Design of Assemblies with Compliant Parts: Application to Datum Flow Chain

by

Gennadiy Goldenshteyn

B. S., Mechanical Engineering University of California at Berkeley, 2000

Submitted to the Department of Mechanical Engineering In Partial Fulfillment of the Requirements for the Degree of

Master of Science in Mechanical Engineering

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2002

MASSACHUSETTS INSTITUTE OF TECHNOLOGY OCT 2 5 2002 LIBRARIES

BARKER

© 2002 Massachusetts Institute of Technology All rights reserved

Signature of Author_____

Department of Mechanical Engineering May 24, 2002

Certified by_____

Dr. Daniel E. Whitney Senior Research Scientist Center for Technology, Policy and Industrial Development Lecturer, Department of Mechanical Engineering Thesis Supervisor

Accepted by_____

Prof. Ain A. Sonin Chairman, Departmental Committee on Graduate Students

.



.

Design of Assemblies with Compliant Parts: Application to Datum Flow Chain

by

Gennadiy Goldenshteyn

Submitted to the Department of Mechanical Engineering In Partial Fulfillment of the Requirements for the Degree of Master of Science in Mechanical Engineering

ABSTRACT

Datum Flow Chain (DFC) is a methodology that provides tools and vocabulary to support design of assemblies. The goal of DFC is to design assemblies that satisfactorily deliver important customer requirement, or KCs. DFC is not able to deal with compliant parts due to inability to determine locational responsibility transfer relationships through a compliant part. Up to now, compliance of parts was ignored in DFC methodology, and compliant parts were assumed to be rigid.

In industry, compliant parts play a significant role in delivering KCs in an assembly. Concepts of flexible and rigid directions of a compliant part are introduced. Two distinct ways of assembling parts, in series and in parallel, are presented. Parallel assembly results in a subassembly that is stiffer that either of its components, while serial assembly does not. Sequence of parallel and serial assembly steps can influence the rigidifying rate of the assembly. Rigidizing step for a compliant part is defined as the first parallel assembly step for the part that results in a rigid subassembly. During the rigidizing step DFC links through the part are established.

KC conflict is a serious challenge to successful assembly design. KC conflict occurs when there are not enough degrees of freedom in an assembly to independently deliver all KCs and is undesirable: delivery of one KC can place delivery of another KC in jeopardy. Method for detection of KC conflict in an assembly using screw theory is presented. Overconstraint of compliant parts, if possible, is an effective way to resolve KC conflict.

Concept of Effective Locating Zone (ELZ) is developed as a tool for determination of locational responsibility transfer relationships through a compliant part. It represents a region of a compliant part where an incoming locator carries all location responsibilities. ELZs are established during the rigidizing step in part's assembly. Two methods for establishment of ELZs are presented: precise determination via a membrane-based model of the part if detailed part information is available and approximation via a beam-based model of the part if detailed part information is not available. Method to properly establish DFC links through a compliant part using ELZs is presented. Finally, all topics of this thesis are united to form a comprehensive DFC-based system for top-down design of assemblies with compliant parts.

Thesis Supervisor: Daniel E. Whitney Title: Senior Research Scientist

ACKNOWLEDGEMENTS:

I would like to thank my advisor Dr. Daniel Whitney for his guidance, support, and patience. He had a profound influence on my life during the last two years, and I have learned much from him, both as a scientist and as a person. I have greatly enjoyed working with him.

I feel greatly indebted to my family. Without a question, none of my accomplishments would be possible without their love, support, and encouragement. No words can truly describe my gratitude to them.

I would like to thank Mary Ann Wright, Craig Moccio, Mike Trygar, Marsha Ryan, and Art Hyde at Ford Motor Company for providing me with valuable industry experience and support. I would like to thank Dr. Chris Magee for helping me find the right audience within Ford. I would also like to thank the Ford-MIT Research Alliance for supporting this work.

It is impossible not to mention great friends I have made at MIT. It is because of them that I will remember this place with great fondness. I thank Alberto, Fredrik, Gaurav, Peng, and Jagmeet for their friendship. Without their ability to both hold a stimulating scientific discussion and come up with a timely joke, these past two years would not have been so enjoyable.

TABLE OF CONTENTS

Abstract	3
Acknowledgements	4
Table of Contents	5
List of Figures	7
1. Introduction	9
1.1 Motivation	9
1.2 Goals	10
1.3 Prior research.	11
1.4 Structure of the Thesis	14
2. Datum Flow Chain	17
2.1 Datum Flow Chain method of modeling assemblies	17
2.2 Key Characteristics	20
2.3 Part features: mates and contacts	22
2.4 Constraint states in the assembly	24
2.5 Mathematical methods used with DFC	25
2.5.1 4x4 Matrix transforms	25
2.5.2 Screw theory	26
2.6 Advantages of top-down design using DFC methodology	27
2.7 DFC and flexible parts	28
2.8 Chapter summary	29
3. Compliant parts in assembly	31
3.1 Use in the industry	31
3.2 Compliance of a part	32
3.2.1 Directions of a compliant part: flexible vs. rigid	32
3.2.2 Types of compliance	34
3.3 Assembly as a removal of degrees of freedom	36
3.4 Different types of assembly of compliant parts: serial vs. parallel	39
3.4.1 Assembly of compliant part and resulting rigidity	41

3.5 Rigidizing assembly step	41
3.5.1 Choosing optimal rigidizing step	44
3.6 Springback	45
3.7 Chapter summary	47
4. KC conflict in assembly	49
4.1 Definition of KC conflict	49
4.2 Causes of KC conflict	50
4.3 "Single part" KC conflict	52
4.3.1 Use of Screw Theory for detection of "single part" KC conflict	53
4.4 Resolving KC conflict	55
4.4.1 Resolving "single part" KC conflict through overconstraint of compliant parts	56
4.5 Chapter summary	57
5. Effective Locating Zone	59
5.1 Defining ELZ	59
5.2 Locational Responsibility Ratio	60
5.2.1 Relationship between ELZ and LRR	61
5.3 Establishing ELZs	62
5.3.1 Minimum acceptable LRR	63
5.3.2 Boundary conditions at locators	64
5.4 Mathematical Methods for ELZ determination	64
5.4.1 Membrane-based approach for precise ELZ determination	65
5.4.2 Approximation of ELZ via beam-based modeling	66
5.5 Linking incoming and outgoing mates through compliant part	75
5.5.1 Influences on size of an ELZ	78
5.6 Springback and ELZs	79
5.7 Chapter summary	81
6. Systematic design of assemblies with compliant parts	85
6.1 Nominal design phase	85
6.2 Constraint Analysis Phase	88
6.2.1 Determination and analysis of ELZs	90
6.3 Variation Design Phase	93

6.4 Chapter summary	94
7. Conclusion	97
7.1 Summary of major points made in the thesis	97
7.2. Scope for future research	99
References	101
Appendix A	107

LIST OF FIGURES

Fig. 2.1 Datum Flow Chain of a typical automotive front end	18
Fig. 3.1 Structural component assembled from two compliant parts	32
Fig. 3.2 Flexible direction of a compliant part and the associated flexible plane	34
Fig. 3.3 Automobile Front End	36
Fig. 3.4 Serial vs. Parallel assembly	40
Fig. 3.5 Effect of assembly sequence on assembly rigidifying rate	42
Fig. 4.1 KC conflict due to not enough degrees of freedom present in the assembly	50
Fig. 4.2 KC conflict due to improper assembly sequence	51
Fig. 5.1 Relationship between ELZ and LRR	62
Fig. 5.2 Boundary conditions at locators	64
Fig. 5.3 ELZ determination for automotive fender	67
Fig. 5.4 Profiles of LRR loss along beams for estimation of ELZ	71
Fig. 5.5. ELZ estimation for automotive fender via beam-based modeling	73
Fig. 5.6. Using ELZ to link incoming and outgoing mates	76
Fig. 5.7. Using ELZ to avoid KC conflict	77
Fig. 5.8. Use of contacts to limit the effect of spring back	80
Fig. 6.1. Design Process Chart for Assemblies with Compliant Parts	86

CHAPTER 1. INTRODUCTION.

1.1 Motivation.

Assemblies are more than a collection of individual parts put together. Rather, assemblies are systems of parts that together perform some function. Assemblies, not individual parts, are what constitute a final product and deliver value to the customer. Despite this, current design in the industry is part-centric, with assembly merely an afterthought. Work in assembly is basically limited to the way parts can be joined together with least problems (see, e.g., [Boothroyd and Dewhurst, 1991]).

Such researchers as [Lee and Thornton, 1996] and [Mantripragada et al., 1996] have brought attention to overall assembly quality and advantages of top-down design. It has been argued that the top-down process can reduce the design time, avoid potential mistakes during initial conceptual design phase, and lead to better, more robust assemblies. A methodology to support top-down design called Datum Flow Chain (DFC) was introduced by [Mantripragada and Whitney, 1998]. It provides a method, together with a vocabulary and a set of symbols, for documenting a location and constraint strategy for the parts and relating that strategy explicitly to the achievement of customer requirements. DFC allows for design of assemblies at the very early stages of product development, and can be deployed while only limited information about individual parts comprising the assembly is known. Prior to now, the work in top-down design, including DFC, focused on rigid parts and ignored challenges and advantages inherent to compliant parts. Compliant parts are in heavy use in the industry – for example, an automotive body is composed of more than 200 compliant sheet metal parts [Ceglarek and Shi, 1995]. Thus, in order to be truly effective, DFC, as any top-down approach, must deal with unique attributes of compliant parts. Prior to now, the DFC dealt with compliant parts by ignoring their compliance and treating them as rigid. As observed by the author, such assumptions limit the usefulness of DFC design methodology.

1.2 Goals.

The goals of this thesis are as follows:

- To better understand the role and behavior of compliant parts in assembly from the point of view of top-down design. Focus will be made on the role compliant parts play in delivery of customer requirements. The unique attributes of compliant parts allow for some advantages in creating a better-performing product, while also presenting some challenges to the process at the same time.
- To extend Datum Flow Chain method to take into account compliance of the parts. This will make the Datum Flow Chain method applicable to a wider range of assemblies. Tools and vocabulary to do so will be developed in this thesis.
- Develop methods to deal with part compliance at early stages of product development. Such methods should be easy to carry out and require as little part knowledge as possible in order to be used as early as possible.

1.3 Prior research.

Recently, increased attention to overall assembly quality has driven a trend to top-down design. Researchers such as [Ulrich and Eppinger, 1995], [Lee and Thornton, 1996], and [Suh, 1990] emphasize requirements-driven top-down design process as a way to deliver products that fulfill customer demands satisfactory manner. Importance of geometrical design of assemblies in early phases of top-down design has been pointed out in [Mantripragada et al., 1996], [Whitney et al., 1999], and [Johannesson and Soderberg, 2000]. It was argued this approach can reduce potential mistakes during initial design phase, resulting in decreased variations in assembly.

Previous research in top-down assembly has relied upon the addition theorem of variance (see, e.g. [Bjørke, 1989]) when dealing with parts, regardless of their rigidity or compliance. [Bjørke, 1989] introduced formal methods to predict variations in assemblies composed of perfectly rigid parts. However, in his statistical study of production data, [Takezawa, 1980] observed that, for assembly of compliant parts, the conventional addition theorem of variance is no longer valid, as the components can deform and change dimensions during assembly. [Shiu et al., 1997] and [Shalon et al, 1992] noted that part deformation due to compliance is among frequent causes of problems in complex assemblies. [Donald and Pai, 1990] pointed out the differences of compliant and rigid work pieces during assembly as well.

Bulk of previous research dealing with compliant parts has focused on manufacturing matters: fixturing and handling of compliant parts, issues in robotic assembly of

compliant work pieces, etc. [Menassa an DeVries, 1991] presented an approach for optimization of deformable work piece fixture that minimizes the amount of deformation in the part. [Rearick et al., 1993] found inconsistencies in [Menassa an DeVries, 1991] and modified their approach to present a technique for design and evaluation of fixtures for compliant parts. [Lee and Haynes, 1987] proposed a finite element model of the fixturing system analysis for prismatic parts. Based on this model, many approaches to optimal fixture design for compliant sheet metal parts have been developed (see, e.g., [Youcef-Toumi et al., 1988] and [Cai et al., 1996]. Of special interest is the presentation in [Cai et al., 1996] of the "N-2-1" locating principle for use with compliant parts. [Ceglarek et al., 1999], [Li et al., 2000] and [Ceglarek, 2001] extended the compliant part fixture design methods to material handling of compliant parts by adding movability conditions. [Chang and Gossard, 1997] presented a graphic approach for multi-station assembly of compliant parts.

[Villarreal and Asada, 1991] have studied robotic assembly of compliant work pieces and have introduced the concept of Buffer Zones as a geometric representation of the effects of part compliance. [Kosuge, 1995] and [Newell and Khodabandehloo, 1995] exploited various methods to approximate shapes of deformed parts during robotic assembly. [Kraus and McCarragher, 1996] and [Kraus and McCarragher, 1997] present a method for planning and control of robotic assembly tasks involving flexible parts. [Henrich et al., 1999] and [Schmidt and Henrich, 2001] develop a method to model robotic manipulation of deformable linear objects (one-dimensional compliant parts). [Yuen and

Bone, 1996] investigate the problem of maintaining proper contact states between compliant parts during robotic assembly.

[Liu and Hu, 1995] proposed use of finite element analysis to predict variation of compliant parts. [Liu et al., 1996] and [Liu and Hu, 1997] expanded this idea and developed methodology for variation simulation of assemblies of compliant parts using finite element analysis. As a result of that research, Compliant Assembly Variation Analysis (CAVA) software has been developed at University of Michigan [Hu and Iyer, 2000]. [Camelio et al., 2001] expanded the methodology of [Liu et al., 1996] and [Liu and Hu, 1997] to model variation propagation of compliant part assemblies through a multi-station assembly process. [Merkley, 2000] has combined statistical tolerance analysis with finite element structural analysis. [Soman, 2000] has presented a method to statistically characterize a population of surfaces of compliant parts; the outputs of this method can be used with finite element analysis to predict assembly forces and part distortions. [Bihlmaier, 2000] and [Tonks, 2001] propose inclusion of nodal covariance in the statistical analysis of flexible assemblies in an attempt to greatly reduce computation times of finite element models.

All these methods are sufficient for analysis of manufacturing and assembly operations involving compliant parts in existing designs. However, use of these methods requires extensive part knowledge that is usually not available at early phases of product design. Thus they are not readily useable with strategic top-down design of assemblies.

Currently, no methods to support top-down design of assemblies with compliant parts are known to the author.

1.4 Organization of the Thesis.

This thesis is organized as follows. Chapter 2 will introduce Datum Flow Chain method for strategic design of assemblies. It will be stressed that the assembly design should be carried out prior to individual part design in order to ensure that overall assembly attributes important to the customer, called Key Characteristics (KCs), are satisfied. DFC method's role in such top-down design process will be demonstrated. Vocabulary and associated mathematical tools will be presented, and the inability of these tools to deal with challenges of compliant parts will be discussed.

