Optimal Design of Fabric Formed Concrete Beams

by

Marwan Sarieddine

Bachelor of Engineering in Civil and Environmental Engineering
American University of Beirut, 2013

Submitted to the Department of Civil and Environmental Engineering
in Partial Fulfillment of the Requirements for the Degree of

MASTER OF ENGINEERING IN CIVIL AND ENVIRONMENTAL ENGINEERING

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2014

Signature of Author: Signature redacted
Department of Civil and Environmental Engineering
May 9, 2014

Certified by: Signature redacted
John A. Ochsendorf
Professor of Architecture and Civil and Environmental Engineering
Thesis Supervisor

Accepted by: Signature redacted
Heidi M. Nepf
Chair, Departmental Committee for Graduate Students

Optimal Design of
Fabric Formed Concrete Beams

by

Marwan Sarieddine

Submitted to the Department of Civil and Environmental Engineering

on May 9, 2014 in partial fulfillment of the requirements for the

degree of Master of Engineering in Civil and Environmental Engineering.

ABSTRACT

The topic of fabric formwork has emerged as a response to the rising need for material efficient designs that also incorporate attractive aesthetic and construction related features. The thesis approaches the topic of the optimization of the design of fabric formed concrete beams. The thesis proposes two methods: an analytical optimization method and a feasible region method. The optimum design of fabric formed reinforced concrete beams is discussed first and a sample output of the optimum design based on minimizing the cost of a cross-section is produced. A relatively direct design process based on simple polynomials is established that can conveniently guide designers to produce optimal designs. Based on sample results, savings of up to 55% in material cost could be accomplished using fabric formed beams.

The subject of pre-stressed fabric formed beams is then approached using the two methods. Certain additional complexities are explained and some simplifications are done in order to arrive at an optimum design using the feasible region method.

Thesis Supervisor: John A. Ochsendorf.

Title: Professor of Architecture and Civil and Environmental Engineering

Acknowledgements

This thesis benefited from the brief yet helpful communications with Professor Mark West at the University of Manitoba, Canada. First, I would really like to thank Dr. Caitlin Mueller for patiently helping me in producing this thesis. Second, I would like to thank Professor John Ochsendorf for his guidance and support. Third, I would also like to thank Dr. Pierre Ghisbain for his assistance on this thesis as well as his help in all the courses I took in this program. Moreover, I acknowledge the input and helpful criticism of everyone in the structures design lab.

I am truly thankful for Benjamin Jenett for interesting conservations, inspirational criticism and constant support. Moreover I am truly grateful to Alexis Ludena who managed to connect me with the right resources here at MIT and who always supplied me with a pragmatic and realistic criticism on most of my crazy ideas. I am also truly appreciative to my brother Bashir for always making me think twice about things in life. Lastly, a dear thanks goes to my parents Afif and Rajaa Sarieddine for their love and support.

Table of Contents

Symbols and Notations

$A = Area\ of\ Cross - Section$

$A_s = Area\ of\ non - prestressed\ reinforcement$

$A_p = Area\ of\ prestressed\ reinforcement$

$E_c = Modulus\ of\ Elasticity\ of\ Concrete$

$E_p = Modulus\ of\ elasticity\ of\ Steel$

$n = \frac{Ep}{Ec} = modulus\ of\ elasticity\ ratio$

$x_0 = Half\ of\ the\ top\ breadth\ of\ the\ cross - section$

$C = Compression\ Force$

$\epsilon_c = Concrete\ Strain$

$f_c' = Concrete\ compressive\ strength$

$f_r = Concrete\ tensile\ strength\ (rupture)$

$I = Second\ moment\ of\ Inertia$

$Z = Section\ modulus$

$M_{DL} = Moment\ due\ to\ dead\ load$

$M_{LL} = Moment\ due\ to\ live\ load$

$M_{\{total\}} = Total\ service\ moment = M_{DL} + M_{LL}$

$M_u = Ultimate\ moment = 1.2 * M_{DL} + 1.6 * M_{LL}$

$f_y = Tensile\ Strength\ of\ mild\ reinforcement\ Steel$

$f_p = Tensile\ Strength\ of\ prestressed\ reinforcement$

$p = Non - prestressed\ reinforcement\ ratio = \frac{A_s}{A}$

$p_b = Prestressed\ reinfocement\ ratio = \frac{A_p}{A}$

$T_0 = Tension\ in\ the\ fabric$

$\rho = Density\ of\ concrete$

$g = Gravitational\ acceleration$

$l = Length\ of\ fabric$

$K = Complete\ elliptic\ integral\ of\ the\ first\ kind$

$F = Incomplete\ elliptic\ integral\ of\ the\ first\ kind$

$E = Incomplete\ elliptic\ integral\ of\ the\ second\ kind$

$x_s = The\ x - coordinate\ of\ the\ profile\ of\ the\ cross - section.$

$ys = The\ y - coordinate\ of\ the\ profile\ of\ the\ cross - section$

1. Introduction

The thesis tackles the specific topic of the optimal design of fabric formed concrete beams. Two main methods are introduced and then sample results are produced. The optimum design of fabric formed reinforced concrete beams is discussed first and finally the design of pre-stressed fabric formed beams is examined.

1.1 Flexible Formwork

The essence of the revolution in structural freedom for Nervi (1956) "consists in the possibility of realizing structures that are in perfect conformity to statical needs and visually expressive of the play of forces within them." The transformation from pure prismatic shapes to shapes that yield themselves to the stresses has been facilitated by the introduction of new forms of formwork. Fabric formwork expands the limits of architectural expression by allowing the concrete to be closer to its fluid nature (West & Araya, 2009). Fabric formwork has the potential to allow the designer to consider new forms that couldn't be achieved using wooden forms.

The topic of fabric formwork has emerged as an attempt to produce aesthetically pleasing forms, which also enhance material efficiency. Orr (2012) explains the significance of decreasing material use from an embodied energy perspective. Embodied energy is defined as the total energy utilized in the construction process and does not include the added energy consumed in the operation of a structure (Orr, 2012). The plateau of efficiency in building technology performance has increased the relative importance of embodied energy as shown by (Orr, 2012). The material efficiency of fabric formwork was evaluated by Garbett, Darby, & Ibell (2010) and (Lee, 2010) who compared the material use in fabric formwork to that in prismatic rectangular beams and has estimated a saving of up to 40 percent in terms of material.

Shown in Figure 1.1 is a sample fabric formed concrete beam that is designed to follow the shape of a bending moment diagram of a simply supported beam subjected to uniform loading. Producing a feasible design option has been well studied (Lee, 2010), and (Orr, 2012). However to maximize the role of the fabric formed beam, this thesis seeks an optimum design for both reinforced and pre-stressed fabric formed beams.

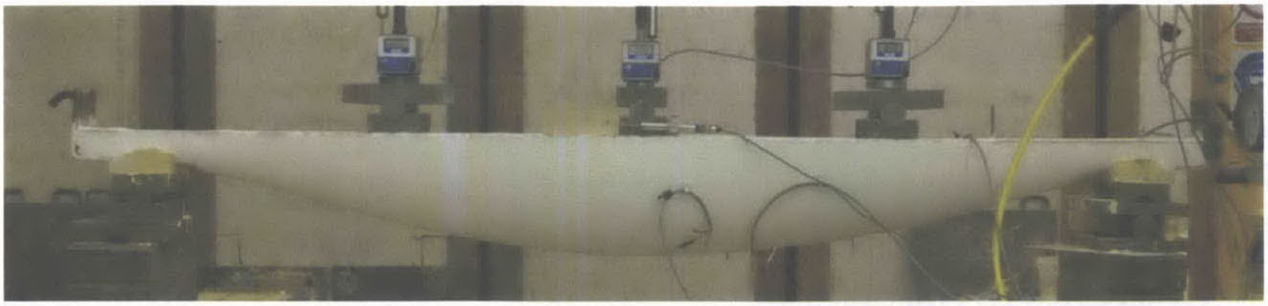

Figure 1.1 Picture taken from (Bailiss, 2006) showing a three point test of a fabric formed beam

1.2 Existing Work in Optimized Concrete Design

The use of constrained minimization for the design of prismatic reinforced concrete sections has already been explored using various methods. For reinforced concrete beam design, Guerra & Kiousis (2006) explored the design optimization of reinforced concrete structures using a nonlinear programming algorithm, which searches for a minimum cost solution that satisfies ACI 2005 code requirements for axial and flexural loads. Others have relied on various algorithms in order to minimize an objective function related to the volume or cost of a design. For instance (Marano , Fiore, & Quaranta, 2014) used a differential evolution algorithm to find the optimum minimum cost design of pre-stressed concrete beams and (Al-Gahtani, Al-Saadoun , & Abul-Feilat, 1995) performed a single objective optimization of partially pre-stressed beams. However the topic of fabric formed beams has not yet been approached using minimization algorithms that aim at minimizing a cost objective function.

The most relative work concerning the topic of optimizing reinforced fabric formed beams has been approached by Garbett, Darby, & Ibell (2010). The optimum design of the fabric formed beams is suggested as a stepwise method that is deduced from empirical relations derived by Bailiss (2006). The paper suggests the methodology to select the depth, and top breadth of a fabric cross-section according to strength based design. It also concluded that up to 55 % of the material can be saved using fabric formed beams when compared to rectangular prismatic beams (Garbett, Darby, & Ibell , 2010).

The optimization models do not so far deal with a mathematical model for a constrained problem of fabric formed beams. The optimization that was carried out by Garbett, Darby, & Ibell (2010) does not rely on analytical relationships and however is based on empirically derived formulae. This makes the method followed by the paper inapplicable in the following scenarios:

- If the top breadth is less than 10% of the total perimeter
- If the cross-section area is less than 10% of the top breadth multiplied by the section depth.

The inapplicability of the optimization method in these scenarios is not extremely significant as certain limits to the ratio of breadth to perimeter need to be specified for constructability purpose (Garbett, Darby, & Ibell , 2010).

1.3 Problem Statement

In contrast with the described existing work, this thesis focuses on producing an optimal design of fabric formed concrete beams using analytically derived relationships. More importantly the thesis attempts to explore how the design is affected by introducing material cost to the model.

The concept of pre-stressed fabric formed beams has been mentioned by Orr (2012) however the feasibility of the design and the methodology to optimize the design of pre-stressed fabric formed beams has still not been tackled. This might be due to certain complexities that arise in the constructability of pre-stressed fabric formed beams and also in establishing analytical relationships that describe various cross-section parameters. The development of the optimization model for reinforced concrete design according to analytically derived relationships is an attempt to establish similar relations for pre-stressed concrete design.

The thesis develops a methodology, which could be used by engineers to design optimal fabric formed beams. However the methods explained below have certain limitations since the relations below tackle the problem of a simply supported beam subjected to uniform loading conditions. Moreover the model below assumes a basic cross-sectional shape of a fabric draped between two boundaries and therefore doesn't look closely at other cross-sections that might be pinched or constrained using wooden formwork. However a methodology for various cross-sections is stated in the concluding part of the thesis as a guide for designing fabric formed beams with a greater sense of freedom.

1.4 Organization of thesis

A literature review of the topic of fabric formed reinforced beams is introduced in Chapter 2 which looks mostly at different form finding processes and certain approximations which need to be made for strength based design. Chapter 3 illustrates the approach to the problem of optimal fabric formed reinforced concrete design. The non-linear constrained optimization is approached by two models: a non-linear programming model and a feasible region model. Chapter 4 details an application of the methodology to a specific example. Chapter 5 introduces the pre-stressed design optimization and the new considerations to be taken into account. It also discusses some results of the optimization method. Chapter 6 concludes with certain recommendations as well as possible future recommendations.

2. Background

In order to explain the optimization model used for the design of fabric formed beams, three main principles need to be illustrated: the process of form-finding that is to be used in the following thesis, and an approximate model for strength based design of the cross-section after it has dried, finally an evaluation of the cost of a fabric formed cross-section.

2.1 The form-finding process

In order to get a close approximation of the area of the cross-section of a fabric-formed beam a form-finding process developed by Iosilevskii has been adopted (Iosilevskii, 2009). This method basically deals with the problem of form finding from a static equilibrium perspective. After the concrete has settled in the fabric, the main forces that exist in a free body diagram shown in Figure 2.1 are the hydrostatic pressure and the tension. The final shape of the fabric could be determined using two parameters only: the top width of the beam as well as the length of the fabric that is draped between the width. (Iosilevskii, 2009) looks at an infinitesimal element and uses the equilibrium relations as well as symmetry considerations to solve a set of two differential equations. Figure 2.1 below illustrates Iosilveskii's model:

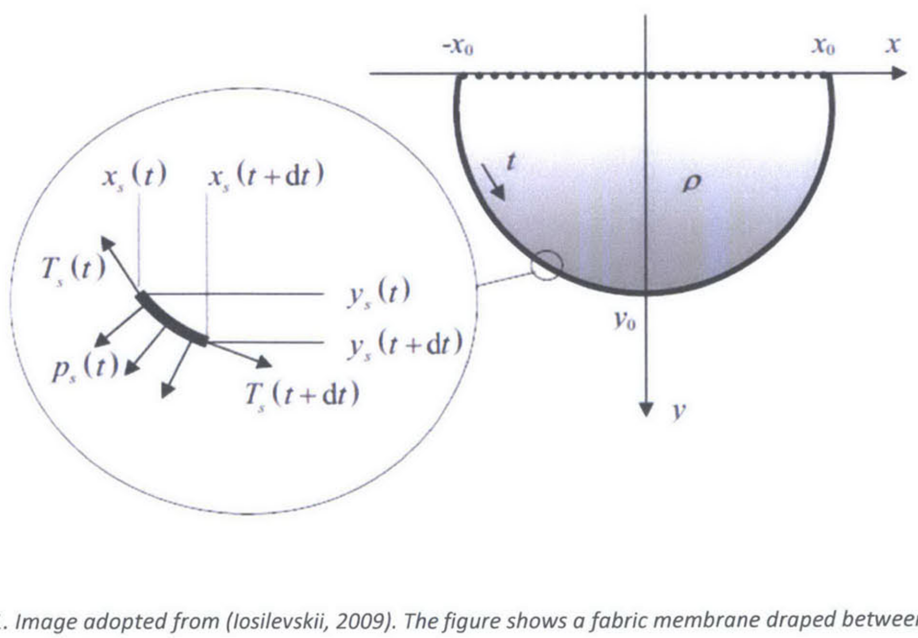

*Figure 2.1. Image adopted from (Iosilevskii, 2009). The figure shows a fabric membrane draped between a total width of 2*x0. A close up on an infinitesimal element shows the tension and the pressure force, i.e. the forces in the equilibrium formulation.*

The final deduced equations by (Iosilevskii, 2009) which describe the tension needed, the profile of the fabric, and the area of the cross-section are respectively shown on the following page:

$$T_0 = \frac{\rho g l^2}{4K^2(k)} \quad \text{(eq.1)}$$

Where:

T_o: Tension in the fabric (lb)

ρ: Density of concrete (psf)

g: gravitational acceleration (ft/s^2)

l : length of fabric used (in)

K: a complete elliptic integral of the first kind

where a complete elliptic integral is a special case of the incomplete elliptic integral with $\theta = \frac{\pi}{2}$

$$\frac{x_s}{l} = \frac{E(\theta_s, k)}{K(k)} - \frac{1}{2}\frac{F(\theta_s, k)}{K(k)} \quad \text{(eq.2)}$$

$$\frac{y_s}{l} = \frac{k}{K(k)} cos\theta_s \quad \text{(eq.3)}$$

Where:

k: the solution of the following equation: $\frac{E(k)}{K(k)} = \frac{1}{2} + \frac{x0}{l}$

x_0: half of the top width of the concrete cross-section

x_s: the x-coordinate of the profile of the cross-section (in)

y_s: the y-coordinate of the profile of the cross-section (in)

F: an incomplete elliptic integral of the first kind

E: an incomplete elliptic integral of the second kind

Where the incomplete elliptic integral of the first kind is given by the equation below:

$$F(\theta,k) = \int_0^{\theta} \sqrt{1-k^2(sin\tau)^2}^{\,-1} d\tau \qquad \text{(eq.4)}$$

And the incomplete elliptic integral of the second kind is given by the following equation:

$$E(\theta,k) = \int_0^{\theta} \sqrt{1-k^2(sin\tau)^2}\, d\tau \qquad \text{(eq.5)}$$

The area of the concrete section could be estimated by the following derived solution:

$$A = l^2 \frac{k\sqrt{1-k^2}}{K^2(k)} \qquad \text{(eq.6)}$$

Where:

A: area of the cross-section of the fabric formed beam (in^2).

In the above derivation the tension in the fabric is assumed to be constant throughout the membrane. The above model has been validated by (Foster, 2010) through comparison to empirical relations developed by (Bailiss, 2006). Figure 2.2 shows the accuracy of Iosilevskii's mathematical model compared to a physical model from Orr (2012).

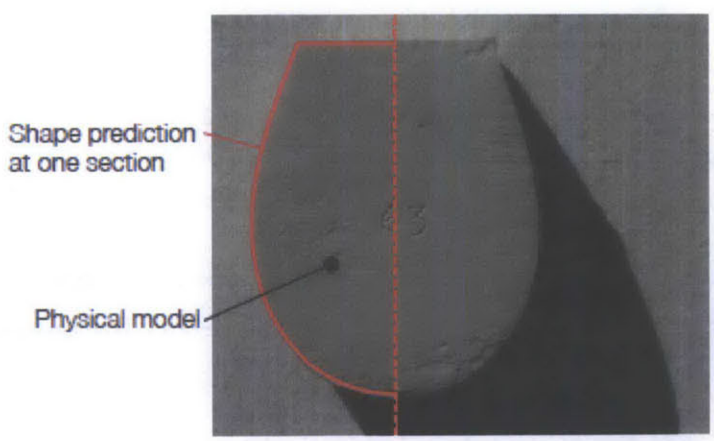

Figure 2.2 Shape prediction precision of the form-finding method. Picture taken from (Orr, 2012),

(Veenendaal, 2008) has developed an accurate form-finding method using dynamic relaxation, however this method tends to computationally be more complex as it relies on feedback from a finite element software and therefore the methodology could not be simplified easily to be directly used in the optimization model. Therefore the Veenendaal model will not be used in the optimization approach. However this entails that the designer will have to make various cross-section cuts as the method of form finding by Iosilevskii (2009) is based solely on two dimensional slices of the beam cross-section.

