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Modeling Water, Climate, Agriculture, and the Economy

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CHAPTER 5

Modeling Water, Climate, Agriculture, and the Economy

This chapter describes the two models used in this integrated modeling framework designed for this study. The first model is a hydro-economic optimization model that takes a variety of inputs (for example, agronomic information, irrigation system data, and water inflows) to generate the optimal crop production across the provinces (subject to a variety of physical and political constraints) in the Indus Basin for every month of the year. This simulation of the hydro-economy of the Indus Basin Irrigation System (IBIS) will respond to changing climate factors and allow the evaluation of different possible investments in terms of their economic contribution. The second model is a computable general equilibrium (CGE) model for the Pakistan macro-economy. This integration helps to better illuminate how changes in climate may impact the macro-economy and different household groups through the agriculture sector. This chapter describes how these two models are integrated and are used to examine investment scenarios in chapter 6.
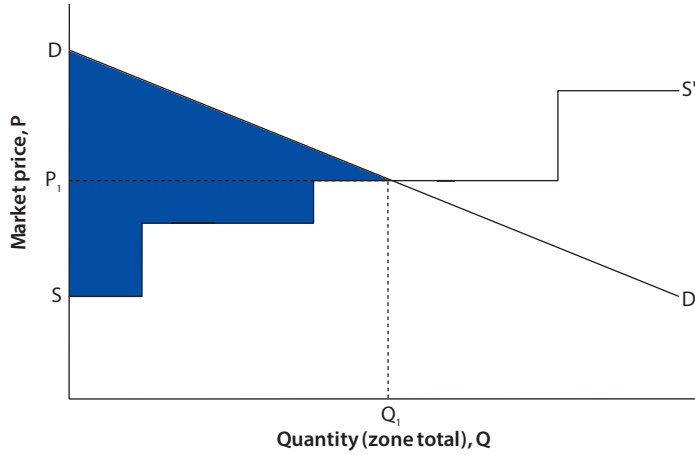
Section 1: Indus Basin Model Revised (IBMR-2012)

This section describes the basic design and the latest modifications and updates to the Indus Basin Model Revised (IBMR-2012). This model was originally developed in Ahmad, Brooke, and Kutcher (1990) and Ahmad and Kutcher (1992) for the purposes of investment planning for the Water and Power Development Authority (WAPDA). Details of this model can be found in these documents.

Economic Objective

The IBMR primarily covers the provinces of Punjab and Sindh. Only a small percentage of the total command area of the IBIS is in Balochistan and North-West Frontier Province (NWFP). Each province contains one or more agro-climatic zones (ACZs), which are further subdivided based on the cropping pattern, land characteristics, and climatic condition. Twelve ACZs are currently

Figure 5.1 The Zonal Supply-Demand Relationship in IBMR-2012

used in the IBMR (see descriptions in appendix A). The overall objective function of the IBMR is to maximize the consumer and producer surplus (CPS) for the entire IBIS. Figure 5.1 shows a zonal supply-demand relationship for an individual crop commodity.

The supply function SS' is built from the embedded farm production models in the overall optimization model. The step-wise nature of the supply curve reflects different efficient production technologies, groundwater types, and water stresses (for example, semi-mechanized production using fresh groundwater). The downward sloping linear demand function DD' is constructed using data on the baseline (observed) equilibrium quantity, price, and estimated price elasticity (from Ahmad, Brooke, and Kutcher 1990). The linear format for the demand function is defined as:

$$P = a + bQ$$

Given observed P_o and Q_o and elasticity (η)

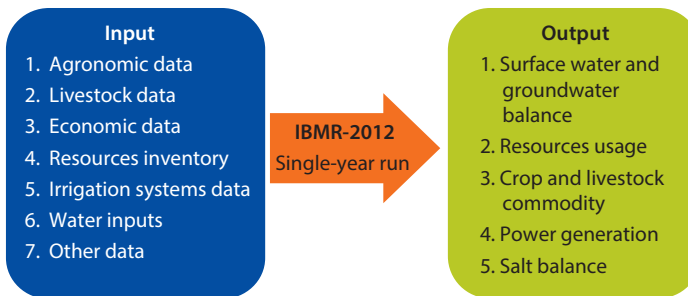$$\eta = dQ\, dP \times P/Q$$

We can solve for the slope (b)

$$b = dP/dQ = (P_o/Q_o)/\eta$$

and the intercept (a)

$$a = P_o - bQ_o$$

The shaded area in figure 5.1 represents the CPS. The objective function of IBMR is to maximize this area. The CPS is a nonlinear function, thus

Figure 5.2 Conceptual Diagram of IBMR

| Input		Output
1. Agronomic data 2. Livestock data 3. Economic data 4. Resources inventory 5. Irrigation systems data 6. Water inputs 7. Other data	**IBMR-2012** Single-year run	1. Surface water and groundwater balance 2. Resources usage 3. Crop and livestock commodity 4. Power generation 5. Salt balance

Note: IBMR-2012 = Indus Basin Model Revised.

the IBMR uses a piecewise linear programming approach to solve it. Although the model prices may fluctuate between zero and the intercept of the demand curve, this is not likely to happen in reality, thus prices are given upper and lower bounds. It is assumed that outside of these bounds inter-zonal trade will exist. However, the model does not actually simulate trade. The IBMR also does not consider international trade explicitly, but it does account for the prices of international exports and imports and adjusts production accordingly.

Figure 5.2 is a conceptual diagram that explains the modeling process of IBMR. With various input datasets, using month as the modeling time step, the results from IBMR will maximize the single-year CPS and generate outputs such as (1) the monthly surface water and groundwater balance in the Indus Basin, (2) resource usage for each production activity, and (3) monthly production of crop and livestock commodities. Side calculations are used to determine the monthly power generation from reservoirs and the salt balances from the optimized system.

The IBMR has a hierarchal spatial structure for the optimization process. A node-link system is used to represent the entire river and canal network. This system provides surface water supply to each ACZ and simulates the agricultural production and consumption at the ACZ level. In each ACZ, the culturable command area (CCA) is defined based on the existing canal diversions. The hierarchal structure of the IBMR and the zone definitions are summarized in appendix A (see table A.1). The maps of the IBMR ACZ and CCA areas are shown in maps 5.1 and 5.2.

Water Balance

The basic water balance unit in IBMR is the ACZ. The model uses the network theory concepts of nodes and arcs to simulate the flows throughout the system of Indus rivers and link canals. Figure 5.3 shows the complete node-link system map for the Indus River Basin. At each node, a water flow decision is made and the water balance for each month is calculated. The IBMR has 47 nodes that represent reservoirs, inflow stations, barrages, confluences of rivers, and