Chapter 3 will deal with the use of compliant parts in assemblies. Main topics that will be covered are the role of the compliant parts in delivering customer requirements and exact nature of compliance. Focus will be on effect of part compliance on the assembly's ability to deliver KCs. Different methods of assembly of compliant parts will be introduced and their effects on KC delivery discussed

Chapter 4 dives into the problem of KC conflict in assemblies. Underlying causes of KC conflict will be discussed. As will be shown, compliant parts can be successfully over constrained in order to resolve certain types of KC conflict. A method for finding such types of KC conflicts will be presented.

Overconstraint of any parts, including compliant parts, results in statically indeterminate assembly and uncertainty in locational and constraint state of parts involved. The need for a method to find locational and constraint responsibility paths through an over constrained compliant parts will shown. Finite element analysis is often used to resolve this issue, but requires extensive part knowledge. As such, it is incompatible with topdown design methodology such as DFC. A simpler method is needed, and is developed in Chapter 5. Concept of Effective Locating Zone (ELZ) of a locator is presented and used to find locational and constraint responsibility paths through a compliant part. Two techniques for finding ELZ are presented. One, a membrane-based technique, requires some part knowledge and uses FE analysis to find precise ELZ. A second technique requires only knowledge about locator types and locations and results in approximations of ELZ.

Chapter 6 unites the results of previous chapters to present DFC-based methodology for designing assemblies with compliant parts. Chapter 7 concludes the thesis by summarizing the main findings of this research and presents the areas for future work.

CHAPTER 2. DATUM FLOW CHAIN

2.1 Datum Flow Chain method of modeling assemblies

Assembly can be thought of as chaining together part-to-part dimensional and constraint relationships. [Mantripragada and Whitney, 1998] presented Datum Flow Chain (DFC) method to capture such relationships and provide a set of corresponding vocabulary and mathematical tools. DFC methodology supports top-down design of products: through use of the DFC the designer can achieve the satisfactory design of the overall assembly by establishing part-to-part constraint and dimensional relationships <u>prior</u> to detailed part design. DFC captures designer's intent, and while it can be effective as an analysis tool to solve existing assembly problems, its best use is as a strategic design tool in early stages of product development. DFC methodology should not be confused with design <u>for</u> assembly. It is strategic design <u>of</u> assemblies.

DFC is a directed acyclic graphical representation of an assembly with nodes representing the parts and arcs representing mates between them. Effectively, a DFC defines the locational responsibility and constraint transfer relationships between parts in assembly as well as role of any other equipment that is used in the assembly process such as fixtures, hand-held tools, measuring machines, etc. A DFC identifies the part-to-part relationships that convey dimensional control and identifies the hierarchy that determines which parts or fixtures define the location of other parts. DFC also contains information on the mating features that perform the function described above and the type of motion constraint applied by each feature. A DFC can be drawn on a wide range of applications. A part level DFC helps see what goes on inside a part and can be used to examine relationships between different locating features within the part. A DFC can be drawn across a set several parts, perhaps to solve an existing localized assembly problem. A DFC can also be drawn for complex assemblies in their entirety, ranging in complexity from a simple household appliance to a complete automobile front end (for an example, see Fig. 2.1). The latter use is an effective tool in early stages of product design.



Fig. 2.1. Datum Flow Chain of a typical automotive front end. Key Characteristics (Sec. 2.2) are represented by double red lines, mates (Sec. 2.3) by arrows (with the direction of the arrow representing the transfer of locational and constraint responsibility), and contacts (Sec. 2.3) by the dashed lines.

Use of the DFC method requires several assumptions about the assembly process. Each assembly operation completely locates the part being assembled with respect to existing parts in the assembly or an assembly fixture. Only after the part is completely located is it fastened to the remaining parts in the assembly. Thus, all subassemblies are assumed to be properly constrained before the assembly process can continue. This is important in order to rationalize the assembly process and to make incomplete DFC's make sense. An incomplete DFC represents a partially completed assembly. If the parts in a partially completed assembly are not completely constrained, by each other or by fixtures, it is not reasonable to expect that they will be in a proper condition for receipt of subsequent parts, in-process measurements, transport, or other actions that may require an incomplete assembly to be dimensionally coherent and robust. If needed, a fixture can be added to make the subassembly properly constrained.

As previously mentioned, DFC is acyclical. Cycles, or loops, in a DFC would mean that a part locates itself once the entire cycle is traversed and hence are not permitted. From this, we can infer that there must be a single root node in a DFC, a part that is not constrained by any other part in an assembly. This represents the part from which the assembly process begins, a base part or a fixture.

It is clear that a particular DFC is heavily related to assembly sequence. A single set of parts can possibly yield a number of distinct DFCs. Furthermore, a single DFC can potentially generate multiple assembly sequences. Software developed at MIT uses DFC

as an input and generates as an output a set of possible assembly sequences, using the assumptions described above [Baldwin et al., 1991].

A designer can use DFC methodology to choose optimal part-to-part relationships, as well as the optimal assembly sequence, to achieve best possible overall product. Such product delivers all the important qualities desired by the customer. These qualities are referred to as Key Characteristics.

2.2 Key Characteristics

[Lee and Thornton, 1996] define Key Characteristic (KC) as an important customer-lever property of a product or assembly that is in danger of not being achieved due to variation at the fabrication or assembly level. A KC is said to be "delivered" by the assembly if the desired property is achieved within some specified tolerances. If this characteristic is not delivered, the product's attractiveness to a customer is reduced. Goal of top-down design using DFC methodology is to develop an assembly that robustly delivers all of the KCs.

KC's that are readily observed by a customer are called "customer-level KCs". Anything that is important to a customer can be considered a KC. Several examples of customer level KC's in an automobile include satisfactory fit and finish between exterior body panels, satisfactory engine performance, and satisfactory door closing effort.

Every customer-level KC has associated with it one or more "assembly-level KCs". These are certain geometric relationships between parts in the assembly that must be achieved in order for a customer-level KC to be delivered. For example, satisfactory appearance of automobile body exterior requires that appropriate margins and flushness between body panels be achieved. In [Moccio, 2002], close relationship between automobile ride and handling and a geometric relationship between parts of automotive sub frame is developed. In order to deliver satisfactory engine performance, certain geometric relationships between the cylinder walls and the piston is needed. Acceptable door closing effort depends on geometric relationship between the body seal and the door inner panel. Similar flow-down to geometric assembly-level KCs can be made from most customer-level KCs.

Assembly-level KC's are dependent on manufactured parts making up each KC delivery chain. These parts have "manufacturing-level KC's" associated with them – certain dimensions that parts must meet in order to deliver acceptable assembly-level KC when put together. Such manufacturing-level KC's are more commonly referred to as part tolerances.

In the remainder of this thesis, term KC will refer to assembly-level KC. In a DFC diagram, a KC is typically identified by a double red line. DFC clearly identifies a complete location and constraint path through the whole assembly connecting two parts, whose geometric relationship defines the KC.

2.3 Part features: mates and contacts

In an assembly, parts are joined by assembly features. A typical part has multiple assembly features joining it with other parts in the assembly. Not all of these features transfer location and dimensional constraint, and it is essential to distinguish the ones that do from the ones that do not and provide other functions such as strength. Based on this distinction, assembly features are split into two types: contacts and mates.

Mates are assembly features that convey dimensional location and constraint from one part to another. On a DFC diagram, a mate is represented by an arc with an arrow, with the direction of the arrow inferring the dimensional responsibility – the node (part) at the tail end of arrow locates the node (part) at the arrowhead. DFC documents KC delivery by tracing a chain of mates between the two parts defining the KC.

A mate can transfer constraint and location in any number and any combination of the six degrees of freedom (three rotational and three rotational). For example, a square peg fitting into a square hole constrains all six degrees of freedom, while a round peg fitting into a round hole constrains five– one rotational degree of freedom is left free. Total of 17 distinct features have been catalogued in [Adams, 1998], with their constraint properties documented.

Based on assembly sequence, relative to a specific part mates can be either incoming or outgoing. The direction of the mate depends on the assembly sequence associated with

the particular DFC being examined. A change in an assembly sequence can reverse the direction of the mate arrow.

Contacts are assembly features that are not intended as part of the locating scheme. These features can have other important functions, such as attachment or reinforcement, but are not intended by the designer to convey dimensional location and constraint from one part to another. Since common sense dictates that a part should be properly constrained or located in space before any reinforcements or fasteners can be added, the incoming mates to a part should be made before the contacts. This becomes even more important when dealing with compliant parts, as the location of contacts themselves might become important – this will be discussed in later chapters.

On a DFC diagram, a contact is typically represented by a dashed line.

Mates are explicitly defined by the designer. However, a common problem is that a feature that is intended to be a contact ends up acting as a mate. This occurs when a contact switches from passive acceptance of location asserted by other features (mates) to active assertion of location itself. This can happen for a number of reasons. For example, overtorquing of a threaded fastener, which, if properly torqued, does not induce constraint, can distort the location of parts involved. A designer should be aware of the potential of contacts unintentionally becoming mates, and ensure that this does not happen.

2.4 Constraint states of assembly

Constraint is a critical property of a design. Based on the mates involved in the assembly, a resulting product will be under-, over-, or properly constrained. Six degrees of freedom – three translational and three rotational – are inherent in any part. A part is considered properly constrained if every one of these degrees of freedom is constrained once and only once. Properly constrained assembly is composed entirely of properly constrained parts. Properly constrained assembly is robust to variations and should be a goal of the designer.

Both under- and over constraint may be required for functionality, with examples including freely-rotating crankshaft of an engine (under constraint) and preloaded ball bearing sets (over constraint). Presence of either under- or over constraint, when not required for function, will result in an assembly that is not robust to variations. Under constraint will lead to random variations in the assembly that may cause non-delivery of some KCs. Over constraint will ether cause random variation (this is more commonly referred to as the redundant constraint, with a classic example being the four-legged stool) or induce local inference between parts, resulting in local stresses.

Compliant parts have additional degrees of freedom, due to their flexibility, which can be used to relieve over constraint. As a matter of fact, over constraint of compliant parts is a common practice used to relive KC conflict and will be discussed in great detail in later chapters.

2.5 Mathematical methods used with DFC

2.5.1 4x4 Matrix transforms

Connective models of assembly such as DFC require matrix transformations to locate the parts with respect to each other. Each part is assumed to have a base coordinate frame with mating features on parts each having their own frame. An assembly is treated as a chain of these feature frames, and each transformation allows to go from frame to frame and thus from part to part. The mathematical representation takes the form of a 4x4 matrix transform. The transform, *T*, takes the following form:

$$T = \begin{bmatrix} A & d \\ O & 1 \end{bmatrix}$$

where A is a 3x3 rotation matrix, d is a 3x1 displacement vector, and O is a 1x3 vector of zeros. Two parts, having base coordinate frames A and B, being joined together by a mating feature with its coordinate frame F, is represented as

$$T_{AB} = T_{AF} * T_{FB}$$

where T_{ij} is the 4x4 transform between frame *i* and frame *j*, expressed in frame *i* coordinates.

[Whitney et al., 1994] extended the variation analysis based on 4x4 matrix transform approach to statistical analysis of GD&T. This approach of calculating accumulated variation in the part location due to variations in part-level dimensions combines the variations in the location of assembly features by multiplying the matrix transforms representing the variations along the tolerance delivery chain. The tolerance delivery chain reflects the intent of the design team which is captured by DFC. This approach can be used to perform all three types of variation analysis methods (worst-case, statistical and Monte Carlo).

2.5.2 Screw theory

[Adams 1998] and [Adams and Whitney, 1999] propose use of Screw Theory to determine constraint properties of assemblies. Screw Theory was first proposed as a tool to analyze mechanisms by [Waldron, 1966], who presented the concepts of twist- and wrench-matrix. Twist-matrix is a collection of screws that represents relative motions between two rigid bodies and is of the form

 $\mathbf{T} = [\boldsymbol{\omega}_{\mathbf{x}}, \, \boldsymbol{\omega}_{\mathbf{y}}, \, \boldsymbol{\omega}_{\mathbf{z}}, \, \mathbf{v}_{\mathbf{x}}, \, \mathbf{v}_{\mathbf{y}}, \, \mathbf{v}_{\mathbf{z}}]$

where the first triplet represents the angular velocity of one body with respect to a base coordinate frame of the second body. The second triplet represents the velocity, in the reference frame of the second body, of a point on the first body. Wrench-matrix is a reciprocal of the twist-matrix and is a collection of screws that represents constraints exerted by one body on other. It is of the form

 $W = [f_x, f_y, f_z, m_x, m_y, m_z]$

with the first triplet representing force that can be resisted by the wrench, while the second triplet represents moment.

[Konkar, 1993] developed an algorithm to intersect the twist-matrices to find the degrees of freedom of a body under multiple constraints. [Adams 1998] has adapted Konkar's algorithm for use with DFC and has tabulated wrench and twist matrices of 17 mating features.

2.6 Advantages of top-down design using DFC methodology

Advantages of top-design are clear and have been pointed out in Chapter 1. Using DFC to assist with top-down design yields several other important advantages that deserve to be mentioned here. Value of DFC lies in its ability to completely describe complex assemblies in their entirety, graphical nature, and ease of use.

Interaction among parts is easier to understand in graphical manner. Thus, DFC can be used to effectively communicate location and constraint information between various collaborators involved in the design of the product. It can be used to easily and logically evaluate effect of alternative locating schemes of parts. Impact of outsourcing of various components can also be evaluated, and suppliers can be involved in discussion of locating scheme.

DFC can document the locating and constraint strategy for all components of an assembly. Through DFC, locating and constraint schemes are captured in a relatively short amount of time and with only minimal knowledge about the parts involved. Using DFC, superior locating schemes can be captured and book shelved for future use.

Finally, DFC analysis helps identify critical part tolerances to which special attention should be paid. Information about any particular KC and in any particular direction can be isolated for ease of analysis. Part tolerances should be chosen in such a manner as to guarantee KC delivery and should be driven by tolerances on KC's.

2.7 DFC and flexible parts

Currently, use of DFC methodology requires the designer to assume that all parts in the assembly are rigid. It is the goal of this thesis to present methods that make DFC applicable to assemblies containing compliant parts as well.

Under rigid assumptions, each part is completely located once its position and orientation in the three dimensional space are determined. Hence, if position and orientation of one point on the part are known, the location of all points on the part can be determined. Clearly, this is not the case with compliant parts. A compliant part, due to the nature of compliance, can be thought of as having additional degrees of freedom in its flexible direction (please see section 3.3 for definition of and discussion on flexible direction of a compliant part).

Part compliance presents the problem of properly identifying the "location responsibility paths" traveling through compliant parts. The cornerstone of DFC – the ability to identify what part is responsible for what other part – is simple with rigid parts: a single set of incoming mates is responsible for all outgoing mates. However, with a compliant part, a DFC alone cannot be used to develop clear relationships between incoming and outgoing mates in the part's compliant direction. Without ability to identify what incoming mate is responsible for each outgoing mate, KC delivery chains cannot be drawn through compliant parts.

