An Approach For Structural Maintenance Scheduling Using Time Variant Reliability Analysis of Serviceability and Ultimate Strength Models

by

Zachary J. Malinoski

B.S., Naval Architecture and Marine Engineering (1996)

United States Coast Guard Academy

Submitted to the Department of Ocean Engineering and the Department of Civil and Environmental Engineering in Partial Fulfillment of the Requirements for the Degrees of

Master of Science in Ocean Engineering
and
Master of Science in Civil and Environmental Engineering

at the
Massachusetts Institute of Technology
June 2002

Signature of Author...
Department of Ocean Engineering and Civil and Environmental Engineering
May 10, 2002

Certified by ....
David V. Burke
Senior Lecturer, Department of Ocean Engineering
sis Supervisor

Certified by ....
Jerome J. Connor
Professor of Civil and Environmental Engineering
Thesis Supervisor

Accepted by ....
Henrik Schmidt
Chairman, Departmental Committee on Graduate Studies
Department of Civil and Environmental Engineering

Accepted by ....
Oral Buyukozturk
Chairman, Departmental Committee on Graduate Studies
Department of Civil and Environmental Engineering

An Approach For Structural Maintenance Scheduling Using Time Variant Reliability Analysis of Serviceability and Ultimate Strength Models

by

Zachary J. Malinoski

Submitted to the Department of Ocean Engineering and the Department of Civil and Environmental Engineering in partial fulfillment of the requirements for the degrees of Master of Science in Ocean Engineering and Master of Science in Civil and Environmental Engineering.

Abstract

Developing a maintenance schedule for commercial and military vessels requires the integration of maintenance or replacement strategies for multiple, vastly different systems. Numerous systems such as electronics, pumps, and engines are currently maintained or inspected on an operating hours or mean time to failure (MTTF) basis. Presented herein is an approach whereby the reliability of ship structural elements and grillages, including corrosion effects, is evaluated over time using a variety of current component strength models. This approach allows for structural maintenance schedules based on two evaluation criteria; namely, the probability of failure of the component as determined from one of a number of strength models and the probability that the component's thickness is less than a minimum value.

Acknowledgements

I would first like to thank Dr. David Burke and Professor Jerome Connor for their patience and guidance in working with me on this research. To my parents, friends, and family, I beg their forgiveness of the many birthdays, special occasions, phone calls, and visits I have missed as a result of my time spent completing this work. I look forward to making it up to you. And most of all, my thanks, admiration, and love to my wife, Sarah, who sacrificed the most during my studies here. She spent many hours alone while I isolated myself to finish this thesis and worked very hard to help in any way she could. I cannot begin to express how grateful I am.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

In the fifteen-year period between 1980 and 1995, a study of marine casualties revealed more than 150 bulk carriers were lost at sea with some 1200 crew [13]. Of these 150, nearly 60% percent were over 15 years old. The RINA study considered it likely that many of the aged vessels suffered from widespread general corrosion and fatigue cracking. A 1989 study of offshore accidents indicated that structural damage initiated 9.4% of tanker accidents [21]. Losses of this magnitude over time tend to initiate drastic changes in the regulation of vessel design, construction, and operation.

Among these regulatory changes are increased inspections with varying degrees of detail. Currently, U.S. military surface combatants are dry-docked every 2-3 years, while ships classed by the American Bureau of Shipping (ABS) are required to dry-dock twice in a five-year period [1]. With the increasing costs of vessel construction, owners and operators must capitalize on their vessel's time in service to maximize profits over the life cycle of the vessel. Since dry-dock examinations take anywhere from 3 days to 1 month, vessel owners and operators are generally at odds with regulatory bodies' frequent inspection schedules. The goal of underwriters, classification societies, flag states, and vessel owners, therefore, is to seek sound structural standards while minimizing vessel downtime and expense for inspections, maintenance, and repair [12]. Time variant reliability (TVR) techniques provide a tool useful for meeting this goal. By tailoring TVR to ship specific conditions and environments, drydocks, maintenance, and inspections can be scheduled at times where the structural reliability drops below a threshold value, and resources for these activities may be concentrated in the areas most likely to have problems. Such a tailored schedule could potentially save vessel owners and operators significant operating costs over the service life.

Building upon traditional time invariant reliability principles, the limit state defining structural reliability may be made a function of time. Critical times for maintenance or inspection may then be identified as those times where the reliability over the vessel's life cycle falls below established target reliability levels. By incorporating repair actions, a set of critical points are determined whereby a suitable maintenance/inspection schedule can be tailored to ship specific conditions.

Benefits of time variant reliability analysis, taking into account the effects of corrosion, include the existence of two evaluation criteria: maintaining a minimum level of reliability and maintaining a minimum thickness in the structural elements. Classification societies typically require a minimum thickness for structural elements, especially hull and tank plating. ABS generally requires plate replacement when the thickness degrades to 75% of original scantling. Having established limits on both criteria through either probabilistic methods or experience, these criteria can be evaluated continually over the life cycle of the vessel to determine which will be dominant in deciding inspection schedules.

Presented herein is an approach to conducting a time variant structural reliability analysis for the purpose of estimating maintenance and inspection schedules prior to a vessel being placed into service. Included is a review of time variant reliability theory and an explanation of the corrosion and structural strength models used for the test case. A numerical example of various structural elements follows.

Chapter 2

Time Variant Reliability

2.1 Introduction

Structures are considered safe when the resistance to loading is greater than the loading applied. This can be generalized in the form of a limit state equation

$$Z = R - L \tag{2.1}$$

$$Z > 0 \tag{2.2}$$

where Z is the limit state function, R is the structural strength or resistance, and L is the applied load. Failure of the structure occurs when (2.2) is not satisfied. If the resistance and applied load were known, constant values, determining structural reliability would be a simple matter of subtracting two values. Unfortunately, numerous uncertainties infiltrate such a simple limit state, making it essential to design structures with adequate resistance to ensure safety.

Structural resistance is defined by an analytic or empirical function of any number of random, physical parameters such as the relevant dimensions and material properties. These parameters are generally uncertain due to manufacturing tolerances and human error and therefore add uncertainty to the actual value of the resistance. In reliability theory, these parameters are modeled as random variables that can be adequately described by the first and second moments (mean and variance, respectively) of a known probability density function (pdf). The resistance is then a function of n random variables

$$R = R(X_1, X_2 \ldots X_n) \tag{2.3}$$

whose joint probability density function is given by

$$P_R(r) = \int_{over\,Re} \cdots \int f_{x_1} f_{x_2} \cdots f_{x_n} \, dx_1 \, dx_2 \cdots dx_n \tag{2.4}$$

where Re is the region of X_i that defines an equivalent event in R, assuming that X_i are statistically uncorrelated [4].

The applied loading is a function of the environment or varying load conditions and thus inherently random. For ship structures, the dominant, applied loading results from the combined vertical bending moment (CVBM) due to a stillwater loading condition and waves. Over the short term, the CVBM may be modeled by stationary, stochastic processes [15]. The probability density of the CVBM reaching a particular moment, M, is then represented by the Rayleigh distribution

$$f_{CVBM}(M) = \frac{M}{\alpha^2} \exp\left[-\frac{1}{2}\left(\frac{M}{\alpha}\right)^2 \right] \qquad M \geq 0 \tag{2.5}$$

where M is the moment and α is the scale parameter [8]. The structural resistance, typically in units of stress, can be converted into units of moment by

$$M_R = \frac{\sigma_R \, I_z}{y} = \sigma_R \, SM \tag{2.6}$$

where *SM* is the section modulus, y is the distance from the neutral axis, and I_z is the moment of inertia about the transverse neutral axis. The applied load is now a function of two or three random variables. The probability density function of M_R can be evaluated using (2.4).

In a likewise manner, the expressions for R and L can be substituted into (2.1) so that Z is now a function of numerous random variables. The probability distribution function can again be evaluated by (2.4). However, direct integration of this expression can be very difficult and near impossible as the number of random variables increases.

A common procedure is to use intermediate methods for evaluating (2.4) for R and L, leaving Z in terms of only two random variables. First order reliability methods (FORM) and direct Monte Carlo simulation are essential tools for this process. A detailed explanation of FORM and Monte Carlo simulation can be found in [4] and [2] and will not be presented here. However, it is important to note their respective assumptions and shortcomings.

Given a function of n random variables, FORM techniques use the first mean and variance of a random variable to obtain the mean and variance of the function. When the function is not linear with respect to the random variables, the first order approximations of mean and variance can introduce significant error into the mean and variance of the function [4]. If the function is in terms of variables with non-normal or a mixed set of distributions, the mean and variance of each variable must be transformed into the mean and variance of an equivalent normal distribution [2]. Variables whose distributions differ greatly from the normal can also introduce significant error.

Monte Carlo simulation generates *N* random numbers for each random variable according to its known probability distribution function, mean, variance, or other distribution parameters. The function is then evaluated *N* times, until the mean and variance of the function achieve convergence [4]. Various data analysis techniques are then applied to determine the probability distribution that best fits the simulated data. Depending upon the behavior of the function, convergence may require a large number of cycles, *N*, potentially making this technique computationally expensive. With the advent of new computer technology, however, Assakaf [2] and Atua [3] have shown that convergence can be achieved for most structural resistance formulations with *N* approximately equal to 5,000.

Applying one of the above techniques to *R* and L now leave *Z* as a linear function of two random variables with known probability distributions, means, and variances. If the probability density functions of *R* and *L* are plotted as in Figure 1-1, it can be quickly determined visually whether or not there is risk for failure, i.e. (2.2) is not satisfied. If the plots were such that there was no overlap between *R* and *L*, then the probability of failure, P_f, is zero. Otherwise, any overlap of the two plots indicates that a probability of failure does in fact exist. The reliability, complement of P_f, can then be determined as follows

$$\text{Reliability} = 1 - \int_0^{\infty} \left[\int_0^{\eta} p_R(\xi)\, d\xi \right] p_L(\eta)\, d\eta \qquad (2.7)$$

where η is a dummy variable of integration [8].

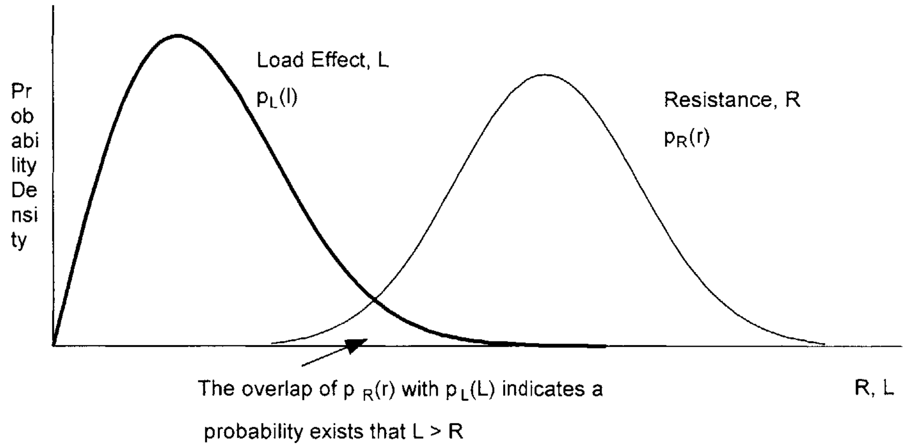

Figure 2-1 Example probability distributions of structural resistance and the load effect

2.2 Extreme Value Implementation of TVR

Determining structural reliability over the lifecycle of a vessel introduces a number of additional difficulties. The random nature of the ocean environment complicates the evaluation of the applied load's probabilistic characteristics. General corrosion of structural members is a complicated random process of time sequences where corrosion has been prevented, initiated, and arrested. A corrosion model and its behavior are covered in chapter 3. Due to these varying conditions over time, the mean, variance, and distribution functions of R and L, as well as those of their own underlying random variables, may change so that (2.1) is now a function of time:

$$Z = Z(t) = R(t) - L(t) \tag{2.8}$$

The method of evaluating reliability presented in 2.1 must be modified to adjust for this time dependency.

The assumption that the CVBM is adequately modeled by stationary processes presented in 2.1 does not hold over long time intervals, such as the 20 to 30 year service lives expected of most large vessels. Calculating the extreme value of the combined vertical bending moment over the life cycle requires detailed information regarding the vessel's operating profile such as time in port, time at sea, and transit routes, as well as likely courses and speeds. Obtaining an accurate result is a daunting task requiring the designer to predict operating conditions before the vessel is placed into service.

Ayyub [5] and Hughes [8] recommend the use of the Lifetime Weighted Sea (LWS) method. The LWS method accounts for the non-stationary behavior of the CVBM through a numerical algorithm whereby the ship response is calculated over a range of sea states, courses, and speeds weighted by a relative frequency of occurrence for each combination [8]. Research has indicated that the vertical bending moment generally follows a Weibull extreme value distribution function [11, 23]

$$P_{CVBM}(M \geq M_E) = \exp\left[-\left(\frac{M}{\gamma}\right)^{\alpha}\right] \tag{2.9}$$

Mansour and Jensen [9] developed a Microsoft® Excel™ worksheet based on an LWS-type algorithm that calculated the extreme CVBM given a vessel's length, breadth, draft, and block coefficient (C_b) over time and fit the data to the Weibull extreme value distribution.

By adopting extreme value analysis methods, failure is now defined as the first upcrossing from the safe state, $Z > 0$, into the unsafe state, $Z < 0$ [6]. Where the limit state function for time invariant reliability was represented by (2.1), the threshold, $\zeta(t)$, between safe and unsafe states for time variant reliability is defined by [1]

$$\zeta(t) = R(t) = M_R(t) \tag{2.10}$$

where $M_R(t)$ is the structural resistance (units of moment) of (2.6) including corrosion effects at any given time, t. Since the peaks of the applied load follow the Weibull distribution the upcrossing rate may be written as

$$\nu[\zeta(t)] = \exp\left[-\left(\frac{\zeta(t) - \mu_{M_L}}{\gamma_L}\right)^{\alpha_L}\right] \tag{2.11}$$

where μ_{M_L} is the mean applied moment due to the mean CVBM and γ_L and α_L are the Weibull scale and form parameters, respectively [14]. Given that the threshold $\zeta(t)$ is a function of n random variables, the upcrossing rate at any given time, t, may also be written as

$$\nu[\zeta(t \mid X_1, X_2, \ldots X_n)] = \exp\left[-\left(\frac{\zeta(t \mid X_1, X_2, \ldots X_n) - \mu_{\sigma_L}}{\gamma_L}\right)^{\alpha_L}\right] \tag{2.12}$$

where Xi are the random variables.

[1] Henceforth, the structural resistance will be noted by ζ, and R will denote reliability.

The above expressions enable the structural, time variant reliability over a period [0,T] for a non-stationary process dependent upon n random variables to be written as [19]

$$R(T) = \int_0^{\infty}\int_0^{\infty}\cdots\int_0^{\infty} f_{X_1}(x_1) f_{X_2}(x_2)\cdots f_{X_n}(x_n)\exp\left[-\int_0^{T} \nu[\zeta(t \mid X_1, X_2, \ldots X_n)]dt\right] dX_n \ldots dX_2 dX_1 \tag{2.13}$$

Chapter 3

Incorporating General Corrosion Effects into TVR Theory

3.1 Background

Corrosion engineers have identified three major environments facilitating the inception and growth of general corrosion of metallic structures: the atmosphere, water, and the presence of caustic chemicals in solid, gas, or liquid form [7]. Corrosion prevention is an important part of ship design and maintenance because marine vessels are subject to all three environments over their entire service life. Most ships have to be designed and maintained so as to address each environment separately.

Salt spray interacts with the ship above the wind/water line. Typical prevention measures for this type of atmospheric corrosion include coatings and man-hour-intensive fresh water wash downs. Certain chemical cargos require the installation of special tanks lined with stainless steel or other corrosion resistant material to protect major structural members. Numerous prevention measures, both passive and active, exist to prevent seawater corrosion of the hull and ballast tanks, most notably coatings, sacrificial anodes, and cathodic protection systems [7, 18].

None of these measures have proven fail-safe or maintenance-free over a ship's life cycle. The dynamic nature of vessel motion in a sea state and the localized structural deflections tend to erode and crack tank and hull surface coatings. Sacrificial anodes, though very effective, require routine and often costly maintenance, usually involving drydocking and tank cleaning for inspection and replacement.

Once a failure of the corrosion prevention system has occurred, the rate at which general corrosion develops is dependent on a number of factors. It is well known that the general corrosion of steel, the most common metal used in the construction of ships and offshore structures, depends upon the salinity, oxygen content, pH level and temperature of the water. Frequency of ballasting, wetted surface area, and tank ventilation are additional factors for salt-water ballast tanks. Corrosion of chemical cargo tanks depends largely upon the time exposure to caustic chemicals or gases as well as the effectiveness of ventilation systems [18]. Assuming appropriate care by the crew, weatherdeck and superstructure corrosion due to the atmosphere can generally be controlled or corrected.

Determining the impact of corrosion in terms of a reduction in element thickness is further complicated by the fact that each of the contributing factors mentioned above are themselves functions of time. Salinity, pH, oxygen content, and temperature are direct functions of time, location, or both. The complexity of physical and environmental conditions that influence the initiation and rate of corrosion presents a probabilistic challenge to estimating the effects of corrosion over the life cycle.

3.2 Time dependent corrosion model for TVR

Rather than attempting analytic expressions in terms of a large number of variables, most attempts to model corrosion have been empirical functions of time. Numerous collections of corrosion data indicate a relatively short time period of non-linear wastage followed by a longer period with a nearly constant corrosion rate. A recent and detailed study by Yamamoto [25] of corrosion in different locations of a ship structure confirms this non-linear behavior over time.

Early corrosion models were linear with respect to time. One such attempt modeled corrosion as a set of linear expressions [19],

$$d(t) = \begin{cases} 0.170t & 0 \le t < 1 \\ 0.152 + 0.0186t & 1 \le t < 8 \\ -0.364 + 0.083t & 8 \le t \le 16 \end{cases} \tag{3.1}$$

where *d(t)* is the wastage due to corrosion in mm and t is time in years. This model is depicted in Figure 3-1. Though easy to use, the linear corrosion model lacked the flexibility to account for the time period where the corrosion prevention measures are effective. Due to the need for multiple linear expressions to estimate non-linear behavior, linear formulations such as (3.1) were ill suited for adaptation into time variant reliability methods that spanned the time periods applicable to each individual expression.