For the purposes of the optimization model, a new approach proposed by this thesis is to carry out a polynomial approximation of the Ioseliveskii parameters. This approximation will prove crucial as the optimization problem can then be later simplified to be a function of only the decision variables (x_0/l). In order to ensure universality of the approximation, dimensionless parameters were used. Shown below is an approximation of k as a function of $\frac{2x_0}{l}$ where:

$$k = -16.982\left(\frac{2x0}{l}\right)^6 + 45.557\left(\frac{2x0}{l}\right)^5 - 46.567\left(\frac{2x0}{l}\right)^4 + 22.267\left(\frac{2x0}{l}\right)^3 - 5.2611\left(\frac{2x0}{l}\right)^2 + 0.138\left(\frac{2x0}{l}\right) + 0.8985 \quad \text{(eq.7)}$$

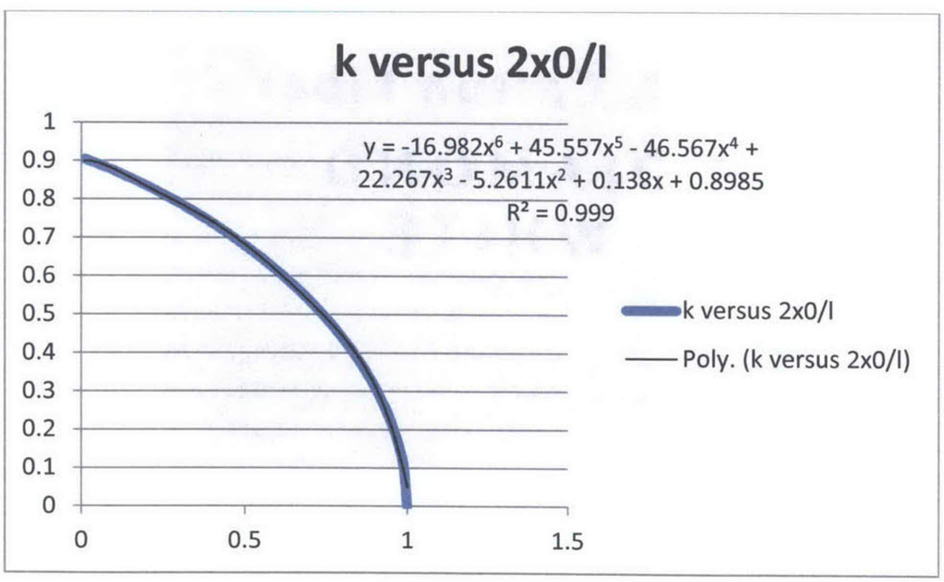

*Figure 2.3 Plot illustrating the polynomial fit between k and 2*xo/l*

K, the complete integral of the first kind could be estimated using a power series, however a polynomial approximation to a lower degree was used where:

$$K = 16.079\,(\mathrm{k})^6 - 37.911\,(\mathrm{k})^5 + 35.039\,(\mathrm{k})^4 - 15.13\,(\mathrm{k})^3 + 3.4482\,(\mathrm{k})^2 - 0.2272(\mathrm{k}) + 1.5717 \qquad \text{(eq.8)}$$

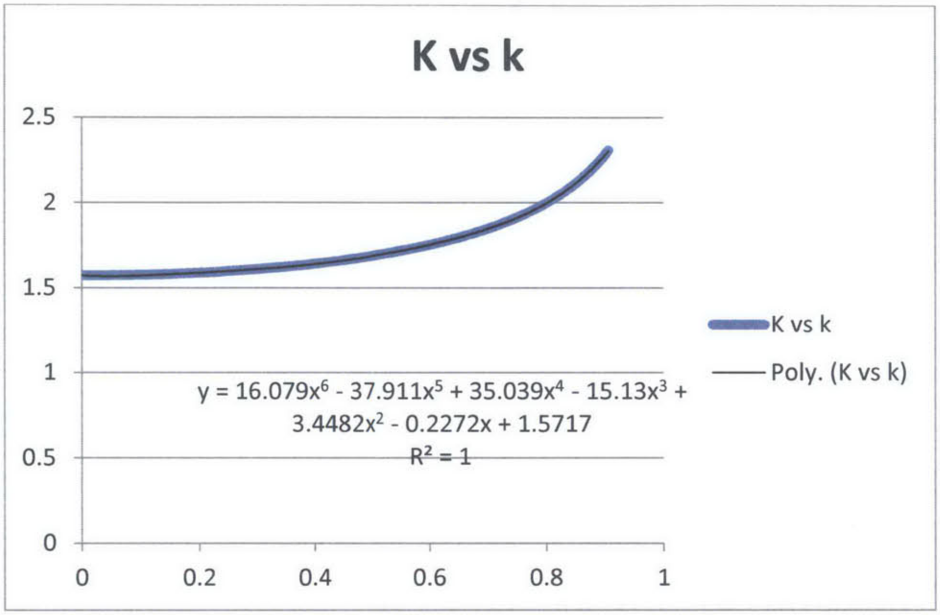

Figure 2.4. Plot illustrating the polynomial fit between K and k

Moreover y_0 (the maximum depth) was estimated as:

$$\frac{y0}{l} = -10.928\left(\frac{2x0}{l}\right)^6 + 29.298\left(\frac{2x0}{l}\right)^5 - 29.953\left(\frac{2x0}{l}\right)^4 + 14.345\left(\frac{2x0}{l}\right)^3 - 3.5258\left(\frac{2x0}{l}\right)^2 + 0.4109\left(\frac{2x0}{l}\right) + 0.3849 \qquad \text{(eq.9)}$$

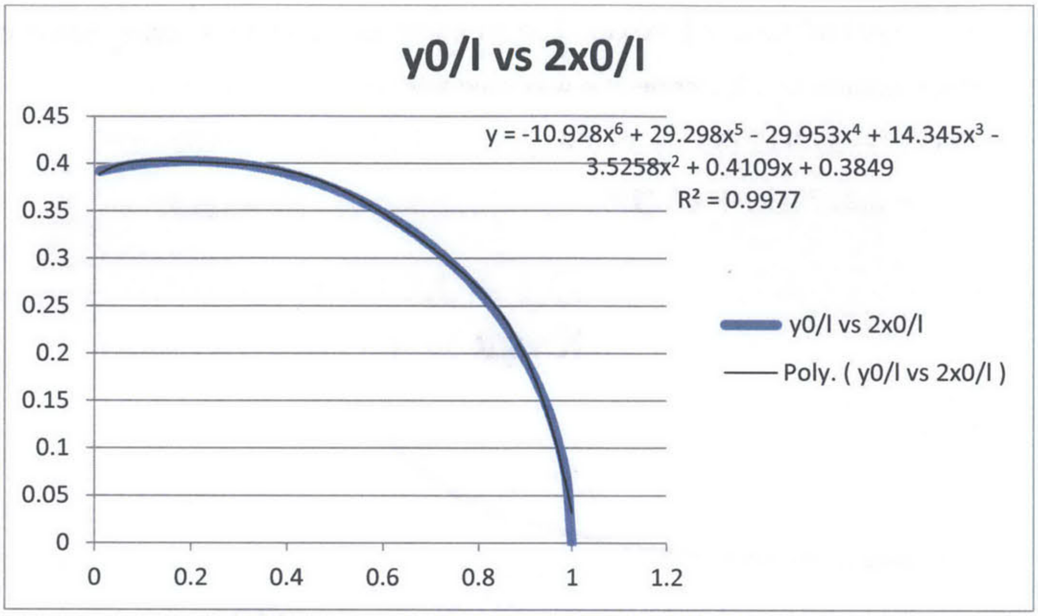

Figure 2.5 Plot showing polynomial fit between yo/l and 2x0/l

The optimization problem therefore is reduced to the constrained minimization of a sixth order polynomial, which simplifies the problem in terms of computational complexity. Having established relations for the form-finding process, an approximation of the beam's strength-based design is explained. The strength-based design will look at the behavior of the cross-section after the concrete has dried and has cracked under ultimate conditions.

2.2 Strength Based Design

The main assumptions introduced in strength-based design are that plane sections remain plane after bending, and the strain distribution could be approximated by a linear strain distribution. The member's moment capacity in this case is provided by the moment couple caused by the moment arm between the compression and tensile force. The centroid of the neutral axis could be calculated using factors recommended by the ACI (2005) code which also illustrated more clearly by (BS EN 1992-1-1: 2004 a) as shown in Figure 2.6. The stress distribution is approximated using an equivalent rectangular stress:

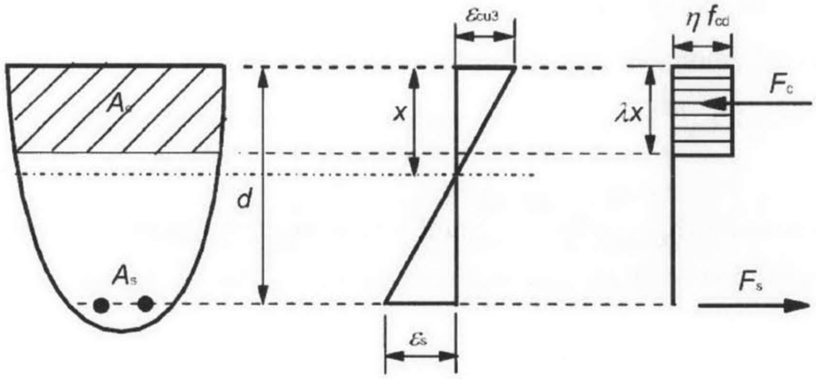

Figure 2.6 Picture Adopted from (BS EN 1992-1-1: 2004 a). Image shows the equivalent rectangular stress block based on a linear strain distribution

The depth of the member's neutral axis can be approximated by assuming that the member will be tension controlled i.e. that the steel would yield before the concrete to avoid brittle failure. The neutral axis depth is then taken to be less than the neutral axis depth at balanced strain conditions and is given by the following equation:

$$c = 0.75 * c_b \ where \ c_b = \frac{87000}{87000 + fy} d \quad \text{(eq.10)}$$

Having found the depth (c) of the neutral axis. The depth of the equivalent rectangular stress block is given by

$$a = B_1 c \ where \ B_1 = 0.85 - \left(\frac{f_c' - 4000}{1000}\right)(0.05) \geq 0.65 \quad \text{(eq.11)}$$

The moment capacity is then given by A.C.I (2005):

$$Moment \ Capacity = \ A_{compression} * f_c'\left(d - \frac{a}{2}\right) = A_{steel} * f_y'(d - \frac{a}{2}) \quad \text{(eq.12)}$$

The minimum reinforcement ratio is given by A.C.I (2005):

$$\rho_{\{min\}} = \ 3 * \frac{\sqrt{f_c'}}{f_y} \quad \text{(eq.13)}$$

The maximum reinforcement ratio is then given by A.C.I (2005):

$$\rho_{\{max\}} = 0.75 * \rho_b \quad \text{(eq.14)}$$

The shear capacity in the model assumed below simplifies the problem by assuming that only 50% of the shear is carried on average by the concrete cross-section:

$$g_5(x) = 50\% * 2 * \sqrt{f'c} * A_{conc} - V_u \text{ (maximum shear constraint)} \quad \text{(eq.15)}$$

This simplification is somewhat conservative and a more accurate measure is given by Garbett,Darby, & Ibell (2010) which however will not be used in the optimization method.

The strength based-design along with the form-finding method are adequate enough to design a beam for a certain moment and shear capacity. However in order to carry out a minimization of the cost of the cross-section, a cost model is looked at more closely.

2.3 Cost Structure

Several authors have dealt with the topic of optimizing the cost of a cross-section and therefore the cost model can be adopted from one of the papers listed in Table 2.1. A more realistic cost model could easily be derived from the local market material costs that do vary according to location and time. For the purpose of the optimization of reinforced concrete beams a cost ratio of 9:1 was taken for the ratio of cost of steel to the cost of concrete. A cost ratio is used instead of the actual cost of the cross-section since the ratio tends to be more constant as the parameters are varied. In addition to that, the designer using the model can input a cost ration that is more representative of the local market in which they are working.

Table 2.1 A list showing some of the cost parameters taken by several authors that tackled the problem of optimization of concrete beams

Cost Parameter	Value and Units	Author(s)
$\frac{Cost\ Post\ Tensioning}{Cost\ Concrete}$	45	(Colin & MacRae, 1984)
$\frac{Cost\ Pre\ Tensioning}{Cost\ Concrete}$	9	
$Cost\ of\ Concrete\ per\ unit\ volume$	$75\ \$/m^3$	(Marano , Fiore, & Quaranta, 2014)
$Cost\ of\ prestressing\ steel\ per\ volume$	$1000 \frac{\$}{ton} * 7.85 \frac{ton}{m^3} = 7850\ \$/m^3$	
$Cost\ of\ concrete\ per\ yard^3$	$80\ \$/yd^3$	(Al-Gahtani, Al-Saadoun , &
$Cost\ of\ prestressing\ steel\ per\ pound$	$1.35\ \$/lb$	

Cost of mild steel per pound	0.385 $\$/lb$	Abul-Feilat, 1995)

The last main material component other than the reinforcement and the concrete to be included in the cost is the cost of the fabric. The common fabric textiles used in the University of Manitoba in CAST under the supervision of Professor Mark West are listed in Table 2.2 according to Orr (2012).

Table 2.2 The industrial name of different geotextiles used as fabric formwork

Fabric Type	Material
LOTRAK 300GT	Polypropelene
Propex 2006	Polypropelene
NOVA Shield	High density Polypropelene

According to (West & Araya, 2009) these fabrics cost less than $1 (U.S.) per meter, and therefore for simplification reasons the cost of the formwork was not taken into account in the objective function of the optimization model. Moreover the fact that the same fabric formwork could be re-used multiple times (Orr, 2012), makes it even less significant to take the cost of fabric into account.

2.4 Summary

This chapter introduced the main methodology for finding the form of a fabric subjected to hydrostatic pressure from a fluid. An original approach is proposed to use polynomial approximations of the dimensionless relationships for the optimization approach. Lastly a cost model was considered, and a cost ratio for the cost of mild reinforcement to the cost of concrete was approximated to be 9:1. The next chapter will introduce the optimization methods for the design of reinforced fabric formed concrete.

3. Methodology

This chapter will explain the two methods developed in this thesis for optimizing a reinforced fabric formed beam. Initially the analytical optimization method is introduced which relies on solving a constrained non-linear optimization problem using Matlab. Then the feasible region method is introduced which is a more direct and approximate method that could be more convenient for designers to use.

3.1 The Analytical Optimization Model

The analytical optimization model uses a non-linear constrained optimization algorithm called "sequential quadratic programming" as a move maker to search for a feasible solution. A MATLAB gradient-based optimization toolbox has been used to solve the optimization problem. This makes use of the 'fmincon' routine (Mathworks, 2014), which is given in more detail in the Appendix. The model aims at minimizing the cost of an area of a cross-section taking into account a fixed cost ratio of 9:1 for mild steel versus concrete. The model formulation as discussed before aims at minimizing the cost of a cross-section. Therefore the objective function is the following:

$$Minimze\ f(x) = A_{conc} + C * A_{steel} \qquad \text{(eq.16)}$$

Subject to the following constraints:

$$g_1(x) = y_{max} - depth_{max}\ (\ a\ maximum\ depth\ constraint) \qquad \text{(eq.17)}$$

$$g_2(x) = M_u - F_{comp} * couple_{arm}(moment\ capacity\ constraint \qquad \text{(eq.18)}$$

$$g_3(x) = \frac{A_{steel}}{A} - \rho_{max}\ (maximum\ reinforcement\ ratio) \qquad \text{(eq.19)}$$

$$g_4(x) = \rho_{min} - \frac{A_{steel}}{A}\ (minimum\ reinforcement\ ratio) \qquad \text{(eq.20)}$$

$$g_5(x) = 50\% * 2 * \sqrt{f'c} * A_{conc} - V_u \ (maximum\ shear\ constraint) \quad \text{(eq.21)}$$

$$g_6(x) = (A_{steel}f_y - f'c * A_{comp}) - (Tolerance)(equilibrium\ constraint) \quad \text{(eq.22)}$$

Where:

C = The cost ratio of steel to concrete related to the steel area.

y_{max} = The maximum depth of the fabric formed beam. (in)

F_{comp} =Resultant Compression Force of an equivalent rectangular stress distribution (lb)

Returning to the optimization method, the decision variables have been chosen to be x_0 (half the top width of the beam) and l (the length of the fabric used) in addition to the reinforcement area A_s. The model outputs a value for x_0 and l, which the designer can then use along with the form-finding methods explained above to get a profile of the fabric formed cross-section. Once multiple cross sections are obtained for different points along the beam's length, the designer can interpolate a three-dimensional shape for the beam and calculate the cost based on the volume.

3.2 Model Limitations

The above model has certain limitations, as it is only applicable to simply-supported fabric formed beams that are under a uniformly distributed loading. In addition to that, the dead load of the beam has not been calculated as a multiple of the self-weight of the beam however has been over-estimated using an approximate equivalent prismatic rectangular beam. Also since the shear behavior of fabric formed beams is complex as illustrated by (Orr, 2012), an estimated portion of 50% of the shear capacity has been considered to be resisted by the concrete while the rest is assumed to be resisted by the shear reinforcement which was also chosen to be neglected in the overall material calculations.

For strength-based design purposes the moment capacity is approximated as follows:
The area of concrete in compression is estimated using the area of the trapezoid defined by the four points shown in Figure 3.1 below:

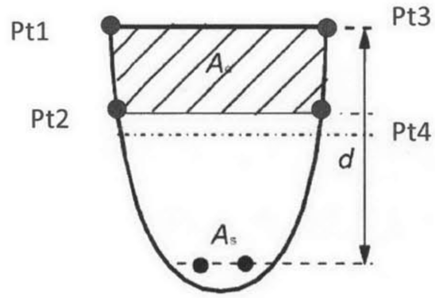

Figure 3.1 Plot shows the four points, Pt1, Pt2, Pt3, and Pt4 which define the four vertices of the trapezoid

3.3 Feasible Solution Method

The problem in tackling the optimization using the above mathematical formulation is that the area of steel could vary between cross-sections, which means that in reality the reinforcement is assumed to be spliced between the cross-sections, which might not be an attractive solution from a fabrication point of view. In addition to that, the optimization only runs for specific upper and lower bounds on the width and depth of the beam that might differ from on case to another based on certain spatial considerations. Therefore a more general approach should also be considered which is why the feasible region methodology is referred to in the following chapter.

Examining the relation between the $\frac{A}{l^2}$ and $\frac{2x_0}{l}$, it could be approximated by the following polynomial which is given by equation 10 and also demonstrated by Figure 3.2 on the following page:

Equation 1. Polynomial relating the dimensionless area parameter to the dimensionless width parameter

$$\frac{A}{l^2} = -7.1921 * \left(\frac{2x0}{l}\right)^6 + 19.221\left(\frac{2x0}{l}\right)^5 - 19.595\left(\frac{2x0}{l}\right)^4 + 9.2215 - 2.0749\left(\frac{2x0}{l}\right)^2 + 0.3744\left(\frac{2x0}{l}\right) + 0.066 \qquad \text{(eq.23)}$$

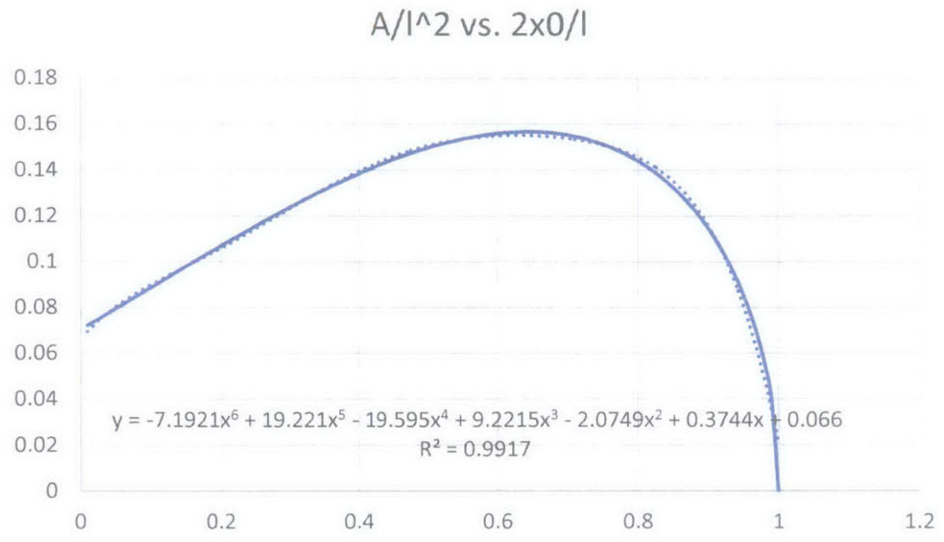

*Figure 3.2 Plot showing the polynomial fit between A/l^2 and 2*x0/l*

It is also known that:

$$Asteel = \rho * A \; => \frac{Asteel}{l^2} = \frac{\rho A}{l^2} \qquad \text{(eq.24)}$$

Where ρ is the reinforcement ratio.

