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Three Quantitative Management Problems in Public Procurement and Decision Procedures for Their Analysis and Solving

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Abstract

Three management problems that a state (or a public administration acting on its behalf) faces in procuring goods and/or services are considered: a) choosing the type of a contract to be awarded and the type of a competitive bidding to determine the winning bid, b) setting the initial price for a contract being the subject of the bidding, and c) designing (or choosing) a set of rules for determining the winning bid by means of the chosen competitive bidding. Mathematical models and decision procedures for analyzing and solving these problems are discussed.

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Keywords: Connected player strategies; disjoint player strategies; game-theoretic models; Nash equilibrium points; public procurement

1. Introduction

Public procurement is an ample source of decision-making problems, and there are three key ones that a public administration faces in procuring goods and/or services. Choosing the type of a contract and the type of a competitive bidding to maximize the chances of successfully

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implementing the contract for procuring goods and/or services under various procurement risks and in line with the existing laws constitutes the first problem. The second problem consists of setting an initial price for the contract to hold the bidding. The third problem is associated with designing a set of new rules for determining the winning bid or with choosing such rules from among available ones to help eliminate or reduce undesirable effects capable of affecting the quality of public procurement such as dumping prices, collusions, forming corrupt ties, etc.

Among practically applicable decision procedures aimed at analyzing and solving the first problem (provided that the estimates of the probabilities of undesirable events to occur are available) the following two are considered: Nonlinear programming tools (applicable under some verifiable natural assumptions about the regularities describing the above probabilities) and elementary matrix analysis techniques (in the simplest cases). In the second problem, an equilibrium price strategy of the public administration in an auxiliary three-person game on polyhedral sets of disjoint player strategies is suggested to use as (or in determining) the initial price of the contract, and the requirements for information needed to develop such games, along with necessary and sufficient conditions for the equilibriums in these games, are discussed. For the third problem, a new set of rules for determining the winning bid is proposed. These rules make dumping prices unreasonable for any bidder while reducing the chances of forming corrupt ties between a bidder and a public administration representative. Under these rules, the chances of every bidder to win the contract at a reasonable price acceptable for the public administration are higher than those of winning at the same price under the rule, where the lowest submitted bid wins.

2. Maximizing the chances of successfully implementing a contract by optimally allocating available financial resources

In awarding contracts to fulfill state, regional, or municipal orders, a public administration should take into account all the risks that may accompany both the placement and the implementation of the corresponding contacts. The pairs “a type of the contract—a type of the competitive bidding to determine the contract winner” form a set of the strategies that the administration can exercise to reduce the risks associated with placing and implementing each contract.

Information about the chances of unfavorable events to impact the fulfillment of a particular order under the chosen strategies of placing and implementing a contract to fulfill the order can be presented in the form of a matrix. The matrix rows correspond to the above strategies, and each strategy is a combination of the type of a contract to be awarded and the type of a competitive bidding to be held to tender this contract. The matrix columns correspond to the unfavorable events that may occur in the course of both the placement and the implementation of the contract. For a particular order, each element of the matrix is the probability of a particular unfavorable event (corresponding to the column in which this element is situated) to occur under a particular strategy (corresponding to the row in which this element is situated) chosen by the administration. It is further assumed that information necessary for calculating all the above probabilities is available, for instance, from expert estimates.

Any public administration that places state, regional, or municipal orders may find itself in one of the following two financial situations: a) It has financial resources to spend to reduce the chances of the unfavorable events to occur, and b) it does not have such financial resources.

Let

i be a strategy that the administration can choose in allocating a particular order, $i \in \overline{1,m}$,

C_j be an unfavorable event that may occur in the course of fulfilling the order, whose occurrence disallows the order to be successfully fulfilled, $j \in \overline{1,n}$,

b be the amount of financial resources that the public administration can use to reduce the chances (probabilities) of the unfavorable events to occur,

$P(A)$ be the probability of the event A,

Q be the event consisting of successfully fulfilling the order,

$\overline{A}$ be the negation of the event A.

