# Vessel Valuation – An Options Approach

by

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## Abstract

In the field of capital budgeting traditionally the widely accepted net-present-value (NPV) technique is used to capture a project's value. However, this approach fails to quantify managerial and operational flexibility and strategic interactions. The underlying analysis deals with the subject of resource allocation or capital budgeting under uncertainty, particularly with the valuation of managerial and operating flexibility as real options. Similar to options on financial assets, real options involve decisions or rights, with no obligation, to acquire or exchange an asset or project for a pre-specified price.

Within the shipping industry the application of real options on operating vessels as strategic decision tools has so far been more or less neglected, since only few players are familiar with the option theory. A charterer operating a vessel may have an agreement with the owner to acquire the ship at some future date, giving him the option, without obligation, to do so. This flexibility to undertake a vessel acquisition provides the charterer with a certain value, depending on the movements of the market.

This paper initially introduces the general option pricing theory applied to financial securities. Furthermore, an alternative way of modeling the stochastic nature of time charter equivalent spot rates for the bulk freight market is presented. It is proposed to abandon the Geometric Brownian motion and, instead, to apply a mean reverting process, such as the Ornstein-Uhlenbeck process, to replicate the freight rates. Based on these findings, closed form option valuation tools are applied to a Panamax vessel type for one specific route, capturing the mean reverting character of the ship's cash flows. The results of the option valuation are discussed considering their practicability. Finally, recommendations for future research are given.

Thesis Supervisor: Professor Paul D. Sclavounos
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Many thanks to the rest of my family, especially my aunt and my uncle, Dorothee and Michael, who where the first within the family to introduce me to the field of Naval Architecture.

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## Biography of the Author

Caspar Andri Largiadèr was born in Winterthur, Switzerland on April 12, 1965. After completing his high school education at the Kantonsschule im Lee, Winterthur, in 1986, Mr. Largiadèr entered the Swiss Air Force to complete his mandatory basic training. His ongoing military education was again at the Swiss Air Force where he attended the corporal education and furthermore a four months training in the field.

In fall 1987 Mr. Largiadèr commenced his studies in the field of Mechanical Engineering at the Swiss Federal Institute of Technology (ETH) in Zurich, Switzerland. Mr. Largiadèr majored in system dynamics and combustion engineering and graduated with a diploma in Mechanical Engineering (dipl. ing. ETH) in January 1993.

Parallel to his university education Mr. Largiadèr attended the Swiss Air Force Academy where he graduated as a second lieutenant in 1992 and then returned for five months to training missions. Mr. Largiadèr's current position in the Swiss Air Force is a company commander in the rank of a first lieutenant.

After completing his military service, Mr. Largiadèr was self-employed for one year in the field of software development. He then joined Andersen Consulting's Zurich office as a consultant where he worked on various projects until June 1997. In the fall of 1997 he attended a two months trainee program with Sociedad Naviera Ultragas, a major Chilean shipping operator engaged in global container transportation, liquid and dry bulk shipment.

Mr. Largiadèr was admitted as a graduate student to the Massachusetts Institute of Technology in January 1998. He selected Naval Architecture and Ocean Systems Management as his majoring field.

Mr. Largiadèr is fluent in German, English and French and has basic knowledge in Spanish.

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# 1 Introduction

A central problem of corporate finance is that of resource allocation. Investments are defined as the act of incurring an immediate cost for the corporation in the expectation of future rewards.[1] Shipping companies, for example, invest in vessels and infrastructure with regard to future payoffs of their assets. However, investment decisions depend on three important characteristics. Primarily an investment is partially or completely irreversible in the sense that once resources have been allocated to a certain project or asset the investment represents a sunk cost. If an investor changes his mind after committing to his initial investment decision he will only be able to partially recover the money spent, e.g. a ship owner will only be able to sell his vessel at the second hand price representing a fraction of his initial outlay.

Secondly, as mentioned earlier, any investment relies on its future rewards; but these rewards are subject to uncertainty, since the market is characterized by change due to micro and macroeconomic factors, such as competitive interactions, political influences, and changes in monetary policies.

Thirdly, the timing of an investment is crucial. Investing in a vessel today may be less rewarding or profitable than having the acquisition of a ship delayed to some time in the future when the market's expectations are to be higher than at present date.

The central question to be answered by a firm is therefore how much to invest and when to invest.

[1] Dixit, Pyndick, p. 3

Traditional discounted-cash-flow (DCF) approaches to appraise a project's net-present-value (NPV) make implicit assumptions about the expected future cash flows of a project and presume management's passive commitment to a static operating strategy. Furthermore the DCF approach assumes that the investment is reversible, that it somehow can be recovered should the market be subject to an unanticipated downward move, or that, if it is irreversible, the investment decision is a now or never decision. Therefore (DCF) approaches do not properly capture management's flexibility to adapt and revise later decisions in response to unexpected (adverse) market developments; the option to invest is not properly captured. If new market information arrives management may have valuable flexibility to alter its initial strategy, such as to defer, expand, contract, or abandon a project at various stages of its operating life. Valuing this managerial flexibility is where "real options" as a strategic management tool come in to play.

In a narrow sense the real options approach is an extension of the financial option theory to options derived from real (non-financial) assets. However, financial options differ from real options in some major characteristics, which makes it difficult to directly apply the financial options theory to the field of real options. The most important differences are the following:

- Typically financial options are standardized contracts between two parties, while real options represent a specific project to be undertaken by a firm.
- Financial options are generally short-lived with one to three years to expiry, while real options may be held from 6 months to 25 years (vessel lifetime).

- Financial options are written on underlying assets that are traded in liquid markets, where the underlying can be bought or shorted, whereas real options again represent unique projects of a single corporation, usually non-tradable in markets.
- Exercising a real option may be a significant project; i.e. exercising an option on a vessel acquisition, whereas exercising a financial option results in a simple financial transaction.

## 1.1 Shipping Options

Financial markets are standardized in the sense that buying a call option on 100 Tschudi&Eitzen shares from bank A is the same as buying the option from bank B.

However, shipping markets are not standardized, since each vessel and also each route on which the vessel is operated contains different attributes. Each vessel varies in size, load factor, speed, yard origin, making it hardly possible to view a specific market segment as being homogenous. Considering two different vessels of the same type (Panamax 70'000 dwt) both built in Korea the same year, although they appear to be identical assets, there can be identified differences in vessel prices due to quality variations.

Within the shipping industry there are three groups of options currently traded:

### A. Options on new-building contracts

A shipowner contracts a vessel from a shipyard. He bargains for an option to contract a second vessel at pre-specified conditions and receives an extension from the shipyard to decide whether to buy the second vessel or not.

## B. Charter-Purchase option

A charterer may charter a vessel on a time charter for x-periods and have the option to purchase the vessel at the end of the charter period, representing a European type options contract.

## C. Option to extend a time charter

A charterer may charter a vessel for x-periods of time and may bargain an option to prolong the charter for another y-periods of time.

## 1.2 Scope of the Analysis

The underlying analysis focuses solely on the valuation of operating vessels, especially on charter purchase options. Charter purchase options written for one or two years are more or less common in the shipping industry; however, the use of such an option as a strategic decision and investment tool over longer periods of time is not very often encountered.

The thesis initially introduces the general principles of options theory applied to financial assets. It then follows the identification of the stochastic process reflecting the nature of the freight rates. Based on the findings a model to value options on freight rates is introduced and discussed. This model is then consequently applied to one type of vessel operating in the bulk segment, a 70'000 [dwt] Panamax carrier, for a specific freight route defined by the Baltic International Freight Futures Exchange (Appendix 1).

Finally, the results are discussed and recommendations for future research projects are made.

# 2 The Basic Nature of Derivatives

A derivative security is a financial instrument whose value depends on the value of one or more underlying basic variables. Derivatives have become more and more important in the field of finance; the most popular derivative instruments that are in use and traded on most exchanges today are futures and options, forward contracts and swaps. The latter are not traded on regular exchanges but between financial institutions and corporations on the so-called over-the-counter (OTC) market.

Derivatives are also referred to as contingent claims. A stock options value for example is a derivative whose value is contingent on the value of its underlying, the stock.

## 2.1 Forward Contracts

A forward contract is a particularly simple derivative, since it denotes an agreement between the holder of the contract and the selling or writing party to buy or sell an asset at a pre-specified price at a certain time in the future. As mentioned earlier, forward contracts are usually traded over the counter. The party that buys the forward contract is said to take a long position in a forward, whereas the other party is said to hold or take a short position. The specified delivery price of the asset is referred to as the delivery price.

The forward price for a certain contract is initially set equal to the spot price of the underlying, which makes the contracts value zero when entering into. Through the time the price of the forward will deviate from its initial delivery price. Forward contracts on the same underlying variable, but with differing times to maturity, are also varying.

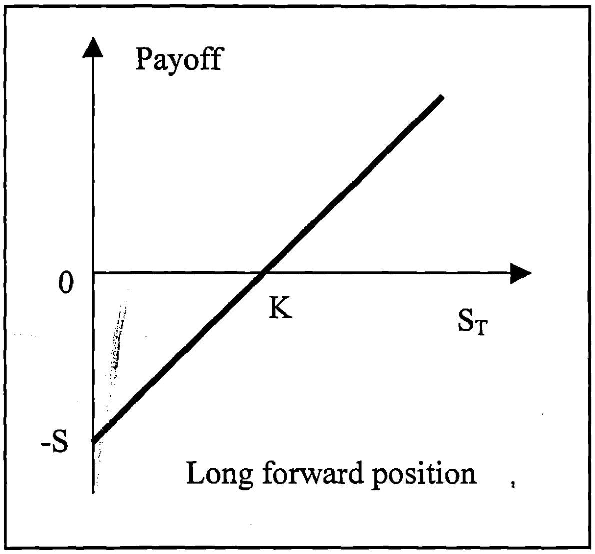
The price of a forward contract on a non-dividend paying underlying security S, maturing at time T, is defined to be:

$$F = Se^{r_f T} \tag{2.1}$$

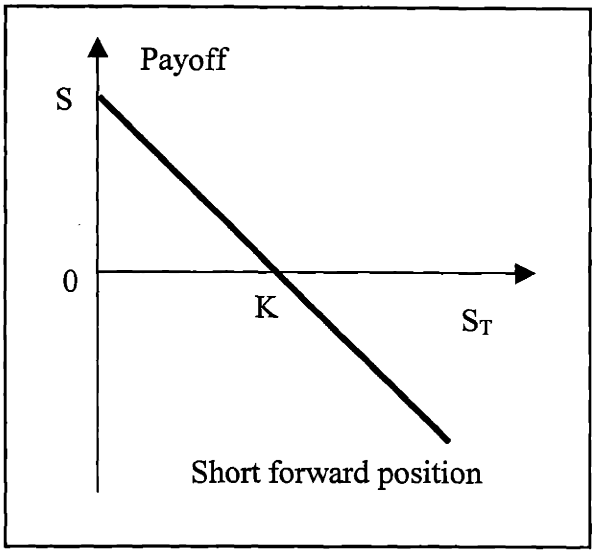
The payoffs from a forward contract are as follows:

Long position: $S_T - K$ (2.2)

Short position: $S_T - K$ (2.3)




[Figure 2.1]




[Figure 2.2]

## 2.2 Futures Contracts

Similar to a forward contract, a futures contract is an agreement between two parties to buy or sell an asset at a certain future time at a given price. But unlike a forward contract, futures are normally traded on exchanges. To make trading of futures contracts possible, the exchange specifies certain standardized features of the contract, since two parties entering a futures contract do not necessarily know each other.[2]

The standardization of a futures contract refers to the underlying asset, the contract size, how prices will be quoted, where delivery of the underlying will be made, and how the price paid will be determined.[3] If the underlying is a commodity, there might be some difference in quality of what is available in the market place; exchanges therefore define different grades referring to the quality of the commodity. The contract size specifies the amount of the underlying to be delivered; too large contracts may be unattractive for traders who wish to hedge relatively small exposure, contracts with a small size may on the other hand be too expensive since there is a cost associated with each contract traded.

Another important difference to the forward contract is that a futures contract requires the investor – at the time he enters the contract – to deposit funds in a, so-called, margin account. The initial down payment is called initial margin and its amount is defined by the brokerhouse. At the end of each trading day the account is adjusted to reflect the investor's gain or loss according to the movement of the underlying's spot price. If the spot price rises above the delivery price the investor realizes a gain, if it drops below the delivery price a loss occurs; a gain will be credited to the margin account the loss debited. This practice is referred to as marking to market.

An important fact is, that under the assumption of constant interest rates over time, the prices of forward and futures contracts on the identical underlying for a certain time to delivery are equal. This fact can be shown by an arbitrage argument.

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[2] Hull, p. 3

### 2.2.1 Futures on Stock Indices

Futures are written on stock indices, currencies, and various commodities. In the case of futures on stock indices the underlying can be seen as a portfolio of stocks that account for the index. These securities forming the index may pay dividends to the holder of the portfolio, which will have to be considered in the futures contract's price. A reasonable approximation for the dividend yield is to assume that dividend payments occur continuously with rate q. The futures contract price F on the index S maturing at time T is then given by:

$$F = Se^{(r_f - q)(T-t)} \tag{2.4}$$

where $r_f$ denotes the risk-free interest rate.

