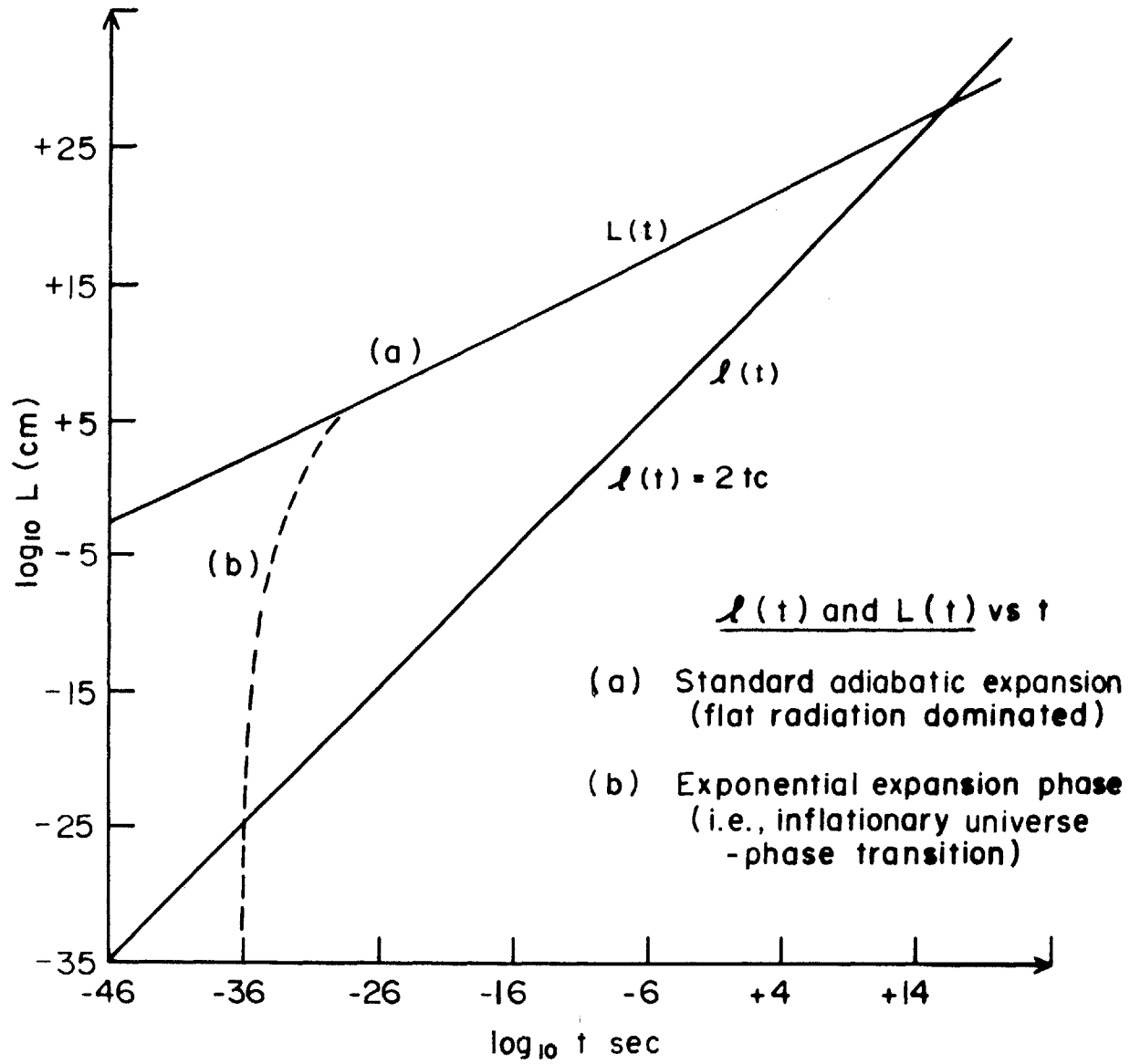
\* in units  $\hbar=c=1$ :  $D \sim 10^2 T^{-2} \text{ GeV}^{-2}$ ,  $\hat{n} = n (6.8 \times 10^{-48}) \text{ GeV}^3$

$$\frac{\dot{R}}{R} \sim 10^{-19} M_0^{1/2} (\text{GeV}) \hat{n}_0^{1/2} (\text{GeV})$$

$$\text{Dilution factor} \sim \frac{\frac{T_{\text{initial}}}{T_{\text{final}}}}{\left(\frac{R_f}{R_0}\right)^3}$$