The DFC cannot take into account use of additional mates needed to properly constrain the compliant part's flexible direction or directions. For example, it is a common industry practice to use more mating features for a compliant part in the flexible direction than would be needed with a rigid part, as in the example of 3-2-1 fixturing method being replaced with N-2-1 (see, e.g., [Rearick, 1993]). Using the screw theory constraint analysis, such schemes would be identified as over constrained and problematic. This is not necessarily the case, as effect of part compliance and its ability to alleviate overconstraint is ignored. In Chapter 5, we will introduce a method to help find proper chains of locational and constraint responsibility transfer.

2.8 Chapter summary

Datum Flow Chain (DFC) is a method to capture part-to-part dimensional relationships in an assembly. DFC method supports top-down design process and is very valuable for strategic design of assemblies. DFC's graphical nature makes it easy to implement and use.

Goal of the DFC method is to design assemblies that robustly deliver all of product's Key Characteristics (KCs). A customer-level KC is an important customer-defined property of a product that is in danger of not being achieved due to variation at the fabrication or assembly level. A customer-level KC can be flowed down to one or more assembly-level KCs. An assembly-level KC is a certain geometric relationship between parts in the assembly. DFC is a directed acyclic graphical representation of an assembly. Links between parts are accomplished through part features. DFC method makes a distinction between two different types of part features: mates and contacts. Mates are features that convey location and constraint from one part to another. These features actively participate in the assembly locating scheme. Contacts are features that do not convey location and constraint from one part to another and are not intended as part of the locating scheme. They can have other important functions, such as reinforcement or attachment.

Two mathematical methods have been shown to be effective with DFC methodology. 4x4 matrix transforms have been used for locational and variation analysis of assemblies. Screw theory has been used for constraint analysis of assemblies.

Currently, DFC methodology is not fully able to analyze assemblies with compliant parts. Part compliance leads to additional degrees of freedom inherent to the part. The presence of these additional degrees of freedom lead to challenges, as well as benefits, to assembly design, which the DFC methodology cannot account for. Rigid assumptions are currently used with DFC. These are not satisfactory for analysis of assemblies with compliant parts.

The next chapter will discuss behavior of compliant parts in an assembly. On the basis of the next chapter's discussion, we will be able to form methods that will allow us to bring assemblies with compliant parts into the scope of DFC.

CHAPTER 3. COMPLIANT PARTS IN ASSEMBLY

3.1 Use in the industry

Compliant parts are widely used in the industry. For example, an automotive body is made up of more than 200 stamped sheet metal parts [Ceglarek and Shi, 1995]. A wide variety of consumer electronics utilize compliant plastic components. It is, thus, of no surprise that assembly operations involving compliant parts compose a major portion of all assembly operations.

Two distinct uses of compliant parts in assemblies have been observed by the author. One of these uses entails compliant part used as a closure – a component that is responsible for enclosing a certain region of space. Examples of closures include automotive fenders and body sides, and airplane skins. Such a use places certain requirements on resultant geometric relationships between the compliant part being assembled and the rest of the assembly. Such relationships are, or assist in delivery of, assembly-level KCs.

The second use of compliant parts is to assemble two or more compliant parts into a rigid subassembly that is then used as a structural, load-bearing component. A good example of such use is joining of two sheet-metal stampings to produce automobile frame rail. Such resulting subassembly of compliant parts must have sufficient strength and rigidity to achieve its function. Geometric relationship between the parts involved is typically not particularly important. Rather, material properties of the resulting subassembly are of interest (please see Fig. 3.1).



Fig. 3.1 Structural component assembled from two compliant parts. Here, two stamped sheet metal parts are assembled together to form a box beam. The geometric relationship between the two parts is not important, as long as they form a box beam of sufficient stiffness. There is a "material property requirement" on the resulting assembly, but no "geometric requirement".

The motivation of this thesis is to develop optimal methods of KC delivery through compliant parts. Therefore, we will focus on the examination and analysis of the first, or "geometric requirement based", use. The second, or "material property requirement", use is best handled during the detailed part-design process and is outside the scope of this thesis.

3.2 Compliance of a part

Many types of compliant parts are in use in the industry. In this section, we will take a closer look at compliance and study several properties of compliant parts.

3.2.1 Directions of a compliant part: flexible vs. rigid

Part compliance is a function of direction. While some parts, such as cloth patterns in the garment industry, are equally compliant in all directions, more typically a compliant part can deform much more readily in certain directions than others. This fact was discussed in [Villarreal and Asada, 1991] in the context of robotic assembly of compliant work pieces.

If an equal force is applied in each direction, a compliant part deforms more readily in certain directions than others. Based on material properties and part shape, certain direction or directions will provide an easy mode of deformation [Panton and Van-Brunt]. Deformation in such directions will be dramatically more pronounced, and relatively small force input will be required to produce large deformation. Such direction is called "flexible" direction of a compliant part. Direction that does not provide an easy mode of deformation is called "rigid" direction of a part. A rigid part has six "rigid" directions.

Concepts of compliance and rigidity are closely dependent on the size scale of deformations due to applied forces. Nothing is perfectly rigid. But, depending on the industry and product, part deformations of certain magnitude are judged negligible. The level of these "negligible" deformations is what distinguishes a rigid direction from compliant. Within the scope in any industry or product, one can safely assume that it does not present a challenge to determine within what range of small deformations a direction is considered "rigid". What's more, parts used in the industry have directions that can easily be qualified as either "flexible" or "rigid" based solely on knowledge of material properties and visual inspection of part shape, as well as designer's experience, as was observed in [Villareal and Asada, 1991].

Plane of the part perpendicular to the flexible direction will be referred to as the "flexible plane" of the part. For example, if a part's flexible direction is z, then the plane that lies in the x-y plane is the part's flexible plane (please see Fig. 3.2).



Fig. 3.2 Flexible direction of a compliant part and the associated flexible planeFlexible plane of the part is a plane perpendicular to its flexible direction.Here, if Z is the flexible direction of the part, the associated flexible plane is the X-Y plane.

3.2.2 Types of compliance

Depending on material properties and part shape, compliant part's flexible direction can be further subdivided into either of two compliance types – "semi-rigid" and "fullycompliant". In flexible direction, if all deformations due to unintended (by designer) external factors – such as gravity or part handling – are non-existent or small enough to be discounted, that flexible direction is called "semi-rigid". In a semi-rigid direction of the part, any part deformation will only occur due to locating features deliberately put in by the designer. In other words, in a semi-rigid direction, any factor affecting the location and shape of the compliant part is intended by the designer. Typically, a part with a "semi-rigid" direction will have at least one rigid direction as well.

If a part in a certain flexible direction can experience deformation due to unintended external forces, such as gravity, in addition to intended locating features, that direction is called "fully-compliant" flexible direction. Distinction between semi-rigid and fully-compliant types of flexible directions depends on material properties and shape of the part, as well as the part's orientation in space relative to factors which can have unintentional influence on the part. Effect of the part's orientation in space will be seen in the example discussed in the next paragraph.

Let us examine the parts in the automotive front end assembly in Fig. 3.3. The fender has a clearly identifiable "flexible" In/Out direction. The remaining two directions – Fore/Aft and Up/Down – are rigid for most typical fender designs. Typical fenders have material properties and shapes that would have the In/Out direction be acting as "fully-compliant" if gravity was acting in the direction parallel to In/Out. Thus, the In/Out direction of the fender would be a "fully-compliant flexible" if the fender was assembled to the rest of the vehicle in such a manner as to allow gravity to act in the In/Out direction – e.g., the car was being positioned on the assembly line on its side. However, the fender is usually assembled with the vehicle positioned in the upright manner, with the gravity acting in the Up/Down direction. With no other unintentional factors affecting the fender in the In/Out direction, the designer can deduce that the fender has "semi-rigid" flexible In/Out direction and two remaining directions are rigid.

A part can have flexible directions of more than one compliance type. For example, depending on its exact shape and material, the plastic bumper cover of Fig. 3.3 might have a rigid In/Out direction, a "semi-rigid flexible" Fore/Aft direction, and a "fully-compliant flexible" Up/Down direction.



Fig. 3.3 Automobile Front End. The three parts in the figure – fender (In/Out), hood (High/Low), and bumper cover (Fore/Aft and Up/Down), all have flexible directions.

Distinction between two types of compliance becomes important when KCs are delivered through the compliant part in the particular flexible direction. It is important to realize the effects of the different behavior of the two types of compliance. Since in a fully-compliant direction unintended part deformations are possible, the part shape in that direction is not fully controlled by the designer. Thus, KC delivery across a fully-compliant direction is less predictable than in a semi-rigid direction. This will influence designer's decision on placement and type of incoming mates to the part in the flexible direction being considered. This topic shall be addressed further when concept of Effective Locating Zone is introduced in Chapter 6.

3.3 Assembly as a removal of degrees of freedom

One way to think about the act of assembly is as removal of degrees of freedom from the part being assembled. During assembly, free degrees of freedom are fixed relative to some known coordinate frame, thus resulting in knowledge about location of the part.
Let us explore this concept by examining removal of degrees of freedom during the traditional 3-2-1 part fixturing method (for more on the 3-2-1 fixturing method itself, see [Lowell, 1982]). Rigid part assumptions are used. Ideally – assuming nominal dimensions – all of the locating surfaces on each plane are engaged simultaneously. So, in the primary plane, the three locating surfaces engage simultaneously, instantly constraining one translational and two rotational degrees of freedom. The three surfaces engage simultaneously, acting as a single feature and instantly constraining one translational and one rotational degrees of freedom. The tertiary case is trivial, as only one locating surface is involved.

However, with manufacturing variations and assembly variations from previous steps, such ideal case of ideal parts is unlikely. Correspondingly, immediate engagement of all locating surfaces in a given dimension will not occur. Let us explore what actually occurs during 3-2-1 fixturing with part variations included. Upon contact of fixture with primary plane of the part, the first locating surface to engage will constrain the translational degree of freedom, second surface to engage will constrain one of the rotational degree of freedom. Similar situation occurs with the secondary part plane. The first locating surface to engage will constrain the translational degree of engage will constrain the translational degree of freedom. Similar situation occurs with the secondary part plane. The first locating surface to engage will constrain the translational degree of freedom, and second surface to engage of freedom. The tertiary plane case is again trivial, due to presence of only one locating surface.

Examining the order of sequential removal of degrees of freedom during assembly highlights the problems with redundant locators and overconstraint. Consider use of 4-2-1 fixturing in place of conventional 3-2-1 approach, with rigid part assumptions still in place. Upon contact of the fixture with the primary plane, the first three locating surfaces to engage the part result in behavior described previously. However, engagement of the fourth locating surface creates problems. Either this fourth locating surface is redundant and plays no role in constraining the part being assembled (thus raising a question about its purpose in the first place), or the fourth locating surface influences the constraint, and correspondingly location, of the part – resulting in an over constrained part – and disturbs the job done by the three locating surfaced already engaged. Such instances result in lack of robustness in the final product. There is ambiguity about the role of each of the features in locating of the part, thus resulting in uncertainty about the final location of the part following assembly. This case can be extended to all situations where redundant locators and overconstraint are present.

Compliant parts present an additional challenge to the analysis of degree of freedom removal. Due to inherent nature of compliance, these parts may have additional degrees of freedom associated with their flexible directions. As each locating surface engages the part's flexible plane, it forces the part to deform. Each of the locating surfaces essentially ends up single-handedly constraining and locating a small region of the part. The size and shape of such regions depends on the interaction between all engaged locators. Situation is further complicated if the flexible direction is of the fully-compliant type, as exterior

factors – usually gravity – influence the shape of the part. N-2-1 fixturing method [Cai et al., 1996], with N>3, is often used to minimize part deformation due to external factors. Use of N locators results in the flexible plane of the part being divided into N number of smaller regions, each controlled by one of the N locators.

3.4 Different types of assembly of compliant parts: serial vs. parallel

[Liu et al., 1996] has split assemblies of compliant parts into two categories: assembly in series and in parallel. Examples of both types of assemblies are shown in Fig. 3.4(a, b). An assembly is serial if the two parts are joined "end-to-end", in such a manner that their flexible planes are added to form resulting subassembly with a larger flexible plane, as in Fig 3.4(a). If the two parts are assembled in such a manner that their flexible planes do not add to form a larger flexible plane of the resulting assembly, as in Fig 3.4(b), the assembly is in parallel. In other words, in serial assembly the flexible planes of parts involved are added to one another, while in parallel assembly they are superimposed on one another.

Fig. 3.4(c) shows an example of a hybrid case, an assembly that features characteristics of both serial and parallel assembly. In such a case, the parts involved should be examined and the decision on assembly type be made depending on the region where KC delivery chain or chains pass through. Chapter 5 will discuss location of points participating in KC delivery and their influence on assembly.



Fig. 3.4 Serial vs. Parallel assembly. a. Serial assembly b. Parallel assembly c. Hybrid assembly

Note that notion of parallel assembly is only valid in case where parts involved have one dimension significantly smaller than the remaining dimensions. This makes sense, as such part shape is inherent to compliant parts with at least one flexible direction. The distinction between serial and. parallel assemblies, while potentially applicable to all parts, is particularly important to the discussion of compliant parts.

The major distinction between the two different types of assembly is in the rigidity of the resulting subassembly: assembly in parallel will result in a subassembly that is more rigid than either of the two parts involved, while the assembly in series will result in a subassembly that is less rigid. One way to recognize the effect of these two assembly methods is by treating the compliant parts as linear springs. If two parts are represented by linear springs with coefficients of stiffness k_1 and k_2 , the resulting coefficients of stiffness for the resulting subassembly are:

$$K_{series} = (1/k_1 + 1/k_2)^{-1}$$

if the assembly is carried out in series, and

 $K_{\text{parrallel}} = k_1 + k_2$

if the assembly is parallel.

3.4.1 Assembly of compliant part and resulting rigidity

Rigidity of the final assembly is a function of nominal design. However, the rate at which this rigidity is achieved is a function of assembly sequence chosen. As was shown in the previous section, choice between serial and parallel assembly steps has large influence on the rigidity of the resulting subassembly. Any nominal design has within it a finite number of serial and parallel assembly steps. By choosing a specific sequence in which these steps are performed, one can then influence the rigidifying rate of the assembly. Let us explore a simple example of one large and two small sheet metal panels (Fig. 3.5). Two possible assembly sequences are shown in Fig. 3.5(a,b), with the resultant graphs of assembly rigidifying in Fog. 3.5(c,d). Not surprisingly, with parallel assembly steps performed first, the assembly is rigidified quicker. Similar conclusion is presented in [Hu, 1997].

3.5 Rigidizing assembly step

Of particular interest to us is the assembly step at which locational and constraint responsibility through a compliant part are set. This is the step after which all of the degrees of freedom of the part have been constrained and the part essentially can be treated as rigid. After such step, part compliance is no longer a factor, and neither

problems nor advantages present due to compliance are of concern. Such a step is called the "rigidizing" assembly step for the part.



Fig. 3.5 Effect of assembly sequence on assembly rigidifying rate. All beams are homogenous, having identical material properties and crossections. Stiffness is measured by clamping the subassembly at one end, applying force **P** at the other end, and measuring resulting end deflection δ . Stiffness is then the ration of **P**/ δ .

a. Process 1: serial assembly step is carried out first, parallel assembly step second b. Process 2: both steps are parallel assemblies

c. Assembly stiffness throughout each process, normalized to final assembly stiffness.

