Atomic Physics II (8.422) Spring 2005
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Problem Set 8
Due Friday April 15

THE DRESSED ATOM

We will derive and examine the dressed states of the atom + field system. We use the Hamiltonian

$$H = \frac{\hbar}{2}\begin{pmatrix} \omega_o & \Omega_1 e^{-i\omega_L t} \\ \Omega_1 e^{i\omega_L t} & -\omega_o \end{pmatrix}$$

where $\hbar\omega_o$ is the separation between the two levels $\mid a >$ and $\mid b >, \omega_L$ is the laser frequency, and $\Omega_1 = eE_o/\hbar < a \mid z \mid b >$ is the Rabi frequency.

1. Guess a solution of the form $\Psi(t) = \binom{b_1 e^{i\omega_\alpha t}}{b_2 e^{i\omega_\beta t}}$. Write the coupled equations from the time-dependent Schrodinger equation. If you choose the frequency correctly, the equations will be steady-state, containing no oscillating terms.

2. The equations from (1) are identical to a two-state system with a Hamiltonian that is a function of the Rabi frequency and the detuning ($\delta_L = \omega_L - \omega_o$) only. Write this Hamiltonian.

3. Write the Hamiltonian from (2) in terms of trig-functions where $\sin 2\theta = \Omega_1/\Omega$ where $\Omega = (\Omega_1^2 + \delta_L^2)^{\frac{1}{2}}$ is the "effective" Rabi frequency.

4. Diagonalize this Hamiltonian to find the eigenvalues and associated eigenvectors.

5. Finally, use these results to find the time-dependent wavefunctions in the Schrodinger picture of our original Hamiltonian. (In part (1) you transformed to a different picture.) These are the time-dependent dressed states and are superpositions of $\mid a >$ and $\mid b >$.

6. Identify the a.c. Stark shift in the dressed states. It appears as a shift in the energy levels of the two dressed states relative to the uncoupled states. What is the Stark shift in the limit $\Omega_1 << \delta_L$. Write the dressed states in this limit.

7. The polarization vector is the expectation value of the dipole operator for the dressed state. It can also be written $\vec{P}(\omega_L, t) = \alpha(\omega_L) E_o \cos(\omega_L t)$. Compare these two expressions, and express $\alpha(\omega_L)$ as a function of $|< a \mid z \mid b >|^2$. Substitute the oscillator strength $f_{ab} = \frac{2m}{\hbar}\omega_o \, |< a \mid z \mid b >|^2$ into the expression and compare it with the known expression $\alpha(\omega_L) = \frac{e^2}{m}\frac{f_{ab}}{\omega_o^2 - \omega_L^2}$. Where do they agree? Comment briefly on the physics of $\omega_L \rightarrow \omega_o$.

8. Let $\omega_L \rightarrow 0$, and write the d.c. polarizability for both expressions. What approximation has been made that accounts for the difference?

9. Let $\mid \Psi(t = 0) > = \mid a >$. Express $\mid \Psi(t) >$ as a linear superposition of the dressed states. What is the probability $P(t)$ that $\mid \Psi(t) >$ will be found in the state $\mid a >$? This shows that the dressed states contain the physics of Rabi "nutation" or oscillation.