### 2.2.2 Futures on Commodities

Looking at futures contracts on commodities we have to distinguish between commodities that are traded solely for investment purposes, such as gold and silver, and commodities that are held primarily for consumption like crude oil, grain, and coffee. In this chapter we will develop the basic relationships between the price of the underlying, the commodity, and the price of the futures contract. The price relationships, assuming again constant risk-free interest rates, is denoted by

$$F = Se^{r_f(T-t)} \tag{2.5}$$

If storage costs for the underlying commodity are taken into account they will reduce the futures contract's price since it is similar to a negative income. The storage costs u can be regarded as a negative dividend yield:

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[3] Hull, p. 17

$$F = Se^{(r_f + u)(T-t)} \tag{2.6}$$

Users of a commodity must feel that there are benefits from the holdership of the physical commodity that are note obtained by the holder of the futures contract. These benefits include the ability to profit from temporary storage of the commodity. These benefits are sometimes referred to as convenience yield provided by the commodity. If the convenience yield is denoted as y then the futures price can be defined by:

$$F = Se^{(r_f + u - y)(T-t)} \tag{2.7}$$

The convenience yield reflects the market's expectations concerning the future availability of the respective commodity. The greater the possibility of storages will occur during the lifetime of the futures contract, the higher the convenience yield will be. If users of the commodity have high inventories, there will be little chance of shortages within the near future and therefore convenience yields tend to be low. Low inventories will tend to high convenience yields.[4]

However, as mentioned at the beginning of the paragraph, futures are traded on commodities that are primarily held for consumption. In the case of these commodities which cannot be regarded as traded securities the return will not be the risk-free interest rate but will have to be replaced by an estimor of the market price of risk. This estimation of the risk premium and the subsequent pricing of futures contracts will be treated in chapter 5.2.

[4] Hull, p. 67

## 2.3 Options

This chapter reviews and discusses the basic concepts and tools of option pricing theory and contingent claims analysis along with the basic financial applications.

Options contracts have been traded on exchanges for a far shorter period of time than futures contracts. Nevertheless, these instruments have become remarkably popular with investors within the past years. An option is defined as the right, without an associated symmetric obligation, to buy or sell a specific asset by paying a pre-specified price on or before a pre-determined date.

The beneficial asymmetry derived from the right to exercise an option only if it is in the option holder's interest to do so – with no obligation to do so – lies at the heart of an options value. Options contracts differ from forward or futures contracts, in the sense that they do not involve an obligation to either buy or deliver an asset in the future at terms agreed upon today. Thus, unlike the potential payoff from futures contracts, which are symmetric with regard to up and down movements of the underlying asset, the payoff to options is asymmetric or one-directional.

The underlying asset of an option may be one of a large variety of financial or real assets. For example there are options traded on individual shares of common stock, stock indices, such as the S&P 500, on various types of bonds, on commodities (oil, metals), on foreign currencies, on various corporate liabilities and on real assets or capital projects etc.

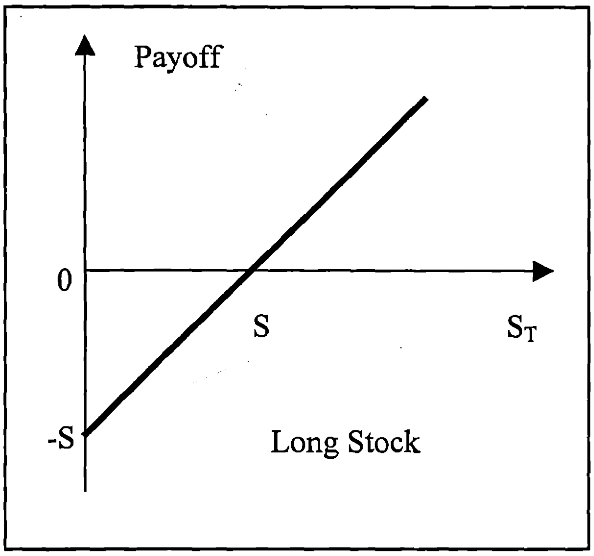
There are two basic types of options contracts: **calls** and **puts**. A **call option** gives the holder the right to buy an asset by a certain date (time to maturity) for a certain pre-specified price (exercise price). A **put option** gives the holder the right to sell an asset by a certain date (expiration date) for a certain specified price (exercise price). If the option – calls or puts – can be exercised before the maturity date (exercise price) it is called an **American option**; if only at maturity, a **European option**.

There are further types of options, such as **Asian options**, and **Bermudan options** that will not be treated in this analysis.

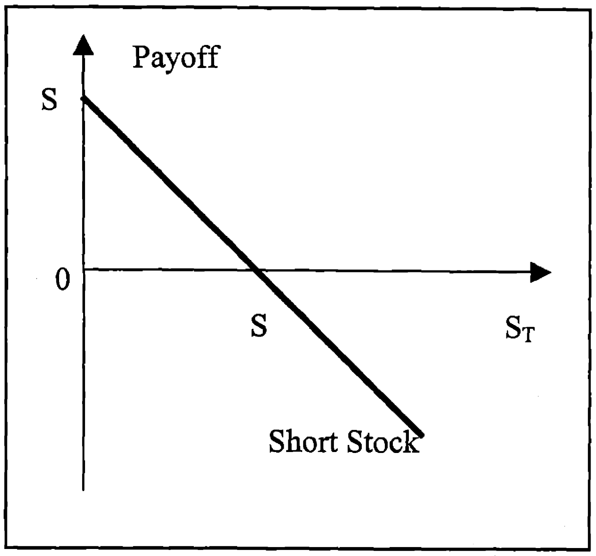
### 2.3.1 Basic Properties of Stock Options

There are several factors affecting the stock options prices. There can also be shown several arbitrage arguments that show the relationship between European option prices, American option prices and the underlying stock price. The most important of these relationships is the put-call parity determining the relationship between European call options prices and European put options prices.

Since we're introducing options on stocks, it is helpful to first look at the payoff diagrams of the underlying, the stock. The following payoff diagrams for a long and short stock position give an insight to the payoffs if a stock is bought or sold at S and held for a certain amount of time.



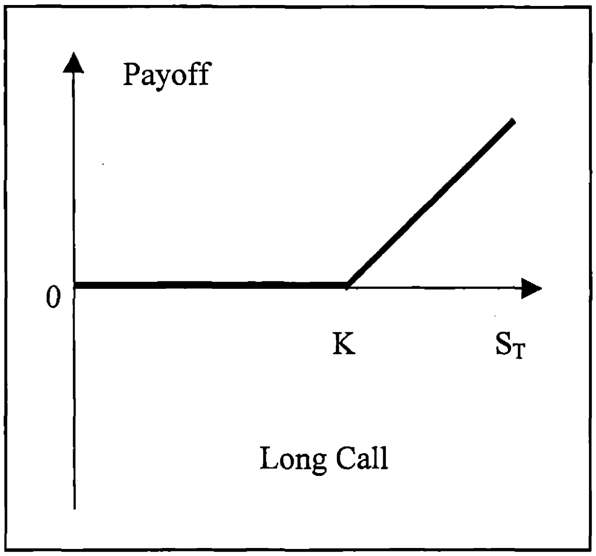

[Figure 2.3]




[Figure 2.4]

Let's first look at the long stock position: if the stock price $S_T$ at the final date happens to be zero ($S_T = 0$), then a long position will have experienced a net loss of S. If $S_T = S$ the position will result in no profit or loss. Generally the net profit will equal ($S_T$-S) for a long position and the reverse for a short position –($S_T$-S). A \$1 increase in $S_T$ results in a \$1 increase for the long position and a \$1 decrease for the short position; therefore the payoff line for a long position in one stock has a positive slope of 1 with a zero profit at $S_T = S$. The reverse refers to a short position in one stock as can be seen in Figures 2.4. The possible gain for a long position is unlimited, whereas the possible loss is limited to (-S).

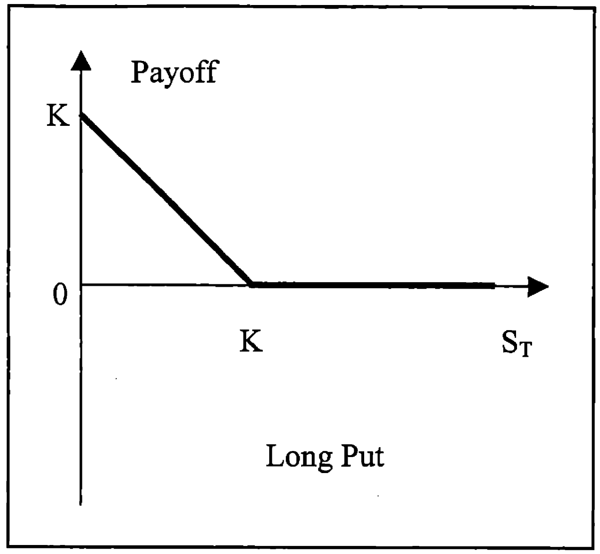
The holder of a call option contract on a stock with price $S_T$, as the underlying asset, has the right to buy the stock at a pre-determined price K. If the holder chooses to exercise the option the cash flow will be [$S_T$-K], if he decides not to exercise his right to buy the underlying, he will receive 0; thus the payoff from a call option contract will always be the maximum of either [ST-K] or 0. Now, since the stock price might end up below K, the holder will not want to exercise the call option and therefore will have a payoff of 0.




[Figure 2.5]

Evidently a long call position is like a long position in the underlying, except that it has the advantage of providing the holder of the option with an insurance again extreme downside movements of the stock price.

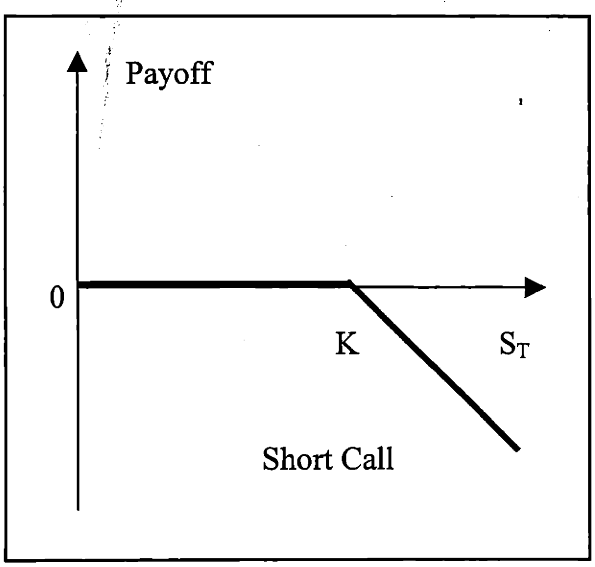
Similarly it is possible to determine the payoff of a put options contract. The holder of the put contract has the right to sell the stock for a predetermined price K. At exercise time the payoff will be [K-$S_T$], assuming the stock price $S_T$ is below K, the exercise price. If the stock price $S_T$ ends up to be above K, and the option is not exercised the payoff of the contract will be 0. Therefore the payoff from a long put options contract is always the maximum of either [K-$S_T$] or 0. The payoff from along put contact can be seen in figure 2.6.

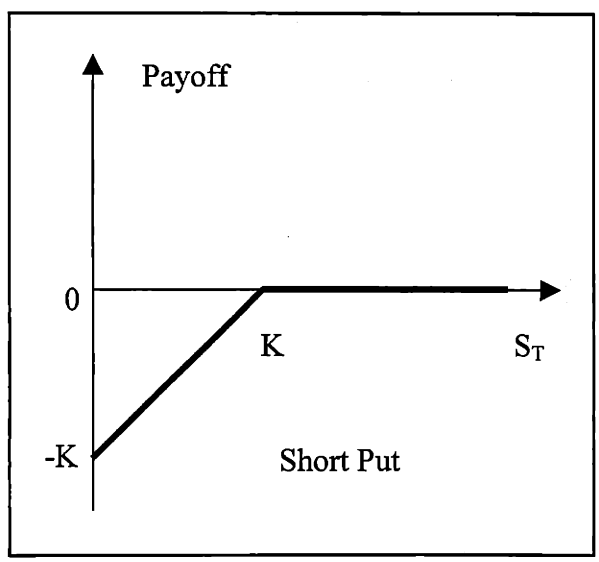



[Figure 2.6]

A purchased or long put is like a short position in the underlying security, except that it affords protection against extreme upside stock price movements.

The reverse positions such as a short position in one call option and a short position in one put contract are the mirror images of figures 2.5 and 2.6.




[Figure 2.7]

[Figure 2.8]

However, both puts and calls have a major disadvantage: the insurance they provide is not for free; the costs incurred are in the form of a premium over parity.

The basic six uncovered or naked positions – long stock, short stock, long call, short call, long put, and short put have been examined. They showed to provide relatively simple payoff patterns. However, there are possibilities of combinations of uncovered positions in order to get covered positions, such as hedges, spreads, and combinations of the two. "A hedge combines an option contract with its underlying stock in such a way that either the stock protects the option against loss or the option protects the stock against loss".[5] A spread combines options of different series but of the same class, where some are bought and others are sold (written).

### 2.3.2 Factors Affecting Options Prices

The major factors affecting the price of a stock option are the following:

- Stock price $S_T$
- Strike price K
- Time to expiration T
- Volatility of the underlying $\sigma$
- Risk-free interest rate $r_f$
- Dividends paid on the underlying D

| Variable (increased, keeping all others constant) | European Call | European Put | American Call | American Put |
|---|---|---|---|---|
| Stock price | + | - | + | - |
| Strike price | - | + | - | + |
| Time to expiration | ? | ? | + | + |
| Volatility | + | + | + | + |
| Risk-free interest rate | + | - | + | - |
| Dividends | - | + | - | + |

[Table 2.9]

If a call option is exercised some time in the future, the payoff will be the amount by which how much the stock price $S_T$ exceeds the strike price, i.e. how far the option is in the money. The call options' value therefore increases if the stock price increases or the strike price decreases. As we saw earlier, the payoff from a call option depends on the difference between the stock price and the strike price $[S_T - K]$. For the holder of the put the values are effected in the opposite way.

The time to expiration doesn't affect the European option price; whereas American contracts are clearly affected by an increase in its lifetime. The owner of a long-life American option gets more opportunities to exercise than the one with a short-life contract. Therefore the long-life option has to be always worth more than the short life.

The volatility of the stock price determines the uncertainty of future stock price movements. As the stock price increases the possibility of the stock doing poorly increases. An investor

[5] Cox, p. 8

therefore requires a premium for the increase of risk which is reflected in an increase of the options price.

