Atomic Physics II (8.422) Spring 2005
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Problem Set 9

Due Friday, April 22

THE MAGNETO-OPTICAL TRAP

1. **1-D MOT**. The motion of an atom in a one-dimensional magneto-optical trap obeys the equation of a damped harmonic oscillator:

$$\ddot{z} + \gamma \dot{z} + \omega^2 z = 0$$

Typical values (for a saturation parameter $S \approx 1$) are $\gamma^{-1} = 100 \mu s$ for the damping time and $\omega = 2\pi \times$ 1 kHz for the trap frequency.

Suppose we are trapping sodium atoms (M = 23 amu, λ = 589 nm) with a laser intensity such that the scattering rate is $10^7 s^{-1}$.

(a) Determine the equilibrium 1-D temperature using the given values for the damping and scattering rates.

(b) What is the size of the trapped cloud at this temperature?

2. **Density Limit in a MOT**. In a 3-D magneto-optical trap, the density of the trapped atoms is limited by a net outward radiation pressure which opposes the trapping force. We can divide the density-dependent photon-pressure force into two parts. First there is a repulsive 'radiation trapping force' due to atoms absorbing photons scattered from other atoms in the trap. Also, there is an attractive 'attenuation force' which is caused by atoms at the side of the cloud attenuating the laser beams, thus creating an intensity imbalance which leads to an inward force.

(a) Show that the radiation trapping force obeys the equation

$$\nabla \cdot \mathbf{F}_R = \frac{6\sigma_L \sigma_R I n}{c},$$

where $\mathbf{I}$ is the intensity of one of the trapping laser beams, n is the number density of atoms in the cloud, σ_L is the cross-section for absorption of the laser beam, and σ_R is the cross-section for absorption of the scattered light. (**Hint**: Find the magnitude of the force between two atoms separated by a distance d, where one atom re-radiates a laser photon and the second atom absorbs it. Now, since this is an inverse-square force, you can use Gauss' law to find $\nabla \cdot \mathbf{F}_R$. Assume that photons are only scattered twice.)

(b) The attenuation force may be obtained simply by replacing σ_R with $-\sigma_L$, so that

$$\nabla \cdot \mathbf{F}_A = -\frac{6\sigma_L^2 In}{c}.$$

Explain why this is so.

(c) The total force is the sum of $\mathbf{F}_R, \mathbf{F}_A$, and the trapping force $\mathbf{F}_T = -\kappa \mathbf{r}$, where κ is the spring constant of the trap. For stability we require that the total force is attractive, $\nabla \cdot \mathbf{F}_{total} < 0$. Find the maximum density of the trapped cloud at a given κ from the condition $\nabla \cdot \mathbf{F}_{total} = 0$.

(d) Give a qualitative argument for whether we expect $\sigma_R = \sigma_L, \sigma_R > \sigma_L$, or $\sigma_R < \sigma_L$. (**Hint**: sketch the absorption and emission spectra for an atom in a strong laser field with a red detuning.)

(e) Suppose that some of the atoms can be put into a 'dark state', so that only a fraction f of the atoms absorb the trapping light. How do F_R, F_A and F_T vary with f? What happens to the limiting density n_{max}? This is the concept of the Dark SPOT trap (PRL **70**, 2253 (1993)).