FIGURE 1



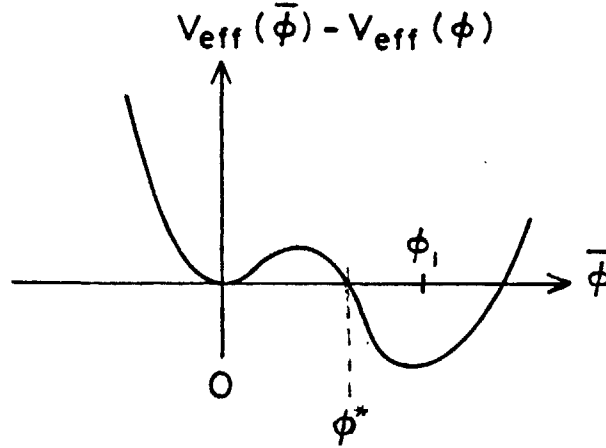


FIG. 2: Effective Potential with Metastable State

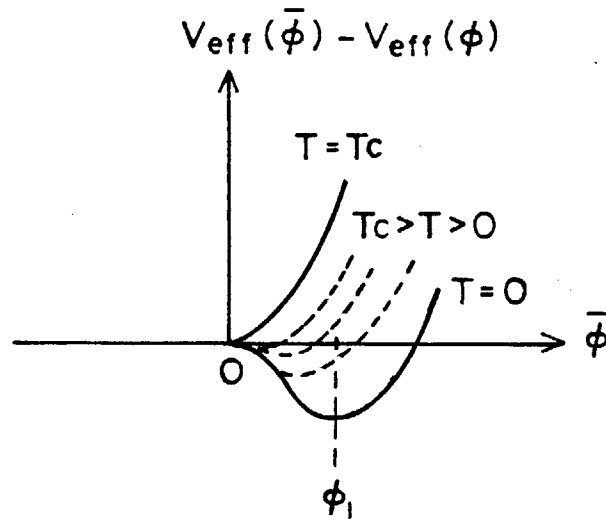


FIG. 3(a): Temperature Dependence of Effective Potential;  
Second Order Transition

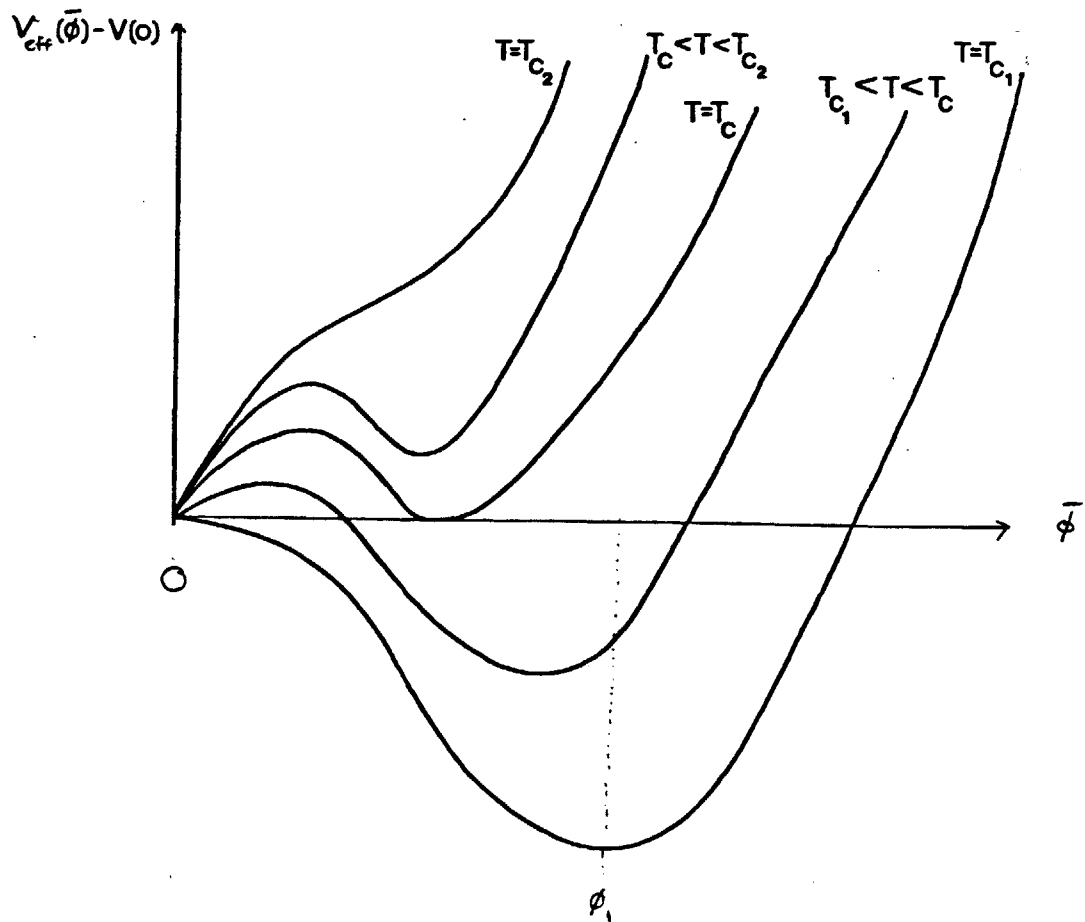


FIG. 3(b): Temperature Dependence of Effective Potential.

