

Service Network Design Optimization for Army Aviation Lift Planning

by

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B.S., Computer Science
United States Military Academy, 2004

Submitted to the Department of Civil and Environmental Engineering
in partial fulfillment of the requirements for the degree of

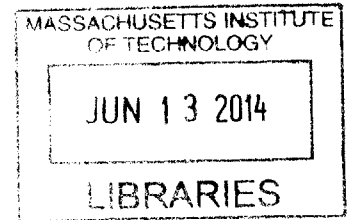
Master of Science in Transportation

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Abstract

The need for optimized aviation lift planning is becoming increasingly important as the United States and her allies participate in the Global War on Terror (GWOT). As part of a comprehensive effort, our nation's fighting forces find themselves conducting operations around the globe, with this trend likely to increase, even as budget constraints limit the number of personnel and amount of equipment that is deployed. While much attention has been given to airline schedule optimization and fleet planning, the challenge of Army Aviation lift planning is unique in that it must be able to adapt to changing requirements and missions on a daily basis. In this thesis, we model Army Aviation lift planning as a service network design problem, and propose two heuristic algorithms, which compare favorably to current human planning systems. Furthermore, we apply these heuristic algorithms to long term asset planning and capacity requirement estimation for future military scenarios, and analyze how passenger flexibility affects the need for capacity.

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Contents

1	Introduction	19
1.1	Research Motivation	19
1.2	Overview of Thesis	20
2	Operational Analysis and Problem Description	23
2.1	History and Organization of U.S. Army Aviation	23
2.2	Army Aviation Missions	26
2.3	Lift Mission Planning and Execution	27
2.3.1	Air Mission Requests	27
2.3.1.1	The Air Mission Request Form	27
2.3.1.2	AMR priorities	31
2.3.1.3	AMR Form Routing	31
2.3.2	Flight Schedule Planning	33
2.3.2.1	Planning Horizon	33
2.3.2.2	Aircraft Availability and Planning Characteristics	34
2.3.3	Tactical Route Planning	35
2.3.4	Mission Execution	35
2.4	Problem Motivation	36
2.5	Problem Statement	36
3	Model Inputs and Network Structure	37
3.1	Aircraft Teams	37
3.1.1	Aircraft Team Constraints	37

3.1.2	Aircraft Team Parameters	38
3.2	Air Mission Requests	39
3.2.1	Decision Window Model	39
3.2.2	AMR Parameters	40
3.3	Network Construction	41
3.3.1	Inter-Base Transport Model	41
3.3.1.1	Static Graph	41
3.3.1.2	Time Periods	42
3.3.1.3	Time-Space Graph	42
3.3.2	Network Inputs	46
4	Model Formulation	49
4.1	Literature Review	49
4.1.1	Multi-Commodity Network Flow Problem	49
4.1.1.1	Arc-Flow Formulation	50
4.1.1.2	Path Flow Formulation	50
4.1.2	Network Design Problem	51
4.2	Formulation as a Service Network Design Problem	55
4.2.1	Decision Variables	55
4.2.2	Assumptions and Constraints	57
4.2.3	Objective Function	58
4.2.3.1	AMR Value	58
4.2.3.2	Aircraft and Passenger Costs	59
4.2.3.3	Objective Function Formulation	60
4.2.4	SNDP _{OPT} Model Formulation	62
4.3	Path-Flow Heuristic using Composite Variables	64
4.3.1	Composite Variable Formulation	65
4.3.2	Path Generation Heuristic Algorithm	66
4.3.2.1	Algorithm Overview	66
4.3.2.2	Algorithm Description	69

4.3.2.3	Path Reduction Techniques	70
4.3.2.4	Important Algorithm Property	71
4.3.3	Set-Packing Problem Formulation	71
4.3.4	Maximum Total Return Algorithm	73
4.3.5	Maximum Marginal Return Algorithm	74
4.3.6	Analytic Solution Comparison	76
4.4	Chapter Summary	76
5	Computational Analysis	79
5.1	User-Defined Parameters	79
5.1.1	Fixed Parameters	79
5.1.2	Variable Parameters	80
5.1.3	Demand Distribution	81
5.2	Sensitivity Analysis	82
5.2.1	Test 1: Increase in Network Size	82
5.2.2	Test 2: Increase in Number of Aircraft Teams	84
5.2.3	Test 3: Increase in Number of AMR legs	86
5.2.4	Test 4: Increase in Number of Passengers per AMR	87
5.2.5	Test 5: Increase in Schedule Tolerance per AMR	88
5.2.6	Summary and Discussion	89
6	Operational Analysis	93
6.1	Comparison Across All Models	93
6.1.1	Test Scenarios	93
6.1.2	Results and Discussion	94
6.2	Operational Scenarios Description	95
6.2.1	Scenario 1	96
6.2.2	Scenario 2	98
6.2.3	Aircraft and Passenger Data	101
6.2.4	Summary of Aircraft Team and AMR Parameters	103
6.3	Heuristic Model Comparison	104

6.3.1	Objective Function Value vs. AMR Legs Requested	104
6.3.2	AMR legs Supported vs. AMR Legs Requested	106
6.3.3	Average Flight and Turn Time per AMR Leg:	108
6.3.4	Average Delay per AMR Leg vs. Number of Supported AMR Legs	110
6.4	Model Comparison to Human Planner	111
6.5	Model Application to Lift Planning	113
6.5.1	Short-Term Planning	113
6.5.2	Long-Term Planning	114
6.5.2.1	Support Level Estimation	114
6.5.2.2	Capacity Requirement Planning	116
6.6	Impact of Increased Schedule Tolerance	119
7	Summary and Future Research	123
7.1	Summary of Contributions and Results	123
7.2	Future Research	125
	Appendix A: Path Generation Heuristic Algorithm Pseudo-Code . .	127
	Appendix B: Sets, Parameters, and Variables	133
	Appendix C: Abbreviations and Acronyms	141
	Bibliography	145

List of Figures

2-1	Full Spectrum CAB	25
2-2	Possible organization of a Combined Arms Brigade Task Force	26
2-3	Example blank AMR form [14]	29
2-4	Example of completed AMR form	30
2-5	The AMR process	32
2-6	AMR approval and routing for Division aviation assets	33
2-7	Planning and execution horizon	34
2-8	Example steady state	35
3-1	Decision window (Boeing [9])	39
3-2	AMR and AMR leg sets	40
3-3	Static graph representation of a network of four HLZs	42
3-4	Flight arc between two nodes in a time-space graph	43
3-5	Ground arc between two nodes in a time-space graph	44
3-6	Aircraft team path in $G(N, A)$ from node $(A, 0)$ to node $(B, 4)$	45
3-7	AMR flow in $G(N, A)$ from node $(A, 0)$ to node $(C, 2)$	45
3-8	Aircraft team path in $G(N, A)$ from node $(A, 0)$ to node $(B, 4)$, with AMR flow	45
4-1	Conversion from AMR priority to value	59
4-2	Example showing the generation of feasible paths	67
4-3	Example showing a feasible path	68
4-4	Comparison of modeling approaches	77

5-1	Computational performance vs. network size	82
5-2	SNDP _{OPT} tractability region ($ F = 1, K = 3$)	84
5-3	Run-time vs. number of identical aircraft teams	85
5-4	Run-time vs. number of different aircraft teams	85
5-5	Run-time vs. number of AMR legs (one aircraft team)	86
5-6	Run-time vs. number of AMR legs (two aircraft teams)	87
5-7	Run-time vs. number of passengers per AMR	88
5-8	Run-time vs. schedule tolerance	89
6-1	Test scenario static graph	94
6-2	Test scenario histogram of supported AMR legs	95
6-3	Operational Scenario 1: satellite overview	96
6-4	Operational Scenario 1: Flight time diagram	97
6-5	Operational Scenario 2: Satellite overview	99
6-6	Operational Scenario 2: Flight time diagram	99
6-7	Steady state support diagram	102
6-8	Probability density function of passenger demand	102
6-9	Objective function value vs. requested AMR legs	105
6-10	Impact of capacity and AMR value function on the solutions of MTRA and MMRA	106
6-11	Number of supported AMR legs vs. number of requested AMR legs .	107
6-12	Impact of capacity and AMR value on relative performance	108
6-13	Average flight and turn time per AMR leg vs. number of supported AMRs	109
6-14	Average AMR delay vs. number of supported AMR legs	110
6-15	Algorithm use as decision and planning support tool	114
6-16	Effect of demand on support level	115
6-17	Estimating support level	115
6-18	Aircraft team steady state	117
6-19	Effect of capacity on support level	118

6-20	Estimating required capacity	118
6-21	Comparing increased schedule tolerance and capacity	119
6-22	Effect of schedule tolerance on support level	120

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List of Tables

2.1	U.S. Army Aviation aircraft	24
2.2	Example Commander's Mission Priority List (CMPL)	31
3.1	Aircraft team parameters	38
3.2	DWM application to the AMR leg travel window	39
3.3	AMR parameters	41
4.1	Composite variable parameters	65
5.1	Fixed parameters	80
5.2	Variable parameters	81
5.3	Randomly generated AMR parameters	81
5.4	Test 1 variable parameters	82
5.5	SNDP _{OPT} computational statistics	83
5.6	Test 2 variable parameters	84
5.7	Test 3 variable parameters	86
5.8	Test 4 variable parameters	87
5.9	Test 5 variable parameters	88
5.10	Computational analysis results	90
6.1	Test scenario parameters	94
6.2	Average number of supported AMRs	95
6.3	Operational Scenario 1: Inter-HLZ flight time (min)	97
6.4	Operational Scenario 1: Base and HLZ configuration	98
6.5	Operational Scenario 2: Inter-HLZ flight time (min)	100

6.6	Operational Scenario 2: Base and HLZ configuration	101
6.7	Aircraft team and AMR parameters	103
6.8	Value function comparison	105
6.9	Heuristic vs. manual solutions	112
6.10	Model Comparison	113
6.11	AMR spill table	116

List of Algorithms

1	Path Generation Heuristic Algorithm (PGHA)	69
2	Maximum Total Return Algorithm (MTRA)	74
3	Maximum Marginal Return Algorithm (MMRA)	75

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Chapter 1

Introduction

1.1 Research Motivation

Army Aviation lift aircraft are necessary to maintain a flexible and dynamic transportation system for contingency operations around the world. As fewer resources constrain our ability to sustain support operations, we must maximize utilization efficiency of aircraft through proper planning. Furthermore, we must be able to adapt to a wide range of geographic locations and missions, while operating in these changing environments.

The author applies his operational experience as an aviation planner at the Army company, battalion, and division levels to the problem of Air Mission Request (AMR) scheduling and aircraft routing, as applicable to a future military scenario.

Two key ideas motivate the research in this thesis. First, it is fairly difficult and time-consuming for a human planner to provide a near-optimal flight schedule, especially if the number of aircraft and requested missions is large. Second, a human planner has only limited capability for assessing the current maximum passenger support level, and for recommending a required aircraft capacity based on a desired support level.

1.2 Overview of Thesis

Chapter 2: *Operational Analysis and Problem Description:* We introduce the reader to Army Aviation and motivate the problem addressed in this thesis. We look at the organization of operational units, their interaction, and how air transportation missions are supported. We present challenges that are currently faced by aviation planners, and motivate the problem of flight scheduling and route planning concerning helicopter lift aviation and Air Mission Requests (AMR).

Chapter 3: *Model Inputs and Network Structure:* We model the geographical layout of Helicopter Landing Zones (HLZ), aircraft characteristics, and passenger demand. Furthermore, we model the flight schedule and aircraft routing information as a time-space graph where aircraft movement is represented by arcs and passenger movement is represented by flow over those arcs.

Chapter 4: *Model Formulation:* We conduct a literature review of relevant network flow formulations, and model the problem of AMR scheduling and aircraft routing as a deterministic Service Network Design Problem (SNDP). We first present an integer programming approach to this problem, SNDP_{OPT} . We then present two composite variable heuristic algorithms: the Maximum Marginal Return Algorithm (MMRA), and the Maximum Total Return Algorithm (MTRA).

Chapter 5: *Computational Analysis:* We compare the run-time of the three models (SNDP_{OPT} , MMRA, MTRA) when different parameters affecting problem size are changed, and show that large operational problems are intractable using SNDP_{OPT} . The heuristic algorithms we propose, however, are tractable and much faster, even when the number of HLZs or time periods considered is large.

Chapter 6: *Operational Analysis:* We use both heuristic algorithms to generate the flight schedule and aircraft routing information for two large operational scenarios, over varying levels of demand, and compare their performance to that of a human

planner. We then look at how these heuristic algorithms can be used to estimate the level of support that can be provided to units, given a fixed number of aircraft, and an estimate of demand. We also look at how they can be used to estimate aircraft requirements based on a desired level of support. Finally, we look at the impact of passenger time-of-day travel flexibility on the number of missions and passengers that can be supported with a fixed number of aircraft.

Chapter 7: *Summary and Future Research:* We summarize the work and contributions of this thesis, and propose several areas of future research.

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Chapter 2

Operational Analysis and Problem Description

2.1 History and Organization of U.S. Army Aviation

Army Aviation is a Maneuver, Fires, and Effects (MFE) branch of the U.S. Army. Originally designated as the U.S. Army Air Corps in 1926, it was re-designated the U.S. Army Air Forces in 1941. In 1946, the Air Force was created and U.S. Army Aviation became a branch of the U.S. Army [14]. Currently, the U.S. Army employs a variety of manned aircraft, both rotary-wing (helicopters) and fixed-wing (planes) (Table 2.1).

Most Army rotary wing aviation units are now organized into Combat Aviation Brigades (CAB), with the exception of some legacy units, training support units, and special operations aviation units. These CABs are generally assigned to an army division. CABs may be designated in two ways: *heavy or full spectrum*. Full Spectrum CABs have one Attack Reconnaissance Battalion (ARB) of AH-64 Apaches, one Attack Reconnaissance Squadron (ARS) of OH-58 Kiowa Warriors, one Assault Helicopter Battalion (AHB) of UH-60 Blackhawks, and one General Support Aviation Battalion (GSAB) containing UH-60 Blackhawks, HH-60 MEDEVAC, and CH-47 Chinooks. Heavy CABs are similar to full spectrum CABs, except they have two ARBs instead of an ARB and an ARS.

Aircraft	Type	Uses
UH-60 Blackhawk	Rotary wing	Utility / multipurpose
UH-72 Lakota	Rotary wing	Utility / multipurpose
UH-1 Huey	Rotary wing	Utility / multipurpose
HH-60 MEDEVAC	Rotary wing	Medical evacuation
CH-47 Chinook	Rotary wing	Cargo / multipurpose
OH-58 Kiowa Warrior	Rotary wing	Attack/reconnaissance
AH-64 Apache	Rotary wing	Attack/reconnaissance
AH-6 Little Bird	Rotary wing	Special operations
MH-60 Blackhawk	Rotary wing	Special operations
MH-47 Chinook	Rotary wing	Special operations
TH-67 Creek	Rotary wing	Trainer
C-12 Huron	Fixed wing	Cargo/transport
RC-12 Huron	Fixed wing	Reconnaissance
C-20 Gulfstream	Fixed wing	Cargo/transport
C-37 Gulfstream	Fixed wing	Cargo/transport
UC-35 Cessna	Fixed wing	Cargo/transport

Table 2.1: U.S. Army Aviation aircraft

In addition to aircraft battalions and squadrons, each CAB has an Aviation Support Battalion (ASB), which augments the maintenance capabilities of its sister battalions and squadrons, and may also have an Air Traffic Services (ATS) Battalion and an Unmanned Aerial Vehicle (UAV) Company. Currently, the active duty Army has eleven divisional CABs and two separate CABs (assigned directly to an Army Corps rather than an Army Division). Figure 2-1 shows a possible organization of units in a Full Spectrum CAB.

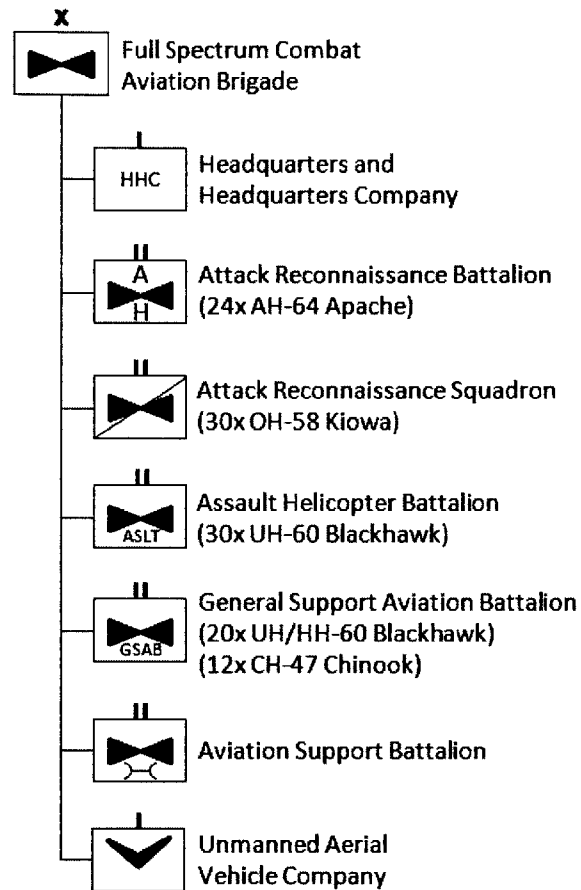


Figure 2-1: Full Spectrum CAB

In order to properly equip units for decentralized operations, CABs can *task organize* their subordinate battalions, giving each one the ability to conduct full spectrum operations. This means that within a brigade, aviation companies are cross-loaded to provide every battalion with a similar set of aircraft and capabilities. The resulting battalions are called *Task Forces* (TF), where each Aviation Task Force has the ability to conduct lift, attack and reconnaissance missions in their assigned areas of operation.

In the operational scenarios considered in this thesis, a brigade or battalion-sized Combined Arms Task Force is geographically spread out over its area of operations. Its subordinate units include several task forces, to include a Full Spectrum Aviation Task Force (Figure 2-2).

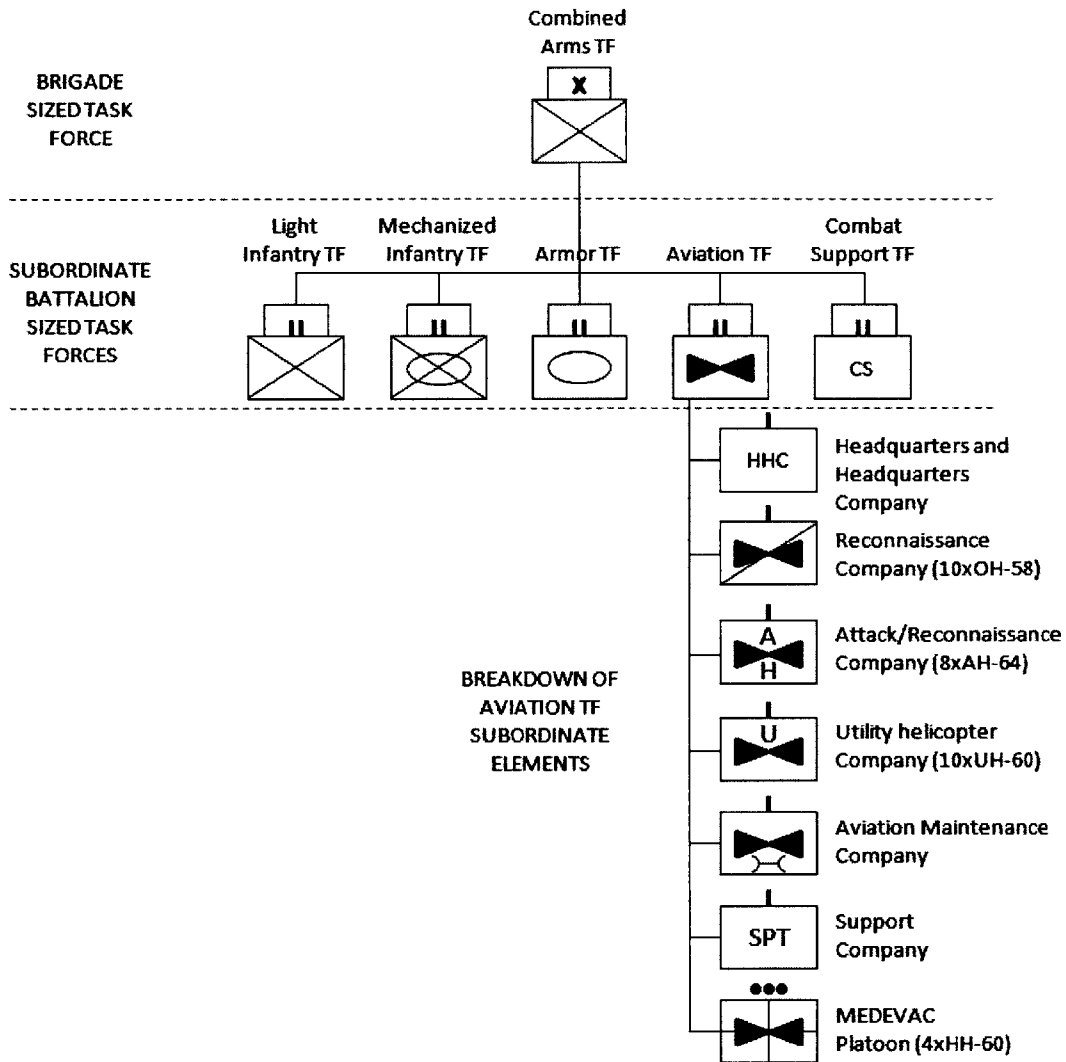


Figure 2-2: Possible organization of a Combined Arms Brigade Task Force

2.2 Army Aviation Missions

Rotary wing assets are unique in that they have a broad spectrum of missions which support the commander's intent. These missions are dictated by unit capabilities and also the current needs of the commander. To ensure that units are meeting training requirements for these missions, all Army units have a Mission Essential Task List (METL) which describes proficiency requirements for individual and collective tasks.

In Army Aviation, the METL is specific to each type of aircraft. AH-64s and OH-

58s conduct attack and reconnaissance missions, convoy escort, and other missions suited to their capabilities. They may also act as an escort to utility (UH-60), cargo (CH-47), and MEDEVAC (HH-60) helicopters.

Utility and Cargo helicopters are considered lift assets since they can carry passengers and cargo. Like attack and reconnaissance helicopters, they are versatile and multi-mission platforms. They are capable of being used in many ways to accomplish a vast array of missions, not simply logistics or troop transport missions. As an example, these units are capable of conducting air assaults, reconnaissance, Signal Intelligence (SIGINT) collection and Command and Control (C2).

In this thesis, we consider logistical transport missions of personnel throughout the area of operation, conducted by utility and cargo helicopters.

2.3 Lift Mission Planning and Execution

In a distributed operations environment, units operate from Forward Operating Bases (FOB) or Combat Outposts (COP). Each FOB or COP usually has a Helicopter Landing Zone (HLZ). An HLZ does not have to be inside a base, and can be designated anywhere. Commanders must generally approve an HLZ before it can be used by ensuring it meets minimum safety requirements, such as distance from obstacles and slope conditions. In an emergency or other unplanned situation, however, it is at the discretion of the Pilot-in-Command (PC) to pick a suitable landing area. The subsection below takes the reader through the process of lift aviation planning as is currently done in a deployed environment, from the submission of a request to the execution of the flight.

2.3.1 Air Mission Requests

2.3.1.1 The Air Mission Request Form

To streamline the request and approval process for Army Aviation lift assets, units must submit an Air Mission Request (AMR) whenever air support is requested. An *AMR* is a paper or electronic form containing information about the requesting unit,

the type of aviation support requested, and all pertinent details necessary for the accomplishment of the mission, such as number of passengers and legs requested.

An *AMR leg* is an individual flight segment of an AMR. It specifies an origin HLZ where the passengers will be picked up and a destination HLZ where the passengers will be dropped off. AMRs may be composed of multiple AMR legs. Multi-leg AMRs have more than one leg for the same day. This could be a round trip, or a multi-stop mission.

Typical information submitted on the AMR is the following, and an example AMR form is shown in Figure 2-3 [14].

- Requesting unit point of contact (POC) and contact information
- Mission description
- Requesting date of travel
- Number of legs, and, for each leg:
 - departure HLZ, and earliest departure time
 - arrival HLZ, and latest arrival time
 - number of passengers
 - amount of cargo (number of pieces, weight, and dimensions)
- Special instructions

AIR MISSION REQUEST (AMR)			
Section 1 - Completed by Requesting Unit/Agency (* Required Entry)			
*Unit/Agency:		*Date of Request:	
POCInfo	Rank/Name	Phone	Email
*Primary			
Alternate			
Mission Data			
*Date of Mission:		Number/Type Aircraft:	
*Purpose/ Scenario/Justification:			
Route of Flight			
Date/Time (ETA)	PZ or LZ (name, grid, or lat long)	Remarks (ground time, load changes, etc.)	
Passenger Information			
*Number (attach manifest):		Uniform/Equipment: <input type="checkbox"/> Duty <input type="checkbox"/> Combat Light <input type="checkbox"/> Combat	
Senior Passenger:		Honors Requested: <input type="checkbox"/> Yes <input type="checkbox"/> No	
Cargo Data			
*Total Weight:		External Loads: <input type="checkbox"/> Yes <input type="checkbox"/> No	Special Handling: <input type="checkbox"/> Yes <input type="checkbox"/> No
Number of Containers:		Height:	Length: Width:
Mission Coordination		Date/Time	Location/Remarks
Initial Air Coordination Meeting			
Air Mission Briefing			
Remarks:			
Section 2 - Completed by Supporting Unit			
AMR Number:		Mission Number:	
AMR Approval/Routing		Status	Initials Date
AMR Received by J35 AVN			
Validated by J35 AVN		<input type="checkbox"/> Yes <input type="checkbox"/> No	
Verified as Authorized by CG		<input type="checkbox"/> Yes <input type="checkbox"/> No <input type="checkbox"/> NA	
CG Authorization is required due to:			
AMR Received by AAOG			
Initially Approved by CDR, AAOG		<input type="checkbox"/> Yes <input type="checkbox"/> No	
AMR Received by 12 th Aviation Battalion			
Mission Assigned to Company/Crew			
Final Mission Status	<input type="checkbox"/> Completed <input type="checkbox"/> Changed <input type="checkbox"/> Cancelled		
Remarks:			

Figure 2-3: Example blank AMR form [14]

For example, a mission may require a group of ten Soldiers to travel from HLZ A to HLZ B in the morning, then travel from HLZ B to HLZ C in the afternoon. Finally, they may wish to return to HLZ A in the evening. An AMR that supports this mission would be comprised of *three AMR legs* (Figure 2-4).

AIR MISSION REQUEST (AMR)			
Section 1 - Completed by Requesting Unit/Agency (* Required Entry)			
*Unit/Agency: 1st Brigade		*Date of Request: 25 May	
POC Info	Rank/Name	Phone	Email
*Primary	CPT Doe		
Alternate			
Mission Data			
*Date of Mission: 1 June		Number/Type Aircraft:	
*Purpose/Scenario/Justification: Routine Movement			
Route of Flight			
Date/Time (ETA)	PZ or LZ (name, grid, or lat long)	Remarks (ground time, load changes, etc.)	
0800	HLZ A	LEG 1	
1000	HLZ B		
1300	HLZ B	LEG 2	
1330	HLZ C		
1800	HLZ C	LEG 3	
1900	HLZ A		
Passenger Information			
*Number (attach manifest): 10		Uniform/Equipment: <input type="checkbox"/> Duty <input type="checkbox"/> Combat Light <input type="checkbox"/> Combat	
Senior Passenger: COL		Honors Requested: <input type="checkbox"/> Yes <input type="checkbox"/> No	
Cargo Data			
*Total Weight: 0 lbs		External Loads: <input type="checkbox"/> Yes <input type="checkbox"/> No	
		Special Handling: <input type="checkbox"/> Yes <input type="checkbox"/> No	
Number of Containers:		Height: Length: Width:	
Mission Coordination		Date/Time	Location/Remarks
Initial Air Coordination Meeting			
Air Mission Briefing			
Remarks:			
Section 2 - Completed by Supporting Unit			
AMR Number:		Mission Number:	
AMR Approval/Routing		Status	Initials Date
AMR Received by J35 AVN			
Validated by J35 AVN		<input type="checkbox"/> Yes <input type="checkbox"/> No	
Verified as Authorized by CG, JFHQ-NCR/MDW		<input type="checkbox"/> Yes <input type="checkbox"/> No <input type="checkbox"/> NA	
CG Authorization is required due to:			
AMR Received by AAOG			
Initially Approved by CDR, AAOG		<input type="checkbox"/> Yes <input type="checkbox"/> No	
AMR Received by 12 th Aviation Battalion			
Mission Assigned to Company/Crew			
Final Mission Status		<input type="checkbox"/> Completed <input type="checkbox"/> Changed <input type="checkbox"/> Cancelled	
Remarks:			

Figure 2-4: Example of completed AMR form

To allow enough time for the approval and planning process, AMRs are typically submitted three or more days in advance. However, as emergencies arise, units may also request expedited approval and planning. Figure 2-5 shows the AMR process from its submission to execution. For high profile missions involving many parties, where the AMR may not provide sufficient information, a Concept of the Operation (CONOP) and Air Mission briefing (AMB) may also be required. The CONOP brief is used to inform the requesting commander of the planning and execution time line

for the mission. The AMB brings together the requesting unit and aviation unit to ensure that everyone is familiar with the mission.