The objective function is given by

$$f = C_1 A + C_2 A_{\{steel\}} = C_1 A + C_2 \frac{pA}{l^2} = A(C_1 + C_2\,\rho\,) \qquad \text{(eq.25)}$$

The dimensionless objective function is given by:

$$\frac{f}{l^2} = \frac{A}{l^2}(C_1 + C_2\,\rho) \qquad \text{(eq.26)}$$

where:

$$f = \left(-7.1921 * \left(\frac{2\text{x}0}{\text{l}}\right)^6 + 19.221\left(\frac{2\text{x}0}{\text{l}}\right)^5 - 19.595\left(\frac{2\text{x}0}{\text{l}}\right)^4 + 9.2215\left(\frac{2\text{x}0}{\text{l}}\right)^3 - 2.0749\left(\frac{2\text{x}0}{\text{l}}\right)^2 + 0.3744\left(\frac{2\text{x}0}{\text{l}}\right) + 0.066\right) * (\text{C}_1 + \text{C}_2\,\rho) \qquad \text{(eq.27)}$$

The moment constraint is then expressed as:

$$M_u \leq A_s f_y (y_0 - \frac{a}{2}) \qquad \text{(eq.28)}$$

$$M_u \leq \phi\rho A\, f_y(y_0 - \frac{a}{2}) \qquad \text{(eq.29)}$$

where:

$$a = 0.85 * 0.75 * \frac{87000}{87000 + fy}(y_0 - 2) \qquad \text{(eq.30)}$$

Therefore dividing both sides by l^3 the equation simplifies to:

$$\frac{M_u}{l^3} \leq \frac{\phi\rho A}{l^2} f_y(\frac{y_0}{l} - 0.31875 * \frac{87000}{87000 + fy} * \frac{(y_0 - 2)}{l}) \qquad \text{(eq.31)}$$

Where y0 -2 could initially be estimated to be 0.9*y0

$$\frac{M_u}{l^3} \leq \frac{\phi\rho A}{l^2} f_y(\frac{y_0}{l} - 0.31875 * \frac{87000}{87000 + fy} * \frac{(0.9 * y_0)}{l}) \qquad \text{(eq.32)}$$

The expressions for $\frac{A}{l^2}$ and $\frac{y_0}{l}$ could then be substituted in the above expression to express the moment capacity problem in terms of two variables only ($x_0$and l). The final expression can therefore be easily computationally processed for plotting using the following polynomial inequality (eq.33):

$$\frac{M_u}{l^3} \leq \phi\rho * \underbrace{\begin{pmatrix} 7.1921 * \left(\frac{2x0}{l}\right)^6 + 19.221\left(\frac{2x0}{l}\right)^5 - 19.595\left(\frac{2x0}{l}\right)^4 + 9.2215\left(\frac{2x0}{l}\right)^3 - 2.0749\left(\frac{2x0}{l}\right)^2 \\ +0.3744\left(\frac{2x0}{l}\right) + 0.066 \end{pmatrix}} * fy * \underbrace{\begin{pmatrix} -10.928\left(\frac{2x0}{l}\right)^6 + 29.298\left(\frac{2x0}{l}\right)^5 - 29.953\left(\frac{2x0}{l}\right)^4 + 14.345\left(\frac{2x0}{l}\right)^3 - 3.5258\left(\frac{2x0}{l}\right)^2 \\ +0.4109\left(\frac{2x0}{l}\right) + 0.3849 \end{pmatrix}} * (1 - 0.31875 * \frac{87000}{87000 + fy})$$

Similarly the shear constraint can be simplified into:

$$0.5 * V_u \leq phi * 2\sqrt{f'cA} \qquad \text{(eq.34)}$$

Dividing both sides with l^2 the expression then can be transformed into an inequality involving only x_0 and l:

$$\frac{0.5V_u}{l^2} \leq 1.7 * \sqrt{f_c'} * \underbrace{\left(\begin{array}{c} -7.1921 * \left(\frac{2x0}{l}\right)^6 + 19.221 \left(\frac{2x0}{l}\right)^5 - 19.595 \left(\frac{2x0}{l}\right)^4 + 9.2215 \left(\frac{2x0}{l}\right)^3 - 2.0749 \left(\frac{2x0}{l}\right)^2 \\ +0.3744 \left(\frac{2x0}{l}\right) + 0.066 \end{array} \right)} \quad \text{(eq.35)}$$

Plotting the constraints at midspan in a set of subplots that share the same x-axis for the length of fabric used, a feasible region could be established. Figure 3.3 shows a sample plot of the constraints polynomials at midspan of a cross-section with a top-breadth of 10 inches. The uppermost plot shows the moment capacity constraint, the one underneath shows the maximum depth constraint and finally the objective function is shown.

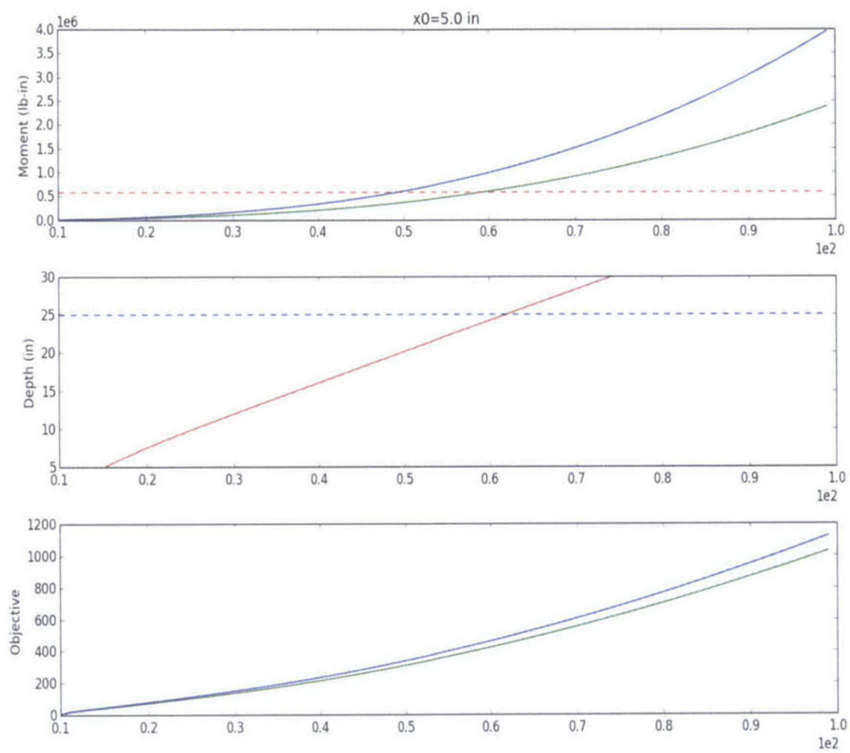

Figure 3.3 A specific case of x0 =5.0, the feasible region is highlighted in blue which satisfies both the maximum depth constraint and the moment capacity

The plot here includes two extremes of reinforcement ratios with the blue curve resembling the moment capacity of a section with maximum reinforcement and the green represents a section with minimum reinforcement. The feasible range for a section with maximum reinforcement is highlighted in the blue zone, whereas the feasible zone for a cross-section with minimum reinforcement is highlighted in green. The feasible range is found by looking first at where the moment capacity is greater than the limit moment, and then second by looking at the graph below in order to make sure that the depth of the cross-section is not greater than the maximum depth specified. The optimal solution can then be found by comparing the value of the objective function for various fabric lengths. It is evident from the above graph that the feasible range for a section with minimal reinforcement is thinner than that of the maximum reinforcement and even though the cost ratio of steel to concrete is 9:1, in the above case having a maximum reinforcement will produce a cheaper cross-section.

After having found the feasible region at midspan, other cross-sections are considered. Figure 3.4 below shows a cut for the same beam at quarter span. An additional constraint is included here which is the shear constraint and that is represented by the third plot. The feasible regions are then found and are highlighted in the curve below.

The following method adds flexibility to the designer as the plots could easily be reproduced by plotting the derived polynomial inequalities. The designer also has the flexibility to keep the same reinforcement ratio or could choose to splice different reinforcements across the span of the beam. Moreover a different of curves could be produced for other choices of top breadths. These decisions are affected by spatial considerations such as span, maximum allowable depth, and width.

In deriving the bending moment constraint, an approximation of 0.9y0 was chosen as the depth of reinforcement. This approximation can then be reassessed after finding the depth of the fabric and the required layers of reinforcement. Therefore this approximation could be made more exact using an iterative process.

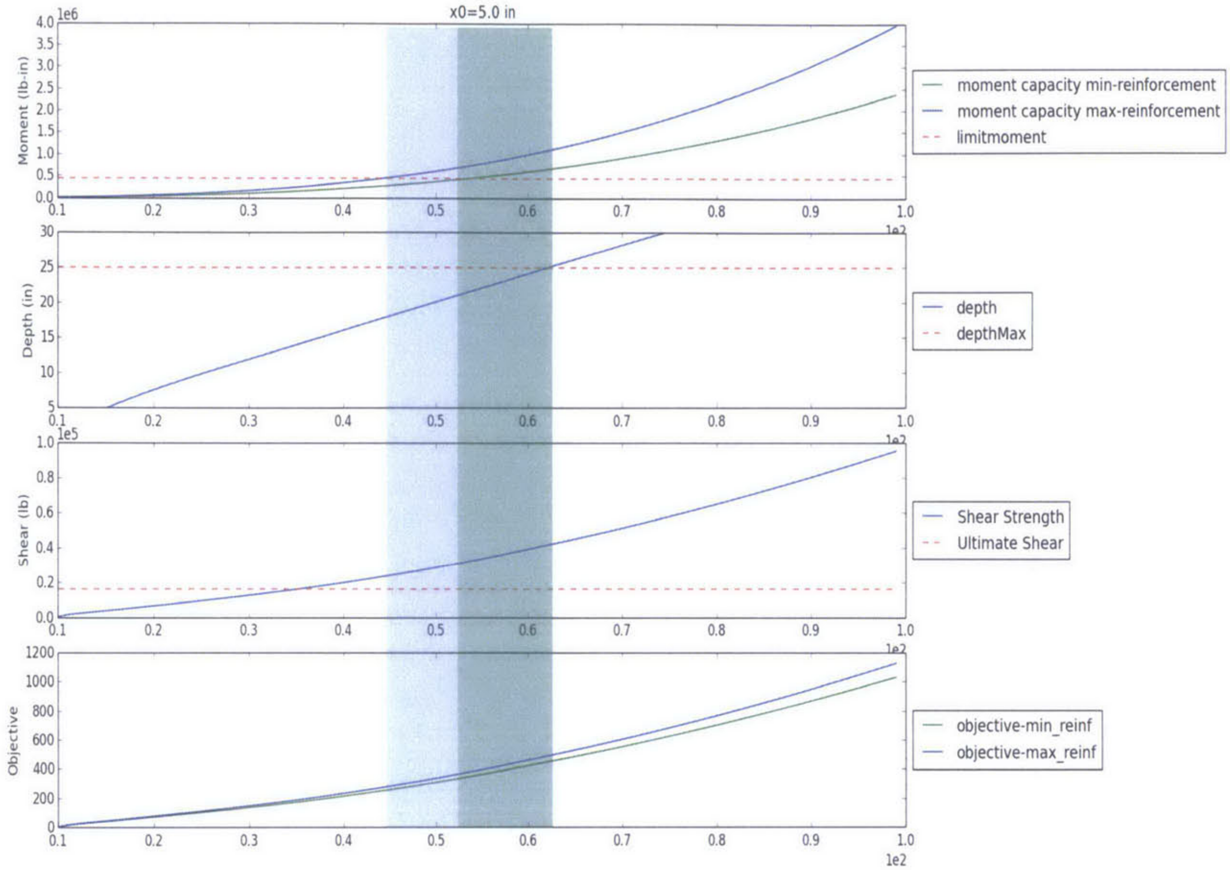

Figure 3.4. A specific case of x0 =5.0, the feasible region is highlighted in blue which satisfies both the maximum depth constraint and the moment capacity

3.4 Summary of Contributions

This chapter has introduced two main methods for the optimization of the cross-section of a fabric formed reinforced concrete beam. The first method is the analytical optimization method that is an original way to deal with the problem from a constrained optimization point of view. The second method is the feasible region method, which might prove to be a more user-friendly method that the designer could conveniently use without having to undergo the same level of computation required by the analytical optimization method. The next chapter will look at a sample result of both of the methods just introduced.

4. Results

This chapter will consider a specific sample problem which is then solved using both the analytical optimization model and the feasible region model. The results are also benchmarked against the output from the model introduced by Garbett, Darby, & Ibell (2010).

4.1 Analytical Optimization Model Results

Following the mathematical formulation introduced in the previous chapter a sample run is carried out on MATLAB and the output cross section is visualized. A sample problem is chosen that resembles a conventional educational problem posed by "Design of reinforced concrete" (McCormac, 1998). The problem is best described by the following input parameters:

$W_{DL} = 100\ psf$ (Dead Load)

$W_{LL} = 50\ psf$ (Live Load)

Tributary Width = 12 ft

Span = 25 ft

$f_c' = 4000.0\ psi$ (Compressive strength of concrete)

$f_y = 60000\ psi$ (Yield strength of steel)

The ultimate load can then be calculated as:

$$W_u = 1.2W_{DL} * TributartyWidth + 1.6 * W_{LL} * TributaryWidth$$
$$= 2400\ lb/ft \qquad \text{(eq.36)}$$

The ultimate moment at mid-span is then:

$$M_u = W_u * \frac{span^2}{8} = 187.5\ kip - ft \qquad \text{(eq.37)}$$

The problem solution is found using the '*fmincon*' routine in MATLAB (Mathworks, 2014), which follows a sequential quadratic programming algorithm to find the constrained minimum. The output at mid-span is the following:

$x_0 = 5.0\ in$ (a top breadth of 10 in)

$l = 41.65\ in$

$A_s = 3.0\ in^2$

depth = 16.6 in

The output of the program is visualized in Figure 4.1. The visualization shows the profile of the fabric formwork along with an equivalent rectangular beam of the same moment capacity which is shown by the dashed lines.

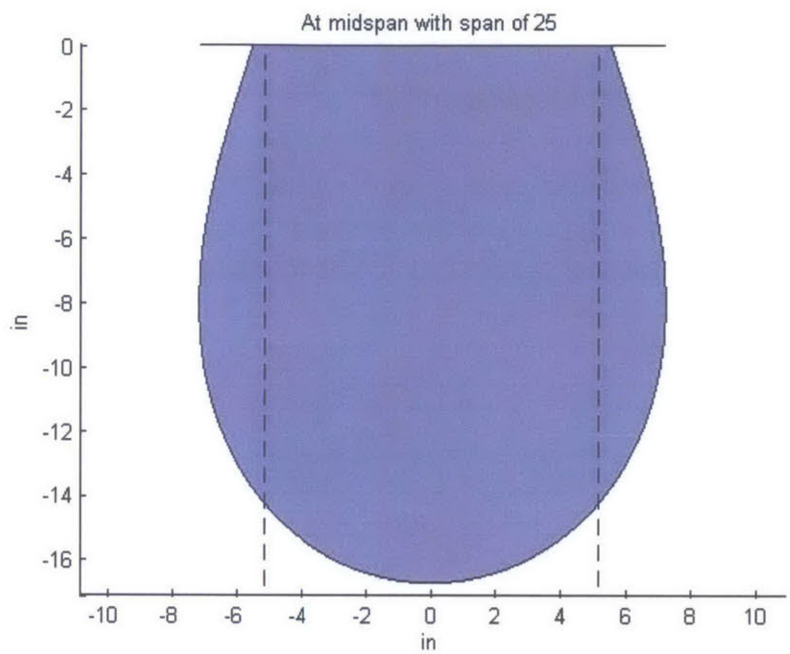

Figure 4.1. A sample output of the optimization software, which shows the profile of the fabric in blue and the equivalent rectangular beam in the dashed line rectangle

The optimum design proposed by (Garbett, Darby, & Ibell , 2010) could be found using the following equation:

$$d = \left(\frac{0.45F_c}{0.67f_{cu} * 0.9 * b}\right) + \left(\frac{M}{F_s}\right) + \left(\frac{\phi}{2}\right) + c \qquad \text{(eq.38)}$$

(Equation adapted from (Garbett, Darby, & Ibell , 2010))

Taking $F_c = A_s f_y$, an area of steel reinforcement could be chosen to be $A_s = 5.44\ in^2$. This specific input will yield the following solution according to (Garbett, Darby, & Ibell , 2010):

$x_0 = 4.0\ in$

$l = 41.6\ in$

$y = 15.2\ in$

However when comparing the Garbett, Darby & Ibell solution to the analytical optimization output it is noticed that the optimum result attempts to find a larger concrete area with a smaller reinforcement ratio in order to decrease the cost of the cross-section, especially since the cost ratio of steel to concrete is taken to be 9:1.

The optimization was then carried out at several cross-sections and at the supports. A workflow shown in Figure 4.2 was established in order to extrapolate the results from a 2D cross-section to a 3D visualization in Rhinoceros 3D©. Profile coordinates from the MATLAB were streamed to an excel file which was then read using Grasshopper® that plotted the coordinates and then interpolated between the cross-sections using the built-in "Loft" command. Finally the difference in volume between the fabric formed beam and the prismatic rectangular beam could be easily calculated using built-in Grasshopper® commands

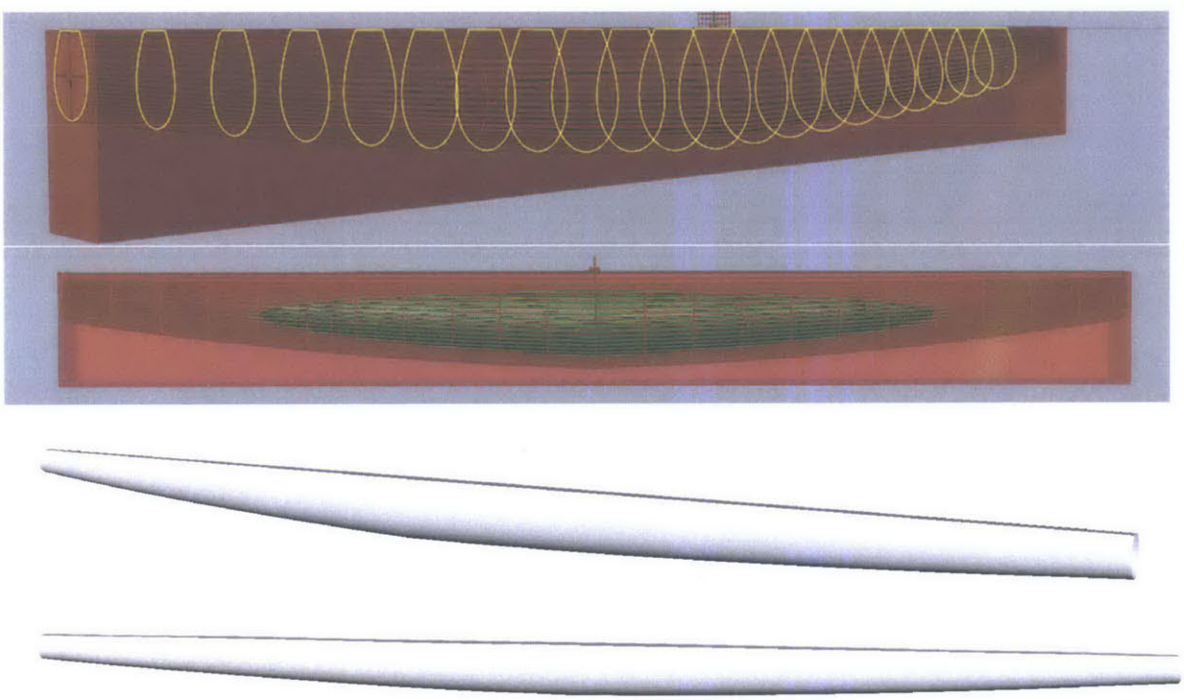

Figure 4.2. Output of the loft of the multiple cross-sections shown above into the final green fabric formed profile shown above. Furthermore an attempt at estimating deflections could be done as a final design check.

4.2 Feasible Region Method Results

Approaching the same problem introduced above using the feasible region method yields the following sample output at mid-span shown in Figure 3.3 below.