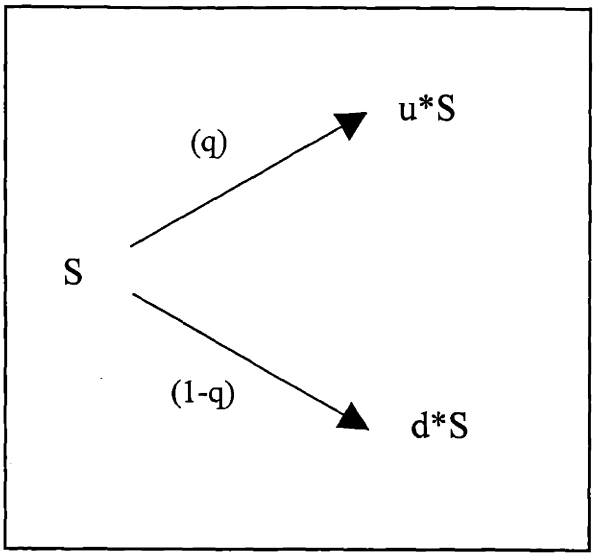
An increase of the risk-free rate results in a higher expected growth rate of the underlying, consequently calls are more likely to be in the money and are therefore worth more, whereas put values are negatively affected by increasing interest rates.

Dividends reduce the stock prices at the ex-dividend date (date after dividends are paid out) resulting in a decrease of call values and an increase of put values.

### 2.3.3 The Basic Valuation Idea: Replicating Portfolio

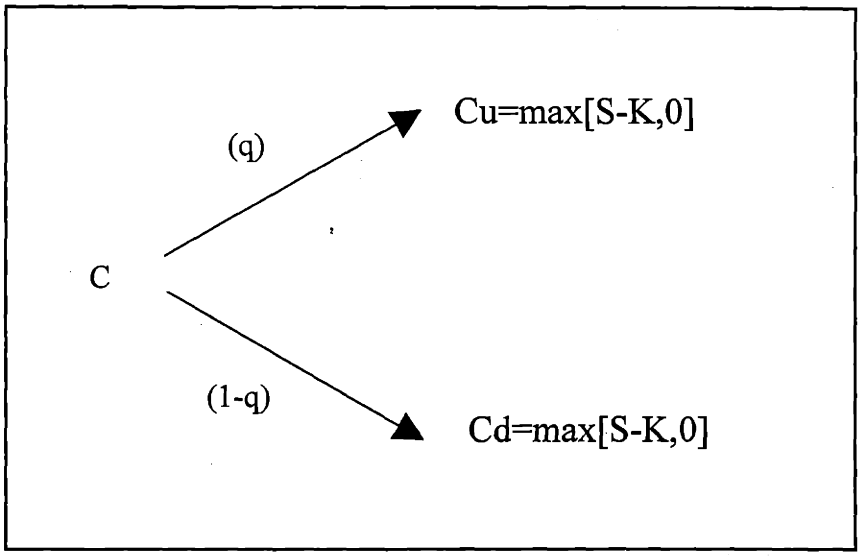
The basic valuation idea for the exact pricing of options is that one can construct a portfolio, consisting of buying a number of shares of the underlying asset (e.g. common stock) and borrowing against them at the risk-free rate (e.g. using zero coupon bonds), that would exactly replicate the future returns of the option in any state. Since the option and the equivalent portfolio would provide the same future returns, avoiding risk-free arbitrage opportunities, they both must sell for the same current price. We can therefore value the option by determining the cost of constructing the replicating portfolio.

Assume that the price of the underlying will move over the next period either up to by a factor u or down by a factor d, with probabilities q and (1-q) respectively. For the first period we would then have the following valuation of the stock:




[Figure 2.10]

The value of the call option over this period would then be the contingent on the price of the underlying stock:



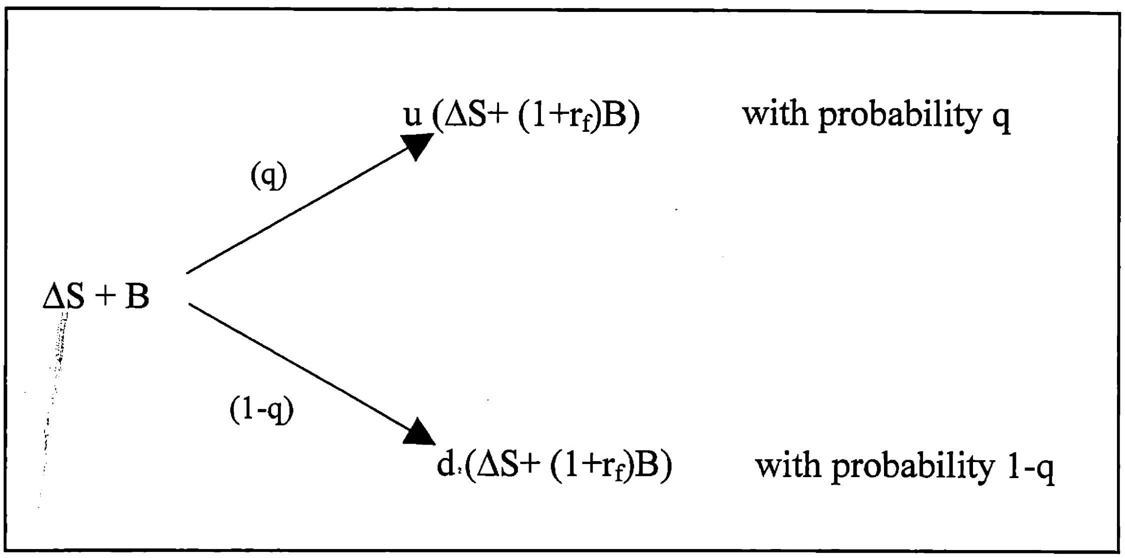

[Figure 2.11]

Where $C_u$ and $C_d$ represent the values of the call option after one period assuming that the underlying stock moves up or down respectively.

The construction of the entire portfolio consisting of buying Δ shares of the underlying stock at its current price S, financed in part by borrowing an amount equal to $B at the risk-free rate for a net cost of (ΔS+B); that is the call option on the underlying has the value of:

$$C = (\Delta S + B) \quad (2.8)$$

Looking at the portfolio after one period, we would have to repay the principal amount borrowed at the beginning with interest (1+r)B. The value of the portfolio at the next period would therefore be:




[Figure 2.12]

If the portfolio offers the same return in each state at the end of the period as the option, then

$$u\Delta S + (1 + r_f)B = C_u \quad (2.9)$$

$$d\Delta S + (1 + r_f)B = C_d \quad (2.10)$$

Solving these two equations for the two unknowns Δ and B gives the following:

$$\Delta = \frac{Cu - Cd}{(u-d)S} \tag{2.11}$$

and

$$B = \frac{uC_d - dC_u}{(u-d)(1+r_f)} \tag{2.12}$$

The number of shares that we need to buy to replicate one option over the next period is known to be the option's delta or the hedge ratio, and is simply obtained in the discussed discrete case as the difference of the option prices divided by the difference of the stock prices.

If there are to be no riskless arbitrage opportunities, the current value of the call C, cannot be worth less than the current value of the portfolio (ΔS+B). If there was an arbitrage opportunity, an investor could make a profit by buying a call and selling the portfolio. Even after one period one could argue that there might be an arbitrage opportunity. An investor, holding the above portfolio will receive after one period an identical payoff to the value of the portfolio after one period. If he decides to exercise now, he will receive a payoff of [S-K] he could, under arbitrage circumstances, take the proceeds and buy the same portfolio realizing a guaranteed profit at no cost. As a consequence no one would be willing to hold calls for more than one period.

This reasoning determines that the value of a call today has to be equal to the calls after one period discounted at the risk-free rate.

$$C = \Delta S + B = \frac{C_u - C_d}{u-d} + \frac{uC_u - dC_d}{(u-d)(1+r_f)} \tag{2.13}$$

Equation (2.12) may then be rewritten in the following manner:

$$C = \frac{pC_u + (1-p)C_d}{(1+r_f)} \tag{2.14}$$

where

$$p = \frac{(1+r_f)-d}{(u-d)} \tag{2.15}$$

is transformed to the risk neutral probability, the probability that prevails in a risk-neutral world where investors are indifferent to risk.

The risk-neutral valuation can be shown be rearranging the portfolio in C - ΔS = B, such that an investor holds a short position in ΔS shares of the underlying stock and buys one call option that would provide an amount of $B; which determines an investment implying a portfolio return at the risk-free rate.

### 2.3.4 The Black-Scholes Option Pricing Formula

Following along the discrete path for n periods, as discussed, we can obtain the path dependent call or put option price by starting at the final node recursively working backwards by discounting the respective option values over one specific period until we reach the current node at time t =0.

The binomial call option pricing may then be expressed as:

$$C = S\Phi[a,n,p'] - K(1+r_f)^{-n}\Phi[a,n,p] \tag{2.16}$$

where Φ denotes the binomial distribution function dependent on

$$p = \frac{(1+r_f)-d}{u-d} \tag{2.17}$$

and

$$p' = \frac{u}{(1+r_f)} p \tag{2.18}$$

$$a \geq \ln(K / Sd^n) / \ln(u / d)$$

Using the same technique for but assuming that the periods n become more frequent due to continuous trading, we get in the limiting case n→∞ and consequently

$$h = \frac{t}{n}$$

approaches 0.

For the limiting case, and assuming a lognormal distribution of the stock prices, the binomial distribution Φ converges with the normal distribution N(x).

For this limiting case Black and Scholes proved there is a closed from solution to price European call and put options.[6]

$$C = SN(x) - K(1+r_f)^t N(x - \sigma\sqrt{t}) \tag{2.19}$$

Where x equals

$$x \equiv \frac{\ln(S / K(1+r_f)^t)}{\sigma\sqrt{t}} + \frac{1}{2}\sigma\sqrt{t} \tag{2.20}$$

The put call parity relationship provides us with an easy relationship to price European put options accordingly.

---

[6] Cox, Rubinstein, p. 205

### 2.3.5 The Put-Call-Parity Relationship

There exists a distinctive relationship between European put and call options for non-dividend paying stock named the **put-call-parity** relationship that can be derived from a simple arbitrage argument.

Consider taking the following positions simultaneously in a European call and put on the same underlying stock with the same exercise price K and time to maturity t: short one call, buy one put, buy the underlying, and borrow $K(1+r_f)^{-t}$ . The present and future cash-flows would then be:

| | Current Date | Expiration Date | Expiration Date |
|---|---|---|---|
| | | S<=K | S>K |
| Short Call | C | -- | K-S |
| Buy Put | -P | K-S | -- |
| Buy Stock | -S | S | S |
| Borrow | $K(1+r_f)^{-t}$ | -K | -K |
| **Total** | | -- | -- |

[Table 2.13]

The portfolio proofs to have zero cash-flows in all possible circumstances. If there are to be no arbitrage opportunities the initial investment also has to be 0:

$$C - P - S + K(1 + r_f)^{-t} = 0 \tag{2.21}$$

Equation (2.21) is known as the put-call-parity relationships for non-dividend paying European options.

# 3 Freight Rate Process

## 3.1 Stochastic Process

Variables whose value changes over time in an uncertain way is said to follow a stochastic process. These processes can be defined as "discrete time" or "continuous time" stochastic processes. A discrete-time stochastic process is one where the value of the variable can only change at certain fixed points in time, whereas a continuous time stochastic process is one where changes of the variable can take place at any time. Stochastic processes can also be classified as continuous variable or discrete variable. In a continuous variable process the underlying variable can take any value within a certain range, whereas in a discrete-variable process, only certain discrete values are possible. [7]

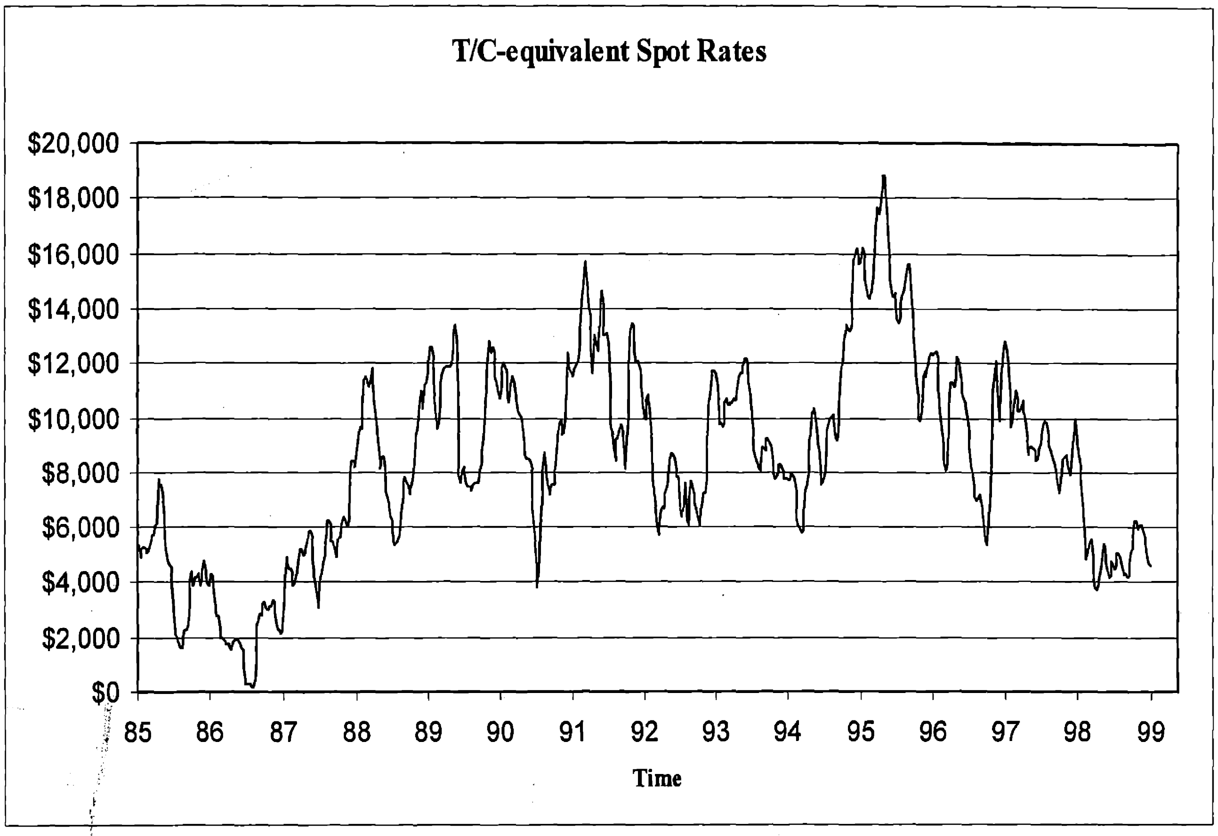
Option theory does not provide an equilibrium price of the underlying asset. Instead, derivative asset analysis shows that in equilibrium there must be certain relationship between the price of the derivative and the underlying asset, given that the underlying asset moves according to certain rules; i.e. following a certain type of stochastic process.