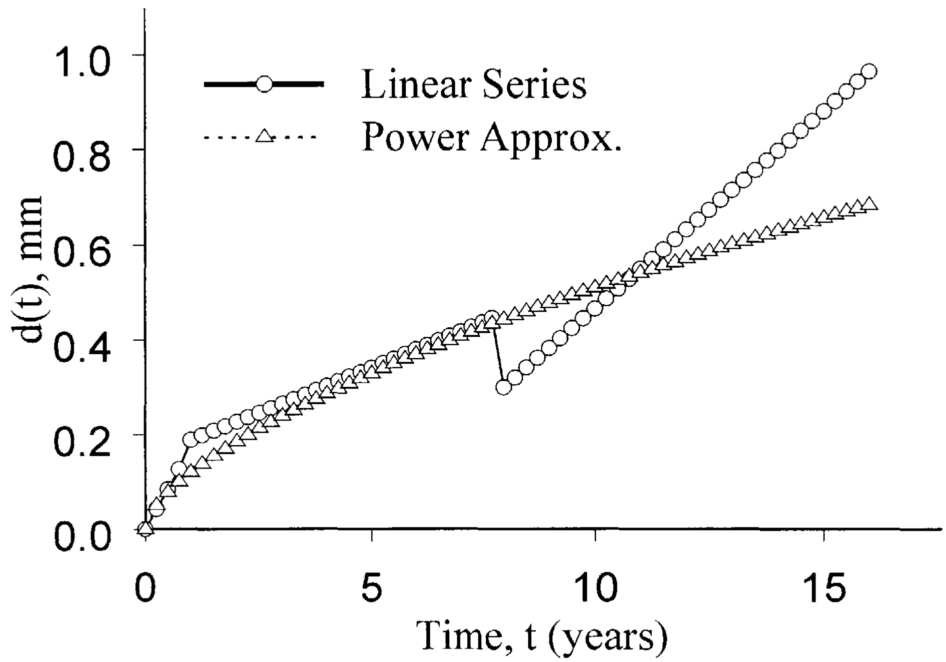

Figure 3-1 Corrosion wastage modeled by a linear series and power approximation

Using the data provided by Yamamoto's study, Soares and Garbatov [19] proposed an exponential expression derived from the solution of the differential equation

$$d_\infty \dot{d}(t) + d(t) = d_\infty \tag{3.2}$$

where d_∞ is the long term corrosion wastage, $d(t)$ is the corrosion wastage at any time t, and $\dot{d}(t)$ is the corrosion rate. The particular solution of (3.2) for a given time period of coating effectiveness, τ_c, is

$$d(t) = \begin{cases} d_\infty(1-\exp[-(t-\tau_c)/\tau_t]), & t > \tau_c \\ 0, & t \le \tau_c \end{cases} \tag{3.3}$$

where τ_t is the transition time. An exponential fit of this data can be found in Figures 3-2 and 3-3 [19].

The benefits of using an exponential function are two-fold. First, the exponential function adequately models observed corrosion behavior. Figures 3-2 and 3-3 clearly indicate a time period of no corrosion, followed by a period of quick corrosion wastage and then leveling off to a steady state. Second, the function can be easily tailored to ship specific data or expectations, maintaining consistency with the idea that the use of time variant reliability methods must be a ship-specific analysis.

Figure 3-4 depicts the concept of three distinct phases of corrosion development. The first region, noted by τ_c, is that time period where the prevention measures remain effective. Statistics have shown τ_c varies in the range of 1.5 to 5.5 years [19]. It is assumed that τ_c is normally distributed. The amount of data supporting this finding is limited, however, resulting in a large coefficient of variance (COV) of 0.4. The second period, noted by the transition time τ_t, is highly variable and uncertain. The data used in Figures 3-2 and 3-3 was fit with τ_t equal to 4.5 years and 3.6 years, respectively. The third period in Figure 3-4 indicates a decreasing corrosion rate, with wastage approaching d_∞, which corresponds to the physical process of renewed corrosion prevention due to a layer rust. Values for the long-term corrosion wastage are also highly variable and uncertain. Figures 3-2 and 3-3 were fit with d_∞ equal to 1.6mm (0.063 in) and 1mm (0.039 in), respectively. The variability in this data suggests that a more realistic value may be the result of long-term linear approximations of about 5mm (0.2 in). It is also assumed that d_∞ follows the normal distribution [19].

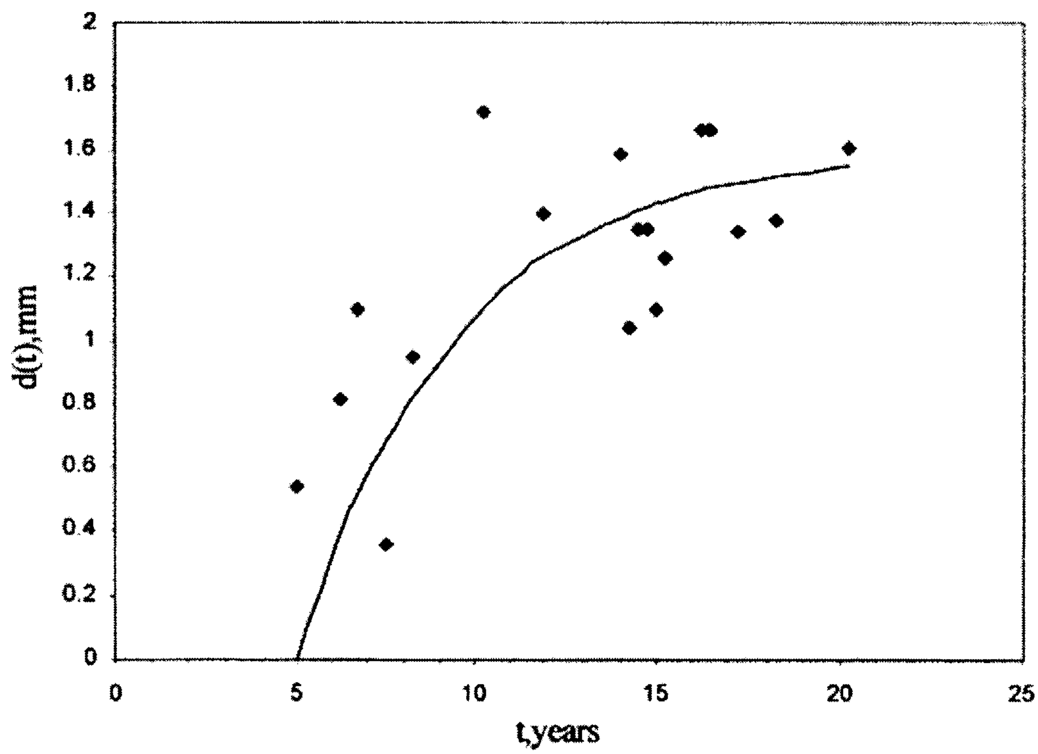

Figure 3-2 Exponential fit of time varying corrosion data for bulk carrier inner bottoms [19].

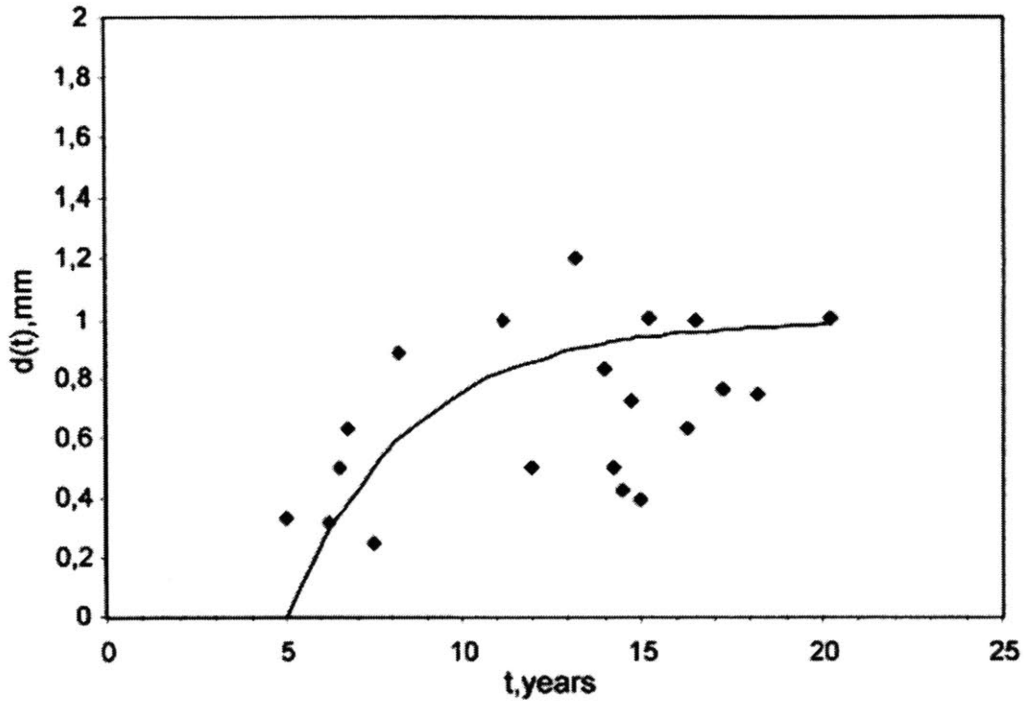

Figure 3-3 Exponential fit of time varying corrosion data for bulk carrier side shells [19].

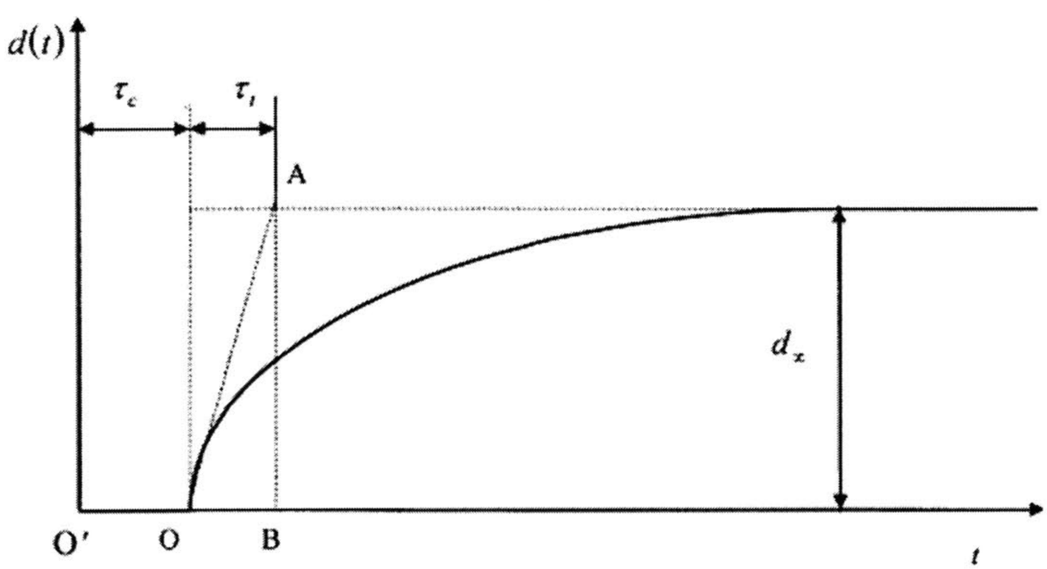

Figure 3-4 Soares/Garbatov corrosion wastage model as a function of time [19].

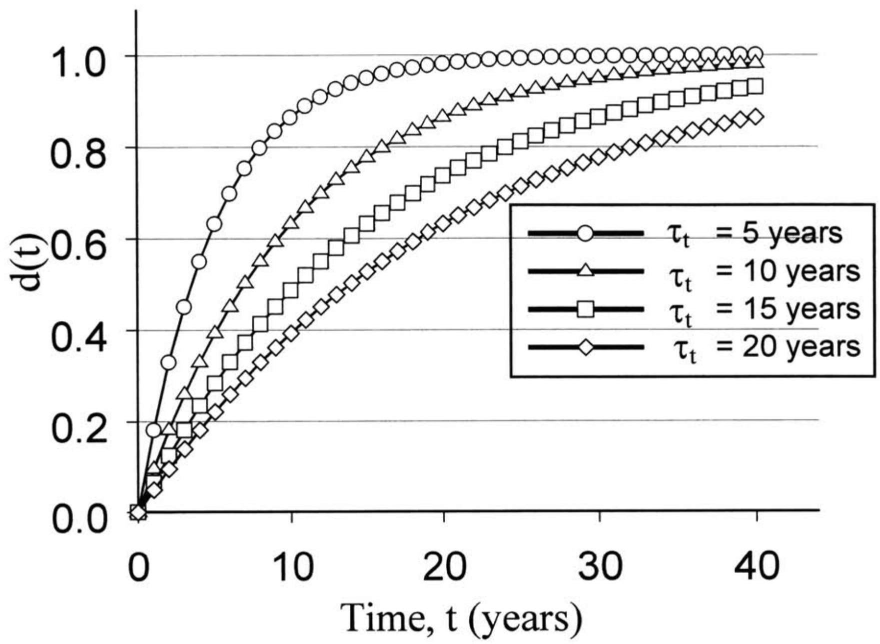

Figure 3-5 Exponential corrosion model sensitivity to the transition time

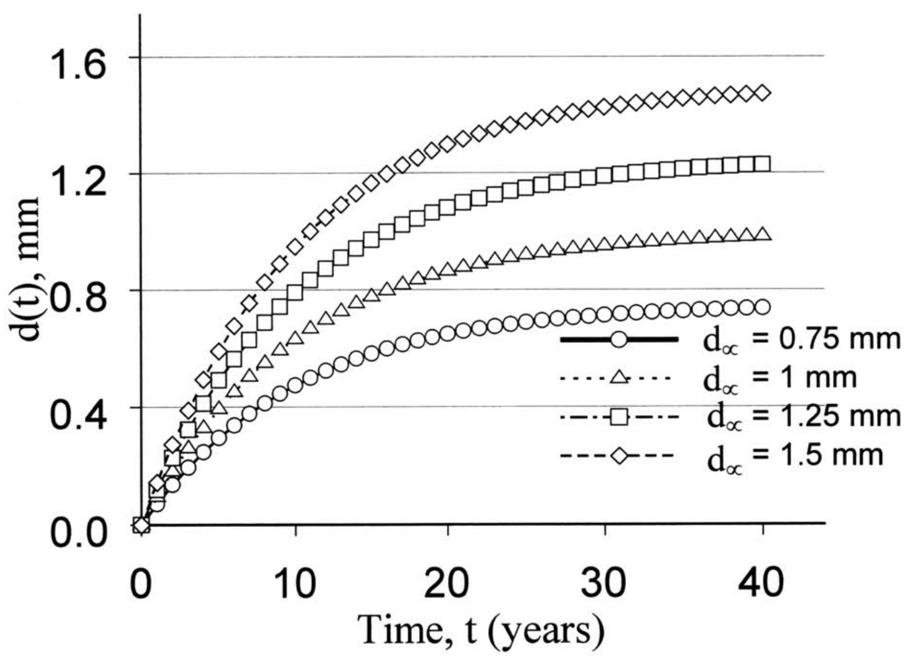

Figure 3-6 Exponential corrosion model sensitivity to d_∞

The influence and sensitivity of τ_t and d_∞ are shown in Figures 3-5 and 3-6. An increase in the transition time results in a lower corrosion rate, approaching a constant rate at higher values of τ_t. Increasing d_∞ increases the impact due to corrosion over time.

Considering this exponential model of corrosion, the thickness of a plate element over time can be written as

$$h(t) = h_0 - d(t) \tag{3.4}$$

where h_0 is the initial element thickness. However, $\zeta(t)$ in (2.10) is reduced not only by local corrosion accounted for in σ_R of (2.6) but also by corrosion over the entire cross section, culminating in a reduced section modulus, *SM.* Incorporating these different corrosion effects, (2.10) can be written as a function of time

$$\zeta(t) = M_R(t) = \sigma_R(t)\, SM(t) \tag{3.5}$$

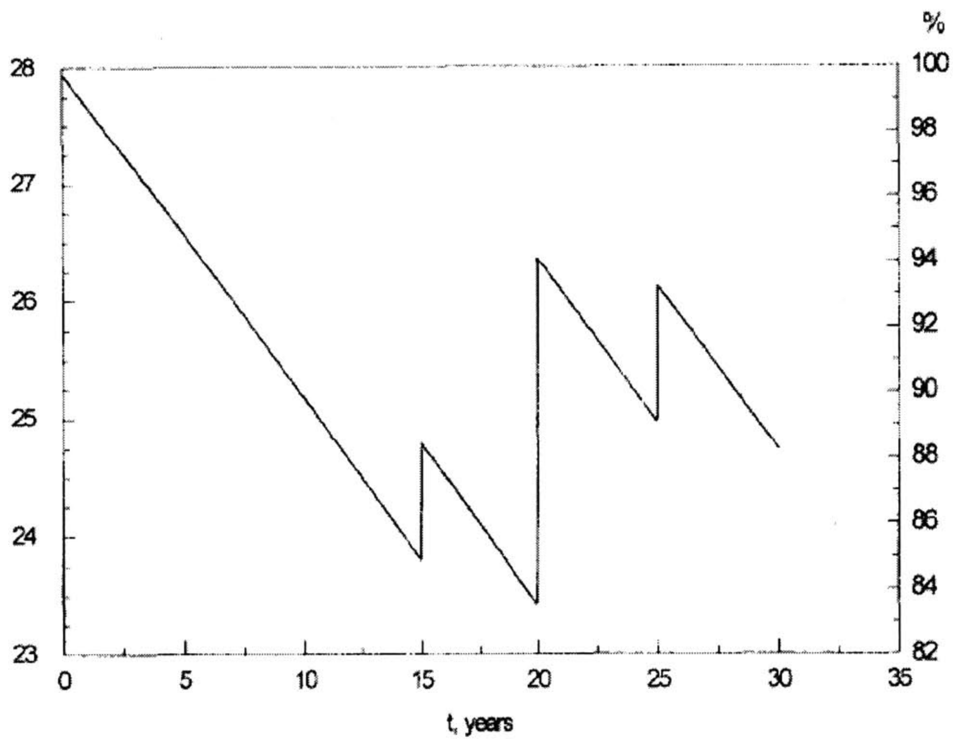

Figure 3-7 Mean value of the section modulus, SM, over time [18]

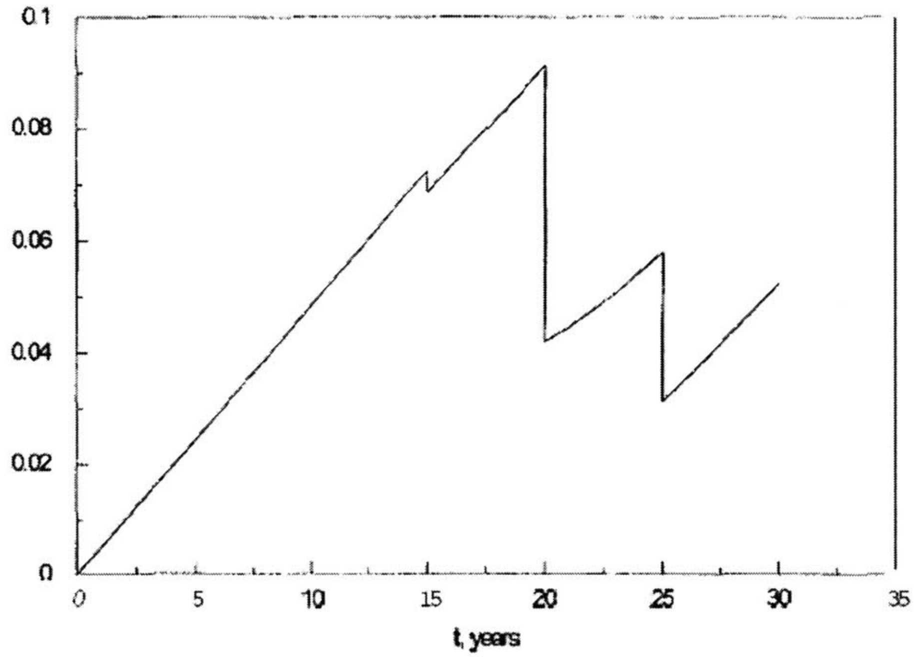

Figure 3-8 Standard deviation of the section modulus over time [18]

Soares and Garbatov present a detailed method for determining the time dependency on and probabilistic characteristics of the section modulus by using FORM methods [18]. In short, Soares and Garbatov determine *SM(t)* allowing for different corrosion rates over the cross section. Figures 3-7 and 3-8 show the mean value and variance, respectively, of the section modulus over time for a tanker midship cross section used by Soares and Garbatov. This method depends heavily on ship specific information and thus will vary over ship types. Using the data from their study, the section modulus is estimated to decrease one percent per year following an assumed normal probability density function. The section modulus over time can be written as

$$SM(t) = (1 - 0.01\,t)SM_0 \qquad (3.6)$$

where SM_0 is the initial section modulus of the structure. It is believed that this estimate will be conservative for most ship types [18].