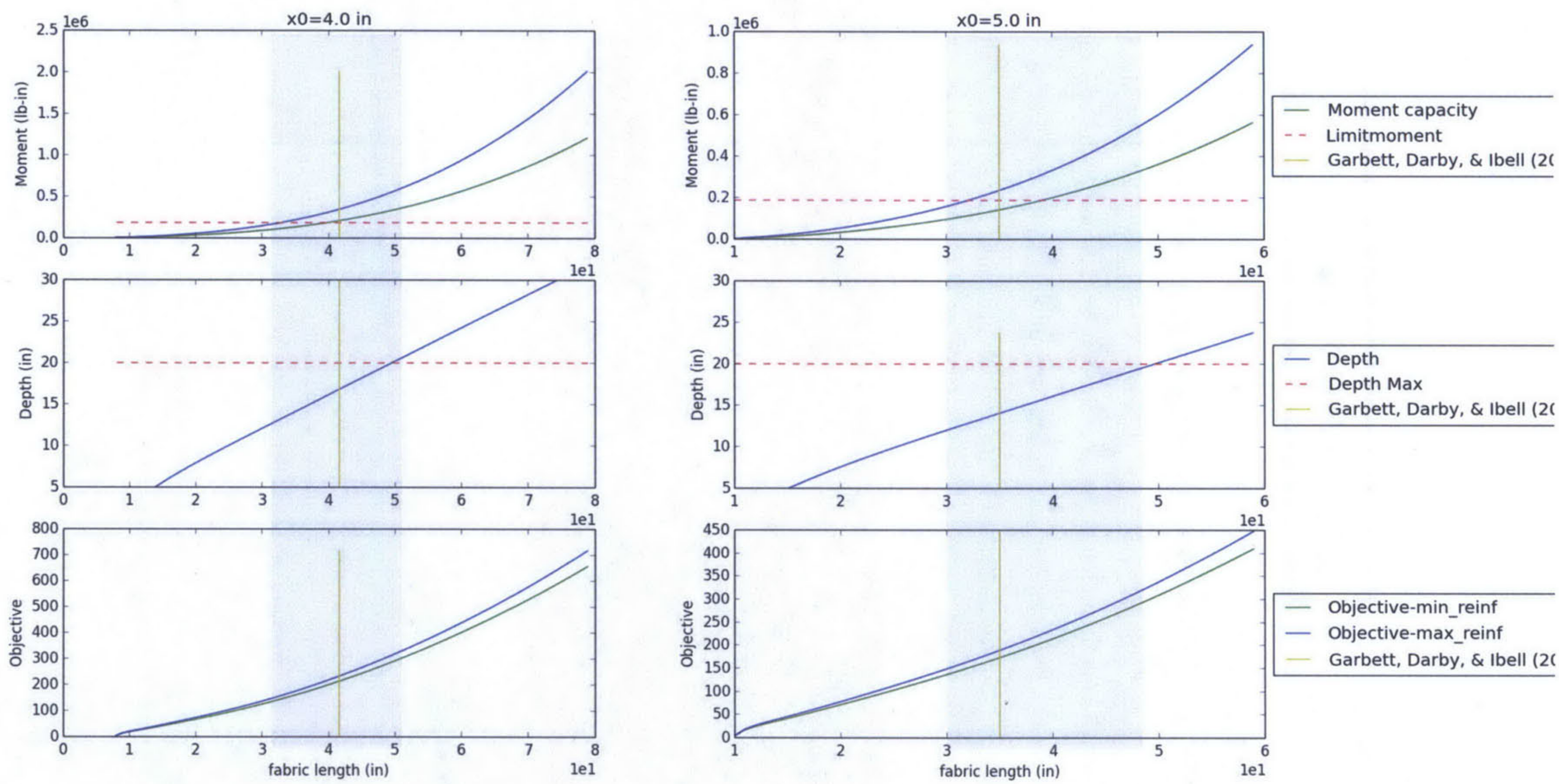

Figure 1.3 Two specific cases of x0 =5.0, and x0=10.0, the feasible region is highlighted in blue which satisfies both the maximum depth constraint and the moment capacity

The above graphs show two shaded feasible regions for a top breadth of 10 in and 20 in. The solution proposed by Garbett, Darby, & Ibell (2010) which is computed above lies within the feasible range. The optimal solution can be found by comparing the value of the objective function for various top breadths by plotting out the polynomial inequalities in the same order proposed above.

Table 4.1 below shows the various solutions given by the different models for the fabric formed beam.

Table 4.1. A summary of the results given by the two methods benchmarked against a solution given by Garbett, Darby & Ibell

Model	Top Breadth (in)	Length of fabric (in)	Maximum depth (in)	Steel Reinforcement Area (in^2)	Relative Cost in terms of the unit cost of Concrete
Analytical Optimization Method	10 in	41.65 in	16.6 in	3.00 in^2	285*C
Garbett, Darby, & Ibell Method	8 in	41.6 in	15.2 in	5.44 in^2	287*C
Feasible Region Method	8 in	41 in	15 in	5.40 in^2	281*C

Table 4.1 shows that the analytical optimization method is slightly better than the solution chosen using the method of Garbett, Darby & Ibell (2010). It is also noted from Table 4.1 above that the feasible region method is slightly less conservative and this might be due to the reason that in approximating the bending moment constraint, an estimate of 0.9 y_0 was chosen as the depth of reinforcement, which only provides a cover of $0.9 * 16\ in = 1.6\ in$ that is slightly smaller than the recommended cover of about 2 in.

A rough estimate of material savings between a fabric formed beam and an equivalent rectangular beam is done using the feasible region method at three cross-section locations and by fixing the width at 10 in. The volume of the fabric formed beam is compared to an equivalent beam of the same moment capacity at mid-span of width 10 in and depth 13.75. The plots used to estimate the length of the fabric required are shown in Figures A.3, A.4, and A.5 in the Appendix. Lofting between the three cross-sections and making use of symmetry the total 3D profile is found and is shown in Figure A.6 in the Appendix.

Location	Top Breadth(in)	Length of fabric (in)	Maximum depth (in)
Mid-span	10.0	32	12.75
Quarter span	10.0	27	10.6
Supports	10.0	25	9.75

Where the Total Volume of Fabric Formed Beam = 18697 in^3

Total Volume of Rectangular Beam = 40278 in^3

The relative volume of the two beams is $\frac{18697}{40278}$ =0.46 which indicates a saving of up to 54 % in volume.

The relative cost of the fabric formed to rectangular beam=$\frac{186967in^3*1+3.00\ in^3*288in*9}{40278in^3+3.00\ in^3*288in*9} = 0.41$ which is equivalent to a savings of about 59% in terms of cost. The estimate is probably a slightly optimistic result since the length of rebar is slightly larger because of the curved distance additionally as stated before the approximation of $0.9y_0$ needs to be reassessed. Moreover additional costs such as bending the rebar are neglected in this study. However on average a savings of about 55% could be expected. This estimate is becoming more realistic with the advancements in the research of pre-fabrication of the pre-cast beams achieved by C.A.S.T in the University of Manitoba under the supervision of Professor Mark West.

4.3 Summary

The results chapter has benchmarked the two methods proposed for the optimum design of fabric formed beams with the method of Garbett, Darby, & Ibell (2010). The chapter also proposes a rough estimation of about 55 % in cost savings. This approximation is not exact since only a total of three cross-sections were chosen and due to other assumptions which have been previously explained. The next chapter will consider the topic of pre-stress design from a similar perspective to which the problem of reinforced concrete design was approached.

5. Pre-stressed Design

5.1 Introduction

The design of pre-stressed fabric formed beams has still not been detailed and therefore a basic study that deals with a basic cross-section of a fabric draped between a void in a forming table is discussed. At first the optimization problem is formulated and several additional complexities are discussed. The computational complexities make the analytical optimization method unattractive and therefore it will not be used. Then a method similar to the feasible region method discussed above is explained and will be relied on to produce the results.

5.2 Analytical Optimization method

The optimization problem had been set up in the following manner:

Minimize

$$f(x) = C_1 A + C_2 * A_p \qquad \text{(eq.39)}$$

Subject to:

$$g_1(x) = yo - depthMax \;\; (maximum\ depth\ constraint) \qquad \text{(eq.40)}$$

$$g_2(x) = -\frac{P_i}{A} - \frac{M_{DL} * 12}{Z_{top}} + \frac{P_i * e}{Z_{top}} - f_r \;\; (initial\ conditions\ tension\ check\ at\ top\ of\ beam) \qquad \text{(eq.41)}$$

$$g_3(x) = \frac{P_i}{A} + \frac{M_{DL} * 12}{Z_{bottom}} - \frac{P_i * e}{Z_{bottom}} - f_c \;\; (initial\ conditions\ compression\ check\ at\ bottom\ of\ beam) \qquad \text{(eq.42)}$$

$$g_4(x) = -\frac{P_i * \mu}{A} - \frac{M_{Total} * 12}{Z_{bottom}} + \frac{P_i * e * \mu}{Z_{bottom}} - f_r \;\; (final\ conditions\ tension\ check\ at\ bottom\ of\ beam) \qquad \text{(eq.43)}$$

$$g_5(x) = -\frac{P_i * \mu}{A} - \frac{M_{Total} * 12}{Z_{top}} + \frac{P_i * e * \mu}{Z_{top}} - f_c \; (final\ conditions\ compression\ check\ at\ top\ of\ beam) \quad \text{(eq.44)}$$

$$g_6(x) = M_u - 0.9 * P_i * \mu * \frac{d}{12} (moment\ capacity\ under\ ultimate\ stress\ conditions) \quad \text{(eq.45)}$$

$$g_7(x) = f_c' * CompressionArea - P_i * \mu \; (equilibrium\ costraint) \quad \text{(eq.46)}$$

Where:

$P_i = initial\ pre - stressing\ force$

$Z_{bottom} = Section\ modulus\ with\ respect\ to\ the\ bottom\ most\ fiber$

$Z_{top} = Section\ modulus\ with\ respect\ to\ the\ top\ fiber$

$\mu = pre - stress\ loss\ ratio$

The decision variables in the above model are the following:

$x_0: half\ the\ top\ breadth\ of\ the\ fabric$

$l : the\ length\ of\ the\ fabric\ used$

$A_p: the\ area\ of\ pre - stressing\ force$

$P_i: the\ prestressing\ force$

The main complexity in the above optimization lies in the fact that the problem is computationally tedious as an update of the transformed section properties of the fabric is required for each specific area of reinforcement used. A sample run of the optimization program does hundreds of function evaluations, which takes a considerable time to compute around 15 minutes on a commercial laptop. Moreover that fact that there exists five decision variables makes the solution subjective since for every solution a lower and upper bound on the variables such as the maximum and minimum depth, width, pre-stressing area, and pre-stressing force magnitude needs to be specified which will differ from one case to another. In order to reach a more holistic approach solution a method is developed that follows a similar logic as the feasible region method introduced above.

5.3 Feasible Region Method

In order to reduce the complexity of the problem at hand, a relationship is sought that combines the various parameters and allows the designer to visualize a feasible region that is defined using two variables only x_0 (half the top breadth) and l (length of the fabric).

In the case of reinforced concrete design, the strength-based approach assumed a cracked section analysis. Whereas in the case of pre-stressed concrete, the section is analyzed prior to cracking and therefore a transformed section analysis is carried out. The section modulus (Z) could be thought of as being related to the area of the cross-section, the area of pre-stress reinforcement used, as well as the location of the pre-stress reinforcement.

A simple program that estimates the moment of inertia, the area of the concrete section and the transformed concrete section has been established with the code attached in the Appendix. Basically the code divides the cross-section into small trapezoids and finds the section properties by aggregating the properties of the small trapezoids.

The transformed section properties are found by widening the cross-section at the location where the pre-stress reinforcement is added by a value of (n-1)*Ap/2 on each side, where n= Ep/Ec. A sample transformed cross-section is shown in Figure 5.1 below.

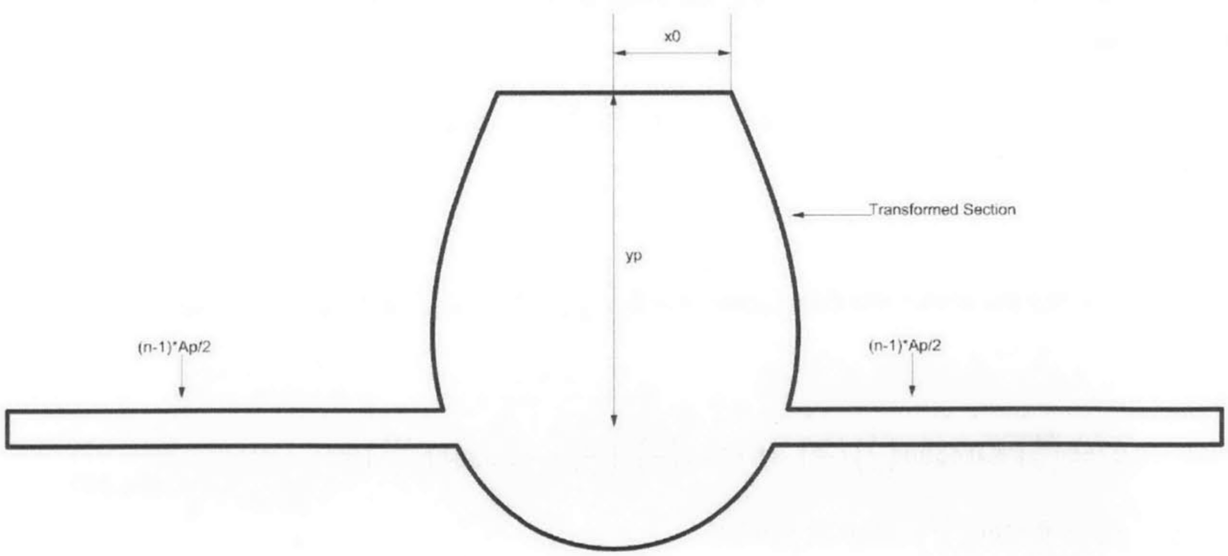

Figure 5.1 A Transformed cross-section of a pre-stress fabric formed beam with the location of reinforcement at a depth of yp

The code was then validated by comparing the results to those produced by Grasshopper®, and an average error of 0.02% was achieved in terms of the accuracy of the moment of inertia evaluation. The small error therefore indicates that the step size chosen is reasonable and that the method is accurate.

In order to deduce a dimensionless relationship that links between section modulus to the area of fabric and the depth of pre-stressed reinforcement, a range of possible ratios of 2x0/l were given to the program along with varying the position of reinforcement. The relationships shown below are based on fixing the area of pre-stressed reinforcement at a ratio of 0.5% and a ratio of 0.15%.

Another dimensionless relationship was also established using the same method of polynomial extrapolation to link the eccentricity to the area of fabric and ratio of depth of reinforcement to the maximum depth of the cross-section. The output of the code was filtered and all the cases containing negative eccentricity were removed as it would not be a desired case in pre-stressed design.

The graphs shown below reveal that the point cloud of results is carefully described by a proper pattern with minimal noise. For simplification reasons a high order polynomial approximation was taken in order to get reasonable results with minimal complexity.

As shown in Figure 5.2 a relation linking the dimensionless parameterZ_{top}/l^3 to $A/l^2$and y_p/l is the following:

$$\frac{Z_{top}}{l^3} = -0.0004 + 0.1882\left(\frac{A}{l^2}\right)\left(\frac{y_p}{l}\right) + 0.1810\left(\frac{A}{l^2}\right)^2 - 0.3212\left(\frac{y_p}{l}\right)^4 + 0.6212\left(\frac{y_p}{l}\right)^5 - 5.4088\left(\frac{A}{l^2}\right)^3\left(\frac{y_p}{l}\right)^2 \text{ with } R^2 = 0.945 \qquad \text{(eq.47)}$$

A relation linking the dimensionless parameter Z_{bottom}/l^3 to $A/l^2$and y_p/l is the following

$$\frac{Z_{bottom}}{l^3} = 0.0015 - 0.0766\left(\frac{A}{l^2}\right) + 0.4500\left(\frac{A}{l^2}\right)\left(\frac{y_p}{l}\right) + 0.6576\left(\frac{A}{l^2}\right)^2\left(\frac{y_p}{l}\right) + -3.3384\left(\frac{A}{l^2}\right)^2\left(\frac{y_p}{l}\right) - 6698\left(\frac{A}{l^2}\right)\left(\frac{y_p}{l}\right)^2 - 0.1488\left(\frac{y_p}{l}\right)^3 \text{ with } R^2 = 0.906 \qquad \text{(eq.48)}$$

A relation developed between the dimensionless parameter $\frac{ecc}{l}$ to $\frac{A}{l^2}$ and $\frac{y_p}{y_0}$ which is the following:

$$\frac{ecc}{l} = -0.0425 + 3.6055\left(\frac{A}{l^2}\right)\left(\frac{y_p}{y_0}\right) - 23.7150\left(\frac{A}{l^2}\right)^2 + 130.4407\left(\frac{A}{l^2}\right)^3 - 431.6834\left(\frac{A}{l^2}\right)^4\left(\frac{y_p}{y_0}\right) \text{ with } R^2 = 0.908 \qquad \text{(eq.49)}$$

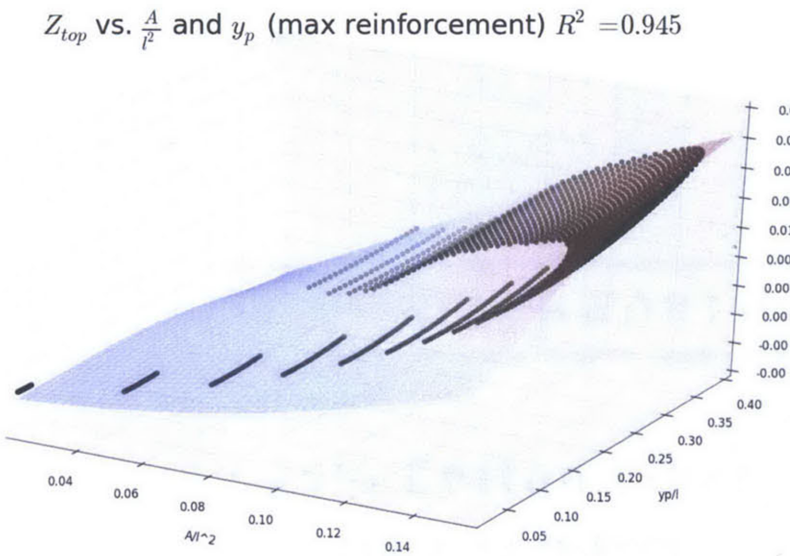

Figure 5.2 A polynomial surface that approximates a relation between the various points which each represent a dimensionless top section moduli for a unique cross-section

Similarly a relation linking the dimensionless parameter Z_{top}/l^3 to A/l^2 and y_p/l for a minimum reinforcement of 0.15 % is expressed as:

$$\frac{Ztop}{l^3} = -0.0004 - 0.3223\left(\frac{A}{l^2}\right)^2 + 3.99650\left(\frac{A}{l^2}\right)^3 + 0.7354\left(\frac{A}{l^2}\right)\left(\frac{yp}{l}\right) - 4.4289\left(\frac{A}{l^2}\right)^2\left(\frac{y_p}{l}\right) - 0.0739\left(\frac{y_p}{l}\right)^2 + 2.8848\left(\frac{A}{l^2}\right)^2\left(\frac{y_p}{l}\right)^2 \text{ with } R^2 = 0.931 \qquad \text{(eq.50)}$$

A regression is derived that links the dimensionless parameter Z_{bottom}/l^3 to A/l^2 and y_p/l. However in this case the R^2 is slightly lower that 0.9 and could be better approximated for future purposes. The final expression then is:

$$\frac{z_{bottom}}{l^3} = 0.0006 - 0.02\left(\frac{y_p}{l}\right) - 36.5481\left(\frac{A}{l^2}\right)^5 + 4.9971\left(\frac{A}{l^2}\right)^2\left(\frac{y_p}{l}\right)^3 + 1.0607\left(\frac{A}{l^2}\right)\left(\frac{y_p}{l}\right) - 0.8934\left(\frac{A}{l^2}\right)^2 + 7.6885\left(\frac{A}{l^2}\right)^3 - 5.6349\left(\frac{A}{l^2}\right)^2\left(\frac{y_p}{l}\right) - 0.1310\left(\frac{A}{l^2}\right)^3 \text{ with } R^2 = 0.864 \quad \text{(eq.51)}$$

A similar relation developed between the dimensionless parameter $\frac{ecc}{l}$ to $\frac{A}{l^2}$ and $\frac{y_p}{y_0}$ which is the following:

$$\frac{ecc}{l} = -0.0067 - 17.1298\left(\frac{A}{l^2}\right)^2\left(\frac{y_p}{y_0}\right)^3 - 2.6493\left(\frac{A}{l^2}\right)\left(\frac{y_p}{y_0}\right) + 1.6140\left(\frac{A}{l^2}\right)^2 + 6.6989\left(\frac{A}{l^2}\right)\left(\frac{y_p}{y_0}\right)^2 - 0.0623\left(\frac{y_p}{y_0}\right)^3 \text{ with } R^2 = 0.911 \quad \text{(eq.52)}$$

5.4 Results

A sample run of the problem introduced above is done for a top width of 20 in and the following results are produced. Comparing for the same moment and using a width of 20 in, the feasible region is found by first looking at the uppermost graph that shows the stresses at initial conditions, in order to be feasible, the tensile stress needs to be less than the fracture strength of concrete and the compressive stresses need to be lower than the compressive strength of concrete. In the case of a 0.5 % of reinforcement a total depth of approximately 24 in is needed for a 0.5 % pre-stressed design compared to a 17 in depth that is required for a reinforced concrete beam of 3% reinforcement ratio. In the case of a 0.15% reinforcement ratio, no feasible region could be found, as the depth required is more than the maximum depth constraint. The above results indicate therefore that perhaps the current cross-section is not ideal when it comes to dealing with pre-stressed design since the eccentricity increase due to an increase in depth of the cross-section is offset by a lowering of the cross-section centroid location as the fabric is allowed to sag even further. This leads into a search of certain cross-sections that guarantee a larger eccentricity i.e. that have a larger area of concrete allocated in the compression zone.