## 3.2 Freight Rates

In a time series forecast the interest lies in making some statements about the value of the series at some future time T. The analyzed underlying variable $S_T$ denotes the time charter equivalent spot rate, the daily price that is paid by a charterer to the ship-owner (rate calculations are discussed in chapter 4.4)

[7] Hull (1997), p. 209

The following Figure 3.1 displays the weekly averaged T/C-equivalent spot rates S(t) over a time period from 1985 to 1998 on a weekly averaged basis.




[Figure 3.1]

However, for all option valuations on vessels data on a daily basis was used in order to enhance the accuracy of the forecasts and simulations (see Appendix 2).

To fully characterize the data for the period from 1985 to 1998 Table 3.2 provides descriptive statistical measures for the entire data set.

| Variable | N | Mean | Median | STDEV | Min | Max |
|---|---|---|---|---|---|---|
| $S_T$ | 743 | 8399.605 | 8348.999 | 3587.694 | 131.555 | 18796.341 |
| $\ln(S_T/S_{T-1})$ | 742 | 0.00038 | -0.00145 | 0.10963 | -1.13977 | 1.15433 |
| $S_T-S_{T-1}$ | 742 | 2.344 | -10.587 | 544.358 | -2555.503 | 2574.664 |

[Table 3.2]

Where $S_T$ ;denotes the discrete T/C-equivalents; $\ln(S_T/S_{T-1})$ the logarithmic returns from the TC-equivalents; and $S_T-S_{T-1}$ is the per period difference in the T/C-equivalents.

The total number of observations on a weekly averaged basis used are 743. The TC-equivalents show a great volatility throughout the observed period especially in the late 80's and early 90's. The maximum rate of $18'796.30 can be observed in May 1995, whereas the minimum T/C-equivalent of $131.55 is to be observed in July 1986. The weekly volatility of the entire period of 14 years is $3587.70 as measured by the standard deviation of the entire series.

The logarithmic price relatives will be used for the estimation whether the TC-equivalent spot rates follow a Geometric Brownian motion or not. The maximum relative downward price change of 113 percent occurred in May 1986 the maximum upward change of nearly 115 percent was to be observed in August1996.

The rate difference ($S_T-S_{T-1}$) will be used for the test whether the T/C-equivalents are mean reverting. The maximum downward difference occurred in September 1989 the maximum upward change at the end of 1996.

## 3.3 The Markov, Wiener and Ito Process

The Markov process is a stochastic process where the historical values of a variable are irrelevant for the prediction of the future. Predictions of the future value of the measured variable relies solely on probability distributions and on the variable's value today.

One example of Markov distributed values are stock prices; there future predictions are based on the actual price and assumed probability distributions.

A Wiener process is a specific type of Markov stochastic process that can be described as follows:

$$\Delta z = \varepsilon * \sqrt{\Delta t} \tag{3.1}$$

where ε is a random normally distributed variable with mean 0 and standard deviation 1 N(0,1). The Wiener process can be regarded as adding noise or variability to the path followed by the underlying variable.

An Ito process is another class of a stochastic process which describes the behaviour of a variable over time. The Ito process is a generalized Wiener process where the variables a and b are functions of the value of the underlying variable, x, and the time, t. This porcess can algebraically be expressed as follows:

$$dx = a(x,t)dt + b(x,t)dz \tag{3.2}$$

Both the expected drift rate a and the variance are subject to changes over time.[8] The term dz represents again a Wiener process.

[8] Hull, p. 215

Generally we can say that the value of any derivative (contingent claim) is a function of the stochastic variables of the underlying asset and of time. The general application of Ito's lemma to a function G dependent on x and t shows that G follows the process

$$dG = (\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2)dt + \frac{\partial G}{\partial x}bdz \tag{3.3}$$

where dz is again the same Wiener process as in (3.2).[9] Therefore G also follows an Ito process with a drift rate

$$\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2 \tag{3.4}$$

and a variance rate of

$$(\frac{\partial G}{\partial x})^2 b^2 \tag{3.5}$$

A complete proof of Ito's lemma is not given in this paper and would be beyond the scope of the thesis.

## 3.4 Process Identification

Parameters enabling us to fully describe the price process depends on the process selected. Two types of processes will be considered for the underlying analysis and for the valuation of vessels: the Geometric Brownian motion, which is the well known process describing the

[9] Hull, p. 220

behavior of stock prices, and the Mean Reverting process, used to model interest rate movements.

### 3.4.1 Geometric Brownian Motion

The basic assumption made by Black and Scholes (1973) was that the stock price follows a Geometric Brownian Motion, which is a specific case of the Ito process described in section 3.1.

One could argue that a stock price follows a generalized Wiener process with constant expected drift rate and constant variance rate. This assumption would not capture the basic requirement of an investor that the percentage return from a stock be independent of the stock's price. Say, an investor requires 12% expected return of a specific stock independently of its price being \$20 or \$50. Therefore the assumption of a constant expected drift rate needs to be replaced by expressing the expected percentage return as a proportion of the stock price. If the variance of the stock price is always zero this leads to the following relationship

$$dS = \mu S dt \tag{3.6}$$

with constant μ, or

$$\frac{dS}{S} = \mu dt$$

The expected stock price at time T will therefore be

$$S = S_0 e^{\mu t} \tag{3.7}$$

where $S_0$ is the stock price at time zero.

In practice however the stock price movement shows volatility. The instantaneous variance rate of S is defined as $\sigma^2 S^2$. Applying Ito's lemma it can be shown that S follows an Ito process with instantaneous expected drift rate $\mu S$ and instantaneous variance rate $\sigma^2 S^2$. This relationship can be expressed as:

$$dS = \mu S dt + \sigma S dz \tag{3.8}$$

or

$$\frac{dS}{S} = \mu dt + \sigma dz \tag{3.9}$$

The above shown relationship is the most widely used model of stock price behavior. By using Ito's lemma it can be furthermore shown that the incremental change of the stock price is lognormally distributed.

$$\ln S_t - \ln S_{t-1} \approx N[(\mu - \frac{\sigma^2}{2})\Delta t, \sigma\sqrt{\Delta t}] \tag{3.10}$$

or

$$\ln(\frac{S_t}{S_{t-1}}) \approx N[(\mu - \frac{\sigma^2}{2})\Delta t, \sigma\sqrt{\Delta t}] \tag{3.11}$$

By integrating (3.8) we will find that

$$S(T) = S(t)e^{(\mu - \frac{\sigma^2}{2})(T-t) + \sigma(Z(T) - Z(t))} \tag{3.12}$$

with an expected value of

$$E[S(T)] = S(t)e^{\mu(T-t)} \tag{3.13}$$

and variance

$$Var[S(T)] = S(t)^2 e^{2\mu(T-t)} (e^{\sigma^2 (T-t)} - 1) \tag{3.14}$$

### 3.4.2 Mean Reversion

The hypothesis of mean reverting cash flows have been tested in a number of markets.

Interest rates have appeared to follow a mean reverting process, which means that they have a drift pulling the rate back to some long-term level.[10] Cox, Ingersoll and Ross tried to model the stochastic behavior of interest rates by applying a mean reverting model. The modelling of interest rates has many similar properties as the mean reverting model of spot rates such as the volatility being a function of the interest rate or spot rate level respectively.

In the maritime field Bjerksund and Ekern provided pioneering work in the field of spot rate modeling. They postulate that the spot rates follow an Ornstein-Uhlenbeck process. The Ornstein-Uhlenbeck process has mean reverting properties. The main argument for choosing a mean reverting process to model the freight rates is because of vessel capacity adjustment. High profitability triggers exit from lay-ups and also new ordering. Low profitability may make ship owners to slow steam or to lay up their vessels and if the market expectations are very poor shipowners may even decide to sell their vessels to scrap them. Therefore, high freight rates imply increased supply, which as a consequence depresses freight rates, and low freight rates reduce supply of capacity and contribute to a recovery of the market. This hypothesis is underlined by the fact that there is an inverse relationship between freight rates and scrap and lay-up tonnage.

The Ornstein-Uhlenbeck process is described by the following relationship:

$$dS(t) = k(\alpha - S(t))dt + \sigma dz(t) \tag{3.14}$$

where S(t) is the spot rate, k is the adjustment factor, the factor pulling the rate back to its equilibrium level $\alpha$. The factor $\sigma$ is the standard deviation and dz is the increment of the Brownian Motion. The sign of the drift term is determined by the difference of the equilibrium level $\alpha$ and the instantaneous spot rate S(t). The adjustment factor k determines the speed of adjustment of the spot rate to the equilibrium level; therefore a high value of k would mean a rapid adjustment back to the equilibrium.

The value of the stochastic process of (5.14) at any future date T, is given by the following relationship[11]:

$$S(T) = e^{-k(T-t)}S(t) + (1 - e^{-k(T-t)})\alpha + \sigma e^{-k(T-t)} \int_t^T e^{k(T-t)} dZ(t) \tag{3.15}$$

where S(t) is normally distributed with expected value

$$E_0[S(T)] = e^{-k(T-t)}S(t) + (1 - e^{-k(T-t)})\alpha \tag{3.16}$$

and variance of

$$Var_0[S(T)] = \frac{\sigma^2}{2k}(1 - e^{-2k(T-t)}) \tag{3.17}$$

[10] Hull, p.

To interpret of the speed of adjustment factor k one can see that

$$T = \frac{\ln(2)}{k}$$

gives the half time at which the expected value $E_0[S(t)]$ is half way between the long range mean $\alpha$ and the current value of S(t).

# 4 Data

## 4.1 Vessel Data

For the underlying analysis a Panamax Bulk carrier is chosen as the representative vessel, which is operated on a route defined as number 1 by the Baltic International Freight Futures Exchange (BIFFEX). This vessel type inherited its name from the ability to pass through the Panama Canal with its respective beam and draft measures.

The vessels deadweight tonnage of the ship is 70'000 [dwt] and its load capacity is, according to the BIFFEX specifications for route 1, 55'000 [tons] of grain.

This shipping segment has been fairly homogenous throughout the seventies and eighties; major changes have been in the propulsion such as that the Panamax vessels have become more fuel efficient. Vessels built in the seventies consume roughly 20 percent more fuel than the ones built in the eighties.

[11] Trigeorgis, Bjerksund and Ekern, p. 209

specific port. They include harbor dues, pilotage, tug boats, dockage and wharfage – if necessary -, line handlers and agency fees. These charges vary from port to port.

For the BIFFEX route 1 the Mississippi river basin was chosen as the US Gulf port, the vessels origin (see Appendix 1). As the destination port Rotterdam was selected as the discharging port on the European continent, assuming minor cost differences between Amsterdam, Rotterdam and Antwerp.

| **Average Port Costs New Orleans** | **Average Port Costs Rotterdam Rotterdam [US$]** |
|---|---|
| 62'000.- | 54'000.- |

[Table 4.2]

## 4.4 Time Charter Equivalent Spot Rates

A time charter equivalent spot rate reflects the net rate per day by the charterer to hire a vessel from a shipowner. The rate covers crew and operational expenses including financial costs.

According to shipowners it seems to be difficult to find a trading route that which provides a representative round trip voyage in ballast. A ship may be fixed travelling from port A to port B, but may be returning empty or only partly carrying bulk.

The BIFFEX route 1 (US Gulf – ARA) was chosen because of its high frequency on the transatlantic trade, although there is not much grain export from the Continent to the US.

The Baltic Freight Index (BFI) is a weighted average of ten major bulk cargo routes. For each route a separate index is calculated on route and cargo specific parameters. The difficulty in using wide ranging historical data from the BFI is that the routes and its parameters have been revised several times. The only route being calculated on the same basis is route number 1 for which daily data is available from 1985 to 1998.

The following table referring to the route specifications is provided by BIFFEX.

| |
|---|
| **Route 1.** |
| 1 Port US Gulf/Antwerp, Rotterdam, Amsterdam, 55,000 tonnes 10 per cent light grain stowing 55ft., free in and out. 10 days Sundays, holidays excepted. Laydays 10 days forward from date of Index, canceling maximum 30 days forward from date of Index. 3.75 per cent total commissions. |
| Nominal Weighting =10% |

[Table 4.3]

It can be inferred from the table that the cargo is 55'000 tons of grain (+/- 5%) from either the US Gulf to either Amsterdam, Rotterdam or Antwerp. Free in and out indicates that the charterer actually pays for the loading and the discharging of the vessel. The charterer is allowed 10 days to load and discharge the cargo; whereas times do not count on Sundays and on holidays, since port labor is idle on these days. The contract also specifies certain days for which the vessel can start loading and if the vessel is not ready within these days, the charterer may cancel the contract. The last term specifies the broker's commission.

The spot market quantities of cargo are quoted in US$/ton. Assuming a spot price of US$/ton 14.-, a bulk carrier with loading capacity of 55'000 tons would have a voyage gross revenue of US$ 770'000.- . For a ship owner facing the decision whether to operate in the spot market versus operating the vessel on a time charter, the time charter equivalent spot rate is the comparable rate. The latter is relatively easy to calculate and it reflects the net revenue to the ship owner who has to cover for the vessel's fixed and variable costs excluding the fuel and port charges.