First Order Transition

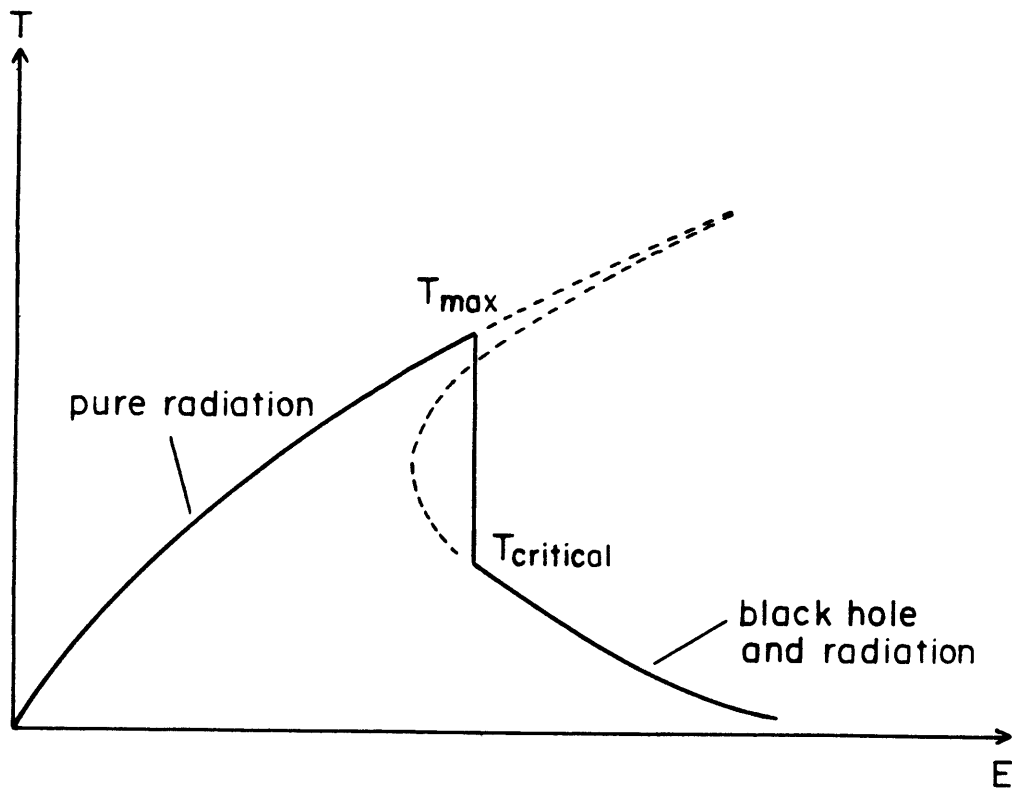


FIG. 4:  $T$  vs.  $E$  for an enclosure of fixed volume  $V$  showing Black Hole-Radiation transition