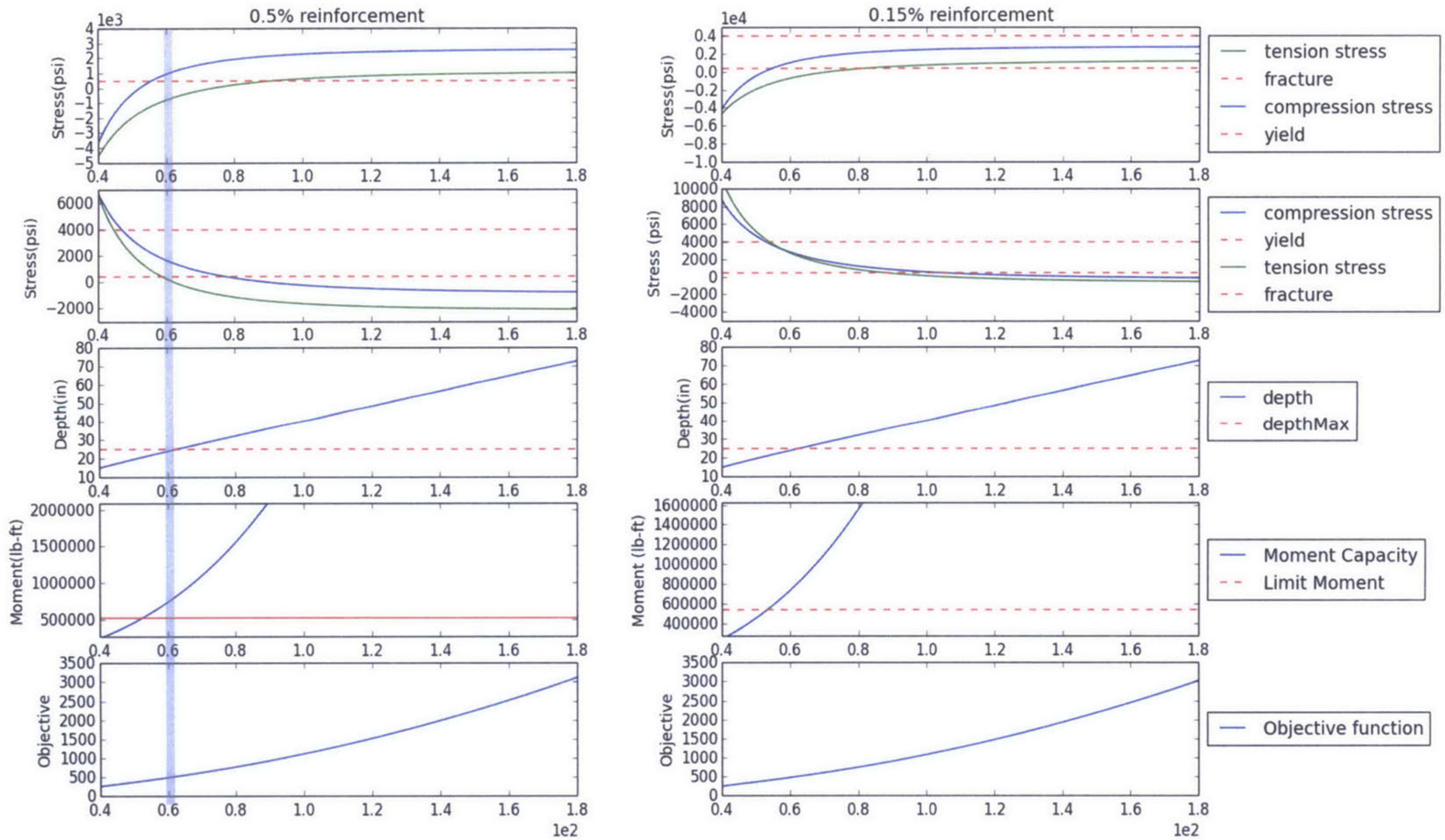

Figure 5.3 A sample output of the feasible region method for a pre-stress ratio of 0.5 and 0.15 %

5.5 Summary

A methodology of a feasible region method is proposed which uses dimensionless polynomial relationships amongst the variables in order to express the problem in terms of the decision variables of x_0 and l. The output of the method on the sample problem however indicates counter-intuitive results that hint at the need for a better cross-section for pre-stressed concrete beams, which will be discussed in the next chapter.

6. Conclusions

This chapter summarizes the contributions of this thesis and discusses the potential implications for future work. In order to fully deal with the optimization problem, a background overview the topic of fabric formed reinforced beams is introduced in Chapter 2. Chapter 3 then explains the approach to the problem of optimal fabric formed reinforced concrete design using two main methods. Chapter 4 illustrates a sample output of the two methods. Chapter 5 introduces the pre-stressed design optimization proposes a feasible region solution method. It also discusses some results of the optimization method.

6.1 Summary of Contributions

Both the analytical optimization method and the feasible region method produce similar results as shown in Chapter 4. The results show that a material savings of up to 55% is possible using the feasible region method. The methodology of the feasible region method is simpler than the analytical optimization method and could also be used by designers who might want to explore the optimization of more complex cross-sections. A similar methodology to the feasible region introduced above could be followed, which is summarized in Figures 6.1 and 6.2 that show a flow chart of the main steps to design reinforced and pre-stressed fabric formed beams assuming that the form-finding method doesn't radically change.

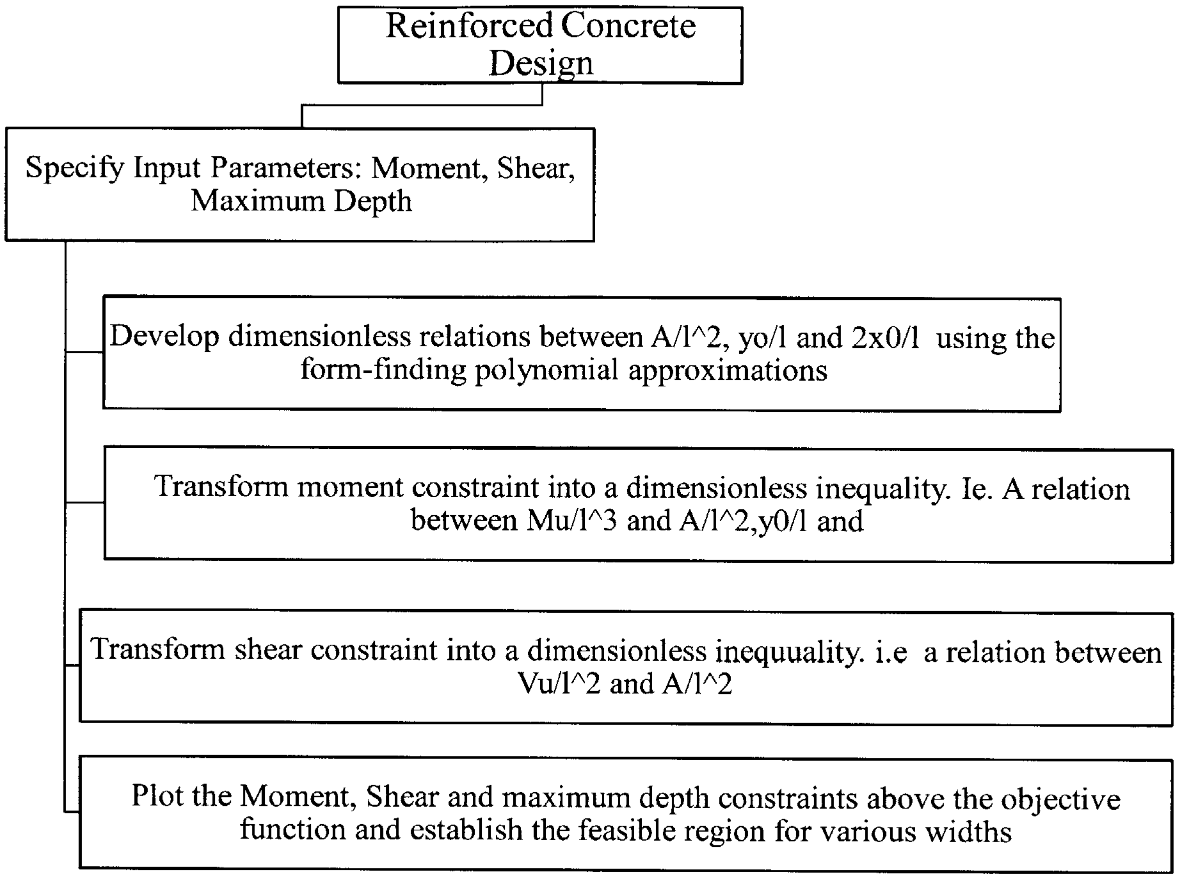

Figure 6.1 A flow chart showing the main steps needed to design an optimum cross-section of fabric in reinforced concrete design

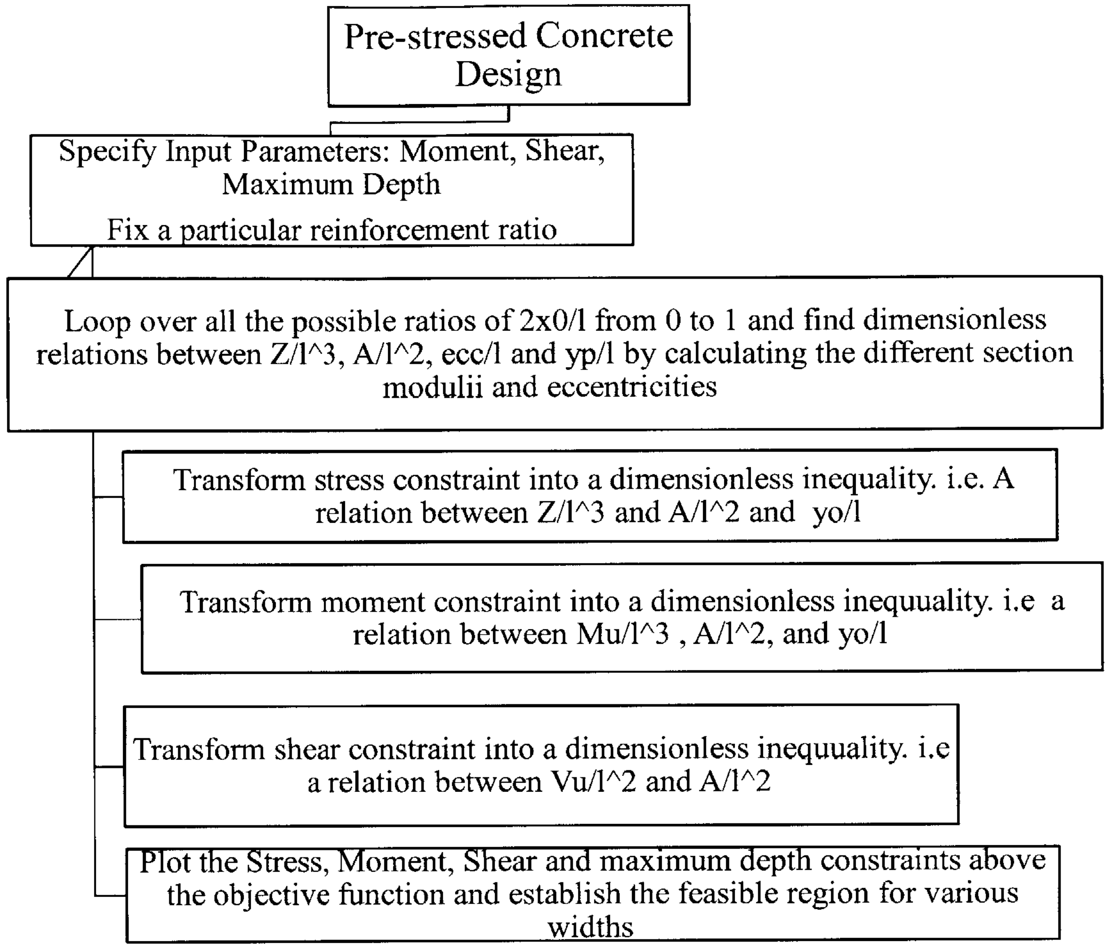

Figure 6.2. A flow chart showing the main steps needed to design an optimum cross-section of fabric in prestressed concrete design

6.2 Directions for future work

There are two key areas for future work which are proposed. First, based on the results from Chapter 5, it is necessary to look at alternative cross-sections for pre-stressed fabric formed beams. Second, it is important to consider constructability and feasibility of fabrication of the pre-stressed fabric formed beams.

The desired cross-section for pre-stressed fabric formed beams might resemble the one introduced by (Garbett, Darby, & Ibell , 2010) and discussed in (Orr, 2012). The cross-section below guarantees a considerable eccentricity and at the same time the same form-finding methods explained above could be implemented to predict the shape. Therefore the same methodology followed for the feasible region method could be implemented on the cross-section shown below. A general methodology summarizing the feasible region method is presented by Figure 6.3.

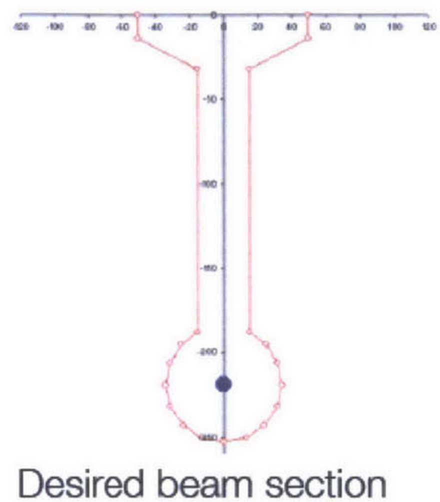

Figure 6.3. Picture taken from (Orr, 2012)

Since the cross-section is fixed above the bulb shape achieved by the fabric, the above polynomials could be simply modified in order to re-assess the feasibility of using pre-stress in fabric formed beams that hint at a better potential to make use of a large eccentricity.

In terms of constructability and feasibility of fabrication, regardless of the shape of the beam decided on, and in order to ensure a zero eccentricity at the supports of the beam, the pre-stressing jack will have to be located at the centroid of the cross-section. This might introduce certain conflicts as to the feasibility of joining two beams together, however a solution such as providing a countersink in the beam at the supports might be proposed. The below solution is inspired from the idea of stitching two precast beams on-site as proposed by (Orr, 2012) and shown in Figure 6.4. A speculation on how the construction might be implemented is shown in Figure 6.5.

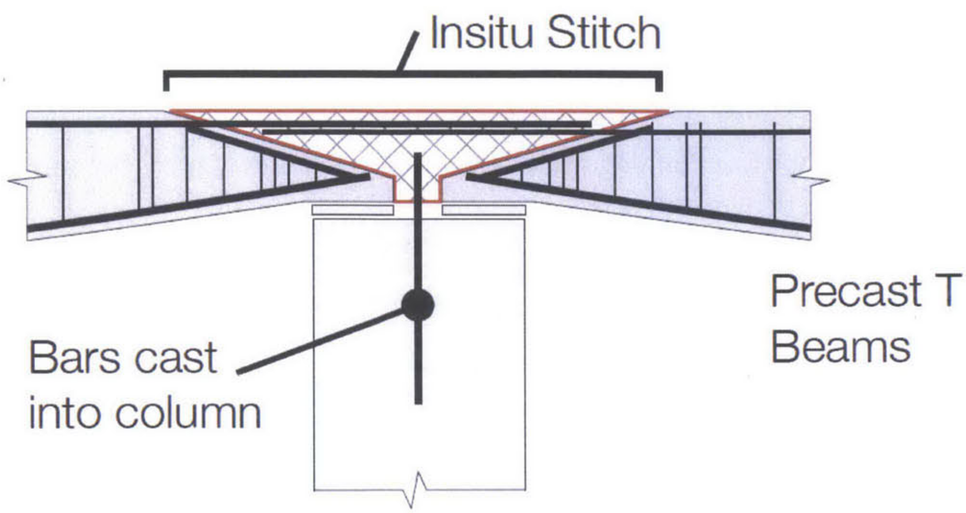

Figure 6.4. Picture taken from (Orr, 2012) which shows an insitu stitch of two precast concrete beams that meet on a column

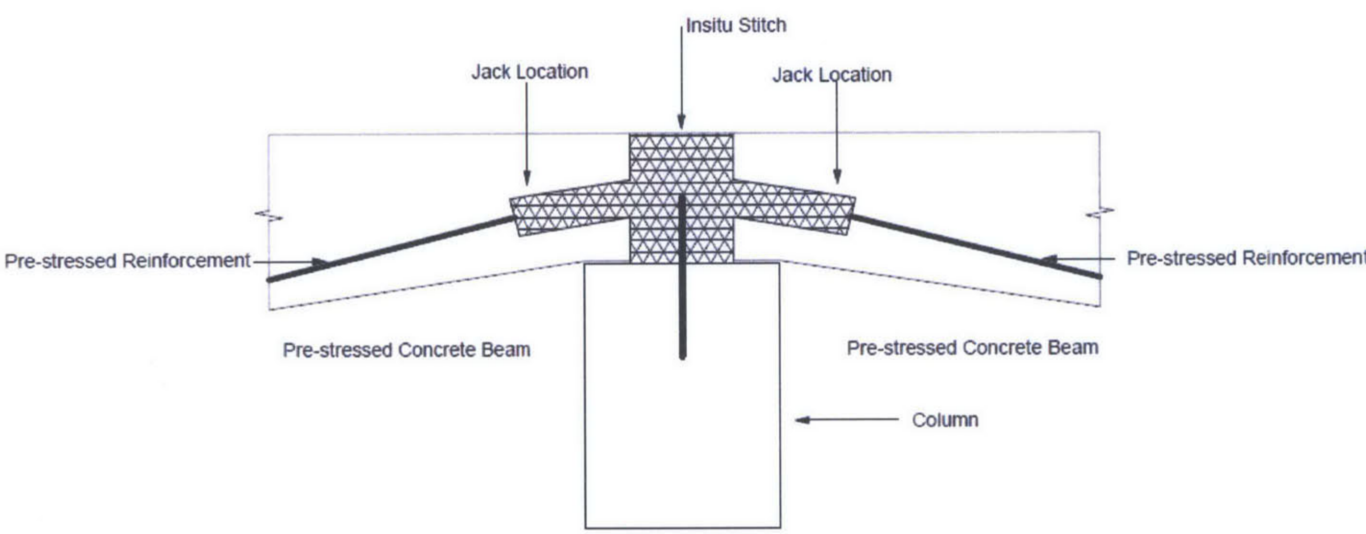

Figure 6.5. Picture inspired from (Orr, 2012) which shows an insitu stitch of two pre-stressed concrete beams with a countersink to allow space for the pre-stressed jacks at the supports.

6.3 Concluding Remarks

The thesis advances the design of fabric formed beams in order to reach well-defined methods to optimize the design of reinforced and fabric formed beams. The topic of fabric formwork is increasingly proving to be a feasible solution to ensure material efficiency. The importance of flexible formwork is in the fact that it encompasses various features that are becoming more attractive in the near future:

- Relatively lighter pre-cast beams to be transported onto site,
- Increasing the aesthetic beauty of the structure
- Ensuring that minimal material is used.

Bibliography

A.C.I. (2005). Proposed manual of standard practice for detailing reinforced concrete structures. Report of American Concrete Institute, Committee 315, A.J.*A* .

Al-Gahtani, A., Al-Saadoun , S., & Abul-Feilat, E. (1995). DESIGN OPTIMIZATION OF CONTINUOUS PARTIALLY PRESTRESSED CONCRETE BEAMS. *hm/~uren & Slrucrvre~ , 55* (2), 365-370.

Bailiss, J. (2006). Fabric Formed Concrete Beams: Design and Analysis. *MEng Thesis*. University of Bath.

BS EN 1992-1-1: 2004 a. (n.d.). Eurocode 2: Design of concrete structures- Part 1-1: General rules and rules for buildings.

Colin, M. , & MacRae, A. (1984). OPTIMIZATION OF STRUCTURAL CONCRETE BEAMS. *Journal of Structural Engineering, 110* (7).

Foster, R. (2010). Form Finding and Analysis of fabric formed concrete beams. *MEng Thesis*. University of Bath, Department of Architecture and Civil Engineering.

Garbett, J., Darby, A., & Ibell, T. (2010). Optimised beam design using innovative fabric-formed concrete. *Advances in Structural Engineering, 13* (5), 849-860.

Guerra , A., & Kiousis, P. (2006). Design optimization of reinforced concrete structures. *Computers and Concrete , 3* (5), 313-334.

Iosilevskii, G. (2009). Shape of a Soft Container Under Hydrostatic Load. *Journal of Applied Mechanics , 77* (1).

Lee, D. (2010). Study of construction methodolgy and structural behaviour of fabric-formed form-efficient reinforced concrete beam. The University of Edinburgh.

Marano , G. C., Fiore, A., & Quaranta, G. (2014). Optimum design of prestressed concrete beams using constrained differential evolution algorithm. *Structural and Multidisciplinary Optimization , 49* (3), 441-453.