The TC-equivalent is calculated as follows:

$$T/C = \frac{(Spot[US\$/ton]*Capacity[ton])*(1-Commission[\%]) - Bunkers[US\$] - PortCharges[US\$]}{RoundTrip[days]} \quad (4.1)$$

The following example indicate the application of conversion formula (4.1):

| | | | |
|---|---|---|---|
| **Gross Revenue:** | 55'000 [tons] *14.- [US$/ton] = | | $770'000.- |
| **Variable Costs:** | | | |
| 1. Bunkers: | Laden: | 31.5 [tons/day]*18 [days]*100 [US$/ton]= | $56'700.- |
| | Ballast: | 29.0 [tons/day]*16 [days]*100 [US$/ton]= | $46'400.- |
| | Port: | 1.2 [tons/day]* 15 [days]*100 [US$/ton]= | $1'800.- |
| | | 0.8 [tons/day]* 15 [days]*100 [US$/ton]= | $1'200.- |
| 2. Port Charges: | | | |
| 3. Commission: | US$770'000.-*(0.0375) = | | $ 28'875.- |
| Total Variable Costs | | | $134'975.- |
| **Net Revenue:** | | | **$635'025.-** |
| **TC-Equivalent spot rate per day:** | | | **$12'959.70** |

The daily TC-equivalent spot rate is found by dividing the net revenue by the total number of days per voyage.

## 4.5 Time Charter

Shipowners may charter their vessel out on various lengths and fix the price of the charter for a certain amount of time. A time charter can therefore be considered as being a forward contract on the spot rates, which gives the ship owner the advantage to hedge the volatility of the spot market. Generally time charters have a duration from 3 months to the life time of the vessel. The owners are typically responsible for manning and maintenance, whereas the charterers cover trading related costs such as bunkers, port costs etc.

The number of time charters fluctuate according to the rates: when rates are low clearly no time charters have been reported, whereas in times of high rates the number of time charters reach the peak.

## 4.6 Regression Estimates for T/C-Equivalent Rates

### 4.6.1 Geometric Brownian Motion

We will first analyze the T/C-equivalent spot rates to see whether a log normal distribution is supported by the data . We therefore assume that the natural logarithms of the rate ratios, $\ln(S_t/S_{t-1})$, are normally distributed. The T/C-equivalent spot rates will be regressed by their lagged values.[12] The null hypothesis for $\beta_1 = 0$ will be tested in order to see whether a percentage return the previous period has any influence on the percentage return of the present period. The relationship for the regression can be algebraically expressed as

$$\ln(\frac{S_t}{S_{t-1}}) = \beta_0 + \beta_1 \ln(\frac{S_{t-1}}{S_{t-2}}) + \varepsilon_t \qquad (4.2)$$

The following regression results were found on the one hand for the entire period from 1985 to 1998 and then also for the sub-periods 1985-1987, 1988-1990, 1991-1993, 1994-1996, 1997-1998 respectively (Table 4.4).

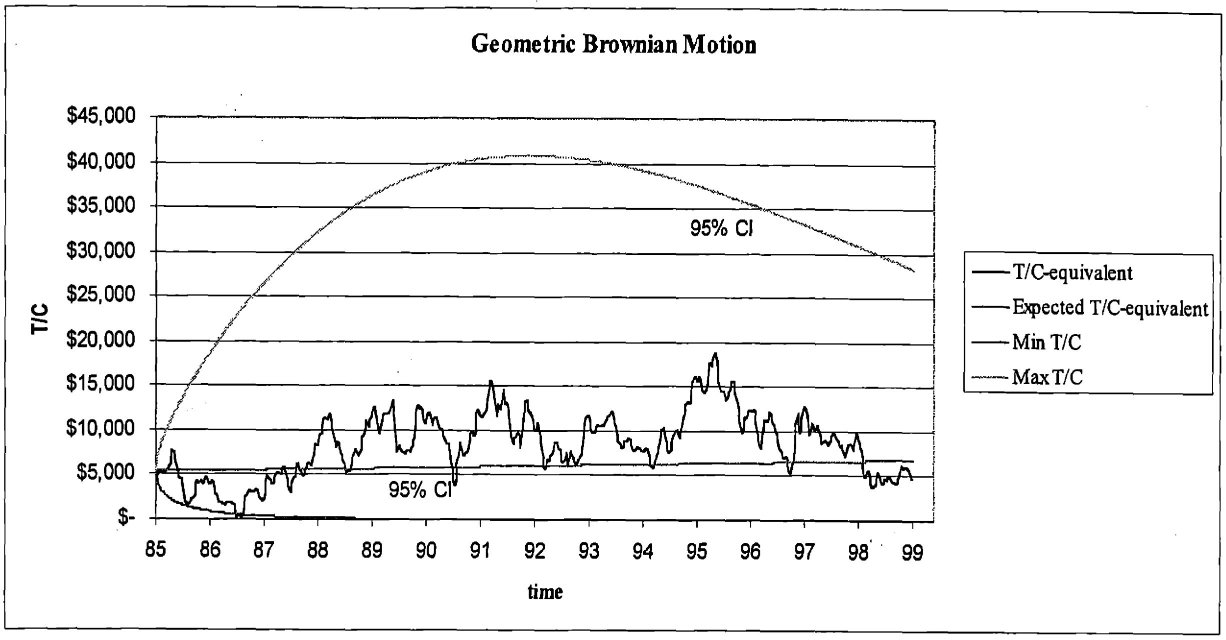
| Period | N | β1 | T | σ | $R^2$ |
|---|---|---|---|---|---|
| 1985-1998 | 741 | 0.5022 | 15.7808 | 0.0947 | 0.2520 |
| 1985-1987 | 155 | 0.4776 | 6.7236 | 0.1799 | 0.2281 |
| 1988-1990 | 155 | 0.5947 | 9.1289 | 0.0590 | 0.3526 |
| 1991-1993 | 157 | 0.3996 | 5.4386 | 0.0569 | 0.1602 |
| 1994-1996 | 156 | 0.6819 | 11.5560 | 0.0449 | 0.4644 |
| 1997-1998 | 118 | 0.5840 | 7.7020 | 0.0498 | 0.3384 |

[Table 4.4]

As can be seen in Table 4.4 the regression never supports the hypothesis for a lognormal distribution since the t-test shows a value always greater than $t>1.96$, since the confidence interval of 95% lies at $t=1.96$. Considering all sub-periods the hypothesis can always be rejected at a 95% confidence level.

Figure 4.5 shows again the T/C-equivalent spot rates over the entire analyzed period from 19985 to 1998. The expected T/C-equivalent spot rate is plotted for the entire period; which clearly shows that a Geometric Brownian motion based model does not reflect the true nature of the spot rates; it may be used as a very rough approximation to value options on vessels with the standardized Black-Scholes equation.

[12] J. Tvet, p.184




[Figure 4.5]

### 4.6.2 Mean Reversion

The following linear regression has been run in order to estimate the parameters for the discrete version of the Ornstein-Uhlenbeck process (3.14)

$$S_t - S_{t-1} = \alpha k - kS_{t-1} + \varepsilon_t \tag{4.3}$$

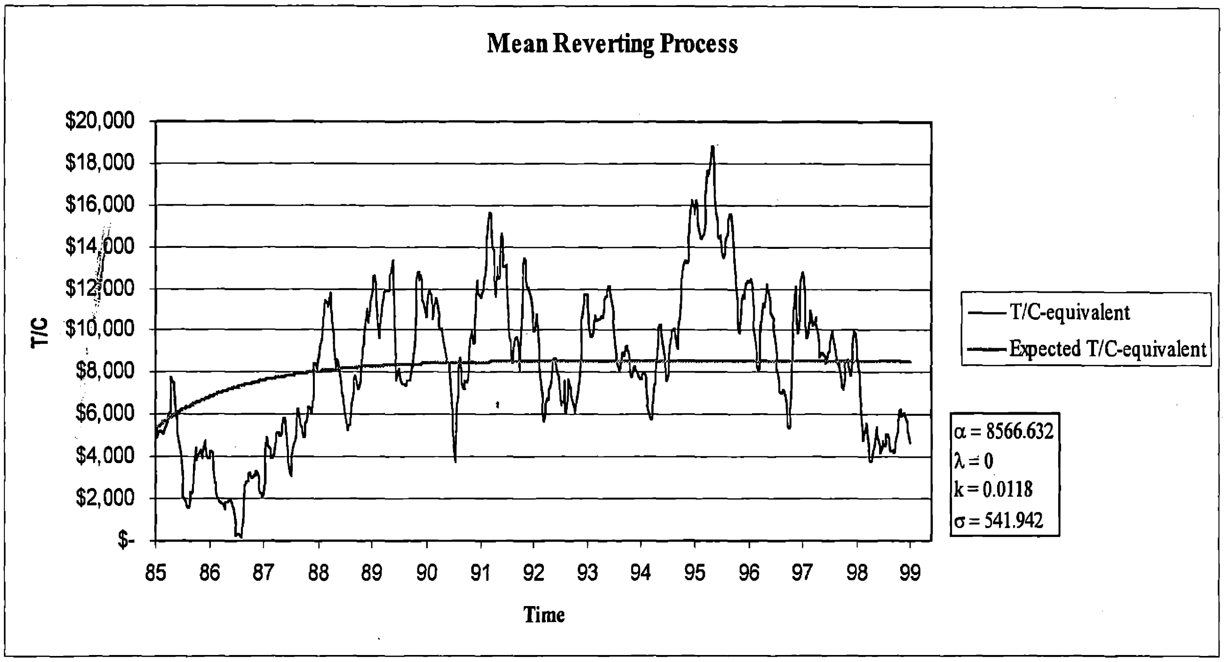
The regression results are reported in Table 4.6

| Period | N | αk | K | α | σ |
|---|---|---|---|---|---|
| 1985-1998 | 742 | 101.0922 | 0.011801 | 8566.6319 | 541.9426 |
| 1985-1987 | 156 | 61.2052 | 0.010886 | 5622.2969 | 423.1737 |
| 1988-1990 | 155 | 375.1589 | 0.037603 | 9976.8793 | 611.5075 |
| 1991-1993 | 157 | 281.9321 | 0.031145 | 9052.2353 | 588.6900 |
| 1994-1996 | 156 | 260.8711 | 0.020134 | 12956.9854 | 610.9955 |
| 1997-1998 | 118 | 241.8969 | 0.040743 | 5937.1445 | 397.2943 |

[Table 4.6]

The regression for period 1985 to 1998 does not strongly support a mean reverting behavior of the spot rates since the speed of adjustment factor k = 0.0118 is rather low. The rate is pulled back by this factor to the mean α on a weekly basis. For the further analysis a mean of α=$8566.32.- is used, which is a rather high level considering the lows in the period of 1985-1987. The regression volatility σ shows the volatile nature of this business.

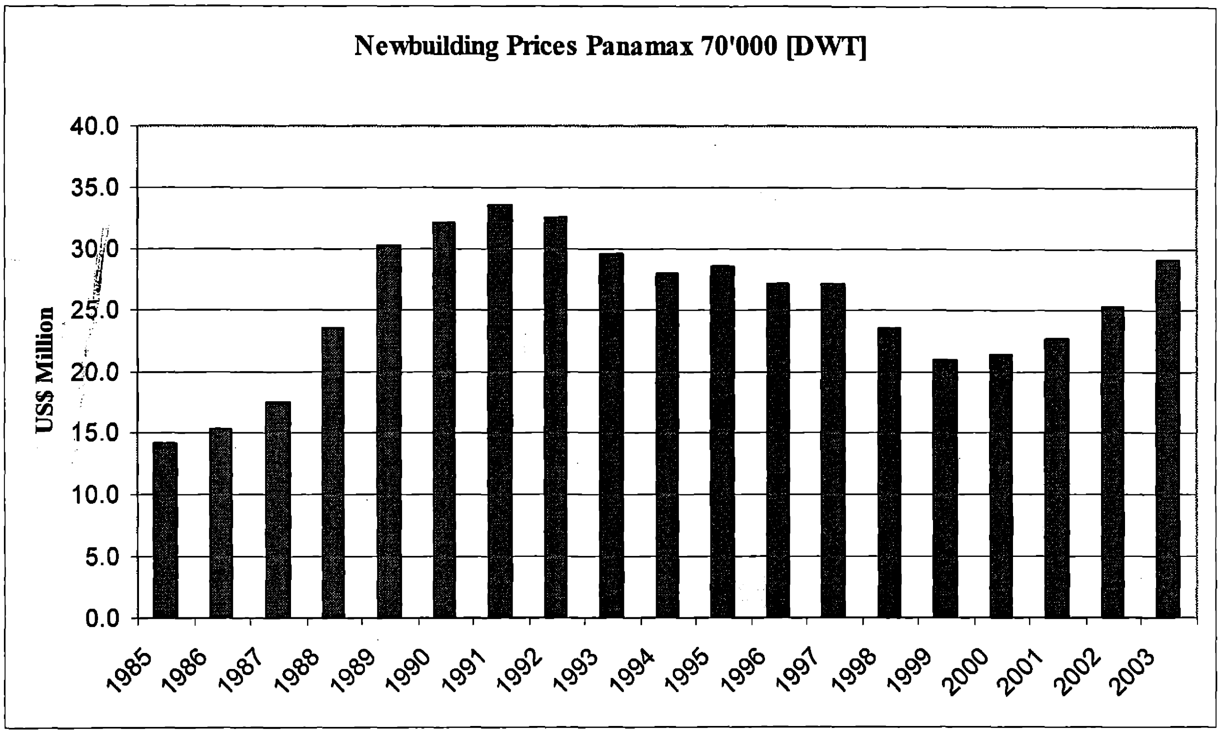
The regression estimates for period 1985-1987 and period 1988-1990 determine a far too high mean. This can only be explained that these short periods do not support a mean reverting behavior of the spot rates; a period over a longer time therefore has to be considered. The entire sample from 1985-1998 seems more or less to have a mean reverting character although k is low as can be seen in Figure 4.7.