By performing parallel assembly steps prior to serial assembly steps, the assembly stays more rigid during the assembly and achieves its final rigidity state quicker.

Rigidizing step is of critical importance for analysis utilizing DFC methodology. During this step, all locational and constraint responsibility paths – which will become links of the DFC – are established. Prior to this step, these paths cannot be established as the part is compliant and has under constrained degrees of freedom. Following this step, these paths will not be disturbed as the part is now essentially rigid and cannot be deformed by

subsequent assembly steps. Thus, all locational and constraint responsibility paths essential for top-down design with DFC are established during the rigidizing step.

Rigidizing step is the first assembly step where the compliant part is assembled in parallel with another part or subassembly to produce a rigid result. Typical rigidizing steps include a compliant part being added to a rigid subassembly, such as a fender installed to the rigid car body frame. Another example is two compliant parts being brought together, with a rigid fixture providing for all locating and constraint, to form a rigid component. Examples of this included assembly of compliant sheet metal stampings into an automotive rail assembly and assembly of door inner and door outer panels into an automobile door.

Rigidizing step could be difficult to pinpoint for some parts in analysis of existing assemblies. Clearly, a compliant part added to a much more rigid frame is a rigidizing step, but such clear-cut steps are not necessarily always present. The step involving a much more rigid frame could be lacking, or if the compliant part was involved in previous assembly operations, it might be possible that one of the previous steps was in fact a "rigidizing" step. The problem of clearly identifying the rigidizing step in early stages of product development is further exacerbated by the lack of detailed knowledge about parts involved – their exact geometries and material properties.

The solution to this dilemma is quite obvious and lies within the spirit of top-down design – the designer should establish the rigidizing step in early product design stage.

Just as designer can use DFC methodology to devise the exact paths of KC delivery, the designer can also plan the exact assembly step at which locational and constraint responsibility paths through a compliant part are made. The selection of the rigidizing step thus becomes a tool of the designer, with the rigidizing step being chosen to optimize KC delivery.

In certain cases, a compliant part may never undergo a rigidizing step, and retain at least one flexible direction. In such cases, no KCs are to be delivered through the part in its flexible direction. It can be argued that in these cases the designer either makes a mistake (which, of course, must be corrected), is not concerned with that direction since no KCs are delivered through it, or actually desires compliance in order to deliver a certain function. For example, the diving board requires a flexible direction in its final assembled state in order to function properly.

3.5.1 Choosing optimal rigidizing step

Chapter 4 will establish overconstraint of a compliant part as a means of resolving KC conflict. In order to do so, the part must remain as compliant as possible until the final step involving the part in KC delivery – assuming that KCs are delivered in the part's flexible direction. Thus, the optimal rigidizing step would be the assembly operation in which the KC's are delivered through the part and ideally should be the last step involving the part and creation of new DFC links.

If no KCs are delivered in part's flexible direction or directions – either KCs involving the part are delivered through its rigid direction, or the part is not involved in KC delivery at all – the selection of the rigidizing step is irrelevant from the point of DFC method. However, if the part is involved in delivering any sort of structural function, it would be a good idea for a part to be rigidized early, though this area is beyond the scope of this thesis.

Care should be paid to insure that the step designated by designer as rigidizing is indeed the step that rigidizes the part. During detailed part design (following the overall assembly design and establishment of rigidizing steps), it should be ensured that no steps prior to the designated rigidizing step can unintentionally "rigidize" the part too early. In addition, the rigidizing step itself should be examined to make sure that it provides sufficient rigidity in the resultant assembly to properly "rigidize" the part.

3.6 Springback

Spring back is a well-known problem occurring during assembly of compliant parts. Effect of spring back on the resulting assembly has been discussed by such researchers as [Hu, 1997] and [Chang and Gossard, 1997]. Spring back occurs when fixtures, used in the assembly to constrain a part in its flexible direction, are removed.

When the part is over constrained in its flexible direction, it deforms due to the manufacturing and assembly variations present. As a result, internal stress is stored within the deformed part. In an assembly where no fixtures are used and all part-to-part

relationships are achieved through features located on parts themselves, this stress remains locked in the part after the assembly process. If fixtures are used, removal of the fixture, and the constraint imposed by it, results in the stored energy in the part forcing it to spring back to achieve a state of least stored energy. This results in loss of location set by the fixture and leads to unpredictable variation in final location of regions of the part. Exact shape of the part affected by spring back can be determined through finite element analysis combined with statistical variation analysis – see, e.g., [Liu and Hu, 1997], [Bihlmaier, 2000], or [Tonks, 2001]. However, such approach requires detailed knowledge about the parts and is not feasible in early stages of product design.

One way to alleviate the effect of spring back is to ensure that no KCs are delivered through the part in regions where spring back can be expected. Then, effect of spring back can be ignored. Another way of alleviating the effect of spring back is via strategic placement of contacts, which remain after the fixture carrying the mates is removed. It is assumed that no spring back occurs at the point of contacts – in other words, the points of attachment and reinforcement are not deformed. The goal of such use of contacts is to minimize actual spring back at points of KC delivery

More on containing the effect of spring back on design of assemblies with compliant parts will be discussed in Chapters 5 and 6.

3.7 Chapter summary

Compliant parts have two uses in the assembly. The one that is of interest to DFC methodology is their use as a closure – a component that is responsible for enclosing a certain region of space. This use of a compliant part places a geometric requirement between the part and the other parts in an assembly. The second use has compliant parts assembled into structural components, and places material properly requirements on the resulting subassembly. The analysis of this use is beyond the scope of this thesis.

Part compliance is a function of direction. A part can have rigid and flexible directions. Flexible direction is one that provides for an easy mode of deformation, while a rigid direction is one that does not. A compliant part is one that has at least one flexible direction. Flexible directions can be one of two types: semi-rigid or fully-compliant. In a semi-rigid direction, the designer intends any factor affecting the location and shape of the part. In a fully-compliant direction, factors not intended by the designer, such as gravity, can influence part location and shape as well.

Assembly can be thought of as removal of part's degrees of freedom. Using this idea, it is easy to see additional degrees of freedom inherent to a compliant part in its flexible direction.

Two types of assemblies of compliant parts have been identified: parallel and serial. Parallel assembly results in a subassembly that is stiffer than either of its components, while serial assembly does not. Sequence of parallel and serial assembly steps can influence the rigidifying rate of the assembly.

Rigidizing step for a compliant part is defined as the first parallel assembly step for the part that results in a rigid subassembly. This step is extremely important to DFC methodology, as this is the step during which all locational and constraint responsibility paths through the compliant part are established.

Springback is well known problem in assembly of compliant parts. Spring back can distort the location of mates on a part and endanger robust delivery of KCs. To minimize the effect of spring back, one can either make sure that no KCs are deliver through regions affected by springback. One can also alleviate the effect of spring back through strategic placement of contacts.

The next chapter deals with the problem of KC conflict in assembly. One of the benefits of compliant parts in an assembly is the ability to over constrain them in order to resolve KC conflict.

CHAPTER 4. KC CONFLICT IN ASSEMBLY

4.1 Definition and consequences of KC conflict

Assemblies achieve their KCs when parts are in correct relative position and orientation, within specified tolerances. Parts achieve their position and orientation by being joined to each other by mating features. Thus, each KC is essentially delivered by a chain of mates. DFC captures such a KC delivery chain, and if used early in the product development cycle, allows the designer to specifically design the KC delivery chain to achieve optimal KC delivery. An optimal, robust design calls for each KC to be delivered independently from others. In such a design, the delivery of any one KC is tailored in such a manner that it will not disturb the delivery of other KCs. If this is not possible, KC conflict arises.

Please note that KCs are associated with specific directions. All discussion in this chapter regarding KC conflict assumes that the conflict exists in a direction shared by all KCs involved in the conflict. If this were not the case, KC conflict would not exist as KC delivery chains in different directions can be adjusted independently.

KC conflict is undesirable, as it leads to a non-robust product and potential high manufacturing costs [Soderberg and Johannesson, 1999]. An assembly exhibiting KC conflict is, in essence, coupling design parameters and functional requirements that is heavily discouraged in axiomatic design [Suh, 1990]. The disadvantages of this are clear – in order to insure delivery of one KC, others might be sacrificed. All KCs cannot be delivered independently, and variation in one KC can depend on variations in other KCs.

For example, a car door has two important, coupled KCs – the "appearance" KC between door outer skin and the body side, and the "seal" KC between the door inner skin and the body seal. Because of limitations in door assembly process, these two KCs are coupled. Since each KC cannot be delivered by an individually tailored chain, a choice must be made. One KC will independent and can be delivered by a specifically tailored chain, while the KC will be dependent, and its delivery is in jeopardy.

4.2 Causes of KC conflict

Presence of KC conflict indicates lack of necessary number of free degrees of freedom in the assembly needed to independently achieve all KCs desired. This can occur for two reasons: 1) the design calls for more KCs than the number of degrees of freedom actually present in the assembly (Fig. 4.1); or 2) enough degrees of freedom are present in the assembly, but use of improper assembly sequence resulted in the degrees of freedom getting used up prematurely (Fig. 4.2).



Fig. 4.1 KC conflict due to not enough degrees of freedom present in the assembly. Only one slip joint is present in the assembly by which to adjust two KCs into compliance. Both KC delivery chains share links. Change of the assembly sequence does not resolve KC conflict.



Fig. 4.2. KC conflict due to improper assembly sequence. The parts of Fig. 4.1 have been rearranged so that there is now two slip joints in the assembly. There is now enough degrees of freedom to independently adjust both KCs into compliance. However, if the wrong assembly sequence is used (Process 1), one of the degrees of freedom is used up prematurely, resulting in KC conflict similar to that in Fig. 4.1.

Possibility of potential KC conflict can be inferred from the examination of Datum Flow Chain for the whole assembly. If no links are shared by two or more KC delivery chains, KC conflict will not occur. Such a case is an example of an ideal design, also known as non-coupled design in axiomatic design methodology.

Such non-coupled designs are not always possible. If all links of one KC belong to other KC delivery chains, as is the case in the Fig. 4.1, KC conflict is unavoidable. In axiomatic design, this is the case of a completely coupled design.

If some, but not all, links in the DFC are shared by two or more KC delivery chains, at least one potential assembly sequence has been observed to exist in all examples of this type encountered so far such that resulting assembly will be free of KC conflict. Presence of at least one independent link in the KC delivery chain allows for independent delivery of that KC – assuming that link does not get constrained prematurely. In axiomatic design, such case is referred to as decoupled design, and requires a certain order of operations – assembly sequence – to be successful. This is the case illustrated by Process 2 in Fig. 4.2.

An important point to note is that an assembly sequence that would help avoid KC conflict might not be feasible. For example, if fixture F1 of Process 2 in Fig. 4.2 were prohibitively expensive to build or maintain, Process 2 would not be feasible. This is what's going on in the above-mentioned car door example. An assembly sequence that first attaches the door inner panel to body side to deliver the "seal" KC, and then attaches door outer panel to the door inner panel to complete the door assembly and deliver the "appearance" KC, would resolve KC conflict. Unfortunately, this sequence is not feasible.

4.3 "Single part" KC conflict

KC conflict in assembly is easy to spot by the presence of so-called "single-part KC conflict". "Single part" KC conflict describes a situation when more than one KC involving the same part is achieved in a single assembly step. Here, single assembly step is defined as joining of a single part or subassembly to the main assembly. In single part KC conflict, the "single part" in question is the shared link belonging to multiple KC delivery chains.

A good example of single part KC conflict is the previously mentioned car door example. The two KCs associated with the car door –the "appearance" KC between door outer skin and the body side, and the "seal" KC between the door inner skin and the body seal – are both achieved in a single assembly step. Another example is assembly of automobile fender (seen in Fig. 3.3) to the rest of the vehicle. A fender typically has KCs with the hood in In/Out and Up/Down directions and with the front door in the In/Out and Fore/Aft directions. Both of these KCs are delivered in a single assembly step.

4.3.1 Use of Screw Theory for detection of "single part" KC conflict

[Whitney et al., 1999] proposed the use of screw theory to determine if sufficient degrees of freedom are present in an assembly to independently deliver all KCs. Here we present an alternate method of using screw theory to identify KC conflict by detecting presence of "single-part" KC conflict. KC represents a certain geometric relationship between parts in assembly that must be met and, thus, can be thought of as a "virtual" constraint in the assembly. The idea of KC as a constraint is exploited for detection of KC conflict by screw theory. Each KC has certain directions associated with it. The two parts involved in the KC can be thought of as being "virtually" connected by a feature that induces constraint in directions associated with the KC. A wrench matrix can then be formed for each KC.

For each assembly sequence, one can then examine each assembly step, one-by-one. For each step where more than one KC is delivered, all wrench matrices associated with those KCs are to be intersected. If "overconstraint" is indicated, "single part" KC conflict has

been identified. Let us illustrate this process with the example of fender mentioned previously. Start by forming wrench matrices for the two KCs involved (Fore/Aft direction corresponds to x, In/Out to y, and Up/Down to z):

Fender/hood KC in In/Out and Up/Down directions:

EDU» W1= $[0\ 1\ 0\ 0\ 0\ 0; 0\ 0\ 1\ 0\ 0\ 0]$ W1 = $0\ 1\ 0\ 0\ 0\ 0$ $0\ 1\ 0\ 0\ 0$

Fender/front door KC in Fore/Aft and In/Out directions:

EDU» W2=[1 0 0 0 0 0;0 1 0 0 0 0] W2 = 1 0 0 0 0 0 0 1 0 0 0 0

Now, intersect the two wrench matrices in the manner described in [Adams, 1998]:

```
EDU» T1=recip(W1)
T1 =
   1
      0
          0
              0
                  0
                      0
  0
      1
          0
                      0
              0
                  0
  0
      0
          1
              0
                  0
                      0
          0
                  0
  0
      0
              1
                      0
EDU» T2=recip(W2)
T2 =
  1
      0
          0
              0
                  0
                      0
  0
      1
          0
              0
                  0
                      0
  0
      0
              0
          1
                  0
                      0
  0
      0
          0
              0
                  0
                      1
EDU» TU=[T1;T2]
TU =
  1
      0
          0
                  0
                      0
              0
  0
      1
          0
              0
                  0
                      0
  0
      0
          1
              0
                  0
                      0
  0
      0
          0
              1
                  0
                      0
  1
      0
          0
              0
                  0
                      0
  0
      1
          0
              0
                  0
                      0
  0
      0
          1
              0
                  0
                      0
  0
      0
          0
              0
                  0
                      1
EDU» Wrench=recip(TU)
```

Wrench = 0 1 0 0 0 0 EDU»

Resultant intersection of wrenches indicates overconstraint in y (In/Out) direction, which clearly identifies single part KC conflict in In/Out. Similar procedure could be repeated for all steps in the assembly sequence that result in KC delivery.

The result obtained above is obvious from simple visual inspection. However, the value of this method lies in its ability to be used as part of assembly sequence analyzer, helping catch presence of KC conflict and evaluate alternative assembly sequences. For assemblies involving large number of assembly steps, this would be a powerful tool. The ability to clearly determine the step where KC conflict appears is also of value in helping to determine optimal rigidizing step for compliant parts.