3.3 Implementing the Time Dependent Corrosion Model into TVR

Recalling that the time period of corrosion prevention, τ_c, is a random variable, there exists a possible time period where the structural strength, ζ, is not a function of time. The randomness of τ_c, presents two distinct, conditional reliabilities, i.e. reliability with corrosion and reliability without corrosion [19]. The existence of these two conditional probabilities must be accounted for in the general TVR theory presented in Chapter 2.

Considering the event of failure after corrosion has developed, the expressions for the upcrossing rate and the TVR, remain essentially unchanged, restricted only by the time. Using the subscript *a* to denote expressions based upon initiated corrosion, (2.12) and (2.13) may be written as:

$$\nu_a[\zeta(t \mid X_1, X_2, \ldots X_n)] = \exp\left[-\left(\frac{\zeta(t \mid X_1, X_2, \ldots X_n) - \mu_{\sigma_L}}{\gamma_L}\right)^{\alpha_L}\right] \quad t \geq \tau_c \tag{3.7}$$

$$\begin{aligned} R_a(T) = \int_0^\infty \int_0^\infty \cdots \int_0^\infty f_{X_1}(x_1) f_{X_2}(x_2) \cdots f_{X_n}(x_n) \cdots \\ \exp\left[-\int_0^T \nu[\zeta(t \mid X_1, X_2, \ldots X_n)] dt\right] dX_n \ldots dX_2 dX_1 \quad t \geq \tau_c \end{aligned} \tag{3.8}$$

Because the structural strength is no longer a function of time while corrosion prevention systems remain effective, the time dependency of (3.7) may be removed. Using the subscript *b* to denote expressions based upon no initiated corrosion, (3.7) and (3.8) may be written as [19]:

$$\nu_b[\zeta(X_1, X_2, \ldots X_n)] = \exp\left[-\left(\frac{\zeta(X_1, X_2, \ldots X_n) - \mu_{\sigma_L}}{\gamma_L}\right)^{\alpha_L}\right] \quad t < \tau_c \tag{3.9}$$

$$\begin{aligned} R_b(T) = \int_0^\infty \int_0^\infty \cdots \int_0^\infty f_{X_1}(x_1) f_{X_2}(x_2) \cdots f_{X_n}(x_n) \cdots \\ \exp\left[-T\, \nu[\zeta(t \mid X_1, X_2, \ldots X_n)]\right] dX_n \ldots dX_2 dX_1 \quad t < \tau_c \end{aligned} \tag{3.10}$$

The TVR over the lifecycle can now be determined in terms of the above expressions so as to address the two major probabilistic events:

$$R(t) = \left[1 - F_{\tau_c}(t)\right] R_b(t) + \int_0^t R_b(\tau_c)\, R_a(t - \tau_c)\, d\tau_c, \quad t \in [0, T] \tag{3.11}$$

where $F_{\tau_c}(t)$ is the cumulative probability density function for tc and T is the expected service life of the structure.

Chapter 4

Serviceability and Ultimate Strength Models for TVR

4.1 Introduction

Time variant reliability methods applied to hull girder ultimate strength models have been presented by a number of authors [4,5,12,22]. This research extends TVR methods to evaluation at the elemental level, namely unstiffened and stiffened panels.

The successful implementation of a Load and Resistance Factor Design (LRFD) code in the civil engineering community has spawned extensive research into a similar design code for marine structures. In support of this initiative, Atua [3] and Assakaf [2] compiled a number of serviceability and ultimate strength models and then assessed their effectiveness for use in the time invariant-based LRFD codes for ships.

The strength models presented by Atua and Assakaf range from highly complex, theoretical models to simple, empirical expressions. Although the theoretical models may be more accurate, they may also be more uncertain due to the sheer number of variables. A balance must therefore be achieved between the model's accuracy, applicability, and simplicity, all of which are desired qualities for implementation into TVR. The strength models used for this thesis are those recommended by Atua and Assakaf for LRFD use and are detailed below.

4.2 Implemented Strength Models

Serviceability Strength Model for Plates Under Uniaxial Compression Stress

The buckling serviceability stress σ_s of an unstiffened panel under uniaxial compression is given by

$For\ a/b > 1.0$

$$\sigma_s = \begin{cases} \sigma_Y \dfrac{\pi^2}{3\left(1-\nu^2\right)B^2} & if\ \sigma_s \le \sigma_{pr} \\ \sigma_Y \dfrac{\left[\dfrac{\pi^2}{3\left(1-\nu^2\right)B^2}\right]^2}{\left[\dfrac{\pi^2}{3\left(1-\nu^2\right)B^2}\right]^2 + c} & if\ \sigma_s > \sigma_{pr} \end{cases}$$

$For\ a/b < 1.0$

$$\sigma_s = \begin{cases} \sigma_Y \left(\alpha + \dfrac{1}{\alpha}\right) \dfrac{\pi^2}{12\left(1-\nu^2\right)B^2} & if\ \sigma_s \le \sigma_{pr} \\ \sigma_Y \left[1 - \dfrac{c}{\left(\alpha + \dfrac{1}{\alpha}\right)^2 \left[\dfrac{\pi^2}{12\left(1-\nu^2\right)B^2}\right] + c}\right] & if\ \sigma_s > \sigma_{pr} \end{cases} \tag{4.1}$$

where

$$c = \frac{\sigma_{pr}(\sigma_Y - \sigma_{pr})}{\sigma_Y^{\ 2}} \tag{4.2}$$

$$B = \frac{b}{h}\sqrt{\frac{\sigma_Y}{E}}, \text{ the plate slenderness ratio} \tag{4.3}$$

σ_Y is the yield stress, ν is Poisson's ratio, α is the aspect ratio, σ_{pr} is the proportional limit stress, *b* is the plate breadth, *a* is the length of the plate, *h* is the plate thickness, and *E* is Young's modulus.

Ultimate Strength Model for Plates under Uniaxial Compression Stress

The ultimate buckling stress σ_u of an unstiffened panel under uniaxial compression is given by

$$For\ a/b > 1.0$$

$$\sigma_u = \begin{cases} \sigma_Y\sqrt{\dfrac{\pi^2}{3(1-\nu^2)B^2}} & if\ B \ge 3.5 \\ \sigma_Y\left(\dfrac{2.25}{B} - \dfrac{1.25}{B^2}\right) & if\ 1.0 \le B < 3.5 \\ \sigma_Y & if\ B < 1.0 \end{cases} \tag{4.4}$$

$$For\ a/b < 1.0$$

$$\sigma_u = \sigma_Y\left[\alpha C_u + 0.08(1-\alpha)\left(1+\frac{1}{B^2}\right)\right]^2$$

where

$$C_u = \begin{cases} \sqrt{\dfrac{\pi^2}{3(1-\nu^2)B^2}} & if\ B \ge 3.5 \\ \left(\dfrac{2.25}{B} - \dfrac{1.25}{B^2}\right) & if\ 1.0 \le B < 3.5 \\ 1 & if\ B < 1.0 \end{cases} \tag{4.5}$$

It is important to note that this model implicitly accounts for the observed tendencies of plate elements to behave as structures that are neither fully clamped nor fully simply supported.

Ultimate Strength Model for Stiffened Plates Under Uniaxial Compression

The ultimate stress due to uniaxial compression of a longitudinally stiffened panel may be written as

$$\sigma_u = \begin{cases} m\sigma_Y\left[0.5+0.5\left(1-\dfrac{a}{r\pi}\sqrt{\dfrac{\sigma_Y}{E}}\right)\right] & for\ \dfrac{b}{h} \le 45 \\ m\sigma_Y\left[0.5+0.5\left(1-\dfrac{a}{r\pi}\sqrt{\dfrac{\sigma_Y}{E}}\right)\right]\left[1-0.007\left(\dfrac{b}{h}-45\right)\right] & for\ \dfrac{b}{h} > 45 \end{cases} \tag{4.6}$$

where the radius of gyration, r, for the plate and stiffener is given by

$$r = \sqrt{\frac{I}{A}} \tag{4.7}$$

and m is a coefficient depending on the level of imperfections and residual stresses in the panel. For an average amount of imperfections and residual stress, $m = 1.0$.

Chapter 5

Numerical Results and Analysis

The use of TVR as a maintenance and repair-scheduling tool is demonstrated on a section of keel structure on a frigate. Details of the relevant physical parameters and the probabilistic characteristics of those modeled as random variables are given in Table 5-1. A Microsoft® Excel™ worksheet provided by Mansour and Jensen [9] was used to determine the Weibull fit parameters. The scale parameter, γ_L, was found to be 15.9x10^6 ft-lbf, and the exponent, α_L, was found to be 0.8923. Soares [14] proposes the mean applied load over the lifecycle, μ_{ML}, to be the stillwater bending moment of approximately 50x10^6 ft-lbf.

The TVR expression incorporating corrosion effects is applied to the serviceability and ultimate strength models presented in Chapter 4. The results are displayed in Figure 5-1 together with the effects of corrosion on the original plate scantling.

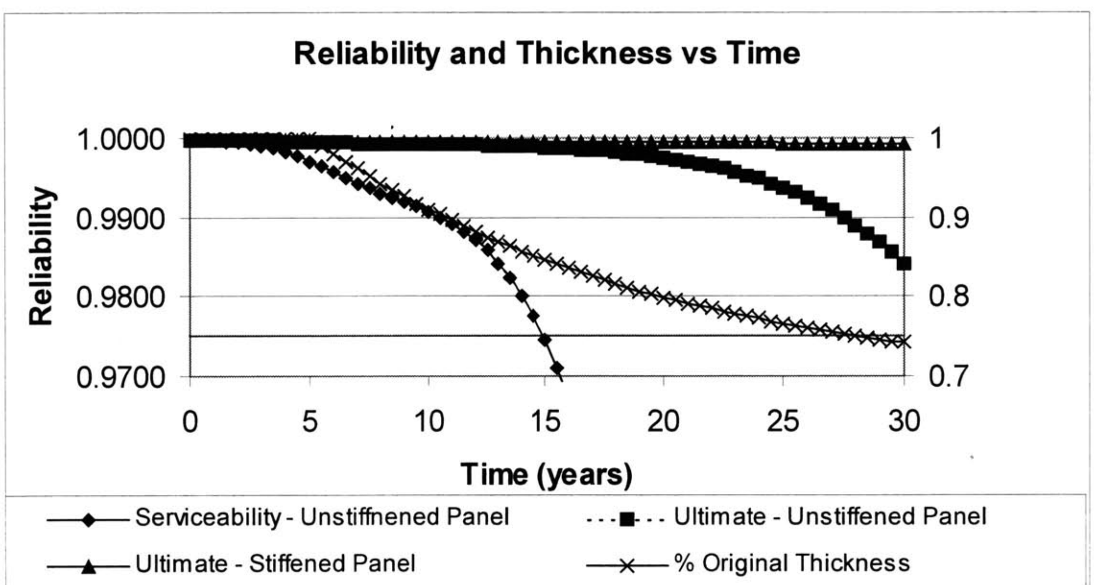

Figure 5-1 Corrosion impact on reliability and thickness over time

The results show explicitly the existence of two evaluation criteria, namely maintaining sufficient structural reliability and maintaining a minimum thickness. Assuming the plate is to be replaced when the remaining material is 75% of the original thickness, repair would be necessary at year 15. Given the average 2.5 year inspection interval for most commercial ships and military surface combatants, six inspections would be carried out before the repair. In contrast, four inspections would be held, with detection at year 16, given a four year inspection interval.

Determining whether or not there is sufficient reliability is difficult to assess when presented in terms of percent probability. Time-invariant reliability formulations such as those found in Load and Resistance Factor Design codes are often established to maintain a target reliability index. This index is generally specific to groups of similar types of structures and is calibrated to existing structures that were built to an established design code and proved reliable over its service life. For most ship structural components, target reliability levels are between 2 and 5 [2].

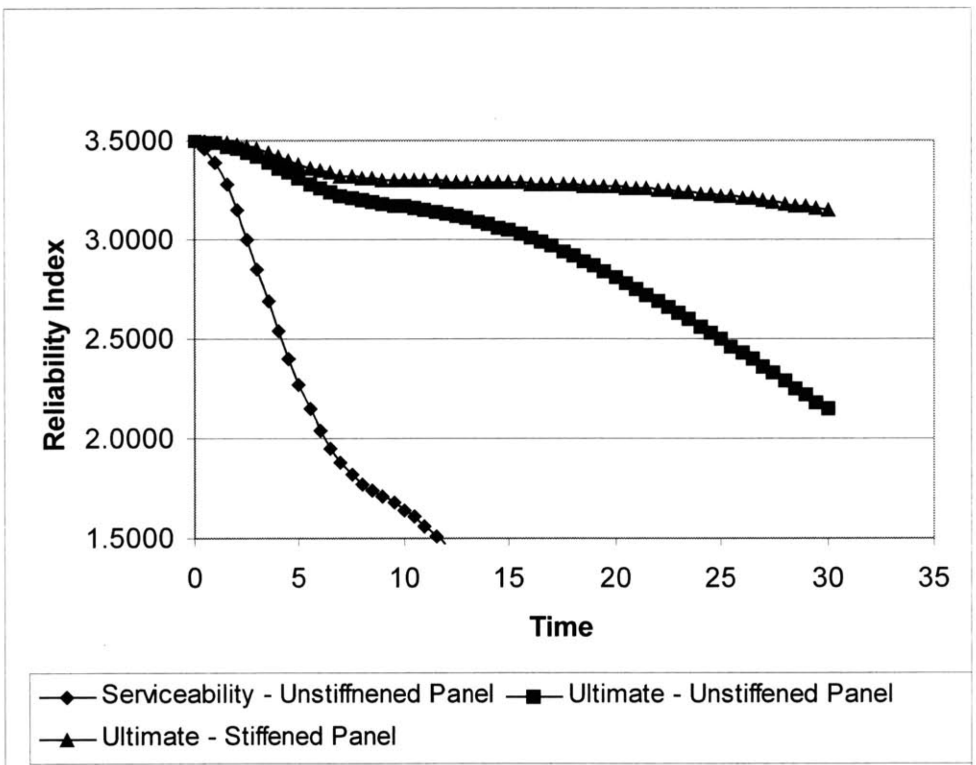

Figure 5-2 Reliability Index of panels over time.

Material/Ship Variables	Nominal	Mean	COV	Distribution
Plate Thickness, h (in)	0.375	nominal	0.0172/h	normal
Plate Length, a (ft)	8	nominal	0.106/ (a-0.037)	normal
Plate Breadth, b(in)	27	nominal	0.093/ (b-0.013)	normal
Stiffener Web Depth, d_w (in)	7.89	nominal	0.0187	normal
Stiffener Web Thickness, t_w (in)	0.17	nominal	0.0904	normal
Stiffener Flange Breadth, f_w (in)	3.94	nominal	0.0161	normal
Stiffener Flange Thickness, t_f (in)	0.205	nominal	0.0917	normal
Ship Length, L (ft)	408	nominal	0.000408	normal
Ship Depth, D (ft)	30	nominal	0.000326	normal
Ship Breadth, B (ft)	46.96	nominal	0.000185	normal
Ship Draft, T (ft)	17			deterministic
Block Coefficient, C_b	0.6			deterministic
Yield Strength-mild steel, σ_y (ksi)	34	37.3	0.068	lognormal
Young's modulus, E (ksi)	29696	nominal	0.0179	normal
Poison's Ratio, ν	0.3			deterministic
Distance from NA, Y (ft)	17	nominal	0.0085	lognormal
Corrosion Prevention Eff., τ_c (yrs)	5	nominal	0.4	normal
Transition Time, τ_t (years)	15.2			deterministic
Wastage Limit, d_{inf} (in)	0.2	nominal	0.3	normal
Mean Bending Moment, P_T (ft-lb)	$50*10^6$			
Distance from NA, Y (ft)	17	nominal	0.0085	lognormal

Table 5-1 Relevant parameters and their probabilistic characterstics.

Of the three components/conditions evaluated, the reliability of serviceability strength for unstiffened panels in Figure 5-2 nears the index limit of 2.0 at the 6-year mark, less than one-fourth of the time at which corrosion wastage reached its limit in Figure 5-1. The reliability of the ultimate strength for unstiffened and stiffened panels is satisfactory well after the mandatory plate replacement period as dictated by the corrosion wastage limit. Reaching the target reliability level for serviceability strength does not necessarily denote a mandatory inspection checkpoint, since a small degree of permanent set is permissible in unstiffened panels. It is, however, a good indicator that inspection resources should utilized in this area to evaluate potential working in the area.

It is not feasible for vessel owners to schedule structural maintenance and inspection activities on the element or component level. To be effective, the structural TVR of a wide variety of component types from a variety of locations should be evaluated whereby a representative picture of the reliability and corrosion effects of the entire structural system may be created.