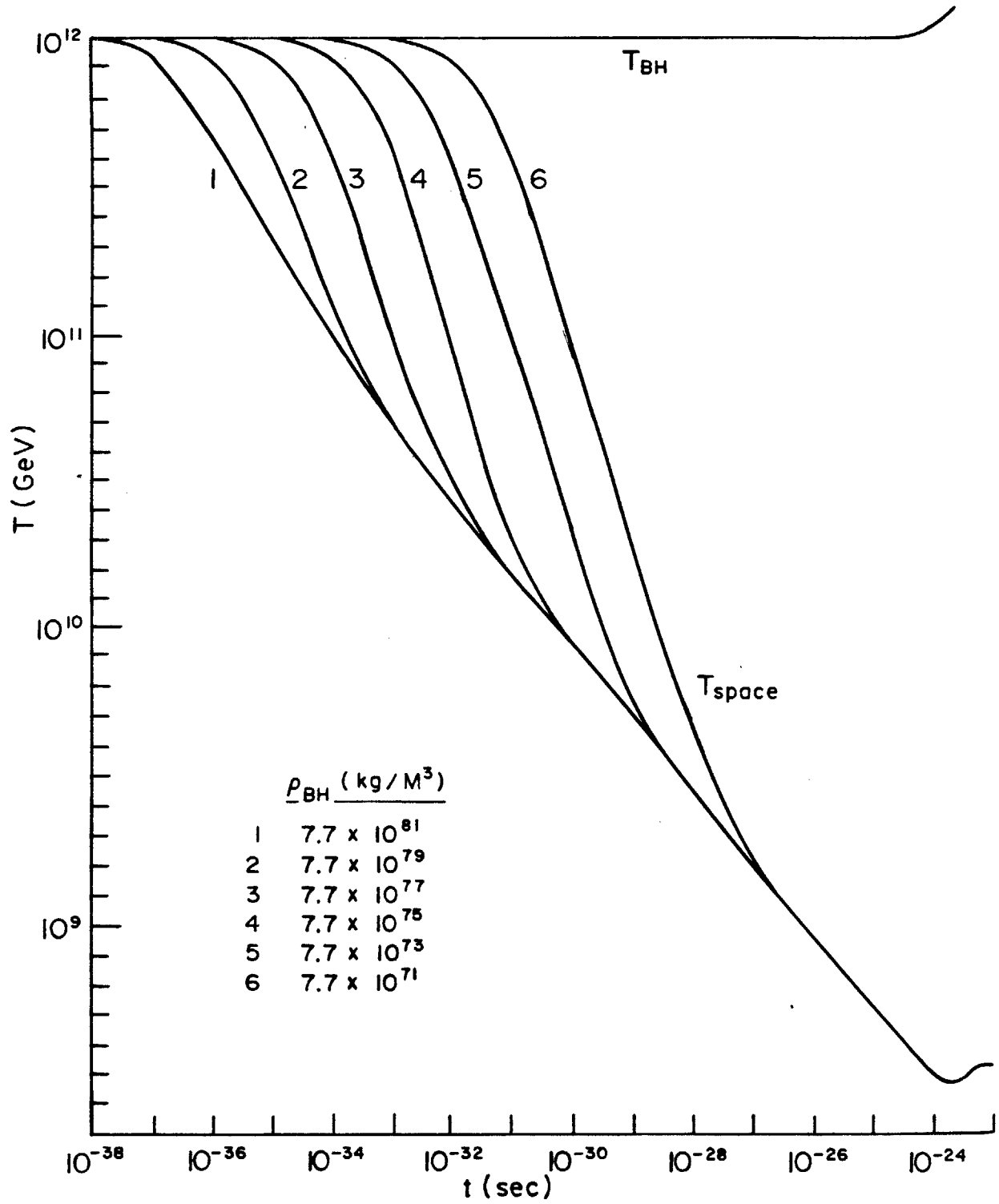


FIG.5(a):  $T_{space}$  and  $T_{BH}$  vs. time for  $M^0=10^{-2}$ kg.