4.4 Resolving KC conflict

Several methods have been used in the industry to help resolve KC conflict. One is to accept presence of KC conflict, but tighten tolerances where needed so that all KCs, dependent as well as independent, will be achieved in a satisfactory manner. Such brute force approach is usually very expensive and difficult to implement.

KC conflict can be avoided in some cases by redesigning the product in such a manner that some of the problematic KCs disappear or are reduced in importance. For example, in automobile design, by changing some styling elements or by introducing so-called gimps (non-structural, decorative parts designed to hide potential problem areas) to the design, one can potentially eliminate or downgrade the importance of some of the problematic KCs. A common instance of this is use of thick rubber gimps around the headlights of automobiles – thus removing the lamp/hood interface from the list of important KCs. These techniques are potentially expensive (the above mentioned gimps around the headlights, for example, are far from cheap, and do not perform any other function other than hiding the troublesome lamp/hood interface), can result in a product with less customer appeal than before, and might result in other problems (the use of rubber gimps requires an additional assembly operation).

More practical approach to resolving KC conflict is to over constrain compliant parts in the assembly.

4.4.1 Resolving KC conflict through overconstraint of compliant parts

Researchers have found value in using part compliance to an advantage. For example, [Villareal and Asada, 1991] take advantage of workpiece compliance to simplify robot task planning process during assembly. [Kraus and McCarragher, 1996] and [Kraus and McCarragher, 1997] exploit component flexibility to achieve cheaper and simpler robotic manipulation of compliant components. [Donald and Pai, 1990] utilize part compliance for design of "snap-fastener"-type devices. [Panton et al., 1998] use techniques of differential geometry to develop a method of fitting a compliant part to a surface.

If in the case of "single-part" KC conflict, the part in question is compliant and KC conflict is in the part's flexible direction (as is the case in the fender example of section

4.3.1), the designer can take advantage of part compliance and choose to over constrain that part to satisfactorily resolve KC conflict. This approach essentially exploits additional degrees of freedom inherent to compliant parts. The part is "forced" by locators (incoming mates) into whatever shape is needed to satisfactorily achieve multiple KCs in the same step. This practice, of course, is only useful if the problematic KCs are delivered through flexible direction or directions of a compliant part.

This technique must be applied during the rigidizing assembly step for the part. If the part is over constrained prior to rigidizing, it will be able to spring back and all points located in the step will be lost during subsequent assembly steps. If the part is over constrained later than the rigidizing step, the effect will be the same as over constraining a rigid part. Non-robust assembly will result and KC conflict will not be resolved.

Over constraining the compliant part creates a problem of predicting what incoming mate carries locational and constraint responsibility for what outgoing mate. As discussed in Section 2.7, this problem cannot be solved by simple kinematic analysis and poses serious challenge to top-own design with DFC. A concept of Effective Locating Zone (ELZ) will be introduced in the next chapter to resolve this issue.

4.5 Chapter summary

KC conflict occurs when there are not enough degrees of freedom in an assembly to independently deliver all KCs. This is undesirable, as it leads to a non-robust product: delivery of one KC can place delivery of another KC in jeopardy. KC conflict occurs for

two reasons: 1) the design calls for more KCs than the number of degrees of freedom actually present in the assembly; or 2) enough degrees of freedom are present in the assembly, but use of improper assembly sequence resulted in the degrees of freedom getting used up prematurely. An important point to note is that an assembly sequence that would help avoid KC conflict might not be feasible.

KC conflict in assembly can be spotted by the presence of so-called "single-part KC conflict" – a situation when more than one KC involving the same part is achieved in a single assembly step. Screw theory can be used for systematic detection of single-part KC conflict in an assembly.

A number of ways exist for dealing with KC conflict. These include tightening tolerances to ensure delivery of dependent KCs and redesigning the product to eliminate or reduce the importance of problematic KCs. These methods are often expensive and difficult to implement. If a compliant part participates in delivery of problematic KCs in the part's flexible direction, a more practical approach is to resolve KC conflict by over constraining that compliant part. This approach takes advantage of additional degrees of freedom inherent to the part due to its compliance.

Over constraining the compliant part creates a problem of predicting what incoming mate carries locational and constraint responsibility for what outgoing mate. A concept of Effective Locating Zone (ELZ) will be introduced in the next chapter to resolve this issue.

CHAPTER 5. EFFECTIVE LOCATING ZONE

5.1 Defining ELZ

Various schemes of graphical representations of effects of part compliance have been presented by such researchers as [Henrich et al., 1999], [Kraus and McCarragher, 1996], and [Villareal and Asada, 1991], to name just a few. These attempts focused on better understanding of behavior of compliant parts for the purposes of developing superior assembly processes. Here, we present a concept of Effective Locating Zone, or ELZ, with a goal of better understanding behavior of compliant parts in an assembly and the role part compliance plays in assembly's ability to deliver Key Characteristics.

ELZ is a property of a part and is defined separately for each incoming mate in each of the part's flexible directions. It represents a region of a compliant part, on part's flexible plane, where that mate carries location and constraint responsibilities. ELZ is direction specific; a single mate will have distinct ELZs in different directions. Notation $ELZ_{a,b}$ refers to ELZ of locator *a* in flexible direction *b*. Note that concept of ELZ has any practical meaning only in flexible direction of the part. In absence of part compliance, i.e. in a rigid direction, ELZ of any incoming mate is the whole part.

Concept of ELZ allows for clear determination of locational and constraint responsibility paths through flexible directions of a compliant part. Incoming mate *a* is responsible in direction b for all outgoing mates originating in ELZ_{*a,b*}. This makes possible for formation of part-level DFCs on compliant parts and complete assembly DFCs through compliant parts.

5.2 Locational Responsibility Ratio

Locational Responsibility Ratio, or LRR, at a certain point is defined as the amount of locational responsibility due to a specific locator in a certain flexible direction, relative to other locators acting on the same part the same direction. Notation $LRR_{a,b,c}$ refers to relative locational responsibility at point *c* in direction *b* due to locator *a*.

LRR_{*a,b,c*} is defined as a ratio of y_c/y_{max} , where y_{max} is an arbitrary induced displacement in direction *b* in position of incoming locator *a* relative to all other locators acting on the part in *b*, and y_c is the resulting displacement in *b* at point *c*. At c=a, LRR_{*a,b,c*} =1, as y_c = y_{max} . As LRR is calculated at points away from *a*, LRR<1 and decreases as *c* moves away from *a* towards other locators acting in *b*. Other locators are essentially "disturbing" the locational work done by *a* at *c*. LRR>1 and increases as *c* moves from *a* away from other locators acting in *b*. At any other locator *d*, which also acts in *b*, LRR_{*a,b,d*}=0. Note, that concept of LRR does not make sense in rigid directions of the part, as LRR=1 everywhere in absence of part compliance.

 y_{max} can be chosen arbitrarily, as the value of y_{max} does not influence the resulting LRR. $y_{max}=1$ is a choice that greatly simplifies calculations of LRR and ELZ. Complimentary to LRR is the concept of "LRR loss". LRR_{*a,b,c*} loss is defined as absolute difference between LRR_{*a,b,a*} and LRR_{*a,b,c*} and represents the disturbance of locational work of mate *a* at point *c* in direction *b* by other locators acting on the part: LRR_{*a,b,c*} loss = $|\text{LRR}_{a,b,a} - \text{LRR}_{a,b,c}|$. Since LRR_{*a,b,a*}=1, LRR_{*a,b,c*} loss = $|1 - \text{LRR}_{a,b,c}|$, and LRR_{*a,b,c*} = $1 \pm \text{LRR}_{a,b,c}$ loss.

5.2.1 Relationship between ELZ and LRR

An ELZ is bounded by contours of equal LRR loss and can be cropped by part edges (Fig. 5.1). All points within ELZ have LRR loss equal to or less than its boundary. If locator *a* is surrounded by other locators on the flexible plane perpendicular to direction *b*, all points on the boundary of ELZ_{*a,b*} have constant LRR=1-LRR loss (Fig. 5.1(a)). If, however, in the flexible plane perpendicular to *b* locator *a* is facing other incoming mates only at some directions and edges of the part at other directions (Fig. 5.1 (b)), edges of ELZ_{*a,b*} "facing" other locators have constant LRR=1-LRR loss, while edges of ELZ "away" from other locators have constant LRR=1+LRR loss.

With two identical locators, and an infinite, homogeneous part, points of constant LRR form straight lines perpendicular to a line passing through each of the two locators. Points equidistant from both of the locators have LRR=0.5 for each of the locators. With number of locators increasing beyond two, contours of constant LRR, and correspondingly ELZs, will depend on the relative positions and types of the locators, as well as part shape and material properties.



Fig. 5.1. Relationship between ELZ and LRR. An ELZ is bounded by lines of equal LRR loss
a. Incoming mate *a* is "surrounded" by other incoming mates; all points on boundary of ELZ_{a,b} have constant LRR=1-LRR loss.
b. Incoming mate *a* is facing other incoming mates in some directions but not in others; ELZ_{a,b} is bounded by points of equal LRR=1-LRR loss on the boundary "facing" other incoming mates and is bounded by points of equal LRR=1+LRR loss on boundaries "away" from other incoming mates. ELZ can also be cropped by part edges.

5.3 Establishing ELZs

All ELZs on the part are established during the "rigidizing" step in part's assembly.

Following the discussion of Chapter 3, ELZs established in this step remain unchanged

through the remainder of the assembly process. Note, that incoming mates cannot be

made after the "rigidizing" step, due to assumptions about properly constrained

subassemblies placed on acceptable assembly sequences in Chapter 2.

ELZs are only meaningful for incoming mates. Thus, any process of establishing ELZs must be done after an assembly sequence is chosen, so that incoming mates can be distinguished from outgoing mates.

Two critical pieces of information about each incoming mate must be known before ELZs for the part can be determined. These are (1) maximum acceptable values of LRR loss composing ELZ border, and (2) boundary conditions appropriate at each locator.

5.3.1 Maximum acceptable LRR loss

Each incoming mate's ELZ depends on the value of LRR loss defining its boundary. For every incoming mate, the designer must establish a maximum acceptable LRR loss (or alternately an minimum acceptable LRR) in each of the flexible directions. This value will determine the boundary of the ELZ. The value of maximum acceptable LRR loss must be chosen so as to ensure delivery of KCs – smaller LRR loss leads to more certainty about location of points within the associated ELZ and more robust delivery of KCs whose delivery chains pass through that ELZ. Once maximum acceptable LRR loss is established by designer, ELZ can be determined. Note, that every incoming mate into the part can have distinct maximum acceptable LRR loss assigned to it. Also, each incoming mate can have distinct maximum acceptable LRR loss in each of the flexible directions it is acting.

5.3.2 Boundary conditions at locators

Boundary conditions applied at each incoming mate depend on what kind of degrees of freedom the mating feature can constrain in the flexible plane. Mates can be split into two types – those that can constrain both translation and rotation, and those that can constrain translation only. Examples of the first type include large locating surfaces, while examples of the latter include small locating surfaces and flanges perpendicular to the part's flexible plane. We shall call those mates that can constrain rotation "encastered", and those that cannot "simple" (please see Fig. 5.2).



Fig. 5.2. Boundary conditions at locators.a. Locator can constrain translation in z only. This is a "simple" mate.b. Locator can constrain both translation in z and rotation around x and y. This is an "encastered" mate.

5.4 Methods for ELZ determination

After assembly sequence, boundary conditions, and maximum acceptable LRR loss for each incoming mate and flexible direction are decided upon by the designer, ELZs for the part can be determined. There are two procedures available for ELZ determination: membrane-based approach for precise ELZ determination, useful if detailed knowledge about the part is present, and approximation of ELZ via beam-based modeling of the part for cases where detailed part information is not available.

5.4.1 Membrane-based approach for precise ELZ determination

With detailed knowledge about part shape and material properties, one can use force/stiffness solver, such as a Finite Element Analysis package, to determine precise ELZ for each locator. The following procedure is to be followed in every flexible plane of the part (each flexible plane is to be analyzed separately):

- Model the part as a membrane. At the location of incoming mates, input boundary conditions based on whether or not the locator is able to constrain rotation in addition to translation.
- 2. Choose one of the incoming mates. Displace the part at the point of the mate in the flexible direction by y_{max} . As previously mentioned, y_{max} is arbitrary; value of 1 is recommended to simplify calculation.
- 3. Examine resulting contours of displacement. These are contours of LRR. The ELZ is the region bounded by points of equal maximum acceptable LRR loss as designated by the designer. If the mate being examined is surrounded in all directions by other incoming mates, its ELZ is then is the region bounded by lines of equal minimum acceptable LRR. If the mate is not surrounded in all directions by other incoming mates (i.e., part of the ELZ will be facing the edge of the part, see Fig. 5.1), its ELZ is then is the region bounded by lines of equal minimum acceptable LRR, and (2-LRR), as well as possibly part edges.
- 4. Repeat steps 2 and 3 for all incoming mates.

The above procedure should be repeated for all of the flexible planes of the part.

Example of an automotive fender is used to illustrate the above process (Fig. 5.3). ABAQUS software package is used for finite element analysis. Value of 0.3 is chosen as maximum acceptable LRR loss for all incoming mates. Fig. 5.3(a) shows a portion of complete assembly DFC (full DFC is seen in Fig. 2.1) featuring all incoming mates, and the degrees of freedom they constrain, into the fender. Fig. 5.3(b) shows part-level DFC of the fender, providing additional information and illustrating locations of all incoming mates. As discussed previously, fender has a single flexible y-direction. Figs. 5.3(d-g) show resulting $ELZ_{a,y}$'s for all locators. These resulting ELZs are then used to establish proper links between incoming and outgoing mates in the flexible direction of the part (please see Section 5.5).

5.4.2 Approximation of ELZ via beam-based modeling

At early stages of product development, detailed information about part shape and material properties is not always known. Thus, the method introduced in the previous section cannot always be used. In such cases, ELZs can be approximated via beam-based modeling of the part. This method allows for estimation of ELZs in absence of part material or detailed shape information, and with only incoming mate information known.



Fig. 5.3. ELZ determination for automotive fender.

Membrane-based approach is used to determine ELZs for an automotive fender. The fender has a lone flexible y-direction a. Portion of assembly DFC showing incoming mates into the fender.

b. Fender part-level DFC showing location of incoming mates.

The shotgun tool consists of two pins, fitting into a hole and a slot on fender. This is shown as one feature on assembly DFC, but as two features on detailed part-level DFC



Fig. 5.3. ELZ determination for automotive fender.
ABAQUS finite element analysis package is used for calculation. Value of 0.3 is chosen for maximum acceptable LRR loss for all incoming mates.
c. Mesh model of the fender.
d. Resulting ELZ in y-direction for bolster incoming mate
e. Resulting ELZ in y-direction for bodyside incoming mate



Fig. 5.3. ELZ determination for automotive fender.
ABAQUS finite element analysis package is used for calculation. Value of 0.3 is chosen for maximum acceptable LRR loss for all incoming mates.
f. Resulting ELZ in y-direction for pin-hole feature of shotgun tool incoming mate g. Resulting ELZ in y-direction for pin-slot feature of shotgun tool incoming mate

In such approach, part is represented by a collection of beams joining pairs of points of contacts of incoming mates. Beams pass through the points of mates and extend to the part edges. Separate analysis is carried out in each of the flexible planes.