2.3.1.2 AMR priorities

The Commander’s Mission Priority List (CMPL) guides aviation planners in deciding which AMRs to schedule if too many are submitted for a given time period (Table 2.2). AMRs that are scheduled are said to be *supported*. Because of the volume of AMRs received, planners are not always able to accommodate all requests, and AMRs must sometimes go *unsupported*, despite being initially approved by the aviation mission approval authority. If an AMR is unsupported, the requesting unit must then submit another AMR for an alternate date, or appeal the decision to their higher headquarters.

AMR mission	Priority
Downed Aircraft Recovery	1 (highest)
Emergency leave	2
General Officer movement	3
Military Working Dog	4
Critical Equipment Repair	5
Religious Services	6
O-6 Colonel or equivalent	7
R&R leave	8
other	9 (lowest)

Table 2.2: Example Commander’s Mission Priority List (CMPL)

2.3.1.3 AMR Form Routing

First, the requesting unit submits the AMR to their higher headquarters, who acts as the first line of approval and review. The AMR is then sent to the aviation mission approval authority, whose headquarters control the use of the aircraft. Once the AMR has been approved, it is sent to the subordinate aviation headquarters for support (Figure 2-5). Following approval, the supported unit has Direct Liaison Authorized (DIRLAUTH) with the supporting aviation unit. If an aviation unit is the requester of

an AMR, it must still be submitted to the aviation mission approval authority, unless it can be executed without affecting the AMRs that have already been approved and the mission risk does not necessitate higher headquarters approval.

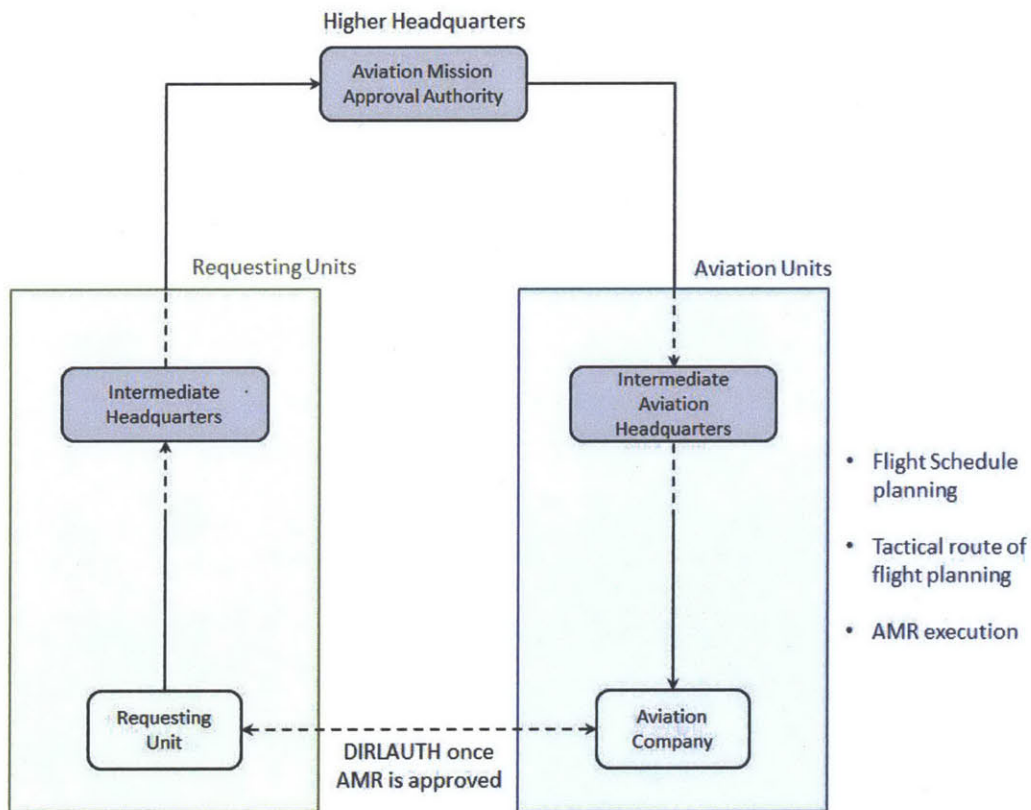


Figure 2-5: The AMR process

Figure 2-6 shows the AMR process for the case where the mission approval authority is the Division G3. In this example, a company requesting lift support submits an AMR to its battalion S3 Air, in the training and operations section of the battalion headquarters. The battalion S3 Air reviews the AMR and submits it for approval to the Brigade Aviation Element (BAE), which is the aviation planning cell inside a non-aviation brigade. It is then reviewed and submitted to the Division G3 for approval. Following approval, it is forwarded to the aviation brigade planning cell, who reviews and assigns the AMR to one of its subordinate battalions (or Task Forces) for support. The aviation battalion planning cell assigns the AMR to one of its subordinate companies and plans the flight schedule accordingly.

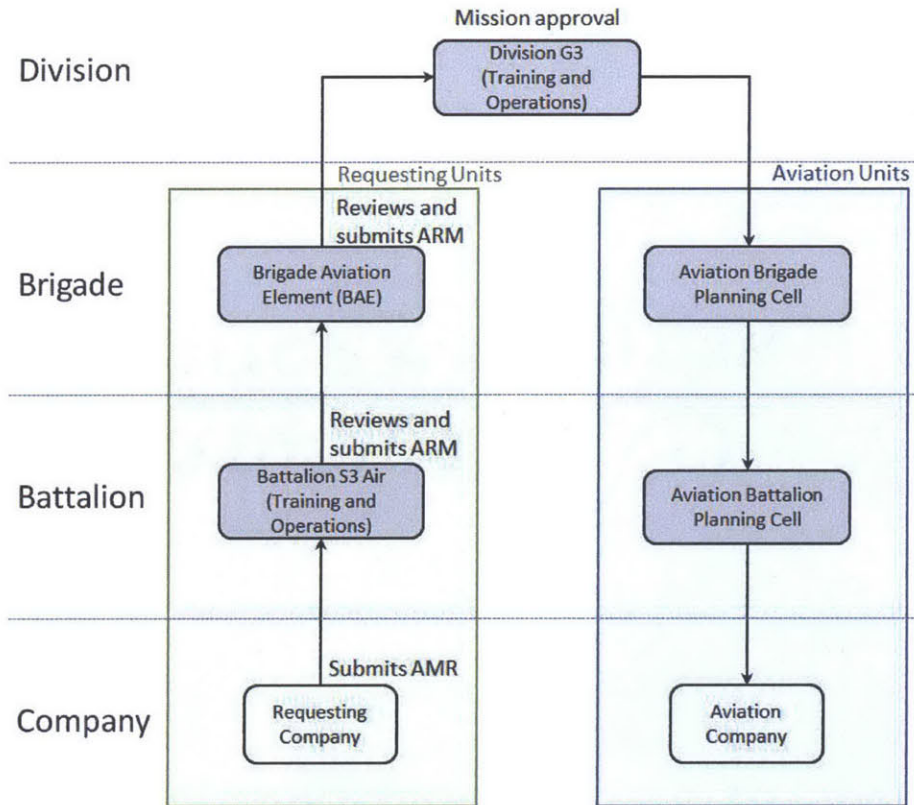


Figure 2-6: AMR approval and routing for Division aviation assets

2.3.2 Flight Schedule Planning

Once an AMR is approved, the aviation unit subordinate to the mission approval authority ensures that the flight schedule reflects support of the approved AMR, and that it is executed. A flight schedule is planned and published so that all aircraft crews and AMR requesters know at what time AMRs will be supported, and what the route of flight will be.

2.3.2.1 Planning Horizon

The flight schedule is planned in the *Future Operations* (FUOPS) section of the aviation battalion S3 or brigade S3 (Training and Operations section). The planning horizon is generally just a few days for routine AMRs, but can be as long as several weeks for high profile ones that require additional planning or coordination, a CONOP, or an AMB.

Figure 2-7 shows a series of rolling 72 hour planning cycles, with each followed by a 24 hour execution window. In this example, flights that support AMRs scheduled for day $n + 3$ are planned between day n and day $n + 2$. AMRs that begin and end on different calendar days are planned for the day in which the mission begins.

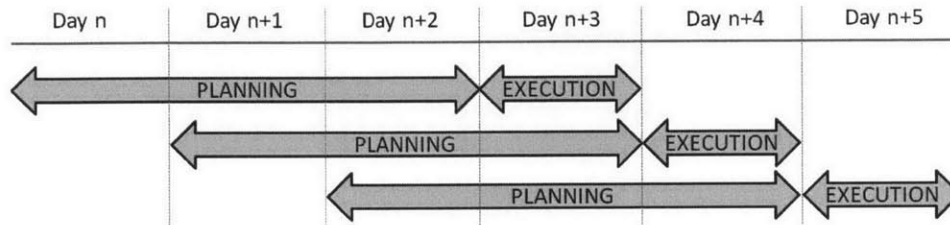


Figure 2-7: Planning and execution horizon

2.3.2.2 Aircraft Availability and Planning Characteristics

To give aviation planners the ability to allocate aviation resources several days out in support of AMRs, a recurring availability of aircraft called the *aircraft steady state* (Figure 2-8) is agreed upon by the aviation units who maintain and crew the aircraft, and the aviation mission approval authority who controls their use. This agreement allows independent flight schedule development to be handled at the battalion level or higher. Aviation companies provide aircraft for specific periods of time every day, and the times during which aircraft can be used generally remains constant from day to day. To account for maintenance and crew requirements, the steady state only promises what commanders can guarantee on a daily basis. Thus, if a commander has ten helicopters, he may only provide six each day, if he anticipates rotating four through maintenance. Some aviation units may have a separate *surge capability* steady state which can provide more flight time for short durations, usually several days. Some aviation units may also dedicate aircraft teams in the steady state to support specific types of AMRs or missions.

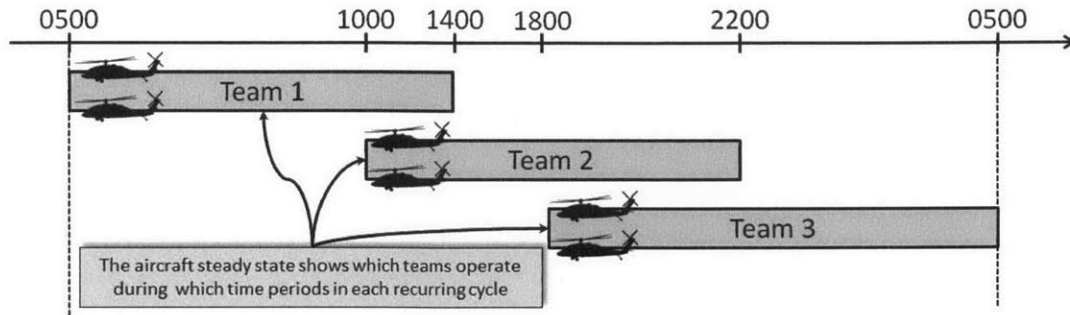


Figure 2-8: Example steady state

Aircraft planning characteristics, such as maximum time aloft for each aircraft type, average travel time between HLZs, and carrying capacity, are made by the aviation planners so that the flight schedule can be planned independently from the tactical route of flight. These can be calculated using the appropriate training manuals or agreed upon between the aviation planners and the aviation companies. Each planning cell holds a daily flight scheduling meeting so that the aviation companies who will be flying the actual missions can agree with the flight schedule and supported AMRs.

2.3.3 Tactical Route Planning

Given the flight schedule and list of AMRs being supported, aircrews plan the tactical route of flight. While the flight schedule provides the legs that the aircrews must fly, they must still plan way-points, headings, and altitude for each leg of the flight, taking environmental factors, enemy activity, and crew experience into consideration. Aircrews may plan different tactical routes between two HLZs, due to these factors.

2.3.4 Mission Execution

On the day of execution, supported units rendezvous with aircraft teams at the time and HLZs designated on the flight schedule. Real-time mission monitoring and contingency planning is conducted by the Current Operations (CUOPS) section in the Tactical Operations Center (TOC) at the Battalion level and higher. Thus, if weather

or maintenance delays occur, the CUOPS section will reroute aircraft, or cancel flights and/or AMRs.

2.4 Problem Motivation

With an anticipated downsizing of forces, we can anticipate the need to support future operations around the globe with fewer resources. Furthermore, the trend towards distributed operations puts additional stress and strain on aviation units. Since aviation units are most often centralized in one hub of a designated geographical area, maintenance activities and crew rotation are facilitated, but availability of lift assets is strained since they must travel farther to service the entire area of operation.

Thus, proper planning is crucial to accomplishing mission objectives. Demand considerations need to be carefully evaluated when determining the lift capacity to allocate to each area of operations.

2.5 Problem Statement

Faced with these growing challenges, the optimization of resources is extremely important to the future of Army Aviation. In this thesis, we consider the problem of scheduling and route planning for Army Aviation lift assets with the following goals:

1. Develop a model to provide near optimal scheduling and routing information applied to the current AMR process.
2. Given a level of aircraft capacity, determine how many AMRs can be supported on a daily basis.
3. Determine what aircraft capacity is needed to satisfy different probabilistic levels of demand.
4. Determine the effect of increased passenger flexibility on the number of AMRs that can be supported.

Chapter 3

Model Inputs and Network Structure

The purpose of this chapter is to describe how the operational characteristics of the aircraft, the AMR demand, and the geographic distribution of HLZs are modeled as inputs to the models presented in Chapter 4.

3.1 Aircraft Teams

During military operations, helicopters generally fly as a team of two or more, for cover and self-rescue purposes. We consider only teams of two and model them as a single entity.

3.1.1 Aircraft Team Constraints

Aircraft team constraints fall into two categories: physical and operational. Physical constraints include maximum flight time due to fuel, and maximum weight limit. Other physical constraints, such as the fuel burn rate depend on environmental conditions (altitude and temperature) and gross weight. Since these factors are not constant, a conservative estimate of average fuel burn rate will be used.

Operational constraints are imposed due to Standing Operating Procedures (SOP). During visual meteorological conditions (VMC), rotary wing flight crews must plan for a minimum of 20 minutes reserve at cruise burn rate during all phases of flight [13]. In addition, most SOPs limit the actual daily flight time per crew-member to

eight hours. Following a period of eight hours of rest, crews may once again fly for eight more hours. This flight hour limit can change depending on the mode of flight: “day”, “night unaided”, “night-vision goggles”, or a combination of these. There are also weekly and monthly flying hour limits which dictate the maximum flight time per aviator before a mandatory 24 or 48 hour rest period must be taken. Finally, there are duty day constraints for aircrews, where an aircrew can only be “on call” for a certain period of time (usually 12 hours) before they must be given 12 hours off. These flight hour and duty day restrictions were developed to ensure that aircrews do not suffer from acute or chronic fatigue and can operate the aircraft safely.

3.1.2 Aircraft Team Parameters

Each aircraft team f has associated parameters which are shown in Table 3.1. A set F is created to contain all available aircraft teams.

<i>Set</i>	<i>Element</i>	<i>Parameters</i>	<i>Description</i>	
F	f	γ	number of aircraft teams	
		<i>aircraft team</i>		
		u^f	passenger capacity	
		τ_{CYCLE}^f	flying hour limit per planning cycle	
		τ_{FLIGHT}^f	flying hour limit on each full tank of fuel	
		τ_{FUEL}^f	average refuel time required	
		τ_{LOAD}^f	average loading/unloading time	
		τ_{SD}^f	minimum time required to shutdown	
		\bar{c}^f	cost of flight arc per time period	
		\underline{c}^f	cost of ground arc per time period	
		t_{START}^f	first time period that team is available	
		t_{END}^f	last time period that team is available	

Table 3.1: Aircraft team parameters

3.2 Air Mission Requests

3.2.1 Decision Window Model

The Decision Window Model (DWM) was introduced by the Boeing Airplane Company [9] and can be used to model the passenger flexibility of an AMR. An AMR travel window, given separately for each leg of the AMR and bounded by the earliest departure and latest arrival times, is the time frame during which the AMR leg must be supported. This is similar to the decision window of a commercial airline passenger, in that the decision window is the time frame within which a traveler is willing to travel. In the decision window model, the perceived travel time (arrival time minus departure time) is known as *delta-T* (ΔT), and the traveler's flexibility is his *schedule tolerance* (Figure 3-1).

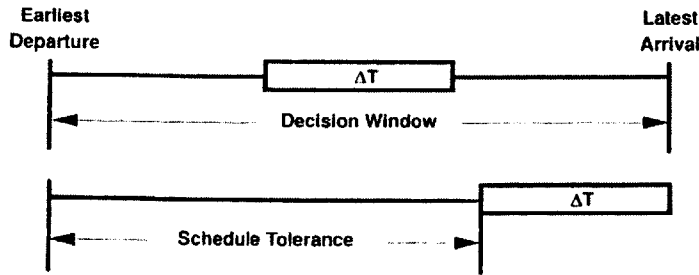


Figure 3-1: Decision window (Boeing [9])

ΔT^k represents the shortest possible flight time needed to accomplish leg k of an AMR, and the schedule tolerance ϕ^k is the degree of flexibility that passengers have on either end of their AMR leg (Table 3.2). A leg with $\phi^k = 0$ means that, if supported, leg k must be supported exactly at the requested times.

<i>Airline Passenger Decision Theory</i>		<i>AMR Leg k</i>	
ΔT	Perceived travel time	ΔT^k	Shortest flight time for AMR leg k
	Schedule tolerance	ϕ^k	Schedule tolerance of AMR leg k
	Decision window	$\Delta T^k + \phi^k$	Travel window of AMR leg k

Table 3.2: DWM application to the AMR leg travel window

We assume that AMRs are submitted for the preferred day, that the window for

each leg is equal to or wider than the shortest travel time, and that all departure and arrival times within the travel window are satisfactory to the traveler. There is generally more passenger demand than can be supported by the number of aircraft teams. Therefore, an AMR leg with a larger decision window has more flexibility and is more likely to be supported. As such, users of the system balance mission priorities and their own flexibility when determining an acceptable travel window for their passenger request. AMRs submitted with *hard* times (no flexibility) are usually of high priority and in support of a high ranking officer or an important event.

3.2.2 AMR Parameters

Recall from Chapter 2 that each AMR may be comprised of several AMR legs. We assume in the following chapters that for each AMR, all or none of its legs must be supported (since AMR legs from the same AMR support the same mission, this avoids partial support). Different teams, however, may support different AMR legs from the same AMR.

Each AMR is modeled as a set L of AMR legs k . \mathcal{L} is the set of all AMRs $\{L_1, L_2, \dots, L_l\}$ and K is the set of all AMR legs $\{k_1, k_2, \dots\}$ that can be supported. Each AMR leg k has a parent AMR L , to which it is assigned (Figure 3-2).

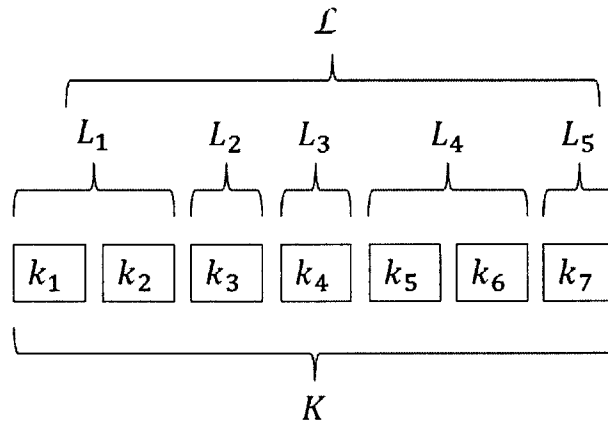


Figure 3-2: AMR and AMR leg sets

Using the set of AMRs from Figure 3-2, a feasible solution could be the support

of AMR legs $\{k_1, k_2, k_5, k_6, k_7\}$ since AMRs L_1, L_4 , and L_5 would be fully supported. However, $\{k_1, k_3, k_7\}$ would *not* be a feasible set of supported AMR legs since AMRs L_1 and L_2 only receive partial support.

Each AMR and the legs that compose it have associated parameters which are collected or generated from the AMR form, and are shown in Table 3.3.

<i>Set</i>	<i>Subset</i>	<i>Element</i>	<i>Parameters</i>	<i>Description</i>
\mathcal{L}	L	k	<i>air mission request</i>	
			η^L	AMR mission number
			ρ^L	number of AMR legs
			<i>air mission request leg</i>	
			η^L	parent AMR mission number
			η^k	AMR leg number
			w^k	number of passengers
			v^k	value of AMR leg
			θ^{k-}	starting HLZ
			θ^{k+}	ending HLZ
			t^{k-}	earliest departure time from starting HLZ
			t^{k+}	latest arrival time at ending HLZ
			c^k	cost of flight arc per time period
			ΔT^k	direct flight time of AMR
			ϕ^k	schedule tolerance

Table 3.3: AMR parameters

3.3 Network Construction

3.3.1 Inter-Base Transport Model

3.3.1.1 Static Graph

In order to model the geographic distribution of HLZs, we construct a network, or *static graph* (Figure 3-3). The network models physical distances between HLZs. Each HLZ is an individual node in the graph, with arcs between every pair of nodes and an associated flight time.

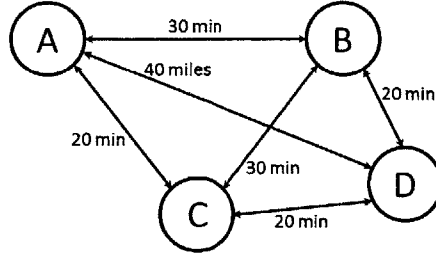


Figure 3-3: Static graph representation of a network of four HLZs

3.3.1.2 Time Periods

The total time period that we will model is divided into m smaller time periods (t_0, t_1, t_2, \dots) of equal duration τ . T is the set of all time periods. The unit time period τ reflects a reasonable duration with which flight time and ground time can be modeled. A typical value for τ , for the scenarios addressed in this thesis, is 10 or 15 minutes.

3.3.1.3 Time-Space Graph

The static graph is extended to include time periods in a *time-space graph* $G(N, A)$. Nodes represent the position of an aircraft team in a specific time period, and are indexed by $(HLZ, time\ period)$. Arcs no longer represent an average time distance between bases as in the static graph, but rather a movement through the network of an aircraft team or an AMR. The time-space graph is an acyclic network where flow is always from left to right, or *forward in time*. In our formulations, the following arcs are used:

Flight Arcs: Flight arcs connect nodes that represent different HLZs. For example, if an aircraft team leaves HLZ A in time period t_1 and arrives at HLZ B in time period t_3 , then we introduce a flight arc between nodes $(A, 1)$ and $(B, 3)$ to represent that portion of the aircraft team schedule and route of flight (Figure 3-4). The horizontal travel distance of a flight arc in time periods is the average amount of time for the aircraft team to fly from the departure to the arrival HLZ.

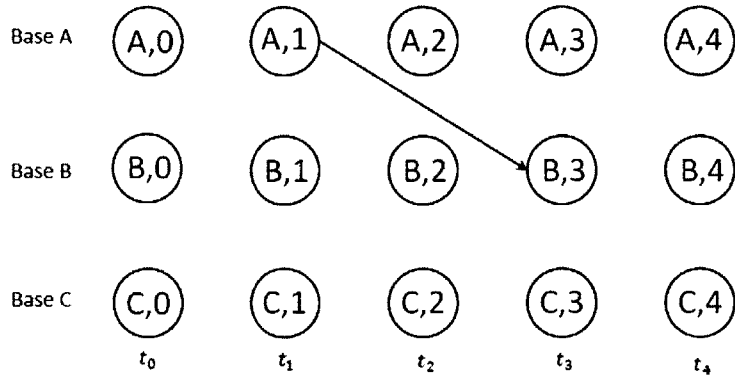


Figure 3-4: Flight arc between two nodes in a time-space graph

Ground Arcs: Ground arcs connect nodes representing the same HLZ. They model *turn time*, which is the amount of time an aircraft team spends on the ground between flight legs. We distinguish four types:

- *Waiting arcs* represent waiting time on the ground of an aircraft team with engines running.
- *Shutdown arcs* represent waiting time on the ground with engines shut down.
- *Loading/unloading arcs* represent the time spent picking up or dropping off AMRs.
- *Refueling arcs* represent the time spent refueling.

Any activity that occurs at the same base over a period of time is modeled with a ground arc. In Figure 3-5, we add a ground arc of length two time periods between nodes $(B, 1)$ and $(B, 3)$ to represent the refueling of an aircraft team.

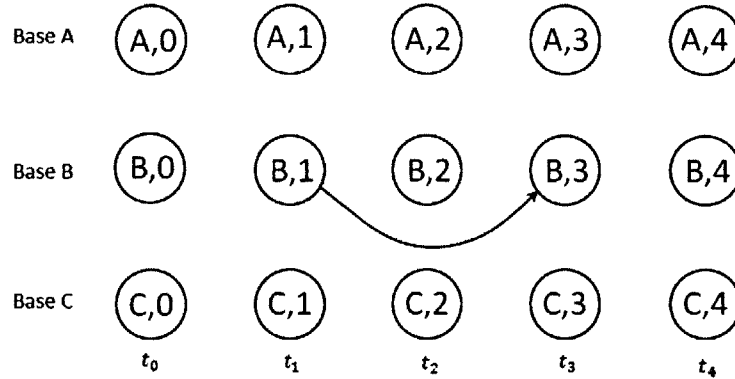


Figure 3-5: Ground arc between two nodes in a time-space graph

Aircraft Team Paths An aircraft team path is a set of connected arcs through the time-space graph. It represents an aircraft team schedule and route of flight. As long as arcs are chosen from the feasible set of arcs and no operational constraints (fuel available, etc) are violated, each possible combination of arcs represents a different feasible path for that aircraft team.

1. Arcs describe the movement and activities (flight, waiting, shutdown, loading/unloading, and refuel) of a specific aircraft team. Each path represents an aircraft team schedule and route of flight. Since arcs model the transition of a team in space and/or time, flight and ground arcs in the time-space graph are specific to the operational characteristics of each aircraft team. Each aircraft team has a path through $G(N, A)$. Figure 3-6 shows an aircraft team that starts at HLZ A in time period t_0 and leaves for HLZ C in time period t_1 , arriving in period t_2 . The aircraft team then departs in period t_3 and arrives at base B in time period t_4 .
2. AMR flow on aircraft team arcs describe the specific route of flight of each AMR. AMR flow is always constrained by the capacity of aircraft teams flowing between the same nodes. Figure 3-7 shows an AMR that starts at HLZ A in time period t_0 and leaves on an aircraft team for HLZ C in time period t_1 , arriving in period t_2 . The AMR is finished unloading from the aircraft team in time period t_3 .

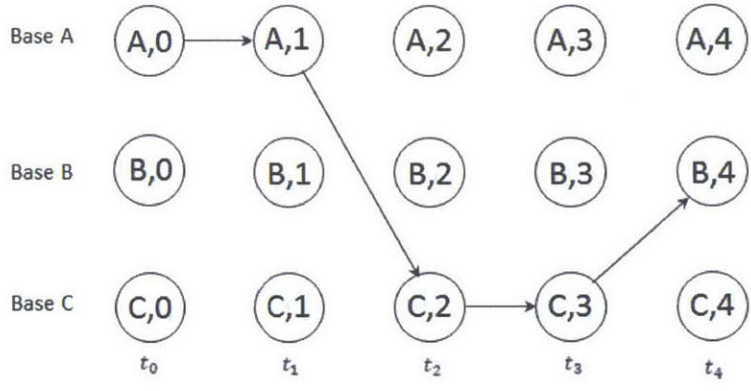


Figure 3-6: Aircraft team path in $G(N, A)$ from node $(A, 0)$ to node $(B, 4)$

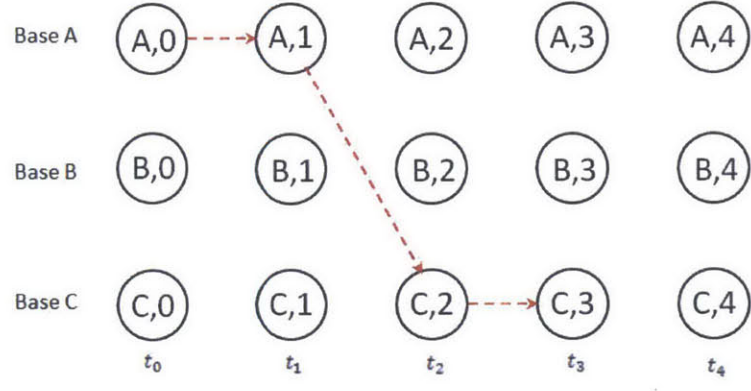


Figure 3-7: AMR flow in $G(N, A)$ from node $(A, 0)$ to node $(C, 2)$

A complete aircraft team path describes both the aircraft team arcs and AMR flow on those arcs, for every AMR that is scheduled to fly on that aircraft team (Figure 3-8).