Mathworks. (2014, May). *Non Linear constrained optimization using fmincon. Retrieved from MATLAB R2014a.* Retrieved from http://www.mathworks.com/help/optim/ug/fmincon.html?s_tid=doc_12b

McCormac, J. (1998). *Design of reinforced concrete.* Calif.: Addison-Wesley.

Nervi, P. L. (1956). *Structures: by Pier Luigi Nervi ; translated by Giuseppina and Mario Salvadori ; foreword by Mario Salvadori.* (Giuseppina , & M. Salvadori, Trans.) New York: F.W. Dodge.

Orr, J. (2012). Flexible formwork for concrete structures. *Thesis (Doctor of Philosophy (PhD))* . University of Bath.

Veenendaal, D. (2008). Evolutionary Optimization of Fabric Formed Structural Elements. *Masters Thesis*. University of Delft.

West, M., & Araya, R. (2009). FABRIC FORMWORK FOR CONCRETE STRUCTURES AND ARCHITECTURE. *International Conference on Textile Composites and Inflatable Structures* (pp. 1-4). Barcelona: B. Kröplin and E. Oñate, (Eds) CIMNE.

Appendix

Validation of Form-Finding Modeling

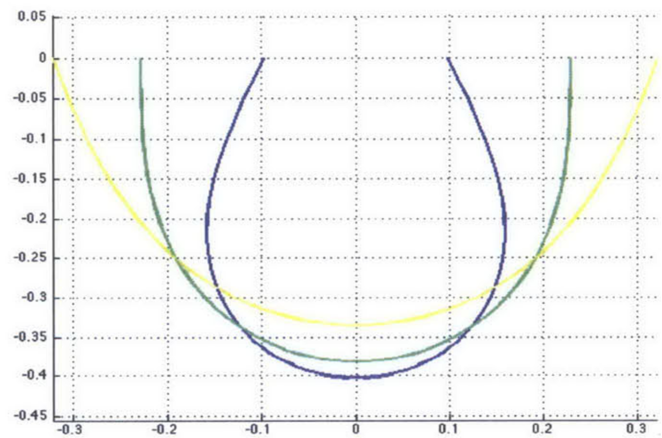

Figure A.1 Matlab output of the model used in this thesis

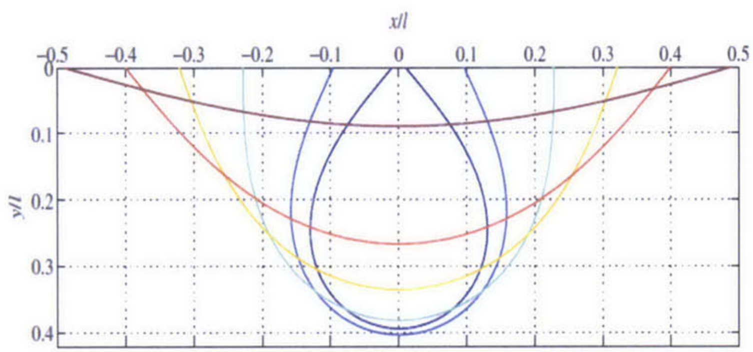

Figure A.2 Picture taken from Iosilevskii (2009)

Properties of Fabric Cross-section

```
function [area_transformed,Ztop,Zbottom,eccentricity,Yt,xs,ys]=fab_propertiesf(rx,l,yp,Ap,n)

%

% The following function calculates the section properties of the concrete section as well as of the transformed section.

%

% rx is  the ratio of the top width (2x0) and the fabric length l

% yp is the position of the prestress reinforcement taken relative to the top most compression fiber

% Ap is the area of pre-stressed reinforcement used

% n is the ratio of the modulus of elasticity of pre-stressed steel to concrete

%estimating k and K from the polynomial approximations

k=-16.982*(rx)^6+45.557*(rx)^5-46.567*(rx)^4+22.267*(rx)^3-5.2611*(rx)^2+ 0.138*(rx)+0.8985;

K=16.079*k^6 - 37.911*k^5 + 35.039*k^4 - 15.13*k^3 + 3.4482*k^2 - 0.2272*k + 1.5717;

% choosing the number of iterations

niter=100;

% Initializing the parameters

thetas=linspace(-pi/2,0,niter);

m=k^2;

xs=zeros(niter,1);

ys=zeros(niter,1);

area_trapezoid=zeros(niter,1);

centroid_trapezoid=zeros(niter,1);

moment_trapezoid=zeros(niter,1);

% populating the profile of the fabric (xs, and ys)

for i=1:niter

  F=ellipticF(thetas(i),m);

  Es=ellipticE(thetas(i),m);

  xs(i)=-l*((Es/K)-0.5*(F/K));
```

```
  ys(i)=l*((k/K)*cos(thetas(i)));
end


% calculating the area, centroid and moment of inertia using parallel axis theorem, and  by splitting the section
into small trapezoids

for j=2:niter
  a=2*xs(j-1);
  b=2*xs(j);
  h=ys(j)-ys(j-1);
  area_trapezoid(j)=(a+b)*h/2;
  centroid_trapezoid(j)= h*(2*a+b)/(3*(a+b))+ys(j-1);
  moment_trapezoid(j)= h^3*(a^2+4*a*b+b^2)/(36*(a+b));


end


%------ Finding the aggregate properties of the Concrete Section
area= sum(area_trapezoid);
centroid= (centroid_trapezoid'*area_trapezoid)/area;
moment=moment_trapezoid+area_trapezoid.*(centroid_trapezoid-centroid.*ones(niter,1)).^2 ;%Parallel Axis
theorem
moment=sum(moment);
Yt=centroid;
Yb=ys(length(ys))-centroid;
Zt=moment/Yt;
Zb=moment/Yb;
% Expanding the profile on each side with an area of (n-1)*Ap/2 in order to get a transformed section
%---- Transformed Section Properties
delta=abs(ys-yp.*ones(niter,1));
```

```
markp=find(delta==min(delta));


if markp ~= length(ys)
   htrans=ys(markp+1)-ys(markp);
   atrans=0.5*((n-1)*Ap/htrans-(xs(markp)-xs(markp+1)));
   btrans=atrans+xs(markp)-xs(markp+1);
else
   htrans=ys(markp)-ys(markp-1);
   atrans=0.5*((n-1)*Ap/htrans-(xs(markp)-xs(markp-1)));
   btrans=atrans+xs(markp-1)-xs(markp);
end
% Finding the aggregate properties of the transformed section.
areatrans= htrans*(atrans+btrans)/2;
centroidtrans=htrans*(2*atrans+btrans)/(3*(atrans+btrans))+ys(markp);
momenttrans= htrans^3*(atrans^2+4*atrans*btrans+btrans^2)/(36*(atrans+btrans));


area_transformed=2*areatrans+area;
centroid_transformed=(centroid*area+2*areatrans*centroidtrans)/(area+2*areatrans);


momentfabric=sum(moment_trapezoid+area_trapezoid.*(centroid_trapezoid-
centroid_transformed.*ones(niter,1)).^2) ;%Parallel Axis theorem
moment_transformed=2*(momenttrans+areatrans*(centroidtrans-centroid_transformed)^2)+momentfabric;


Yt=centroid_transformed;
Yb=ys(length(ys))-centroid_transformed;
Ztop=moment_transformed/Yt;
Zbottom=moment_transformed/Yb;
```

```
eccentricity=yp-centroid_transformed;
```