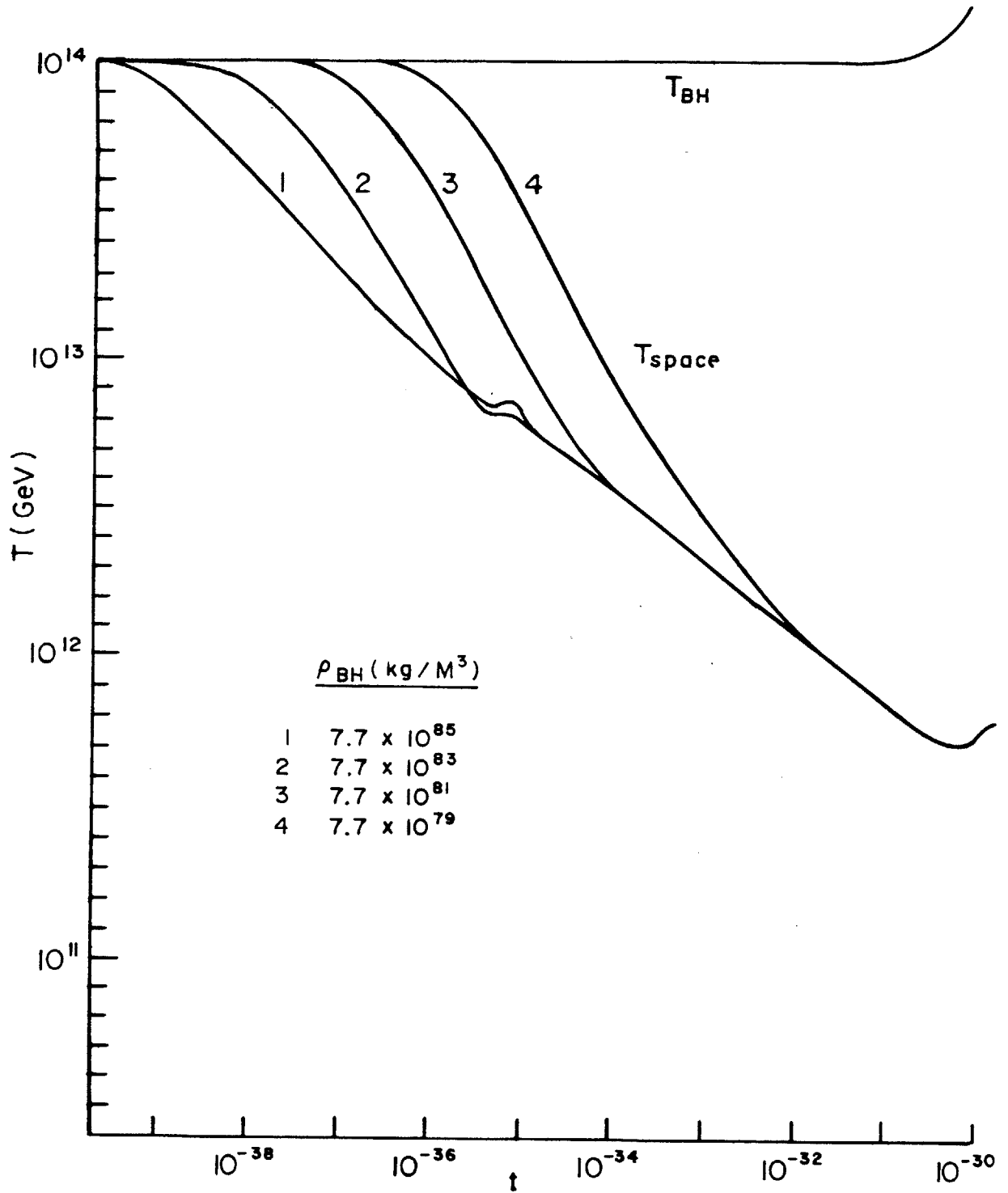


FIG. 5(b):  $T_{space}$  and  $T_{BH}$  vs. time for  $M^0 = 10^{-2}$  Kg.