[Figure 4.7]

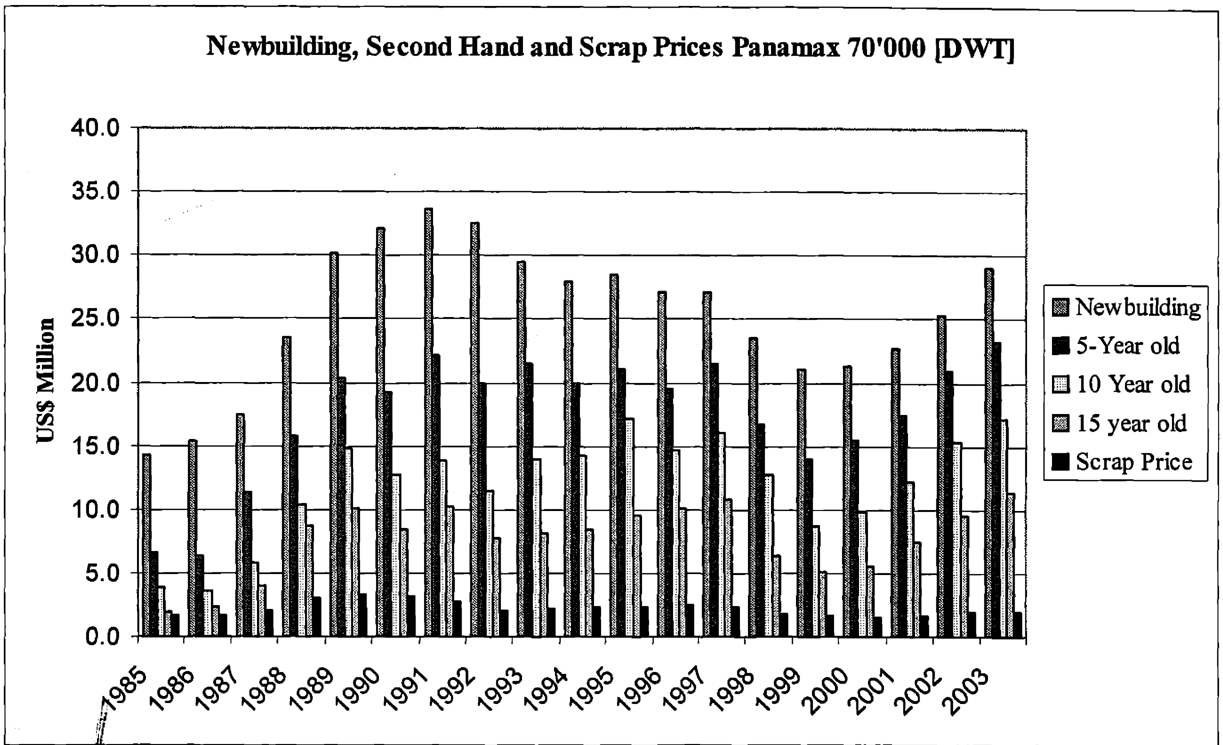
The results in the table show the importance of choosing the "right" input parameters, since there can be observed a great difference in the mean reverting parameters α between the analyzed periods. The equilibrium level can be interpreted as the rate paying off a new-building. As mentioned earlier, if the spot rates tend to be very low like in the periods from 1985-1987 then no new vessels will be contracted, since the capital costs will then not be covered.

New-building prices have been fluctuating greatly over the relevant periods. As seen in figure 4.8 although the relevant data available is only on an annual basis.




[Figure 4.8]

The second hand and scrap prices follow a similar pattern in that they also increase with the spot rates (Figure 4.9).




[Figure 4.9]

# 5 The Valuation of Operating Vessels

## 5.1 Value of a Time Charter

As mentioned earlier a time charter contract (T/C-contract) is an agreement where the shipowner sells to the charterer the right but not the obligation to operate a ship for a fixed period of time. The terms of the contract specify hat costs are to be paid by which party. The spot freight rate S(t) is assumed to be a time charter equivalent, and assumed to be paid continuously by the chartering party. If both the charterer and the shipowner are risk neutral, the time charter would be equal to the expected spot rates discounted by the risk free rate. In a world where the parties have different risk preferences the same relationship will hold after adjusting for the risk premium.

The value of a time charter is given by:

$$V_0[\int_0^T S(t)dt] = A(T, r+k)S(0) - B(T) \tag{5.1}$$

where A(T) and B(T) are defined as[13]

$$A(t) = \frac{1-e^{(r+k)T}}{r+k}$$

$$B(t) = \alpha * (\frac{1-e^{-(r+k)T}}{r+k} - \frac{1-e^{-rT}}{r})$$

They denote the risk adjusted annuity factors

[13] Trigoergis, Bjerksund and Ekern, p. 214

The value of the time charter as given by equation (5.1) is the integral of the spot rates from t=0 to t=T. This can be expressed as the spot rate multiplied by an annuity factor A(t) and less the risk adjusted equilibrium rate $\alpha*$ multiplied by the difference in the two annuity terms, one adjusted for k, less one term discounting with the risk free rate.

The risk adjusted equilibrium rate can be expressed as

$$\alpha^* = \alpha - \frac{\sigma\lambda}{k} \qquad (5.2)$$

where $\sigma$ is the standard deviation (in absolute figures) and $\lambda$ is the risk premium. Note that the equilibrium rate is equal to the risk adjusted equilibrium rate when investors are risk neutral, i.e. the risk premium $\lambda$ is 0.

## 5.2 Market Price of Risk

Generally traded securities like stocks, bonds and precious metals are held by several people solely for investment purposes. In contrary, most commodities are not traded securities and therefore there has to be made a distinction between the price behavior of underlying variables that define prices of traded securities and of those that are not traded. When an underlying variable is the price of the traded security, the risk neutral valuation shows that the investor is risk neutral concerning the relationship of the underlying variable and the price of the derivative. The risk is completely offset by the portfolio consisting of either a long position in the derivative and a short position in the security that both have equal value – or vice versa. Regarding non-traded securities the risk attitudes of an investor becomes important.

Since we can't use the risk-neutral valuation to price freight rates as it is in the case for tradable goods, we have to determine the market price of risk or a risk premium. The risk premium determines the how much investors are willing to pay above or below the risk free rate to hold a position in freight rates.

Considering a derivative that depends on the value of a single variable – in our case the underlying spot freight rate – that follows a Geometric Brownian motion:

$$\frac{dV}{V} = \alpha dt + \sigma dz \tag{5.3}$$

The differential equation for defining the derivative F is:

$$dF = \alpha * Fdt + \sigma Fdz \tag{5.4}$$

The risk adjusted expected growth rate of the underlying V is then defined as:

$$\alpha* = \alpha - \lambda\sigma \tag{5.5}$$

where λ is the market price of risk, σ the annualized volatility and α the expected growth rate of the underlying V.

In the case of freight futures contracts the expected payoff in an adjusted risk-neutral world then becomes:

$$F = e^{-r(T-t)}\hat{E}(F_T) \tag{5.6}$$

Where E denotes the expected value in a risk-neutral world, i.e. where the growth rate of the underlying is

$$F_T = Se^{(\alpha-\lambda\sigma)(T-t)}$$

To value a derivative, i.e. a futures contract, it is necessary to calculate the expected payoff in a risk-neutral world conditional on the particular path followed by r. Therefore the value today of the derivative, that pays off ft at a final time T is the future value discounted at r:

$$F = \hat{E}[e^{-\bar{r}(T-t)}(F_T)] \tag{5.7}$$

with r being the average risk-free interest rate between T and t.

An estimator of the market price of risk α* can be obtained by comparing futures prices with different maturity on the non-traded security at a specific date:

$$\alpha^* = \ln(\frac{F_{t1}}{F_{t2}}) \tag{5.8}$$

where λ, the market price of risk, can be calculated by the equation (5.5).

In the case of mean reverting processes the estimation of risk premiums is more difficult to undertake. I have therefore used data from relevant literature which estimates $\lambda = -0.114$.

## 5.3 Vessel Valuation

As mentioned in the introduction, purchase options are traded frequently in shipping industry. Charterers may charter vessels on a period time charter, and negotiate for a purchase option at the end of the charter period.

The value of a vessel is assumed to be a function of the spot rates. This means that the current value of a vessel can be expressed as a long bare-boat charter. Equation (5.1) provides the value of a time charter contract from t=0 to t=T. The valuation of a vessel will therefore be similar to a European call option: the value of a time charter less the present value of the future operating costs and less the present value of the expected second-hand or scrap value, which can be considered as the equivalent to the exercise price.

The time charter value is:

$$V_0[\int_0^T S(t)dt] = A(T, r+k)S(0) - B(T)$$

The difficulty of this valuation is the input of the optimal lifetime T of the option. As mentioned earlier one of the advantages of option theory is the flexibility that it provides. An owner will most likely scrap or lay up his vessel when the cost of operating the vessel exceeds revenues, even though the vessel may still be in condition to trade for a few more years.

### 5.3.1 Operating and Maintenance Costs

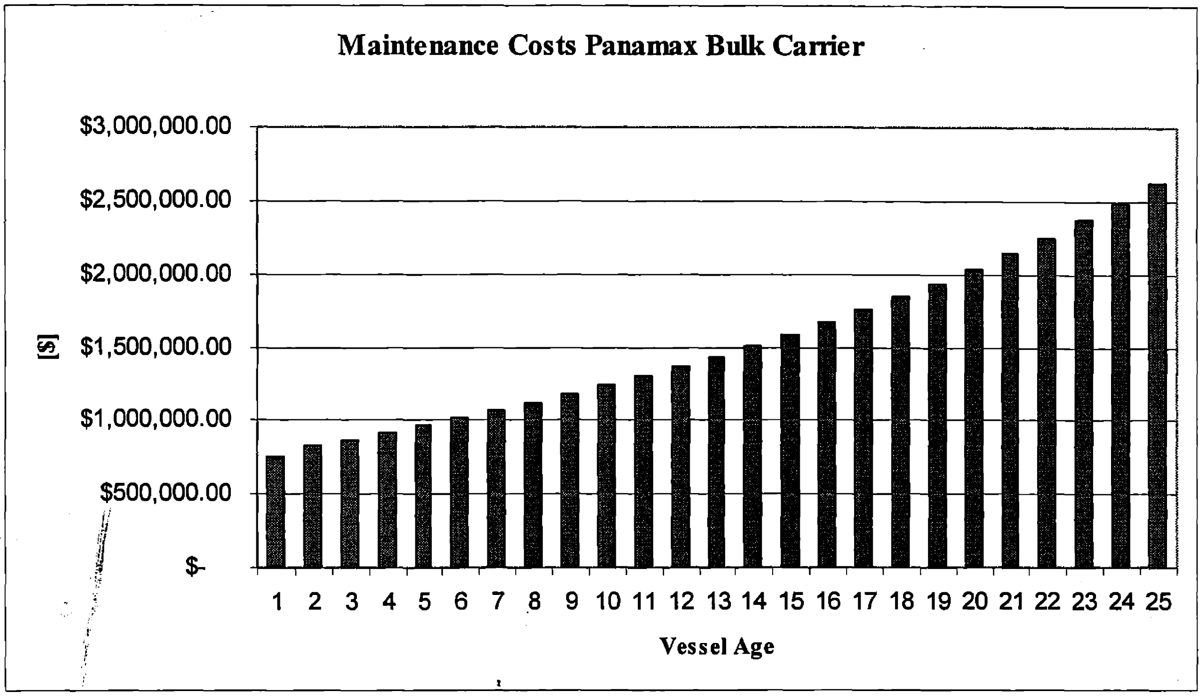
Many operators in the shipping industry claim that maintenance cost as a function of time, is exponential. The structure of the cost curve is important to the operators who are faced with an aging fleet. As their vessels become older, they will constantly have to make a decision whether to scrap the vessel or upgrade for further trading.

It is difficult to evaluate operating costs and to predict them. Overall costs will vary both with size and the age of the vessel.

For the underlying analysis an exponential growth rate for the cost structure over the life time of the vessel is chosen. Approximate maintenance cost (total operating cost less crew cost and insurance) including docking expenses is set at time t=0 to US$ 750'000.-. The cost is expected to grow annually at a rate of 5%.

Therefore the maintenance cost can be expressed as follows:

$MC = \$750'000 * e^{.05*T}$




[Figure 5.1]

The vessel life time is assumed to be 25 years.

## 5.3.2 Crew Cost

Crew cost is a major cost item, and many owners have identified this cost element as a deciding factor for flagging out. Some countries have recently given dispensations on manning, allowing owners to either replace national crew with cheaper third world labor or to offer their national ratings new contracts which transfer the burden of social security payments and tax to the employee. However, it is more common to take fuller advantage of the flexibility offered by an international register and employ a cheap crew from a low-wage economy.

The following table gives a range of monthly crew costs:

| | |
|---|---|
| 10 North European Officers/15 North European Ratings | US$ 65'000.- |
| 5 North European Officers/ 5 Filipino Officers/15 Filipino Ratings | US$ 39'000.- |
| 10 Filipino Officers/15 Filipino Ratings | US$ 23'500.- |

[Table 5.2]

The majority of shipowners sail their vessels under flag of convenience and operate their vessels with a combination of nationalities concerning their crew. Therefore, for the underlying analysis an average monthly crew cost of US$42'500.- will be assumed.

### 5.3.3 Insurance Costs

According to Tschudi&Eitzen the annual insurance coverage for a Panamax bulk vessel is approximately US$60'000.-. This is a rough figure, since many shipowners cover more than one vessel and discounts are made for an entire fleet coverage.

### 5.3.4 Interest

For the analysis it is assumed that the shipping company operating the vessel has a debt to equity ratio of 50 percent and a cost of debt of 5 percent per annum. The weighted average cost of capital is set to be 10 percent annually, which represents the relevant opportunity cost of capital.