Map 5.1 Indus River and IBMR Agro-Climatic Zones in Pakistan
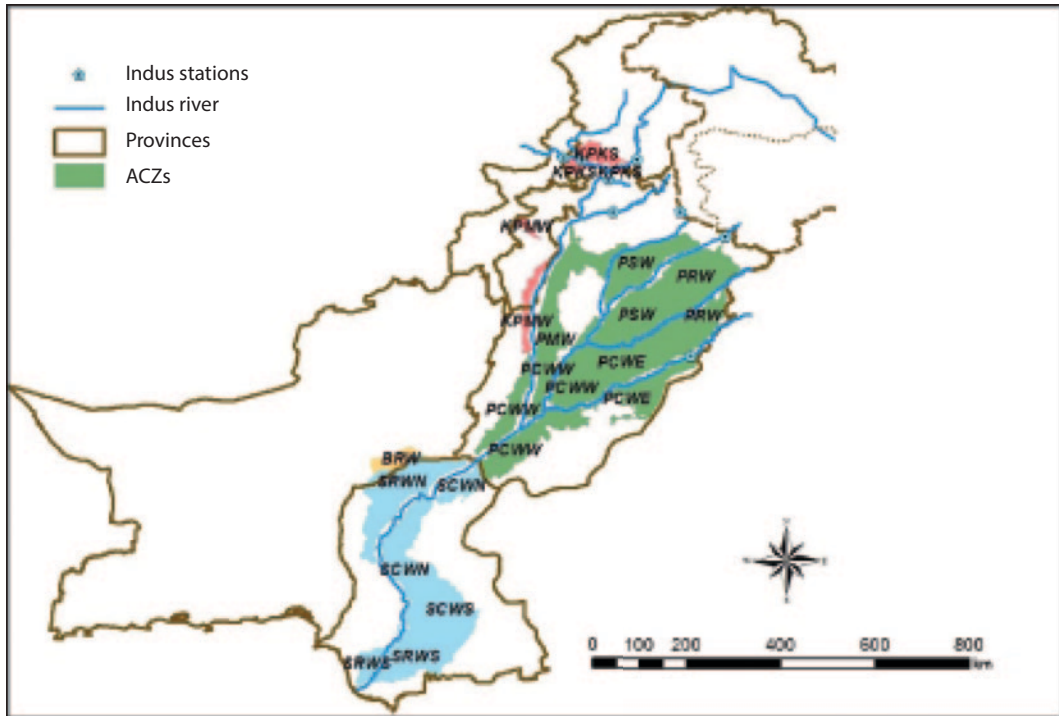
Note: ACZ = agro-climatic zones, IBMR = Indus Basin Model Revised. Details of each ACZ are given appendix A.

the terminus of the Arabian Sea. Forty-nine sinks (terminal nodes) represent diversions to irrigation canals. Finally, 110 arcs represent river reaches and link canals between nodes. Flows along these river reaches are simulated with losses and gains from river bank storage. The IBMR also considers the efficiency of manmade surface channels, such as canals and watercourses for irrigation purposes, which are drawn from river reaches. Figure 5.4 provides a complete sketch of the various losses included in the surface water balance. The stream recharge to groundwater is computed as river seepage and treated as a loss from surface water. The canal water diversion efficiency and watercourse diversion efficiency are considered losses from surface water and as additions to groundwater.

In the IBMR the residual moisture in the root zone is explicitly modeled and represents a potential source of water for crops. That is, the root zone water balance is also the crop water requirement water balance. Thus, crop water needs are met from precipitation, canals, groundwater wells, and the moisture in the root zone (also known as "sub-irrigation"). An evaporation parameter in the model is used to define the sub-irrigation water available to plants. The IBMR assumes that 60 percent of the evaporation from groundwater can be absorbed by crops. Figure 5.5 illustrates this water balance and demonstrates how surface and groundwater interact.

Map 5.2 Indus River and IBMR Canal Command Areas in Pakistan
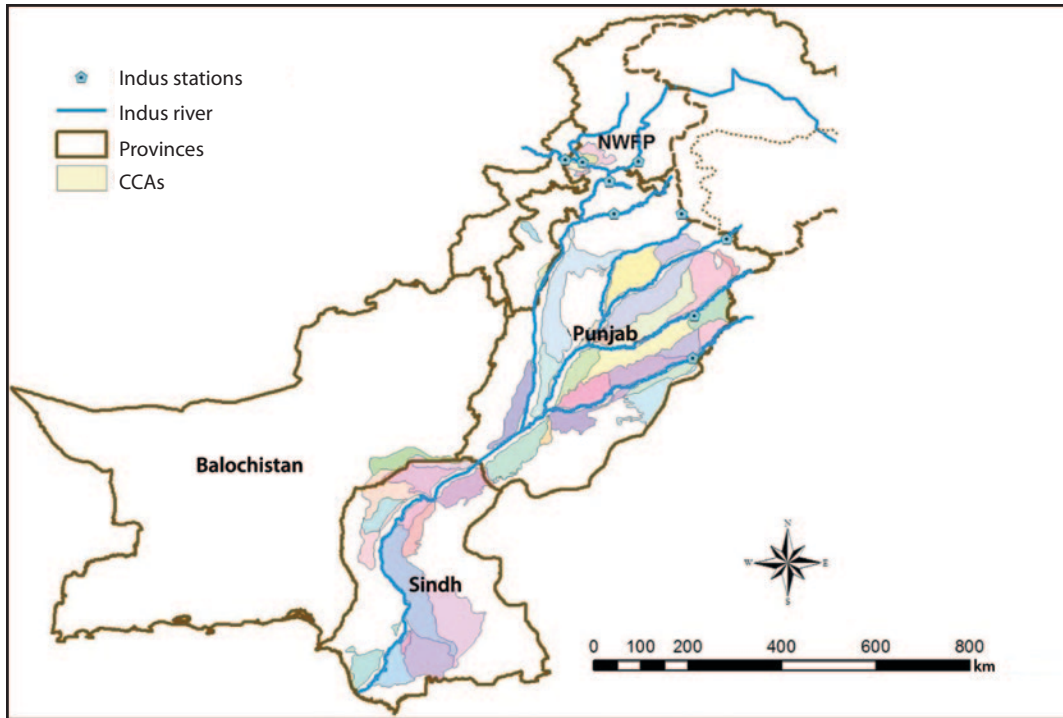
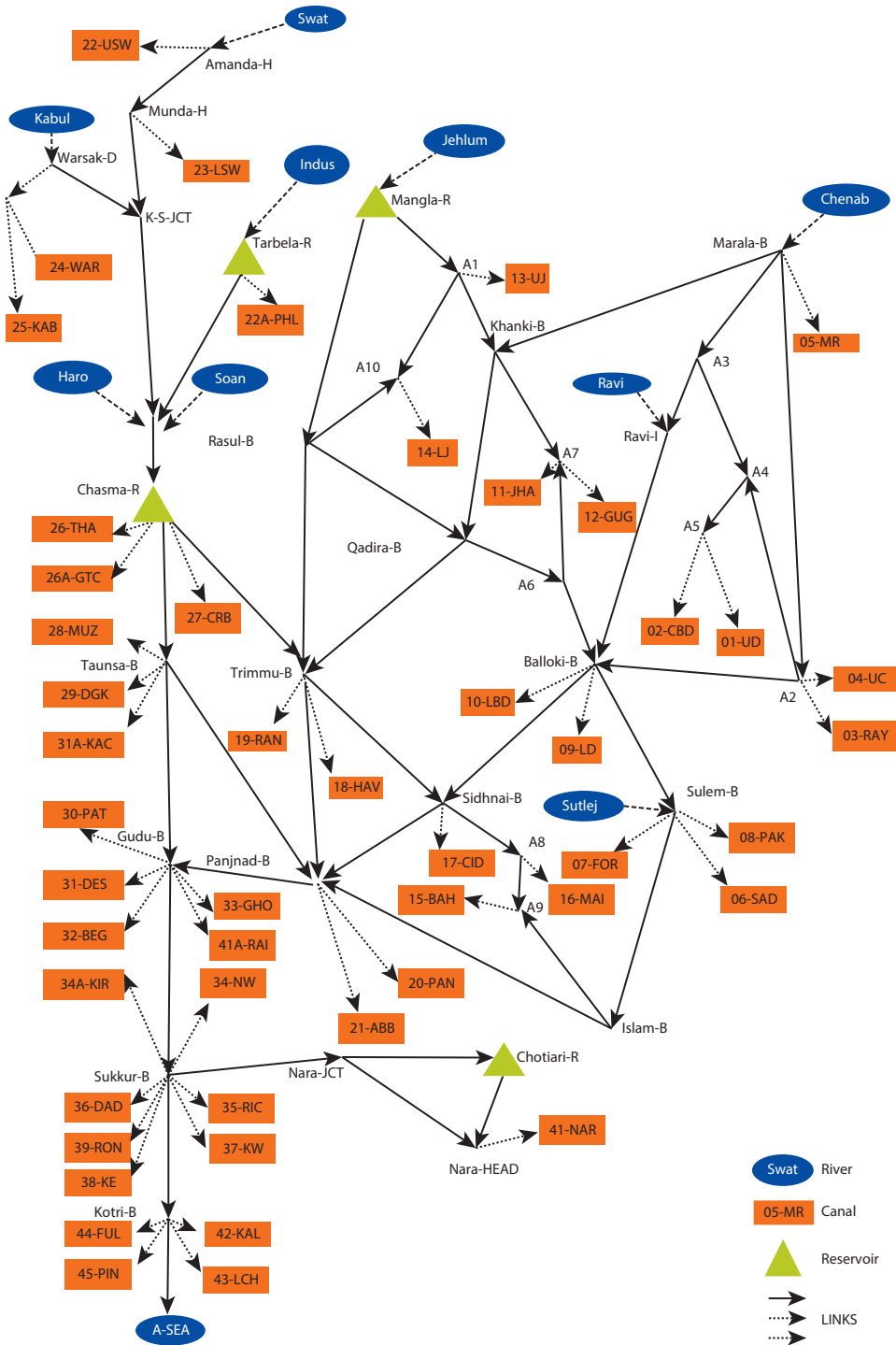*Note:* IBMR = Indus Basin Model Revised.