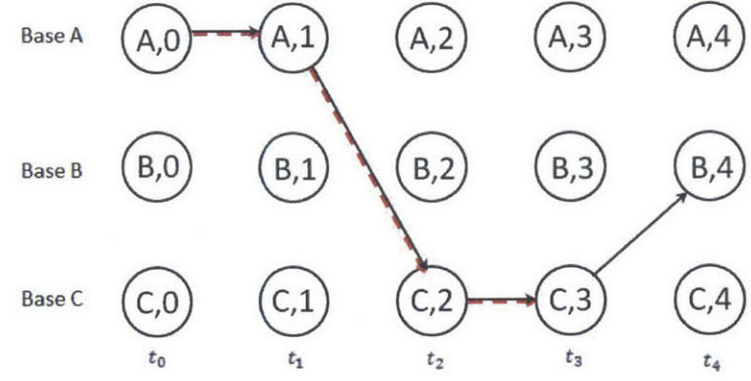


Figure 3-8: Aircraft team path in $G(N, A)$ from node $(A, 0)$ to node $(B, 4)$, with AMR flow

3.3.2 Network Inputs

A number of variables and sets are defined to help formulate the problem in the next chapter.

- β Total number of HLZs in the area of operations;
- B Set of all HLZs i indexed from 1 to β ;
- B_{FUEL} Set of HLZs that have refuel capability for helicopters;
- N The set of all nodes in the time-space graph. For every HLZ in B and time duration in T , we create a node and index it by (b, t) , where b is the index of the HLZ in B , and t is the index of the time period in T ;
- N_{FUEL} Subset of all nodes that have rotary wing refuel capability, $N_{FUEL} \subseteq N$;
- a An arc indexed by origin node and destination node $\{(o, t_i), (d, t_j)\}$, indicating that it begins at node (o, t_i) and ends at node (d, t_j) , and is of duration $\tau_a = t_j - t_i$, where $j > i$;
- A Set of all arcs in the time-space graph;
- A^f Set of all arcs in the time space graph for aircraft team $f \in F$. Within the set A^f , we have the following:
 - $\overline{A^f}$ Set of air arcs in the time-space graph for aircraft team f . Air arcs are defined between nodes (o, t_i) and (d, t_j) where $t_j > t_i$, $t_j - t_i = \text{flight time from } o \text{ to } d \text{ for team } f$, and $o \neq d$.
 - $\underline{A^f}$ Set of all ground arcs in the time-space graph for aircraft team f . A ground arc is defined as an arc between nodes (o, t_i) and (d, t_j) where $t_j > t_i$, and $o = d$. Within the set of all ground arcs, we have the following:
 - A_{WAIT}^f Set of all waiting arcs of length τ . Waiting arcs are defined between all pairs of nodes $\{(o, t_i), (o, t_{i+1})\}$;
 - $A_{WAIT,t}^f$ Set of all waiting arcs of length τ that begin at time t ;

A_{SD}^f	Set of all shutdown arcs. Shutdown arcs are defined between all pairs of nodes $\{(o, t_i), (o, t_i + \tau_{SD}^f)\}$;
$A_{SD,t}^f$	Set of all shutdown arcs that begin at time t ;
A_{FUEL}^f	Set of all refueling arcs of length τ_{FUEL}^f . Refueling arcs are defined between all pairs of nodes $\{(o, t_i), (o, t_i + \tau_{FUEL}^f)\}$;
$A_{FUEL,t}^f$	Set of all refueling arcs of length τ_{FUEL}^f that begin at time t ;
A_{LOAD}^f	Set of all loading/unloading arcs of length τ_{LOAD}^f . Loading/unloading arcs are defined between all pairs of nodes $\{(o, t_i), (o, t_i + \tau_{LOAD}^f)\}$;
$A_{LOAD,t}^f$	Set of all loading/unloading arcs of length τ_{LOAD}^f that begin at time t ;
A_n^{f-}	Subset of arcs in A^f that leave node n ;
A_n^{f+}	Subset of arcs in A^f that enter node n ;
A_b^{f-}	Subset of arcs in A^f that leave HLZ b ;
A_b^{f+}	Subset of arcs in A^f that enter HLZ b ;
A_t^f	Subset of arcs in A^f that enter or exit any node at time t ;
A_t^{f-}	Subset of arcs in A^f that exit any node at time t ;
A_t^{f+}	Subset of arcs in A^f that enter any node at time t ;
$A_{[t_i t_j]}^f$	Subset of arcs in A^f that begin at or before time period t_i and end at or after time period t_j ;

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Chapter 4

Model Formulation

This chapter proposes three approaches to solving the problem of AMR scheduling and aircraft team route planning. After conducting a review of relevant literature, we propose an arc flow formulation to a service network design problem (SNDP) adapted to our specific problem. Then, we consider two additional heuristic approaches.

4.1 Literature Review

In this section, we explore previous work and relevant research on multi-commodity network flow (MCNF) problems and network design problems (NDP) in order to lay the foundation for our models.

4.1.1 Multi-Commodity Network Flow Problem

As opposed to the minimum cost flow problem (MCF), which models the flow of one commodity, the MCNF models the flow of several commodities. Different commodities have different origins and destinations, and may have different physical characteristics (Ahuja et. al. [1]).

We explore two formulations of the Multi-Commodity Network Flow (MCNF) problem. The first is an arc-flow formulation approach to solving the optimal flow of commodities over a fixed network. The second is a path-flow formulation of the same MCNF problem. In these formulations, we assume that each commodity has

one supply and one demand node.

4.1.1.1 Arc-Flow Formulation

Let x_{ij}^k be a decision variable which determines the amount of commodity k that flows over arc (i, j) . We define u_{ij} as the capacity of arc (i, j) , and c_{ij} as the unit cost of flow on arc (i, j) . We define b_i^k as the supply or demand of commodity k at node i . If there is supply of commodity k at node i , $b_i^k > 0$. If there is demand for commodity k at node i , $b_i^k < 0$. Ahuja et. al. [1] present the following arc-flow LP formulation for the MCNF problem.

$$\text{MCNF}_{\text{ARC}} = \min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k \quad (4.1)$$

s.t.

$$\sum_{k \in K} x_{ij}^k \leq u_{ij} \quad \forall (i, j) \in A \quad (4.2)$$

$$\sum_{j:(i,j) \in A} x_{ij}^k - \sum_{j:(j,i) \in A} x_{ji}^k = b_i^k \quad \forall i \in N, k \in K \quad (4.3)$$

$$x_{ij}^k \geq 0 \quad \forall (i, j) \in A, k \in K \quad (4.4)$$

The objective function (4.1) minimizes the cost of flow through the network. Constraints (4.2) restrict the flow of commodities $k \in K$ over every arc to the capacity of that arc. Constraints (4.3) ensure flow balance at each node. Constraints (4.4) ensure that the flow of commodities is non-negative. The structure of the MCNF_{ARC} problem is such that the number of individual arcs drives the complexity of the the problem.

4.1.1.2 Path Flow Formulation

An alternative way to model the MCNF problem is to assign flow to paths in the network, as opposed to individual arcs. Each commodity k is assigned to a path,

p . Each path is composed of a set of connected arcs, which denotes a specific flight sequence. The decision variable x_p^k is the fraction of commodity k that is assigned to path p . Let $\pi_{ij}^p = 1$ if arc (i, j) is assigned to path p , otherwise let $\pi_{ij}^p = 0$. Let w^k be the flow requirement for commodity k . Let P be the set of all paths. We draw from Ahuja et. al. [1], Koepke [11], and Nielsen [12] to formulate the following path-based MCNF problem.

$$\text{MCNF}_{\text{PATH}} = \min \sum_{k \in K} \sum_{p \in P} c_p^k x_p^k \quad (4.5)$$

s.t.

$$\sum_{k \in K} \sum_{p \in P} x_p^k w^k \pi_{ij}^p \leq u_{ij} \quad \forall (i, j) \in A \quad (4.6)$$

$$\sum_{p \in P} x_p^k = 1 \quad \forall k \in K \quad (4.7)$$

$$x_p^k \geq 0 \quad \forall k \in K, p \in P \quad (4.8)$$

The objective function (4.5) minimizes the cost of flow through the network. Constraints (4.6) restrict the flow of commodities k over every arc to the capacity of that arc. Constraints (4.7) ensure that each commodity is transported in full. Constraints (4.8) ensure that the flow of commodities is non-negative on all paths. The structure of the MCNF problem presents both benefits and challenges to obtaining an optimal solution. As opposed to the arc flow formulation, there are fewer constraints, but more decision variables, since the number of possible paths will generally be greater than the number of arcs in the network. This means that the complexity is driven by the number of feasible paths, which can increase exponentially as the size of the network increases.

4.1.2 Network Design Problem

A problem that builds on elements of the MCNF problem is the NDP. In the MCNF problem, the underlying network is fixed and we only optimize the flow of commodi-

ties. In the NDP, on the other hand, we wish to find a time-space network $G(N, A)$ and the optimal flow over it (Ahuja et. al. [1]). Therefore, the NDP deals with *simultaneously* finding the structure of the network and the flow of commodities on that network so that a cost function is minimized. An NDP formulation has *network design decision variables* for the structure of the network, and *path decision variables* for the flow of commodities. Ahuja et. al. [1] present the following arc-flow NDP formulation (some notation changed for consistency with previously declared variables).

$$\text{NDP}_{\text{ARC}} = \min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k + \sum_{(i,j) \in A} c_{ij} y_{ij} \quad (4.9)$$

s.t.

$$\sum_{j \in N} x_{ij}^k - \sum_{j \in N} x_{ji}^k = \begin{cases} 1 & \text{if } i \text{ is origin node of commodity } k \\ -1 & \text{if } i \text{ is destination node of commodity } k \\ 0 & \text{otherwise} \end{cases}$$

$$\forall k \in K, i \in N \quad (4.10)$$

$$\sum_{k \in K} w^k x_{ij}^k \leq u_{ij} y_{ij} \quad \forall (i, j) \in A \quad (4.11)$$

$$x_{ij}^k \leq y_{ij} \quad \forall (i, j) \in A, k \in K \quad (4.12)$$

$$x_{ij}^k \geq 0 \quad \forall (i, j) \in A, k \in K \quad (4.13)$$

$$y_{ij} \in \{0, 1\} \quad \forall (i, j) \in A \quad (4.14)$$

Each commodity $k \in K$ has a supply and demand node, and a flow requirement w^k . x_{ij}^k is the fraction of commodity k that flows on arc (i, j) , and y_{ij} is a binary

decision variable indicating whether arc (i, j) is installed in the network. The capacity of arc (i, j) is u_{ij} .

The objective function (4.9) minimizes the cost of the network and flow over the network. c_{ij}^k is the cost of sending commodity k on arc (i, j) , and c_{ij} is the fixed cost for installing arc (i, j) in the network. Constraints (4.10) are flow balance constraints. Constraints (4.11) ensure that the flow of commodities over each arc does not exceed the capacity of that arc. Constraints (4.12) are forcing constraints that ensure that there is no flow on arcs that are not installed in the network. Constraints (4.13) ensure non-negative flow, and constraints (4.14) ensure that each arc is either installed or not installed in the network.

This formulation is particularly useful when the network is not known. Transportation problems are often modeled by a variant of the NDP, called the service network design problem (SNDP). The SNDP is a specific form of NDP, where design decision variables are forced to have balance of flow.

We reformulate the NDP formulation presented by Ahuja et. al. [1], as an arc-flow SNDP. We use the aircraft team, AMR, and network structure defined in Chapter 3. There is a set F of aircraft teams f . A is the set of all arcs on which AMRs can flow, and A^f is the set of arcs specific to vehicle f . We assume that we are given the origin and destination node of each aircraft team f . Each AMR leg $k \in K$ has an origin and destination node, and a flow requirement, w^k . We define $x_{ij}^{k,f}$ as the fraction of AMR leg k that flows on arc $(i, j) \in A^f$, and y_{ij}^f as a binary decision variable, which indicates if arc (i, j) is installed in the network. The capacity of arc (i, j) is u_{ij}^f . We define $c_{ij}^{k,f}$ as the cost of sending AMR leg k on arc (i, j) , and c_{ij}^f as the fixed cost for installing arc $(i, j) \in A^f$ in the network.

$$\text{SNDP}_{\text{ARC}} = \min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k x_{ij}^{k,f} + \sum_{f \in F} \sum_{(i,j) \in A^f} c_{ij}^f y_{ij}^f \quad (4.15)$$

s. t.

$$\sum_{f \in F} \sum_{j \in N} x_{ij}^{k,f} - \sum_{f \in F} \sum_{j \in N} x_{ji}^{k,f} = \begin{cases} 1 & \text{if } i \text{ is origin node of AMR leg } k \\ -1 & \text{if } i \text{ is destination node of AMR leg } k \\ 0 & \text{otherwise} \end{cases} \quad \forall f \in F, \forall i \in N \quad (4.16)$$

$$\sum_{j \in N} y_{ij}^f - \sum_{j \in N} y_{ji}^f = \begin{cases} 1 & \text{if } i \text{ is origin node of vehicle } f \\ -1 & \text{if } i \text{ is destination node of vehicle } f \\ 0 & \text{otherwise} \end{cases} \quad \forall f \in F, \forall i \in N \quad (4.17)$$

$$\sum_{f \in F} \sum_{k \in K} w^k x_{ij}^{k,f} \leq \sum_{f \in F} u_{ij}^f y_{ij}^f \quad \forall (i,j) \in A \quad (4.18)$$

$$x_{ij}^{k,f} \leq y_{ij}^f \quad \forall (i,j) \in A, k \in K, f \in F \quad (4.19)$$

$$x_{ij}^{k,f} \geq 0 \quad \forall (i,j) \in A, k \in K, f \in F \quad (4.20)$$

$$y_{ij}^f \in \{0, 1\} \quad \forall (i,j) \in A, f \in F \quad (4.21)$$

The objective function (4.15) minimizes the cost of the network (aircraft routes of flight) and the flow of AMRs over it. Constraints (4.16) ensure flow balance of each commodity through every node. Constraints (4.17) ensure flow balance of each vehicle through every node. Constraints (4.18) ensure that the flow of commodities over each arc does not exceed the capacity of vehicles using that arc. Constraints (4.19) restricts each commodity to flowing on arcs that have been installed. Constraints (4.20) ensure non-negative flow of commodities, and constraints (4.21) ensure that each arc (i, j) is either installed or not installed.

4.2 Formulation as a Service Network Design Problem

In this section, we develop an SNDP formulation, SNDP_{OPT} , to address the problem of AMR scheduling and aircraft routing.

4.2.1 Decision Variables

Our formulation takes an arc-flow approach to solving the SNDP for aircraft team arcs and AMR flow, given deterministic inputs. In this formulation, commodity path flow variables are called *AMR flow variables*, because they represent the flow of AMRs through the network. Similarly, network design decision variables are called *aircraft team arc variables*. We define all variables used in this formulation below.

AMR flow decision variables

$$\theta^L = \begin{cases} 1 & \text{if AMR } L \text{ is supported} \\ 0 & \text{otherwise} \end{cases}$$

$$\delta^k = \begin{cases} 1 & \text{if AMR leg } k \text{ is supported} \\ 0 & \text{otherwise} \end{cases}$$

$$x_a^{k,f} = \begin{cases} 1 & \text{if AMR leg } k \text{ is supported on aircraft } f \text{ on arc } a \\ 0 & \text{otherwise} \end{cases}$$

$\zeta^k = \text{total travel time of AMR leg } k$

$\zeta_{start}^k = \text{start of travel time of AMR leg } k$

$\zeta_{end}^k = \text{end of travel time of AMR leg } k$

We introduce *dummy AMR flow variables* to ensure that all missions are supported, either in real terms or by dummy variables. AMR flow on dummy variables do not add value or cost to the objective function. Furthermore, flow is not constrained for dummy variables other than for flow-balance. If an AMR is supported with dummy variables, then it is *unsupported* on the flight schedule.

$$\hat{x}_a^k = \begin{cases} 1 & \text{if AMR leg } k \text{ flows on arc } a \\ 0 & \text{otherwise} \end{cases} \quad (\text{dummy variable})$$

Aircraft team arc decision variables

$$y_a^f = \begin{cases} 1 & \text{if aircraft team } f \text{ is scheduled on flight arc } a \\ 0 & \text{otherwise} \end{cases}$$

$$g_a^f = \begin{cases} 1 & \text{if aircraft team } f \text{ is scheduled on waiting arc } a \\ 0 & \text{otherwise} \end{cases}$$

$$v_a^f = \begin{cases} 1 & \text{if aircraft team } f \text{ is scheduled on loading/unloading arc } a \\ 0 & \text{otherwise} \end{cases}$$

$$r_a^f = \begin{cases} 1 & \text{if aircraft team } f \text{ is scheduled on refueling arc } a \\ 0 & \text{otherwise} \end{cases}$$

$$s_a^f = \begin{cases} 1 & \text{if aircraft team } f \text{ is scheduled on shutdown arc } a \\ 0 & \text{otherwise} \end{cases}$$

In order to formulate the problem, the structure of the time-space graph and additional variables are needed:

D $(|N| \times |A|)$ incidence matrix for AMR flow variables, where:

$$\mathbf{D}(n, a) = \begin{cases} 1 & \text{if arc } a \text{ starts at node } n \\ -1 & \text{if arc } a \text{ ends at node } n \\ 0 & \text{otherwise} \end{cases} \quad \forall n \in N, a \in A$$

D^f $(|N| \times |A^f|)$ incidence matrix for aircraft team flow variables, *specific to each aircraft team f*, where:

$$\mathbf{D}^f(n, a) = \begin{cases} 1 & \text{if arc } a \text{ starts at node } n \\ -1 & \text{if arc } a \text{ ends at node } n \\ 0 & \text{otherwise} \end{cases} \quad \forall n \in N, a \in A^f$$

E $(\gamma \times |N|)$ matrix indicating departure and arrival node for aircraft teams, where:

$$\mathbf{E}(f, n) = \begin{cases} 1 & \text{if team } f \text{ starts at node } n \\ -1 & \text{if team } f \text{ ends at node } n \\ 0 & \text{otherwise} \end{cases} \quad \forall n \in N, f \in F$$

B $(\rho \times |N|)$ matrix indicating demand data where:

$$\mathbf{B}(k, n) = \begin{cases} w^k & \text{if AMR leg } k \text{ starts at node } n \\ -w^k & \text{if AMR leg } k \text{ ends at node } n \\ 0 & \text{otherwise} \end{cases} \quad \forall n \in N, k \in K$$

J $(|\mathcal{L}| \times |K|)$ matrix where:

$$\mathbf{J}(L, k) = \begin{cases} 1 & \text{if leg } k \text{ belongs to AMR } L \\ 0 & \text{otherwise} \end{cases} \quad \forall L \in \mathcal{L}, k \in K$$

M very large constant

4.2.2 Assumptions and Constraints

The SNDP_{OPT} formulation models both the physical and operational aircraft constraints of the aircraft teams discussed in Section 1.1, and the AMR constraints

discussed in section 2.1. In addition, we ensure integrality of aircraft team and AMR flow through the network. We assume that the airspeed, loading/unloading time and refuel time of each aircraft team and the carrying capacity remain constant throughout the day. We also assume that HLZs have sufficient capacity for helicopter teams to land, load/unload, refuel or wait, as the number of aircraft teams that we consider in a typical planning window is not high enough to congest a typical HLZ. Furthermore, if we use a conservative estimate of flight times and refuel times, the schedule will be flexible enough to accommodate multiple landings or refueling operations at the same HLZ in the same time period.

4.2.3 Objective Function

The objective function establishes a tradeoff between value and cost. In order to compare the three models, we must ensure that they measure value and cost in the same way.

4.2.3.1 AMR Value

While AMRs are given a priority level, the Commander's Mission Priority List (CMPL) does not specify the nature of the tradeoff between AMRs of different priority, *tradeoff* being when we support several low priority AMRs at the expense of a high priority AMR. Since the CMPL is used as a guide by planners, its application can be quite subjective. A human planner uses experience and additional knowledge of current operations to determine how the tradeoff between AMRs of different priority is handled.

We separate AMR priorities into two tiers (several more may be used). The values given to AMRs in the higher tier are orders of magnitude higher than those given to AMRs in the lower tier. This prevents very high priority missions from being unsupported at the expense of a collection of lower priority missions. At the same time, there is flexibility within each tier to tradeoff AMRs. It allows AMR priorities to be modeled more realistically, and tailored to each commander's guidance. In this thesis, we consider six priority levels, with AMRs of priority 1-2 assigned to the first

tier, and AMRs of priority 3-6 assigned to the second tier (Figure 4-1).

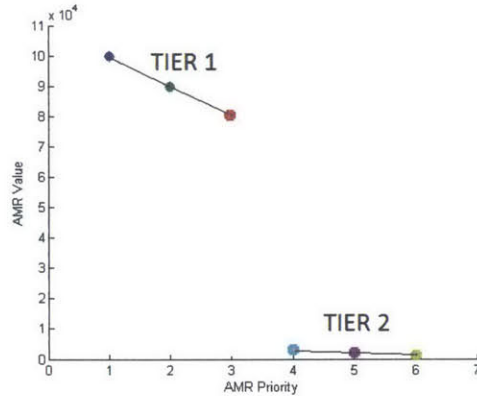


Figure 4-1: Conversion from AMR priority to value

The priority for each AMR L is translated into a value, v^L , which is earned if the AMR is supported, and is the sum of individual AMR leg values, v^k :

$$v^L = \sum_{k \in L} v^k \quad \forall L \in \mathcal{L}$$

For the purpose of our formulations, we divide the total value of an AMR equally between all of its legs, and add a constraint to ensure that AMRs must be supported in their entirety.

4.2.3.2 Aircraft and Passenger Costs

In the objective function, the total cost is dictated by the costs of aircraft usage, \bar{c}^f and \underline{c}^f , and the cost of passenger movement, c^k . The cost values that we consider are not a strictly monetary, but are relative to the scale of AMR values, v^L .

Aircraft Costs.

While we still want to gain the most value from the AMRs supported, we want to do so for the lowest cost in terms of aircraft hours. Thus, given two solutions with the same supported AMR value, the solution with the fewest flight hours should be selected, which minimizes aircraft costs.

The issue of weighting flight versus ground arcs presents itself as well. We choose to weight the cost of airborne time versus ground time according to the average fuel

burn rate in each mode, as the scheduled maintenance and crew costs are the same in either mode:

- *Scheduled maintenance* is dependent only on total flight hours, which Army regulations define as the time when the aircraft leaves the ground for the first time to the time when the engine(s) are shut down [4]. Therefore, there is no additional required maintenance cost for an aircraft that flies one hour, than for one that waits on the ground at engine idle for an hour.
- Crew costs are not increased with more flight hours, since these aircraft are operated by military crews.

Passenger Cost.

The passenger cost of an AMR is modeled as the total travel time. The cost of travel time, however, is subjective and depends on individual preferences, lost productivity and individual schedule tolerance. We assume that passengers prefer to spend the least amount of time flying as possible. Therefore, the minimum cost to passengers is simply ΔT^k for AMR leg k . If AMR leg k is being scheduled with total travel time above ΔT^k , the path will have a higher passenger cost, proportional to the additional flight time.

We choose not to add a cost penalty due to transshipment of passengers during an AMR leg because transshipment will already add additional required ground time, which will be counted in the total travel time cost. Thus, any transshipment of passengers is already penalized in the model.

4.2.3.3 Objective Function Formulation

We now propose an objective function that formalizes the tradeoff between the AMR value and cost. We assume an aviation planner prioritizes solutions as follows:

- First, maximize the total value of supported AMRs.
- If several solutions are of equal value, then select the one with the lowest aircraft cost.

- If several solutions are still equal, select the one with the lowest passenger cost.

$$\begin{aligned}
\text{Objective Function} &= \sum_{k \in K} v^k \delta^k - \\
&\sum_{f \in F} \left(\sum_{a \in \overline{A^f}} \bar{c}^f y_a^f \tau_a + \sum_{k \in K} c^k \zeta^k + \underline{c}^f \left(\sum_{a \in A_{WAIT}^f} g_a^f \tau_a + \sum_{a \in A_{LOAD}^f} v_a^f \tau_a + \sum_{a \in A_{FUEL}^f} r_a^f \tau_a \right) \right)
\end{aligned} \tag{4.22}$$

The following variables are restated for convenience:

- v^k is the value of AMR k
- δ^k is an indicator variable equal to 1 if AMR leg k is supported and 0 if not.
- \bar{c}^f is the cost of one time period of a flight arc for aircraft team f .
- \underline{c}^f is the cost of one time period of a ground arc for aircraft team f .
- y_a^f is a binary decision variable indicating whether aircraft team f has flight arc a in its route.
- g_a^f is a binary decision variable indicating if aircraft team f has waiting arc a in its route.
- v_a^f is a binary decision variable indicating if aircraft team f has loading/unloading arc a in its route.
- r_a^f is a binary decision variable indicating if aircraft team f refueling arc a in its route.
- τ_a is the duration of arc a . c^k is the passenger cost of one time period for AMR leg k .
- ζ^k is the total travel time of AMR leg k .