Generation of Relationships and Results

The following python code generates the dimensionless relationships between the section moduli, eccentricity, area, and top width of the fabric cross –section. It also generates the plots based on the feasible region method.

```
# coding: utf-8

# Author: Marwan Sarieddine

# Reinfoced Concrete Design

# In[1]:

#Minimizing graphically:

import numpy as np

import matplotlib.pyplot as plt

#%pylab inline

ratiowidth= np.arange(0,1,0.0001)

x=ratiowidth

ratioArea=-7.1921*x**6 + 19.221*x**5 - 19.595*x**4 + 9.2215*x**3 - 2.0749*x**2 + 0.3744*x + 0.066

# $$ \frac{As}{l^2}= \rho *ratioArea $$

# Dimensionless Objective Function

# $$ \frac{f}{l^2}= C_1 ratioArea+ C_2*As/l^2 $$

# $$ \frac{f}{l^2}= ratioArea(C_1+ C_2 \rho) $$

# In[2]:

C1=1

C2=9

pmin=0.018 # minimum reinforcement

pmax=0.030 # maximum reinforcement



# In[17]:

from matplotlib.font_manager import FontProperties

# Now taking real values for Vu, Mu , l^2, x0 ;

Mu=187500.0 # lb-ft

Vu=0;

x0=4.0;
```

```
# for midspan only 1 real constraint :

# now trying x0, and l combination

l=np.arange(2.0*x0,20.0*x0,1);

x0=x0*np.ones([np.shape(l)[0]])

depthMax=20.0*np.ones([np.shape(l)[0]])

x=2.0*x0/l

Area=l**2*(-7.1921*x**6 + 19.221*x**5 - 19.595*x**4 + 9.2215*x**3 - 2.0749*x**2 + 0.3744*x + 0.066)

depth=l*(-10.928*x**6 + 29.298*x**5 - 29.953*x**4 + 14.345*x**3 - 3.5258*x**2 + 0.4109*x + 0.3849)

phi=0.9

fy=60000.0

RHS1=phi*pmin*fy*Area*(depth-0.31875*(87000.0)/(87000.0+fy)*(0.9*depth))/(12.0) # y0-2 estimated as 90% of y0

RHS2=phi*pmax*fy*Area*(depth-0.31875*(87000.0)/(87000.0+fy)*(0.9*depth))/(12.0)

fmin=Area*(C1+C2*pmin)

fmax=Area*(C1+C2*pmax)

fig=plt.figure()

ax1=plt.subplot2grid((3,10),loc=(0,0),colspan=4)

ax2=plt.subplot2grid((3,10),loc=(1,0),colspan=4,sharex=ax1)

ax3=plt.subplot2grid((3,10),loc=(2,0),colspan=4,sharex=ax1)

ax1.plot(l,RHS1,color='g',label='moment capacity min-reinforcement')

ax1.plot(l,RHS2,color='b')

ax1.plot(l,(Mu)*np.ones([np.shape(l)[0]]),color='r',label='limitmoment',ls='dashed')

ax1.plot(41.6*np.ones([np.shape(l)[0]]),np.linspace(0,max(RHS2),np.shape(l)[0]),label='Garbett (2010)',color='y')

ax1.set_ylabel('Moment (lb-in)')

# Put a legend to the right of the current axis

ax1.set_title('x0=4.0 in')

ax1.ticklabel_format(scilimits=(-1, 1))

ax2.plot(l,depth,color='b',label='depth')

ax2.plot(l,depthMax,color='r',label='depthMax',ls='dashed')

ax2.plot(41.6*np.ones([np.shape(l)[0]]),np.linspace(0,max(depth),np.shape(l)[0]),label='Garbett (2010)',color='y')

ax2.set_ylim(5,30)

ax2.set_ylabel('Depth (in)')

ax3.plot(l,fmin,color='g',label='objective-min_reinf')

ax3.plot(l,fmax,color='b',label='objective-max_reinf')
```

```
ax3.plot(41.6*np.ones([np.shape(l)[0]]),np.linspace(0,max(fmax),np.shape(l)[0]),label='Garbett (2010)',color='y')

ax3.set_ylabel('Objective ')

ax3.set_xlabel('fabric length (in)')

x0=5.0;

# for midspan only 1 real constraint :

# now trying x0, and l combination

l=np.arange(2.0*x0,12.0*x0,1);


x0=x0*np.ones([np.shape(l)[0]])

depthMax=20.0*np.ones([np.shape(l)[0]])


x=2.0*x0/l

Area=l**2*(-7.1921*x**6 + 19.221*x**5 - 19.595*x**4 + 9.2215*x**3 - 2.0749*x**2 + 0.3744*x + 0.066)

depth=l*(-10.928*x**6 + 29.298*x**5 - 29.953*x**4 + 14.345*x**3 - 3.5258*x**2 + 0.4109*x + 0.3849)

RHS4=phi*pmin*fy*Area*(depth-0.31875*(87000.0)/(87000.0+fy)*(0.9*depth))/(12.0) # y0-2 estimated as 90% of y0

RHS5=phi*pmax*fy*Area*(depth-0.31875*(87000.0)/(87000.0+fy)*(0.9*depth))/(12.0)

fmin=Area*(C1+C2*pmin)

fmax=Area*(C1+C2*pmax)

ax4=plt.subplot2grid((3,10),loc=(0,5),colspan=4)

ax5=plt.subplot2grid((3,10),loc=(1,5),colspan=4,sharex=ax4)

ax6=plt.subplot2grid((3,10),loc=(2,5),colspan=4,sharex=ax5)

ax4.plot(l,RHS4,color='g',label='Moment capacity')

ax4.plot(l,RHS5,color='b')

ax4.plot(l,(Mu)*np.ones([np.shape(l)[0]]),color='r',label='Limitmoment',ls='dashed')

ax4.plot(35.0*np.ones([np.shape(l)[0]]),np.linspace(0,max(RHS5),np.shape(l)[0]),label='Garbett, Darby, & Ibell (2010)',color='y')

ax4.set_ylabel('Moment (lb-in)')

ax4.legend(loc='center left', bbox_to_anchor=(1, 0.5))

ax4.set_title('x0=5.0 in')

ax4.ticklabel_format(scilimits=(-1, 1))

ax5.plot(l,depth,color='b',label='Depth')

ax5.plot(l,depthMax,color='r',label='Depth Max',ls='dashed')

ax5.plot(35.0*np.ones([np.shape(l)[0]]),np.linspace(0,max(depth),np.shape(l)[0]),label='Garbett, Darby, & Ibell (2010)',color='y')

ax5.legend(loc='center left', bbox_to_anchor=(1, 0.5))
```

```
ax5.set_ylabel('Depth (in)')

ax5.set_ylim(5,30)

ax6.plot(l,fmin,color='g',label='Objective-min_reinf')

ax6.plot(l,fmax,color='b', label='Objective-max_reinf')

ax6.plot(35.0*np.ones([np.shape(l)[0]]),np.linspace(0,max(fmax),np.shape(l)[0]),label='Garbett, Darby, & Ibell (2010)',color='y')

ax6.set_ylabel('Objective')

ax6.legend(loc='center left', bbox_to_anchor=(1, 0.5))

ax6.set_xlabel('fabric length (in)')

plt.show()

# In[4]:

from matplotlib.font_manager import FontProperties

# Now taking real values for Vu, Mu , l^2, x0 ;

#span =36 ft

# tributary width = 18 ft

Mu=437400.0 # lb-ft

Vu=32400;

x0=5.0;

fc=4000;

# for midspan only 1 real constraint :

# now trying x0, and l combination




l=np.arange(2.0*x0,20.0*x0,1);

x0=5.0*np.ones([np.shape(l)[0]])

depthMax=25.0*np.ones([np.shape(l)[0]])

x=2.0*x0/l

Area=l**2*(-7.1921*x**6 + 19.221*x**5 - 19.595*x**4 + 9.2215*x**3 - 2.0749*x**2 + 0.3744*x + 0.066)

depth=l*(-10.928*x**6 + 29.298*x**5 - 29.953*x**4 + 14.345*x**3 - 3.5258*x**2 + 0.4109*x + 0.3849)

phi=0.9

fy=60000.0

RHS1=phi*pmin*fy*Area*(depth-0.31875*(87000.0)/(87000.0+fy)*(0.9*depth))/(12.0) # y0-2 estimated as 90% of y0

RHS2=phi*pmax*fy*Area*(depth-0.31875*(87000.0)/(87000.0+fy)*(0.9*depth))/(12.0)
```

```
RHSshear=Area*1.7*np.sqrt(fc)

LHSshear=0.5*Vu*np.ones([np.shape(l)[0]])

fmin=Area*(C1+C2*pmin)

fmax=Area*(C1+C2*pmax)

fig=plt.figure()

ax1=plt.subplot2grid((4,10),loc=(0,0),colspan=4)

ax2=plt.subplot2grid((4,10),loc=(1,0),colspan=4,sharex=ax1)

ax3=plt.subplot2grid((4,10),loc=(2,0),colspan=4,sharex=ax1)

ax4=plt.subplot2grid((4,10),loc=(3,0),colspan=4,sharex=ax1)

ax1.ticklabel_format(scilimits=(-1, 1))

ax3.ticklabel_format(scilimits=(-1, 1))

ax1.plot(l,RHS1,color='g',label='moment capacity min-reinforcement')

ax1.plot(l,RHS2,color='b',label='moment capacity max-reinforcement')

ax1.plot(l,(Mu)*np.ones([np.shape(l)[0]]),color='r',label='limitmoment',ls='dashed')

ax1.set_ylabel('Moment (lb-in)')

# Put a legend to the right of the current axis

ax1.set_title('x0=5.0 in')

ax1.legend(loc='center left', bbox_to_anchor=(1, 0.5))

ax2.plot(l,depth,color='b',label='depth')

ax2.plot(l,depthMax,color='r',label='depthMax',ls='dashed')

ax2.set_ylim(5,30)

ax2.set_ylabel('Depth (in)')

ax2.legend(loc='center left', bbox_to_anchor=(1, 0.5))

ax3.plot(l,RHSshear,color='b',label='Shear Strength')

ax3.plot(l,LHSshear,color='r',label='Ultimate Shear',ls='dashed')

ax3.set_ylabel('Shear (lb)')

ax3.legend(loc='center left', bbox_to_anchor=(1, 0.5))

ax4.plot(l,fmin,color='g',label='objective-min_reinf')

ax4.plot(l,fmax,color='b',label='objective-max_reinf')

ax4.set_ylabel('Objective')

ax4.legend(loc='center left', bbox_to_anchor=(1, 0.5))

"""

x0=10.0;
```

```
# for midspan only 1 real constraint :

# now trying x0, and l combination

l=np.arange(2.0*x0,10.0*x0,1);

x0=10.0*np.ones([np.shape(l)[0]])

depthMax=25.0*np.ones([np.shape(l)[0]])


x=2.0*x0/l

Area=l**2*(-7.1921*x**6 + 19.221*x**5 - 19.595*x**4 + 9.2215*x**3 - 2.0749*x**2 + 0.3744*x + 0.066)

depth=l*(-10.928*x**6 + 29.298*x**5 - 29.953*x**4 + 14.345*x**3 - 3.5258*x**2 + 0.4109*x + 0.3849)

RHS4=phi*pmin*fy*Area*(depth-0.31875*(87000.0)/(87000.0+fy)*(0.9*depth))/(12.0) # y0-2 estimated as 90% of y0

RHS5=phi*pmax*fy*Area*(depth-0.31875*(87000.0)/(87000.0+fy)*(0.9*depth))/(12.0)

RHSshear=Area*1.7*np.sqrt(fc)

LHSshear=0.5*Vu*np.ones([np.shape(l)[0]])

fmin=Area*(C1+C2*pmin)

fmax=Area*(C1+C2*pmax)

ax5=plt.subplot2grid((4,10),loc=(0,5),colspan=4)

ax6=plt.subplot2grid((4,10),loc=(1,5),colspan=4,sharex=ax5)

ax7=plt.subplot2grid((4,10),loc=(2,5),colspan=4,sharex=ax5)

ax8=plt.subplot2grid((4,10),loc=(3,5),colspan=4,sharex=ax5)

ax5.ticklabel_format(scilimits=(-1, 1))

ax7.ticklabel_format(scilimits=(-1, 1))

ax5.plot(l,RHS4,color='g',label='Moment capacity')

ax5.plot(l,RHS5,color='b')

ax5.plot(l,(Mu)*np.ones([np.shape(l)[0]]),color='r',label='Limitmoment',ls='dashed')

ax5.set_ylabel('Moment (lb-in)')

ax5.legend(loc='center left', bbox_to_anchor=(1, 0.5))

ax5.set_title('x0=10.0 in')

ax6.plot(l,depth,color='b',label='Depth')

ax6.plot(l,depthMax,color='r',label='Depth Max',ls='dashed')

ax6.legend(loc='center left', bbox_to_anchor=(1, 0.5))

ax6.set_ylabel('Depth (in)')

ax6.set_ylim(5,30)

ax7.plot(l,RHSshear,color='b',label='Shear Strength (lb)')
```

```
ax7.plot(l,LHSshear,color='r',label='Ultimate Shear (lb)',ls='dashed')

ax7.legend(loc='center left', bbox_to_anchor=(1, 0.5))

ax7.set_ylabel('Shear (lb)')

ax8.plot(l,fmin,color='g',label='Objective-min_reinf')

ax8.plot(l,fmax,color='b', label='Objective-max_reinf')

ax8.set_ylabel('Objective ')

ax8.legend(loc='center left', bbox_to_anchor=(1, 0.5))

"""

plt.show()

# In[19]:

#--------------------------------Results Validation --------------------------------------------

l=41.6

x0=4.0

x=2.0*x0/l

fc=4000.0 #in2

Area=l**2*(-7.1921*x**6 + 19.221*x**5 - 19.595*x**4 + 9.2215*x**3 - 2.0749*x**2 + 0.3744*x + 0.066)

depth=l*(-10.928*x**6 + 29.298*x**5 - 29.953*x**4 + 14.345*x**3 - 3.5258*x**2 + 0.4109*x + 0.3849)

As= 0.03*Area #in2

print As

d= (0.45*As*fy)/(0.67*fc*0.9*2.0*x0)+(187500.0*12/(As*fy))+(1.0/2)+(0.2)


print "depth= " +str(depth) #in

print "Garbett depth= " +str(d)

#------------------------------------------------------------------------------------------------------

x=2*5.0/35.0

l=35.0

fc=4000 #in2

Area=l**2*(-7.1921*x**6 + 19.221*x**5 - 19.595*x**4 + 9.2215*x**3 - 2.0749*x**2 + 0.3744*x + 0.066)

depth=l*(-10.928*x**6 + 29.298*x**5 - 29.953*x**4 + 14.345*x**3 - 3.5258*x**2 + 0.4109*x + 0.3849)

As= 0.020*Area #in2

print As

d= (0.45*As*fy)/(0.67*fc*0.9*2*10.0)+(187500.0*12/(As*fy))+(1.0/2)+(0.2)

print "depth= " +str(depth) #in
```

```
print "Garbett depth= " +str(d)

x=2*5.0/32.0

l=35.0

fc=4000.0 #in2

Area=l**2*(-7.1921*x**6 + 19.221*x**5 - 19.595*x**4 + 9.2215*x**3 - 2.0749*x**2 + 0.3744*x + 0.066)

depth=l*(-10.928*x**6 + 29.298*x**5 - 29.953*x**4 + 14.345*x**3 - 3.5258*x**2 + 0.4109*x + 0.3849)

As= 0.020*Area #in2

print As




# ## Prestressed Concrete Design

# In[5]:

import statsmodels.api as sm

import scipy.stats as stats

import pandas as pd

import numpy as np

import matplotlib.pyplot as plt

import csv

from mpl_toolkits.mplot3d import Axes3D

from matplotlib import cm

from matplotlib.ticker import LinearLocator, FormatStrFormatter

# In[6]:

with open('/Users/marwan/Downloads/Zt_Ratio1.csv','r') as f:

    data=[row for row in csv.reader(f.read().splitlines())]

Ztop=np.asarray(data)

Ztop=Ztop.flatten()

Ztop=Ztop.astype(np.float)

with open('/Users/marwan/Downloads/Zb_ratio1.csv','r') as f:

  data=[row for row in csv.reader(f.read().splitlines())]

Zbottom=np.asarray(data)

Zbottom=Zbottom.flatten()

Zbottom=Zbottom.astype(np.float)
```

```
with open('/Users/marwan/Downloads/Area_ratio1.csv','r') as f:
  data=[row for row in csv.reader(f.read().splitlines())]
Ratio_Area=np.asarray(data)
Ratio_Area=Ratio_Area.flatten()
Ratio_Area = Ratio_Area.astype(np.float)
with open('/Users/marwan/Downloads/Yp_ratio1.csv','r') as f:
  data=[row for row in csv.reader(f.read().splitlines())]
Ratio_Yp=np.asarray(data)
Ratio_Yp=Ratio_Yp.flatten()
Ratio_Yp = Ratio_Yp.astype(np.float)
with open('/Users/marwan/Downloads/ecc_ratio1.csv','r') as f:
  data=[row for row in csv.reader(f.read().splitlines())]
ecc=np.asarray(data)
ecc=ecc.flatten()
ecc = ecc.astype(np.float)
with open('/Users/marwan/Downloads/yp_yo.csv','r') as f:
    data=[row for row in csv.reader(f.read().splitlines())]
ypyo=np.asarray(data)
ypyo=ypyo.flatten()
ypyo=ypyo.astype(np.float)
with open('/Users/marwan/Downloads/Zt_ratio2.csv','r') as f:
    data=[row for row in csv.reader(f.read().splitlines())]
Ztop2=np.asarray(data)
Ztop2=Ztop2.flatten()
Ztop2=Ztop2.astype(np.float)
with open('/Users/marwan/Downloads/Zb_ratio2.csv','r') as f:
    data=[row for row in csv.reader(f.read().splitlines())]
Zbottom2=np.asarray(data)
Zbottom2=Zbottom2.flatten()
Zbottom2=Zbottom2.astype(np.float)
with open('/Users/marwan/Downloads/ecc2.csv','r') as f:
```

```
    data=[row for row in csv.reader(f.read().splitlines())]
ecc2=np.asarray(data)
ecc2=ecc2.flatten()
ecc2=ecc2.astype(np.float)
x=Ratio_Area
y=Ztop
z=Ratio_Yp
e=ecc
e2=ecc2
# In[7]:
X = sm.add_constant(zip(x*z,x**2,z**4,z**5,x**3*z**2), prepend=True) #Add a column of ones to allow the calculation of the intercept
k=1380
print y[k]
print  -0.0004+ 0.1882*x[k]*z[k]+0.1810*x[k]**2-0.3212*z[k]**4+ 0.6212*z[k]**5+-5.4088*x[k]**3*z[k]**2
#fit the model
results = sm.OLS(y, X).fit()
print results.summary()


# In[8]:
fig = plt.figure()
ax = fig.gca(projection='3d')
xs=np.linspace(min(x),max(x),50)
zs=np.linspace(min(z),max(z),50)
X, Z = np.meshgrid(xs, zs)
Y=-0.0004+ 0.1882*X*Z+0.1810*X**2-0.3212*Z**4+ 0.6212*Z**5+-5.4088*X**3*Z**2
surf = ax.plot_surface(X, Z, Y, rstride=1, cstride=1, cmap=cm.coolwarm,
        linewidth=0, antialiased=False,alpha=0.2)
ax.scatter(x,z,y,c='k')
ax.set_xlabel('A/l^2')
ax.set_xlim3d(min(Ratio_Area),max(Ratio_Area))
ax.set_ylabel('yp/l')
ax.set_title(r"$Z_{top}$ vs. $\frac{A}{l^2}$ and $y_p$ (max reinforcement) $R^2=0.945$", fontsize=28, color='k')
```

```
ax.zaxis.set_major_locator(LinearLocator(10))

ax.set_ylim3d(min(Ratio_Yp),max(Ratio_Yp))

ax.zaxis.set_major_formatter(FormatStrFormatter('%.02f'))

#fig.colorbar(surf, shrink=0.5, aspect=5)

plt.show()

# In[10]:

x=Ratio_Area

y=Zbottom

z=Ratio_Yp

X = sm.add_constant(zip(x,x*z,x**2,x**2*z,x*z**2,z**3), prepend=True) #Add a column of ones to allow the calculation of the intercept

k=1230

print y[k]

print 0.0015-0.0766*x[k]+0.4500*x[k]*z[k]+0.6576*x[k]**2-3.3384*x[k]**2*z[k]+0.6698*x[k]*z[k]**2-0.1488*z[k]**3

#fit the model

results = sm.OLS(y, X).fit()

print results.summary()

# In[11]:

fig = plt.figure()

ax = fig.gca(projection='3d')

ax.set_title(r" $Z_{bottom}$ vs. $ \frac{A}{l^2}$ and $y_p$ (max reinforcement) $R^2$=0.906", fontsize=28, color='k')

xs=np.linspace(min(x),max(x),50)

zs=np.linspace(min(z),max(z),50)

X, Z = np.meshgrid(xs, zs)

Y=0.0015-0.0766*X+0.4500*X*Z+0.6576*X**2-3.3384*X**2*Z+0.6698*X*Z**2-0.1488*Z**3

surf = ax.plot_surface(X, Z, Y, rstride=1, cstride=1, cmap=cm.coolwarm,linewidth=0, antialiased=False,alpha=0.2)

ax.scatter(x,z,y,c='k')

ax.set_xlabel('A/l^2')

ax.set_xlim3d(min(Ratio_Area),max(Ratio_Area))

ax.set_ylabel('yp/l')

ax.zaxis.set_major_locator(LinearLocator(10))

ax.set_ylim3d(min(Ratio_Yp),max(Ratio_Yp))

ax.zaxis.set_major_formatter(FormatStrFormatter('%.02f'))

#fig.colorbar(surf, shrink=0.5, aspect=5)
```

```
plt.show()

# In[12]:

# correlating eccentricity to ypyo

z=ypyo

X = sm.add_constant(zip(x*z,x**2,x**3,x**4*z), prepend=True) #Add a column of ones to allow the calculation of the intercept

k=150

print e[k]

print -0.0425+3.6055*x[k]*z[k]-23.7150*x[k]**2+130.4407*x[k]**3-431.6834*x[k]**4*z[k]

#fit the model

results = sm.OLS(e, X).fit()

print results.summary()

# In[13]:

fig = plt.figure()

ax = fig.gca(projection='3d')

xs=np.linspace(min(e),max(e),50)

zs=np.linspace(min(z),max(z),50)

X, Z = np.meshgrid(xs, zs)

Y=-0.0425+3.6055*X*Z-23.7150*X**2+130.4407*X**3-431.6834*X**4*Z

surf = ax.plot_surface(X, Z, Y, rstride=1, cstride=1, cmap=cm.coolwarm,linewidth=0, antialiased=False,alpha=0.3)

ax.scatter(x,z,e,c='k')

ax.set_xlabel('A/l^2')

ax.set_xlim3d(min(Ratio_Area),max(Ratio_Area))

ax.set_ylabel('yp/yo')

ax.zaxis.set_major_locator(LinearLocator(10))

ax.set_ylim3d(min(ypyo),max(ypyo))

ax.zaxis.set_major_formatter(FormatStrFormatter('%.02f'))

ax.set_title(r" Eccentricity vs $\frac{yp}{y_0}$ and $\frac{A}{l^2}$ $R^2=0.908$ maximum reinforcement", fontsize=28, color='k')

#fig.colorbar(surf, shrink=0.5, aspect=5)

plt.show()

# In[14]:

# correlating Ztop2 to Area and Yp

x=Ratio_Area

y=Ztop2
```

```
z=Ratio_Yp

X = sm.add_constant(zip(x**2,x**3,x*z,x**2*z,z**2,z**2*x**2), prepend=True) #Add a column of ones to allow the calculation of the intercept

k=1354

print y[k]

print -0.0004-0.3223*x[k]**2+3.99650*x[k]**3+0.7534*x[k]*z[k]-4.4289*x[k]**2*z[k]-0.0739*z[k]**2+ 2.8848*z[k]**2*x[k]**2

#fit the model

results = sm.OLS(y, X).fit()

print results.summary()

# In[15]:

#Plotting Correlation:

fig = plt.figure()

ax = fig.gca(projection='3d')

ax.set_title(r" $Z_{top}$ vs. $ \frac{A}{l^2}$ and $y_p$ (minimum reinforcement) $R^2$=0.931", fontsize=28, color='k')

xs=np.linspace(min(x),max(x),50)

zs=np.linspace(min(z),max(z),50)

X, Z = np.meshgrid(xs, zs)

Y=-0.0004-0.3223*X**2+3.99650*X**3+0.7534*X*Z-4.4289*X**2*Z-0.0739*Z**2+ 2.8848*Z**2*X**2

surf = ax.plot_surface(X, Z, Y, rstride=1, cstride=1, cmap=cm.coolwarm,linewidth=0, antialiased=False,alpha=0.2)

ax.scatter(x,z,y,c='k')

ax.set_xlabel('A/l^2')

ax.set_xlim3d(min(Ratio_Area),max(Ratio_Area))

ax.set_ylabel('yp/yo')

ax.zaxis.set_major_locator(LinearLocator(10))

ax.set_ylim3d(min(Ratio_Yp),max(Ratio_Yp))

ax.zaxis.set_major_formatter(FormatStrFormatter('%.02f'))

#fig.colorbar(surf, shrink=0.5, aspect=5)

plt.show()

# In[17]:

# correlating Zbottom2 to Area and Yp

x=Ratio_Area

y=Zbottom2

z=Ratio_Yp
```

```
X = sm.add_constant(zip(z,x**5,x**2*z**3,x*z,x**2,x**3,x**2*z,z**3), prepend=True) #Add a column of ones to allow the calculation of the intercept

k=160

print y[k]

print  0.0006-0.0200*z[k]-36.5481*x[k]**5+4.9971*x[k]**2*z[k]**3+1.0607*x[k]*z[k] -0.8934*x[k]**2 +7.6885 *x[k]**3 -5.6349*x[k]**2*z[k]-0.1310*z[k]**3

#fit the model

results = sm.OLS(y, X).fit()

print results.summary()

# In[18]:

#Plotting Correlation Zbottom2 to Area and Yp:

fig = plt.figure()

ax = fig.gca(projection='3d')

ax.set_title(r" $Z_{bottom}$ vs. $ \frac{A}{l^2}$ and $y_p$ (minimum reinforcement) $R^2$=0.864", fontsize=28, color='k')

xs=np.linspace(min(x),max(x),50)

zs=np.linspace(min(z),max(z),50)

X, Z = np.meshgrid(xs, zs)

Y= 0.0006-0.0200*Z-36.5481*X**5+4.9971*X**2*Z**3+1.0607*X*Z -0.8934*X**2 +7.6885 *X**3 -5.6349*X**2*Z-0.1310*Z**3

surf = ax.plot_surface(X, Z, Y, rstride=1, cstride=1, cmap=cm.coolwarm,linewidth=0, antialiased=False,alpha=0.2)

ax.scatter(x,z,y,c='k')

ax.set_xlabel('A/l^2')

ax.set_xlim3d(min(Ratio_Area),max(Ratio_Area))

ax.set_ylabel('yp/yo')

ax.zaxis.set_major_locator(LinearLocator(10))

ax.set_ylim3d(min(Ratio_Yp),max(Ratio_Yp))

ax.zaxis.set_major_formatter(FormatStrFormatter('%.02f'))

#fig.colorbar(surf, shrink=0.5, aspect=5)

plt.show()

# In[20]:

# correlating ecc2 to Area and Yp

x=Ratio_Area

y=ecc2

z=ypyo

X = sm.add_constant(zip(x**2*z**3,x*z,x**2,x*z**2,z**3), prepend=True) #Add a column of ones to allow the calculation of the intercept
```

```
k=150

print e[k]

print  -0.0067-17.1298*x[k]**2*z[k]**3-2.6493*x[k]*z[k]+1.6140*x[k]**2+6.6989*x[k]*z[k]**2-0.0623*z[k]**3

#fit the model

results = sm.OLS(y, X).fit()

print results.summary()

# In[22]:

#Plotting Correlation Zbottom2 to Area and Yp:

fig = plt.figure()

ax = fig.gca(projection='3d')

ax.set_title(r" $Eccentricity $ vs. $ \frac{A}{l^2}$ and $\frac{y_p}{y_0}$ (minimum reinforcement) $R^2$=0.911", fontsize=28, color='k')

xs=np.linspace(min(x),max(x),50)

zs=np.linspace(min(z),max(z),50)

X, Z = np.meshgrid(xs, zs)

Y= -0.0067-17.1298*X**2*Z**3-2.6493*X*Z+1.6140*X**2+6.6989*X*Z**2-0.0623*Z**3

surf = ax.plot_surface(X, Z, Y, rstride=1, cstride=1, cmap=cm.coolwarm,linewidth=0, antialiased=False,alpha=0.2)

ax.scatter(x,z,y,c='k')

ax.set_xlabel('A/l^2')

ax.set_xlim3d(min(Ratio_Area),max(Ratio_Area))

ax.set_ylabel('yp/yo')

ax.zaxis.set_major_locator(LinearLocator(10))

ax.set_ylim3d(min(ypyo),max(ypyo))

ax.zaxis.set_major_formatter(FormatStrFormatter('%.02f'))

#fig.colorbar(surf, shrink=0.5, aspect=5)

plt.show()

# constraints:

# $$g_1(x): y_o \le depthMax $$ below horizontal line if plot depth versus length.

# $$g_2(x): -\frac{P_i}{A} - \frac{M_{DL}*12}{Z_{bottom}} + \frac{P_i*e}{Z_{bottom}} \le fr $$

# Rest of constraints same form.

# $$g_5(x): 0.9*P_i* \mu * d/12  \ge M_u $$

# ####Conducted Cases for 2x0/l from 0 to l and from yp/y0 from 0 to 1

# ####for max-reinforcement by ACI

# ####for over max-reinforcement
```

```
# ####for no reinforcement  3 cases taken into account

# Pi=factor*A and therefore first term cancels out.

# MDL is a constant

# Zbottom is a function of yo and A which can be written as a function of x0 and therfore can draw a graph.

# eccentricity = yp-ycentroid / and therfore should also be a function of x0 and I.

# simplified $g_2(x)$ is then :

#    C1*Z + C2 + constant*A*ecc <= Z units in^3 * lb/in^2 lb-in  lb/in^2*in^2*in <= in^3 *lb.in^2

#    C1*f(A,yo)+C2*A*f(yo) <= f(A,yo)

# simplif y into

#    C1*f(x0)+C2*f(xo) <= f(xo) and plot it through xo vs I

#

# In[63]:


#setting up code :

fig=plt.figure()

ax1=plt.subplot2grid((5,10),loc=(0,0),colspan=4,sharex=ax1)

ax2=plt.subplot2grid((5,10),loc=(1,0),colspan=4,sharex=ax1)

ax3=plt.subplot2grid((5,10),loc=(2,0),colspan=4,sharex=ax1)

ax4=plt.subplot2grid((5,10),loc=(3,0),colspan=4,sharex=ax1)

ax5=plt.subplot2grid((5,10),loc=(4,0),colspan=4,sharex=ax1)

ax6=plt.subplot2grid((5,10),loc=(0,5),colspan=4)

ax7=plt.subplot2grid((5,10),loc=(1,5),colspan=4,sharex=ax6)

ax8=plt.subplot2grid((5,10),loc=(2,5),colspan=4,sharex=ax6)

ax9=plt.subplot2grid((5,10),loc=(3,5),colspan=4,sharex=ax6)

ax10=plt.subplot2grid((5,10),loc=(4,5),colspan=4,sharex=ax6)

MDL=300000.0 # lb-ft

Vu=0 # shear at midspan

mu=0.85 # loss-ratio

phi=1.0 # prestressed

fp=230000.0 #ksi yield strength prestress

fr=450.0 #psi strength of rupture

fc=4000.0 #lb/in^2

C1=1.0
```

```
C2=9.0

# need to specify

Ap1= 0 # 2lines opposite boundaries

#yp= 0.5->1 *yo #what if it were a matrix ?

#Ap2= 0.1*A # for now

x0=10.0;

# for midspan only 1 real constraint :

# now trying x0, and l combination

l=np.linspace(4.0*x0,18.0*x0,100);

x0=10.0*np.ones([np.shape(l)[0]])

depthMax=25.0*np.ones([np.shape(l)[0]])

c=np.linspace(0.5,0.9,100)

c=0.9 # for now

x=2.0*x0/l

Area=l**2*(-7.1921*x**6 + 19.221*x**5 - 19.595*x**4 + 9.2215*x**3 - 2.0749*x**2 + 0.3744*x + 0.066)

depth=l*(-10.928*x**6 + 29.298*x**5 - 29.953*x**4 + 14.345*x**3 - 3.5258*x**2 + 0.4109*x + 0.3849)

yp=c*depth

Ztop=(l**3.0)*(-0.0004+ 0.1882*(Area/l**2)*(yp/l)+0.1810*(Area/l**2)**2-0.3212*(yp/l)**4+            0.6212*(yp/l)**5+-
5.4088*(Area/l**2)**3*(yp/l)**2)

Zbottom=(l**3.0)*(0.0015-0.0766*(Area/l**2)+0.4500*(Area/l**2)*(yp/l)+0.6576*(Area/l**2)**2            -
3.3384*(Area/l**2)**2*(yp/l)+0.6698*(Area/l**2)*(yp/l)**2-0.1488*(yp/l)**3)

# diff is that here z=yp/y0 = c

ecc=l*(-0.0425+3.6055*(Area/l**2)*c-23.7150*(Area/l**2)**2+130.4407*(Area/l**2)**3-431.6834*(Area/l**2)**4*c)

# {P_i}/{A} - {M_{DL}*12}/{Z_{bottom}} - {P_i*e}/{Z_{bottom}} < fr

#initial conditions:

#constraint 1 - tension above

# Max reinforcement need to change later

Cnst1=0.75*fp*0.005 #Ap=0.005*A

Cnst2=12*MDL

Cnst3=0.75*fp*0.005


RHS1=(-Cnst1-Cnst2/Ztop+Cnst3*Area*ecc/Ztop)# -Compression prestress uniform - DL compression moment + Eccentricity Tension

LHS1=fr*np.ones([np.shape(l)[0]])

# constraint 2- compression below
```

```
RHS2=(Cnst1-Cnst2/Zbottom+Cnst3*Area*ecc/Zbottom)# Compression prestress uniform - DL tension moment + Eccentricity Compression

LHS2=fc*np.ones([np.shape(l)[0]])

# final conditions:

#constraint 3- compression above

# Max reinforcement need to change later

Mtotal=400000.0

Cnst4=0.75*fp*0.005*mu #Ap=0.005*A

Cnst5=12*Mtotal

Cnst6=0.75*fp*0.005*mu

RHS3=(Cnst4+Cnst5/Ztop-Cnst6*Area*ecc/Ztop)# Compression prestress uniform + total Compression moment - Eccentricity tension

LHS3=fc*np.ones([np.shape(l)[0]])

# constraint 4- tension below

RHS4=(-Cnst4+Cnst5/Zbottom-Cnst6*Area*ecc/Zbottom)# -Compression prestress uniform+ DL Tension moment - Eccentricity compression

LHS4=fr*np.ones([np.shape(l)[0]])

# Maximum Depth Constraint

RHS5=depth

LHS5=depthMax

# Moment Capacity Constraint

Mu=520000.0

RHS6=phi*0.75*fp*0.005*Area*(depth-0.31875*(87000.0)/(87000.0+fp)*(yp))/(12.0)

LHS6=(Mu)*np.ones([np.shape(l)[0]])

f=Area*(C1+C2*0.005)

ax1.set_title('0.5% reinforcement ')

ax1.plot(l,RHS1,color='g',label='tension stress')

ax1.plot(l,LHS1,color='r',label='fracture',ls='dashed')

ax1.plot(l,RHS2,color='b',label='compression stress')

ax1.plot(l,LHS2,color='r',label='yield',ls='dashed')

ax1.set_ylabel('Stress(psi)')

ax1.set_xlim(40,180)

ax1.ticklabel_format(scilimits=(-1, 1))

ax2.plot(l,RHS3,color='b',label='compression stress')

ax2.plot(l,LHS3,color='r',label='yield',ls='dashed')

ax2.plot(l,RHS4,color='g',label='tension stress')
```

```
ax2.plot(l,LHS4,color='r',label='fracture',ls='dashed')

#ax2.set_ylim(-5000,10000)

ax2.set_ylabel('Stress(psi)')

ax3.plot(l,RHS5,color='b',label='depth')

ax3.plot(l,LHS5,color='r',label='depthMax',ls='dashed')

ax3.set_ylabel('Depth(in)')

ax4.plot(l,RHS6,color='b',label='Moment Capacity')

ax4.plot(l,LHS6,color='r',label='Limit Moment')

ax4.set_ylim(0.5*Mu,4*Mu)

ax4.set_ylabel('Moment(lb-ft)')

ax5.plot(l,f,color='b',label='Objective function')

ax5.set_ylabel('Objective')

Ztop2=(l**3.0)*(-0.0004-0.3223*(Area/l**2)**2+3.99650*(Area/l**2)**3+0.7534*(Area/l**2)*(yp/l)-4.4289*(Area/l**2)**2*(yp/l)-
0.0739*(yp/l)**2+2.8848*(yp/l)**2*(Area/l**2)**2)

Zbottom2=(l**3.0)*(0.0006-0.0200*(yp/l)-36.5481*(Area/l**2)**5+4.9971*(Area/l**2)**2*(yp/l)**3+1.0607*(Area/l**2)*(yp/l) -
0.8934*(Area/l**2)**2 +7.6885 *(Area/l**2)**3 -5.6349*(Area/l**2)**2*(yp/l)-0.1310*(yp/l)**3)

ecc2=l*( -0.0067-17.1298*(Area/l**2)**2*(c)**3-2.6493*(Area/l**2)*(c)+1.6140*(Area/l**2)**2+      6.6989*(Area/l**2)*(c)**2-
0.0623*(c)**3)

# {P_i}/{A} - {M_{DL}*12}/{Z_{bottom}} - {P_i*e}/{Z_{bottom}} < fr

#initial conditions:

#constraint 1 - tension above

# Max reinforcement need to change later

Cnst10=0.75*fp*0.0015 #Ap=0.005*A

Cnst11=12*MDL

Cnst12=0.75*fp*0.0015

RHS10=(-Cnst1-Cnst2/Ztop2+Cnst3*Area*ecc2/Ztop2)# -Compression prestress uniform - DL compression moment + Eccentricity Tension

LHS10=fr*np.ones([np.shape(l)[0]])

# constraint 2- compression below

RHS11=(Cnst1-Cnst2/Zbottom2+Cnst3*Area*ecc2/Zbottom2)# Compression prestress uniform - DL tension moment + Eccentricity Compression

LHS11=fc*np.ones([np.shape(l)[0]])

# final conditions:

#constraint 3- compression above

# Max reinforcement need to change later

Mtotal=1.5*MDL
```

```
Cnst4=0.75*fp*0.0015*mu #Ap=0.005*A

Cnst5=12*Mtotal #lb-in

Cnst6=0.75*fp*0.0015*mu

RHS12=(Cnst4+Cnst5/Ztop2-Cnst6*Area*ecc2/Ztop2)# Compression prestress uniform + DL Compression moment - Eccentricity tension

LHS12=fc*np.ones([np.shape(l)[0]])

# constraint 4- tension below

RHS13=(-Cnst4+Cnst5/Zbottom2-Cnst6*Area*ecc2/Zbottom2)# -Compression prestress uniform+ DL Tension moment - Eccentricity
compression

LHS13=fr*np.ones([np.shape(l)[0]])

# Maximum Depth Constraint

RHS14=depth

LHS14=depthMax

# Moment Capacity Constraint

Mu=1.2*Mtotal

RHS15=phi*0.75*fp*0.005*Area*(depth-(0.31875*(87000.0)/(87000.0+fp))*yp)/(12.0)

LHS15=(Mu)*np.ones([np.shape(l)[0]])

f=Area*(C1+C2*0.0015)

ax6.set_title('0.15% reinforcement ')

ax6.plot(l,RHS10,color='g',label='tension stress')

ax6.plot(l,LHS10,color='r',label='fracture',ls='dashed')

ax6.plot(l,RHS11,color='b',label='compression stress')

ax6.plot(l,LHS11,color='r',label='yield',ls='dashed')

ax6.set_ylabel('Stress(psi)')

ax6.set_ylim(-10000,5000)

# Put a legend to the right of the current axis

ax6.legend(loc='center left', bbox_to_anchor=(1, 0.5))

ax6.ticklabel_format(scilimits=(-1, 1))


ax7.plot(l,RHS12,color='b',label='compression stress')

ax7.plot(l,LHS12,color='r',label='yield',ls='dashed')

ax7.plot(l,RHS13,color='g',label='tension stress')

ax7.plot(l,LHS13,color='r',label='fracture',ls='dashed')

ax7.set_ylabel(' Stress (psi)')
```

```
ax7.set_ylim(-5000,10000)

# Put a legend to the right of the current axis

ax7.legend(loc='center left', bbox_to_anchor=(1, 0.5))

ax8.plot(l,RHS14,color='b',label='depth')

ax8.plot(l,LHS14,color='r',label='depthMax',ls='dashed')

ax8.set_ylabel('Depth(in)')

# Put a legend to the right of the current axis

ax8.legend(loc='center left', bbox_to_anchor=(1, 0.5))

ax9.plot(l,RHS15,color='b',label='Moment Capacity')

ax9.plot(l,LHS15,color='r',label='Limit Moment',ls='dashed')

ax9.set_ylabel('Moment (lb-ft)')

# Put a legend to the right of the current axis

ax9.legend(loc='center left', bbox_to_anchor=(1, 0.5))

ax9.set_ylim(0.5*Mu,3*Mu)

ax10.plot(l,f,color='b',label='Objective function')

ax10.set_ylabel('Objective')

ax10.legend()

# Put a legend to the right of the current axis

ax10.legend(loc='center left', bbox_to_anchor=(1, 0.5))

plt.show()
```