Input Data, Equations, Constraints, and Output Data

The input data for the IBMR has been refined over the years by WAPDA and various researchers (Ahmad, Brooke, and Kutcher 1990). Appendix B provides a discussion of the data used to update the model to a 2008 baseline. The input data of IBMR can be categorized as (1) agronomic and livestock data, (2) economic data, (3) resources inventory, and (4) irrigation systems and water data.

Agronomic and Livestock Data

The required agronomic inputs include crop growing period, labor, crop water needs, fertilizer use, draft power requirements, and crop yield and by-products. There are 14 crops in the current version of the IBMR: basmati rice, irrigated rice, cotton, rabi season fodder, gram, maize, mustard and rapeseed, Kharif season fodder, sugarcane, wheat, orchard, potatoes, onions, and chilies. Information on the water requirements for each crop by month and the types of water application (for example, standard, stressed) is also specified. A different future crop mix is not considered in this model. Livestock data include the labor and the feed requirements of each animal. Conversion factors are used to determine the by-products from these animals: meat and milk. The three types of livestock in the IBMR are bullocks, cows, and buffalo.

Figure 5.3 The Node-Link System Map of IBMR

Note: IBMR = Indus Basin Model Revised. Solid line = streamflow. Bold dotted line = stream inflow. Thin dotted line = canal diversions.

Figure 5.4 Surface-Water Balance in IBMR
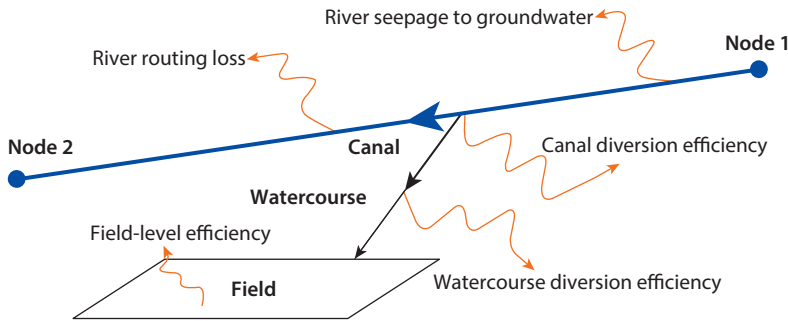
Note: IBMR = Indus Basin Model Revised.

Figure 5.5 Water Balance in IBMR
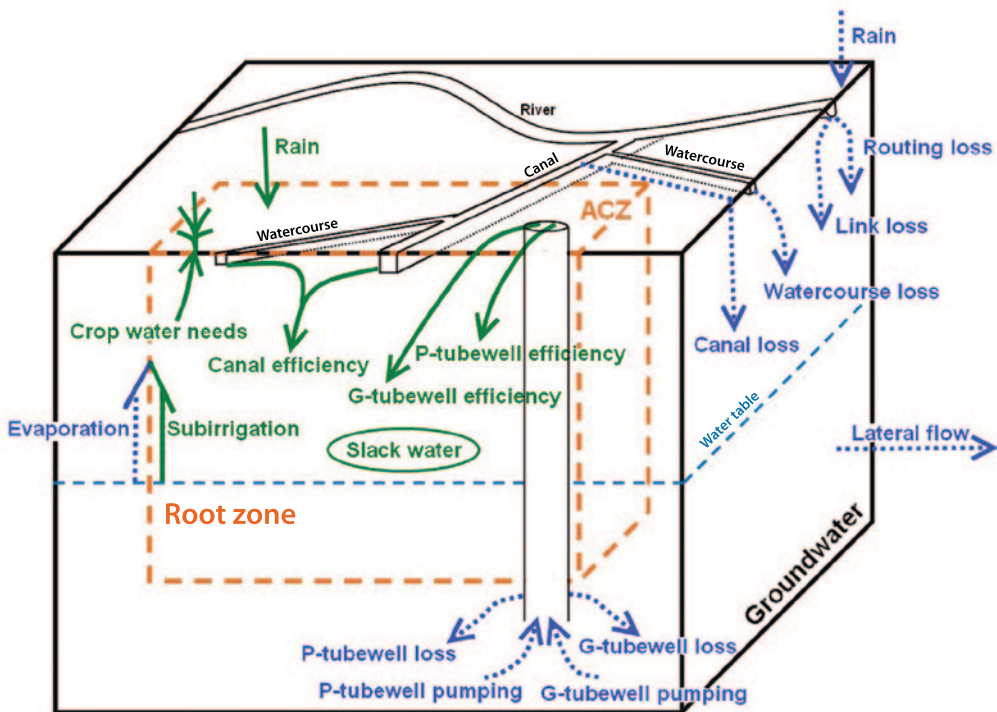
Note: IBMR = Indus Basin Model Revised. The solid lines indicate the root zone water balance components used to supply the crop. The dashed lines represent the groundwater balance components tracked during the simulation runs. All water balance calculations are at the agro-climatic zone (ACZ) scale.

Economic Data

Economic data include estimated demand for each crop and livestock product, market prices for all commodities, fixed cost for meat and orchards, and price elasticities for different commodities. Price elasticities are used from an earlier version of the IBMR. On-farm consumption of grown agriculture crops is included in the model. In the current version of IBMR, consumption crops

are identified (basmati, irrigated rice (irri), gram, maize, mustard and rapeseed, sugarcane, wheat, onion, potatoes and chilies), and the on-farm consumption is calculated as proportions of total production based on earlier farm surveys. Depending on the crop, the percentage of total output consumed varies from about 10–50 percent. Once the ACZ production meets the household consumption requirement, the remaining production is available to the market. Therefore, the demand in IBMR is the residual of production less on-farm consumption and is used to fix the quantity axis of the demand function in figure 5.1.