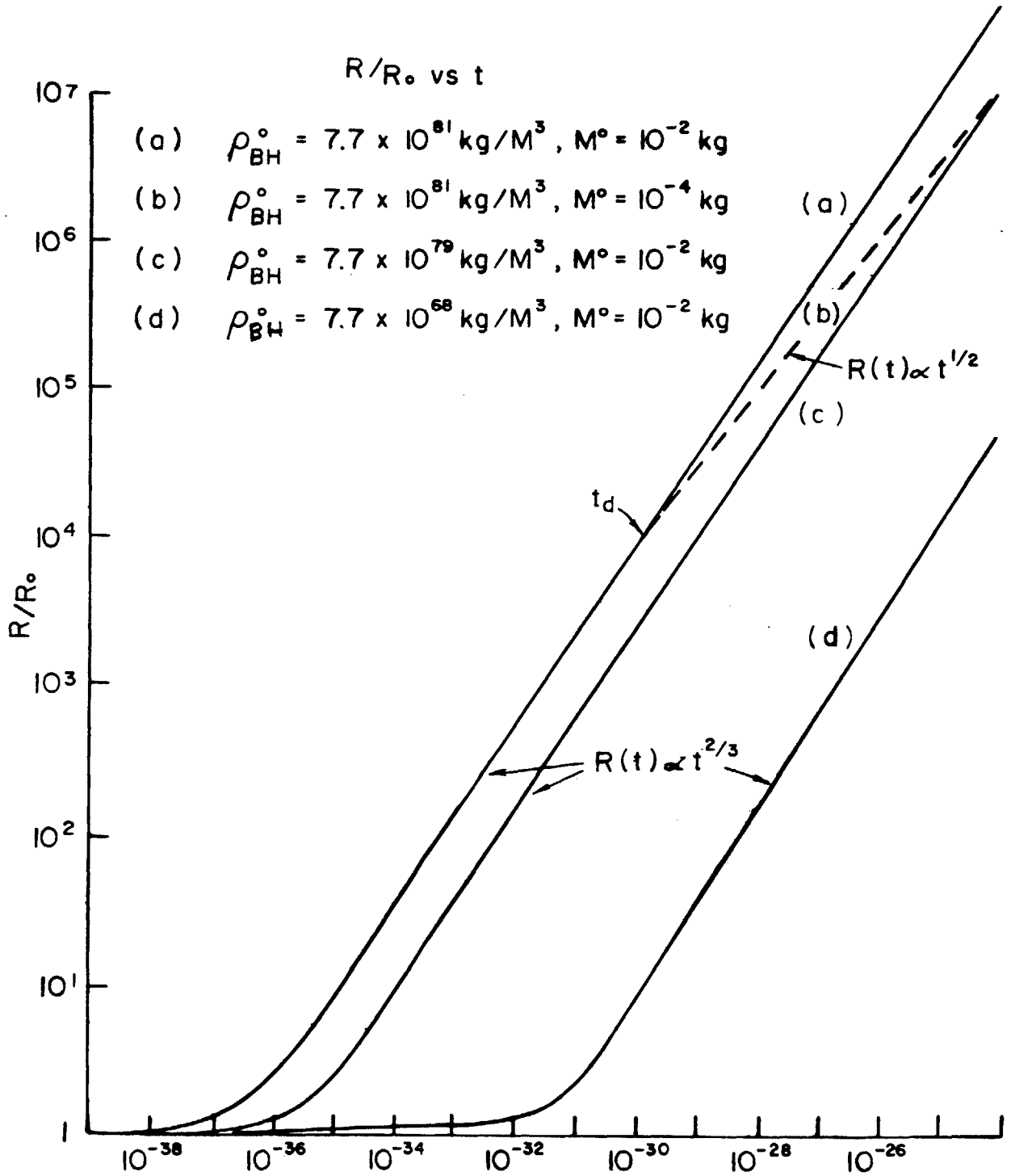
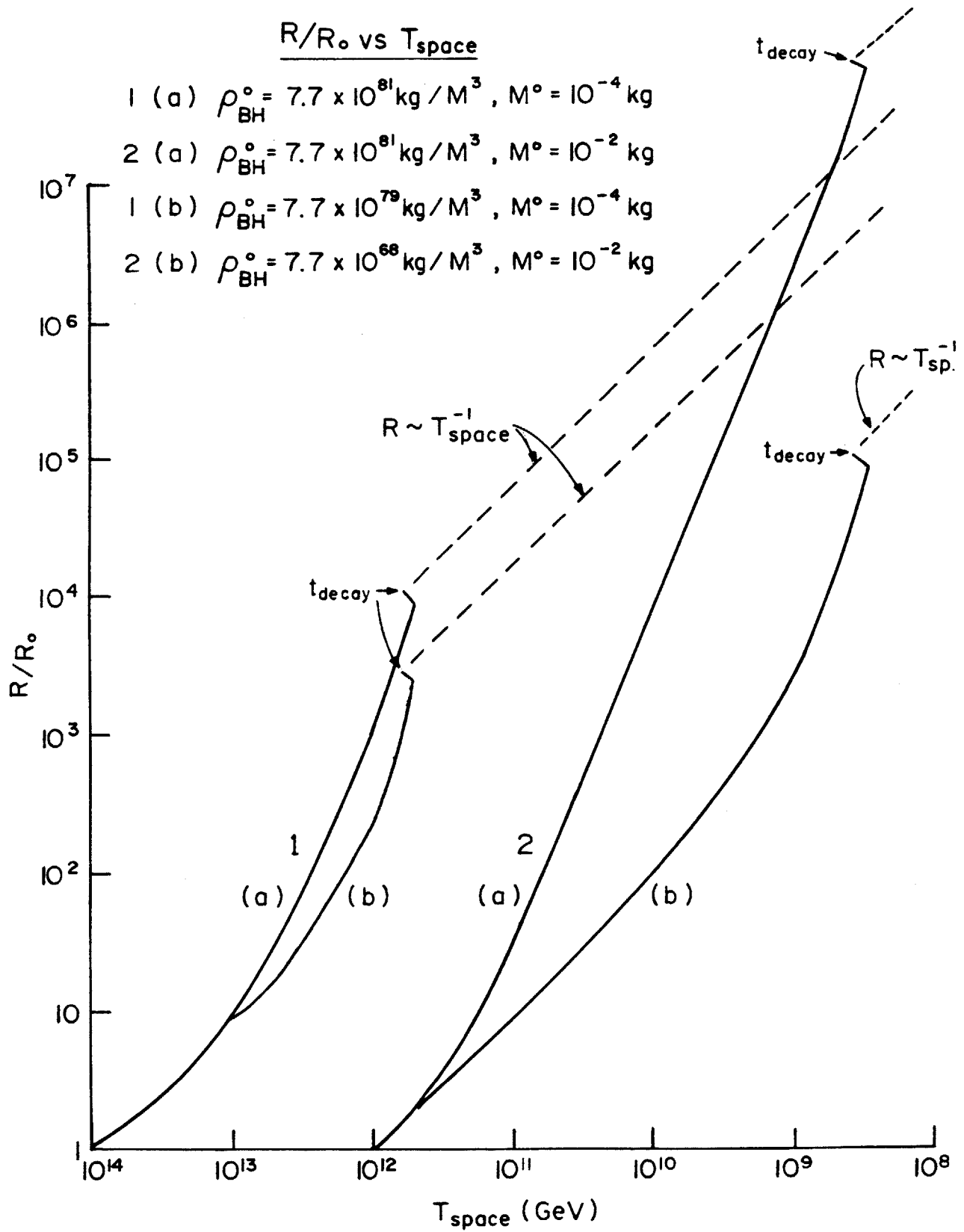