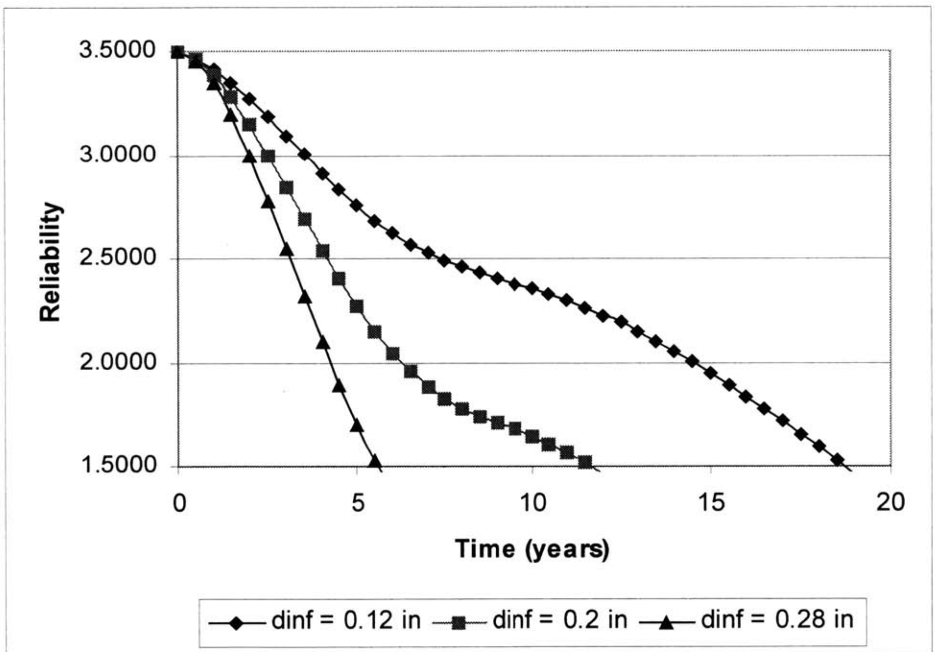

Figure 5-3 Reliability Sensitivity to d_∞

As discussed in Chapter 3, the corrosion parameters, d_∞, τ_t, and τ_c depend on function and location of the structural element. To further demonstrate the usefulness of TVR in structural maintenance and inspection scheduling, a sensitivity study to changes in the corrosion parameters is presented, keeping all of the design variables in Table 5-1 constant except for the corrosion parameter indicated.

A comparison between Figures 5-3 and 5-4 indicates how strongly sensitive reliability is to changes in d_∞. A 0.08 in. increase in d_∞ brings the structural reliability to its limit in a third of the time. The plate thickness reaches its limit in half the time, as shown in Figure 5-4. Shorter inspection intervals may be necessary to ensure structural adequacy. An identical relationship can be established between structural reliability and the transition time, τ_t, as evidenced in Figures 5-5 and 5-6.

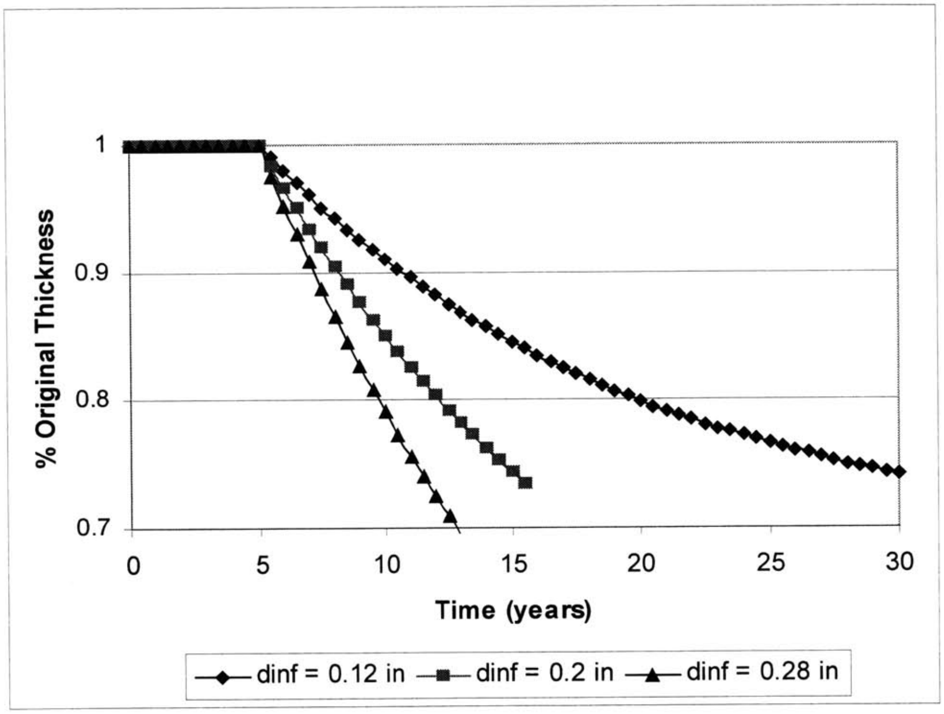

Figure 5-4 Thickness Sensitivity to d_∞.

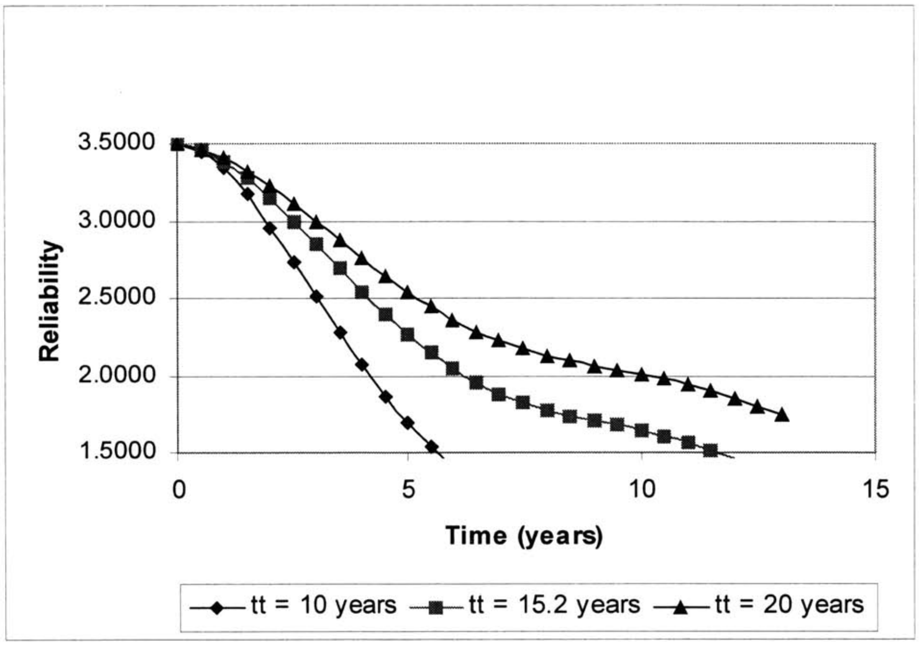

Figure 5-5 Reliability sensitivity to the transition time.

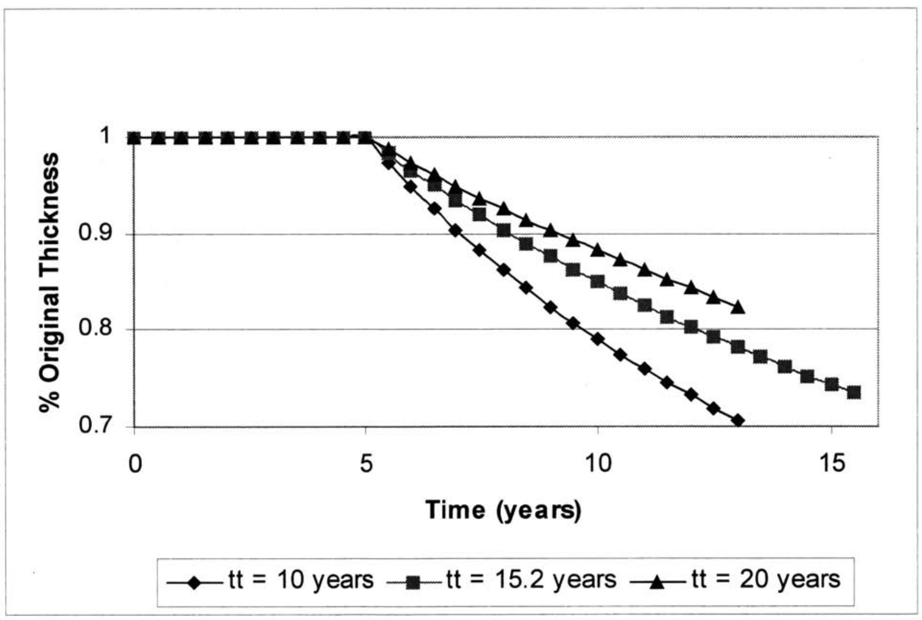

Figure 5-6 Thickness sensitivity to the transition time.

Chapter 6

Conclusions and Recommendations

A number of conclusions may be made regarding the use of time variant reliability techniques for maintenance and inspection scheduling. Perhaps chief of these is recognition of the fact that time variant reliability, incorporating corrosion effects, is only one tool; one piece of the probabilistic puzzle which defines the uncertainty of a structural system to perform safely through its service life. Maintenance and inspection schedules established by the tens, even hundreds, of years' experience by proven engineers should not be tossed aside but rather used as a compass by which to gauge new techniques.

The theory of time variant reliability combined with a newly researched and increasingly popular corrosion model has been presented. A numerical example of time variant reliability demonstrated its usefulness in projecting the expected performance of a ship structure based upon ship-specific, operational profiles. The technique provides two separate evaluation criteria, a plate thickness limit and a minimum reliability index, whereby decisions may be made as to when and how inspections and maintenance should be carried out.

When using probabilistic analysis techniques, an obvious limitation is a model that is a function of highly uncertain random variables. In this application, the corrosion parameters such as d_∞ and τ_c had coefficients of variance of 0.3 and 0.4, respectively. Continued data collection and analysis of general corrosion would help improve the accuracy of time variant reliability methods.

Implementing the proposed time variant reliability method including corrosion effects with other structural effects such as fatigue would provide a broader, and perhaps more accurate, picture of structural performance over time. General corrosion and fatigue are both easily implemented using extreme value theory. To date, general corrosion and fatigue have been approached separately.

An obvious recommendation for future study in the application of time variant reliability to maintenance and inspection schedules would be the application of these techniques to an existing vessel nearing the end of its expected service life and comparing the results to documented maintenance and inspection data. If the evaluation of the time variant reliability of structural elements compares favorably with the maintenance and repair actions conducted over the life cycle, a strong argument could be made for extending the inspection intervals for ship structures based on the ability to adequately predict the extreme value performance over the lifecycle.

Recognizing that increased inspection intervals may not be well received with regulatory bodies responsible for vessel safety, another recommendation would be to evaluate how large the structural elements should be sized so as to avoid maintenance and repair until sometime given time T of the vessel's service life. This could take the form of amplified load and resistance factors in the increasingly popular LRFD design codes.

Bibliography

[1] American Bureau of Shipping (ABS). *Rules for Building and Classing Steel Vessels.* 1999.

[2] Assakaf, Ibrahim Alawi. *Reliability-Based Design of Panels and Fatigue Details of Ship Structures.* Diss. U. of MD, College Park, 1998.

[3] Atua, Khaled Ibrahim. *Reliability-Based Structural Design of Ships Hull Girders and Stiffened Panels.* Diss. U. of MD, College Park, 1998.

[4] Ayyub, Bilal M. and Richard H. McCuen. *Probability, Statistics, and Reliability for Engineers.* Boca Raton: CRC Press, 1997.

[5] Ayyub, Bilal M. and White, G.J. "Life Expectancy of Marine Structures." *Marine Structures* 3 (1990):301-317.

[6] Castillo, Enrique. *Extreme Value Theory in Engineering.* Boston: Academic Press, 1988.

[7] Flinn, Richard A. and Paul K. Trojan. *Engineering Materials and Their Applications.* Boston: Houghton Mifflin Company, 1990.

[8] Hughes, Owen F. *Ship Structural Design.* Jersey City, NJ: The Society of Naval Architects and Marine Engineers, 1988.

[9] Jensen, Jorgen Juncher and Alaa E. Mansour. "Estimation of Long-Term wave-induced Bending Moment Using Closed Form Expressions." (October, 29, 2001): < www.ish.dtu.dk/reports/articles/jjj/estimation_01.htm>.

[10] Liu, Donald (ABS). "Double Hull Tankers...What We Have Learned." American Institute of Marine Underwriters (AIMU) Seminar. New York City, NY. May 7, 2001.

[11] Naess, A. "On the Long-Term Statistics of Extremes." *Applied Ocean Research* 6 (1984):227-228.

[12] Paik, Jeom Kee, et al. "Ship Hull Ultimate Strength Reliability Considering Corrosion" *Journal of Ship Research* 42.2 (1998): 154-165.

[13] Royal Institute of Naval Architects (RINA) "The Search, Assessment, and Survey." *The Naval Architect* (May 1996): 44-48.

[14] Soares, C. Guedes. "Ship Structural Reliability." *Risk and Reliability in Marine Technology* Ed. C.Guedes Soares Rotterdam: A.A.Balkema, 1998. 227-244.

[15] Soares, C. Guedes. "Stochastic Modeling of Waves and Wave Induced Loads." *Risk and Reliability in Marine Technology*. Ed. C.Guedes Soares Rotterdam: A.A.Balkema, 1998. 197-212.

[16] Soares, C. Guedes. "Combination of Primary Load Effects in Ship Structures" *Probabilistic Engineering Mechanics* 7 (1992):103-111.

[17] Soares, C. Guedes and S. Dias. "Probabilistic Models of Still-Water Load Effects in Containers" *Marine Structures* 9 (1996): 287-312.

[18] Soares, C. Guedes and Y. Garbatov. "Reliability Assessment of Maintained Ship Hulls with Correlated Corroded Elements." *Marine Structures* 10 (1997): 629-653.

[19] Soares, C. Guedes and Y. Garbatov. "Reliability of Maintained, Corrosion protected Plates Subjected to Non-linear Corrosion and Compressive Loads" *Marine Structures* 12 (1999): 425-445.

[20] Soares, C. Guedes and A.P. Teixeira. "Structural Reliability of Two Bulk Carrier Designs" *Marine Structures* 13 (2000): 107-128.

[21] Stewart, Mark G. and Robert E. Melchers. *Probabilistic Risk Assessment of Enginneering Systems*. London: Chapmand & Hall, 1997.

[22] Sun, Hai-Hong and Yong Bai. "Time Variant Reliability of FPSO Hulls" *SNAME Transactions* 109 (2001).

[23] Wang, Xiaozhi and Torgeir Moan. "Stochastic and Deterministic Combinations of Still Water and Wave Bending Moments in Ships" *Marine Structures* 9 (1996): 787-810.

[24] Wirsching, Paul H., James Ferensic, and Anil Thayamballi. "Reliability with Respect to Ultimate Strength of a Corroding Ship Hull" *Marine Structures* 10(1997):501-518.

[25] Yamamoto N. "Reliability Based Criteria for Measures to Corrosion." *Proc.17th OMAE'98*, New York, USA: ASME. 1998.

Appendix A

An Excerpt of Jensen and Mansour's [9] Microsoft® Excel-based Extreme Value Bending Momement Estimator

Four Parts

Long-term linear predictions: Determination of the individual peak value distribution						
Standard deviations are calculated using the analytical expressions with adjusted coefficients:						
a, b, c, d, f	st. dev. New :	3000	14.1	0.8	6.9	-0.13
g, h, k, l, m	F(T/L):	-0.75	0.15	0.95	-5.4	1.14
a, b, c, d, f	st. dev. Sikora:	3000	14.8	0.71	7	-0.14

Formula for st. dev. w/o T/L, Cb and Fn dependence: a*exp(-b*u^(-c))*u^(-d-f*u); u=Tz*(g/Lcos(head))^½
F(T/L) (only new RAO): exp(g*v)*(1+h*v)*(k+l*T/L+m*(T/L)^2); v = 4pi^2*T/L/Tz_bar^2
F(Cb) = (1-r)^2+r*(2-r)*s; r = (1-Cb)/(1-s); s = Cbmin = 0.6. F(Fn) =1+n*Fn^2; with n: 3

	Operational diagram:					
Length (m):	V (knots)	Heading	Hs = 1 - 5 m	Hs = 6 - 10 m	Hs = 11 -16 m	
124.4	5	0	0.08	0.079	0.075	Only input
Cb (min=0.6):	5	45	0.118	0.227	0.1	in the yellow
0.6	5	90	0.087	0.095	0.075	boxes on
Draught (m):	5	135	0.093	0.037	0.075	page 1!
5.49	5	180	0.054	0.054	0.05	
Breadth (m):	15	0	0.083	0.083	0.1	
14.31	15	45	0.132	0.136	0.1	
	15	90	0.096	0.128	0.15	
Press	15	135	0.101	0.054	0.075	
	15	180	0.059	0.045	0.1	
Ctrl-Shift A	25	0	0.015	0.0083	0.025	
	25	45	0.023	0.0083	0.025	
to opdate all	25	90	0.019	0.0289	0.025	
results!	25	135	0.021	0.0083	0	
	25	180	0.014	0.0042	0.025	
	35	0	0.0007	0	0	
T/L:	35	45	0.0012	0	0	
0.044131833	35	90	0.001	0	0	
F(Cb):	35	135	0.0012	0	0	
0.6	35	180	0.0005	0	0	
1	Sum		0.9996	0.996	1	
	Sum excl. beam sea		0.7966	0.7441	0.75	
Operational time (years):		20	corresponds to	69490420 peaks		
Effective number of peaks (see Note 2):				54936995		
Bow flare coefficient Cf (DNV definition):			0.3			
Relative number of peaks with whipping (0-1):			0.03	Reasonable values: 0.001 to 0.02		
St. dev. Whipping/rho*g*B*L*Hs^2*Cf/Cb:			0.03	Reasonable values: 0.02 to 0.05		

Note 1:
The DNV North Atlantic scatter diagram is used. No values for Hs>15m.

Note 2:
There is no difference between head and following seas in the present formulations.
The response is zero in beam sea and therefore the operational weight factors are scaled to give 1.00 without beam sea conditions, but the effective number of peaks are scaled down accordingly.

Note 3:
The long-term weighting does account for the difference in wave peak rates in each sea state.