Resources Inventory

Input resources include agricultural workers, tractors, private tubewells, households, animals, and total available irrigated area (cropped land). The farm population is used to compute the labor availability. The IBMR was updated to reflect power requirements (for land preparation) from tractors instead of animals. Almost all power requirements are provided by tractor in the Indus Basin today.

Irrigation Systems and Water Data

When modeling the irrigation system, the basic unit of area is the canal command. All the data availability on these commands must be aggregated to the level of the agricultural model (that is, ACZ). In each ACZ, the cropping pattern and associated technologies are defined and assumed consistent. An ACZ may cut across canal command areas and need not be geographically contiguous; therefore, sub-areas are defined. Fresh and saline groundwater areas are defined and are treated separately; tubewell pumping is allowed in fresh but not in saline areas. The physical characteristics of the canal system are also given, including the culturable command area, canal capacity at the head, canal efficiency, watercourse command efficiencies, and field efficiencies. Expansion of the irrigated area into new areas is not included in this model.

Table 5.1 is the summary of current efficiency used in the model for different provinces. In general, canals have higher efficiency than watercourse channels, since most canals are constructed using impervious concrete. Thus, the average system-wide efficiency is about 35 percent. Since canal and watercourse efficiency is a critical factor related to how much water can be diverted to the field, an efficiency improvement scenario will be discussed in a later section to evaluate the system-wide impacts of efficiency improvement.

Table 5.1 Typical Canal, Watercourse, and Field Efficiency for Different Provinces
percent

Province	Canal	Watercourse	Field
NWFP	0.76	0.59	0.89
Punjab	0.74	0.54	0.87
Sindh	0.80	0.52	0.85
Baluchistan	0.78	0.61	0.83
Indus system-wide	0.76	0.55	0.86

Note: NWFP = North-West Frontier Province.

Water input data include the surface water inflow, historical canal diversions, rainfall, evaporation and sub-irrigation and public tubewell pumping. Data (50-year record) from the nine inflow tributaries into the model (Indus, Chenab, Jhelum, Harro, Kabul, Ravi, Soan, Sutlej, and Swat) are used. The subsequent analysis uses different exceedance probabilities of inflow to assess the system-wide impacts and sensitivity to inflow. The baseline run uses the 50 percent exceedance probability, which equals 132 million acre-feet (MAF) annually available to the IBIS.

The total live storage in the system is about 11.5 MAF currently. Four reservoirs are used in the current model structure: Mangla, Tarbela, Chashma, and Chotiari. The Mangla dam is located on the Jhelum River. The current live capacity has declined to 4.6 MAF from the designed 5.3 MAF due to sedimentation. Tarbela dam on the Indus River is the largest earth-filled dam in the world. Completed in 1974, it was designed to store water from the Indus River for irrigation, flood control, and hydroelectric power generation. The designed storage is 11.3 MAF and the existing live storage is 7.3 MAF. The sedimentation rate is lower than expected, so the lifespan is re-estimated as 85 years to 2060. The Chashma barrage is located on the Indus River. Unlike Mangla and Tarbela, it is not primary used for irrigation but for power generation. The installed capacity of the power station is 184 megawatts (MW), comprising eight bulb-type turbine units of 23 MW capacity each. Chotiari Reservoir is an off-stream reservoir on the Nara and Lower Indus Canal systems. It was built to store water and irrigate the Thar Desert. Both Chashma and Chotiari have live capacity of less than 1 MAF.

Equations and Constraints

In total, 26 General Algebraic Modeling System (GAMS) equations are used in the IBMR to optimize the complex process related to water allocation and economics activities. These equations are categorized into six classes: (1) objective function, (2) economic equations, (3) water balance equations, (4) canal equations, (5) crop equations, and (6) livestock equations. This section will highlight some of the key equations and constraints. The detailed discussion of these equations can be found in Ahmad, Brooke, and Kutcher (1990).

Objective Function

The objective function of the IBMR is to maximize the CPS for the entire basin as described in figure 5.1 and given in equation (5.1). The objective function is only for the agriculture sector and does not include hydropower production or municipal and industrial water consumption. The primary decision variables are production across the agriculture commodity, groundwater type (saline or fresh), and ACZs.

$$
\begin{aligned}
CPS = \Sigma_Z\, \Sigma_G\, \Sigma_C\, Price_{Z,C} \times Production_{Z,G,C} - \Sigma_Z\, \Sigma_G\, Cost_{Z,G} \\
- \Sigma_Z\, \Sigma_C\, Import_{Z,C} - ImaginaryWater + \Sigma_Z\, \Sigma_C\, Export_{Z,C} \quad (5.1) \\
+ \Sigma_M\, \Sigma_N\, WaterValue_{M,N}
\end{aligned}
$$

where Z is index for ACZ, G is index for groundwater type, C is index for crop, M is index for month, and N is index for node or reservoir. *Price × Production* is the total benefit from crop production and livestock production. *Cost* is the total cost for production, *Import* is the total cost for importing crops, *Export* is the total benefit for exporting crops, and *WaterValue* is the value of water that flows to the sea (and not utilized in the system) or stored in reservoirs. This value can be set to reflect the economic benefit of maintaining environmental flows to the sea.

The *ImaginaryWater* parameter in the objective function represents the penalty when there is insufficient water in the network flow model. In reality, production would not cease if there is a shortage of irrigation water. Thus, to prevent infeasible solutions (that is, hard constraints on the water demands) this variable acts as a penalty against the objective value. Therefore, the cost of production will be higher under these circumstances. When this variable is non-zero, it indicates that full irrigation demands are not being met.

Cost Function

The cost function (equation 5.2) contains all the cost for farm crop and livestock production in each ACZ as shown:

$$
\begin{aligned}
Cost_{Z,G} = \sum_Z \sum_C \sum_S \sum_W & (FERT_{Z,C,S,M} + MISCCT_{Z,C,S,M} + SEEDP_{Z,C,S,M} \\
& + TW_{Z,C,S,M} + TRACTOR_{Z,C,S,M}) + \sum_Z \sum_G \sum_A Animal_{Z,G,A} \\
& + \sum_Z \sum_{SEA} PP_{Z,SEA} + \sum_Z \sum_G \sum_M Labor_{Z,G,M}
\end{aligned}
\tag{5.2}
$$

where S is the index for cropping sequence (for example, standard, late, or early planting); W is the index for water stress (for example, standard, light, or heavy stress); A is the index for different animals (cow, bullock, and buffalo); and SEA is the index for season (rabi and kharf). *FERT* is the cost for fertilizer, *MISCCT* is the miscellaneous cost like insecticides and herbicides, *SEEDP* is the cost for seed, *TW* is the energy cost for groundwater pumping, *TRACTOR* is the cost for operating tractors, *Animal* is the fixed cost for livestock, *PP* is the cost for purchased protein concentrates for animals, and *Labor* is the cost for hiring labor.