Assuming that all the events $\overline{C}_j$ are pair-wise independent, one can easily be certain that the following relations hold:

$$P(Q) = \prod_{j=1}^{n} P(\overline{C}_j).$$

In situation b), to calculate the probability $P_i(C_j)$ for each particular strategy $i, i \in \overline{1,m}$ and to choose the strategy that maximizes this probability is the best the administration can do, which means that the administration should calculate the number

$$\max_{i \in \overline{1,m}} P_i(Q) = \max_{i \in \overline{1,m}} P_i\left(\prod_{j=1}^{n} \overline{C}_j\right) = \max_{i \in \overline{1,m}} \prod_{j=1}^{n} P_i(\overline{C}_j), \qquad (1)$$

where $P_i(Q)$ is the probability of successfully fulfilling the order by the winner of a tender (for the right to fulfill this order) under administration strategy i, i.e., under choosing a combination of the bidding procedure for allocating a contract and the type of the contract that correspond to row i of the above matrix, $P_i(\overline{C}_j)$ is the probability that the unfavorable event C_j will not occur if the administration chooses strategy i, $i \in \overline{1,m}$, $j \in \overline{1,n}$.

In situation a), one should find how the probabilities $P_i(\overline{C}_j)$ depend on the amount of financial resources that the administration can spend for reducing these probabilities, assuming that all the probabilities $P_i(C_j),\ i \in \overline{1,m},\ j \in \overline{1,n}$, can be reduced on account of certain activities, each requiring financing.

Let $x_j,\ j \in \overline{1,n}$ be the amount of financing that the administration spends in an attempt to reduce the risk of the unfavorable event C_j (that may affect the fulfillment of a particular order under consideration) to occur $j \in \overline{1,n}$. It is natural to assume that the larger the x_j, (generally) the larger the probabilities $P_i(\overline{C}_j)$ [1]. Also, it is natural to assume that for each $i,\ i \in \overline{1,m},$ there is a certain threshold θ_{ij} such that spending any amount of financial resources exceeding θ_{ij} cannot reduce the probability $P_i(C_j)$ below its value at $x_j = \theta_{ij}$. This "saturation" phenomenon is similar to the well-known one associated with the perception of advertising messages for goods and services [1].

Let $P_{ij}(x_{ij}) = (P_i(C_j))(x_{ij})$ be the functions reflecting the regularities that describe how the probability of the unfavorable event C_j to occur under administration strategy i depends on the financial resources x_{ij} spent to reduce these probabilities. Under the assumptions made, the functions $P_{ij}(x_{ij})$ can be viewed as continuous, monotone functions decreasing (or non-increasing) on the segment $[0, \theta_{ij}]$ and non-decreasing on the subset of real numbers (θ_{ij}, ∞). It is natural to assume that in all the situations that a public administration may face, the inequality

$$b < \min_{i \in \overline{1,m}} \sum_{j=1}^{n} \theta_{ij}$$

holds.

The problem of optimally allocating financial resources to reduce the chances of unfavorable events to disallow the successfully fulfilling of the order under administration strategy $i,\ i \in \overline{1,m}$ can be formulated as follows:

$$\begin{aligned} &\sum_{j=1}^{n} x_{ij} \le b, \\ &0 \le x_{ij} \le \theta_{ij},\ j \in \overline{1,n}, \qquad (2) \\ &\prod_{j=1}^{n} (1 - P_{ij}(x_{ij})) \to \max. \end{aligned}$$

This nonlinear programming problem can be solved by well-developed general nonlinear programming techniques, as well as by those developed for particular types of the functions $P_{ij}(x_{ij})$ that may describe the above probabilities. (Several examples of such particular functions $P_{ij}(x_{ij})$ are considered in [1].)

Thus, in situation b), when the financial resources are absent, one should solve problem (1), whereas in situation a), when the resources are available, one should solve problems (2) and find a number i^* for which the equality

$$\max_{(x_{i^*1},\ldots,x_{i^*n})\in M_{i^*}} \prod_{j=1}^{n}(1-P_{i^*j}(x_{i^*j})) = \max_{i\in\overline{1,m}} \max_{(x_{i1},\ldots,x_{in})\in M_i} \prod_{j=1}^{n}(1-P_{ij}(x_{ij}))$$

holds, where M_i is a set of feasible solutions to problem (2) corresponding to administration strategy $i\in\overline{1,m}$.