FIGURE 6(a)

FIGURE 6 (b)





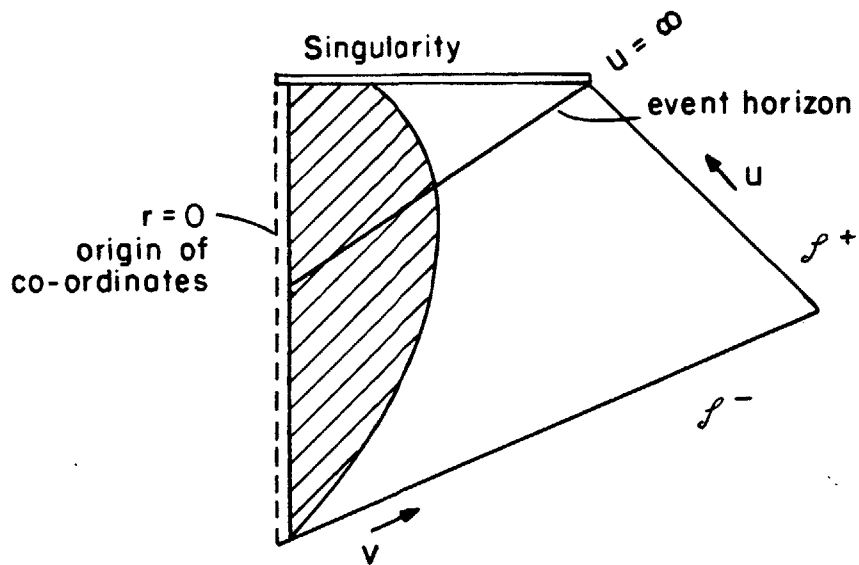


FIG. 7: Penrose Diagram for Spherically Symmetric Collapsing Body

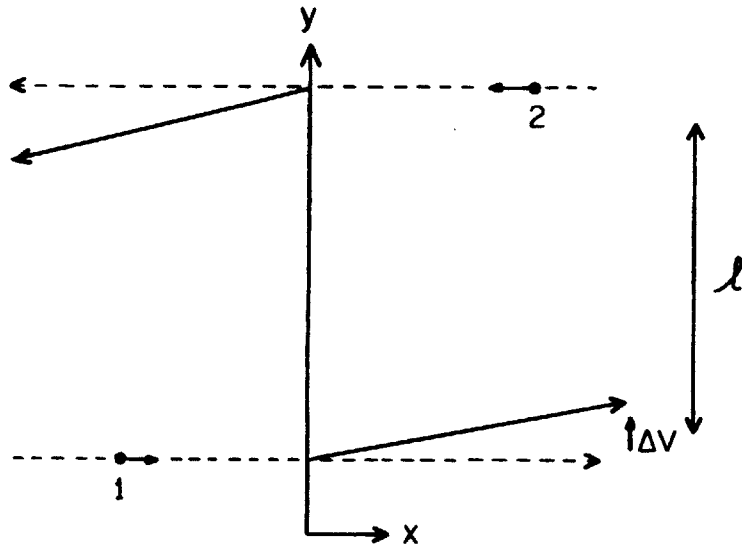


FIG. 8: Gravitational encounter

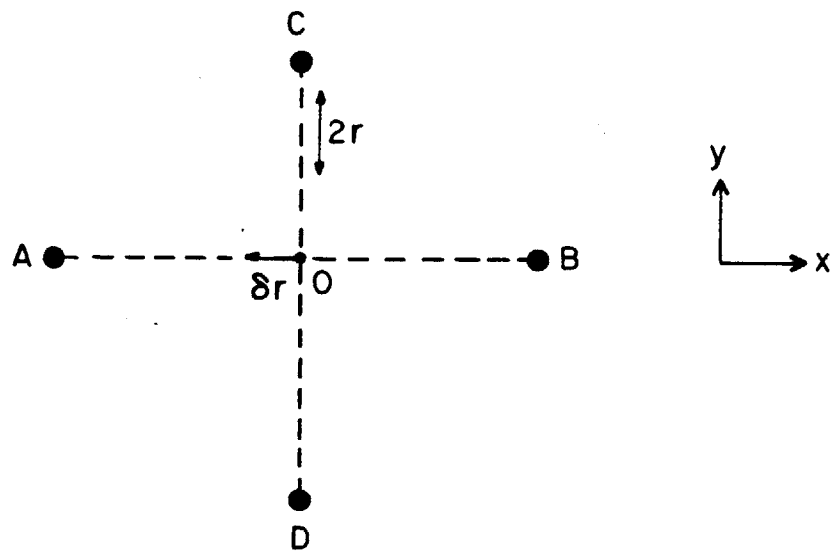


FIG. 9: Black Hole Nearest Neighbour Configuration

Acknowledgement

The list of people to whom I am indebted for having helped me begin, survive, and at times even flourish during my graduate years at M.I.T. is long - corresponding both to my good fortune, and to the importance of people in every aspect of my activity - even physics.

First, because of his most direct connection with the successful completion of this thesis, I thank my supervisor Roscoe Giles. Our relationship began at an important turning point when he helped salvage what was beginning to look like another tragic graduate experience. From that time he has continually unselfishly devoted his time and energies to helping me, as we both took on roles that were new to us. Roscoe is one of the most widely knowledgeable physicists I have met at M.I.T. Moreover, he has provided an excellent role model, demonstrating that qualities of excellence and hard work need not be achieved at the expense of one's humanity.

Next I thank my wife, Kate, for seeing me through every phase of my graduate career. The fact that she experienced along with me some of the best, and worst, times, made the experience more worthwhile. Her faith and support, and perspective through these times also made it all possible.

I would also like to thank the many excellent physicists I have had the good fortune to learn from and work with during my period at M.I.T. Steven Weinberg taught me continually - from introductory quantum field theory to the details of gauge theories. In many ways he indirectly provided directions for my work, by influencing my perspectives in a way which is clearly

demonstrated by this particular body of work. I have also had the good fortune to get to know Sheldon Glashow, who helped remind me that physics can be exciting, fun, and creative, and that at the same time it must and actually does make contact with the physical world. I have also had the opportunity to interact with Alan Guth, from whom I have learned many basics of cosmology, and whose critical comments have often spurred my thinking. Among the many others from whose knowledge and company I have benefitted are included, Ian Affleck, John Preskill, Louis Alvarez Gaumé, Mark Wise, Ken Johnson, David Gross, Claude Bernard, Marc Sher, Nathan Isgur, Jon Machta, Bart Lane, Costas Callias, Joe Lykken, and Manu Paranjape, and the rest of my colleagues among the graduate students. I look forward to being able to continue interacting with all of these people in the future.