The probability that the individual long-term peak values exceed a given value is determined.
As guidance the short-term most probable value among 1000 peaks in a stationary state with

Hs (m):	10	Tz (sec):	14	V (knots):	12	0.1767
in head sea becomes:		154.7172	MNm (using the new formulas of the RAO)			
Max peak value (MNm):	700	The nonlinear range is increased automatically by 50%.				

Figure A-1 Part 1 of Extreme Value Bending Moment Estimator [9].

Scatter diagram: DNV North Atlantic for extreme value calculations. Limited to Hs =15m, Tz = 15 sec.

Hs	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Tz													Total sum:		99894
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	73	5	0	0	0	0	0	0	0	0	0	0	0	0	0
6	1416	356	62	12	2	1	0	0	0	0	0	0	0	0	0
7	4594	3299	1084	318	89	25	7	2	1	0	0	0	0	0	0
8	4937	8001	4428	1898	721	254	85	27	8	3	1	0	0	0	0
9	2590	8022	6920	4126	2039	896	363	138	50	17	6	2	1	0	0
10	839	4393	5566	4440	2772	1482	710	312	128	50	18	7	2	1	0
11	195	1571	2791	2889	2225	1418	791	398	184	80	33	13	5	2	1
12	36	414	993	1301	1212	907	580	330	171	82	37	15	6	2	1
13	6	87	274	445	494	428	311	197	113	59	29	13	6	2	1
14	1	16	63	124	162	160	131	92	58	33	17	8	4	2	1
15	0	3	12	30	45	50	46	35	24	15	8	4	2	1	0
Sum	14687	26167	22193	15583	9761	5621	3024	1531	737	339	149	62	26	10	4

Aux. results

0	0
0	0
0	0
0	0
78	15.6
1849	308.2
9419	1346
20363	2545
25170	2797
20720	2072
12596	1145
6087	507.3
2465	189.6
872	62.29
275	18.33
99894	11006

Average zero upcrossing rate (1/sec):	0.11
Corr. zero upcrossing period Tz (sec):	9.076

Figure A-2 Part 2 of Extreme Value Bending Moment Estimator [9].

rho*g*B*L^2:	2226.759261	MNm/m			3.931446311	0.707106781
sqrt(g/L):	0.280817594	1/sec			0.112721563	1.742254925
Fn/knots:	0.014726305	1/knots				
		Standard deviation/Hs (MNm/m) vs. heading, V=0				
		180	180		135	135
Tz (sec)	Tz_bar	New	Sikora	Tz__bar	New	Sikora
5	1.404087968	3.674026171	5.899164872	1.669751402	4.181485245	6.333599043
6	1.684905562	5.586343716	7.150116873	2.003701682	5.356486725	6.642331738
7	1.965723156	6.634623722	7.466129195	2.337651963	5.637044073	6.259379702
8	2.246540749	6.895233177	7.190050415	2.671602243	5.364300327	5.593297355
9	2.527358343	6.645138759	6.619694493	3.005552523	4.842152886	4.872255627
10	2.808175936	6.131441999	5.945782428	3.339502804	4.253903559	4.200023486
11	3.08899353	5.518457718	5.272744189	3.673453084	3.690419266	3.613519377
12	3.369811124	4.899885424	4.650387685	4.007403365	3.188636071	3.11819646
13	3.650628717	4.322600197	4.097732296	4.341353645	2.757993849	2.706521993
14	3.931446311	3.805946204	3.617841464	4.675303925	2.395415115	2.366803411
15	4.212263905	3.354382173	3.206259012	5.009254206	2.093012445	2.087075496

Speed factors:			3.674026171	4.181485245
Sikora:not used	New	Fn	5.586343716	5.356486725
1.03	1.016264804	0.073631524	6.634623722	5.637044073
1.09	1.146383238	0.220894573	6.895233177	5.364300327
1.15	1.406620104	0.368157622	6.645138759	4.842152886
1.21	1.796975405	0.515420671	6.131441999	4.253903559
			5.518457718	3.690419266
			4.899885424	3.188636071
			4.322600197	2.757993849
			3.805946204	2.395415115
			3.354382173	2.093012445

Hs = 1 - 5 m	P(individual long term peak >		1050 MNm)			
Hs (m):	1	2	3	4	5	Sum
Tz (sec):						
5	0	0	0	0	0	0
6	0	0	1.5234E-270	9.026E-156	4.2738E-103	4.2738E-103
7	0	0	1.525E-192	3.3552E-111	7.3983E-74	7.3983E-74
8	0	0	4.1255E-178	1.1234E-102	5.20157E-68	5.20157E-68
9	0	0	2.9579E-191	7.2871E-110	2.15307E-72	2.15307E-72
10	0	0	5.7866E-224	3.3808E-128	4.9672E-84	4.9672E-84
11	0	0	1.0928E-275	2.5016E-157	1.1521E-102	1.1521E-102
12	0	0	0	1.4847E-198	4.1761E-129	4.1761E-129
13	0	0	0	4.767E-254	1.0021E-164	1.0021E-164
14	0	0	0	0	2.9432E-211	2.9432E-211
15	0	0	0	0	1.1245E-270	1.1245E-270
Sum:	0	0	4.1255E-178	1.1234E-102	5.2018E-68	5.2018E-68

Figure A-3 Part 3 of Extreme Value Bending Moment Estimator [9].

Hs = 6 - 10 m	P(individual long term peak >		1050 MNm)			
Hs (m):	6	7	8	9	10	Sum
Tz (sec):						
5	0	0	0	0	0	0
6	5.0591E-115	0	0	0	0	5.0591E-115
7	2.40058E-82	1.21684E-62	4.88915E-50	2.52624E-41	0	2.52624E-41
8	9.8174E-76	1.87057E-57	8.84888E-46	5.8125E-38	2.03263E-32	2.03264E-32
9	1.1668E-80	7.38406E-61	3.58721E-48	1.2578E-39	1.14435E-33	1.14435E-33
10	8.94574E-94	2.2447E-70	2.44728E-55	3.59953E-45	4.98406E-38	4.98406E-38
11	8.4412E-115	8.99672E-86	4.57514E-67	2.26381E-54	2.05606E-45	2.05606E-45
12	1.1736E-144	1.0381E-107	7.53659E-84	1.31388E-67	4.17797E-56	4.17797E-56
13	6.6008E-185	2.5386E-137	1.5163E-106	1.49771E-85	1.21115E-70	1.21115E-70
14	1.6133E-237	4.7961E-176	3.0355E-136	4.7289E-109	1.05639E-89	1.05639E-89
15	9.8902E-305	1.6178E-225	3.3613E-174	4.2382E-139	4.4419E-114	4.4419E-114
Sum:	9.81752E-76	1.87132E-57	8.88524E-46	5.94081E-38	2.14707E-32	2.14708E-32

Hs = 11 -16 m	P(individual long term peak >		1050 MNm)			
Hs (m):	11	12	13	14	15	Sum
Tz (sec):						
5	0	0	0	0	0	0
6	0	0	0	0	0	0
7	0	0	0	0	0	0
8	7.02174E-28	0	0	0	0	7.02174E-28
9	9.13035E-29	1.26005E-25	4.11388E-23	0	0	4.12649E-23
10	2.74378E-32	1.89086E-28	1.09373E-25	2.29796E-23	0	2.30892E-23
11	2.64716E-38	1.8321E-33	8.50383E-30	5.89292E-27	1.20804E-24	1.21395E-24
12	4.1747E-47	7.6042E-41	4.5746E-36	1.9553E-32	2.01508E-29	2.01703E-29
13	4.09502E-59	6.48473E-51	1.34439E-44	8.50939E-40	7.71416E-36	7.71501E-36
14	6.78759E-75	3.39439E-64	6.46947E-56	2.08309E-49	3.24135E-44	3.24137E-44
15	4.33001E-95	3.39171E-81	1.88975E-70	5.5061E-62	0	5.5061E-62
Sum:	7.93505E-28	1.26194E-25	4.12482E-23	2.29855E-23	1.20807E-24	6.55688E-23

Specify exceedance level (MNm):		1050
Probability(individual long-term peaks>	1050 MNm) :	6.55688E-23
Probability(max long-term peak >	1050 MNm) :	3.55271E-15
Fill in this grey box only if a single value of the probability of exceedence is needed!		
Both the linear and non-linear calculation are performed using the RAO specified in F72:G82		

Figure A-4 Part 4 of Extreme Value Bending Moment Estimator [9].

Appendix B

Sample Mathcad Worksheet for Reliability Calculations

Thesis Calculations

Physical Dimensions (nominal values for random variables):

$h := 0.375$ in $a := 8$ ft $b := 27$ in $d_w := 3$ in $f_w := 3$ in $t_w := \frac{1}{8}$ in $t_f := \frac{1}{8}$ in

$L := 408$ ft $D := 30$ ft $B := 46.96$ ft

Material Properties (random variables, unless otherwise noted):

$F_{yOS} := 34$ ksi $F_{yHS} := 48$ ksi $F_u := 61.6$ ksi $E := 29696$ ksi $\nu := 0.3$ deterministic

Loading/Location Parameters (random variables, unless otherwise noted):

$Y := 17$ ft $P_T := 30 \cdot 10^6$ lbf-ft $c := 0.6$ deterministic

Enter Yield Stress for Ordinary Strength (OS) steel and High Strength (HS) steel:

$\sigma_y := 34$ ksi

Probabilistic Data

Physical Dimensions:

$h_{mean} := h$ in $h_{SD} := 0.0172$ in $a_{mean} := a$ ft $a_{SD} := \frac{0.106}{(a - 0.037)} \cdot a$ ft

$b_{mean} := b$ in $b_{SD} := \frac{0.093}{(b - 0.013)} \cdot b$ in $d_{wmean} := d_w$ in $d_{wSD} := 0.0187 d_w$ in

$f_{wmean} := f_w$ in $f_{wSD} := 0.0161 \cdot f_w$ in $t_{wmean} := t_w$ in $t_{wSD} := 0.0904 t_w$ in

$t_{fmean} := t_f$ in $t_{fSD} := 0.0917 t_f$ in $L_{mean} := L$ ft $L_{SD} := 0.000408 L$ ft

$D_{mean} := D$ ft $D_{SD} := 0.000326 D$ ft $B_{mean} := B$ ft $B_{SD} := 0.000185 B$ ft

$Y_{mean} := Y$ ft $Y_{SD} := 0.0085 Y$ ft

$$Y_{SDln} := \sqrt{\ln\left[1 + \left(\frac{Y_{mean}}{Y_{SD}}\right)^2\right]} \qquad Y_{meanln} := \ln(Y_{mean}) - 0.5 \cdot Y_{SDln}^{\,2}$$

For log normal dist. parameters.

Material Properties:

$\sigma_{ymean} := 37.3$ ksi $\sigma_{ySD} := 2.5364$ ksi

$$\sigma_{ySDln} := \sqrt{\ln\left[1 + \left(\frac{\sigma_{ySD}}{\sigma_{ymean}}\right)^2\right]} \qquad \sigma_{ymeanln} := \ln(\sigma_{ymean}) - 0.5 \cdot \sigma_{ySDln}^{\,2}$$

For log normal dist. parameters.

Figure B-1 Part 1 of Mathcad worksheet

Material Properties (cont.):

$\sigma_{umean} := 61.6$ ksi $\sigma_{uSD} := 2.9568$ ksi

$E_{mean} := 29696$ ksi $E_{SD} := 531.5584$ ksi $\nu := 0.3$ deterministic

Loading Properties:

$P_{Tmean} := P_T$ lbf-ft $P_{TSD} := 0.12 \cdot P_T$ lbf-ft

Corrosion Properties:

$d_{inf} := \frac{4}{7}$ in $d_{SD} := 0.3 \cdot d_{inf}$ in $\tau_{cmean} := 5.5$ yrs $\tau_{cSD} := 0.4 \cdot \tau_{cmean}$ yrs

$\tau_t := 15.2$ yrs

Probability distributions:

"dnorm" is the normal probability distribution function.
"dlnorm" is the lognormal probability density function.

$pdf_h(x) := dnorm(x, h_{mean}, h_{SD})$ $pdf_a(x) := dnorm(x, a_{mean}, a_{SD})$

$pdf_b(x) := dnorm(x, b_{mean}, b_{SD})$ $pdf_{dw}(x) := dnorm(x, d_{wmean}, d_{wSD})$

$pdf_{fw}(x) := dnorm(x, f_{wmean}, f_{wSD})$ $pdf_{tw}(x) := dnorm(x, t_{wmean}, t_{wSD})$

$pdf_{tf}(x) := dnorm(x, t_{fmean}, t_{fSD})$ $pdf_L(x) := dnorm(x, L_{mean}, L_{SD})$

$pdf_D(x) := dnorm(x, D_{mean}, D_{SD})$ $pdf_B(x) := dnorm(x, B_{mean}, B_{SD})$

$pdf_{\sigma y}(x) := dlnorm(x, \sigma_{ymeanln}, \sigma_{ySDln})$ $pdf_E(x) := dnorm(x, E_{mean}, E_{SD})$

$pdf_Z(x) := dlnorm(x, Y_{meanln}, Y_{SDln})$ $pdf_{PT}(x) := dnorm(x, P_{Tmean}, P_{TSD})$

To facilitate numerical integration, apply truncated distributions or modify limits of integration and ensure that the integral ocer the range is 1.

$b_{max} := b_{mean} + 3.5 b_{SD}$ $b_{min} := b_{mean} - 3.5 b_{SD}$ $\int_{b_{min}}^{b_{max}} pdf_b(x)\,dx = 1$

$h_{max} := h_{mean} + 3.5 \cdot h_{SD}$ $h_{min} := h_{mean} - 3.5 \cdot h_{SD}$ $\int_{h_{min}}^{h_{max}} pdf_h(x)\,dx = 1$

$d_{max} := d_{inf} + 3.5 \cdot d_{SD}$ $d_{min} := d_{inf} - 3.5 \cdot d_{SD}$

Figure B-2 Part 2 of Mathcad worksheet

Probability distributions (cont.):

$$pdf_{dinf}(x) := \frac{dnorm(x, d_{inf}, d_{SD})}{(1 - pnorm(0, d_{inf}, d_{SD}))}$$

truncated normal distribution (no negative values allowed)

$$\int_0^{\infty} pdf_{dinf}(x)\,dx = 1$$

$$\sigma_{ymax} := \sigma_{ymean} + 4\sigma_{ySD} \qquad \sigma_{ymin} := \sigma_{ymean} - 3.5 \cdot \sigma_{ySD} \qquad \int_{\sigma_{ymin}}^{\sigma_{ymax}} pdf_{\sigma y}(x)\,dx = 1$$

$$\tau_{cmax} := \tau_{cmean} + 3.5 \cdot \tau_{cSD} \qquad pdf_{\tau c}(x) := \frac{dnorm(x, \tau_{cmean}, \tau_{cSD})}{1 - pnorm(0, \tau_{cmean}, \tau_{cSD})}$$

truncated normal distribution (no negative values allowed)

$$\int_0^{\tau_{cmax}} pdf_{\tau c}(x)\,dx = 1$$

$$E_{max} := E_{mean} + 3.5 \cdot E_{SD} \qquad E_{min} := E_{mean} - 3.5 \cdot E_{SD} \qquad \int_{E_{min}}^{E_{max}} pdf_E(x)\,dx = 1$$

$$P_{Tmax} := P_{Tmean} + 3.5 \cdot P_{TSD} \qquad P_{Tmin} := P_{Tmean} - 3.5 \cdot P_{TSD} \qquad \int_{P_{Tmin}}^{P_{Tmax}} pdf_{PT}(x)\,dx = 1$$

Time Variant Calculations

Moment of Inertia (Izz) $I := 162853$ ft2-in2 As determined from POSSE

Weibull parameters of combined vertical bending moment (SWBM and VWBM) as estimated by Jensen and Mansour, scale parameter and exponent, respectively:

$u := 12.1 \cdot 10^6$ lbf-ft $\quad k := 0.9175$

Converting stress into bending moment using eqn (3.6), use the following:

$$SM(t) := \frac{I}{Y} \cdot (1 - 0.01 \cdot t)$$

Evaluation of Strength Models:

Serviceability Strength Model for Unstiffened Panels Under Uniaxial Compression (Mansour):

Corrosion effects:

$\tau_c := 5$ yrs $\quad pdf_{\tau}(x) := dnorm(x, 5.5, 0.4) \quad F_{\tau c}(x) := pnorm(x, 5.5, 0.4)$

$$d(t, h, d_{inf}) := h \cdot d_{inf} \cdot \left[1 - e^{\frac{-(t)}{\tau_t}} \right]$$

Figure B-3 Part 3 of Mathcad worksheet

Serviceability Strength Model for Unstiffened Panels Under Uniaxial Compression (Mansour) (cont.):

Plate Slenderness Ratio (PSR):

$$PSR\left(t,b,h,\sigma_y,E,d_{inf}\right) := \frac{b}{h-d\left(t,h,d_{inf}\right)}\cdot\sqrt{\frac{\sigma_y}{E}}$$

Plate aspect ratio (α): $\alpha := \dfrac{a}{\frac{b}{12}}$ $\qquad c_1 := 0.24$

Proportional limit stress in compression (Fpr): $\sigma_{pr} := 0.6\cdot\sigma_y\cdot 1000$

a. For $\alpha > 1.0$

$$\zeta_{s1}\left(t,b,h,\sigma_y,E,d_{inf}\right) := \sigma_y\cdot\frac{\pi^2}{3\cdot\left(1-\nu^2\right)\cdot\left(PSR\left(t,b,h,\sigma_y,E,d_{inf}\right)\right)^2}\cdot 1000 \qquad \text{if } \zeta_s \le \sigma_{pr_i}$$

$$\zeta_{s2}\left(t,b,h,\sigma_y,E,d_{inf}\right) := \sigma_y\cdot\frac{\left[\dfrac{\pi^2}{3\cdot\left(1-\nu^2\right)\cdot\left(PSR\left(t,b,h,\sigma_y,E,d_{inf}\right)\right)^2}\right]^2}{\left[\dfrac{\pi^2}{3\cdot\left(1-\nu^2\right)\cdot\left(PSR\left(t,b,h,\sigma_y,E,d_{inf}\right)\right)^2}\right]^2+c_1}\cdot 1000 \qquad \text{if } \zeta_s > \sigma_{pr_i}$$

Time Dependent Structural Resistance:

$$\begin{aligned}\zeta_s\left(t,b,h,\sigma_y,E,d_{inf}\right) := {} & \zeta_{s1}\left(t,b,h,\sigma_y,E,d_{inf}\right)\cdot\left(\zeta_{s1}\left(t,b,h,\sigma_y,E,d_{inf}\right) < \sigma_{pr}\right) \ldots \\ & + \zeta_{s2}\left(t,b,h,\sigma_y,E,d_{inf}\right)\cdot\left(\zeta_{s2}\left(t,b,h,\sigma_y,E,d_{inf}\right) \ge \sigma_{pr}\right)\end{aligned}$$