Surface Water Balance Equation

Water balances in the river network and root zone are the essential mass balances in the IBMR. The surface water balance is related to the river routing process in the IBMR. Equation (5.3) describes the entire river network monthly water balance at each node:

$$
\begin{aligned}
& \sum_I Inflow_I^M + \sum_N RIVERD_N \times TRIB_N^M + \sum_N RIVERC_N \times TRIB_N^{M-1} \\
& \quad + \sum_N RIVERB_N \times F_N^M + \sum_N RIVERC_N \times F_N^{M-1} + \sum_N RCONT_N^{M-1} \\
& \quad - RCONT_N^M + Prec_N^M + Evap_N^M - \sum_N CANALDIV_N^M + ImaginayWater_N^M = 0
\end{aligned}
$$

$$\tag{5.3}$$

where I is the index for inflow node; *Inflow* is the streamflow; *RIVERD* is the routing coefficient for tributaries; *TRIB* is the tributaries' flow; *RIVERC* is the routing coefficient for previous month; *RIVERB* is the routing coefficient for mainstream; F is the mainstream's flow; *RCONT* is the monthly reservoir storage; *Prec* is the rainfall at reservoir; *EVAP* is the evaporation loss at reservoir; *CANALDIV* is the canal diversion; and *ImaginaryWater* is the imaginary surface water needed at nodes.

The root zone water balance at each ACZ in the IBMR is the relationship between the total available water in the root zone and the total crop water requirements, as shown in figure 5.5. The following equation (5.4) describes this balance.

$$Max[(WNR^M_{Z,G,C,S,W} - SUBIRRI^M_{Z,G} \times LAND^M_{Z,G,C,S,W}), 0] \times X^M_{Z,G,C,S,W}$$

$$\leq TW^M_{Z,G} + GWT^M_{Z,G} + WDIVRZ^M_{Z,G} + ImaginaryRWater_{Z,G} \qquad (5.4)$$

where *WNR* is the water requirement from crops, *SUBIRRI* is the sub-irrigation, X is the cropped area, *TW* is the total private tubewell pumping, *GWT* is the public tubewell pumping, *WDIVRZ* is the surface water diversion and *ImaginaryRWater* is the imaginary water at the root zone.

Major Constraints

Canal capacity: The physical canal capacity (of the existing IBIS infrastructure) is used as the upper boundary of canal water diversion in the model. That is, expansion of the irrigated area (beyond the cultivable command) is not considered in this work. Note that this model was previously used to examine the cost-benefit of new irrigated areas as part of WAPDA's Water Sector Investment Planning Study (WAPDA 1990).

Provincial Historical Diversion Accord: Maintaining the 1991 Provincial Accord (described in chapter 2) is another constraint in the model. This water sharing agreement specifies how much water is to be diverted to each province (table 2.1). In order to consider this Accord in the IBMR, the actual monthly canal-wise diversion data from 1991 to 2000 are averaged and utilized as the constraint itself ("DIVACRD"). That is, it is assumed that the average historical diversions to each canal command (intra-provincial allocations) in the aggregate meets the inter-provincial requirements. In an earlier version of the model, this DIVACRD was set as an equality constraint. However, this eliminates the possibility for intraprovincial optimization. In this study, a 20 percent deviation from the monthly canal diversion was allowed, that is, each canal command diversion can be 0.8–1.2 percent of this historical value (while maintaining the physical constraints in the system and the aggregate provincial allocations).

Reservoir operation rule: No complex operation rules have been applied to these reservoirs. Only the upper and lower boundaries of reservoir storage have been set up. This is acceptable given that the model operates for a single year.

Output Data

Every item in the objective function is an output: revenue from production, farm cost, imports, exports, total welfare, value of water in the reservoir, and the flow to the sea. The imaginary parameters can also be checked to see if their values are non-zero (thus, indicating a shortfall in demanded irrigation waters).

The major output that will be examined in this study is the cropped areas for different crops, ACZs, and months. The model also provides detailed information for every combination of cropped area. For example, production can be summed across ACZs or provinces or from monthly to seasonal and annual time scales. The results are also given in each ACZ for areas that are sourced from different groundwater types (fresh and saline). Resources used, such as labor and private tubewells, can also be calculated for each ACZ. The power generation from reservoirs is a by-product from the model and calculated after an optimized solution is found. Finally, the surface water and groundwater balance is determined from the calculations of the monthly canal diversion; monthly node-to-node flow balance; reservoir inflow, outflow, and storage; surface water to the root zone for each ACZ; and groundwater depth and recharge.

Baseline: Year 2008/09

This section presents the baseline performance of the IBMR-2012. All scenario simulations will be in relation to this baseline. Table 5.2 and figure 5.6 show the major outputs from the baseline model. The basin system-wide objective value is PRs 2,850 billion (US$35.62 billion[1]). This is consistent with the agriculture gross domestic product (GDP) for the entire country of US$34.8 billion (World Development Indicators). Punjab has the largest cropped area, followed by Sindh. Surface and groundwater use across the provinces follow closely the DIVACRD constraint. Most groundwater usage is in Punjab. According to NTDC (2010), the annual generated power in 2009 at Tarbela, Mangla, and Chashma was 13.95, 4.79, and 1.09 billion kilowatt-hours (BKWH), respectively. The total power generation is 19.83 BKWH and is consistent with the modeled results.

Table 5.3 shows the revenues across different farm commodities, which vary by province. Basmati, cotton, sugarcane, and wheat generate the most revenue

Table 5.2 Major IBMR Outcome under Baseline Condition

	Objective value (PRs, millions)	Commodity total revenue (PRs, millions)	On-farm costs (PRs, millions)	Cropped area (1,000 acres)	Crop production (1,000 tons)	Power generation (BKWH)
Indus	2,850,099	3,162,371	601,369	48,491	95,138	19.59
Punjab	n.a.	2,430,117	440,965	34,734	65,374	n.a.
Sindh	n.a.	628,036	132,823	11,057	24,905	n.a.
Other		104,218	27,582	2,701	4,859	n.a.

Note: BKWH = billion kilowatt-hours, n.a. = not applicable. "Other" includes Balochistan and North-West Frontier Province (NWFP).

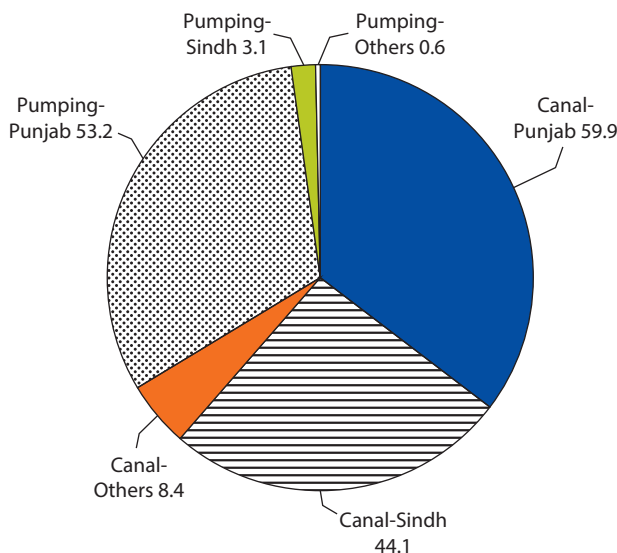**Figure 5.6 Model Irrigation Use by Source and Province**
million acre-feet

**Table 5.3 Commodity Revenue Breakdown for the Baseline Condition**
PRs, millions

	Pakistan	Punjab	Sindh	Others
Basmati	749,694	749,694	0	0
Irrigated rice	170,466	27,733	108,530	34,204
Cotton	674,609	552,092	122,190	327
Gram	36,101	20,860	12,810	2,431
Maize	70,457	44,542	692	25,223
Mus+rap	2,574	1,923	7	645
Sc-mill	245,950	156,249	78,764	10,937
Wheat	418,049	377,301	35,080	5,669
Potatoes	111,421	108,316	682	2,424
Onions	56,891	18,360	37,187	1,344
Chili	35,674	17,685	17,962	27
Cow-milk	144,051	76,608	64,160	3,282
Buff-milk	446,434	278,755	149,973	17,706
Total	3,162,371	2,430,117	628,036	104,218

in Punjab. Irrigated rice and cotton revenues are highest in Sindh. These baseline numbers are consistent with actual agriculture census data for 2008 (see appendix B). Table 5.4 breaks down the on-farm cost for different categories. In general, the primary production costs are labor hired, tractor, and fertilizer use. The following climate change impact and investment analysis will be compared against these results.