For particular types of the functions $P_{ij}(x_{ij})$ in problem (2), one may choose (or develop) solution methods that are more effective than general nonlinear programming techniques. For instance, if the functions $P_{ij}(x_{ij})$ are as follows:

$$P_{ij}(x_{ij}) = \begin{cases} p_{ij}^0(a_{ij}-x_{ij}^{\alpha_{ij}}) < 1, \text{if } 0 \le x_{ij} \le \theta_{ij}, \\ p_{ij}^0(a_{ij}-\theta_{ij}^{\alpha_{ij}}) < 1, \text{if } x_{ij} > \theta_{ij}, i\in\overline{1,m}, j\in\overline{1,n}, \end{cases}$$

where the inequalities $0<\alpha_{ij}<1$, $x_{ij}^{\alpha_{ij}} < a_{ij}$ for $0 \le x_{ij} \le \theta_{ij}$ and $p_{ij}^0 > 0$, $i\in\overline{1,m}$, $j\in\overline{1,n}$ hold, problem (2) for analyzing strategy i takes the form

$$\sum_{j=1}^{n} x_{ij} \le b,$$

$$0 \le x_{ij} \le \theta_{ij}, j\in\overline{1,n},$$

$$\prod_{j=1}^{n}(q_{ij}^0 + p_{ij}^0 x_{ij}^{\alpha_{ij}}) \to \max,$$

where $q_{ij}^0 = 1 - p_{ij}^0 a_{ij}$. In this problem, all the functions in the constraints and the goal function are posynomials or monomials, which may allow one to use ideas of geometric programming in developing techniques for solving the problem [1-3].

3. An approach to finding an initial price for a contract in a competitive bidding

Let a state (public administration) have funds to finance the fulfillments of n projects (orders), and let it consider several potential contractors interested in participating in a competitive bidding to be held to tender n contracts for developing and operating each project (for instance, for operating the objects to be created as a result of the project implementation). To hold the bidding, the state must set an initial price for each contract being the subject of the bidding.

One approach to finding this price consists of determining "the best partner" from among legal entities interested in competing (or "the best pair of partners" if the development and the operation of all the projects are to be done by two different legal entities, respectively). Here, the "best partner" means a partner (or a pair of partners) that would agree to develop as many projects as possible at the volumes and at the prices acceptable to the public administration. Here, the administration is interested in minimizing its total cost associated both with developing and operating all the projects that will be implemented and with the failure to implement some of the projects (or all of them) at the desirable volumes.

Let

m be the number of projects that the state intents to implement within a certain period of time T,

y_i be the volume of work to be done in the framework of project i, $i \in \overline{1,m}$,

x_i be the cost of developing a "unit volume" of project i, $i \in \overline{1,m}$,

u_i be the cost of operating a "unit volume" of project i after the project completion, $i \in \overline{1,m}$,

k be the share of the cost associated with operating each of the completed projects that is to be paid to the project developer according to the contract for developing the project,

b_i be the volume of the state's demand for the objects to be created in the framework of project i, $i \in \overline{1,m}$,

w_i be the penalties associated with the failure to implement project i at the "unit volume" $i \in \overline{1,m}$,

v_i be the expenses associated with operating a "unit volume" of completed project i, $i \in \overline{1,m}$ that the project developer bears according to the contract for developing the project,

s_i be the revenue to be generated by a "unit volume" of project i after the project completion, $i \in \overline{1,m}$,

l be the share of the revenue to be generated by each of the completed projects that is to be received by the project operator according to the contract for operating the projects.

Further, let

$$\boldsymbol{y} = (y_1,..., y_m), \boldsymbol{x} = (x_1,...,x_m), \boldsymbol{u} = (u_1,...,u_m), \boldsymbol{b} = (b_1,...,b_m),$$
$$\boldsymbol{w} = (w_1,..., w_m), \boldsymbol{v} = (v_1,..., v_m), \boldsymbol{s} = (s_1,..., s_m)$$

be vectors in R^m, and let the first three vectors $\boldsymbol{y} \in \Omega, \boldsymbol{x} \in M, \boldsymbol{u} \in H$, where Ω, M, H are polyhedra in R_+^m described by compatible systems of linear inequalities. If the state were to find "the best partner" to develop all the projects and "the best partner" to operate them, it could consider the following three-person game with the payoff functions