4.2.4 SNDP_{OPT} Model Formulation

SNDP_{OPT} =

$$\begin{aligned} & \max \sum_{k \in K} v^k \delta^k - \\ & \sum_{f \in F} \left(\sum_{a \in \bar{A}^f} \bar{c}^f y_a^f \tau_a + \sum_{k \in K} c^k \zeta^k + \underline{c}^f \left(\sum_{a \in A_{WAIT}^f} g_a^f \tau_a + \sum_{a \in A_{LOAD}^f} v_a^f \tau_a + \sum_{a \in A_{FUEL}^f} r_a^f \tau_a \right) \right) \end{aligned} \quad (4.23)$$

s.t.

$$\sum_{k \in K} w^k x_a^{k,f} \leq \sum_{f \in F} u^f y_a^f \quad \forall f \in F, a \in \bar{A}^f \quad (4.24)$$

$$\sum_{a \in \bar{A}^f} y_a^f \tau_a^f + \sum_{a \in A_{WAIT}^f} g_a^f \tau_a^f + \sum_{a \in A_{LOAD}^f} v_a^f \tau_a^f + \sum_{a \in A_{FUEL}^f} r_a^f \tau_a^f \leq \tau_{CYCLE}^f \quad \forall f \in F \quad (4.25)$$

$$\sum_{f \in F} \sum_{a \in A^f} \mathbf{D}(n, a) \left(\sum_{f \in F} x_a^{k,f} w^k + \hat{x}_a^k w^k \right) = \mathbf{B}(k, n) \quad \forall n \in N, k \in K \quad (4.26)$$

$$\begin{aligned} \mathbf{D}^f(n, a) \left(\sum_{a \in \bar{A}^f} y_a^f + \sum_{a \in A_{WAIT}^f} g_a^f + \sum_{a \in A_{FUEL}^f} r_a^f + \sum_{a \in A_{LOAD}^f} v_a^f + \sum_{a \in A_{SD}^f} s_a^f \right) &= \mathbf{E}(f, n) \\ & \forall n \in N, f \in F \end{aligned} \quad (4.27)$$

$$\sum_{t=t_i}^{t_j} \left(\sum_{a \in \bar{A}_{i,t}^f} y_a^f \tau_a + \sum_{a \in A_{WAIT,t}^f} g_a^f \tau_a + \sum_{a \in A_{LOAD,t}^f} v_a^f \tau_a \right) \leq \tau_{FUEL}^f \left(\sum_{t=t_i}^{t_j} \sum_{a \in A_{FUEL,t}^f} r_a^f \tau_a + 1 \right)$$

$$\forall t_i = (t_{START}^f) \dots (t_{END}^f - \tau_{FLIGHT}^f), \forall t_j = (t + t_{FLIGHT}^f) \dots (t_{END}^f), f \in F \quad (4.28)$$

$$\begin{aligned} \sum_{a \in \bar{A}_t^- \cup \bar{A}_t^+} y_a^f &\leq \sum_{a \in A_{LOAD,t}^{f-} \cup A_{LOAD,t}^{f+}} v_a^f + \sum_{a \in A_{FUEL,t}^{f-} \cup A_{FUEL,t}^{f+}} r_a^f + \sum_{a \in A_{SD,t}^{f-} \cup A_{SD,t}^{f+}} s_a^f \\ & \forall t = 1 \dots m-1, f \in F \end{aligned} \quad (4.29)$$

$$\sum_{f \in F} \sum_{a \in \bar{A}_t^-} x_a^{k,f} + \sum_{f \in F} \sum_{a \in \bar{A}_t^+} x_a^{k,f} \leq 1 \quad \forall k \in K \quad (4.30)$$

$$\sum_{f \in F} x_{a_i}^{k,f} + \hat{x}_{a_j}^k \leq 1 \quad \forall a_i, a_j \in A, k \in K \quad (4.31)$$

$$\sum_{k \in K} \sum_{a \in \bar{A}_{(b^{k+}, t^{k+})}^+} x_a^{k,f} \leq M \sum_{a \in A_{LOAD,t}^-} v_a^f \quad \forall t = 1 \dots (t_{END}^f - \tau_{FUEL}^f), k \in K, f \in F \quad (4.32)$$

$$\sum_{k \in K} \sum_{a \in \bar{A}_{(b^{k-}, t^{k-})}^-} x_a^{k,f} \leq M \sum_{a \in A_{LOAD,t}^+} v_a^f \quad \forall t = 1 \dots (t_{END}^f - \tau_{FUEL}^f), k \in K, f \in F \quad (4.33)$$

$$\sum_{k \in K} \delta^k \mathbf{J}(L, k) = \theta^L \rho^L \quad \forall L \in \mathcal{L} \quad (4.34)$$

$$\delta^k = \sum_{f \in F} \sum_{a \in A_{i^{k,dep}}^-} x_a^{k,f} \quad \forall k \in K \quad (4.35)$$

$$\zeta_{start}^k = \sum_{a \in A_{b^k}^-} x_a^{k,f} \quad \forall k \in K, f \in F \quad (4.36)$$

$$\zeta_{end}^k = \sum_{a \in A_{b^k}^+} x_a^{k,f} \quad \forall k \in K, f \in F \quad (4.37)$$

$$\zeta^k = \zeta_{end}^k - \zeta_{start}^k \quad \forall k \in K \quad (4.38)$$

$$x_a^{k,f}, \hat{x}_a^k, y_a^f, g_a^f, w_a^f, s_a^f, r_a^f, \theta^L, \delta^k, \zeta_{start}^k, \zeta_{end}^k, \zeta^k \in \{0, 1\}$$

$$\forall k \in K, a \in A, f \in F, L \in \mathcal{L} \quad (4.39)$$

The objective function (4.23) maximizes the total value of supported AMR legs minus the cost of the aircraft team arcs and passenger flow. Constraints (4.24) restrict the flow of personnel over all flight arcs to the capacity of aircraft going over those arcs. This capacity constraint does not exist on the ground for we assume there is enough space for personnel to load/unload and wait for flights. Constraints (4.25) ensure that the total flight time of each crew does not exceed their allowed maximum

flight time per planning cycle. Flight time is only counted during flight, waiting, loading/unloading, and refueling arcs, and is not counted for shutdown arcs.

Constraints (4.26) conserve the flow of passengers at each node. Constraints (4.27) conserve the flow of aircraft teams at each node. Constraints (4.28) ensure that aircraft follow *fuel-feasible paths*. This is done by ensuring that all path segments of duration greater than τ_{FLIGHT}^f (minus shutdown arcs) have enough refueling arcs. If any paths are not fuel feasible, then they will have at least one path segment that violates one of these constraints. Constraints (4.29) ensure that there are never two successive flight arcs for the same aircraft team.

Constraints (4.30) ensure that AMR legs are never scheduled on two successive flight arcs to prohibit instantaneous transshipment. Constraints (4.31) ensure that AMR legs are either supported or unsupported in their entirety, and that every time period of each supported AMR leg is only supported by one aircraft team. Constraints (4.32) and (4.33) ensure that there is at least a loading/unloading arc, or a refuel arc, before and after every AMR flight arc.

Constraints (4.34) ensure that AMRs are supported or unsupported in their entirety (either all legs supported or none). Constraints (4.35) define an indicator variable for the completion of each AMR leg. Constraints (4.36), (4.37), and (4.38) define the total travel time for each AMR. Constraints (4.39) are binary constraints on the decision variables and ensure that AMRs and aircraft teams are not split up while moving through the network.

4.3 Path-Flow Heuristic using Composite Variables

Solving the SNDP as an integer program presents several issues. First, the number of decision variables and constraints increase exponentially as the number of nodes in $G(N, A)$ increase linearly. Second, the LP-relaxation of this formulation yields fractional aircraft arcs and AMR flow, and provides a poor lower bound on the optimal integer solution (Nielsen [12]).

We adopt a *composite variable* approach to mitigate some of these issues. The

idea is to combine design decision variables and commodity decision variables into a single decision variable. Many authors have conducted extensive research into the use of this approach for optimizing large-scale network design problems. Armacost et. al. [3] use composite variables in formulating express shipment service network design problems. Barnhart et. al. [5] present a model to solve the airline fleet assignment problem using composite variables. Cohn [10] explores techniques that are used to limit the number of composite variables, such as real-world operational rules and the branch-and-price algorithm. As an application of composite variables to Air Mobility Command's (AMC) channel route planning, Nielsen [12] uses a Composite Variable Formulation (CVF) in addition to column generation techniques.

4.3.1 Composite Variable Formulation

To solve the SNDP using composite variables, we define the feasible set of *aircraft team path* composite variables \mathcal{P} , where each variable $p^{f,\lambda}$ in the set defines a specific aircraft team route of flight, and also AMR legs supported by that team. Each path composite variable $p^{f,\lambda}$ belongs to aircraft team f and is indexed by λ , and has value $v^{f,\lambda}$. It has additional parameters, $\mathbf{q}^{f,\lambda}$, $A^{f,\lambda}$ and $A^{k,\lambda}$, which indicate the route of flight and AMR schedule for that path (Table 4.1).

<i>Set</i>	<i>Subset</i>	<i>Element</i>	<i>Parameters</i>	<i>Description</i>
\mathcal{P}	P^f	<i>aircraft team path composite variable</i>		
		$p^{f,\lambda}$	λ	number of path
			$v^{f,\lambda}$	value of path
			$c^{f,\lambda}$	cost of path
			$\mathbf{q}^{f,\lambda}$	vector of dimension $ K $, where $\mathbf{q}^{f,\lambda}(k) = \begin{cases} 1 & \text{if AMR leg } k \text{ is supported} \\ 0 & \text{otherwise} \end{cases}$
			$A^{f,\lambda}$	set of arcs on path for team f
			$A^{k,\lambda}$	set of arcs on path for AMR leg k

Table 4.1: Composite variable parameters

4.3.2 Path Generation Heuristic Algorithm

The problem of scheduling aircraft teams and AMRs remains complex, even after introducing composite variables, since the number of paths to consider in the set \mathcal{P} is very large. As opposed to using generic aircraft routes to solve the Fleet Assignment Model (FAM) (Barnhart [7]), or using the AMC channel route structure to narrow the list of feasible paths (Nielsen [12]), daily AMR scheduling and route planning does not have a set of fixed daily routes to serve as a template for paths. HLZs can be visited by aircraft teams in any order and at any time.

Instead of searching for the entire set of feasible paths \mathcal{P} , we use a heuristic approach to generate a smaller set, \mathcal{P}^* , of good paths for each aircraft team. We further use several complexity reduction techniques (section 4.3.2.3) to reduce the total runtime of the algorithm.

4.3.2.1 Algorithm Overview

We use a recursive path generation heuristic algorithm (PGHA), to generate feasible paths. An example is illustrated in Figure 4-2. The algorithm follows a depth-first with backtracking methodology, using the following rules for branching:

- At each node, check if any time, fuel, or capacity constraints are violated. If no constraints are violated:
 - If no AMR legs are on board, create a branch to return aircraft team to its final destination base, and store completed path in P^f . Backtrack along the tree to the last node where there remains unexplored branches.
 - Create a new branch to pick up each AMR leg that can be supported from the current time and position of the aircraft team.
 - Create a new branch to drop off each AMR leg that is currently on board.
- Otherwise, discard current path, and backtrack along the tree to the last node where there remains unexplored branches.

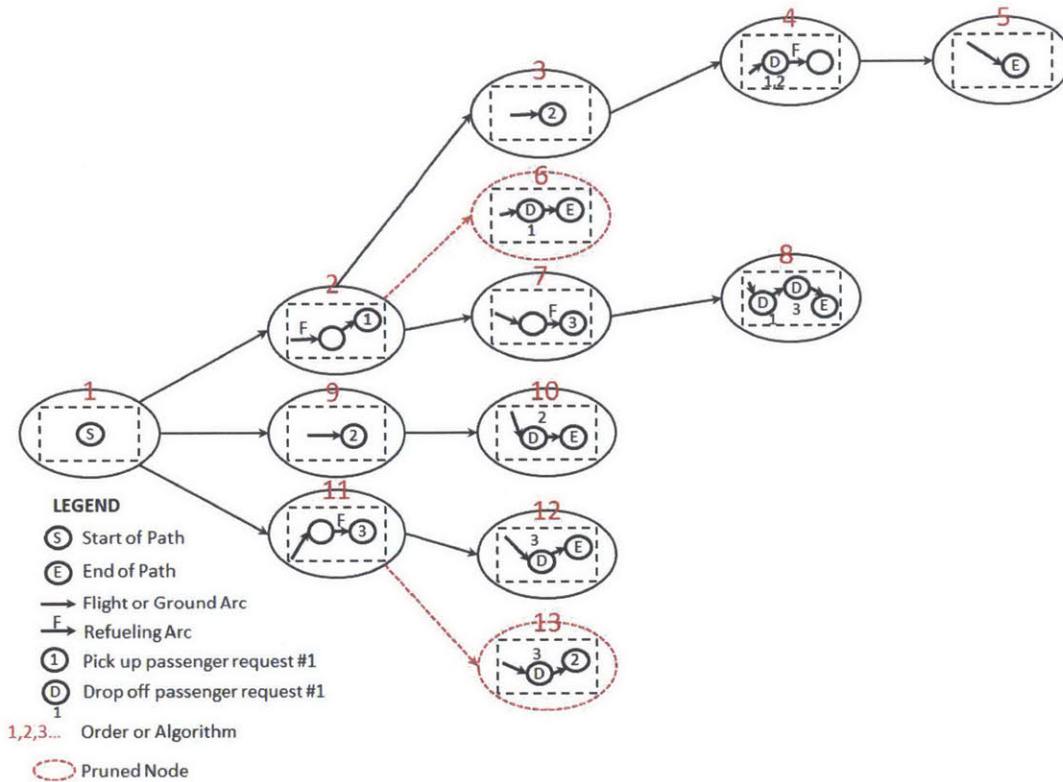


Figure 4-2: Example showing the generation of feasible paths

In Figure 4-2, node (1) is created for an aircraft team start base and time. Next, three alternative paths are considered based on requested AMR legs. In considering the first alternative, we create nodes (2) and (3) to refuel then pick up AMRs 1 and 2. This leads to another decision point where we have two alternatives. We first create nodes (4) and (5), which drops off AMRs 1 and 2 before going to the ending HLZ for the aircraft team.

We now have a feasible aircraft team path described by nodes (1) to (5) which is stored in memory. Then, we consider the previous decision point that had additional alternatives. From node (2), we create node (6). However, let us assume for this example that this path violates a model constraint, such as fuel, time, or aircraft capacity. Node (6) is then pruned from the tree. This process continues until all branches are either completed or pruned. The final result in this example is five feasible aircraft team paths described by unique root-to-leaf paths in the tree.

Figure 4-3 shows how a tree path from root to leaf is decomposed into flight legs for the aircraft team and AMRs, forming an aircraft team route of flight and AMR schedule. AMR passenger flow is obtained from pickup and drop-off information contained in the selected nodes of the tree, and the completed path is stored as a composite variable. The bold arrows represent aircraft arcs through $G(N, A)$ and the thin arrows represent AMR passenger flow on those arcs.

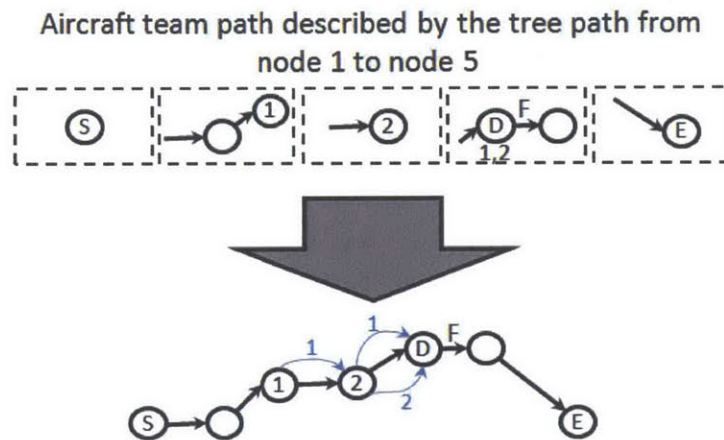


Figure 4-3: Example showing a feasible path

Throughout the execution of the algorithm, a current aircraft team path is kept in memory along with all AMR legs that are supported by that team. The number of possible branches created depends on the number of remaining AMRs that can be supported, constrained by the team's availability window and remaining capacity, and by the number of AMRs currently on board that still need to be taken to their destination HLZ. When no additional AMRs can be picked up or dropped off, the current branch is terminated, the current path is added to the list of feasible paths P^f , and the algorithm returns to a previous decision point in the recursive tree. After completing each decision, the current path is inspected for feasibility, and the current branch is pruned if any constraints are violated. In this way, the algorithm cycles recursively through a tree for each aircraft team f , where paths from root to leaf are feasible.

4.3.2.2 Algorithm Description

The PGHA algorithm is shown in Algorithm 1. The algorithm's main procedure is $\text{PGHA}(f, K)$, which returns a set of feasible paths P^f for team f , given a set of AMR legs K .

Algorithm 1 Path Generation Heuristic Algorithm (PGHA)

```

1  Procedure PGHA( $f, K$ ) {
2      Global variable  $P^f := \{empty\}$ 
3      Local variable  $p := \{empty\}$ 
4      Branch( $f, p, K$ )
5      return  $P^f$ 
6  end procedure }

1  Procedure Branch( $f, p, K$ ) {
2      if no AMR legs in  $K$  {
3          add path segment to  $p$ , from current node to ending node for  $f$ 
4      end if }
5      for all AMR legs  $k$  no on board  $f$  do {
6          if  $k$  can be picked up {
7              add path segment to  $p$ , from current node to earliest departure
              node for  $k$ 
8              add  $k$  to list of AMRs on board  $f$ 
9              if no fuel, time, or capacity constraints are violated {
10                 Branch( $f, p, K$ )
11             end if }
12         end if }
13     end do }
14     for  $k$  on board  $f$  do {
15         if  $k$  can be dropped off {
16             add path segment to  $p$ , from current node to droppoff HLZ of  $k$ 
17             remove  $k$  from list of AMRs on board  $f$ 
18             remove  $k$  from  $K$ 
19             if no more AMRs on board  $f$  {
20                 add path segment to  $p$ , from current node to ending
                node for  $f$ 
21                 if  $p$  has a higher value than other paths executing the
                same AMRs {
22                     remove other paths in  $P^f$  executing the same AMRs
23                     add  $p$  to  $P^f$ 
24                 else if  $p$  is the only path executing its set of AMRs {
25                     add  $p$  to  $P^f$ 
26                 end if }
27             end if }
28             Branch( $f, p, K$ )
29         end if }
30     end do }
31     return
32 end procedure }

```

The recursive procedure used by the algorithm is **Branch**(f, p, K), which constructs the path segments exemplified above in Figure 4-2. If a flight leg requires additional fuel to pick up or drop off an AMR, we solve a shortest path sub-problem to determine the best HLZ for refuel. Its solution provides the refuel HLZ that minimizes the deviation from the current route of flight. Fuel, remaining capacity, path value, and path cost are tracked in the path composite variable at each step of the algorithm, and updated whenever an arc is added to the current path or an AMR leg is completed. The full pseudo-code for the PGHA is provided in Appendix A.

4.3.2.3 Path Reduction Techniques

Several techniques are used to limit the number of branches explored and paths created:

1. We only create path segments needed to directly support AMR legs, refuel, or return to the aircraft home HLZ.
2. The shortest path sub-problem only picks one refuel HLZ, and only when refuel is necessary to support an AMR.
3. When supporting an AMR, the algorithm does not branch to create several different pick-up times, reducing the number of paths created. AMR pickups and drop-offs are scheduled as close to the beginning of the travel window of each leg as is feasible, based on the assumption that aircraft teams maximize their chances of picking up more requests if they do not wait before picking up or dropping off AMRs.
4. We only store in P^f the best path for each feasible sequence of supported AMR legs. For example, if aircraft team path $p^{f,1}$ supports the sequence of AMRs $\{1, 5, 6\}$, and path $p^{f,2}$ supports the sequence of AMRs $\{5, 1, 6\}$, then the path with the highest value will be kept, and the other discarded.

Unlike paths created by solving SNDP_{OPT} , the PGHA does not consider transshipment of an AMR between different aircraft teams in the middle of a leg. While this is feasible in practice, it may make the solution less robust in the face of unforecasted

delays, because of logistics issues that arise in the event of weather or maintenance delays. During the execution phase of the flight schedule, solutions that supports each AMR leg with a single team will not be impacted by delays to other aircraft teams. We also assume that it is better for an AMR to not be supported than to be dropped off at an HLZ other than its destination. Passengers may be left for an extended period of time at a foreign location with little logistics support.

4.3.2.4 Important Algorithm Property

The algorithm PGHA determines a set of feasible paths P^f , given an aircraft team f , and set of AMR legs K . At any given iteration of **Branch**(f, p, K), the procedure calls itself once to pick up every AMR leg available, and once to drop off every AMR leg on board. Therefore, if at a given iteration there are three remaining feasible AMR legs, and two AMR legs on board, the algorithm will explore five new branches. As a result, we can state the following important property of PGHA:

Given **PGHA**(f, K) returns P^f , and **PGHA**(f, K') returns P'^f

$$\text{If } K' \subseteq K, \text{ then } P'^f \subseteq P^f \tag{4.40}$$

This property says that if the set of AMR legs K' is a subset of K , then all paths contained in P'^f are also contained in P^f .

4.3.3 Set-Packing Problem Formulation

Once we select a set of heuristically generated paths \mathcal{P}^* for consideration, we must choose the optimal subset of paths such that each aircraft team is assigned one path at most, and each AMR leg is supported no more than once, and if an AMR leg is supported, all legs from that AMR are also supported. This problem can be formulated as a weighted variant of the *maximum set packing problem* (MSSP), which we call MSPP_H. Given each path has a set of supported AMR legs and associated path value, the set packing problem picks a path for each aircraft team such that all sets of supported AMR legs are pairwise disjoint (in other words, no two share

a common AMR leg), and total value is maximized. Recall from Chapter 2 that we want to ensure that if an AMR is supported, then all AMR legs from that AMR are supported (not necessarily by the same aircraft team). Therefore, we specify this as an additional constraint in the set packing problem.

Decision variables and data structures are defined as follows:

Binary Decision Variables

$$z^{f,\lambda} = \begin{cases} 1 & \text{if path } \lambda \text{ is selected for team } f \\ 0 & \text{otherwise} \end{cases}$$

$$\delta^k = \begin{cases} 1 & \text{if AMR leg } k \text{ is supported} \\ 0 & \text{otherwise} \end{cases}$$

$$\theta^L = \begin{cases} 1 & \text{if AMR } L \text{ is supported} \\ 0 & \text{otherwise} \end{cases}$$

The following notation is restated for convenience:

ρ^L number of AMR legs belonging to AMR L

\mathbf{J} $(|J| \times |K|)$ matrix where:

$$\mathbf{J}(L, k) = \begin{cases} 1 & \text{if leg } k \text{ belongs to AMR } L \\ 0 & \text{otherwise} \end{cases}$$

$\mathbf{q}^{f,\lambda}$ vector of length $|K|$ indicating which AMR legs are supported on path $p^{f,\lambda}$ where:

$$\mathbf{q}^{f,\lambda}(k) = \begin{cases} 1 & \text{if leg } k \text{ is supported on path } p^{f,\lambda} \\ 0 & \text{otherwise} \end{cases}$$

Maximum Set Packing Problem Formulation

$$\text{MSPP}_H = \max \sum_{f \in F} \sum_{p^{f,\lambda} \in \mathcal{P}^f} z^{f,\lambda} (v^{f,\lambda} - c^{f,\lambda}) \quad (4.41)$$

s.t.

$$\sum_{p^{f,\lambda} \in \mathcal{P}^f} z^{f,\lambda} \leq 1 \quad \forall f \in F \quad (4.42)$$

$$\sum_{p^{f,\lambda} \in \mathcal{P}^*} z^{f,\lambda} \mathbf{q}^{f,\lambda}(k) \leq 1 \quad \forall k \in K \quad (4.43)$$

$$\sum_{k \in K} \delta^k \mathbf{J}(L, k) = \theta^L \rho^L \quad \forall L \in \mathcal{L} \quad (4.44)$$

$$z^{f,\lambda} \in \{0, 1\} \quad \forall p^{f,\lambda} \in \mathcal{P}^* \quad (4.45)$$

$$\theta^L \in \{0, 1\} \quad \forall L \in \mathcal{L} \quad (4.46)$$

$$\delta^k \in \{0, 1\} \quad \forall k \in K \quad (4.47)$$

The objective function (4.41) maximizes the value of the total missions flown minus the cost of those missions. Constraints (4.43) ensure that only one path is selected for each aircraft team. Constraints (4.42) ensure that each passenger request is supported at most by one aircraft team. Constraints (4.44) ensure that AMRs are supported or unsupported in their entirety. Constraints (4.47) ensure that paths are either selected or not in their entirety. Constraints (4.46) and (4.47) impose binary constraints on AMR and AMR leg decision variables.

4.3.4 Maximum Total Return Algorithm

The Maximum Total Return Algorithm (MTRA, Algorithm 2) generates a set of feasible paths \mathcal{P}^* by running the PGHA on the entire set of AMR legs, K , for each aircraft team, f , then solving MSPP_H to select the best paths.

Algorithm 2 Maximum Total Return Algorithm (MTRA)

```
1   for each aircraft team  $f \in F$  do {
2       if  $f$  is identical to another aircraft team  $\hat{f}$  for which  $P^{\hat{f}}$  has
           already been computed {
3            $P^f := P^{\hat{f}}$ 
4       else
5            $P^f := \text{PGHA}(f, K)$ 
6       end if }
7   end do }
8   solve  $\text{MSPP}_H$  using the set
    $\mathcal{P}^* = \{P^f, \forall f \in F\}$  which returns  $p^{f,\lambda}$  for each team  $f$ 
```

4.3.5 Maximum Marginal Return Algorithm

The Maximum Marginal Return Algorithm (MMRA) is a greedy algorithm for AMR scheduling and route planning. The intuition behind this algorithm is that when the set of requested AMR legs becomes large, generating paths may require significant computation time due to the recursive nature of the Path Generation Heuristic Algorithm (PGHA). Therefore, instead of calling the PGHA once for each aircraft team on the entire set K , as we do for MTRA, we call it several times on smaller subsets of K . The algorithm works as follows:

- We divide the set of AMRs into subsets, such that the subsets are ordered from highest value AMRs to lowest.
- We generate feasible paths from the first subset, using PGHA, then select a path for each aircraft team using MSPP_H . We discard any AMR legs that are not supported in the current solution.
- The set of supported AMRs is taken from the current solution, and augmented with the next subset of AMRs that have not yet been considered.
- New paths are generated using PGHA, and a new solution is computed using MSPP_H . We discard any AMRs not supported in the newest solution.

- We continue this process until all subsets of AMRs have been added in, and select the last solution generated by MSPP_H as the final solution.

In this manner, we sequentially re-optimize using different sets of AMRs. Any AMRs that improve the objective function are kept, while AMRs that do not are discarded. Once an AMR has been discarded, it is not reconsidered in future iterations. MMRA is shown in Algorithm 3.

Algorithm 3 Maximum Marginal Return Algorithm (MMRA)

```

1   divide set of AMRs  $\mathcal{L}$  into  $n$  pairwise independent ordered value sub-sets  $\mathcal{L}_i$ , such that
       $\mathcal{L} = \mathcal{L}_1 \cup \mathcal{L}_2 \cup \dots \cup \mathcal{L}_n$ ,  $v^{\mathcal{L}_i} \geq v^{\mathcal{L}_{i+1}}$  and each sub-set  $\mathcal{L}_i$  contains all
      AMR legs  $k \in L$  for each AMR  $L \in \mathcal{L}_i$ 
2    $\mathcal{L}^* := \{empty\}$ 
3    $v^* := 0$ 
4    $p^{f*} := \{empty\} \forall f \in F$ 
5   for  $i := 1$  to  $n$  do {
6      $\mathcal{L}' := \mathcal{L}^* \cup \mathcal{L}_i$ 
7     for each aircraft team  $f \in F$  do {
8        $P'^f := \mathbf{PGHA}(f, K')$  where  $K'$  is the set of AMR legs in  $J'$ 
9       solve MSPPH using the set  $\mathcal{P}^* = \{P'^f, \forall f \in F\}$ ,
          returns  $p^{f,\lambda}$  for each team  $f$ 
10    end do }
11    if  $\sum_{f \in F} (v^{f,\lambda} - c^{f,\lambda}) > v^* \{$ 
12       $p^{f*} := p^{f,\lambda} \forall f \in F$ 
13       $v^* := \sum_{f \in F} (v^{f,\lambda} - c^{f,\lambda})$ 
14       $\mathcal{L}^* := \text{set of supported AMR legs } \forall p^{f*}$ 
15    end if }
16  end do }

```

4.3.6 Analytic Solution Comparison

We show below that the solution value provided by MMRA cannot exceed the solution value provided by MTRA.

1. Recall from Property (4.40) that if $K' \subseteq K$, then $P^{f'} \subseteq P^f$, as generated by PGHA. Thus, the set of paths generated in MTRA by PGHA with inputs f and K contains all paths generated in MMRA with inputs f and any subset of K .
2. It follows that if $p^{f,\lambda} \subseteq P^{f'}$, then $p^{f,\lambda} \subseteq P^f$. Every path contained in $P^{f'}$ is also contained in P^f .
3. These results follow for every aircraft team f .
4. Let $Z_{MTRA}^{F,K}$ be the value returned by MTRA, and let $Z_{MMRA}^{F,K}$ be the value returned by MMRA, for a set of aircraft teams F , and a set of AMR legs K .
5. For Given sets F and K , MTRA solves MSPP_H for each set of paths P^f , and MMRA solves MSPP_H for a subset of paths $P^{f'} \subseteq P^f$. It follows that:

$$Z_{MMRA}^{F,K} \leq Z_{MTRA}^{F,K} \quad (4.48)$$

We have shown that for a particular set of aircraft teams and AMR legs, the value returned by MMRA cannot exceed the value returned by MTRA. This makes sense intuitively, as we would expect a solution given by the greedy approach, which successively considers subsets of K , to be lower than an approach that considers all of K .

4.4 Chapter Summary

In summary, we formulated the problem of AMR scheduling and aircraft team route planning for Army Aviation as a *service network design problem*, SNDP_{OPT} , and as *maximum set packing problem*, MSPP_{H} , given a set of heuristically generated aircraft team path composite variables from the PGHA. We then proposed two algorithms to generate a heuristic solution: the Maximum Marginal Return Algorithm and the

Maximum Total Return Algorithm. MMRA is a greedy algorithm that iteratively considers subsets of total demand, while MTRA considers the entire set of AMR demand in one iteration (Figure 4-4).