Table 5.4 Farm Cost Breakdown under Baseline Condition
PRs, millions

	Pakistan	Punjab	Sindh	Others
Seed	38,434	27,035	9,189	2,210
Labor	205,834	145,040	51,970	8,824
Water	47,343	37,711	8,756	876
Protein	2,488	1,988	401	98
Fertilizer	120,082	79,972	33,261	6,849
P-well	45,930	42,942	2,530	458
Livestock	735	432	273	30
Tractor	140,524	105,844	26,442	8,238

Figure 5.7 CGE Conceptual Diagram
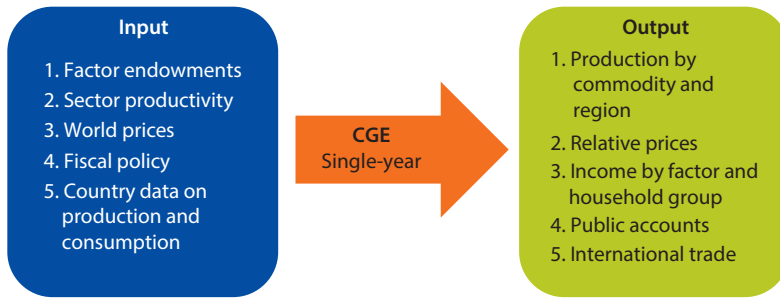
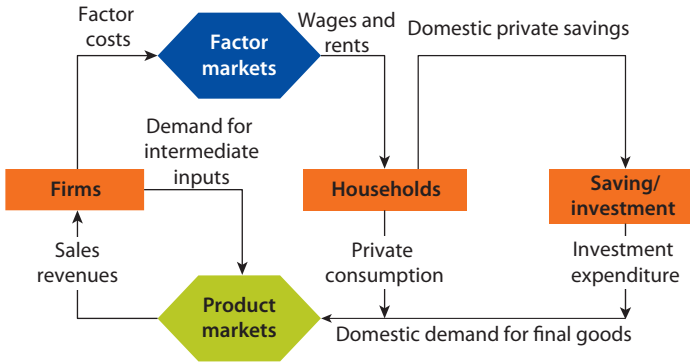*Note:* CGE = computable general equilibrium.

Section 2: Computable General Equilibrium Model

This section provides a brief description of the CGE model. Detailed information can be found at Diao et al. (2011). The inputs for the CGE model include factor endowments (amount of labor, land, and capital); sector-specific productivity; world prices; fiscal policy (tax rates, government expenditure); and updated country-specific data on production (value-added and intermediate use by sector) and consumption (value of consumption for each commodity and household group). The outputs of the CGE include production by commodity and region; relative prices; income by factor and household group; public accounts (for example, public deficit); and international trade (exports and imports by commodity). Figure 5.7 is a conceptual diagram that explains the modeling process of the CGE.

CGE Model Structure

In essence, the CGE model takes into account the interaction between producers and consumers in the economy. The model tracks the selling of goods from firms to households and to other firms, the selling of factor services from households to firms, and the generation of savings that finance the investment in the economy, as shown in figure 5.8. The arrows in the figure track the (explicit or implicit) payments in the countrywide economy. Firms pay wages and rents to

Figure 5.8 Flow in the CGE Model

Note: CGE = computable general equilibrium.

households, and they buy goods produced by other firms (intermediate inputs). Households use their income to buy products from firms and to save, in turn financing investment, that is, an additional demand for firms' product markets. The components of the model follow, going through the behavior of consumers and producers; the introduction of the government, investment, and the foreign sectors; the way the model equilibrates supply and demand in the product and factor markets; and its macro-economic behavior and regional disaggregation.

Consumer and Producer Behavior

Following general equilibrium theory, representative consumers (that is, households) and producers in the model are treated as individual economic agents. Household incomes are determined by the sum of factor income, public transfers, and foreign transfers. Households use their income for consumption and saving. Consumption is allocated over different goods to maximize household's utility[2] subject to their budget constraints. Producers are defined at the sector level (that is, agriculture, industry, services). Each representative producer maximizes profits, taking the prices at which they buy/hire inputs and sells their outputs as given. Following neoclassical theory, constant returns are assumed to scale in production, that is, increasing all the factors by a given percentage leads to increasing the production by the same percentage. In particular, a constant elasticity of substitution (CES) function is used to determine production:

$$X_i = \Lambda_i (\textstyle\sum_f a_{if} \cdot V_{if}^{-\rho_i})^{-1/\rho_i} \tag{5.5}$$

where X is the output quantity of sector i, Λ is a shift parameter reflecting total factor productivity (TFP), V is the quantity used of each factor f (that is, land, labor, and capital) by the firm, and α is a share parameter of factor f employed in the production of good i. The maximization of profits by the sector-specific producers provides the system of factor demand equations used in the model. For example, if the amount sold from a producer increases, it will make him try to hire more workers and acquire more necessary factors for his production.

The model captures the use of intermediate inputs in the production process. In particular, the demand for intermediates is based on fixed physical input-output coefficients.

Government and Investment Demand

The government is treated as a separate agent, with income and expenditures but without any behavioral functions. Total domestic revenues R is the summation of all individual taxes (sales taxes, income taxes, tariffs, and so on). Tax rates are exogenous so that they can be used to simulate policy changes. The government uses its revenues to purchase goods and services (that is, recurrent consumption spending), to make transfers to other actors (households, firms, non-residents) and to save (that is, finance public capital investment). The participation of the government demand in the domestic absorption of the economy is fixed in the model.

There are also no behavioral functions determining the level of investment demand for goods and services. The total value of all investment spending must equal the total amount of investible funds I in the economy. This value is split among different commodities in fixed proportion, as informed by the composition of investment in the National Accounts.

International Trade

Given observed two-way trade between countries for similar goods, we assume imperfect substitution between domestic goods and goods supplied to and from foreign markets. World prices are exogenous, reflecting the perception that the domestic economy does not have significant power to affect the world prices (small country assumption).