Upcrossing rates:

$$\nu_a\left(t,b,h,\sigma_y,E,d_{inf},P_T\right) := \int_0^t e^{-\left(\frac{\zeta_s\left(\tau,b,h,\sigma_y,E,d_{inf}\right)\cdot SM(\tau)-P_T}{u}\right)^k}\,d\tau$$

$$\nu_b\left(t,b,h,\sigma_y,E,d_{inf},P_T\right) := t\cdot e^{-\left(\frac{\zeta_s\left(t,b,h,\sigma_y,E,d_{inf}\right)\cdot SM(t)-P_T}{u}\right)^k}$$

Figure B-4 Part 4 of Mathcad worksheet

Serviceability Strength Model for Unstiffened Panels Under Uniaxia Compression (Mansour) (cont.):

Time Variant Reliabity:

$$R_a(t,b,h,\sigma_y,E,d_{inf},P_T) := \int_{P_{Tmin}}^{P_{Tmax}}\int_0^{d_{max}}\int_{E_{min}}^{E_{max}}\int_{\sigma_{ymin}}^{\sigma_{ymax}} pdf_{PT}(P_T)\cdot pdf_{dinf}(d_{inf})\cdot pdf_E(E)\cdot pdf_{\sigma y}(\sigma_y)\cdot e^{-\nu_a(t,b,h,\sigma_y,E,d_{inf},P_T)}\, d\sigma_y\, dE\, dd_{inf}\, dP_T$$

$$R_b(t,b,h,\sigma_y,E,d_{inf},P_T) := \int_{P_{Tmin}}^{P_{Tmax}}\int_0^{d_{max}}\int_{E_{min}}^{E_{max}}\int_{\sigma_{ymin}}^{\sigma_{ymax}} pdf_{PT}(P_T)\cdot pdf_{dinf}(d_{inf})\cdot pdf_E(E)\cdot pdf_{\sigma y}(\sigma_y)\cdot e^{-\nu_b(t,b,h,\sigma_y,E,d_{inf},P_T)}\, d\sigma_y\, dE\, dd_{inf}\, dP_T$$

$$R(t,b,h,\sigma_y,E,d_{inf},P_T) := (1 - F_{\tau c}(t))\cdot R_b(t,b,h,\sigma_y,E,d_{inf},P_T)\ \dots$$
$$+ \int_0^t R_b(\tau_c,b,h,\sigma_y,E,d_{inf},P_T)\cdot R_a(t-\tau_c,b,h,\sigma_y,E,d_{inf},P_T)\cdot pdf_{\tau c}(\tau_c)\, d\tau_c$$

Ultimate Strength Model for Unstiffened Panels under Uniaxial Compression (Mansour):

Time Dependent Structural Resistance:

$$\zeta_{u1}(t,b,h,\sigma_y,E,d_{inf}) := \sigma_y\cdot\sqrt{\frac{\pi^2}{3\cdot(1-\nu^2)\cdot(PSR(t,b,h,\sigma_y,E,d_{inf}))^2}}\cdot(PSR(t,b,h,\sigma_y,E,d_{inf}) \ge 3.5)\cdot 100($$

$$\zeta_{u2}(t,b,h,\sigma_y,E,d_{inf}) := \sigma_y\cdot\left[\frac{2.25}{PSR(t,b,h,\sigma_y,E,d_{inf})} - \frac{1.25}{(PSR(t,b,h,\sigma_y,E,d_{inf}))^2}\right]\cdot(1.0 \le PSR(t,b,h,\sigma_y,E,d_{inf}) \le 3.5)\cdot 100($$

$$\zeta_{u3}(t,b,h,\sigma_y,E,d_{inf}) := \sigma_y\cdot 1000(PSR(t,b,h,\sigma_y,E,d_{inf}) < 1.0)$$

$$\zeta_u(t,b,h,\sigma_y,E,d_{inf}) := \zeta_{u1}(t,b,h,\sigma_y,E,d_{inf}) + \zeta_{u2}(t,b,h,\sigma_y,E,d_{inf}) + \zeta_{u3}(t,b,h,\sigma_y,E,d_{inf})$$

Upcrossing Rate:

$$\nu_a(t,b,h,\sigma_y,E,d_{inf},P_T) := \int_0^t e^{-\left(\frac{\zeta_u(\tau,b,h,\sigma_y,E,d_{inf})\cdot SM(\tau)-P_T}{u}\right)^k} d\tau$$

Figure B-5 Part 5 of Mathcad worksheet

Ultimate Strength Model for Unstiffened Panels under Uniaxial Compression (Mansour) (cont.):

$$\nu_b(t,b,h,\sigma_y,E,d_{inf},P_T) := t\cdot e^{-\left(\frac{\zeta_u(t,b,h,\sigma_y,E,d_{inf})\cdot SM(t)-P_T}{u}\right)^k}$$

$$R_a(t,b,h,\sigma_y,E,d_{inf},P_T) := \int_{P_{Tmin}}^{P_{Tmax}}\int_0^{d_{max}}\int_{E_{min}}^{E_{max}}\int_{\sigma_{ymin}}^{\sigma_{ymax}} pdf_{PT}(P_T)\cdot pdf_{dinf}(d_{inf})\cdot pdf_E(E)\cdot pdf_{\sigma y}(\sigma_y)\cdot e^{-\nu_a(t,b,h,\sigma_y,E,d_{inf},P_T)}\,d\sigma_y\,dE\,dd_{inf}\,dP_T$$

$$R_b(t,b,h,\sigma_y,E,d_{inf},P_T) := \int_{P_{Tmin}}^{P_{Tmax}}\int_0^{d_{max}}\int_{E_{min}}^{E_{max}}\int_{\sigma_{ymin}}^{\sigma_{ymax}} pdf_{PT}(P_T)\cdot pdf_{dinf}(d_{inf})\cdot pdf_E(E)\cdot pdf_{\sigma y}(\sigma_y)\cdot e^{-\nu_b(t,b,h,\sigma_y,E,d_{inf},P_T)}\,d\sigma_y\,dE\,dd_{inf}\,dP_T$$

$$R(t,b,h,\sigma_y,E,d_{inf},P_T) := (1-F_{\tau c}(t))\cdot R_b(t,b,h,\sigma_y,E,d_{inf},P_T)\ \ldots$$
$$+\int_0^t R_b(\tau_c,b,h,\sigma_y,E,d_{inf},P_T)\cdot R_a(t-\tau_c,b,h,\sigma_y,E,d_{inf},P_T)\cdot pdf_{\tau c}(\tau_c)\,d\tau_c$$

Ultimate Strength Model for Stiffened Panels Under Uniaxial Compression (Herzog):

Preliminary calculations:

$$A(t,b,h,\sigma_y,E,d_{inf},P_T) := b\cdot(h-d(t,h,d_{inf})) + d_w\cdot t_w + f_w\cdot t_f$$ Area

$$A_1(t,b,h,\sigma_y,E,d_{inf},P_T) := b\cdot\frac{h}{2}\cdot(h-d(t,h,d_{inf})) + d_w\cdot t_w\cdot\left(\frac{d_w}{2}+h\right)\ \ldots$$
$$+ f_w\cdot t_f\cdot\left(\frac{t_f}{2}+d_w+h\right)$$ First moment of area

$$y_{NA}(t,b,h,\sigma_y,E,d_{inf},P_T) := \frac{A_1(t,b,h,\sigma_y,E,d_{inf},P_T)}{A(t,b,h,\sigma_y,E,d_{inf},P_T)}$$ Neutral Axis of stiffened panel

$$A_2(t,b,h,\sigma_y,E,d_{inf},P_T) := b\cdot\left(\frac{h}{2}\right)^2\cdot(h-d(t,h,d_{inf})) + d_w\cdot t_w\cdot\left(\frac{d_w}{2}+h\right)^2\ \ldots$$
$$+ f_w\cdot t_f\cdot\left(\frac{t_f}{2}+d_w+h\right)^2$$

$$I(t,b,h,\sigma_y,E,d_{inf},P_T) := A_2(t,b,h,\sigma_y,E,d_{inf},P_T)\cdot y_{NA}(t,b,h,\sigma_y,E,d_{inf},P_T)^2\ \ldots$$
$$+\frac{b\cdot(h-d(t,h,d_{inf}))^3}{12}+\frac{t_w\cdot d_w^{\,3}}{12}+\frac{f_w\cdot t_f^{\,3}}{12}$$ Moment of inertia

Figure B-6 Part 6 of Mathcad worksheet

Ultimate Strength Model for Stiffened Panels Under Uniaxial Compressior (Herzog) (cont.):

$$r(t,b,h,\sigma_y,E,d_{inf},P_T) := \sqrt{\frac{I(t,b,h,\sigma_y,E,d_{inf},P_T)}{A(t,b,h,\sigma_y,E,d_{inf},P_T)}}$$

Time Dependent Structural Resistance:

$$\zeta_{u1}(t,b,h,\sigma_y,E,d_{inf},P_T) := m\cdot\sigma_y\cdot\left[0.5 + 0.5\cdot\left(1 - \frac{a}{r(t,b,h,\sigma_y,E,d_{inf},P_T)\cdot\pi}\cdot\sqrt{\frac{\sigma_y}{E}}\right)\right]\cdot\left(\frac{b}{h} \le 45\right)\cdot 1000$$

$$\zeta_{u2}(t,b,h,\sigma_y,E,d_{inf},P_T) := m\cdot\sigma_y\cdot\left[0.5 + 0.5\cdot\left(1 - \frac{a}{r(t,b,h,\sigma_y,E,d_{inf},P_T)\cdot\pi}\cdot\sqrt{\frac{\sigma_y}{E}}\right)\right]\cdot\left[1 - .007\cdot\left(\frac{b}{h} - 45\right)\right]\cdot\left(\frac{b}{h} > 45\right)\cdot 100($$

Upcrossing Rate:

$$\nu_a(t,b,h,\sigma_y,E,d_{inf},P_T) := \int_0^t e^{-\left(\frac{\zeta_u(\tau,b,h,\sigma_y,E,d_{inf},P_T)\cdot SM(\tau)-P_T}{u}\right)^k} d\tau$$

$$\nu_b(t,b,h,\sigma_y,E,d_{inf},P_T) := t\cdot e^{-\left(\frac{\zeta_u(t,b,h,\sigma_y,E,d_{inf},P_T)\cdot SM(t)-P_T}{u}\right)^k}$$

Time Variant Reliability:

$$R_a(t,b,h,\sigma_y,E,d_{inf},P_T) := \int_{P_{Tmin}}^{P_{Tmax}}\int_0^{d_{max}}\int_{E_{min}}^{E_{max}}\int_{\sigma_{ymin}}^{\sigma_{ymax}} pdf_{PT}(P_T)\cdot pdf_{dinf}(d_{inf})\cdot pdf_E(E)\cdot pdf_{\sigma y}(\sigma_y)\cdot e^{-\nu_a(t,b,h,\sigma_y,E,d_{inf},P_T)}\, d\sigma_y\, dE\, dd_{inf}\, dP_T$$

$$R_b(t,b,h,\sigma_y,E,d_{inf},P_T) := \int_{P_{Tmin}}^{P_{Tmax}}\int_0^{d_{max}}\int_{E_{min}}^{E_{max}}\int_{\sigma_{ymin}}^{\sigma_{ymax}} pdf_{PT}(P_T)\cdot pdf_{dinf}(d_{inf})\cdot pdf_E(E)\cdot pdf_{\sigma y}(\sigma_y)\cdot e^{-\nu_b(t,b,h,\sigma_y,E,d_{inf},P_T)}\, d\sigma_y\, dE\, dd_{inf}\, dP_T$$

$$R(t,b,h,\sigma_y,E,d_{inf},P_T) := (1 - F_{\tau c}(t))\cdot R_b(t,b,h,\sigma_y,E,d_{inf},P_T)\ \ldots$$
$$+\int_0^t R_b(\tau_c,b,h,\sigma_y,E,d_{inf},P_T)\cdot R_a(t-\tau_c,b,h,\sigma_y,E,d_{inf},P_T)\cdot pdf_{\tau c}(\tau_c)\, d\tau_c$$

Figure B-7 Part 7 of Mathcad worksheet

Appendix C

Data and Graphs

The following raw data and graph sets are test runs using the parameters given in ?? with the corresponding set of varying parameters as dictated in Figure D-1 below.

Varying Parameters				
Test number	d_{inf}	τ_t	τ_c	h
1	0.12	15.2	5	0.375
2	0.2	15.2	5	0.375
3	0.28	15.2	5	0.375
4	0.2	10	5	0.375
5	0.2	20	5	0.375
6	0.2	15.2	3	0.375
7	0.2	15.2	7	0.375
8	0.2	15.2	5	0.3125
9	0.2	15.2	5	0.4375

Figure C-1 Table of Parameters varied for sensitivity study.

Test 1

Time, (years)	Serviceability - Unstiffnened Panel		Ultimate - Unstiffened Panel		Ultimate - Stiffened Panel		% Original Thickness
	Reliability	Reliability Index	Reliability	Reliability Index	Reliability	Reliability Index	
0	0.999767	3.500027	0.999767	3.500027	0.999767	3.500027	1
0.5	0.999735	3.465684	0.999762	3.494206	0.999765	3.497698	1
1	0.999681	3.415043	0.999755	3.486639	0.999762	3.494206	1
1.5	0.999594	3.348687	0.999745	3.474997	0.999757	3.488967	1
2	0.999462	3.270106	0.99973	3.460445	0.99975	3.480818	1
2.5	0.999272	3.183086	0.999711	3.441237	0.99974	3.470341	1
3	0.999007	3.092282	0.999685	3.418536	0.999727	3.456953	1
3.5	0.998655	3.001187	0.999654	3.392925	0.99971	3.440655	1
4	0.998209	2.912857	0.999617	3.364985	0.999689	3.422028	1
4.5	0.99767	2.829693	0.999576	3.336463	0.999666	3.402238	1
5	0.997051	2.753441	0.999531	3.308814	0.999642	3.383029	1
5.5	0.996366	2.684319	0.999486	3.282912	0.999617	3.364985	0.99
6	0.995643	2.623092	0.999442	3.259629	0.999594	3.348687	0.98
6.5	0.994924	2.570632	0.999402	3.239838	0.999574	3.335008	0.97
7	0.994223	2.525485	0.999366	3.222958	0.999556	3.32424	0.961
7.5	0.993597	2.489141	0.999335	3.209279	0.999543	3.315508	0.951
8	0.993014	2.458	0.999308	3.197929	0.999533	3.309397	0.943
8.5	0.992477	2.431261	0.999285	3.18847	0.999525	3.305031	0.934
9	0.99192	2.405268	0.999264	3.180321	0.99952	3.302121	0.926
9.5	0.991345	2.380038	0.999245	3.1729	0.999516	3.299792	0.918
10	0.990731	2.354682	0.999226	3.165478	0.999514	3.298337	0.91
10.5	0.990024	2.327251	0.999205	3.157766	0.999512	3.297173	0.903
11	0.989213	2.297784	0.999183	3.149617	0.99951	3.2963	0.896
11.5	0.98825	2.265188	0.999157	3.14074	0.999508	3.295427	0.889
12	0.987078	2.228553	0.999129	3.13099	0.999507	3.294554	0.882
12.5	0.985765	2.190736	0.999096	3.119931	0.999505	3.293389	0.875
13	0.984162	2.148481	0.999058	3.107998	0.999503	3.292225	0.869
13.5	0.982297	2.103679	0.999015	3.094756	0.999501	3.291352	0.863
14	0.980092	2.055658	0.998966	3.080349	0.999499	3.289897	0.857
14.5	0.977471	2.004117	0.99891	3.064633	0.999497	3.288733	0.851
15	0.974453	1.950684	0.998847	3.047608	0.999494	3.287278	0.846
15.5	0.970947	1.894905	0.998775	3.029418	0.999492	3.285822	0.84
16	0.966902	1.837097	0.998693	3.009918	0.999489	3.284367	0.835
16.5	0.962264	1.77758	0.998601	2.989254	0.999486	3.282621	0.83
17	0.95698	1.716662	0.998498	2.967281	0.999482	3.280584	0.825
17.5	0.950998	1.654607	0.998381	2.944216	0.999478	3.278547	0.821
18	0.944271	1.591675	0.99825	2.919987	0.999474	3.276509	0.816
18.5	0.936753	1.528074	0.998103	2.894812	0.99947	3.274181	0.812
19	0.928405	1.464018	0.997938	2.868619	0.999465	3.271562	0.807
19.5	0.919195	1.399676	0.997754	2.841407	0.99946	3.268942	0.803
20	0.909096	1.335206	0.997549	2.813395	0.999455	3.266032	0.799
20.5	0.898092	1.270755	0.99732	2.784582	0.999449	3.263121	0.795
21	0.886176	1.20644	0.997065	2.754969	0.999443	3.25992	0.792

Figure C-2 Test 1 raw data.

Test 1

Time, (years)	Serviceability - Unstiffnened Panel		Ultimate - Unstiffened Panel		Ultimate - Stiffened Panel		% Original Thickness
	Reliability	Reliability Index	Reliability	Reliability Index	Reliability	Reliability Index	
21.5	0.87335	1.142371	0.996782	2.724701	0.999436	3.256428	0.788
22	0.859626	1.07864	0.996468	2.693814	0.999428	3.252644	0.785
22.5	0.845026	1.015333	0.99612	2.662309	0.99942	3.24857	0.781
23	0.829583	0.952518	0.995735	2.630331	0.999411	3.244204	0.778
23.5	0.813337	0.890259	0.995309	2.597808	0.999402	3.239547	0.775
24	0.796337	0.828607	0.99484	2.564884	0.999391	3.234891	0.772
24.5	0.77864	0.767607	0.994323	2.531597	0.99938	3.229652	0.769
25	0.760309	0.707296	0.993754	2.497927	0.999368	3.224122	0.766
25.5	0.741412	0.647703	0.993129	2.46393	0.999355	3.218302	0.763
26	0.72202	0.588853	0.992443	2.429661	0.999341	3.211899	0.76
26.5	0.702208	0.530763	0.991693	2.395118	0.999325	3.205205	0.758
27	0.682052	0.473444	0.990871	2.360321	0.999309	3.19822	0.755
27.5	0.661626	0.416906	0.989974	2.32536	0.999291	3.190944	0.753
28	0.641007	0.361151	0.988994	2.29018	0.999271	3.183086	0.75
28.5	0.620267	0.306181	0.987929	2.254856	0.99925	3.174646	0.748
29	0.599476	0.251991	0.98677	2.219385	0.999227	3.165915	0.746
29.5	0.578703	0.198577	0.985511	2.183788	0.999202	3.156747	0.744
30	0.558011	0.145928	0.984146	2.148063	0.999175	3.146997	0.742

Figure C-2 Test 1 raw data (cont).