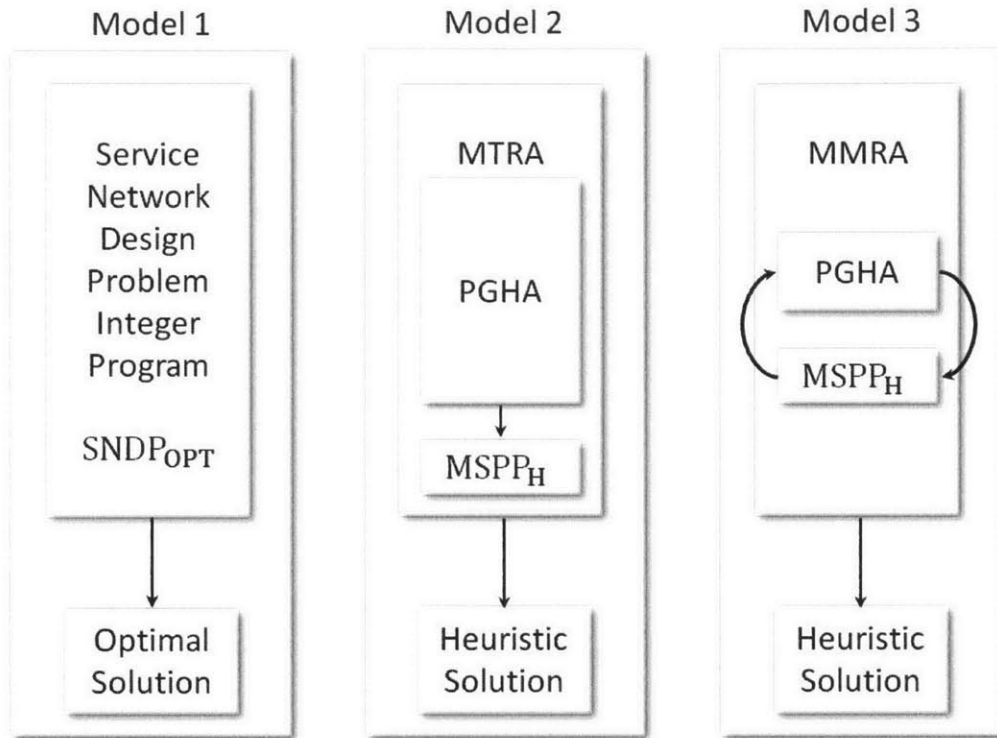


Figure 4-4: Comparison of modeling approaches

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Chapter 5

Computational Analysis

In this chapter, we evaluate the three models described in chapter 4 for computational efficiency. All models were run on an Intel i73770k Windows 8 computer system using Python 2.7 as the programming language, PuLP as the Integer Program modeler and IBM ILOG CPLEX[®] as the commercial Integer Program solver.

5.1 User-Defined Parameters

The models developed in Chapter 4 require the user to input parameters to model the physical characteristics of the aircraft teams, AMRs, and time-space graph $G(N, A)$. Of these parameters, we treat certain ones as *fixed*, and others as *variable*.

5.1.1 Fixed Parameters

Fixed parameters model the duration of a time period, cost parameters, and certain characteristics of an aircraft team. They do not change between test cases, and reflect fixed operational or model data. Table 5.1 shows the list of fixed parameters and their values. We select $\tau = 10 \text{ minutes}$ as the unit time period. All flying, waiting, refueling and shutdown times are modeled in increments of 10 minutes. We model aircraft teams with similar characteristics to a team of UH-60 Blackhawk helicopters. Accordingly, we select the team capacity, v^f , as 40 passengers, and the cost of fuel for one flight time period, \bar{c}^f , equal to the cost of three ground periods ($3\underline{c}^f$). This

models an average fuel consumption rate difference between these two modes. The cost of one flight time period is normalized at 1, with the cost of AMR flow, c^k , set equal to 1/10. This ensures that AMR flow is weighted the least in our objective function. The maximum flight time per tank of fuel, $\bar{\tau}_{FLIGHT}^f$, corresponds to the average flight time available on one tank of fuel at cruising airspeed, without cutting into the required fuel reserve time of 20 minutes in visual meteorological conditions (VMC). We opt for a conservative estimate, and, for our purposes, we assume a constant fuel burn rate in all modes of flight, while in reality aircraft use less when on the ground than when in the air. The refuel time, $\bar{\tau}_{FUEL}^f$, and the passenger loading and unloading time, $\bar{\tau}_{LOAD}^f$, are given average operational values based on the author's experience. The minimum shutdown time, τ_{SD}^f , is set at 30 minutes and can be changed to reflect different operational needs. Since it usually takes up to 15 minutes to shutdown or start an aircraft, aircrews will only want to shutdown if they have at least an additional 15 minutes on the ground. Otherwise, we assume they will stay on board with the engines running and wait.

<i>Parameter</i>	<i>Description</i>	<i>Value</i>
τ	unit time period	10 minutes
c^k	cost of AMR flow per time period	$\frac{1}{10}$
\bar{c}^f	cost of air arc per time period	1
\underline{c}^f	cost of a ground arc per time period	$\frac{1}{3}$
v^f	passenger capacity	40 seats
τ_{SD}^f	minimum shutdown time	30 minutes
$\bar{\tau}_{FLIGHT}^f$	max flight time per tank of fuel	120 minutes
$\bar{\tau}_{FUEL}^f$	refuel time	20 minutes
$\bar{\tau}_{LOAD}^f$	passenger loading/unloading time	10 minutes

Table 5.1: Fixed parameters

5.1.2 Variable Parameters

The overall problem size is dictated by variable parameters. They are changed between tests or between iterations of the same experiment. Table 5.2 shows the list of variable parameters. The Characteristics of $G(N, A)$, such as the number of time

periods, m , and the number of HLZs, β are variable parameters. In addition, we consider the number of aircraft teams, γ , the number of AMR legs, ρ , the number of passengers per AMR, w^k , and schedule tolerance, ϕ^k , as variable parameters. From Chapter 3, the schedule tolerance of an AMR leg is the degree of flexibility that passengers have on either end of their AMR leg, and is defined by the travel window of an AMR leg minus the shortest feasible flight time for that AMR leg.

<i>Parameter</i>	<i>Description</i>
m	number of time periods
β	number of HLZs
γ	number of aircraft teams
ρ	number of AMR legs
w^k	passengers per AMR leg
ϕ^k	schedule tolerance

Table 5.2: Variable parameters

5.1.3 Demand Distribution

Given a certain level of AMR demand, certain AMR parameters are randomly generated. These parameters reflect the specifics of the AMR, such as the number of passengers, the starting and ending HLZs, and travel window. In the test cases in this chapter, AMR parameters for each AMR L and AMR leg k are uniformly distributed as shown in Table 5.3, unless stated otherwise for different test cases.

<i>AMR Parameter</i>	<i>Description</i>	<i>Value</i>
w^k	number of passengers on AMR leg k	$U \sim (1, 20)$
v^k	value of AMR leg k	$U \sim \{10, 9, 8, 0.3, 0.2, 0.1\} \times 10^4$
b^{k-}	starting HLZ of AMR leg k	$U \sim (\text{All HLZs})$
b^{k+}	ending HLZ of AMR leg k	$U \sim (\text{All HLZs})$
t^{k-}	earliest departure time from starting HLZ	$U \sim (\text{All time periods})$
t^{k+}	latest arrival time at ending HLZ	$t^{k-} + \Delta T^k + \phi^k, \text{ s.t. } t^{k+} \leq t_m$
ρ^L	number of legs belonging to AMR L	$U \sim \{1, 2\}$

Table 5.3: Randomly generated AMR parameters

5.2 Sensitivity Analysis

In this section, we conduct a sensitivity analysis to determine the effect of different variable parameters on algorithm run-time.

5.2.1 Test 1: Increase in Network Size

We run SNDP_{OPT} , MTRA, and MMRA on test scenarios of increasing network size, by varying both the number of time periods and the number of HLZs (Table 5.4). We maintain the number of teams, AMR legs, passengers per AMR, and AMR schedule tolerance constant.

<i>Parameter</i>	<i>Description</i>	<i>Value</i>
m	number of time periods	<i>variable</i>
β	number of HLZs	<i>variable</i>
γ	number of teams	1
ρ	number of AMR legs	3
w^k	passengers per AMR leg	8
ϕ^k	schedule tolerance	30 min

Table 5.4: Test 1 variable parameters

Figure 5-1 shows the relative increase in run-time for SNDP_{OPT} per change in these variables

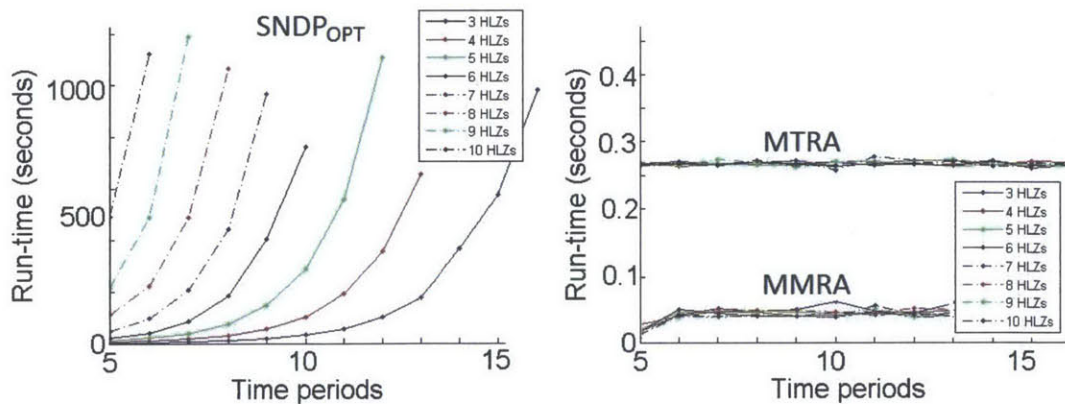


Figure 5-1: Computational performance vs. network size

Solving the service network design problem with SNDP_{OPT} results in an exponen-

tial increase in run-time, when the number of time periods and or number of HLZs is increased (Figure 5-1). While the number of decision variables increases linearly, the number of constraints approximately doubles for every additional time period and HLZ (Table 5.5). We are not able to compute a solution for problems that have over one million constraints, so this model scales poorly to larger problems.

The Maximum Total Return Algorithm (MTRA) and Maximum Marginal Return Algorithm (MMRA), however, show a constant run-time when the number of time periods and HLZs increases, with MMRA showing much faster run-times than MTRA.

<i>Time Periods</i>	<i>HLZs</i>	<i>AMR legs</i>	<i>Decision Variables</i>	<i>Constraints</i>	<i>run-time (sec)</i>
5	3	3	678	5,719	2.02
5	4	3	1,122	16,682	4.87
5	5	3	1,622	35,987	10.50
5	6	3	2,186	66,154	21.14
5	7	3	2,994	126,651	46.06
5	8	3	3,950	222,682	110.82
5	9	3	5,038	366,307	213.94
5	10	3	6266	571,386	491.78
10	3	3	2,638	95,374	31.17
10	4	3	4,002	225,717	102.44
10	5	3	5,562	443,052	285.69
10	6	3	7,336	775,099	763.26
10	7	3	9,484	1,307,986	intractable
11	6	3	8,870	1,139,371	intractable
13	5	3	9,606	1,343,585	intractable
14	4	3	8,322	1,002,952	intractable
15	3	3	6,698	643,613	575.51
16	3	3	7,762	868,881	981.80
17	3	3	8,910	1,150,077	intractable

Table 5.5: SNDP_{OPT} computational statistics

Figure 5-2 shows the time period vs. HLZ tractability region for SNDP_{OPT}, where we consider a problem with one aircraft team and 3 AMR legs. In this case, we consider any problem “intractable” when we are not able to solve it with our current computer configuration (described in the chapter introduction).

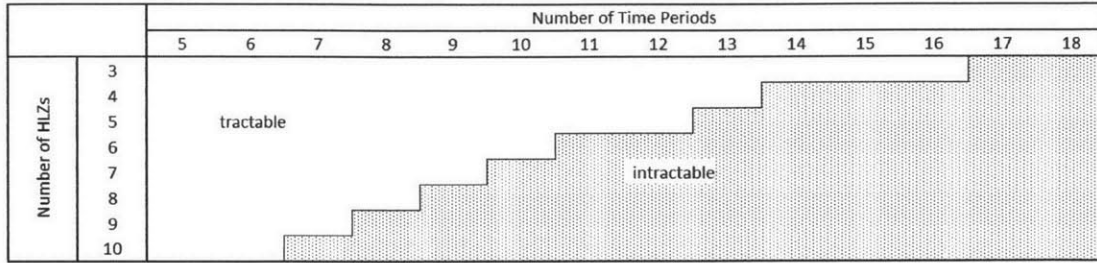


Figure 5-2: SNDP_{OPT} tractability region ($|F| = 1, |K| = 3$)

5.2.2 Test 2: Increase in Number of Aircraft Teams

The run-time of our heuristic algorithms favors scenarios where the aircraft teams have identical characteristics, since MTRA and MMRA only compute a set of feasible paths for each *type* of aircraft team. Therefore, this test is divided into two parts. We first consider increasing the number of identical aircraft teams, and then we consider increasing the number of different aircraft teams. Table 5.6 shows the variable parameters for this test.

<i>Parameter</i>	<i>Description</i>	<i>Value</i>
m	number of time periods	10
β	number of HLZs	3
γ	number of teams	<i>variable</i>
ρ	number of AMR legs	6
w^k	passengers per AMR	10
ϕ^k	schedule tolerance	30 min

Table 5.6: Test 2 variable parameters

Case 1: Increasing the Number of *Identical* Aircraft Teams

Here, we increase the number of identical teams from one to eight, with each aircraft team available during the entire time period considered (Figure 5-3). The run-time for SNDP_{OPT} model increases exponentially for every additional aircraft team. For both MTRA and MMRA, the run-time increases very slowly and nearly linearly.

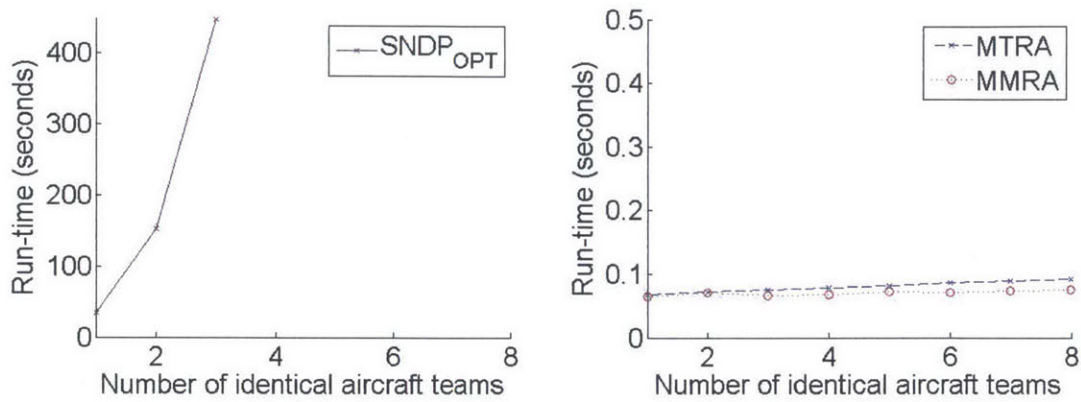


Figure 5-3: Run-time vs. number of identical aircraft teams

Case 2: Increasing the Number *Different* Aircraft Teams

In this case, we increase the number of different aircraft teams, with each aircraft team available during the entire time period considered. The run-time for SNDP_{OPT} is nearly identical to the run-time in Case 1, showing that the characteristics of each aircraft team have little effect on the run-time of this model. For both MTRA and MMRA, the run-time again increases linearly, just as in Case 1. However, the run-time for MTRA is much higher, on average, than in Case 1. This is explained by the fact that the algorithm must compute a new set of feasible paths for each type of aircraft team. The run-time of MMRA remains lower than MTRA, and about the same as in Case 1.

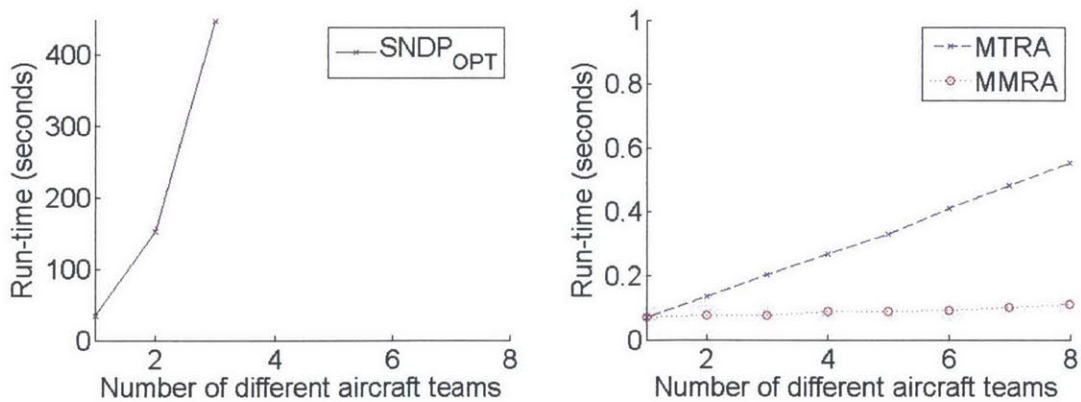


Figure 5-4: Run-time vs. number of different aircraft teams

5.2.3 Test 3: Increase in Number of AMR legs

For this test, we want to determine the effect of increasing the number of AMR legs considered on run-time. Table 5.7 shows the variable parameters defined for this test. We also split this test into two cases to determine the model behavior when one, and then when two *different* aircraft teams are considered.

<i>Parameter</i>	<i>Description</i>	<i>Value</i>
m	number of time periods	10
β	number of HLZs	3
γ	number of teams	1
ρ	number of AMR legs	<i>variable</i>
w^k	passengers per AMR	10
ϕ^k	schedule tolerance	30 min

Table 5.7: Test 3 variable parameters

Case 1: One Aircraft Team

In this case, we increase the number of AMR legs considered from 1 to 14 (Figure 5-5). As a result, the run-time increases for the three models. For SNDP_{OPT} , the average run-time increases somewhat linearly over the range of the experiment. MTRA experiences an exponential growth in run-time, while MMRA run-time increases exponentially, but much slower than MTRA.

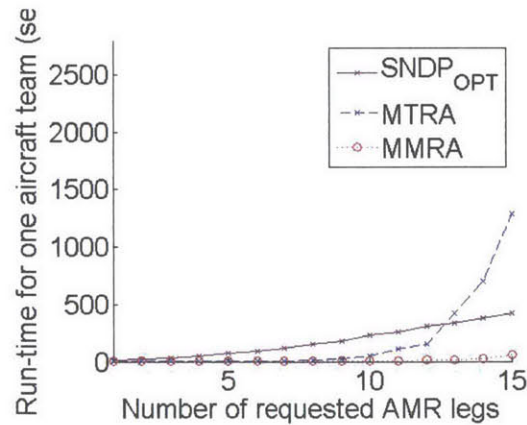


Figure 5-5: Run-time vs. number of AMR legs (one aircraft team)

<i>Parameter</i>	<i>Description</i>	<i>Value</i>
m	number of time periods	10
β	number of HLZs	3
γ	number of teams	1
ρ	number of AMR legs	6
w^k	passengers per AMR leg	<i>variable</i>
ϕ^k	schedule tolerance	30 min

Table 5.8: Test 4 variable parameters

Case 2: Two Aircraft Teams

In this case, we consider two aircraft teams and again increase the number of AMR legs from 1 to 14 (Figure 5-6). The run-time increases for all models (SNDP_{OPT} cannot be solved for cases where demand is greater than 8 AMR legs on the computer running the experiment). However, we note that the rate of increase for the run-time of SNDP_{OPT} is greater than that of MTRA and MMRA, while the run-time of MMRA remains slightly lower than that of MTRA.

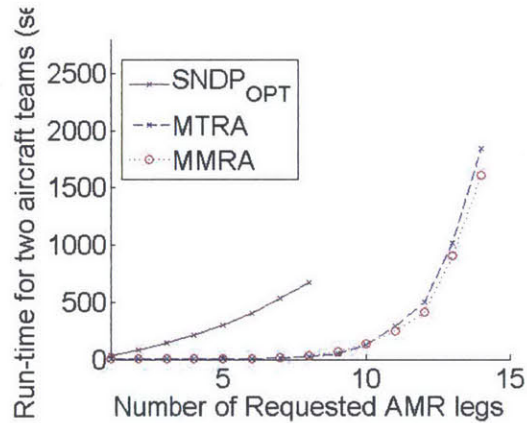


Figure 5-6: Run-time vs. number of AMR legs (two aircraft teams)

5.2.4 Test 4: Increase in Number of Passengers per AMR

For this test, we consider one aircraft team and six AMR legs, and vary the number of passengers per AMR leg (Table 5.8).

Figure 5-7 shows the run-time vs. the number of passengers per AMR. As the number of passengers per AMR leg increases from 1 to 40, the run-time of SNDP_{OPT} remains constant, while the run-time of MTRA and MMRA decreases, on average. This is because the Path Generation Heuristic Algorithm (PGHA), which is used by

MTRA and MMRA, creates additional branches based on remaining aircraft capacity. If AMRs have a higher passenger count, then remaining aircraft capacity is lower, on average, thus fewer recursive branches are created to pick up additional AMRs. In the small time-space graph $G(N, A)$ that we consider, SNDP_{OPT} has a much longer run-time than the other two models. This is expected from the results of Test 1.

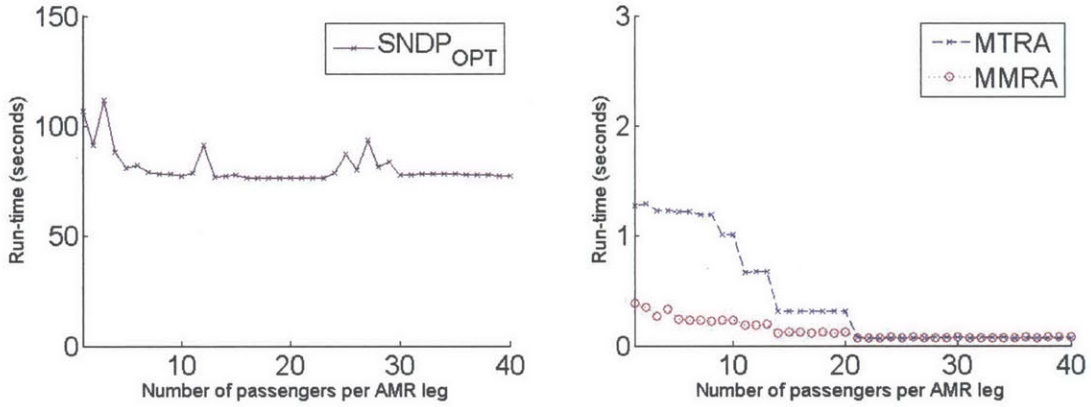


Figure 5-7: Run-time vs. number of passengers per AMR

5.2.5 Test 5: Increase in Schedule Tolerance per AMR

In this test, we change the schedule tolerance of the AMR legs, while maintaining all other fixed and variable parameters constant (Table 5.9).

<i>Parameter</i>	<i>Description</i>	<i>Value</i>
m	number of time periods	10
β	number of HLZs	3
γ	number of teams	1
ρ	number of AMR legs	6
w^k	passengers per AMR	10
ϕ^k	schedule tolerance	<i>variable</i>

Table 5.9: Test 5 variable parameters

Figure 5-8 shows the run-time vs. schedule tolerance. The schedule tolerance does not affect the run-time of SNDP_{OPT} , on average, while the run-time of MTRA and MMRA increases exponentially, albeit at a very low rate. This is because of the

way that PGHA finds feasible paths. If each AMR is available to be picked up and dropped off during a longer time period, then there is a higher probability that it can be supported. Therefore, the number of paths created by PGHA is increased as the schedule tolerance of each AMR increases.

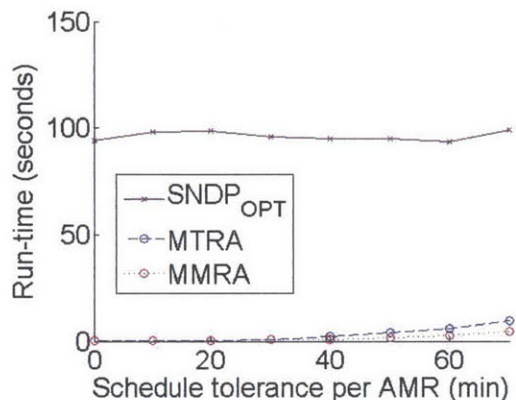


Figure 5-8: Run-time vs. schedule tolerance

5.2.6 Summary and Discussion

Table 5.10 summarizes the qualitative effect of each variable parameter on total run-time. Discussion Points:

- Increasing the number of time periods, m , or the number of HLZs, β , increases the run-time of SNDP_{OPT}, but not of MTRA or MMRA. This is because the Path Generation Heuristic Algorithm (PGHA) only builds route segments with a goal of picking up or dropping off AMRs, so many possible route segments are not considered. On the other hand, SNDP_{OPT} considers every feasible aircraft team arc and AMR flow combination in $G(N, A)$, therefore the number of decision variables and constraints increase at a much faster rate than the size of $G(N, A)$.
- Increasing the number of aircraft teams, γ , or the number of AMR legs, ρ , increases the run-time of all models. The run-time of SNDP_{OPT} increases exponentially, when the number of aircraft teams is increased, while it increased at a linear rate for MTRA and MMRA. The integer program SNDP_{OPT} considers all possible combinations of aircraft arcs and AMR flow, for all aircraft teams.

<i>Parameter</i>	<i>Description</i>	<i>Change</i>	<i>Model</i>	<i>run-time</i>
m	number of time periods	increase	SNDP _{OPT}	increase
			MTRA	no change
			MMRA	no change
set B				
$\beta = B $	number of HLZs	increase	SNDP _{OPT}	increase
			MTRA	no change
			MMRA	no change
set F				
$\gamma = F $	number of aircraft teams	increase	SNDP _{OPT}	increase
			MTRA	increase
			MMRA	increase
set K				
$\rho = K $	number of AMR legs	increase	SNDP _{OPT}	increase
			MTRA	increase
			MMRA	increase
w^k	passengers per AMR	increase	SNDP _{OPT}	no change
			MTRA	decrease
			MMRA	decrease
ϕ^k	schedule tolerance	increase	SNDP _{OPT}	no change
			MTRA	increase
			MMRA	increase

Table 5.10: Computational analysis results

Therefore, as the number of aircraft teams increases linearly, the number of variables and constraints created by SNDP_{OPT} also increases linearly, but as a result the run-time is increased exponentially.

- Increasing the number of passengers per AMR, w^k , does not have a measurable effect on the run-time of SNDP_{OPT}, but decreases the run-time of MTRA and MMRA. SNDP_{OPT} still considers the same number of aircraft arcs and passenger flow for different numbers of passengers per AMR, but MTRA and MMRA, which use the Path Generation Heuristic Algorithm, compute fewer paths because the aircraft teams are more quickly filled to capacity.
- Increasing the schedule tolerance for each AMR, ϕ^k , does not have a measurable effect on the run-time of SNDP_{OPT}, but increases the run-time of MTRA and MMRA. SNDP_{OPT} still considers the same number of aircraft arcs and passenger flow for different numbers of passengers for each AMR, but MTRA and MMRA compute more paths, because a greater number of feasible AMR pickup and drop-

off schedule combinations exists (Chapter 4). This has the effect of increasing the run-time of both MTRA and MMRA exponentially as schedule tolerance is increased linearly.

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Chapter 6

Operational Analysis

This chapter focuses on operational analysis and contains several major sections. We compare the solution that each model provides to the problem of AMR scheduling and aircraft routing, for problem sizes that SNDP_{OPT} is able to solve. Then, we compare the solution of the Maximum Marginal Return Algorithm (MMRA) to that of the Maximum Total Return Algorithm (MTRA). We also compare the solution of both models against one developed manually, similar to what is done in practice. In addition, we discuss how these heuristic algorithms can be used as a tool in the lift planning process. Finally, we look at how changes in schedule tolerance affect service and capacity estimations.

6.1 Comparison Across All Models

In this section, we address problem sizes that all models are able to solve, in order to gain insight into how the heuristic models (MTRA and MMRA) perform, compared to the optimal solution provided by SNDP_{OPT} .

6.1.1 Test Scenarios

We consider a static graph containing three HLZs (Figure 6-1), where one aircraft team is available from HLZ 'C'. Since the number of time periods for our test scenarios will be small, we set the maximum flight time before refueling, τ_{FLIGHT}^f , to eight time

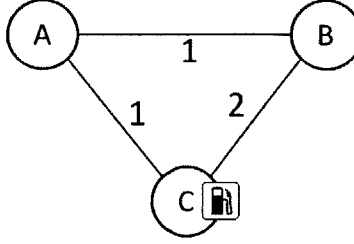


Figure 6-1: Test scenario static graph

periods, or 80 minutes, and the refuel duration, τ_{FUEL}^f , to one time period, or 10 minutes.