Equilibrium Conditions

Full employment and factor mobility across sectors is assumed for labor, and fixed sector-specific employment is assumed for land and capital. This means that when the price of a commodity increases, the producer of the commodity will increase its demand for factors, but only workers (who are mobile) will be able to move into the sector, while the other factors (which are fixed in the sectors they are in) will increase their wage in this sector. Relative prices are determined such that their supplies equal their demands. More specifically, in each commodity, the supply of the good Q equals total demand, composed of consumption by households (C_{ih}), investment (N_i), public sector demand (G_i), intermediate demand by other production sectors ($\sum_{i'}(io_{i'i} \cdot X_i)$), and net exports ($E - M$):

$$Q_i = \sum_h C_{ih} + N_i + G_i + \sum_{i'}(io_{i'i} \cdot X_i) + E - M \qquad (5.6)$$

Macro-Economic Closures

Macroeconomic balance in a CGE model is determined exogenously by a series of "closure rules." The most important of these is the current account balance. While this is a substantive research topic within macroeconomics, it is treated as

an exogenous variable within the single-country open economy CGE model. Either total savings S or total investment I, but not both, should be determined exogenously. This choice is called the "savings-investment" closure. The model is savings driven, with households saving a fixed share of their income and investment I automatically determined by the level of total available savings. Finally, recurrent consumption spending of the government G is a fixed share of domestic absorption, and public savings are endogenously determined by the model. Finally, the original consumer basket is chosen as the model's numeraire that is, the consumer price index (CPI) is fixed.

Regional Production

We disaggregate representative producers and households across sub-national regions, an extension that allows us to reflect spatial heterogeneity in geographic conditions. These are important considerations for agriculture, which depends on agro-ecological and climatic conditions, and for developing countries, where markets are often underdeveloped. Each regional producer has his own production function and associated technology coefficients, and often uses region-specific factors, such as agricultural land. Each regional producer supplies his output to a national product market, with output from each region combined into a composite national good through a CES aggregation function. Households are classified into groups that consider the region where they reside.

CGE Model Social Accounting Matrices Update

One of the main advantages of CGE models over theoretical models is their calibration to detailed empirical data. "Calibration" refers to the process of assigning values to the model's parameters and variables, typically using observed country data. Some of the assumptions that the authors made when specifying the CGE model were done to ease its calibration, since in many cases the data needed for more complex functional forms is unavailable in developing countries. For example, the reason a function is used that is based on constant income elasticities to determine consumer demand is because it requires data that can readily be obtained from household surveys (that is, expenditure shares and income elasticities). More elaborate functions often drop this constant income elasticities assumption, such as in the "Almost Ideal Demand System," but need more detailed data. Calibrating the behavior of more complicated functional forms often involves simply making more assumptions where data are unavailable. The next section describes the data sources and estimation procedures used to calibrate the CGE model.

Social Accounting Matrices

The values of almost all variables and parameters in the CGE model are drawn from a social accounting matrix (SAM; Pyatt and Round 1985; Reinert and Roland-Holst 1997). Constructing a SAM is therefore a fundamental part of developing a CGE model for a country. A SAM is an economy-wide representation of a country's economic structure. It captures all income and expenditure

flows between producers, consumers, the government, and the rest of the world (ROW) during a particular year. Table 5.5 presents the basic structure of a SAM that could be used to calibrate the core model described above. The SAM contains a number of "accounts" representing different agents in the model, including sectors (producers) and households (consumers). The rows and columns of the SAM represent incomes and payments, respectively, from one account to another. As with double-entry accounting, the SAM is a consistent economy-wide database because row and column totals must be equal. In other words, a payment from one account always becomes an income for another. The SAM therefore provides the baseline year equilibrium state for the CGE model.

A SAM is constructed in two stages. During the first stage, data from different sources are entered into each of the SAM's cells. As with the CGE model, the SAM allows for multiple sectors and households. Thus, the "sector", "product," and "household" rows and columns actually contain many subaccounts. The three main data sources for constructing a SAM are national accounts, input-output tables (or supply-use tables), and nationally representative household budget surveys. As shown in table 5.5, national accounts provide information on the composition of GDP at factor cost (that is, sectoral value-added) and by broad expenditure groups at market prices. The technical coefficients (that is, the requirements of inputs produced by other industries to produce a given commodity) in the input-output table are scaled in light of the value-added in the given commodity to estimate intermediate demands. (That is, the matrix that captures the use of goods by a sector of goods produced by other sectors and the same sector).

The SAM also disaggregates government and investment demand across products. The household survey is used to segment labor markets (that is, disaggregate labor income into different groups, such as by education). The survey also defines households' expenditure patterns and the distribution of incomes to representative household groups. Therefore, the survey data is the main determinant of differential income and distributional effects across household groups in the CGE model.

Other databases are used to complete specific cells within the SAM. Government budgets provide information on tax rates, revenues, and expenditures. Although not shown in table 5.5, government budgets (and household surveys) also determine the level and distribution of social transfers (that is, payments from government to households, like pensions and subsidies to the poor). Customs and revenue authorities provide data on imports and exports and their associated tariffs and subsidies. The balance of payments, usually compiled by a country's central bank, is used to populate the external or ROW account, including information on transfer receipts and payments and the current account balance. Finally, sectors in the SAMs are usually disaggregated across subnational regions using information on regional production and technologies from agricultural and industrial surveys. The information about regional production is a key link between the CGE and IBMR models: the CGE takes as input the ratio between production and area that come out of IBMR to update the (total factor) productivity in the CGE model, which is an exogenous parameter. Trade margins,

Table 5.5 General Structure of a Social Accounting Matrix

	Sectors	Products	Factors	Households	Government	Investment	Rest of the world	Total
Sectors		Marketed supply (PD, D)					Export demand (PE, E)[f]	
Products	Intermediate demand (io)[b]			Private consumption (C)[a, c]	Public consumption (G)[a, b, d]	Investment demand (N, ε)[a, b]		Total demand
Factors	Value-added (V, W, Z)[a, c]							Factor income
Households			Income distribution (δ)[c]				Transfers (hw)[c, e, f]	Household income (Y)
Government	Indirect tax (te)[d, f]	Indirect tax (tc, tm)[d, f]	Factor tax (tf)[d]	Income tax (ty)[c, d]			Transfers (rw)[d, e, f]	Total revenues (R)
Savings				Private savings (s)[a, c]	Public savings (FB)[a, d]		Foreign savings (FS)[e]	Total savings (S)
Rest of world		Import supply (PM, M)[f]						Total foreign payments
Total	Gross output (PP, X)	Total supply (P, Q)	Factor payments	Total household spending	Recurrent spending	Total investment (I)	Total foreign receipts	

Note: Main data sources used to populate the SAM: a. national accounts and regional production data; b. input-output tables and industrial surveys; c. household and labor force surveys; d. government budgets; e. balance of payments; and f. customs data and tax revenue authorities.

which are not shown in the table, are estimated using information on producer and consumer prices. Trade margins may also be drawn from input-output or supply-use tables.

Inevitably, inconsistencies occur between data from different sources, which lead to unequal row and column totals in the model's SAMs. Therefore, the second stage of constructing a SAM is to "balance" these totals. This reconciliation of data from disparate sources is similar to a "rebasing" of national accounts. Cross-entropy econometric techniques are used to estimate a balanced SAM (Robinson, Cattaneo, and El-Said 2001). This is a Bayesian approach that uses a cross-entropy distance measure to minimize the deviation in the balanced SAM from the unbalanced prior SAM containing the original data. Constraints such as total and sectoral GDP are imposed during the estimation procedure to reflect narrower confidence intervals around better-known control totals (for example, total GDP). The SAM and its underlying data sources provide almost all of the information needed to calibrate the CGE model. Only the behavioral elasticities remain.