$$\begin{aligned} f_1(\boldsymbol{x},\boldsymbol{y},\boldsymbol{u}) &= <\boldsymbol{y},\boldsymbol{u}> + <\boldsymbol{y},\boldsymbol{x}> + <\boldsymbol{w},\boldsymbol{b}-\boldsymbol{y}> \to \min_{\boldsymbol{y}\in\Omega} \\ f_2(\boldsymbol{x},\boldsymbol{y},\boldsymbol{u}) &= <\boldsymbol{y},\boldsymbol{x}> + k<\boldsymbol{y},\boldsymbol{u}> - <\boldsymbol{v},\boldsymbol{y}> + (1-l)<\boldsymbol{s},\boldsymbol{y}> \to \max_{\boldsymbol{x}\in M} \qquad \text{(Game 1)} \\ f_3(\boldsymbol{x},\boldsymbol{y},\boldsymbol{u}) &= (1-k)<\boldsymbol{y},\boldsymbol{u}> + l<\boldsymbol{s},\boldsymbol{y}> \to \max_{\boldsymbol{u}\in H} \end{aligned}$$

where player 1, which is the state (or the public administration), needs to calculate Nash equilibrium points of the game $(\boldsymbol{y}^*, \boldsymbol{u}^*, \boldsymbol{x}^*)$ and to take component *j* of the vector $\boldsymbol{x}^*$ and component *j* of the vector $\boldsymbol{u}^*$ as the initial prices in the bidding for the contracts associated with developing and with operating project j, respectively.

Proposition. The triple of vectors $(\boldsymbol{y}^*, \boldsymbol{u}^*, \boldsymbol{x}^*)$ is a Nash equilibrium point in Game 1 if and only if the pair of vectors $(\boldsymbol{y}^*, (\boldsymbol{u}^*, \boldsymbol{x}^*))$ is a saddle point in an auxiliary antagonistic game of players A and B with the payoff function

$$<\boldsymbol{y},\boldsymbol{u}> + <\boldsymbol{y},\boldsymbol{x}> - <\boldsymbol{w},\boldsymbol{y}> \qquad \text{(Game 2)}$$

on the sets of player strategies Ω (for player A) and $H \times M$ for (player B). This proposition follows from the definition of a saddle point in an antagonistic game and from that of a Nash equilibrium point in a three-person game.

Game 2 is a particular case of an antagonistic game with the payoff function

$$<\boldsymbol{p},\boldsymbol{x}> + <\boldsymbol{x},\boldsymbol{D}\boldsymbol{y}> + <\boldsymbol{q},\boldsymbol{y}> \qquad \text{(Game 3)}$$

on the polyhedral set $\tilde{M} \times \tilde{\Omega}$, where $\boldsymbol{x} \in \tilde{M}$ and $\boldsymbol{y} \in \tilde{\Omega}$, and both sets are described by compatible systems of linear inequalities. As shown in [4], finding equilibrium points in Game 3 on (generally unbounded) polyhedral sets $\tilde{M}$ and $\tilde{\Omega}$ is equivalent to solving the auxiliary linear programming problems

$$< \boldsymbol{b}, \boldsymbol{z} > + < \boldsymbol{q}, \boldsymbol{y} > \to \max_{(z,y)\in Q},$$
$$< -\boldsymbol{d}, \boldsymbol{t} > + < \boldsymbol{p}, \boldsymbol{x} > \to \min_{(t,x)\in P},$$

forming a dual pair, where

$$Q = \{(z, \boldsymbol{y}) \ge 0 : \boldsymbol{z}\boldsymbol{A} \le \boldsymbol{p} + \boldsymbol{D}\boldsymbol{y}, \boldsymbol{B}\boldsymbol{y} \ge \boldsymbol{d}\},\ P = \{(\boldsymbol{t}, \boldsymbol{x}) \ge 0 : \boldsymbol{t}\boldsymbol{B} \le -\boldsymbol{q} - \boldsymbol{x}\boldsymbol{D}, \boldsymbol{A}\boldsymbol{x} \ge \boldsymbol{b}\},$$

$$\tilde{M} = \{\boldsymbol{x} \in R_+^m : \boldsymbol{A}\boldsymbol{x} \ge \boldsymbol{b}\},\ \tilde{\Omega} = \{\boldsymbol{y} \in R_+^n : \boldsymbol{B}\boldsymbol{y} \ge \boldsymbol{d}\},$$

$\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{D}$ are matrices, and $\boldsymbol{b}, \boldsymbol{d}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{t}, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$ are vectors of corresponding dimensions.