For Test Scenario 1, we consider one UH-60 Blackhawk team over twelve time periods, with a demand of 10 AMR legs . For Test Scenario 2, we consider two UH-60 Blackhawk teams over a ten time periods, with a demand of seven AMR legs (Table 6.1).

<i>Parameter</i>	<i>Description</i>	<i>Test Scenario 1 Value</i>	<i>Test Scenario 2 Value</i>
m	number of time periods	12	10
β	number of HLZs	3	3
γ	number of teams	1	2
ρ	number of AMR legs	10	6
w^k	passengers per AMR	U[1,20]	U[1,20]
ϕ^k	schedule tolerance	100 min	100 min
v^k	leg value	1000	1000

Table 6.1: Test scenario parameters

6.1.2 Results and Discussion

We ran 100 iterations for each scenario, randomizing AMR leg origin and destination nodes uniformly over the time-space graph, $G(N, A)$, subject to a schedule tolerance, ϕ^k , of 100 minutes.

The results for both test scenarios are shown in Figure 6-2. In Test Scenario 1, MTRA achieved a solution within 5% of SNDP_{OPT} in 95% of the cases. MMRA achieved within 5% of SNDP_{OPT} in 50% of the cases. In test Scenario 2, both MTRA and MMRA achieved the same results, within 5% of SNDP_{OPT} in 97% of the cases,

and within 15% of SNDP_{OPT} in 100% of the cases.

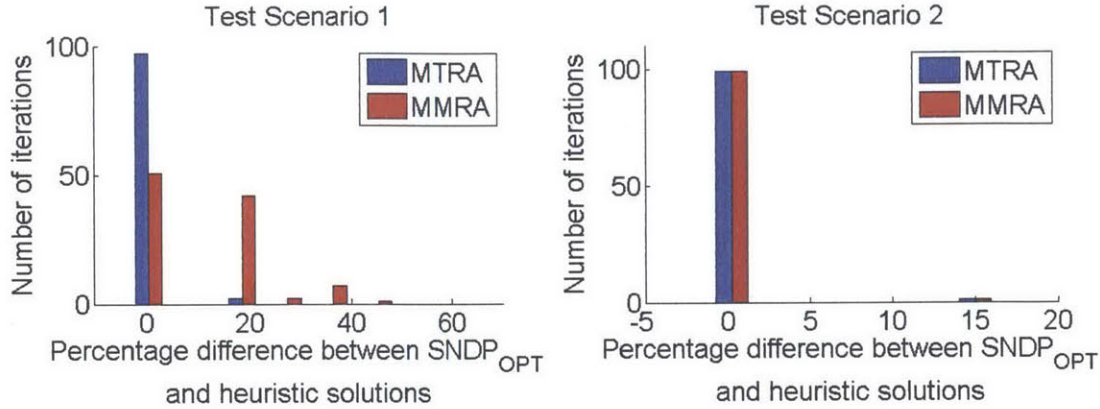


Figure 6-2: Test scenario histogram of supported AMR legs

In both test scenarios, SNDP_{OPT} supported an equal or greater number of AMR legs than MTRA and MMRA. Table 6.2 shows the average number of supported AMR legs of each model, for these test scenarios. On average, the best heuristic algorithm result (MTRA) achieved an average number of supported AMR legs within 1% of SNDP_{OPT} for both small test scenarios.

<i>Model</i>	<i>Test Scenario 1 Average</i>	<i>Test Scenario 2 Average</i>
SNDP_{OPT}	5.62	5.98
MTRA	5.60	5.97
MMRA	5.00	5.97

Table 6.2: Average number of supported AMRs

Solving the service network design problem optimally on a small network $G(N, A)$, with few aircraft teams and AMRs is very fast, but larger networks prove to be very hard for SNDP_{OPT} . While the formulation guarantees an optimal solution, it may be intractable for problems of meaningful size.

6.2 Operational Scenarios Description

We describe two larger, realistic, and quite different scenarios, based on the author's operational experience, which reflect actual geographical conditions in Afghanistan.

We use these scenarios to compare the solutions generated by MTRA and MMRA.

6.2.1 Scenario 1

Geographical Layout

The geography for this scenario is similar to the terrain in the south and western parts of Afghanistan, and corresponds to an area over which an Aviation Battalion Task Force may operate. The elevation changes due to terrain are minimal, and helicopters operate point-to-point between HLZs, assuming sufficient fuel. In this scenario, the Aviation Task Force is located at HLZ Alpha (Figure 6-3), and consists of two lift companies of ten UH-60 Blackhawk helicopters each.

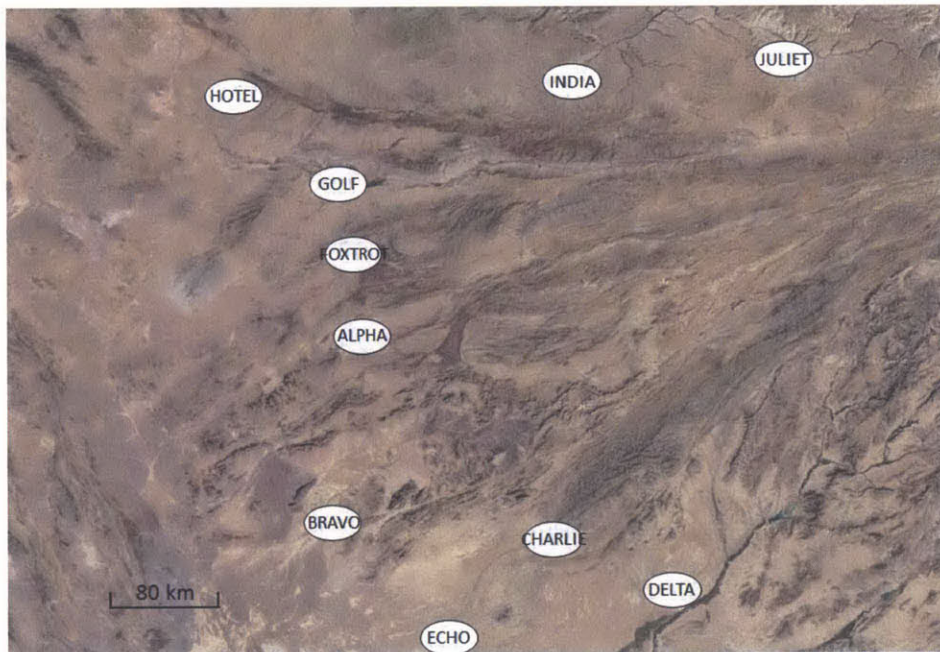


Figure 6-3: Operational Scenario 1: satellite overview

Figure 6-4 shows the flight time in minutes between certain pairs of HLZs. The list of inter-HLZ flight times is shown in Table 6.3. Bold numbers indicate that the direct flight time exceeds the fuel capacity of the UH-60 Blackhawk.

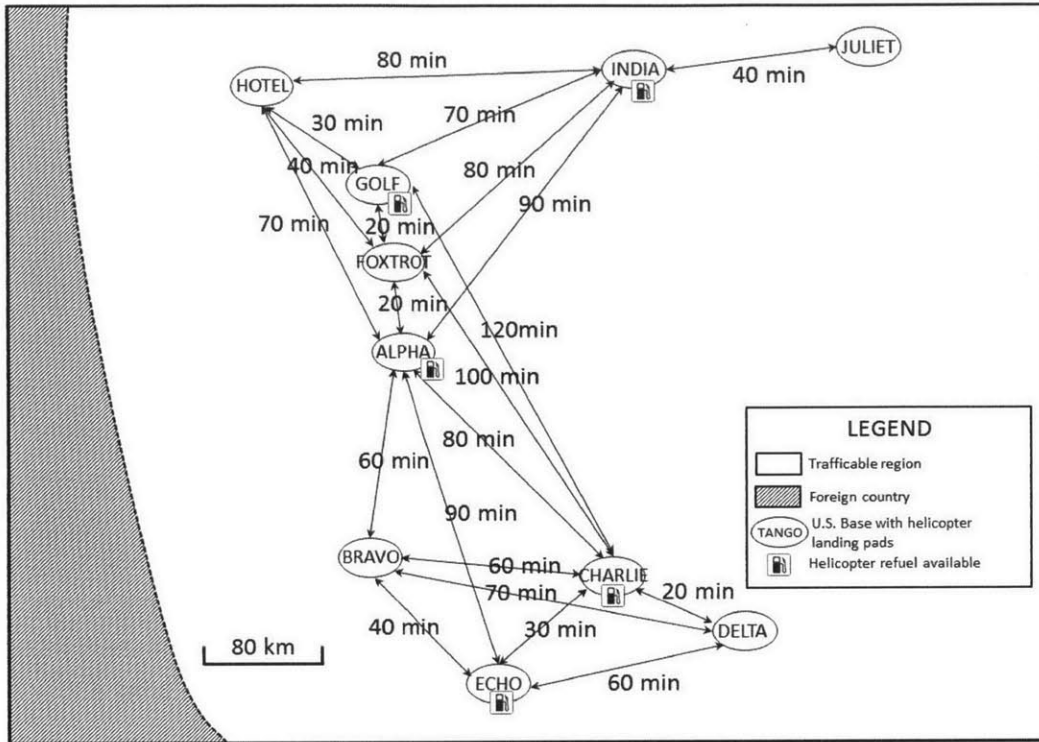


Figure 6-4: Operational Scenario 1: Flight time diagram

	A	B	C	D	E	F	G	H	I	J
A	0	60	80	100	90	20	40	70	90	130
B		0	60	70	40	80	100	120	140	160
C			0	20	30	100	120	140	120	140
D				0	60	120	140	160	140	160
E					0	110	130	160	160	170
F						0	20	40	80	110
G							0	30	70	110
H								0	80	120
I									0	40
J										0

Table 6.3: Operational Scenario 1: Inter-HLZ flight time (min)

HLZ Characteristics

This scenario consists of 10 HLZs, each at a different base (FOB or COP) (Table 6.4).

HLZ Name	Helicopters Stationed	Refuel Capable	Base Population
Alpha	20xUH-60	Yes	1000
Bravo	None	No	200
Charlie	None	Yes	200
Delta	None	No	100
Echo	None	Yes	300
Foxtrot	None	No	100
Golf	None	No	800
Hotel	None	Yes	100
India	None	No	200
Juliet	None	Yes	100

Table 6.4: Operational Scenario 1: Base and HLZ configuration

6.2.2 Scenario 2

Geographical Layout

The geography for this test scenario reflects the terrain in the northeastern parts of Afghanistan (Figure 6-5). Though the geographical distance between bases is shorter, on average, than in Scenario 1, the terrain creates barriers to air travel, forcing helicopters to operate along specific valley corridors. The Aviation Task Force is located at HLZ Alpha, and consists of the same number of aircraft as in Scenario 1. Figure 6-6 shows the flight time in minutes between certain pairs of HLZs.

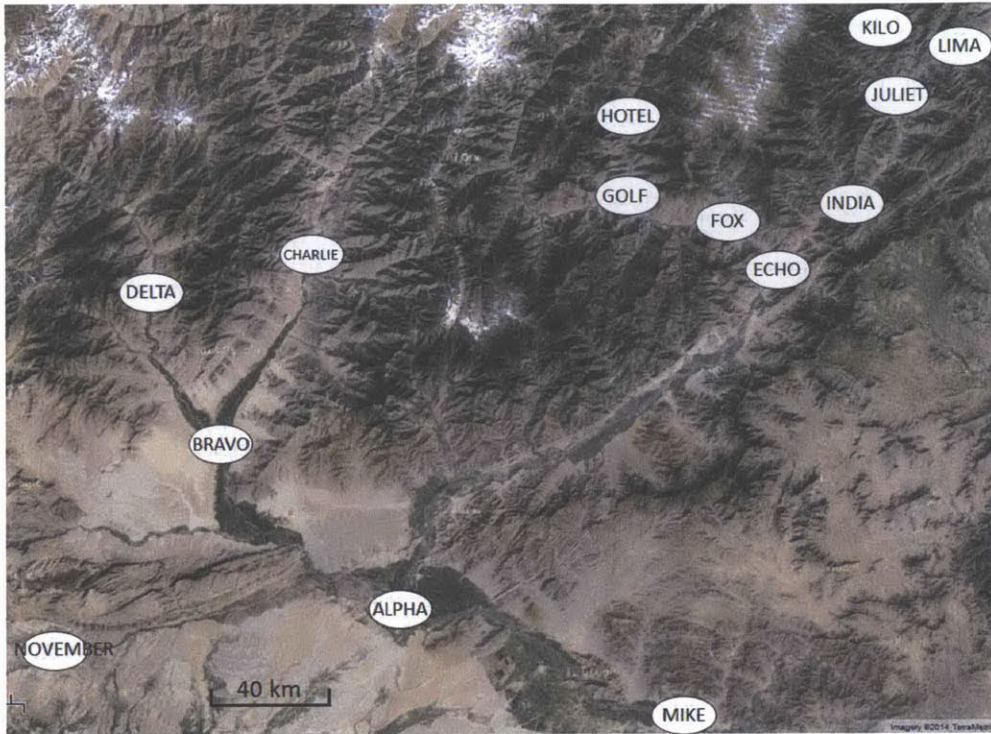


Figure 6-5: Operational Scenario 2: Satellite overview

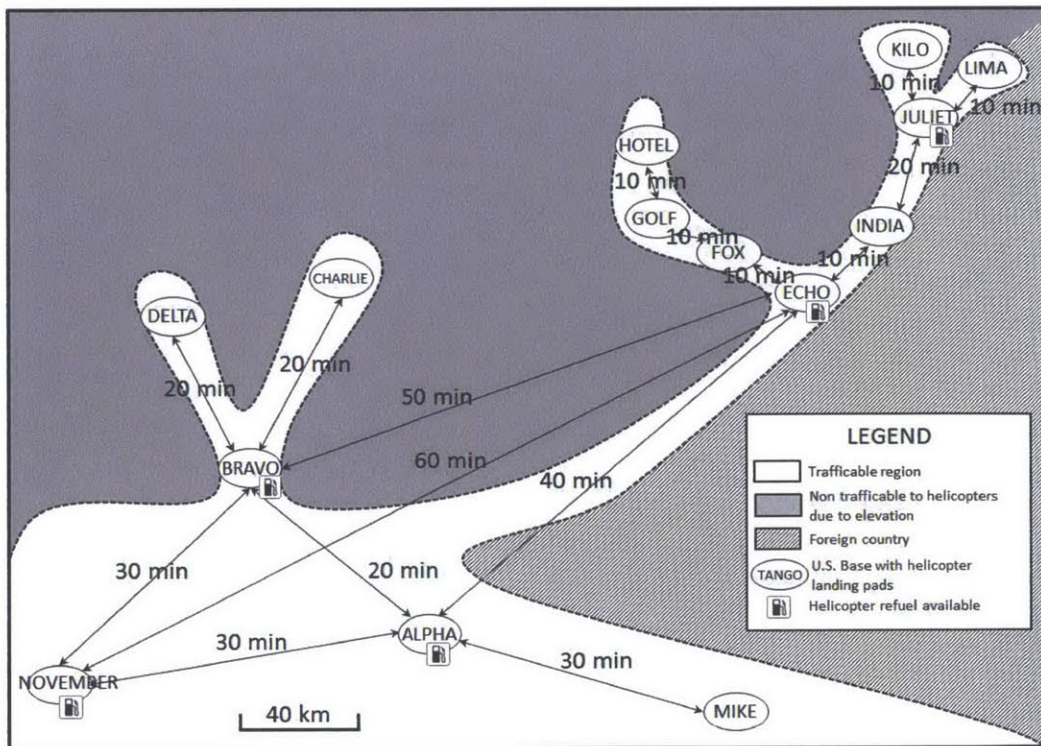


Figure 6-6: Operational Scenario 2: Flight time diagram

The list of inter-HLZ flight times is shown in Table 6.5.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
A	0	20	40	40	40	50	60	70	50	70	80	80	30	30
B		0	20	20	50	60	70	80	60	80	90	100	50	30
C			0	40	70	80	90	100	80	100	110	110	70	50
D				0	70	80	90	100	80	100	110	110	70	50
E					0	10	20	30	10	30	40	40	70	60
F						0	10	20	20	40	60	60	80	70
G							0	10	30	50	60	60	90	80
H								0	40	60	70	70	100	90
I									0	20	30	30	80	70
J										0	10	10	100	90
K											0	20	120	110
L												0	120	110
M													0	50
N														0

Table 6.5: Operational Scenario 2: Inter-HLZ flight time (min)

HLZ Characteristics

This scenario models HLZ and base characteristics in the same manner as Scenario

1. These are shown in Table 6.6.

HLZ Name	Helicopters Stationed	Refuel Capable	Base Population
Alpha	20xUH-60	Yes	700
Bravo	None	Yes	400
Charlie	None	No	100
Delta	None	No	100
Echo	None	Yes	300
Foxtrot	None	No	100
Golf	None	No	100
Hotel	None	No	100
India	None	No	100
Juliet	None	Yes	400
Kilo	None	No	100
Lima	None	No	100
Mike	None	No	200
November	None	Yes	200

Table 6.6: Operational Scenario 2: Base and HLZ configuration

6.2.3 Aircraft and Passenger Data

AMR Execution Window

The daily AMR execution window we consider for both operational test scenarios is a 20 hour period (Figure 6-7). The aircraft steady state provides five UH-60 Blackhawk helicopter teams, each available for a 10 hour block of time. Flight crews must be available for a ten hour continuous period of time, but have different flight hour restrictions during that time. Teams 1 and 2 fly during daylight hours, so our assumed SOP allows them up to 8 hours of flight time. Team 3 flies a combination of day and night vision goggles, so they are allowed up to 7 hours of flight time. Teams 4 and 5 are flying primarily night vision goggles, with some day-time flying, so they are allowed up to 6 hour of flight time during their shift.

Flights are continuously supported from 0600 to 0200 the next day, with a 4 hour reset period built in for scheduled maintenance. With a total of twenty aircraft stationed together in each scenario, the remaining aircraft, beyond the 10 in service here, are assumed to be in maintenance, participating in deliberate combat operations,

or used as spares to ensure steady state availability.

Aircraft Team Steady State																											
Time	5:00	6:00	7:00	8:00	9:00	10:00	11:00	12:00	13:00	14:00	15:00	16:00	17:00	18:00	19:00	20:00	21:00	22:00	23:00	0:00	1:00	2:00	3:00	4:00			
Aircraft Availability				Team 1: 2xUH-60 (8 flt hrs avail.)																							
				Team 2: 2xUH-60 (8 flt hrs avail.)																							
							Team 3: 2xUH-60 (7 flt hrs avail.)																				
													Team 4: 2xUH-60 (6 flt hrs avail.)														
														Team 5: 2xUH-60 (6 flt hrs avail.)													

Figure 6-7: Steady state support diagram

AMR Parameters

We assume that demand originating and terminating at each HLZ is directly proportional to the population density around that HLZ (Figures 6.4 and 6.6). Since AMR leg demand is more likely to originate in the morning and in the afternoon, simulating day-trip behavior, we assume a bimodal distribution of demand with local maxima at 9:00 and 17:00. We also assume a number of passengers for each AMR leg that is uniformly distributed over [1,20].

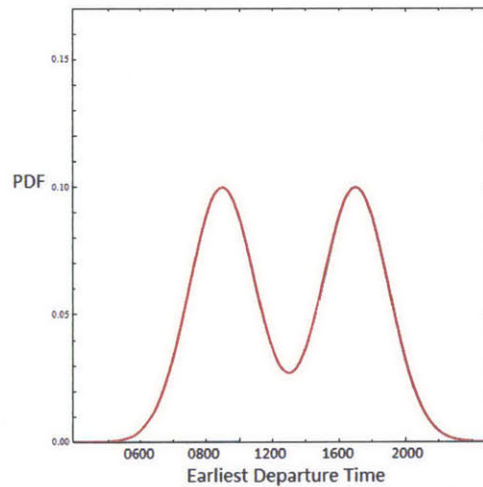


Figure 6-8: Probability density function of passenger demand

6.2.4 Summary of Aircraft Team and AMR Parameters

A summary of aircraft team and AMR parameters used for Scenarios 1 and 2 is shown in Table 6.7.

<i>Set</i>	<i>El.</i>	<i>Parameter</i>	<i>Description</i>	<i>Value</i>	
<i>F</i>		γ	number of aircraft teams	5	
	<i>f</i>	<i>aircraft team</i>			
		v^f	passenger capacity	40	
		τ_{CYCLE}^f	flying hour limit per planning cycle	see figure 6-7	
		τ_{FLIGHT}^f	flying hour limit on each full tank of fuel	120 min	
		τ_{FUEL}^f	average refuel time required	20 min	
		τ_{TURN}^f	average turn time	10 min	
		τ_{SD}^f	minimum time required to shutdown	30 min	
		\bar{c}^f	cost of flight arc per time period	1	
		\underline{c}^f	cost of ground arc per time period	$\frac{1}{3}$	
		t_{START}^f	first time period that team available	see figure 6-7	
t_{END}^f	last time period that team is available	see figure 6-7			
<i>Set</i>	<i>El.</i>	<i>Parameter</i>	<i>Description</i>	<i>Value</i>	
<i>K</i>		ρ	number of AMR legs	30	
	<i>k</i>	<i>air mission request leg</i>			
		w^k	number of passengers	$U \sim [1, 20]$	
		v^k	value of AMR leg	tiered (see Chap. 4)	
		b^{k-}	starting HLZ	prop. to pop. size	
		b^{k+}	ending HLZ	prop. to pop. size	
		t^{k-}	earliest departure time	see Figure 6-8	
		t^{k+}	latest arrival time	$t^{k+} + \Delta T^k + \phi^k$	
		c^k	cost of flight arc per time period	$\frac{1}{3}$	
		ΔT^k	shortest flight time of AMR leg	distance($b^{k-} \rightarrow t^{k+}$)	
		ϕ^k	schedule tolerance of AMR leg	100 minutes	

Table 6.7: Aircraft team and AMR parameters

6.3 Heuristic Model Comparison

In the context of Army Aviation lift planning, an optimal flight schedule and aircraft routing maximizes a function of AMR value, and aircraft and passenger cost. The models described in Chapter 4 maximize first and foremost the value of total AMRs supported. Then, if two solutions are of equal value, they select the one with the lowest aircraft cost. Finally, if two solutions are still of equal value, they select one with the lowest passenger cost.

In this section, we take the perspective of three different parties to look at different aspects of the solution.

- The Commander is interested in the *objective function value vs. number of AMR legs requested, and number of AMR legs supported vs. number of AMR legs requested.*
- Maintenance personnel are interested in the *average flight and turn time per AMR leg vs. number of AMR legs supported.*
- AMR passengers are interested in the *average AMR delay vs. number of AMR legs supported.*

We define the following terms:

The *support level* is the average number of AMR legs that can be supported, given a fixed aircraft capacity. The *support rate* is the number of supported AMRs divided by the total number of requested AMR legs. Aircraft *load factors* are the number of passengers divided by the number of seats. In airline terms, passengers that want to fly but are not supported are said to be *spilled* [8]. We use this term to describe *non-supported* AMRs in our model.

6.3.1 Objective Function Value vs. AMR Legs Requested

As AMR demand increases, MTRA is able to achieve a slightly higher objective function value than MMRA (Figure 6-9). A higher objective function for MTRA is expected from our analytical analysis of the objective function values in Chapter 4.

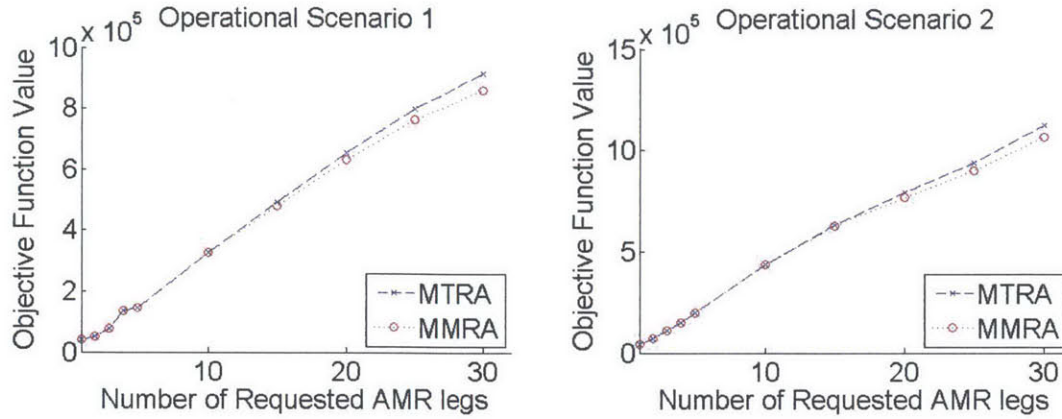


Figure 6-9: Objective function value vs. requested AMR legs

The gap between MTRA and MMRA objective functions increases as the number of requested AMR legs increases, but is also small. In fact, the greedy algorithm, MMRA, performs almost as well as MTRA in these two scenarios.

Further study is conducted using Operational Scenario 1, and different value functions (*Tiered, Equal, Linear*) shown in Table 6.8.

AMR Priority	AMR Value		
	Case 1 (Tiered)	Case 2 (Equal)	Case 3 (Linear)
1	100,000	1,000	6,000
2	90,000	1,000	5,000
3	80,000	1,000	4,000
4	3,000	1,000	3,000
5	2,000	1,000	2,000
6	1,000	1,000	1,000

Table 6.8: Value function comparison

Figure 6-10 shows the impact that the chosen value function and aircraft capacity have on the objective function value. As aircraft capacity increases, the gap between the performance of MMRA and MTRA decreases, on average, for the same level of demand. With a greater aircraft capacity relative to demand, MMRA performs almost as well as MTRA.

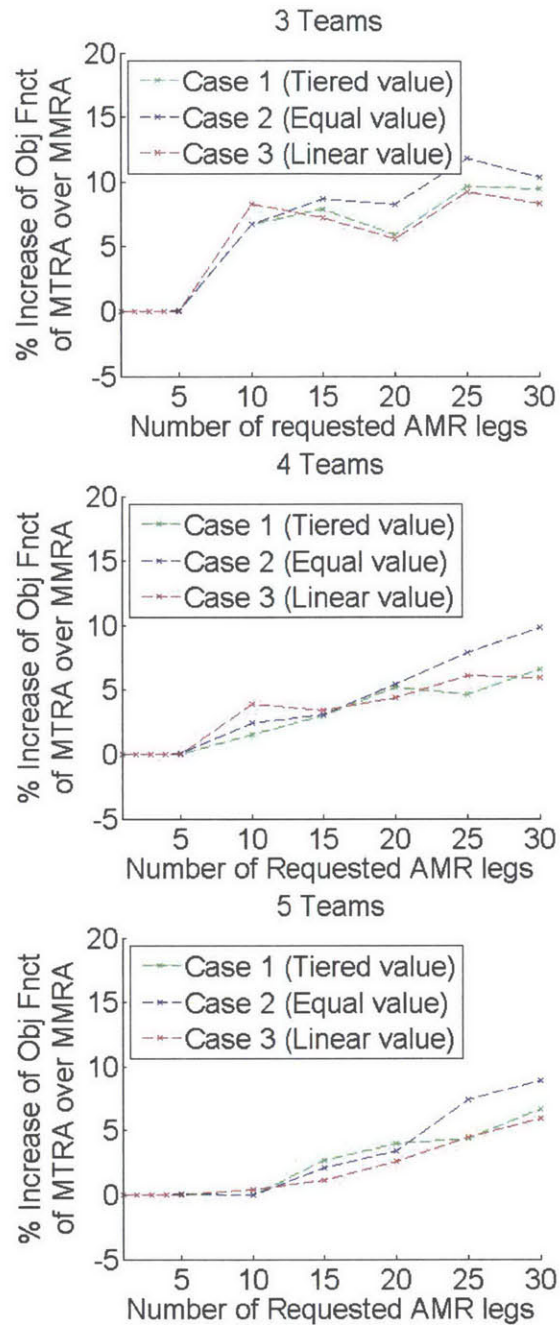


Figure 6-10: Impact of capacity and AMR value function on the solutions of MTRA and MMRA

6.3.2 AMR legs Supported vs. AMR Legs Requested

Figure 6-11 shows the average number of supported AMR leg vs. the number of requested AMR legs. As demand increases, the marginal number of additional supported AMRs decreases.