One by one, each of the incoming mates is displaced by an arbitrary y_{max} in the flexible direction (y_{max} =1 simplifies calculations). Using simple beam theory and specific

constraints imposed by the boundary conditions on each beam, it is possible to derive equations governing the deflection in these beams. The deflection along each beam is then evaluated, and the ratio of y_c/y_{max} can be calculated along each beam. Knowing the LRR loss profile in each beam resulting from an induced displacement y_{max} to the mate's location, ELZ for that mate can then be estimated. This procedure is then repeated for all of the incoming mates. Example at the end of the section will provide clear step-by-step description of the process.

Equations of LRR loss profile along the beam are established in the following manner. In order to attain a certain displacement at one of the two beam supports, force F is applied at the support. The beam is deflected into a shape needed to achieve state of lowest internal stress [Frisch-Fay, 1962], [Rumman, 1991]. Using Bernoulli-Euler beam theory, deflection of the beam is y(x)=f(F,l,x,E,I), where x is position along the beam, *l* is the distance between the two supports, E is the Young's modulus, and I is the rotational moment of the beam cross-sectional area. y_{max} occurs at one the supports, at x=l; $y_{max}=y(l)=f(F,l,l,E,I)$. Assuming homogeneous material properties and constant crosssection throughout the length of the beam, i.e., E and I remain constant, LRR= $y(x)/y_{max}=f(l,x)$. Based on the specific boundary conditions existing at the supports, the following equations governing distribution of LRR loss along the beam are derived (Appendix A contains detailed derivations for each set of boundary conditions):

LRR
$$loss = \frac{x}{l}$$
, boundary conditions: simple at x=0 and at x=l (5.1(a))

LRR loss = $\frac{x(3l^2 - x^2)}{2l^3}$, boundary conditions: simple at x=0, encastered at x=l (5.1(b))

LRR loss =
$$\frac{x^2(3l-x)}{2l^3}$$
, boundary conditions: encastered at x=0, simple at x=l (5.1(c))

LRR loss =
$$\frac{x^2(3l-2x)}{l^3}$$
, boundary conditions: encastered at x=0 and at x=l (5.1(d))

LRR loss profiles are plotted in Fig. 5.4. By inserting the pre-determined value of maximum acceptable LRR loss into the equations above and solving for x (or by using results of Fig. 5.4), estimated boundary of ELZ in the direction of the beam can be determined. The procedure will be made more clear with the example at the end of the section.



LRR loss profiles

Fig. 5.4 Profiles of LRR loss along beams for estimation of ELZ. Please note how encastered locators (which can constrain rotation) give raise to a larger ELZ with the equivalent LRR loss when compared to simple locators

The assumption of constant E and I along each of the beams means that we assume that the part being examined has homogeneous material properties and is of uniform area moment in its flexible plane. This assumption limits the accuracy of the ELZ estimate achieved with the described approach. For instance, if part contains reinforced regions, such as tailor-welded blank stampings, the beam-based model does not give an accurate estimate. Similar problem arises if the part has cutouts – i.e., regions of zero thickness. However, it can still be used with satisfactory results at early stages of product design, when detailed information about the part is not available. As more information about the part becomes available, membrane-based method of Section 5.4.1 can be used to check, and correct if needed, the results of the beam-based model.

In the following example (Fig. 5.5), the beam-based approach is used to estimate ELZs for the same fender that examined in the previous section. Value of 0.3 is chosen as maximum acceptable LRR loss for all incoming mates. Fig. 5.5(a) shows part-level DFC of the fender, featuring all incoming mates, and the degrees of freedom they constrain. Locations of all incoming mates are isolated, boundary conditions according to locator type are established (Fig. 5.5(b)), and mates are joined by lines representing deformable beams (Fig. 5.5(c)). These beams are then portioned according to the boundary corresponding to maximum acceptable LRR loss (Fig. 5.5(d)). Plots in Fig. 5.4 are used for this calculation. Next, lines are constructed passing through the points marked off the previous step (Fig. 5.5(e)). Estimated ELZ is then the region bounded by the lines constructed in the previous step and part edges (if known) (Fig. 5.5(f)). Fig. 5.5(g) shows estimated ELZs for all incoming mates of the fender.


Fig. 5.5. ELZ estimation for automotive fender via beam-based modeling.
Beam-based approach is used to estimate ELZs for an automotive fender. The fender has a lone flexible y-direction

a. Fender part-level DFC showing incoming mates.
b. Locations of incoming mates acting in y-direction on the fender
c. All incoming mates connected by lines representing deformable



Fig. 5.5. ELZ estimation for automotive fender via beam-based modeling.
One incoming mate will be used to demonstrate the procedure for ELZ estimation. This procedure is to be repeated for all incoming mates.
d. On all beams joining the mate to other incoming mates, mark off appropriate points representing the boundary of maximum acceptable LRR loss. For this example, value of 0.3 is chosen for maximum acceptable LRR loss for all incoming mates. Plots in Fig. 5.4 are used for calculation.
e. Construct lines passing through pairs of points marked in the previous step. f. Estimated ELZ is the region bounded by the lines constructed in previous step, and part edges (if known).



Fig. 5.5. ELZ estimation for automotive fender via beam-based modeling.g. Repeating the above procedure for all incoming mates, all estimated ELZs are formed.Value of 0.3 is chosen as maximum acceptable LRR loss for all incoming mates.

Comparing the results obtained via beam-based modeling to the ELZs determined by precise membrane-based approach (Figs. 5.3(d-g)), we see that our model gives fairly good approximations for all ELZs. A number of reasons for inaccuracies in the model exist, the main of which are:

- We are using flat beams to model a curved part. Curved beams could be used, but this requires knowledge of part curvature, which is likely absent at early phases of design.
- Beam-based approximation ignores effects of a large wheel well cut out in the fender.

5.5 Linking incoming and outgoing mates through compliant part.

Goal of ELZs is to provide for establishment and identification of proper chains of locational and constraint responsibility transfer through compliant parts by linking incoming and outgoing mates. This allows for creation of DFCs though a compliant part in its flexible direction. For the purposes of this section, outgoing mates also include points of KC delivery.

By the definition of the ELZ, an incoming mate essentially carries all locational and constraint responsibility for everything originating within its ELZ. Any outgoing mate originating within some incoming mate's ELZ gets its location from that incoming mate (please see relationship between incoming mate a and outgoing mate c in Fig. 5.6). If an outgoing mate originates in a region of the part not included in any ELZ (as in the case of outgoing mate d in Fig. 5.6), constraint and locational responsibility links to that mate of KC cannot be established. So, each outgoing mate on the compliant part must originate within an established ELZ. Outgoing mate cannot originate in a region of intersecting ELZs as that would result in overconstraint and uncertainty about which incoming mate carries locational and constraint responsibility for the outgoing mate. This situation is illustrated in Fig. 5.6 by outgoing mate e.



Fig. 5.6. Using ELZ to link incoming and outgoing mates

z is a flexible direction of a compliant part. Incoming mates *a* and *b* have set up $ELZ_{a,z}$ and $ELZ_{b,z}$, respectively. Outgoing mate *c* originates within $ELZ_{a,z}$; *a* carries all locational responsibility for *c* in direction *z*. DFC link from *a* to *c* can be drawn. Outgoing mate *d* originates outside of established ELZs. Thus, it is unknown what incoming mate carries locational responsibility for *d*; no DFC link to *d* can be drawn. Outgoing mate *e* originates within an intersection of $ELZ_{a,z}$ and $ELZ_{b,z}$. It is not certain which incoming mate carries locational responsibility for *e*; DFC link to *e* cannot be drawn. In order to avoid KC conflict, each incoming mate's ELZ should spawn but a single outgoing mate. If more than one outgoing mates originate in a single ELZ, KC conflict results. In other words, KC conflict is avoided if each outgoing mate originates within a unique ELZ, as shown in Fig. 5.7.



Fig. 5.7. Using ELZ to avoid KC conflict
z is a flexible direction of a compliant part. Incoming mates a, has set up ELZ_{a,z}.
a. Outgoing mates c and d both originate within ELZ_{a,z}; a carries locational responsibility for both c and d in direction z. KC conflict arises.
b. Incoming mate b is added, resulting in an appearance of an additional ELZ. Now outgoing mates c and d each originate within a unique ELZ; KC conflict is avoided.

Size or location of an ELZ might need adjustment in order to properly deliver proper chains of locational and constraint responsibility transfer from incoming mates to outgoing mates. Perhaps an ELZ might need to be enlarged or moved to include origin point of a certain outgoing mate whose location on the part is predetermined due to functional requirements. On the other hand, ELZ might need to be reduced in size or shifted to avoid KC conflict or overlap with another ELZ and resulting overconstraint.

5.5.1 Influences on size of an ELZ

The following factors have the biggest effect on position and size of an ELZ and can adjusted by designer to deliver a successful assembly: type of locator (based on boundary conditions imposed), locations of locator, and minimum acceptable LRR associated with the mate.

Types of locators that constrain rotation in addition to translation lead to greater area of ELZ than those that constrain translation alone. This is evident from examination of equations governing distribution of LRR along the beams. For example, for the same LRR loss, Eq. 5.1(c) gives a larger solution for x than Eq. 5.1(b). This effect can also be observed by comparing LRR loss profiles in Fig. 5.4.

Smaller minimum acceptable LRR for the mate, and correspondingly larger acceptable LRR loss, directly leads to larger ELZ. Minimum acceptable LRR should never be allowed to become so low as to sabotage the design and jeopardize the delivery of KCs. LRR is directly related to desired tolerance in the KC delivered through the part – larger LRR leads to smaller LRR loss and higher certainty in locational transfer – hence tighter tolerance on the delivered KC.

Location of a incoming mate on a part has dual effect on the resulting ELZ. By changing the location of the mate, one directly changes the location of the epicenter of the ELZ. In addition, by changing the spacing between the incoming mates (this is l in Eqs. 5.1(a-d), one also changes the size of the ELZ. As the distance between two incoming mates is

increased, size of ELZs of both of the mates will increase as well, and vice-versa. This seems counterintuitive at first, but makes sense upon closer inspection. With greater distance between locators, effect of one in disturbing the ELZ of another is lessened. This effect is proportional to *l*, with the exact relationship depending on boundary conditions provided by locators.

5.6 Springback and ELZs

If the assembly is carried out using fixtures, and ELZ is set up by an incoming mate that originates on the fixture, removal of that fixture will result in spring back and deformation of that ELZ. As previously mentioned, there are two ways to deal with spring back – ignore it, or limit its effect through use of contacts.

Ignoring springback is a possibility if no outgoing mates originate in the ELZ of the removed incoming mate. However, if outgoing mates (remember, outgoing mates also include points of KC delivery) do originate in this ELZ, springback will disturb the outgoing mate and cannot be ignored. Contacts made within ELZ can be used to "retain" the ELZ originally set up by removed incoming mate.

Please remember that contacts are always made after mates. It is assumed that there is no springback at the point of contact – in other words the attachment (weld, threaded fastener, other type of contact) does not deform. Thus, contacts can be treated as "enforcers" of the locational conditions set up by the mate. From this, it follows that placement of contacts should be established only after the determination of ELZs. If at

least one contact is present with in the ELZ subject to springback, the effect of springback will be lessened. Larger number of contacts will alleviate the effect of spring back further.

In order to insure that the point of origin of outgoing mate is not disturbed by springback, best practice is to surround the outgoing mate with contacts. If two contacts are used, they ideally should be placed in such a way that the outgoing mate lies on to the line joining the two contacts (Fig. 5.8(a)). If more than two contacts can be used, outgoing mate should originate within a region surrounded by the contacts (Fig. 5.8(b)). If only a single contact can be used, it should be placed as close to the outgoing mate as possible.



Fig. 5.8. Use of contacts to limit the effect of spring back.
Contacts can be used to alleviate the effect of springback on outgoing mates originating within an ELZ of a incoming mate that is removed with a fixture.
a. If only two contacts can be placed within the affected ELZ, they should be located in such a manner that the outgoing mate lies on a line joining the two contacts.
b. If more than two contacts can be used within the ELZ, they should be located in such a manner that the outgoing mate lies in a region surrounded by the contacts.

5.7 Chapter summary

Effective Locating Zone (ELZ) is a property of a compliant part and is defined separately for each incoming mate in each of the part's flexible directions. It represents a region of a compliant part, on part's flexible plane, where that mate carries locational and constraint responsibilities. ELZ is direction specific; a single mate will have distinct ELZs in different directions.

ELZ allows for clear determination of locational and constraint responsibility paths through flexible directions of a compliant part. Incoming mate is responsible for all outgoing mates originating in its ELZ.

Locational Responsibility Ratio (LRR) at a certain point is defined as the amount of locational responsibility due to a particular incoming mate in a certain flexible direction, relative to other locators acting on the same part the same direction. Complimentary to LRR is LRR loss; LRR loss represents the disturbance of locational work of a particular incoming mate by other incoming mates acting on the part. An ELZ is bounded by contours of equal LRR loss. ELZ can also be cropped by part edges.

An ELZ is established during the rigidizing step in part's assembly. ELZs are only meaningful for incoming mates. Thus, any process of establishing ELZs must be done after an assembly sequence is chosen, so that incoming mates can be distinguished from outgoing mates.

Two factors affect the ELZ: maximum acceptable LRR loss chosen by the designer to denote the particular ELZ boundary and type and location of the incoming mate. The value of maximum acceptable LRR loss must be chosen so as to ensure satisfactory delivery of KCs – smaller LRR loss leads to more certainty about location of points within the associated ELZ and more robust delivery of KCs whose delivery chains pass through that ELZ. Based on its constraint properties in the flexible direction, incoming mates can be divided into two types of locators: encastered and simple. Encastered locators can constraint both rotation and translation, while simple locators can constrain translation only. Generally, encastered locators lead to larger ELZs than simple locators, all else being equal.

ELZs can be precisely determined via a membrane-based model of the part and force/stiffness solver such as finite element analysis. This method is applicable when detailed part information is present. If detailed part information is not available, ELZs can be approximated via a beam-based model of the part. This approach makes an assumption of a homogenous part properties and uniform area moment in its flexible plane. The results of the beam-based model approximation, while very useful in early design stages, should be checked by the membrane-based approach later on.

Once ELZs are determined, locational responsibility paths (DFC links) through the part can be established. An incoming mate carries all locational responsibility for an outgoing mate originating with its ELZ. Each outgoing mate should, thus, originate with an

established ELZ. Outgoing mate should not originate within a region of intersecting ELZs. In order avoid KC conflict, each ELZ should spawn only a single outgoing mate.

An ELZ's shape and location can be influenced by adjusting type and location of an incoming mate and changing the value of minimum acceptable LRR. This should be done to ensure creation of acceptable links between incoming and outgoing mates through the flexible direction of the compliant part.

In ELZs created by mates that are affected by spring back, effect of spring back can be alleviated through strategic placement of contacts. This requires "surrounding" outgoing mates originating within the affected ELZ by contacts.

In Chapter 6, methods introduced in this chapter will be united with other topics presented in this thesis to create a systematic approach to design of assemblies with compliant parts.