Sample Results of the Feasible Region Method

Sample Output at Support

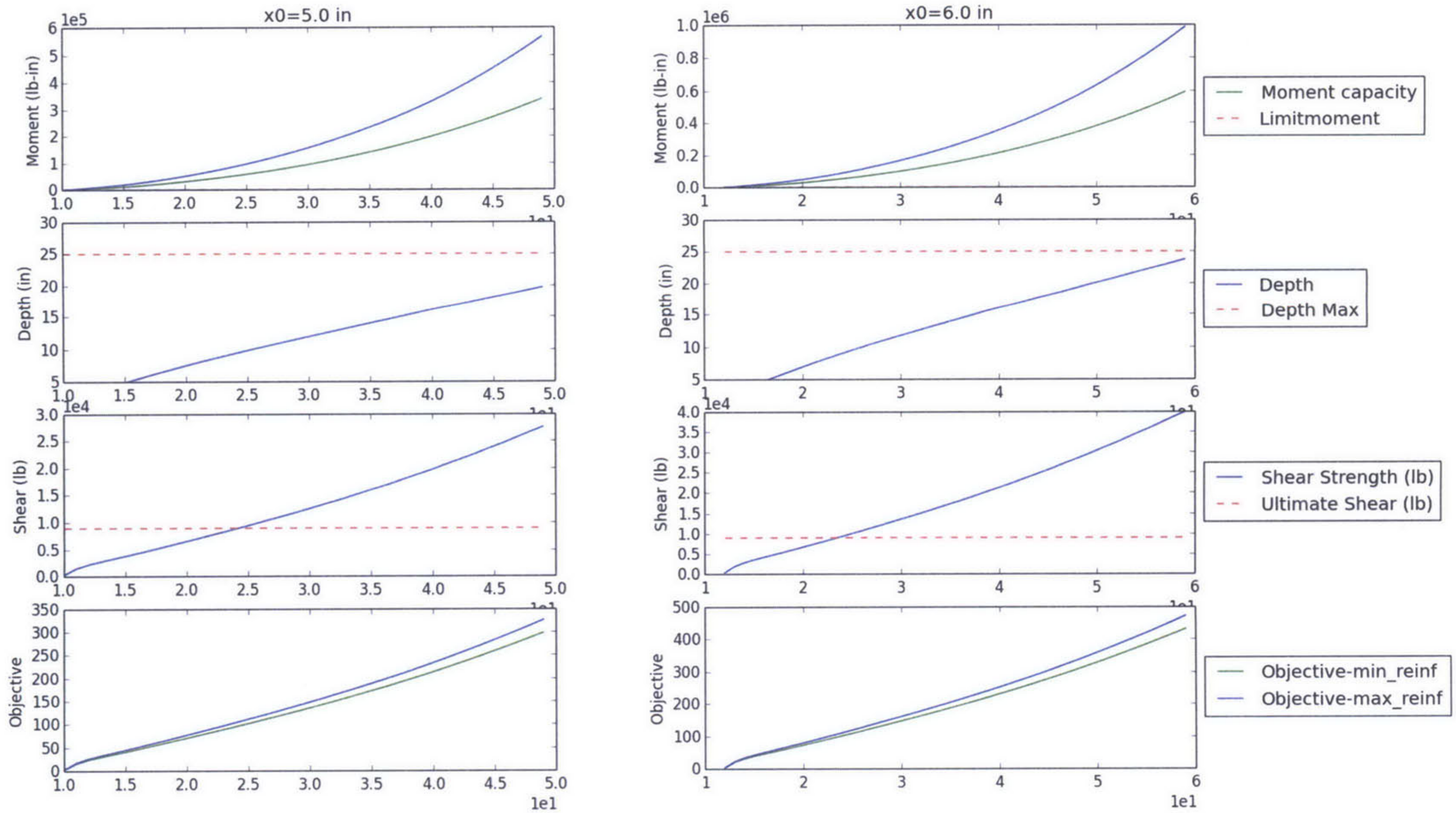

Figure A.3. The feasible region results for x0= 5.0 in and 6.0 with the shear and moment constraints at the supports

Sample Output at Quarter Span

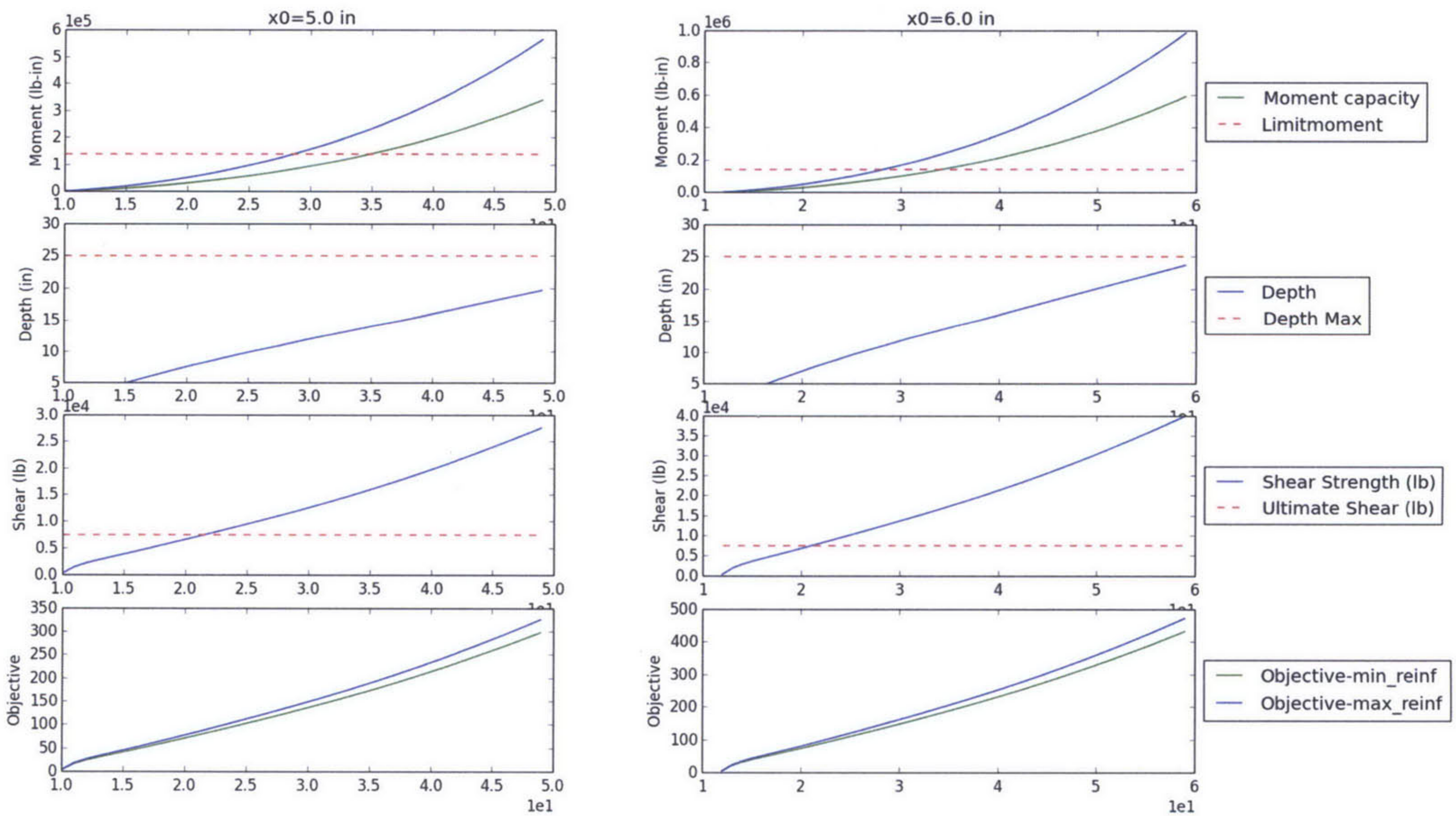

Figure A.4. The feasible region results for x0= 5.0 in and 6.0 with the shear and moment constraints at quarter span

Sample Output at Mid-span

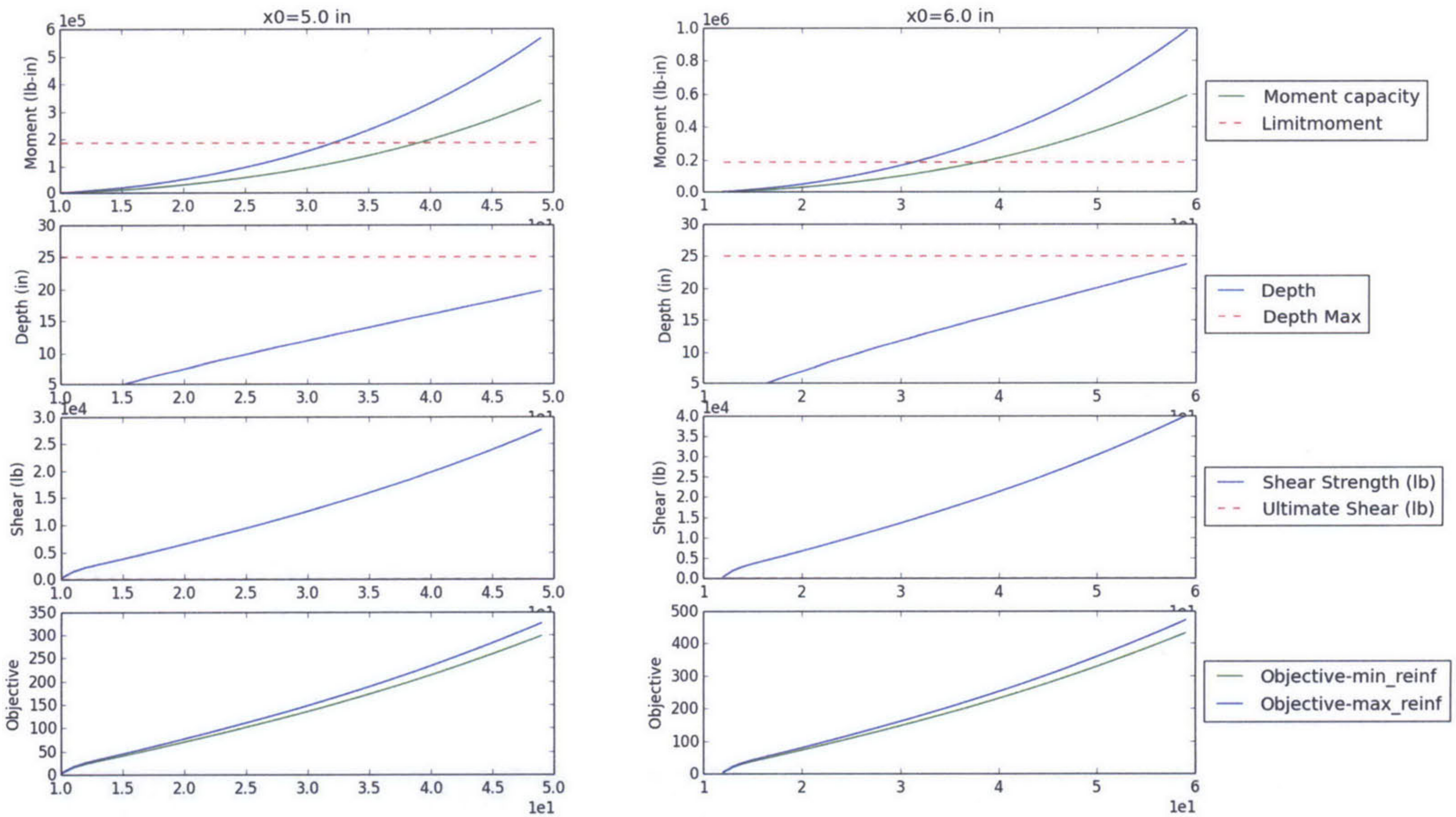

Figure A.5. The feasible region results for x0= 5.0 in and 6.0 with the shear and moment constraints at mid-span.

Figure A.6. 3D visualization using Rhinoceros 3D© of the fabric formed beam shown in green with the equivalent rectangular beam shown by the red box.