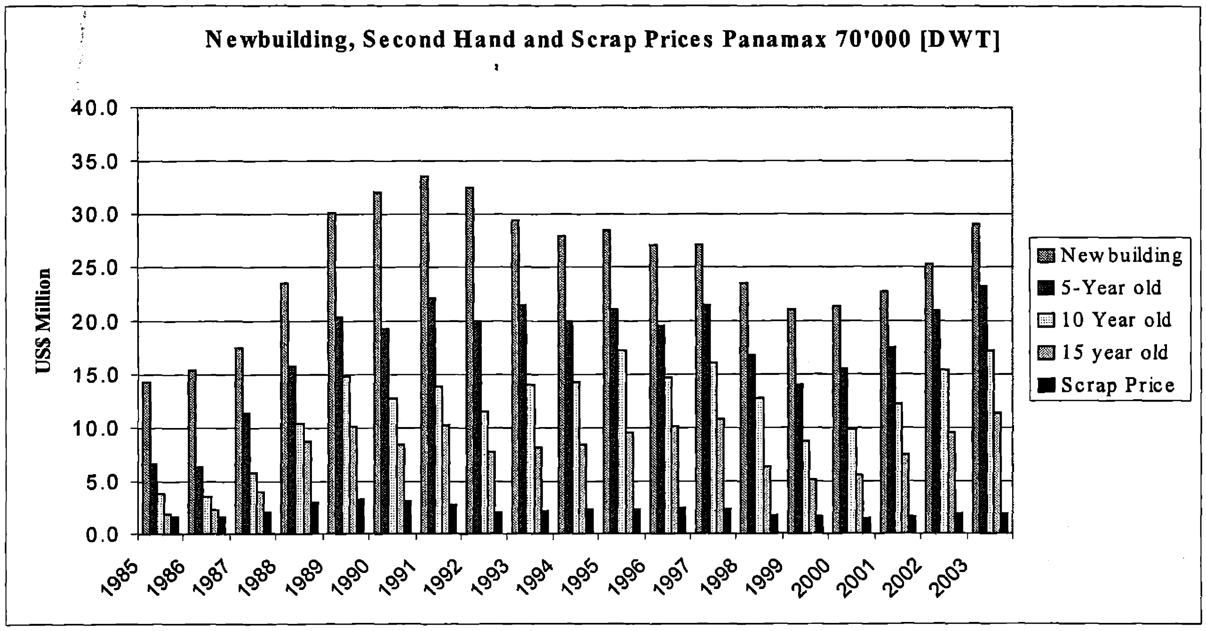
### 5.3.5 Depreciation

The vessel is assumed to be linearly depreciated over its lifetime of 25 years with a final scrap value of $ 1'600'000.- (1999 value taken as basis for calculations).

### 5.3.6 Secondhand and Scrap Values

As mentioned in the previous section, the relationship between earning capacity and cost levels becomes more critical as a vessel gets older. Operating costs rise, service speeds tend to fall and the incidence of both foreseen and unforeseen repair time increase. Therefore, as vessels become older the cost factor will play a more significant role in the scrapping decision. Scrapping is then likely to occur when net revenue (gross income – operating and maintenance cost) becomes negative.

The following chart displays the secondhand and scrap values for the 70'000 [dwt] Panamax vessel used for the analysis.kkkk




[Table 5.3]

### 5.3.7 Option Values

The second hand data available for the 70'000 [dwt] Panamax bulk carrier is for 5, 10, 15, and 25 years. The respective cases for charter purchase options for a 5, 10, 15, 25 year old vessel will be determined.

#### 5.3.7.1 Charter Purchase Option at 5 Years

The valuation of a charter purchase option will be based on the estimated parameters of Appendix 2. The market price of risk $\lambda$ is assumed to be -.114[14] and the vessel is evaluated considering a purchase after 5 years. The initial time charter equivalent spot rate is set to be \$11'000.-. The annual weighted average cost of capital is assumed to be 10 percent (WACC).

According to the estimated parameters the net revenues over 5 years are:

$$V_0[\int_0^{1825} S(t)dt] = (631.92 \times \$11'000 - (-20'975'590.83)) = \$27'926'735.38$$

In order to get the present value of the option the determination of the discounted cash-flows has to be undertaken. The cash flows are calculated according to the following relationship:

$$CF = \text{Revenues} - Costs - Interest - Depreciation$$

assuming zero taxes, no capital expenditures, and no changes in net working capital. The present value of costs associated with the vessel are determined as follows:

The present value of the maintenance costs discounted at 10% for 5 years is

[14] Bjerksund and Ekern

PV = $3,245,153.88

The present value of insurance cost, assuming a discount rate of 10%, for 5 years is

PV= $227,447.21

The present value of the crew cost, assuming a discount rate of 10%, is

PV= $1,933,301.25

The present value of the interest rate is discounted at 10% for 5 years is

PV = $651,967.39

The present value of depreciation is discounted at 10% for 5 years is

PV = $2,941,650.53

And the present value of the 5-years second hand price ($, 10% p.a. 5 years) is calculated to be

PV = $8,692,898.52

The value of the option therefore is the value of the time charter (revenues) less the total cost

**Option Value = $6,905,957.87**

### 5.3.7.2 Charter Purchase Option at 10 Years

The valuation of a charter purchase option will again be based on the estimated parameters of Appendix 2. All other parameters, such as risk premium, opportunity cost of capital, risk-free interest rate, and time charter equivalent spot rate are identical to the five year case.

According to the estimated parameters the net revenues over 10 years are.

$$V_0[\int_0^{3650} S(t)dt] = (674.71 \times \$11'000 - (-42'943'348.99)) = \$50'365'187.83$$

The present value of the maintenance costs discounted at 10% for 10 years is

PV = $5,860,317.08

The present value of insurance cost, assuming a discount rate of 10%, for 10 years is

PV= $368,674.03

The present value of the crew cost, assuming a discount rate of 10%, is

PV= $3,133,729.22

The present value of the interest rate is discounted at 10% for 10 years is

PV = $6,451,795.46

The present value of depreciation is discounted at 10% for 10 years is

PV = $4,768,184.07

And the present value of the 5-years second hand price (10% p.a. 10 years) is calculated to be

PV = $3,354,226.62

The value of the option therefore is the value of the time charter (revenues) less the total cost

**Option Value = $ 26,428,261.36**

### 5.3.7.3 Charter Purchase Option at 15 Years

Following the now familiar path, we may determine the value of an option with duration of 15 years by

$$V_0[\int_0^{5475} S(t)dt] = (677.61 \times \$11'000 - (-57'188'415.04)) = \$64'642'126.78$$

The value of the option therefore is the value of the time charter (revenues) less the total cost

**Option Value = $37,251,729.69**

### 5.3.7.4 Charter Purchase Option at 25 Years

Assuming holding an option over the vessel's entire lifetime (25 years) its value would then be

$$V_0[\int_0^{9125} S(t)dt] = (677.81 \times \$11'000 - (-71'593'589.56)) = \$79'049'605.71$$

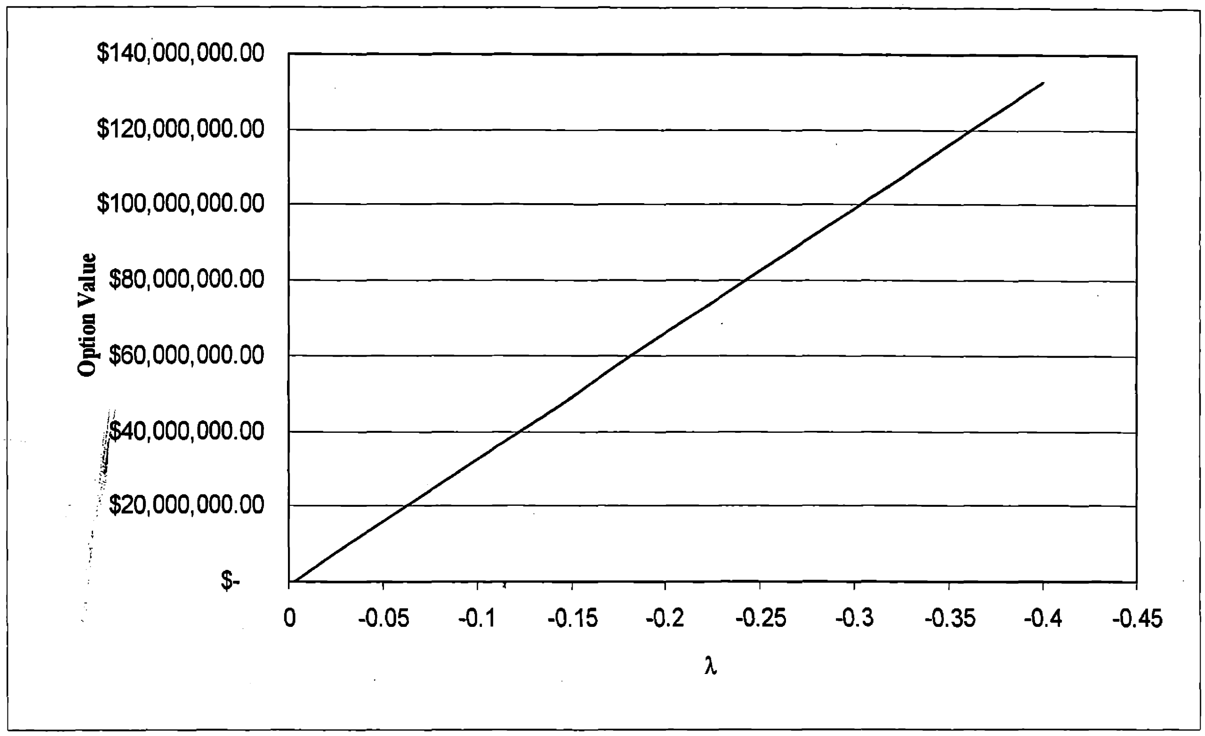
The value of the option therefore is the value of the time charter (revenues) less the total cost

**Option Value = $46,220,329.32**

### 5.3.8 Sensitivity Analysis of Options

Having analyzed the different possibilities of charter purchase options, it is now possible to undertake a sensitivity analysis and observe the parameters that affect the option value over its lifetime for the 15-year case.

#### 5.3.8.1 Changes in Market Price of Risk (λ)

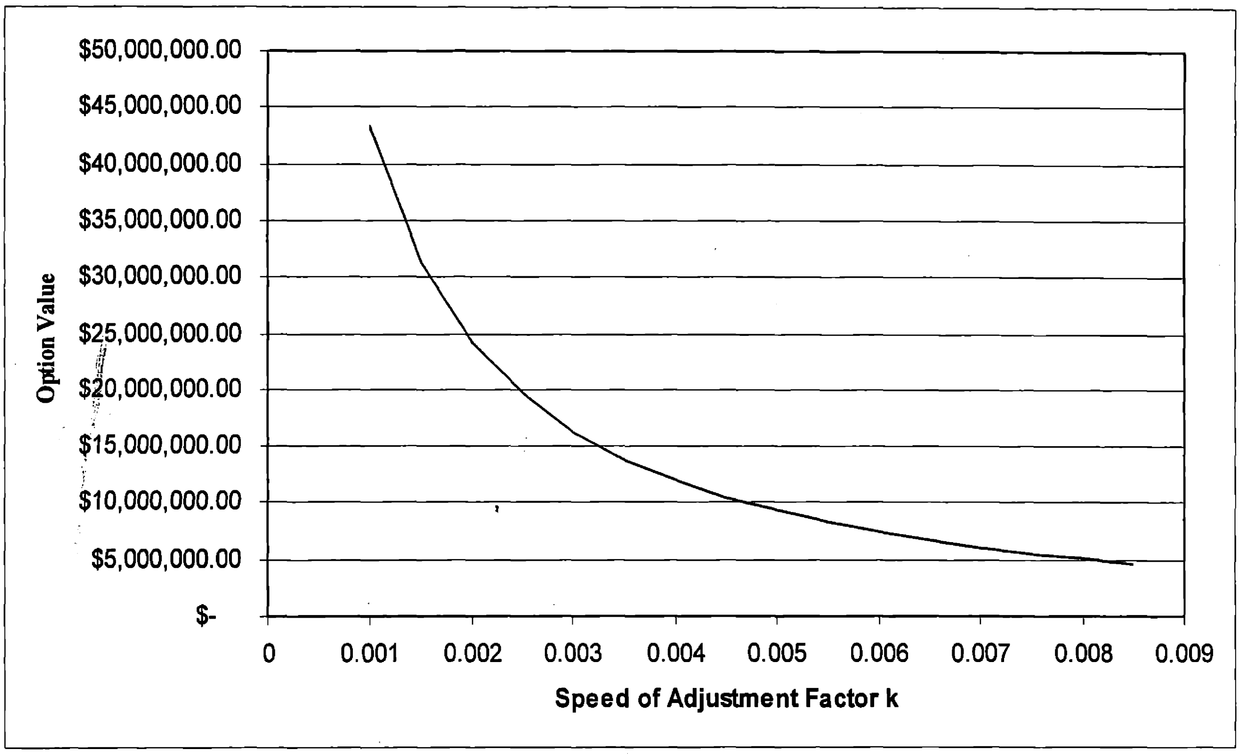



[Figure 5.4]

The market price of risk $\lambda$ was set to $-0.114$, according to Bjerksund and Ekern, for the determination of the option values on the Panamax bulk carrier. Figure 5.4 shows the influence of an increasing $\lambda$, which can be interpreted as an increase of the premium a shipowner will receive for holding a vessel in a highly volatile market. The higher market price of risk, the more the adjusted mean α* will increase and therefore the cash-flows for the owner of the ship will be adjusted upwards.