CHAPTER 6. SYSTEMATIC TOP-DOWN DESIGN OF ASSEMBLIES WITH COMPLIANT PARTS

This chapter will summarizes the results of previous chapters and unify concepts developed in this thesis to present a comprehensive system for top-down design of assemblies with compliant parts. [Whitney et al., 2001] presented a procedure for top-down design methodology using DFC under rigid part assumptions. Here, we will add to that methodology to be compatible with the compliant parts using the concepts of this thesis.

[Whitney et al., 2001] identifies three phases of top-down design: nominal design phase, constraint analysis phase, and variation design phase. Compliant parts present unique challenges and opportunities that are best treated during the nominal design and constraint analysis phases. At the variation design phase, one must be aware of part compliance, but the strategy for variation analysis does not deviate from that proposed in [Whitney et al., 2001] due to the presence of compliant parts.

Fig. 6.1 presents all the steps that may be required in a top-down design process. These steps will be discussed in detail below.

6.1 Nominal design phase

Top-down design of any assembly starts with identification of customer-level KCs. It is important to remember that KCs are not mere customer preferences, but qualities that are



Fig. 6.1. Design Process Chart for Assemblies with Compliant Parts

judged critical to customer satisfaction. Once customer-level KCs are determined, they must be interpreted and flowed down to assembly level KCs. Based on functional requirements of the parts in assembly, part involved in immediate KC delivery can be recognized and important geometric relationships between these parts, which compose assembly-level KCs, are identified. It is important to clearly identify the direction or directions associated with each assembly-level KC. By treating each assembly-level KC as a "virtual constraint", one can use screw theory notation (a wrench matrix) to make a record of each KC and its associated directions, as was shown in Section 4.3.1.

With KCs identified, a designer then draws preliminary DFCs. Each DFC should be tailored to delivering a particular KC in the best possible manner. Designers are perfectly free to try innovative locating schemes, although function dictates approximate location of parts in the assembly and may limit potential locating schemes.

At this point, decision should also be made on whether or not fixtures and various other tools will be used during the assembly for locating purposes. This decision depends on many factors, of which optimal possible locating scheme is only one. For example, in certain applications it might be more cost effective to use a set of precise fixtures rather than manufacture parts with precisely toleranced assembly features. Other factors such as assembly worker ergonomics and existing assembly plant conditions might figure in the decision.

The next step is to define assembly features – mates – between parts that will realize the constraint and transfer location between parts in such a manner as to deliver the designer's intent expressed by the DFC. Upon completion of this step, the DFC should be complete and should show all delivered KCs and all mates, including those that involve fixtures and other equipment, participating in KC delivery. Thought should be given to those parts that might require contacts such as fasteners and reinforcements. These contacts should be added to the DFC and clearly distinguished from mates.

Decisions made in this stage may be subject to change based on the analysis carried out in the next phase. Also, more than one potential DFC can be created, and selected based on the later analysis.

6.2 Constraint Analysis Phase

The first step is to examine the DFC for compliant parts involved in the KC delivery. If part shape and material information needed to precisely determine part compliance are unknown, which is likely at early stages of product design, legacy knowledge and designer's experience should be relied upon. All compliant parts involved in KC delivery in their flexible directions should be identified, and the compliance type noted.

Constraint properties of the design expressed in the DFC are analyzed next. Mates making up the DFC should be examined, to ensure that they deliver a properly constrained assembly. As indicated in Section 2.5.2, screw theory has been successfully used for DFC constraint analysis. Under- and over constraints required for functionally should be excepted, as well as compliant parts over constrained in their flexible directions. If the constraint analysis shows improper constraint resulting from improper mate selection, this mistake should be corrected by going back to the mate definition step.

Properly constrained DFC is then used to generate possible assembly sequence or sequences. Assembly sequences formed from DFC have properly constrained subassemblies, meaning that assembly cannot proceed to the next step if it is under constrained (unless required for functionality). This also requires that all mates be made prior to contacts.

With assembly sequence chosen, KC conflict analysis should be carried out. "Single part KC conflict" detection method using screw theory is described in Section 4.3.1. If KC conflict is detected, alternative assembly sequences should be explored. If all feasible assembly sequences fail to resolve the KC conflict, the designer has following choices, in the order of desirable to undesirable:

- 1. If a compliant part is involved in delivery of conflicting KCs, it might be possible to over constrain the compliant part to resolve the KC conflict.
- 2. Major changes to the mating features, including possible inclusion of fixtures into the assembly, might be considered in order to produce an assembly sequence free of KC conflict. This would involve going back to the nominal design phase.
- Conflicting KCs can be prioritized, and independence assigned to KCs based on their priority. This would insure delivery of independent, higher priority KCs, but would compromise the delivery of dependent, lower priority KCs.

The assembly sequence must then be examined in order to identify the rigidizing step for all the compliant parts. At this point, constraint analysis of compliant parts should take place. If compliant parts are over constrained during or prior to the rigidizing step, over constraint does not present a problem. However, if any compliant parts are over constrained after the rigidizing step, the effect is the same as over constraint of a rigid part and should be avoided. Either an alternative assembly sequence can be tried, or mates should be redefined back at the mate definition step of nominal design phase.

If the compliant part participates in KC delivery and is over constrained in an attempt to resolve KC conflict, the rigidizing step should be the assembly operation in which the KCs are delivered through the part. Ideally, this should be the last step involving the part and creation of new DFC links. Otherwise, the rigidizing step should occur as close to the first KC delivery step involving the part as possible. If this is not feasible with assembly sequence in consideration, an alternative assembly sequence should be considered.

6.2.1 Determination and analysis of ELZs

Last part of the constraint analysis phase is determination and analysis of ELZs. Remember, ELZs are established during the rigidizing step and are retained for the remainder of the assembly.

For every compliant part participating in KC delivery in each of its flexible directions, maximum acceptable LRR loss for every incoming mate is chosen by the designer. LRR is directly related to desired tolerance in the KC delivered through the part – larger LRR leads to smaller LRR loss and higher certainty in locational transfer – hence tighter tolerance on the delivered KC. Minimum acceptable LRRs should be tighter for fullycompliant directions than for semi-rigid directions, in order to negate potentially damaging effect of unanticipated external factors.

ELZ for each incoming mate can then be formed using techniques developed in Section 5.4. Designer should then check on whether or not each outgoing mate originates in an established ELZ. This includes outgoing mates made at all stages of assembly at or after the rigidizing step. If an outgoing mate originates outside an established ELZ, either the location of outgoing mate or shape of the ELZ should be adjusted until the outgoing mate originates with an ELZ. Similarly, an outgoing mate cannot originate in a region of intersecting ELZs as that would result in over constraint and uncertainty in which incoming mate is responsible for the outgoing mate. The designer has following choices of solving these issues, in the order of desirable to undesirable:

- 1. Point of engagement of an outgoing mate can be moved to reside within an existing ELZ.
- Point of engagement of an incoming mate or mates can be moved. As discussed in Section 5.5, this will have dual effect on the ELZ: it will move the location of the ELZ epicenter, and possibly alter the ELZ shape.
- 3. Type of one of the incoming mates can be changed, from one that supports rotational motion to one that does not, and vice versa. This would involve redefining the mates back at the mate definition step of nominal design phase.

4. Minimum acceptable LRR can be adjusted to change the size of ELZ. This is a compromise, as it can have negative effect on robustness of KC delivery, as mentioned previously.

If the compliant part participates in KC delivery and is over constrained in an attempt to resolve KC conflict, multiple outgoing mates will originate on the part. In order to alleviate KC conflict, each outgoing mate should originate in a unique ELZ. If this condition is not met, either the location of one of the outgoing mates participating in the delivery of the problematic KC or shape of the ELZ should be adjusted until it is met. The designer has four methods of doing so, identical to the above paragraph:

- 1. Relocate the point of engagement of the outgoing mate to within a single ELZ.
- 2. Change shape of the ELZs by adjusting the location of point of engagement of one of the incoming mates.
- 3. Type of one of the incoming mates can be changed, from one that supports rotational motion to one that does not, and vice versa.

4. Minimum acceptable LRR can be adjusted to change the shape of the ELZ. If none of the above techniques leads to a desirable result, conflicting KCs will have to be prioritized, and independence assigned to KCs based on their priority. The drawbacks of KC prioritization have been discussed earlier.

With ELZs successfully established, the last step is to check for possibility of springback. If likelihood of springback exists, contacts should be added to each ELZ affected, in order to ensure the each outgoing mate's location stays intact. This is done in the manner described in Section 5.6.

6.3 Variation Design Phase

The purpose of the variation design phase is to ensure the robustness of the DFC and the assembly sequence chosen as result of nominal design and constraint analysis phases. Based on the desired assembly-level tolerance specifications of each KC, designer can allocate tolerances to the mates in its delivery chain.

Variation analysis should be carried out to ensure that constraint state of assembly does not change under allowed variations. This serves as a check that features intended to be mates remain mates and features indeed to be contacts remain contacts under all possible scenarios. If this does not result in a satisfactory result, the designer would need to redefine certain clearances in the design.

If KC conflict was encountered and not resolved during constraint analysis phase, the KCs were prioritized. Now, the designer will have to either add clearance or relax the tolerance specifications on lower priority KCs.

Finally, the designer needs to perform a conventional tolerance analysis for each KC to ensure that it is being delivered a high enough percentage of time. If compliant parts participate in KC delivery, finite element structural analysis should be combined with traditional statistical tolerance analysis for this purpose, as described in [Liu and Hu, 1997] and [Merkley, 2000].

6.4 Chapter summary

Systematic approach to top down design of assemblies with compliant parts is presented. The complete design process is summarized in Fig. 6.1. The approach divides the design process into three phases: nominal design, constraint analysis, and variation design.

During the nominal design phase, customer-level KCs are identified and flowed down to assembly level KCs. Preliminary DFCs are drawn, and mates are chosen to create an assembly that satisfactorily delivers all KCs. Compliant parts involved in KC delivery are noted.

During the constraint analysis phase, constraint properties of the design are analyzed. Properly constrained DFCs are then used to generate possible assembly sequences. KC conflict analysis is carried out. If available, compliant parts can be over constrained to resolve KC conflict. Assembly sequences with proper rigidizing step are chosen.

For all compliant parts participating in KC delivery in their flexible directions, ELZs are determined for all incoming mates. Designer then should check if all outgoing mates properly originate in unique ELZs. If not, either the problematic outgoing mate or the ELZ should be adjusted.

With ELZs successfully established, check for possibility of springback is carried out. If likelihood of springback exists, properly located contacts should be added to each ELZ affected.

During the variation design phase, robustness of the chosen assembly design is checked. Variation analysis for the design is carried out. Based on the results of variation analysis, tolerances are allocated and clearances are specified. If compliant parts participate in KC delivery, finite element analysis should be combined with traditional statistical tolerance analysis.

The next chapter will present a summary of major topics discussed in the thesis and present subjects for future research.

CHAPTER 7. CONCLUSION

7.1 Summary of major points of the thesis

Chapter 1 presented the benefits of the of top-down design process, in which the assembly design is carried out prior to detailed part design. At the same time it was shown that present methodologies of top-down design are not completely compatible with assemblies containing compliant parts, as challenges and benefits inherent to compliant parts are ignored.

Chapter 2 presented Datum Flow Chain methodology for designing assemblies. Concept of Key Characteristics was introduced. It was shown how customer-level KCs can be flowed down to geometrical assembly-level KCs. DFC documents KC delivery by composing a chain of parts connected by mates and expresses designer's intent concerning geometrical relationships of parts in assembly. Advantages of top-down design with DFC were discussed. Inability of DFC to deal effectively with part compliance was presented as a major drawback of the methodology.

Chapter 3 dealt with part compliance and its effect on assembly design. Role of compliant parts in KC delivery was discussed. Concepts of flexible direction and flexible plane of a compliant part were introduced. Nature of compliance was studied, and two types of flexible directions were presented: semi-rigid and fully-compliant. All part deformations in a semi-rigid direction occur due to mating features intended by designer, while in a fully-compliant direction part deformations due to unintended external sources

are also possible. View of assembly process as removal of degrees of freedom was presented. Two types of assembly of compliant parts – in parallel and in series – were presented, and the effect of assembly type on resulting assembly stiffness was discussed. It was shown how order of parallel and serial assembly steps in assembly sequence influences the rigidifying rate of assembly. Concept of rigidizing step was introduced as the assembly step when a compliant part becomes effectively rigid. It was shown how locational and constraint responsibility chains through a compliant part in a flexible direction are established during the rigidizing step. Difficulty in determining such chains due to part compliance was talked about. Method to choose the optimal rigidizing step was presented. Spring back was discussed, and ways to negate its effects on KC delivery were presented.

Chapter 4 discussed KC conflict in assembly. KC conflict describes a situation when not all KCs in an assembly can be delivered independently. Underlying causes of KC conflict were discussed. Method to detect KC conflict using Screw Theory was presented. Ways to resolve KC conflict were discussed, and over constraint of a compliant part was presented as method to alleviate KC conflict.

Chapter 5 presented tools for determination of locational and constraint responsibility chains through a compliant part in a flexible direction. Concepts of Effective Locating Zone and Locational Responsibility Ratio were presented. Relationship between ELZ and LRR was established. Two methods for determination of ELZs – precise ELZ determination via finite element analysis of membrane model of the part, useful if

detailed part information is available, and approximation of ELZ via beam-based model of the part, useful at early phases of design when detailed part information id lacking – were presented. Ways to create locational and constraint responsibility chains through a compliant part by linking incoming and outgoing mates through an ELZ were introduced. Ways to influence shape and location of an ELZ were discussed. Effects of spring back on an ELZ were talked about, and ways to negate these effects through use of contacts were presented.

Chapter 6 unified all topics discussed in the thesis to present a DFC-based systematic methodology for design of assembles with compliant parts. Coherent scheme of design steps forming a complete design procedure was outlined. Specific steps dealing with part compliance were discussed, using tools and terminology developed in the thesis.

7.2. Scope for future work

Designation of a "rigidizing" step for a compliant part was shown to be of extreme importance in the design of assemblies involving compliant parts. Work in integration of "rigidizing" step analysis with assembly sequence generation and analysis would be of great importance.

Relationship between tolerances on KC and minimum acceptable LRR is a very important one. Designation of minimum acceptable LRR leads to size of ELZ, which, in turn, affects the chains of locational and constraint responsibility between incoming and outgoing mates. Work needs to be carried out in order to determine optimal minimum acceptable LRR for every incoming locator and compliant part based on tolerances on the KC. Statistical data needs to be gathered and examined, and links should be determined between LRR, part tolerances, and successful KC delivery.

Software for automatic generation and instantaneous analysis of ELZs would be very valuable. Such software would need to include provisions for evaluating the effect of springback in assemblies that utilize fixtures. Substantial benefit would be realized from integration of this software with DFC creation and analysis software.