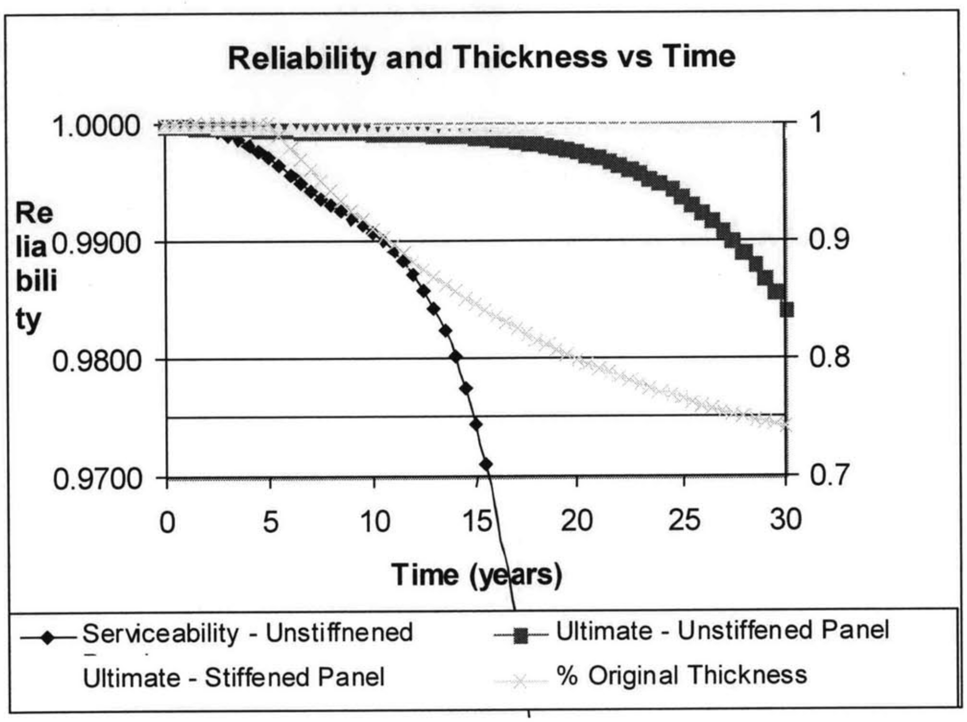

Figure C-3 Test 1 graph.

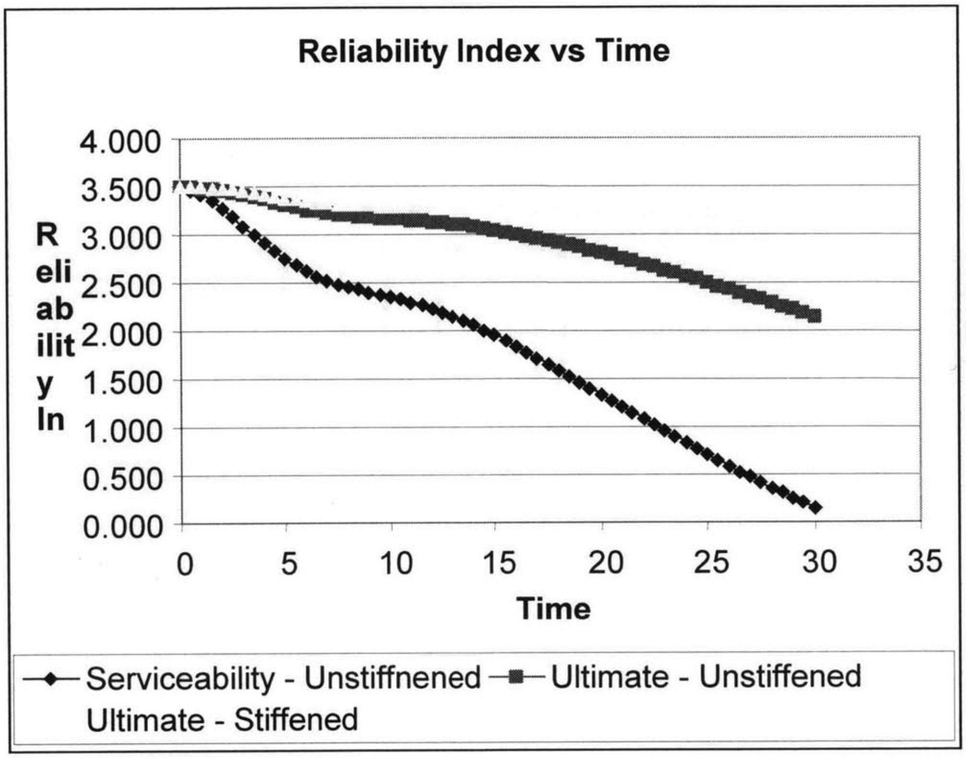

Figure C-4 Test 1 graph.

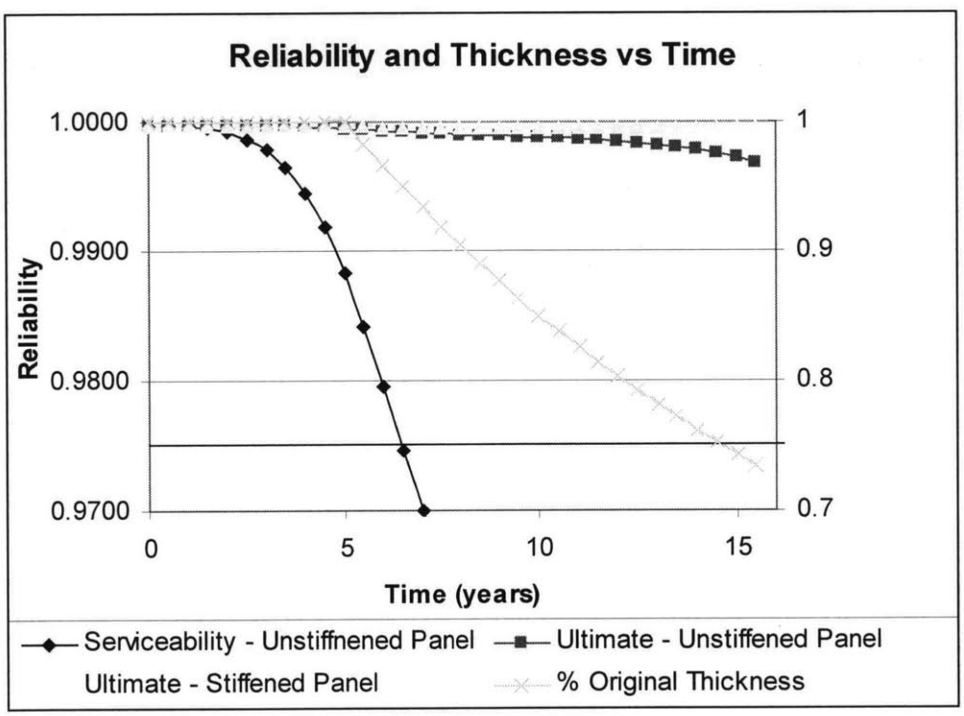

Figure C-5 Test 2 graph.

Test 2

Time, (years)	Serviceability - Unstiffnened Panel		Ultimate - Unstiffened Panel		Ultimate - Stiffened Panel		% Original Thickness
	Reliability	Reliability Index	Reliability	Reliability Index	Reliability	Reliability Index	
0	0.9998	3.5000	0.9998	3.5000	0.9998	3.5000	1
0.5	0.9997	3.4599	0.9998	3.4942	0.9998	3.4977	1
1	0.9996	3.3865	0.9998	3.4849	0.9998	3.4942	1
1.5	0.9995	3.2800	0.9997	3.4715	0.9998	3.4890	1
2	0.9992	3.1474	0.9997	3.4529	0.9998	3.4808	1
2.5	0.9986	2.9996	0.9997	3.4284	0.9997	3.4703	1
3	0.9978	2.8458	0.9997	3.3987	0.9997	3.4570	1
3.5	0.9964	2.6921	0.9996	3.3644	0.9997	3.4407	1
4	0.9945	2.5428	0.9996	3.3266	0.9997	3.4220	1
4.5	0.9918	2.4010	0.9995	3.2876	0.9997	3.4025	1
5	0.9884	2.2695	0.9994	3.2489	0.9996	3.3830	1
5.5	0.9842	2.1498	0.9993	3.2122	0.9996	3.3650	0.983
6	0.9795	2.0439	0.9993	3.1784	0.9996	3.3487	0.966
6.5	0.9747	1.9543	0.9992	3.1487	0.9996	3.3353	0.95
7	0.9700	1.8802	0.9991	3.1228	0.9996	3.3242	0.934
7.5	0.9657	1.8211	0.9990	3.1009	0.9995	3.3158	0.919
8	0.9620	1.7747	0.9990	3.0821	0.9995	3.3097	0.904
8.5	0.9589	1.7378	0.9989	3.0661	0.9995	3.3053	0.89
9	0.9560	1.7062	0.9989	3.0523	0.9995	3.3021	0.877
9.5	0.9532	1.6763	0.9988	3.0392	0.9995	3.3001	0.863
10	0.9499	1.6441	0.9988	3.0264	0.9995	3.2986	0.85
10.5	0.9460	1.6074	0.9987	3.0133	0.9995	3.2975	0.838
11	0.9412	1.5645	0.9986	2.9984	0.9995	3.2963	0.826
11.5	0.9351	1.5147	0.9986	2.9814	0.9995	3.2957	0.814
12	0.9275	1.4577	0.9985	2.9622	0.9995	3.2946	0.803
12.5	0.9183	1.3937	0.9984	2.9396	0.9995	3.2937	0.792
13	0.9072	1.3235	0.9982	2.9139	0.9995	3.2925	0.782
13.5	0.8939	1.2476	0.9980	2.8846	0.9995	3.2916	0.772
14	0.8784	1.1672	0.9978	2.8514	0.9995	3.2905	0.762
14.5	0.8606	1.0832	0.9976	2.8145	0.9995	3.2890	0.752
15	0.8405	0.9965	0.9972	2.7741	0.9995	3.2876	0.743
15.5	0.8181	0.9080	0.9968	2.7300	0.9995	3.2861	0.734

Figure C-6 Test 2 raw data.

Test 3

Time, (years)	Serviceability - Unstiffnened Panel		Ultimate - Unstiffened Panel		Ultimate - Stiffened Panel		% Original Thickness
	Reliability	Reliability Index	Reliability	Reliability Index	Reliability	Reliability Index	
0	0.9998	3.5000	0.9998	3.5000	0.9998	3.5000	1
0.5	0.9997	3.4523	0.9998	3.4936	0.9998	3.4977	1
1	0.9996	3.3504	0.9998	3.4831	0.9998	3.4942	1
1.5	0.9993	3.1907	0.9997	3.4668	0.9998	3.4890	1
2	0.9986	2.9910	0.9997	3.4430	0.9998	3.4808	1
2.5	0.9972	2.7720	0.9997	3.4104	0.9997	3.4703	1
3	0.9946	2.5461	0.9996	3.3682	0.9997	3.4570	1
3.5	0.9898	2.3206	0.9995	3.3181	0.9997	3.4407	1
4	0.9822	2.1005	0.9994	3.2620	0.9997	3.4220	1
4.5	0.9707	1.8914	0.9993	3.2020	0.9997	3.4028	1
5	0.9554	1.7000	0.9992	3.1400	0.9996	3.3833	1
5.5	0.9373	1.5324	0.9990	3.0767	0.9996	3.3650	0.976
6	0.9184	1.3942	0.9987	3.0159	0.9996	3.3490	0.952
6.5	0.9009	1.2869	0.9985	2.9583	0.9996	3.3353	0.93
7	0.8864	1.2077	0.9982	2.9044	0.9996	3.3242	0.908
7.5	0.8751	1.1506	0.9979	2.8567	0.9995	3.3161	0.887
8	0.8661	1.1081	0.9976	2.8159	0.9995	3.3097	0.866
8.5	0.8582	1.0724	0.9973	2.7809	0.9995	3.3053	0.846
9	0.8501	1.0370	0.9970	2.7518	0.9995	3.3024	0.827
9.5	0.8407	0.9971	0.9968	2.7266	0.9995	3.3004	0.809
10	0.8288	0.9493	0.9966	2.7040	0.9995	3.2986	0.791
10.5	0.8139	0.8924	0.9963	2.6803	0.9995	3.2975	0.773
11	0.7956	0.8259	0.9960	2.6540	0.9995	3.2966	0.756
11.5	0.7735	0.7504	0.9956	2.6225	0.9995	3.2957	0.74
12	0.7477	0.6672	0.9951	2.5839	0.9995	3.2948	0.724
12.5	0.7182	0.5775	0.9944	2.5370	0.9995	3.2940	0.709
13	0.6854	0.4829	0.9935	2.4817	0.9995	3.2928	0.694

Figure C-7 Test 3 raw data.

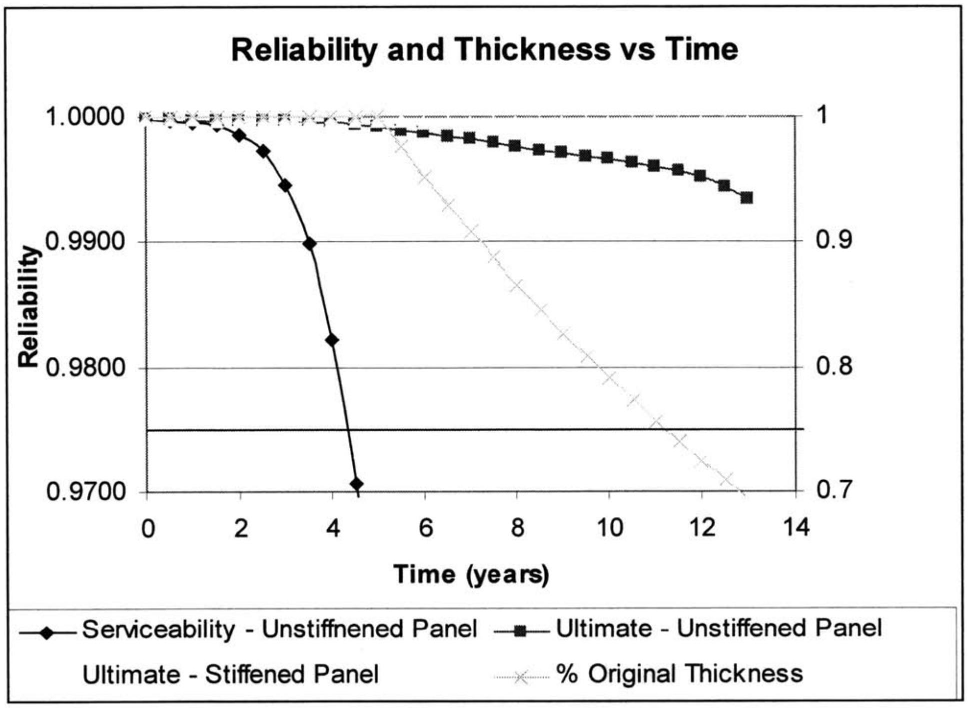

Figure C-8 Test 3 graph.

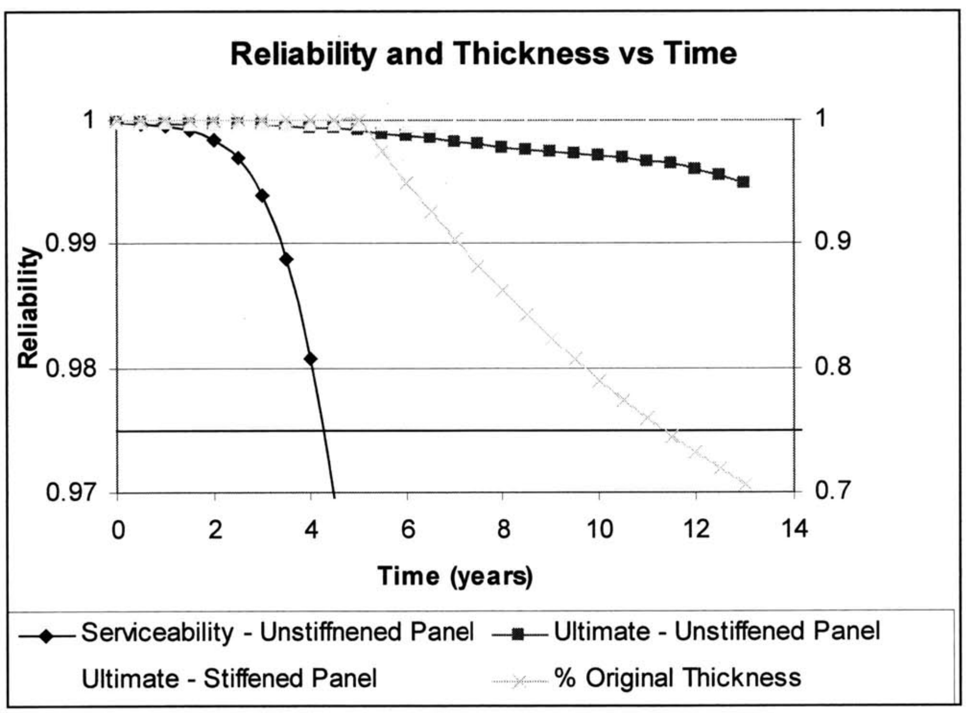

Figure C-9 Test 4 graph.