To develop Game 3, one needs information about the potential contractors interested in developing the projects and in operating them (after completing the contracts) to describe the polyhedra M, Ω, H, along with the polyhedra $\tilde{M}$ and $\tilde{\Omega}$ [5]. This information is needed to estimate the production potential of the contractors, which the public administration should do for all the prospective contractors to be invited, as well as for all the legal entities expected to participate in the bidding. Examples of developing the polyhedra $\tilde{M}$ and $\tilde{\Omega}$ in industrial, agricultural, and transportation systems are presented in [5].

4. The description of a set of rules for determining the winner in a competitive bidding for a contract

Throughout the rest of the article, only tender procedures in the form of sealed-bid auctions are the subject of consideration.

If all the bidders were reputable, experienced companies, capable of completing a contract (the subject of the auction) timely and in line with the quality requirements, the risks of the auction organizer associated with both placing and implementing the contract would substantially be reduced. So designing sets of rules for determining the auction winner that would reduce both risks seems important. One such set of rules, proposed in [6] and further developed in [7], makes the auction attractive for both the auction organizer and the bidders.

The idea underlying the rules is as follows: the auction organizer guarantees to the winner the contract price to be within the segment $[kx, x]$, where x is the (unknown to the bidders) reserve price, and the value of k, $0 < k < 1$ is announced. The auction winner is determined as follows:

a) if all the submitted prices do not exceed kx, then the bidder who has submitted the price that is either the closest to kx (among all the submitted prices) or coincides with kx is declared the winner, and the winning price is kx,

b) if all the submitted prices are not lower than kx, then the bidder who has submitted the price that is either the closest to kx (among all the submitted prices) or coincides with kx is declared the winner, and the winning price is the price submitted by the winner,

c) if some of the bidders have submitted the prices that do not exceed kx, whereas the others have submitted the prices that are not lower than kx, rule a) applies, and the winning price is kx,

d) if several bidders have submitted the same winning price, the winner is determined by an additional procedure, whereas the winning price is determined by either rule a) or rule b),

e) if all the submitted prices exceed x, the sealed-bid auction is considered as failed.

It turns out that proceeding from a) a probabilistic evaluation of the chances that each of n bidders will submit its price within a certain range, and b) the reserve price for the contract, by solving some mathematical programming problems [5-6], the auction organizer can determine both x and k that minimize the probability $P(T)$ of the winning price to exceed kx. In these problems the function $P(T)$ takes one of the two forms

$$P(T) = \prod_{i=1}^{n} \min(1, \frac{\overline{h}_i^f - kx}{\overline{h}_i^f - \underline{h}_i^f}) - \prod_{i=1}^{n} \min(1, \frac{\overline{h}_i^f - x}{\overline{h}_i^f - \underline{h}_i^f}),$$

and

$$P(T) = \prod_{i=1}^{n} \min(1, \frac{\overline{h}_i^f - kx}{\overline{h}_i^f - \underline{h}_i^f}).$$

Here, a) h_i^f is the price for the contract that the auction organizer believes that bidder i, say, a firm, is likely to submit as its bid, b) h_i^f can vary within the segment $\underline{h}_i^f \le h_i^f \le \overline{h}_i^f$, and c) h_i^f is a continuous, uniformly distributed random variable with the probability density function $p(h_i^f)$, where

$$p(h_i^f) = \begin{cases} 1/(\overline{h}_i^f - \underline{h}_i^f), \text{if } \underline{h}_i^f < h_i < \overline{h}_i^f; \\ 0, \text{otherwise.} \end{cases}$$

The rules possess the following feature: The chances of every bidder to win the contract at the price kx are higher that those of winning it at this price under the rule, where the lowest bid always wins the contract, whereas the chances of every bidder to win the contract at a price exceeding kx are lower than those of winning it at the price kx. Thus, the proposed rules are advantageous for both the bidders and the auction organizer [6].