REFERENCES

- Adams, J. D., "Feature-Based Analysis of Selective Limited Motion in Assemblies", S.M. Thesis, Massachusetts Institute of Technology, Mechanical Engineering Department, Cambridge, MA, 1998
- Adams J. D., and Whitney D. E., "Application of Screw Theory to Motion and Constraint Analysis of Mechanical Assemblies", <u>Proceedings of the IEEE Intl.</u> <u>Symposium On Assembly and Task Modeling</u>, Porto, Portugal, July 1999
- Baldwin, D. F., Abell, T. E., Lui, M.-C., De Fazio, T. L., and Whitney, D. E., "An Integrated Computer Aid for Generating and Evaluating Assembly Sequences for Mechanical Products, <u>IEEE J. Automation and Robotics</u>, v 7, no 1, pp 78-94, 1991
- Bihlmaier, B. F., "Tolerance Analysis of Flexible Assemblies Using Finite Element and Spectral Analysis", <u>ADCATS 2000 Conference</u>, Utah, June 2000
- Bjørke O., Computer-aided Tolerancing, 2nd edition, ASME Press, New York, 1989
- Boothroyd, G., and Dewhurst, P., "Product Design for Assembly", Boothroyd Dewhurst, Inc., Rhode Island, 1991
- Cai, W., Hu, S. J., Yuan, J. X., "Deformable Sheet Metal Fixturing: Principles: Algorithms, and Simulations", <u>ASME Journal of Engineering for Industry</u>, Vol. 118, August, pp. 318-324, 1996
- Camelio, J. A., Hu, S. J., and Ceglarek, D., "Modeling Variation Propagation of Multi-Station Assembly Systems with Compliant Parts", <u>Proceedings of 2001 ASME</u> <u>Design Engineering Technical Conference</u>, Pittsburgh, Pennsylvania, September, 2001
- Ceglarek, D. and Shi, J., 1995, "Dimensional Variation Reduction for Automotive Body Assembly", <u>Manufacturing Review</u>, Vol. 8, pp. 139-154
- Ceglarek, D., Li, H. F., and Tang, Y., "Modeling and Optimization of Fixture for Handling Compliant Sheet Metal Parts", <u>1999 ASME International Mechanical</u> <u>Engineering Congress & Exposition</u>, DE-Vol. 103, pp. 89-100, 1999
- Ceglarek, D., Li, H. F., and Tang, Y., "Modeling and Optimization of End Effector Layout for Handling Compliant Sheet Metal Parts", <u>Journal of Manufacturing</u> <u>Science and Engineering</u>, Vol. 123, pp. 473-480, 2001
- Chang, M., and Gossard, D. C., "Modelling the Assembly of Compliant, Non-Ideal Parts", <u>Computer-Aided Design</u>, Vol. 29, no. 10, pp. 701-708, 1997

- Donald, B. R. and Pai, D. K., "On the motion of Compliantly-Connected Rigid Bodies in Contact, Part II: A System for Analyzing Designs for Assembly", <u>Proceedings of the 1990 IEEE International Conference on Robotics and Automation</u>, Cincinnati, Ohio, May 1990
- Frisch-Fay, R., "Flexible Bars", Washington Butterworths, 1962
- Henrich, D., Ogasawara, T., and Wörn, H., "Manipulating Deformable Linear Objects

 Contact States and Point Contacts", <u>1999 IEEE International Symposium on</u> <u>Assembly and Task Planning</u>, Porto, Portugal, July 1999
- Hu, S. J. and Dr. K. Iyer, K., "Variation Modeling and Design for Compliant Assemblies", <u>ADCATS 2000 Conference</u>, Utah, June 2000
- Hu, J., "Stream-of-Variation Theory for Automotive Body Assembly", <u>Annals of</u> <u>CIRP</u>, Vol. 46/1, 1997
- Johannesson H. and Soderberg R., "Structure and Matrix Models for Tolerance Analysis from Configuration to Detail Design", <u>Research in Engineering Design</u>, Vol. 12, pp. 112-125, 2000
- Konkar R., "Incremental Kinematic Analysis and Symbolic Synthesis of Mechanisms", Ph. D. Dissertation, Stanford University, Stanford CA, 1993
- Kosuge K., "Manipulation of a flexible object by dual manipulators", <u>Proceedings of</u> the 1995 IEEE International Conference on Robotics and Automation, pp. 318-323, 1995
- Kraus, W. and McCarragher, B. J., "Force Fields in the Manipulation of Flexible Materials", <u>Proceedings of the 1996 IEEE International Conference on Robotics and</u> <u>Automation</u>, Minneapolis, April 1996
- Kraus, W. and McCarragher, B. J., "Case Studies in the Manipulation of Flexible Parts Using a Hybrid Position/Force Approach", <u>Proceedings of the 1997 IEEE</u> <u>International Conference on Robotics and Automation</u>, Albuquerque, New Mexico, April 1997
- Lee, D. and Thornton A., "The Identification and Use of Key Characteristics in the Product Development Process", <u>Proceedings of the ASME Design Theory and</u> <u>Methodology Conference</u>, Irvine, California, August 1996
- Lee, J., Long, Y., and Hu, S. J., "Robustness Evaluation for Compliant Assembly Systems", <u>ASME Design for Manufacturing Conference</u>, Baltimore, 2000
- Lee, J., and Haynes, L., "Finite Element Analysis of Flexible Fixturing Systems", <u>ASME Journal of Engineering for Industry</u>, Vol. 109, pp. 134-139, 1987

- Li, H., Ceglarek, D., and Shi, J., "A Dexterous Part-Holding Model for Handling Compliant Sheet Metal Parts", Japan-USA Symposium, Ann Arbor, Michigan, July, 2000
- Liu, S. C. and Hu, S. J., "An Offset Finite Element Model and Its Applications in Predicting Sheet Metal Assembly Variation", <u>International Journal of Machine Tools</u> and <u>Manufacture</u>, Vol. 35, No. 11, pp. 1545-1557, 1995
- Liu, S. C., Hu, S. J., and Woo, T. C., "Tolerance Analysis for Sheet Metal Assemblies", <u>Transactions of the ASME</u>, vol. 118, pp. 62-67, 1996
- Liu, S. C. and Hu, S. J., "Variation Simulation for Deformable Sheet Metal Assemblies Using Finite Element Methods", <u>ASME Journal of Engineering for</u> <u>Industry</u>, Vol. 129, pp. 368-374, 1997
- Liu, S. C. and Hu, S. J., "Sheet Metal Joint Configurations and Their Variation Characteristics", <u>Journal of Manufacturing Science and Engineering</u>, Vol. 120, pp. 001-008, 1998
- Lowell, W. F., "Geo-metrics II: Dimensioning and Tolerancing", <u>ANSI/ASME</u> <u>Standard</u>, Y13.5M, pp. 35-52, 1982
- Mantripragada, R., Cunningham, T., and Whitney, D. E., "Assembly Oriented Design: A New Approach to Designing Assemblies", <u>Fifth IFIP WG 5.2 Workshop</u> on Geometric Modeling in Computer-Aided Design, Airlie, Virginia, 1996
- Mantripragada R. and Whitney D.E., "The Datum Flow Chain: A Systematic Approach to Assembly Design and Modeling", <u>Research in Engineering Design</u>, Vol. 10, pp. 150-165, 1998
- Menassa, R., and DeVries, W., "Optimization Methods Applied to Selecting Support Positions in Fixture Design", <u>ASME Journal of Engineering for Industry</u>, Vol. 113, pp. 412-418, 1991
- Merkley, K., "Breakthroughs in Tolerance Analysis of Compliant Assemblies", <u>ADCATS 2000 Conference</u>, Utah, June, 2000
- Moccio, C., "Using Datum Flow Chain and Design Structure Matrix to Manage Team Structure and Integration", S.M. Thesis, Massachusetts Institute of Technology, System Design and Management Program, Cambridge, MA, 2002
- Newell, C. G. and Khodabandehloo, K., "Modeling flexible sheets for automatic handling and lay-up of composite components", <u>Journal of Engineering Manufacture</u>, pp. 423-432, 1995

- Noort, A, Hoek, G. F. M., and Bronsvoort, W. F., "Integrating Feature-Based Part and Assembly Modeling", to be published in *Computer-Aided Design*, 2002
- Panton, S., Lissington, T. R., and Duncan, J. L., "A Differential Geometry Model of Assembly of Compliant Developable Sheet Components", <u>ASME Symposium on</u> <u>Assembly Methods and Assembly Modeling</u>, Anaheim, California, November, 1998
- Panton, S. and Van-Brunt, B., "A Differential Geometry Model of the Assembly of Compliant Sheet Metal Components, unpublished
- Rearick, M. R., Hu, S. J., Wu, S. M., "Optimal Fixture Design for Deformable Sheet Metal Workpieces", <u>Transactions of NAMRI/SME</u>, Vol. XXI, pp. 407-412, 1993
- Rumman, W. S., "Statically Indeterminate Structures", John Wiley and Sons, New York, 1991
- Schmidt, T.W., and Henrich, D., "Manipulating Deformable Linear Objects: Robot Motions in Single and Multiple Contact Points", <u>Proceedings of the 4th IEEE</u> <u>International Forum on Assembly and Task Planning</u>, Fukuoka, Japan, May 2001
- Shalon, D., Gossard, D., Ulrich, K., and Fitzpatrick, D., "Representing Geometric Variations in Complex Structural Assemblies on CAD systems", <u>ASME Advances in</u> <u>Design Automation</u>, Vol. 44, No.2, pp. 121-132, 1992
- Shigley, J.,E. and Mischke, C. R., "Mechanical Engineering Design", 5th Edition, McGraw-Hill Inc., New York, 1989
- Shiu, B., Ceglarek, D., and Shi, J., "Flexible Beam-Based Modeling of Sheet Metal assembly for Dimension Control", <u>Transactions of NAMRI</u>, vol. XXV, pp. 49-54, 1997
- Soderberg, R. and Johannesson, H., "Tolerance Chain Detection by Geometrical Constraint Based Coupling Analysis", <u>Journal of Engineering Design</u>, Vol. 10, November, pp. 5-24, 1999
- Soman, S., "Modeling Surface Variations For Flexible Assemblies", <u>ADCATS 2000</u> <u>Conference</u>, Utah, June 2000
- Suh N. P., "The Principles of Design", Oxford, New York, 1990
- Takezawa, N., "An Improved Method for establishing the Process-Wise Quality Standard", <u>Reports of Statistical Application Research</u>, Union of Japanese Scientists and Engineers, Vol. 27, pp. 63-76, 1980
- Thornton, A. C., "A Mathematical Framework for the Key Characteristic Process", <u>Research in Engineering Design</u>, Vol. 11, pp. 145-157, 1999

- Tonks, M., "Tolerance Analysis of Flexible Assemblies", <u>ADCATS 2001</u> <u>Conference</u>, Utah, June 2001
- Ulrich K. T. and Eppinger S. D., "Product Design and Development", McGraw-Hill Inc., New York, 1995
- Villarreal, A. and Asada, H., "A Geometric Representation of Distributed Compliance for the Assembly of Flexible Parts", <u>Proceedings of the 1991 IEEE</u> <u>International Conference on Robotics and Automation</u>, Sacramento, California, April 1991
- Waldron, K. J, "The Constraint Analysis of Mechanisms", <u>Journal of Mechanisms</u>, Vol. 1, pp. 101-114, 1966
- Whitney D. E., Gilbert O., and Jastrzebski M., "Representation of Geometric Variations Using Matrix Transforms for Statistical Tolerance Analysis in Assemblies", <u>Research in Engineering Design</u>, Vol. 6, pp. 191-210, 1994
- Whitney D. E., Mantripragada R., Adams J. D., and Cunningham T., "Use of Screw Theory to Detect Multiple Conflicting Key Characteristics", <u>Proceedings of the</u> <u>ASME Design Engineering Technical Conferences</u>, Las Vegas, September 1999
- Whitney D. E., Mantripragada R., Adams J. D., and Rhee S. J., "Designing Assemblies", <u>Research in Engineering Design</u>, Vol. 11, pp. 229-253, 1999
- Whitney D. E., Shukla G. and Stefan Von-Praun, "A Design Procedure Applicable to Different Classes of Assemblies", ASME 2001 Design Engineering Technical Conference, Pittsburgh, PA, 2001
- Youcef-Toumi, K., Liu, W., and Asada, H., "Computer-Aided Analysis of Reconfigurable Fixtures and Sheet Metal Parts for Robotic Drilling", <u>Robotics &</u> <u>Computer-Integrated Manufacturing</u>, Vol. 4, pp. 387-393, 1988
- Yuen, K.-M. and Bone, G. M., "Robotic Assembly of Flexible Sheet Metal Parts", <u>Proceedings of the 1996 IEEE International Conference on Robotics and Automation</u>, Minneapolis, April 1996

APPENDIX A.

Derivation of equations governing LRR loss profiles for beam-based model for ELZ estimation:

l is the length of the beam between the supports, F is the resultant force acting on the beam due to the induced displacement, E is Young's modulus of the beam, and I is beam's rotational moment of inertia.

Boundary conditions: simple at x=0, simple at x=l



$$y(x) = y_{\max} \frac{(l-x)}{l}$$
$$LRR = y(x)/y_{\max} = \frac{(l-x)}{l} = 1 - \frac{x}{l}$$
$$LRR \ loss = 1 - LRR = \frac{x}{l}$$

Boundary conditions: simple at x=0, encastered at x=l



From [Frisch-Fay, 1962]:

$$y(x) = \frac{F}{6EI} \left(-x^3 + 3xl^2 - 2l^3 \right)$$

$$y_{\text{max}} = \frac{F}{6EI} \left(-2l^3 \right) = -\frac{F}{3EI} l^3$$

$$LRR = y(x) / y_{\text{max}} = \frac{\left(x^3 - 3xl^2 + 2l^3 \right)}{2l^3}$$

$$LRR \ loss = 1 - LRR = 1 - \frac{\left(x^3 - 3xl^2 + 2l^3\right)}{2l^3}$$
$$LRR \ loss = \frac{x(3l^2 - x^2)}{2l^3}$$

Boundary conditions: encastered at x=0, simple at x=l

From [Frisch-Fay, 1962]:

$$y(x) = \frac{F}{6EI} \left(-x^{3} - 2lx^{2} - 2l^{3} \right)$$

$$y_{\text{max}} = -\frac{F}{3EI} l^{3}$$

$$LRR = y(x) / y_{\text{max}} = \frac{\left(x^{3} + 2lx^{2} + 2l^{3} \right)}{2l^{3}}$$

$$LRR \ loss = 1 - LRR = 1 - \frac{\left(x^{3} + 2lx^{2} + 2l^{3} \right)}{2l^{3}}$$

$$LRR \ loss = \frac{x^{2}(3l - x)}{2l^{3}}$$

Boundary conditions: encastered at x=0, encastered at x=l



From [Frisch-Fay, 1962]:

$$y(x) = \frac{F}{24EI} \left(-2x^3 + 3x^2l - l^3 \right)$$

$$y_{\text{max}} = -\frac{F}{24EI} l^3$$

$$LRR = y(x) / y_{\text{max}} = \frac{\left(2x^3 - 3x^2l + l^3\right)}{l^3}$$

$$LRR \ loss = 1 - LRR = 1 - \frac{\left(2x^3 - 3x^2l + l^3\right)}{l^3}$$

$$LRR \ loss = \frac{x^2(3l - 2x)}{l^3}$$

4430-35

.