Behavioral Elasticities and Other External Data

Behavioral elasticities are needed for the consumption, production, and trade functions. The demand function requires information on income elasticities and the marginal utility of income with respect to income (Frisch parameter, see Frisch 1959). Marginal budget shares (the fraction of consumption that a given household allocates to each commodity) are derived by combining the estimated income elasticities with the average budget shares drawn directly from the SAM. The income elasticities in this case are based on a set of priors given by the price elasticities of different crops from IBMR and a cross-entropy process. Trade elasticities determine how responsive producers and consumers are to changes in relative prices when deciding to supply goods to or purchase goods from foreign markets. Higher elasticities are expected when substituting between more homogenous products, such as maize and copper. Lower elasticities are expected for more differentiated product categories, such as chemicals and machinery. In Pakistan, most of the Armington elasticities (the elasticity of substitution between domestic and imported commodities) are in the range of 2–3, with the following exceptions: chemicals (0.5), cement (0.5), petroleum (0.8), and manufacturing (0.5). In most developing countries, the data needed to econometrically estimate country-specific elasticities do not exist—at least not in an appropriate form (Arndt, Robinson, and Tarp 2002). The elasticities governing factor substitution in the production rarely exist for developing countries. In the absence of reliable country-specific estimates, we assume inelastic factor substitution for most activities in the [0.75, 0.90] range.

Social Accounting Matrix for Pakistan

Building an updated SAM for Pakistan was started from Pakistan SAM 2001–02 developed by Dorosh, Niazi, and Nazli (2006). Given that the most recent available input-output table is for 1990–91, and the published national accounts

on an even earlier input-output table (1985), in preparing the 2001–02 Pakistan SAM it was necessary to construct a consistent set of accounts for production and value-added by sector based on the 1991 input-output table. These accounts then formed the base upon which factor and household accounts were disaggregated.

Four major types of accounts are distinguished in the 2001–02 Pakistan SAM: (1) activities, (2) commodities, (3) factors of production, and (4) institutions (households, government, and the rest of world), including an aggregate institutional savings-investment account, which collects all the savings in the economy and uses them to finance investment. The production accounts describe the values of commodities (goods and services) and inputs into each production activity along with payments to factors of production (land, labor, water, and capital). Commodity accounts show the components of total supply in value terms, domestic production, imports, indirect taxes and marketing margins; and total demand, intermediate input use, final consumption, investment demand, government consumption, and exports. Factor accounts describe the sources of factor income (value-added in each production activity) and how these factor payments are further distributed to the various institutions in the economy (households of different types, enterprises, government, and the ROW). Accounts for institutions record all income and expenditures of institutions, including transfers between institutions. Savings of the different institutions and investment expenditures by commodities are given in the savings-investment accounts.

Data Used to Build the 2008 Pakistan SAM
The 2008 SAM uses data mainly from the following sources:

- 2008 Macroeconomic Aggregates
- 2001–02 National Accounts (value-added for 15 sectors)
- 1990–91 Input-Output Table (97 sectors)
- 2001–02 Pakistan Integrated Household Survey (consumption disaggregation)
- 2001 Pakistan Rural Household Survey (household income disaggregation)
- 2001–02 Pakistan Economic Survey (sector/commodity data on production, prices, trade)

Structure of the 2008 Pakistan SAM
The 2001–02 Pakistan SAM will now be updated into a 2008 Pakistan SAM. (Appendix C lists the accounts of the 2008 Pakistan SAM). The SAM includes 63 activities, 36 of which are national and the remaining, regional. For the agricultural activities, returns to land and own-family labor are disaggregated by region (Punjab, Sindh, and Other Pakistan) and by size of farm: small (0–12.5 acres), medium (12.5–50 acres), and large (50 acres plus) farms (defined according to area cultivated, not land ownership). Of the 27 factors of production specified, 23 involve only agricultural production: 8 types of agricultural labor; 12 types of land, water, livestock capital, and 3 types of other agricultural capital.

This detailed treatment of rural factors and agriculture in the SAM reflects the primary objective of constructing the SAM: to better understand the relationship between agricultural performance and rural income growth in the context of imperfect rural factor markets. Fifteen of the 19 household categories are rural agricultural households, split according to amount of land cultivated (large farm, small farm, landless) and region (Sindh, Punjab, and Other Pakistan). Non-farm households, both rural and urban, are split into poor and non-poor, according to their 2000–01 per capita household expenditures, with poor households defined as those with a per capita expenditure of less than 748 PRs per month per capita (22.67 percent of urban households). Non-farm rural households, defined as rural households for which the main occupation of the head of household is not crop or livestock farming, form the last household group, accounting for 19.8 percent of total population. The structure and accounts in the 2008 Pakistan SAM can be found in appendix C.

Integration of the Two Models

As this brief description shows, the CGE-SAM modeling approach can provide a much wider range of agro-economic linkages than can be addressed with the IBMR. The IBMR and CGE as described here have not been jointly used to date, and there are interesting analytical challenges in linking them (refer to Yu et al. 2010, for a comparable study in Bangladesh). There are many potential linkages between the IBMR and CGE models. In this study, IBMR outputs are exported to the CGE model to compute a wider and more complex array of potential agro-economic impacts in Pakistan. Two of the primary outputs from the IBMR are crop production and cropped area. To evaluate the impact of climate change and the effect of adaptation investments on GDP, Ag-GDP and other economic outputs, crop production and cropped area are passed directly to the CGE model. The production and cropped area for the baseline and subsequent climate and investment scenarios are used to update the (total factor) productivity in the CGE model. All results from the CGE are relative to the 2008 baseline. The results from these analyses are presented in chapter 6. Figure 5.9 illustrates how the two models are integrated.

Model Limitations

The models used here are among the best mathematical representations available of the physical and economic responses to exogenous future climate risks. Both IBMR-2012 and CGE-2008 are single-year models. That is, a comparative statics approach is taken here in examining future scenarios instead of a dynamic approach. In IBMR-2012, a simple measure for environmental consideration is used; groundwater dynamics are limited; energy production is a by-product of the system optimization; only one dimension of food security, food self-supply, is examined; no flooding damage is considered and no detailed cost-benefit analysis of investment scenarios are conducted. In CGE-2008, only one composition

Figure 5.9 Integration of IBMR and CGE
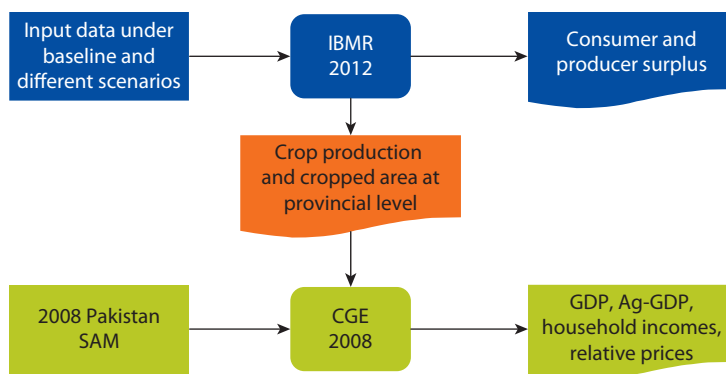
Note: CGE = computable general equilibrium, IBMR = Indus Basin Model Revised, SAM = social accounting matrix.

of Pakistan economy (current condition) is used; the world sugar, wheat, and other prices would not change due to climate change in the future. As in all modeling approaches, uncertainty exists as to parameters that may not be known with precision and functional forms that may not be fully accurate. Thus, careful sensitivity analysis and an understanding and appreciation of the limitations of these models are required. Further collection and analysis of critical input and output observations (for example, snow and ice data) will enhance this integrated framework methodology and future climate impact assessments.

Notes

1. US$1 = PRs 80 (2009).
2. In particular, through a Stone-Geary utility function.

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