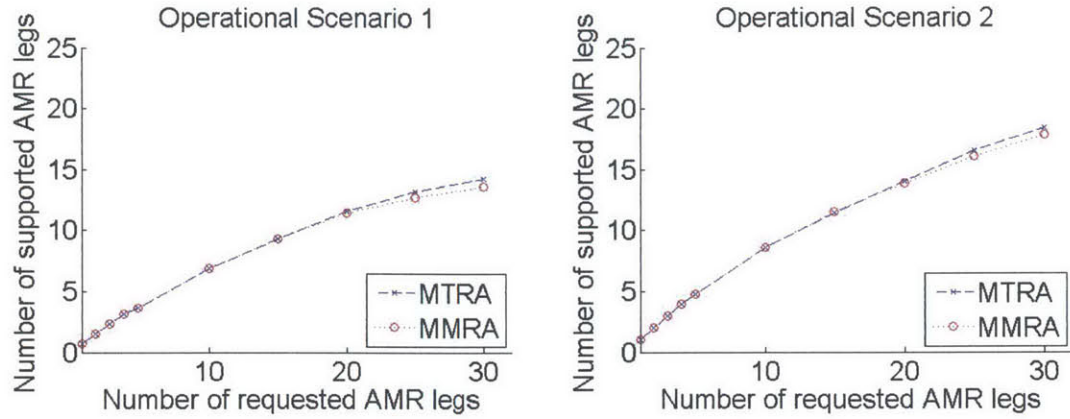


Figure 6-11: Number of supported AMR legs vs. number of requested AMR legs

Using Operational Scenario 1 and Table 6.8, we examine the effect of aircraft capacity and the AMR value function on the number of AMRs supported (Figure 6-12). Interestingly, Case 1 (tiered value function) results in the lowest number of supported AMRs, on average. We can attribute this to the fact that only half of the AMR legs are of high value, thus the algorithm attempts to support these first above all others (with no trade-off between the first and second tiers), lowering the available capacity for low priority AMRs. Furthermore, as aircraft capacity increases, the gap between the number of AMR legs supported by MMRA and MTRA decreases, on average, for the same level of demand. This follows the analysis in the previous section where, with a larger aircraft capacity relative to demand, MMRA performs almost as well as MTRA.

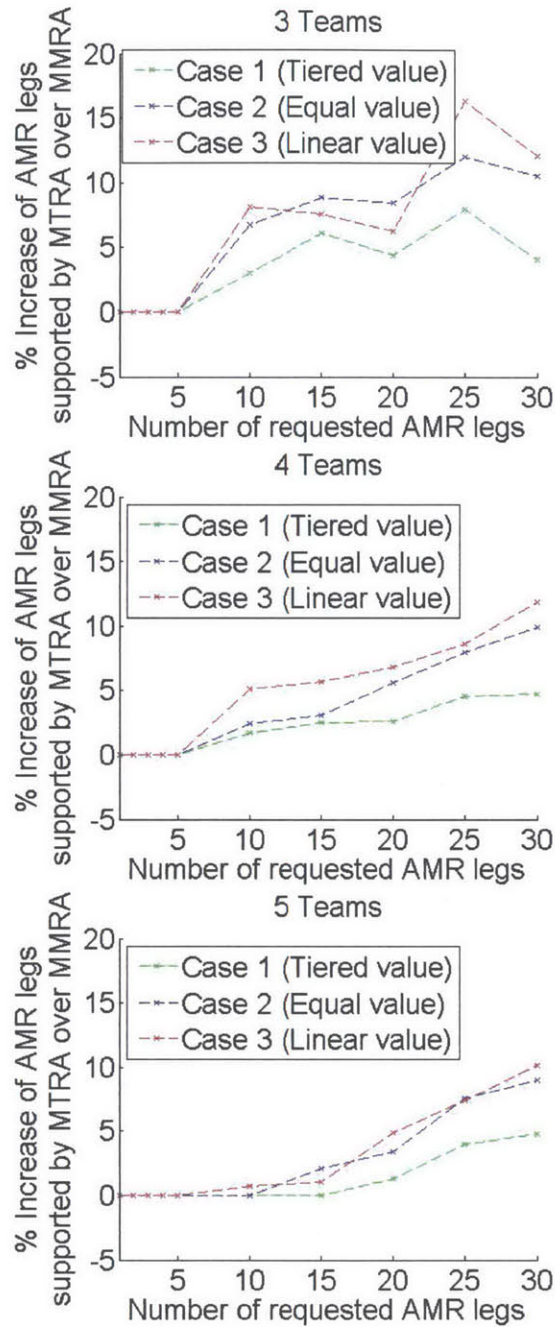


Figure 6-12: Impact of capacity and AMR value on relative performance

6.3.3 Average Flight and Turn Time per AMR Leg:

Figure 6-13 shows the average flight and turn time per supported AMR leg, as a function of the number of supported AMR legs. Again, there is little difference between the results of each model. Each model experiences a slight decrease in average

flight and turn time per AMR, until four or five AMR legs are supported. Then, as the number of supported AMR legs increases, the average flight and turn time per AMR leg increases before leveling off. We see a decrease again, when the number of supported AMR legs becomes high. A reasonable explanation for the early decrease is that the additional flight time from the first and last flight legs causes a high average flight and turn time per AMR leg when the number of AMR legs supported is low. These flight legs are needed for the aircraft teams to go to the first pick-up HLZ and return from the last drop-off HLZ. As the number of supported AMR legs increases, we assume passengers may be less likely to have a direct flight, so the average flight and turn time per AMR leg increases.

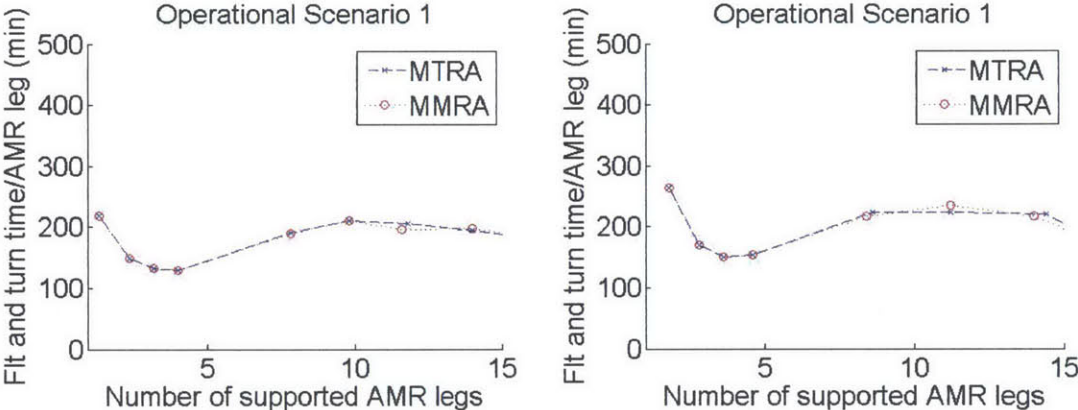


Figure 6-13: Average flight and turn time per AMR leg vs. number of supported AMRs

A likely explanation for the later decrease in average flight and turn time per AMR leg, as the number of supported AMR legs becomes large, appears to be the presence of a *short haul bias*. Consider an example where there are two requested AMR legs of equal value. The minimum direct flight time, ΔT^k , needed to support the first is twice that of the second. If unable to support both AMR legs, the algorithm will favor the shorter leg because it results in the same value earned, for a lower aircraft and passenger cost. It is biased in favor of the shorter leg since it returns more value per flight time. Trading off of AMRs in favor of shorter legs does not mean that the objective function is poorly designed. By favoring shorter legs when demand exceeds capacity, we are able to support a higher total value of AMRs. This is a reminder that

weighting of AMR values should be carefully designed to reflect what is important to the planner. One could weight AMRs of equal value according to their flight distance to remove the short haul bias, but this may not necessarily reflect the priorities set forth in the CMPL.

6.3.4 Average Delay per AMR Leg vs. Number of Supported AMR Legs

The average AMR delay is calculated by subtracting the shortest possible travel time (ΔT^k), from the actual travel time of each AMR leg. The total delay that an AMR leg can experience is limited by the schedule tolerance, ϕ^k . Figure 6-14 shows the average AMR delay, in minutes, experienced per AMR leg as the number of supported AMR legs increases.

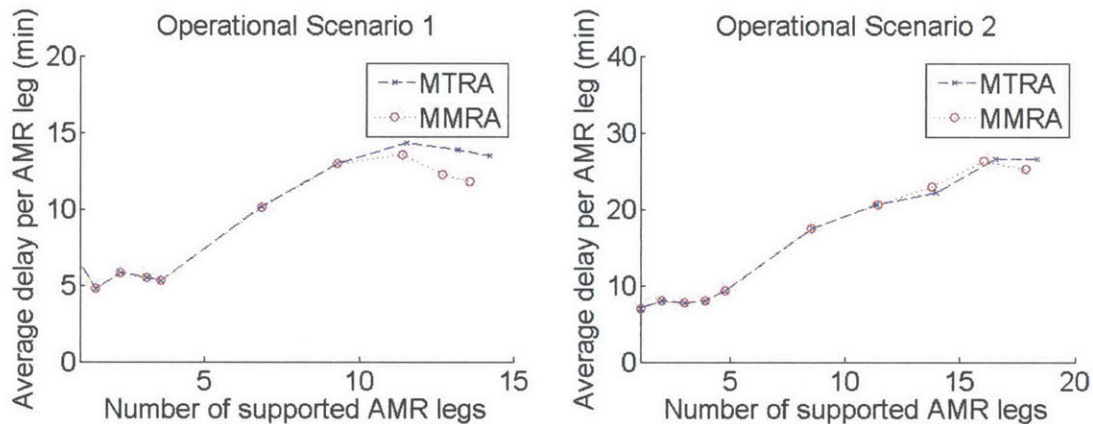


Figure 6-14: Average AMR delay vs. number of supported AMR legs

The average delay per supported AMR behaves in the same manner for both models. On average, however, MTRA experiences a slightly higher average delay. Interestingly, as AMR demand becomes large, the average delay per AMR begins to stabilize or decrease slightly.

This decrease or leveling off of average AMR delay may be explained by the presence of a *low passenger bias*. Given two AMR legs of the same value, the algorithm will favor the one with the fewest passengers since it increases the likelihood that additional AMRs can be supported. To remove the low passenger bias, one would have to change the AMR value scheme, and weight the value of each AMR leg by the

number of passengers it contains. Again, this may not reflect the priorities set forth in the CMPL.

6.4 Model Comparison to Human Planner

To assess the applicability of these models in an operational scenario, it is necessary to see how they perform compared to a human planner. The author, using his operational experience, planned several test cases by hand, and compared the solution to that of each heuristic algorithm.

We first ordered AMR legs by departure time and created a feasible solution, favoring high value AMR legs. Then, we attempted to include additional AMR legs into the schedule, which sometimes created the need to swap AMR legs between aircraft teams or substantially changes routes of flight.

Some aspects that make manual planning difficult are the following:

- It is hard to look at the big picture, in terms of what is being requested, when the number of AMR legs is large.
- Fuel considerations mean the total flight time between fuel stops must be tracked as the schedule is built.
- Aircraft capacity considerations require available seats to be tracked as the schedule is built.
- Multi-leg AMRs add complexity since we must support all legs of an AMR or none.

For these reasons, it is often much easier and faster to sort AMR legs by departure time and create a sequential plan without considering all possible combinations. That plan is then updated with additional AMR legs, if possible. This also reflects actual planning conditions, since additional AMR legs are routinely received by the planning cell after the AMR schedule and aircraft routing has been created, requiring constant updates.

For this experiment, we used Operational Scenario 1, with an AMR demand of ten single leg AMRs, ten 2-leg AMRs, and a schedule tolerance of 3 hours. Table 6.9 shows the difference in solution between the human plan and the heuristic algorithm plans. For the human solution, we timed the first solution obtained, as well as the best solution obtained.

Iteration 1		
<i>Model</i>	<i>Solution</i>	<i>Time (sec)</i>
MTRA	898,846	1034
MMRA	797,830	252
Human (first attempt)	789,851	920
Human (best attempt)	898,835	1450

Iteration 2		
<i>Model</i>	<i>Solution</i>	<i>Time (sec)</i>
MTRA	746,255	146
MMRA	715,150	25
Human (first attempt)	689,360	850
Human (best attempt)	745,368	1120

Iteration 3		
<i>Model</i>	<i>Solution</i>	<i>Time (sec)</i>
MTRA	845,240	340
MMRA	818,230	55
Human (first attempt)	756,421	710
Human (best attempt)	846,188	1280

Table 6.9: Heuristic vs. manual solutions

Three iterations of this experiment were timed to compare with the solution run-times of MTRA and MMRA. Both MTRA and MMRA had a significant advantage in terms of run-time. In two out of three cases, both MMRA and MTRA were able to achieve a higher solution value than the first flight schedule developed manually.

In all cases, with enough time, the human planner was able to generate a better solution than MMRA, and support the same or a better value of AMR legs than MTRA.

A human planner is capable of solving some complex scheduling problems in very

good time. With the help of a map, some routes are intuitively more efficient than other. The difference between the human plan and the plan generated by the heuristic algorithms, therefore, was most notably the time needed to reach a solution.

We summarize both the computational results and operational analysis of the three models and the human planning method in Table 6.10.

<i>Model</i>	<i>Run-Time</i>	<i>Solution Quality</i>
SNDP _{OPT}	Slow / Intractable	Best
MTRA	Medium	Medium
MMRA	Fastest	Worst
Human (first attempt)	Medium	Medium
Human (best attempt)	Slow	Medium/Best

Table 6.10: Model Comparison

6.5 Model Application to Lift Planning

6.5.1 Short-Term Planning

The Maximum Total Return Algorithm and the Maximum Marginal Return Algorithm can be directly integrated into the lift planning process. They are scalable over a large operational area, incorporating multiple aircraft teams and AMRs. Input parameters can also be adjusted to reflect changing operational constraints.

As a *short term planning support tool*, they can propose very good solutions to Army Aviation planners based on deterministic AMR demand. These algorithms generate a complete AMR schedule and flight routes for the desired day. Combined with a human planner's ability to consider more factors than are modeled in these algorithms, they may be used by the FUOPS section of an Aviation Brigade of Battalion Task Force to help human planners plan the AMR schedule and actual routes of flights (Figure 6-15).

As a *decision support tool*, they can help the mission approval authority determine which AMRs can be supported, given existing resources.

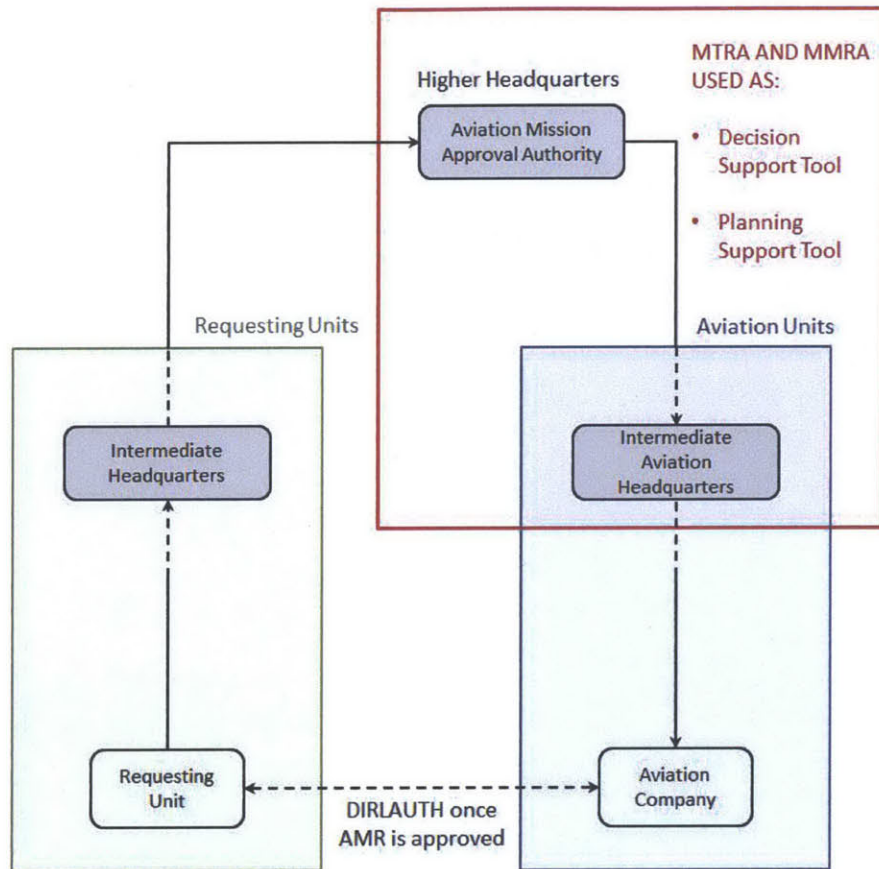


Figure 6-15: Algorithm use as decision and planning support tool

6.5.2 Long-Term Planning

6.5.2.1 Support Level Estimation

As a *long-term planning support tool*, we can use MTRA or MMRA to estimate the number of AMR legs that can be supported on a daily basis. Consider the results shown in Figure 6-16, based on the aircraft steady state and AMR parameter distribution given in Operational Scenario 1 (Section 6.2.2). As the AMR demand is increased, the rate at which additional AMR legs are supported begins to decrease. For small numbers of AMRs requested, we are able to support almost 100% of requests. The spill in these cases is negligible.

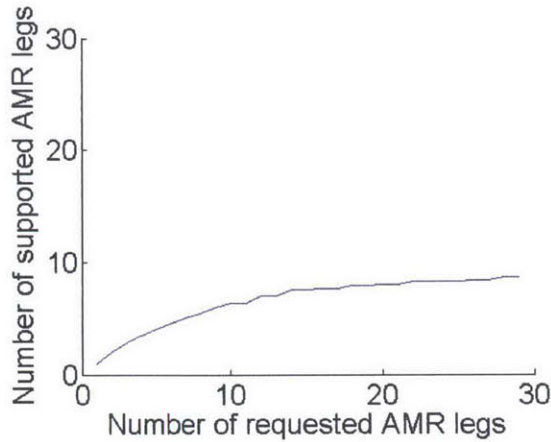


Figure 6-16: Effect of demand on support level

However, after approximately five AMR legs (for an average of 50 total passengers moved) in this scenario, the marginal number of AMR legs supported per additional AMR leg requested begins to decrease.

Given a current aircraft steady state configuration, we can estimate, for a given demand, the corresponding average support level and support rate. In this example, we can support seven AMR legs (70% of demand) if daily demand is estimated to be 10 AMR legs (Figure 6-17). Inherent variability of demand may push the support rate higher or lower on some days. In this example, the support rate drops below 70% with a demand higher than 10 AMR legs.

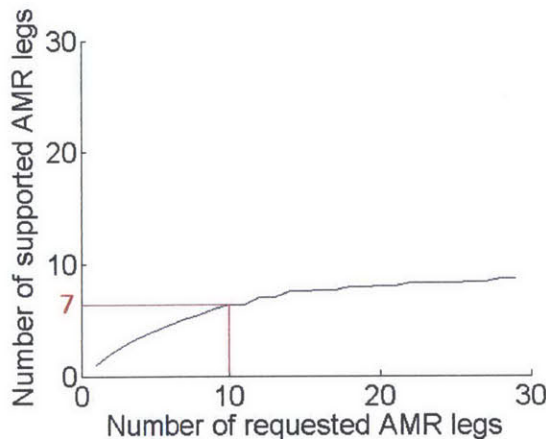


Figure 6-17: Estimating support level

We are able to estimate the spill rate, as a function of AMR demand and current

aircraft capacity. Continuing with our example, for 2 AMR legs, on average, the current aircraft team configuration can support all requested missions (Table 6.11). Beyond this level of demand, there will be an average spill rate as indicated in the table.

<i>Legs Requested</i>	<i>Legs Supported</i>	<i>Spill</i>	<i>Spill Rate</i>
2	2	0	<1%
5	4	1	20%
8	6	2	25%
10	7	3	30%
15	9	6	40%
20	10	10	50%

Table 6.11: AMR spill table

6.5.2.2 Capacity Requirement Planning

Capacity requirement planning is an important part of planning any contingency operation. For a distributed operations scenario of similar size to the operational scenarios considered in this thesis, the heuristic algorithms can be very useful in assisting human planners in determining the number of air assets required.

While an aircraft steady state can be configured in a number of ways, we make the following assumptions:

- There is a finite number of aircraft teams available, which depends on the total number available in the Task Force as well as housing for the crew, maintenance, and parking considerations.
- We use 24 hour planning cycle, with the aircraft team steady state configuration covering certain periods as dictated by the commander.
- Demand is more likely to originate in the morning and late afternoon.
- Separate planning allocates additional aircraft needed for other lift missions such as air assault, VIP support, and spares. Therefore we only find a required steady state applied to AMR support.

It is sensible to overlap multiple teams during periods of peak demand, if demand exceeds the capacity of one team. Figure 6-18 shows the different aircraft steady states used in this analysis, from three teams up to eight teams.

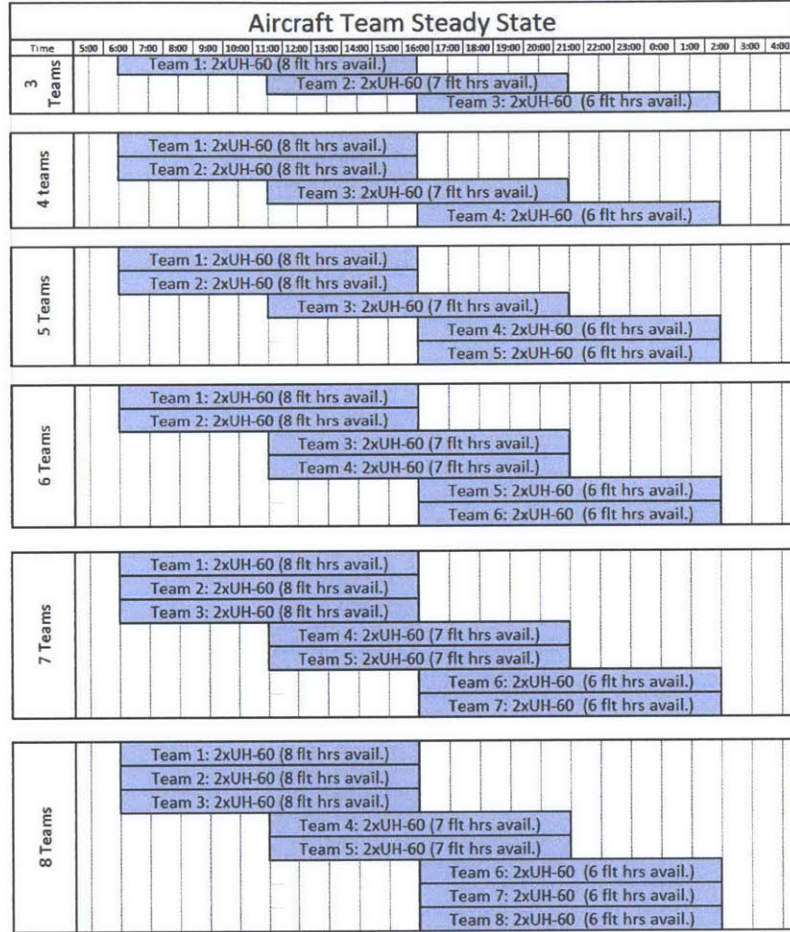


Figure 6-18: Aircraft team steady state

Capacity Estimation

Figure 6-19 shows the average support level vs. demand, given an assumed AMR leg schedule tolerance, ϕ^k , of 180 minutes (Operational Scenario 1). The dotted line represents a 100% average support rate of requested AMRs.

Regardless of the number of teams and their configuration, as the number of requested AMR legs increases, the average support rate begins to decrease. Each line represents a support level by a different aircraft steady state from Figure 6-18.

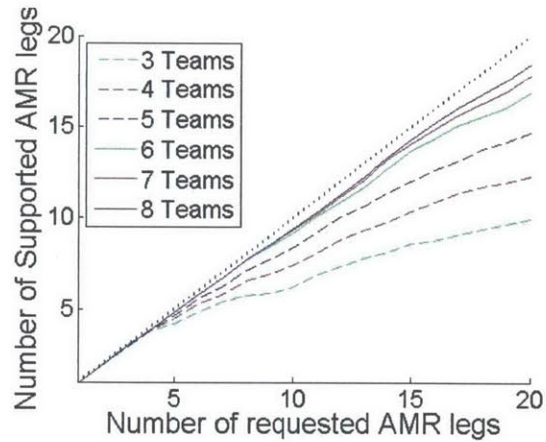


Figure 6-19: Effect of capacity on support level

With each curve, we can determine the estimated spill rate, given the demand. In this example, if we assume a daily mission requirement of 10 AMR legs for an average of 100 total passengers, we can interpret Figure 6-20 as follows:

- If demand is assumed to be 11 AMR legs per day, six aircraft teams can support on average 10 AMR legs, for an average spill rate of 9%
- If demand is assumed to be 12 AMR legs per day, five aircraft teams can support on average 10 AMR legs, for an average spill rate of 20%
- If demand is assumed to be 14 AMR legs per day, four aircraft teams can support on average 10 AMR legs, for an average spill rate of 29%

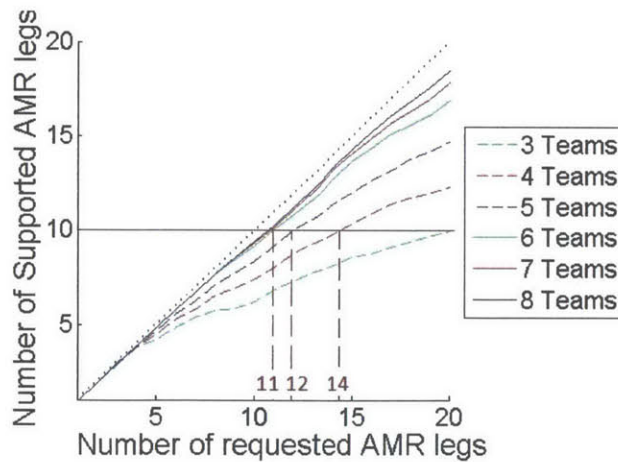


Figure 6-20: Estimating required capacity

6.6 Impact of Increased Schedule Tolerance

We can assume that increased schedule tolerance also increases, on average, the objective function value and number of AMR legs that can be supported, since larger AMR travel window allows for more paths to be created by the Path Generation Heuristic Algorithm. For Operational Scenario 1, increasing the average schedule tolerance of all requested AMR legs has a positive effect on the number of AMR legs that can be supported (Figure 6-21). Interestingly, in this scenario, the support level increase, due to increasing the schedule tolerance from three to four hours, is larger than the support level increase gained from increasing the number of teams from three to four.

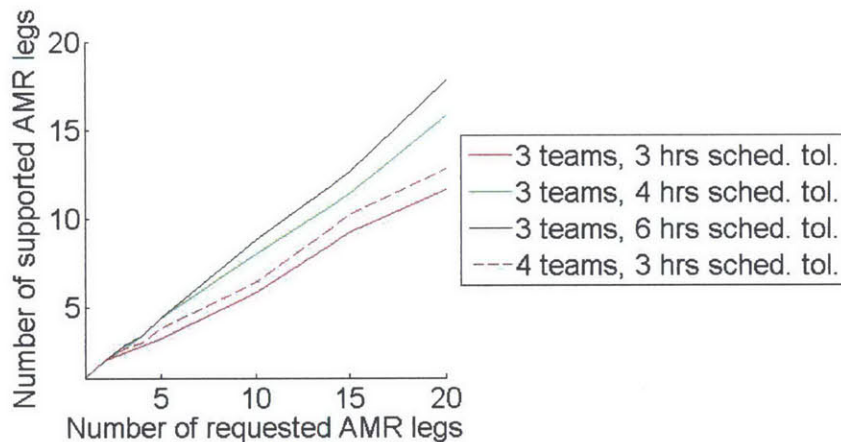


Figure 6-21: Comparing increased schedule tolerance and capacity

In another experiment, we see that the level of increase in support level caused by an increase in schedule tolerance depends on the aircraft capacity provided and the level of demand (Figure 6-22). In every scenario that we considered, using different capacities and AMR demand, *the most gains were achieved in the beginning* as schedule tolerance was increased. As we further increased the passenger schedule tolerance, we observed a *decreasing* marginal return of AMR legs supported.

The increase in supported AMR legs was most pronounced for cases where AMR demand was high. Intuitively, if aircraft teams provide sufficient capacity for the current level of demand, then increasing schedule tolerance will have very little effect on the objective function. If, on the other hand, aircraft teams cannot support

some AMR legs because of timing constraints, then increasing the schedule tolerance (creating a larger feasible travel window) for each AMR leg will have a greater effect on the number of supported AMR legs.