### 5.3.8.2 Changes in Speed of Adjustment Factor (k)

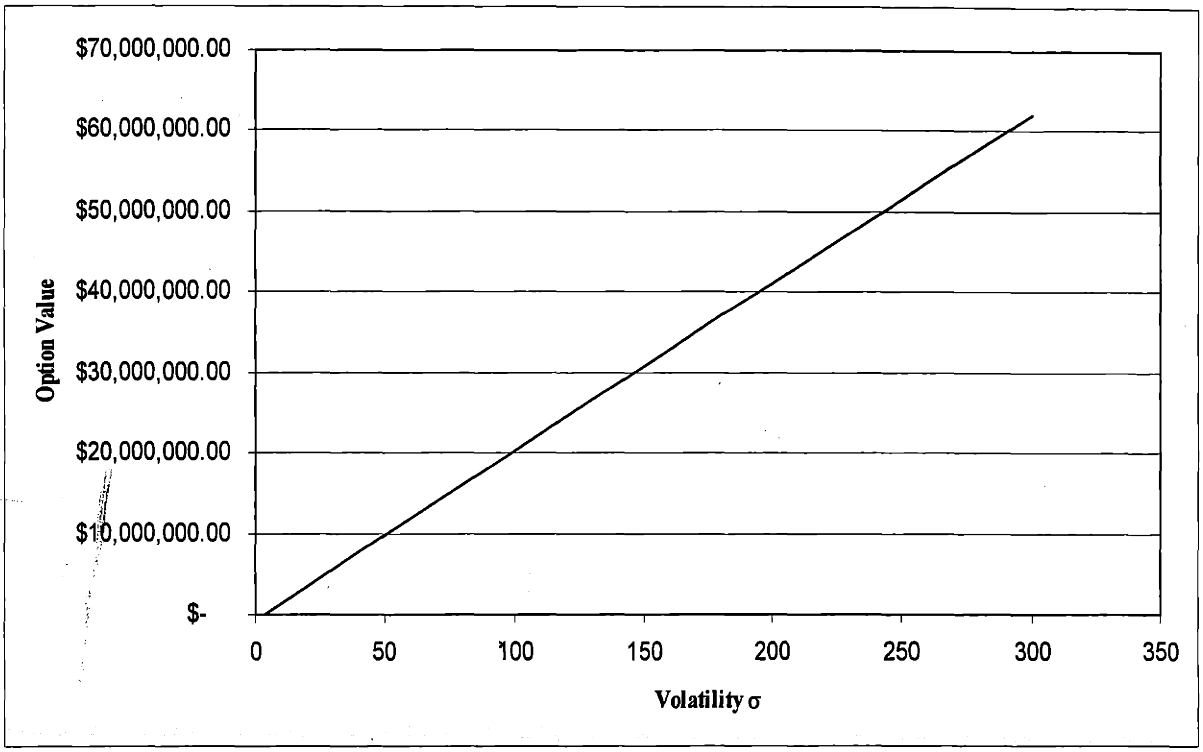
Figure 5.5 shows the change in the speed of adjustment factor k and its effect on the option value. As can be seen, an increase of k leads to a significant decrease in the option value, which converges towards zero.




[Figure 5.5]

### 5.3.8.3 Changes in Volatility (σ)

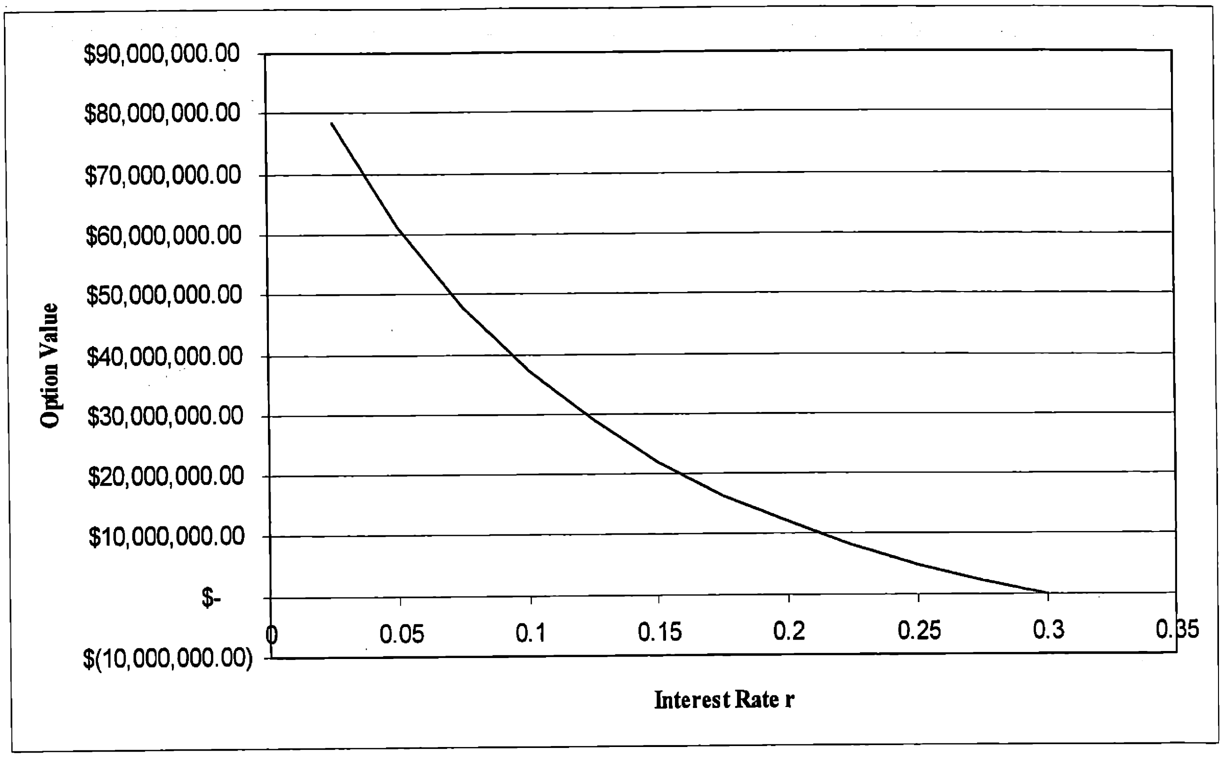
The modeled case of a purchase option on the described vessel assumes no changes in volatility of the T/C-equivalent spot rates, which, in reality, is never the case. The following figure shows the change in the option value as an approximately linear function of the changes in volatility.




[Figure 5.6]

### 5.3.8.4 Changes in Interest Rates

The change of interest rates does have an impact on the option value since the contract has a duration of several years. Figure 5.7 shows the change in the option value as a function of the change in the interest rate.


$90,000,000.00
$80,000,000.00
$70,000,000.00
$60,000,000.00
$50,000,000.00
$40,000,000.00
$30,000,000.00
$20,000,000.00
$10,000,000.00
$-
$(10,000,000.00)
Option Value
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
Interest Rate r

[Figure 5.7]

# 6 Conclusions

The empirical findings of the underlying analysis indicate that a mean-reverting process is clearly more accurate in capturing the stochastic nature of time charter equivalent spot rates as shown in figure 4.7. A correctly determined freight rate process has major implications on the valuation of options on operating vessels.

The underlying valuation method, using an Ornstein-Uhlenbeck process, neglects the fact that the spot rates might fall below a certain level resulting in losses by operating the vessel. In such a case the vessel would be laid up or, given a certain age, even scrapped. This in a sense “invalidates” the application of options over periods longer than one to two years, since the prediction of freight rates is an impossible task.

Taking the possibility of low freight rates into account, a more accurate model would have to consider thresholds where the ship is laid up and thresholds where it is again reactivated, involving the rather significant layup and reactivating costs. However, this will lead to more complicated mathematical models where no closed form solutions are available and therefore would have to be solved using a numerical approach.

A major drawback applying the Ornstein-Uhlenbeck process to model freight rates is that the model has no downside limitations, e.g. freight rates may adopt negative values. This problem could be resolved by using a geometric mean reverting model that prevents the freight rates from falling below zero.

The analysis is reduced to a single vessel and a single BFI route due to the fact that this was the only consistent data available. It would therefore be interesting to conduct future research for other vessel classes such as Capesize, and Handymax. Furthermore, a challenging project could be the extension of the analysis to the tanker segment, which represents a large and competitive market driven by even more micro- and macroeconomic parameters.

My discussion with major players within the shipping industry revealed that there are few companies, who understand and recognize the importance of the contingent claim analysis and the options theory. It seems difficult for many of them to capture the general understanding of terms such as volatility, market price of risk and as a consequence their impact on an option value. The volatility of the business and the time span over which options are currently signed, indicate that the application of options to assets such as vessels may be extremely valuable as a strategic decision tool. Hopefully this study will be a step to broaden the knowledge of option theory within the shipping community and support major players in their strategic management decisions.

# Appendix 1

## The Baltic Freight Index Routes

The Baltic Freight Index (BFI) is an index (rather like the Financial Times Index) of voyage and time charter rates in the dry bulk shipping market. It is composed of voyage/timecharter rate assessments averaged and weighted according to a pre-determined formula.

**Route 1**

1 Port US Gulf/Antwerp, Rotterdam, Amsterdam, 55,000 long tons 10 per cent light grain stowing 55ft., free in and out. 10 days Sundays, holidays excepted. Laydays 10 days forward from date of Index, canceling maximum 30 days forward from date of Index. 3.75 per cent total commissions.

Nominal Weighting =10%

**Route 1A**

Basis a 70,000 deadweight Baltic Panamax aged not over 15 years with a 3.0 million cu.ft. grain, LOA maximum 230m and capable of about 14 knots laden on 30 fuel oil and no diesel at sea, for a transatlantic round of 45/60 days on the basis of delivery and redelivery Skaw/Passero range. Loading 15-20 days ahead in the loading area. Cargo basis grain, ore or coal, or similar. 3.75 per cent total commissions.

Nominal Weighting =10%

**Route 2**

1 Port US Gulf/1 no combo port South Japan, 52,000 long tons 5 per cent heavy soya sorghum, free in and out 11 days Sundays, holidays excepted. Laydays 10 days forward from date of Index, canceling maximum 30 days forward from date of Index. 3.75 per cent total commissions.

Nominal Weighting =10%

**Route 2A.**

Basis a 70,000 deadweight Baltic Panamax aged not over 15 years with a 3.0 million cu.ft. grain, LOA maximum 230m and capable of about 14 knots laden on 30 fuel oil and no diesel at sea, basis delivery Skaw/Passero range for a trip via US Gulf to the Far East, redelivery Taiwan/Japan range, duration 50/60 days. Loading 15-20 days ahead in the loading area. Cargo basis grain, ore or coal, or similar. 3.75 per cent commissions.

Nominal Weighting =10%

**Route 7.**

Port Hampton Roads excluding Baltimore/Rotterdam,110,000 tonnes 10 per cent coal, free in and out and trimmed 35,000 tonnes, Sundays, holidays included loading/25,000 tonnes Sundays holidays included discharge. Laydays 10 days forward from date of Index, canceling maximum 30 days forward from date of Index. 3.75 per cent total commissions.

Nominal Weighting =7.5%

**Route 9.**

Basis a 70,000 deadweight Baltic Panamax aged not over 15 years, with a 3.0 million cu.ft. grain, LOA maximum 230m and capable of about 14 knots laden on 30 fuel oil and no diesel at sea, delivery Japan/Korea range. for a trip via US West Coast/British Columbia range, redelivery Skaw/Passero range, duration 50/60 days. Loading 15/20 days ahead in the loading area. Cargo basis grain, coal, petcoke or similar. 5 per cent total commissions.

Nominal Weighting =10%

**Route 10.**

Tubarao/Rotterdam, 150,000 long tonnes 10 per cent more or less iron ore, 6 days Sundays, holidays included, laydays 15 days forward from date of Index, canceling maximum 30 days forward from date of Index. 3.75 per cent total commissions.

Nominal Weighting =7.5%

**Route 14.**

Tubarao/Beilun and Baoshan, 140,000mt 10 per cent iron ore 18m swad, scale load/30,000mt shinc discharge. Laydays 25 days forward from date of index, canceling maximum 40 days forward from date of index. 3.75 per cent total commission.

Nominal Weighting =7.5%

**Route 15.**

Richards Bay/Rotterdam 140,000mt 10 per cent coal, scale load/40,000 mt shinc discharge. Laydays 25 days forward from date of index, canceling 40 days forward from date of index. 3.75 per cent total commission.

Nominal Weighting =7.5%

The weighting of each route is subject to continuous change. Unfortunately BIFFEX doesn't publish the relationship on how the total index is calculated based on the indices of each specific route.

# Appendix 2

Descriptive statistics for the T/C-equivalent spot rates based on daily data reveal the following results:

| | Mean | Median | StDev | Min | Max |
|---|---|---|---|---|---|
| **1985-1998** | 8423.30 | 8376.79 | 3604.98 | 115.23 | 18994.86 |
| **1985-1987** | 3941.99 | 4177.08 | 1859.33 | 8704.06 | 2625.90 |
| **1988-1990** | 9377.99 | 9434.44 | 2179.58 | 3449.19 | 13441.15 |
| **1991-1993** | 9830.80 | 9658.32 | 2364.46 | 5643.89 | 15784.73 |
| **1994-1996** | 11336.81 | 11132.52 | 3303.19 | 5295.70 | 18994.86 |
| **1997-1998** | 7195.49 | 7383.27 | 2326.94 | 3735.94 | 12813.89 |

The regression analysis based on daily T/C-equivalent spot rates to determine the parameters for the Geometric Brownian motion yielded to the following results:

| | N | $\beta 1$ | t | $\sigma$ | $R^2$ |
|---|---|---|---|---|---|
| **1985-1998** | 3527 | 0.4045 | 26.2620 | 0.0369 | 0.1630 |
| **1985-1987** | 751 | 0.3946 | 11.7500 | 0.0716 | 0.1550 |
| **1988-1990** | 756 | 0.3329 | 9.6968 | 0.0244 | 0.1100 |
| **1991-1993** | 759 | 0.3732 | 11.0600 | 0.0202 | 0.1392 |
| **1994-1996** | 756 | 0.6882 | 26.0496 | 0.0116 | 0.4736 |
| **1997-1998** | 505 | 0.6569 | 19.5100 | 0.0127 | 0.4309 |

The regression analysis based on daily T/C-equivalent spot rates to determine the parameters for the mean reverting process (Ornstein-Uhlenbeck) yielded to the following results:

| | N | αk | K | α | σ |
|---|---|---|---|---|---|
| **1985-1998** | 3528 | 10.0350 | 0.0012 | 8264.9933 | 181.2820 |
| **1985-1987** | 752 | 4.8212 | 0.0003 | 18601.0713 | 153.3866 |
| **1988-1990** | 756 | 48.0592 | 0.0047 | 10331.6751 | 221.0829 |
| **1991-1993** | 759 | 34.2153 | 0.0040 | 8553.6395 | 215.7273 |
| **1994-1996** | 756 | 27.6706 | 0.0018 | 15002.2037 | 160.7102 |
| **1997-1998** | 505 | 25.2837 | 0.0057 | 4403.5091 | 111.5950 |

All calculations referring to option valuations on vessels were based on the data provided by the table containing daily mean reverting parameters.

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