In addition I thank a long list of people who have supported and encouraged me. These include my excellent teachers throughout my academic career, and other academic colleagues who have gone out of their way to show their faith in me, including especially Mike Casper and Maurice Careless. As well I thank my close friends whose constant encouragement has been vital. I also thank the towns of Sackville, N.B. and Amherst, Nova Scotia (and Mrs. Kelley for her back room), where much of this work was done, and Mark Hale at M.I.T. and Paula Constantine at Harvard for typing assistance. Oh yes, thanks to Sir James Jeans, and Stanislaw Lem for inspiration.

Finally, I thank my family for the environment in which I was fortunate to grow.

Biographical Note

I was born in New York City on May 27, 1954 - from which I escaped three months later - my parents having the wisdom to move to Canada. I attended primary and secondary school in Toronto, Ontario, and entered Carleton University in Ottawa in 1972. In 1973 I left university and spent a year doing independent research on the social history of the Communist Party of Canada during the depression. I returned to university in 1974, and graduated with a double honors degree in Mathematics and Physics in 1977, receiving a Carleton University Senate Medal for outstanding academic achievement. I began my doctoral studies in Physics at M.I.T. in that year. In January 1980 I married Katherine Kelley. During my period at M.I.T. in addition to my research I spent four years as a teaching assistant and recitation instructor, and ran a T.V. program in physics for undergraduate students. I also served on the Executive Committee of the Graduate Student Association, and on several institute committees and student groups.

My outside professional activities have included public science education at the Ontario Science Center, writing and lecturing on the threat of Nuclear War, and involvement in professional societies including the American Physical Society, and the Canadian Association of Physicists. I was on the board of directors of the latter in 1977. On July 1, 1982 I began my tenure as a Junior Fellow of the Society of Fellows at Harvard University.