Also, the proposed rules a) do not discourage reputable potential bidders from submitting the bids that reflect their values of the contract and their ability to implement it with the required quality at the submitted price, since dumping prices is unprofitable for all the bidders, and b) encourage all the potential bidders to study both the market and their competitors in the bid. However, the proposed rules are vulnerable to forming corrupt ties, since once a particular bidder learns about the value of x, this bidder can submit kx as the bid.

A slight modification of the above rules can make them less vulnerable to potential corrupt activities. That is, if x is the reserve price, the winning price is guaranteed to be within the segment $[kx, x]$, where the value of k, $0 < k < 1$ is announced, in just the same way it takes place under rules a) - e). However, the winner is determined as follows:

f) if all the submitted prices do not exceed kx, and not all of them are the same, then the bidder who has submitted (or a bidder from among those who have submitted) the price that is either next smaller than the closest to kx price (if the price kx has not been submitted by the bidders) or next smaller than kx (if the price kx has been submitted by at least one of the bidders) is declared the winner, and the winning price is kx,

g) if all the submitted prices are not lower than kx, and not all of them are the same, then the bidder who has submitted (or a bidder from among those who have submitted) the price that is either next greater than the closest to kx price (if the price kx has not been submitted by the bidders) or next greater than kx (if the price kx has been submitted by at least one of the bidders) is declared the winner, and the winning price is the price submitted by the winner,

h) if at least one from among all the submitted prices is smaller than kx, whereas at least one from among the other submitted prices is greater than kx, the winner is either determined by rule f) from among the bidders who have submitted the prices not exceeding kx (if at least two different prices not exceeding kx have been submitted) or the participant who has submitted (or a participant from among those who have submitted) the price that is smaller than kx is declared the winner (if only one price smaller than kx has been submitted), and the winning price is kx in both cases,

i) if several bidders have submitted the same winning price, the winner is determined by an additional procedure, whereas the winning price is determined by rules f) - h),

j) if all the submitted prices exceed x, the sealed-bid auction is considered as failed.

One can be certain that under the same natural assumptions that hold for rules a) - e), rules f) - j) possess the same above feature as do rules a) - e) while keeping the submission of dumping prices unprofitable for each auction participant. However, unlike rules a) - e), rules f) - j) make unreliable any guarantees to win the auction that the auctioneer can give to an individual bidder, and thus they reduce the chances of forming corrupt ties between the auctioneer and an individual bidder in one-step, sealed-bid auctions (though they can nether block nor reduce the chances of forming corrupt ties among the auctioneer and more than one bidder or between the auctioneer and a cartel acting as a collective auction participant).

4. Concluding remarks

1. Though all the considered management problems are those arising in public procurement, decision procedures proposed for their analysis and solving are applicable in other fields, in particular, in public-private partnership, in planning advertising campaigns, and in a variety of sealed-bid auctions not associated with public procurement.

2. Game 2 is the simplest among those on polyhedral sets that can be used in determining the initial prices for all the contracts to be awarded as a result of the bidding. If the contract developer and the company that will operate the "project results" can affect the choice of more than one vector out of the vectors $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}$, the considered problem of finding the initial prices for all the contracts that a public administration wants to implement becomes a game on a polyhedral set of connected vector strategies [8].

3. Game problems, both considered and referred to in this article, are those in which sets of player strategies are those of prices and volumes and have the form of linear inequalities. Information about the coefficients in these inequalities is always either available or can be obtained by every public administration from the past experience or from expert estimates.

4. The proposed rules for determining the winner of a sealed-bid auction serve two particular goals: a) to discourage dumping prices by those potential contractors that may be interested in disallowing their reputable competitors to win a particular contract, and b) to discourage public administration representatives from establishing corrupt ties with participants of the tender. However, from the author's viewpoint, these rules reflect no more than a possible approach to achieving the above goals, which is not an easy task. Both the potential of this approach and mathematical features of the proposed rules, as well as those of other possible approaches to achieving these goals, should be a subject of further studies.

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