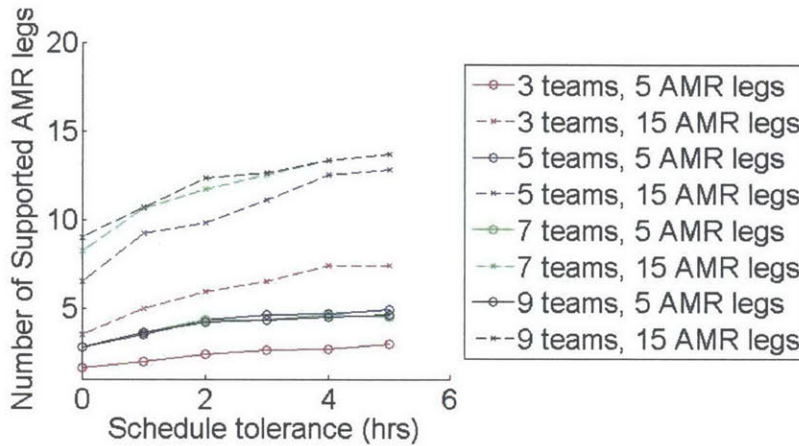


Figure 6-22: Effect of schedule tolerance on support level

This has important implications for Army Aviation lift planning. If we consider a distributed operations scenario with limited resources, an alternate way to increase the number of AMR missions supported, aside from increasing the available resources, is to require an increased schedule tolerance from requesting units. In fact, in practice this is already being done when the number of resources constrain the operational mission.

As discussed in Chapter 2, the process of AMR submission, approval, and scheduling is much more dynamic than how it appears. With Direct Liaison Authorized (DIRLAUTH), aviation planners can contact units to request flexibility in their AMR in order to ensure support. When faced with the prospect of shifting their mission rather than canceling, units are often able to adjust their schedule. While the AMR was not originally submitted with a large schedule tolerance, the process of communication between planner and requester leads to a de facto increased schedule tolerance, in some cases.

Furthermore, this concept is applied in Army Aviation lift planning through a different type of mission support: ring routes. The analogy can be made between a ring route for aircraft teams and a bus route. Weekly routes are published ahead of

time, and units can sign up for available space. Sometimes this is done through the AMR process with a specification on the AMR form that this request is for a ring route, and sometimes units have stand-alone web programs or separate processes in place specifically designed for ring route requests. By incorporating a ring route into the schedule, however, units are able to constrain low priority demand to the ring route, and reduce the number of AMRs submitted.

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Chapter 7

Summary and Future Research

We have shown that the problem of Air Mission Request (AMR) scheduling and aircraft route planning, when modeled as a service network design problem (SNDP), can become intractable with an comprehensive integer programming approach. We have also shown that a tractable heuristic model can perform very well compared to a human aviation planner. Both the Maximum Total Return Algorithm (MTRA) Maximum Marginal Return Algorithm (MMRA) provide an effective and flexible tool to solving this problem. While the work in this thesis looked primarily at a specific part of the overall set of Army Aviation missions, we feel that the work and methods used are an important step towards future work in this area.

7.1 Summary of Contributions and Results

An overview of the contributions and findings of this thesis is outlined below:

- We propose an integer programming service network design problem formulation, SNDP_{OPT} , to the challenge of AMR scheduling and route planning. We derive the inputs to this model from the AMR form, the geographical layout and characteristics of Helicopter Landing Zones, and the aircraft steady state supply provided.
- We propose two heuristic algorithms to solve the service network design formulation (MTRA, MMRA). Both make use of the Path Generation Heuristic

Algorithm (PGHA) in order to derive a feasible set of composite variables representing aircraft team paths and AMR flow. MMRA is a greedy algorithm which considers AMRs in order from highest to lowest value. MTRA is a comprehensive algorithm which looks at all AMRs at once.

- We show that problems of realistic size are intractable for SNDP_{OPT} , using a computer configuration and software that would normally be accessible to an aviation planner. MTRA and MMRA, on the other hand, are tractable for problems of this size.
- We show that solutions generated by MTRA are within 5% of optimal in 95% of the cases for problem sizes that the three models can solve.
- We show that solutions generated by MTRA are marginally better than those generated by MMRA. Solutions generated by MTRA, however, come at a higher cost in terms of computation time. We show that solutions generated by MTRA and MMRA both have advantages over a human plan, in solution time and aircraft and passenger cost.
- We discuss how these heuristic algorithms can be used as a short-term planning tool to help aviation planners develop the AMR schedule and flight routes, and how they can be used as a decision-support tool for the mission approval authority to determine which AMRs can be supported.
- We propose a method to estimate the AMR support level that can be sustained, given an estimate of demand and an acceptable AMR spill rate. This was demonstrated on an operational scenario using MTRA.
- We propose a method to estimate capacity requirements, given a required support level. We see that this estimate also depends on an estimate of demand and acceptable spill rate.
- We look at the impact of AMR schedule tolerance on support level. We conclude that increasing the schedule tolerance by only a marginal amount can lead to significant gains in support. In fact, the gains achieved outperform gains achieved

by increasing the number of aircraft teams in some cases. We relate this concept back to the current operational model of a ring route being used in the field by aviation units, to increase their AMR support level without increasing their steady state capacity.

7.2 Future Research

Below, we propose several areas of work and research that would not only be beneficial to the problem modeled in this thesis, but also to military aviation optimization in general.

- *Test and Validate models in this thesis on actual operational data.* While military passenger movement data is classified in most cases, we feel that this is the best way to validate the efficiency of the models. In most cases, these records are kept for several years or more. Actual operational data would help validate parameter assumptions and demand probability distributions that were used. Even though the author has made every effort to provide the most realistic operational scenarios, only a true set of operational data can be used to validate the model performance against actual human performance.
- *Extend models to include cargo.* In this thesis we modeled AMR demand as a number of passengers. The number of bags and cargo carried by passengers was accounted for in the seat capacity parameter, by decreasing the number of available seats on an aircraft team by the average number of bags and cargo carried by a full aircraft team. However, another model could use the additional cargo information on the AMR form and generate a mapping of passengers and cargo to available space on each type of aircraft team.
- *Extend models to include risk.* The work in this thesis concentrated primarily on using deterministic inputs to generate the flight schedule and aircraft team routing. This analysis does not take into account the risk level of the flight schedule, or the enemy situation. Future work could develop a risk function associated with every HLZ based on the threat level and number of landings

performed. The objective function for each model would take into account the maximum level of risk tolerated by the commander.

- *Conduct research on the application of MTRA on developing an n-day redistribution of AMRs.* The models in this thesis were run on the assumption that AMRs submitted for a certain day need to be resubmitted if they are not supported in the flight schedule. While this is generally true in practice, many aviation planners have taken the initiative to redistribute certain AMRs on neighboring days where the passenger load is lower. Future work could look at the solution improvement achieved by allowing redistribution of certain AMRs to other days, which is in effect increasing the schedule tolerance to several days or more.
- *Develop a dynamic reallocation algorithm based on the heuristic models in this thesis.* Once the flight schedule and aircraft routing has been completed in the Future Operations (FUOPS) section, execution monitoring of the plan is conducted in the Current Operations (CUOPS) section in the Tactical Operations Center (TOC). Throughout the day, additional missions arise that require a reallocation of resources. Extensions of the models discussed in this thesis could look at AMR cancellation procedures based on priority and cost. Depending on the current time, some AMRs have already been executed, some are in the process of being executed, and some have yet to be executed. A new algorithm could recommend cancellations and rerouting of aircraft in order to support new high priority missions, by looking at where aircraft physically are located in space and time, and the combined effect of rerouting aircraft and canceling AMRs.

Appendix A

Path Generation Heuristic Algorithm Pseudo-Code

```
1  Procedure PGHA( $f, K$ ) {
2    Global variable  $P^f := \{empty\}$ 
3    Local variable  $p := \{empty\}$ 
4    Branch( $f, p, K$ )
5    return  $P^f$ 
6  end procedure }

1  Procedure Branch( $f, p, K$ ) {
2     $K := K - \{AMR \text{ legs in } K \text{ that must end before the current time}\}$ 
3    if  $K == \{empty\}$  {
4      Gotoend( $f, p$ )
5    end if }
6     $KLIST := \{AMR \text{ legs in } K \text{ that are not on board in } f\}$ 
7    for  $k \in KLIST$  do {
8      if current capacity and current time permit picking up  $k$  {
9        if current fuel permits picking up  $k$  {
10          $[f_1, p_1, K_1error] := \mathbf{Gotostart}(f, p, k, K)$ 
11         if  $error == 0$  {
12           Branch( $f_1, p_1, K$ )
13         end if }
14         else {
15            $[f_2, p_2, error] := \mathbf{Getfuel}(f, p, \mathbf{Shortestfuel}(f, o))$ 
16           where  $o$  is the origin HLZ of  $k$ 
17           if current capacity and time permit picking up  $k$ 
18           and  $error == 0$  {
19              $[f_3, p_3, K_3error] := \mathbf{Gotostart}(f_2, p_2, k, K)$ 
20             if  $error == 0$  {
21               Branch( $f_3, p_3, K_3$ )
22             end if }
23           end if }
24         end if }
25       end do }
26     for  $k$  on board  $f$  do {
27       if current time permits dropping off  $k$  on time {
28         if fuel permits dropping off  $k$  {
29            $[f_1, p_1, K_1, error] := \mathbf{ExecuteAMR}(f, p, K, k)$ 
```

```

29         if no more AMRs on board  $f$  {
30             Gotoend( $f_1, p_1$ )
31         end if }
32         Branch( $f_1, p_1, K_1$ )
33     else {
34         [ $f_2, p_2, error$ ] := Getfuel( $f, p, Shortestfuel(f, d)$ )
           where  $d$  is the destination HLZ of  $k$ 
35         if current capacity and time permit dropping off  $k$ 
           and  $error == 0$  {
36             [ $f_3, p_3, K_3, error$ ] := ExecuteAMR( $f_2, p_2, K, k$ )
37             if  $error == 0$  {
38                 if no more AMRs on board  $f$  {
39                     Gotoend( $f_3, p_3$ )
40                 end if }
41                 Branch( $f_3, p_3, K_3$ )
42             end if }
43         end if }
44     end if }
45 end do }
46 end do }
47 return
48 end procedure }

1 Procedure Gotostart ( $f, p, k, K$ ) {
2     [ $f, p, K, error$ ] := RemoveAMRs( $f, p, K$ )
3     if  $k$  is the first AMR leg of the day {
4         if the pickup HLZ of  $k$  is the current HLZ {
5             add shutdown arc to  $p$ , from current time to earliest
           pickup time of  $k$ 
6         else {
7             add shutdown arc to  $p$ , from current HLZ to pickup HLZ of  $k$ 
8             add flight arc to  $p$ , from current HLZ to pickup HLZ of  $k$ 
9         end if }
10    else if this is not the first pickup of the day {
11        if the pickup HLZ of  $k$  is the current HLZ {
12            if  $\tau_{SD}^f$  is greater than the time remaining until the earliest
           pickup time of  $k$  {
13                if current HLZ is refuel capable and  $f$  not have full fuel
           on board{
14                    add refuel arc to  $p$ 
15                    add passenger flow to  $p$ 
16                end if }
17                add shutdown arc to  $p$ , from current time to earliest
           pickup time of  $k$ 
18                add passenger flow to  $p$ , from current time to earliest
           pickup time of  $k$ 
19            else {
20                if there is enough time to refuel before the earliest pickup
           time of  $k$  {
21                    add refueling arc arc to  $p$ 
22                    add passenger flow to  $p$ 
23                    add waiting arc to  $p$ , from current time to earliest
           pickup time of  $k$ 
24                    add passenger flow to  $p$ , from current time to earliest

```

```

25         pickup time of  $k$  {
26     else {
27         add waiting arc to  $p$ , from current time to earliest
28         pickup time of  $k$  {
29             add passenger flow to  $p$ , from current time to earliest
30             pickup time of  $k$  {
31                 end if }
32             end if }
33     else { (the current HLZ is not the pickup HLZ of  $k$ )
34         if current time is before earliest pickup time of  $k$  {
35             if destination refuel capable and there is enough time to
36             refuel before earliest pickup time of  $k$  {
37                 add flight arc to  $p$ , from current HLZ to pickup
38                 HLZ of  $k$ 
39                 add passenger flow to  $p$ , from current HLZ to pickup
40                 HLZ of  $k$ 
41                 add refuel arc to  $p$ 
42                 add passenger flow to  $p$ 
43             else if current HLZ refuel capable and there is enough
44             time to refuel before earliest pickup time of  $k$  {
45                 add refuel arc to  $p$ 
46                 add passenger flow to  $p$ 
47                 add flight arc to  $p$ , from current HLZ to pickup
48                 HLZ of  $k$ 
49                 add passenger flow to  $p$ , from current HLZ to pickup
50                 HLZ of  $k$ 
51             else {
52                 add flight arc to  $p$ , from current HLZ to pickup
53                 HLZ of  $k$ 
54                 add passenger flow to  $p$ , from current time to earliest
55                 pickup time of  $k$ 
56             end if }
57         end if }
58     if the time until earliest pickup of  $k$  is greater
59     than  $\tau_{SD}^f$  {
60         add shutdown arc to  $p$ , from current time to earliest
61         pickup time of  $k$ 
62     else {
63         add waiting arc to  $p$ , from current time to earliest
64         pickup time of  $k$ 
65         add waiting arc to  $p$ , from current time to earliest
66         pickup time of  $k$ 
67     end if }
68     else { (current time is not before earliest pickup time of  $k$ )
69         add flight arc to  $p$ , from current HLZ to pickup
70         HLZ of  $k$ 
71         add passenger flow to  $p$ , from current time to earliest
72         pickup time of  $k$ 
73     end if }
74     end if }
75     end if }
76     add AMR leg  $k$  on board  $f$ 
77     if the current capacity of  $f$  is negative, or any AMRs on board are
78     past their latest arrival time {
79         error := 1
80     }

```

```

61     end if }
62     [ $f, p, K, error$ ] := RemoveAMRs( $f, p, K$ )
63     return [ $f, p, K, error$ ]
64 end procedure }

1  Procedure Gotoend ( $f, p, K$ ) {
2      if we are at the ending base for team  $f$  {
3          add shutdown arc to  $p$ , from current HLZ to ending HLZ for  $f$ 
4      else if enough fuel to fly to ending HLZ {
5          add flight arc to  $p$ , from current HLZ to ending HLZ for  $f$ 
6      else {
7          [ $f, p, error$ ] := Getfuel( $f, p, Shortestfuel(f, d)$ )
           where  $d$  is the destination HLZ of AMR leg  $k$ 
8          add flight arc to  $p$ , from current HLZ to ending HLZ for  $f$  {
9      end if }
10     if no fuel or time constraints are violated form team  $f$  {
11         if  $p$  supports the same AMR legs as other paths in  $P^f$  {
12             if  $p$  is more valuable than these paths {
13                 add  $p$  to  $P^f$ 
14             else {
15                 add  $p$  to  $P^f$ 
16             end if }
17         end if }
18     return
19 end procedure }

```

```

1  Procedure Shortestfuel( $f, b$ )
2       $a$  := current HLZ of team  $f$ 
3       $q$  :=  $a$ 
4       $mindistance$  :=  $M$  (where  $M$  is a very large constant)
5      for each refuel-capable HLZ  $i$ , do {
6          if remaining fuel allows travel to  $i$  {
7              if Distance( $a, i$ ) + Distance( $i, b$ ) <  $mindistance$  {
8                   $mindistance$  := Distance( $a, i$ ) + Distance( $i, b$ )
9                   $q$  :=  $i$ 
10             end if }
11         end do }
12     end do }
13     return  $q$ 

```

```

1  Procedure RemoveAMRs( $f, p, K$ ){
2      for all AMRs  $k$  on board  $f$  do {
3          if current HLZ is the destination HLZ of  $k$  {
4              if current time is before latest arrival time of  $k$  {
5                  remove  $k$  from on board team  $f$ 
6                  remove  $k$  from  $K$ 
7                  add value of  $k$  to  $p$ 
8              else {
9                   $error$  := 1
10             end if }
11         end if }
12     end do }

```

```

13     if any AMRs were unloaded {
14         add unloading arc to  $p$ 
15         add passenger flow to  $p$ 
16     end if }
17     return [ $f, p, K, error$ ]
18 end procedure }

1 Procedure ExecuteAMR( $f, p, K, k$ ) {
2     [ $f, p, K, error$ ] := RemoveAMRs( $f, p, K$ )
3     add flight arc to  $p$ , from current HLZ to destination HLZ of AMR leg  $k$ 
4     add passenger flow to  $p$ , from current HLZ to destination HLZ of
    AMR leg  $k$ 
5     [ $f, p, K, error$ ] := RemoveAMRs( $f, p, K$ )
6     return [ $f, p, K, error$ ]
7 end procedure }

1 Procedure Getfuel( $f, p, q$ ) {
2     add flight arc to  $p$ , from current HLZ to HLZ  $q$ 
3     add passenger flow to  $p$ , from current HLZ to HLZ  $q$ 
4     add refueling arc to  $p$ 
5     add passenger flow to  $p$ 
6     return [ $f, p$ ]
7 end procedure }

```

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Appendix C

Sets, Parameters, and Variables

Aircraft Teams

<i>Set</i>	<i>Elements</i>	<i>Parameters</i>	<i>Description</i>
<i>F</i>	<i>f</i>	γ	number of aircraft teams
		<i>aircraft team</i>	
		u^f	passenger capacity
		τ_{CYCLE}^f	flying hour limit per planning cycle
		τ_{FLIGHT}^f	flying hour limit on each full tank of fuel
		τ_{FUEL}^f	average refuel time required
		τ_{LOAD}^f	average loading/unloading time
		τ_{SD}^f	minimum time required to shutdown
		\bar{c}^f	cost of flight arc per time period
		\underline{c}^f	cost of ground arc per time period
		t_{START}^f	first time period that team is available
		t_{END}^f	last time period that team is available

Air Mission Requests

<i>Set</i>	<i>Subset</i>	<i>Element</i>	<i>Param.</i>	<i>Description</i>	
\mathcal{L}	L	<i>air mission request</i>			
			η^L	AMR mission number	
			ρ^L	number of AMR legs	
		k	<i>air mission request leg</i>		
				η^L	parent AMR mission number
				η^k	AMR leg number
				w^k	number of passengers
				v^k	value of AMR leg
				b^{k-}	starting HLZ
				b^{k+}	ending HLZ
				t^{k-}	earliest departure time from starting HLZ
				t^{k+}	latest arrival time at ending HLZ
				c^k	cost of flight arc per time period
				ΔT^k	direct flight time of AMR
				ϕ^k	schedule tolerance

Path Composite Variables

<i>Set</i>	<i>Subset</i>	<i>El.</i>	<i>Param.</i>	<i>Description</i>
\mathcal{P}	P^f	<i>aircraft team path composite variable</i>		
		$p^{f,\lambda}$	λ	number of path
			$v^{f,\lambda}$	value of path
			$c^{f,\lambda}$	cost of path
			$\mathbf{q}^{f,\lambda}$	vector of dimension $ K $, where $\mathbf{q}^{f,\lambda}(k) = \begin{cases} 1 & \text{if AMR leg } k \text{ is supported} \\ 0 & \text{otherwise} \end{cases}$
			$A^{f,\lambda}$	set of arcs on path for team f
			$A^{k,\lambda}$	set of arcs on path for AMR leg k

Network Data

- β Total number of HLZs in the area of operations;
- B Set of all HLZs i indexed from 1 to β ;
- B_{FUEL} Set of HLZs that have refuel capability for helicopters;
- N The set of all nodes in the time-space graph. For every HLZ in B and time duration in T , we create a node and index it by (b, t) , where b is the index of the HLZ in B , and t is the index of the time period in T ;
- N_{FUEL} Subset of all nodes that have rotary wing refuel capability, $N^{FUEL} \subseteq N$;
- a An arc indexed by origin node and destination node $\{(o, t_i), (d, t_j)\}$, indicating that it begins at node (o, t_i) and ends at node (d, t_j) , and is of duration $\tau_a = t_j - t_i$, where $j > i$;
- A Set of all arcs in the time-space graph;
- A^f Set of all arcs in the time space graph for aircraft team $f \in F$. Within the set A^f , we have the following:
- $\overline{A^f}$ Set of air arcs in the time-space graph for aircraft team f . Air arcs are defined between nodes (o, t_i) and (d, t_j) where $t_j > t_i$, $t_j - t_i = \text{flight time from } o \text{ to } d \text{ for team } f$, and $o \neq d$.
- $\underline{A^f}$ Set of all ground arcs in the time-space graph for aircraft team f . A ground arc is defined as an arc between nodes (o, t_i) and (d, t_j) where $t_j > t_i$, and $o = d$. Within the set of all ground arcs, we have the following:
- A_{WAIT}^f Set of all waiting arcs of length τ . Waiting arcs are defined between all pairs of nodes $\{(o, t_i), (o, t_{i+1})\}$;
- $A_{WAIT,t}^f$ Set of all waiting arcs of length τ that begin at time t ;
- A_{SD}^f Set of all shutdown arcs. Shutdown arcs are defined between all pairs of nodes $\{(o, t_i), (o, t_i + \tau_{SD}^f)\}$;

$A_{SD,t}^f$	Set of all shutdown arcs that begin at time t ;
A_{FUEL}^f	Set of all refueling arcs of length τ_{FUEL}^f . Refueling arcs are defined between all pairs of nodes $\{(o, t_i), (o, t_i + \tau_{FUEL}^f)\}$;
$A_{FUEL,t}^f$	Set of all refueling arcs of length τ_{FUEL}^f that begin at time t ;
A_{LOAD}^f	Set of all loading/unloading arcs of length τ_{LOAD}^f . Loading/unloading arcs are defined between all pairs of nodes $\{(o, t_i), (o, t_i + \tau_{LOAD}^f)\}$;
$A_{LOAD,t}^f$	Set of all loading/unloading arcs of length τ_{LOAD}^f that begin at time t ;
A_n^{f-}	Subset of arcs in A^f that leave node n ;
A_n^{f+}	Subset of arcs in A^f that enter node n ;
A_b^{f-}	Subset of arcs in A^f that leave HLZ b ;
A_b^{f+}	Subset of arcs in A^f that enter HLZ b ;
A_t^f	Subset of arcs in A^f that enter or exit any node at time t ;
A_t^{f-}	Subset of arcs in A^f that exit any node at time t ;
A_t^{f+}	Subset of arcs in A^f that enter any node at time t ;
$A_{[t_i t_j]}^f$	Subset of arcs in A^f that begin at or before time period t_i and end at or after time period t_j ;

D $(|N| \times |A|)$ incidence matrix for AMR flow variables, where:

$$\mathbf{D}(n, a) = \begin{cases} 1 & \text{if arc } a \text{ starts at node } n \\ -1 & \text{if arc } a \text{ ends at node } n \\ 0 & \text{otherwise} \end{cases} \quad \forall n \in N, a \in A$$

\mathbf{D}^f ($|N| \times |A^f|$) incidence matrix for aircraft team flow variables, *specific to each aircraft team f* , where:

$$\mathbf{D}^f(n, a) = \begin{cases} 1 & \text{if arc } a \text{ starts at node } n \\ -1 & \text{if arc } a \text{ ends at node } n \\ 0 & \text{otherwise} \end{cases} \quad \forall n \in N, a \in A^f$$

\mathbf{E} ($\gamma \times |N|$) matrix indicating departure and arrival node for aircraft teams, where:

$$\mathbf{E}(f, n) = \begin{cases} 1 & \text{if team } f \text{ starts at node } n \\ -1 & \text{if team } f \text{ ends at node } n \\ 0 & \text{otherwise} \end{cases} \quad \forall n \in N, f \in F$$

\mathbf{B} ($\rho \times |N|$) matrix indicating demand data where:

$$\mathbf{B}(k, n) = \begin{cases} w^k & \text{if AMR leg } k \text{ starts at node } n \\ -w^k & \text{if AMR leg } k \text{ ends at node } n \\ 0 & \text{otherwise} \end{cases} \quad \forall n \in N, k \in K$$

\mathbf{J} ($|\mathcal{L}| \times |K|$) matrix where:

$$\mathbf{J}(L, k) = \begin{cases} 1 & \text{if leg } k \text{ belongs to AMR } L \\ 0 & \text{otherwise} \end{cases} \quad \forall L \in \mathcal{L}, k \in K$$

$\mathbf{q}^{f,\lambda}$ vector of length $|K|$ indicating which AMR legs are supported on path $p^{f,\lambda}$ where:

$$\mathbf{q}^{f,\lambda}(k) = \begin{cases} 1 & \text{if leg } k \text{ is supported on path } p^{f,\lambda} \\ 0 & \text{otherwise} \end{cases}$$

AMR Flow Decision Variables

$$\theta^L = \begin{cases} 1 & \text{if AMR } L \text{ is supported} \\ 0 & \text{otherwise} \end{cases}$$

$$\delta^k = \begin{cases} 1 & \text{if AMR leg } k \text{ is supported} \\ 0 & \text{otherwise} \end{cases}$$

$$x_a^{k,f} = \begin{cases} 1 & \text{if AMR leg } k \text{ is supported on aircraft } f \text{ on arc } a \\ 0 & \text{otherwise} \end{cases}$$

$$\hat{x}_a^k = \begin{cases} 1 & \text{if AMR leg } k \text{ is scheduled on arc } a \\ 0 & \text{otherwise} \end{cases} \quad (\text{dummy variable})$$

ζ_{start}^k = start of travel time of AMR leg k

ζ_{end}^k = end of travel time of AMR leg k

ζ^k = total travel time of AMR leg k

Aircraft Team Arc Decision Variables

$$y_a^f = \begin{cases} 1 & \text{if aircraft team } f \text{ is scheduled on flight arc } a \\ 0 & \text{otherwise} \end{cases}$$

$$g_a^f = \begin{cases} 1 & \text{if aircraft team } f \text{ is scheduled on waiting arc } a \\ 0 & \text{otherwise} \end{cases}$$

$$v_a^f = \begin{cases} 1 & \text{if aircraft team } f \text{ is scheduled on loading/unloading arc } a \\ 0 & \text{otherwise} \end{cases}$$

$$r_a^f = \begin{cases} 1 & \text{if aircraft team } f \text{ is scheduled on refueling arc } a \\ 0 & \text{otherwise} \end{cases}$$

$$s_a^f = \begin{cases} 1 & \text{if aircraft team } f \text{ is scheduled on shutdown arc } a \\ 0 & \text{otherwise} \end{cases}$$

Composite Variable Path Decision Variable

$$z^{f,\lambda} = \begin{cases} 1 & \text{if path } \lambda \text{ is selected for team } f \\ 0 & \text{otherwise} \end{cases}$$

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Appendix B

Abbreviations and Acronyms

AHB	Assault Helicopter Battalion
AMB	Air Mission Brief
AMC	Air Mobility Command
AMR	Air Mission Request
ARB	Attack Reconnaissance Battalion
ARS	Attack Reconnaissance Squadron
ASB	Aviation Support Battalion
ATS	Air Traffic Services
BAE	Brigade Aviation Element
C2	Command and Control
CAB	Combat Aviation Brigade
CMPL	Commander's Mission Priority List
CONOP	Concept of the Operation
COP	Combat Outpost
CUOPS	Current Operations (section)

CVF	Composite Variable Formulation
DIRLAUTH	Direct Liaison Authorized
DWM	Decision Window Model
FAM	Fleet Assignment Model
FOB	Forward Operating Base
FUOPS	Future Operations (section)
GSAB	General Support Aviation Battalion
GWOT	Global War on Terror
HLZ	Helicopter Landing Zone
MCF	Minimum Cost Flow Problem
MCNF	Multi-Commodity Network Flow Problem
MEDEVAC	Medical Evacuation
METL	Mission Essential Task List
MFE	Maneuver, Fires, and Effects
MMRA	Maximum Marginal Return Algorithm
MSPP	Maximum Set Packing Problem
MTRA	Maximum Total Return Algorithm
NDP	Network Design Problem
PC	Pilot-in-Command
PDF	Probability Density Function
PGHA	Path Generation Heuristic Algorithm
POC	Point of Contact

S3	Training and operations section of battalion or brigade headquarters
SIGINT	Signal Intelligence
SNDP	Service Network Design Problem
SOP	Standing Operating Procedures. The military uses Standing Operating Procedures, rather than Standard Operating Procedures, because a military SOP refers to a unit's unique procedures, and may not be the same across all units.
TF	Task Force
TOC	Tactical Operations Center
TTPs	Techniques, Tactics and Procedures
UAV	Unmanned Aerial Vehicle
UAS	Unmanned Aerial Systems
VMC	Visual Meteorological Conditions

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