Test 4

Time, (years)	Serviceability - Unstiffnened Panel		Ultimate - Unstiffened Panel		Ultimate - Stiffened Panel		% Original Thickness
	Reliability	Reliability Index	Reliability	Reliability Index	Reliability	Reliability Index	
0	0.999767	3.5000	0.999767	3.5000	0.999767	3.5000	1
0.5	0.99972	3.4506	0.999762	3.4936	0.999765	3.4977	1
1	0.999582	3.3405	0.999752	3.4831	0.999762	3.4942	1
1.5	0.999237	3.1696	0.999736	3.4657	0.999757	3.4890	1
2	0.998463	2.9603	0.99971	3.4407	0.99975	3.4808	1
2.5	0.996884	2.7353	0.999671	3.4069	0.99974	3.4703	1
3	0.993921	2.5075	0.999615	3.3638	0.999727	3.4570	1
3.5	0.988811	2.2839	0.999538	3.3126	0.99971	3.4407	1
4	0.980752	2.0695	0.999436	3.2564	0.999689	3.4220	1
4.5	0.969238	1.8697	0.999306	3.1972	0.999666	3.4028	1
5	0.954504	1.6902	0.999147	3.1372	0.999642	3.3833	1
5.5	0.937679	1.5356	0.998957	3.0776	0.999617	3.3650	0.974
6	0.920661	1.4095	0.998744	3.0219	0.999594	3.3490	0.949
6.5	0.90527	1.3122	0.99851	2.9699	0.999574	3.3353	0.926
7	0.892528	1.2401	0.998273	2.9242	0.999557	3.3242	0.903
7.5	0.882467	1.1874	0.998038	2.8842	0.999543	3.3158	0.882
8	0.874381	1.1473	0.997818	2.8506	0.999533	3.3097	0.862
8.5	0.86714	1.1130	0.997618	2.8226	0.999526	3.3053	0.843
9	0.859626	1.0786	0.997436	2.7988	0.999521	3.3024	0.824
9.5	0.85091	1.0403	0.997265	2.7780	0.999517	3.3004	0.807
10	0.84012	0.9950	0.997094	2.7582	0.999514	3.2986	0.79
10.5	0.826814	0.9416	0.996906	2.7377	0.999512	3.2975	0.774
11	0.810597	0.8801	0.996678	2.7142	0.999511	3.2966	0.759
11.5	0.791322	0.8110	0.996393	2.6869	0.999509	3.2957	0.745
12	0.768957	0.7354	0.996027	2.6544	0.999508	3.2948	0.732
12.5	0.743588	0.6544	0.995551	2.6159	0.999506	3.2940	0.719
13	0.715468	0.5694	0.99494	2.5717	0.999504	3.2928	0.706

Figure C-10 Test 4 raw data.

Test 5

Time, (years)	Serviceability - Unstiffnened Panel		Ultimate - Unstiffened Panel		Ultimate - Stiffened Panel		% Original Thickness
	Reliability	Reliability Index	Reliability	Reliability Index	Reliability	Reliability Index	
0	0.999767	3.5000	0.999767	3.5000	0.999767	3.5000	1
0.5	0.999733	3.4634	0.999762	3.4942	0.999765	3.4977	1
1	0.999668	3.4040	0.999755	3.4855	0.999762	3.4942	1
1.5	0.999553	3.3222	0.999743	3.4738	0.999757	3.4890	1
2	0.999363	3.2218	0.999727	3.4575	0.99975	3.4808	1
2.5	0.999061	3.1090	0.999705	3.4366	0.99974	3.4703	1
3	0.998606	2.9903	0.999676	3.4110	0.999727	3.4570	1
3.5	0.997951	2.8705	0.99964	3.3816	0.99971	3.4407	1
4	0.997052	2.7536	0.999596	3.3499	0.999689	3.4220	1
4.5	0.995881	2.6422	0.999545	3.3173	0.999666	3.4025	1
5	0.994435	2.5386	0.99949	3.2849	0.999642	3.3830	1
5.5	0.992732	2.4437	0.999433	3.2547	0.999617	3.3650	0.987
6	0.990835	2.3589	0.999375	3.2273	0.999594	3.3487	0.974
6.5	0.988855	2.2854	0.999321	3.2035	0.999574	3.3353	0.961
7	0.986933	2.2242	0.999271	3.1831	0.999557	3.3242	0.949
7.5	0.985078	2.1722	0.999227	3.1659	0.999543	3.3158	0.937
8	0.983394	2.1295	0.999188	3.1517	0.999533	3.3097	0.926
8.5	0.981879	2.0942	0.999154	3.1397	0.999525	3.3050	0.914
9	0.980478	2.0637	0.999123	3.1291	0.99952	3.3021	0.903
9.5	0.979103	2.0356	0.999094	3.1193	0.999517	3.3001	0.893
10	0.977608	2.0067	0.999064	3.1097	0.999514	3.2986	0.882
10.5	0.975885	1.9753	0.999031	3.0997	0.999512	3.2975	0.872
11	0.973806	1.9399	0.998994	3.0886	0.99951	3.2963	0.862
11.5	0.971216	1.8990	0.998952	3.0763	0.999509	3.2954	0.852
12	0.968016	1.8524	0.998901	3.0622	0.999507	3.2946	0.843
12.5	0.96407	1.8000	0.998841	3.0463	0.999505	3.2934	0.833
13	0.959206	1.7415	0.99877	3.0283	0.999503	3.2925	0.824
13.5	0.953327	1.6780	0.998685	3.0079	0.999502	3.2914	0.815
14	0.946237	1.6094	0.998584	2.9853	0.999499	3.2902	0.807
14.5	0.937707	1.5358	0.998463	2.9602	0.999497	3.2890	0.798
15	0.927707	1.4589	0.998319	2.9325	0.999495	3.2876	0.79
15.5	0.91605	1.3790	0.998148	2.9023	0.999492	3.2861	0.782
16	0.90263	1.2967	0.997944	2.8695	0.999489	3.2844	0.774
16.5	0.88737	1.2127	0.997704	2.8343	0.999486	3.2826	0.767
17	0.870232	1.1275	0.997418	2.7966	0.999483	3.2809	0.759
17.5	0.851218	1.0417	0.99708	2.7566	0.999479	3.2788	0.752
18	0.830374	0.9556	0.996681	2.7145	0.999475	3.2768	0.745
18.5	0.807791	0.8698	0.996211	2.6703	0.999471	3.2745	0.738
19	0.7836	0.7844	0.995659	2.6244	0.999466	3.2721	0.732
19.5	0.757972	0.6998	0.995013	2.5767	0.999461	3.2695	0.725
20	0.731107	0.6162	0.994257	2.5276	0.999456	3.2666	0.719

Figure C-11 Test 5 raw data.

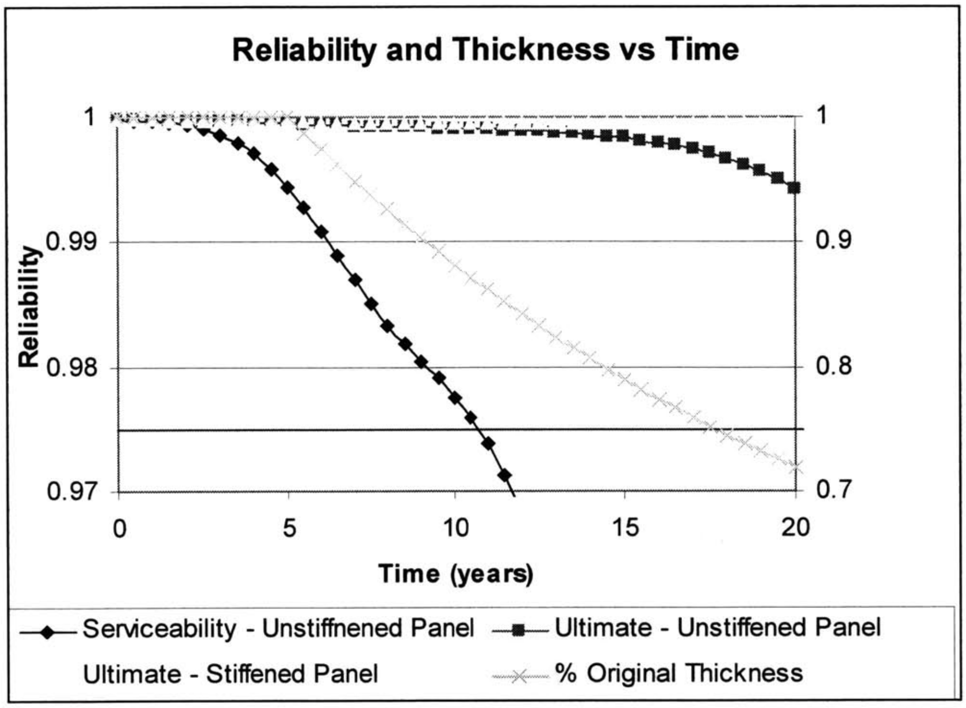

Figure C-12 Test 5 graph.

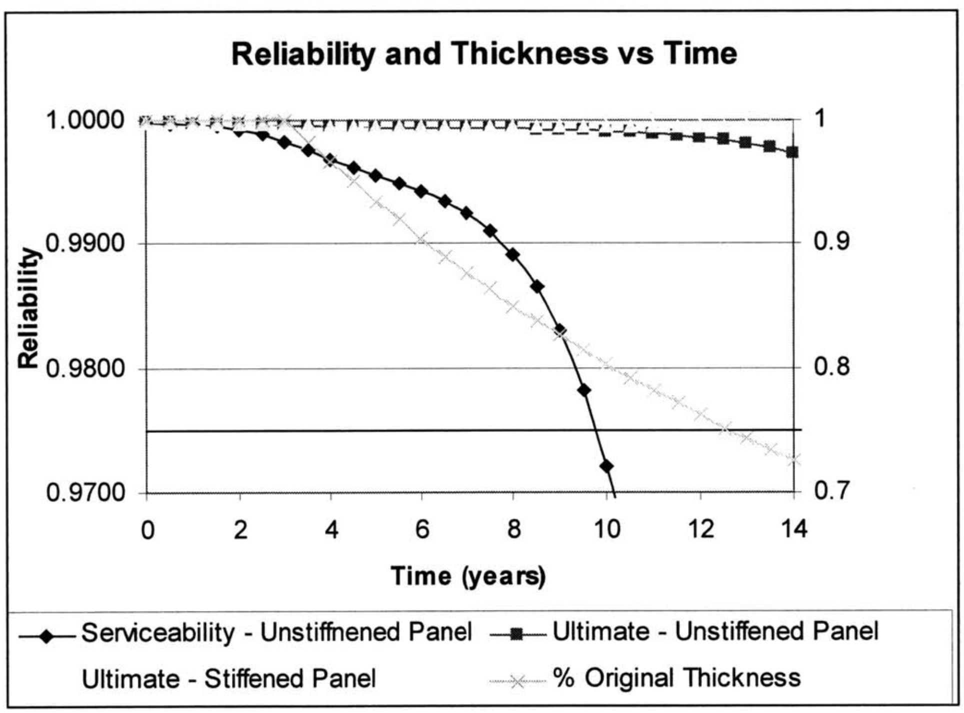

Figure C-13 Test 6 graph.

Test 6

Time, (years)	Serviceability - Unstiffnened Panel		Ultimate - Unstiffened Panel		Ultimate - Stiffened Panel		% Original Thickness
	Reliability	Reliability Index	Reliability	Reliability Index	Reliability	Reliability Index	
0	0.9998	3.5000	0.9998	3.5000	0.9998	3.5000	1
0.5	0.9997	3.4599	0.9998	3.4925	0.9998	3.4960	1
1	0.9996	3.3877	0.9997	3.4785	0.9998	3.4878	1
1.5	0.9995	3.2849	0.9997	3.4564	0.9997	3.4727	1
2	0.9992	3.1626	0.9997	3.4244	0.9997	3.4488	1
2.5	0.9988	3.0352	0.9996	3.3865	0.9997	3.4191	1
3	0.9982	2.9158	0.9996	3.3469	0.9996	3.3871	1
3.5	0.9975	2.8110	0.9995	3.3114	0.9996	3.3586	0.983
4	0.9968	2.7254	0.9995	3.2835	0.9996	3.3365	0.966
4.5	0.9961	2.6599	0.9994	3.2631	0.9996	3.3213	0.95
5	0.9954	2.6071	0.9994	3.2492	0.9995	3.3126	0.934
5.5	0.9948	2.5647	0.9994	3.2393	0.9995	3.3079	0.919
6	0.9942	2.5240	0.9994	3.2314	0.9995	3.3056	0.904
6.5	0.9934	2.4786	0.9994	3.2232	0.9995	3.3044	0.89
7	0.9924	2.4272	0.9993	3.2142	0.9995	3.3039	0.877
7.5	0.9910	2.3645	0.9993	3.2035	0.9995	3.3033	0.863
8	0.9891	2.2934	0.9993	3.1904	0.9995	3.3027	0.85
8.5	0.9865	2.2113	0.9993	3.1749	0.9995	3.3018	0.838
9	0.9830	2.1190	0.9992	3.1567	0.9995	3.3010	0.826
9.5	0.9783	2.0190	0.9991	3.1354	0.9995	3.3004	0.814
10	0.9721	1.9125	0.9991	3.1109	0.9995	3.2995	0.803
10.5	0.9642	1.8012	0.9990	3.0827	0.9995	3.2986	0.792
11	0.9542	1.6865	0.9989	3.0507	0.9995	3.2975	0.782
11.5	0.9418	1.5698	0.9987	3.0150	0.9995	3.2963	0.772
12	0.9268	1.4522	0.9985	2.9754	0.9995	3.2951	0.762
12.5	0.9090	1.3347	0.9983	2.9321	0.9995	3.2940	0.752
13	0.8884	1.2179	0.9980	2.8854	0.9995	3.2928	0.743
13.5	0.8649	1.1026	0.9977	2.8351	0.9995	3.2911	0.734
14	0.8387	0.9893	0.9973	2.7820	0.9995	3.2896	0.725

Figure C-15 Test 6 raw data.

Test 7

Time, (years)	Serviceability - Unstiffnened Panel		Ultimate - Unstiffened Panel		Ultimate - Stiffened Panel		% Original Thickness
	Reliability	Reliability Index	Reliability	Reliability Index	Reliability	Reliability Index	
0	0.9998	3.5000	0.9998	3.5000	0.9998	3.5000	1
0.5	0.9997	3.4593	0.9998	3.4948	0.9998	3.4983	1
1	0.9996	3.3865	0.9998	3.4866	0.9998	3.4960	1
1.5	0.9995	3.2788	0.9997	3.4750	0.9998	3.4925	1
2	0.9992	3.1439	0.9997	3.4593	0.9998	3.4884	1
2.5	0.9986	2.9922	0.9997	3.4389	0.9998	3.4831	1
3	0.9977	2.8317	0.9997	3.4127	0.9997	3.4762	1
3.5	0.9962	2.6675	0.9996	3.3813	0.9997	3.4680	1
4	0.9938	2.5030	0.9996	3.3446	0.9997	3.4581	1
4.5	0.9904	2.3403	0.9995	3.3033	0.9997	3.4465	1
5	0.9854	2.1809	0.9994	3.2585	0.9997	3.4337	1
5.5	0.9786	2.0264	0.9993	3.2110	0.9997	3.4203	1
6	0.9699	1.8787	0.9992	3.1627	0.9997	3.4057	1
6.5	0.9591	1.7398	0.9991	3.1140	0.9997	3.3912	1
7	0.9465	1.6120	0.9989	3.0659	0.9996	3.3769	1
7.5	0.9328	1.4969	0.9987	3.0197	0.9996	3.3632	0.983
8	0.9186	1.3959	0.9985	2.9741	0.9996	3.3504	0.966
8.5	0.9049	1.3102	0.9983	2.9312	0.9996	3.3391	0.95
9	0.8924	1.2393	0.9981	2.8919	0.9996	3.3289	0.934
9.5	0.8814	1.1821	0.9978	2.8547	0.9996	3.3202	0.919
10	0.8721	1.1364	0.9976	2.8213	0.9995	3.3129	0.904
10.5	0.8643	1.0999	0.9974	2.7921	0.9995	3.3068	0.89
11	0.8577	1.0699	0.9972	2.7654	0.9995	3.3018	0.877
11.5	0.8517	1.0437	0.9969	2.7419	0.9995	3.2978	0.863
12	0.8459	1.0189	0.9968	2.7216	0.9995	3.2946	0.85
12.5	0.8398	0.9935	0.9966	2.7031	0.9995	3.2922	0.838
13	0.8330	0.9660	0.9964	2.6867	0.9995	3.2899	0.826
13.5	0.8251	0.9351	0.9962	2.6711	0.9995	3.2882	0.814
14	0.8160	0.9001	0.9960	2.6557	0.9995	3.2870	0.803
14.5	0.8053	0.8609	0.9959	2.6399	0.9995	3.2855	0.792
15	0.7931	0.8171	0.9956	2.6231	0.9995	3.2844	0.782
15.5	0.7791	0.7691	0.9954	2.6042	0.9995	3.2829	0.772
16	0.7633	0.7170	0.9951	2.5833	0.9995	3.2817	0.762
16.5	0.7458	0.6613	0.9948	2.5594	0.9995	3.2806	0.752
17	0.7266	0.6026	0.9943	2.5325	0.9995	3.2791	0.743
17.5	0.7057	0.5410	0.9938	2.5027	0.9995	3.2774	0.734
18	0.6834	0.4772	0.9932	2.4698	0.9995	3.2759	0.725

Figure C-16 Test 7 raw data.

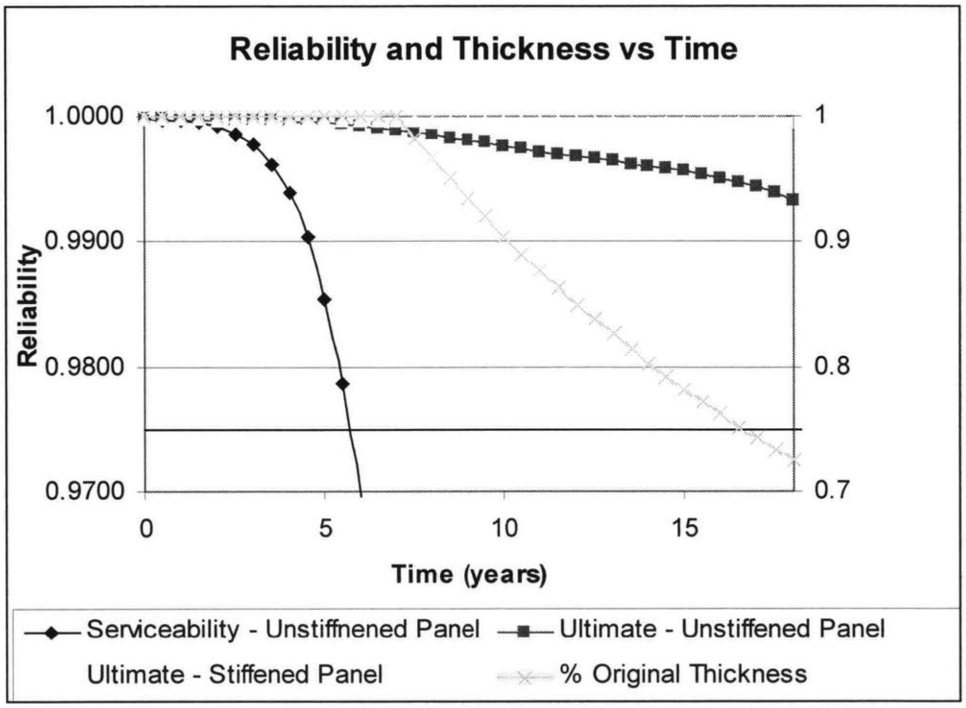

Figure C-17 Test 7 graph.

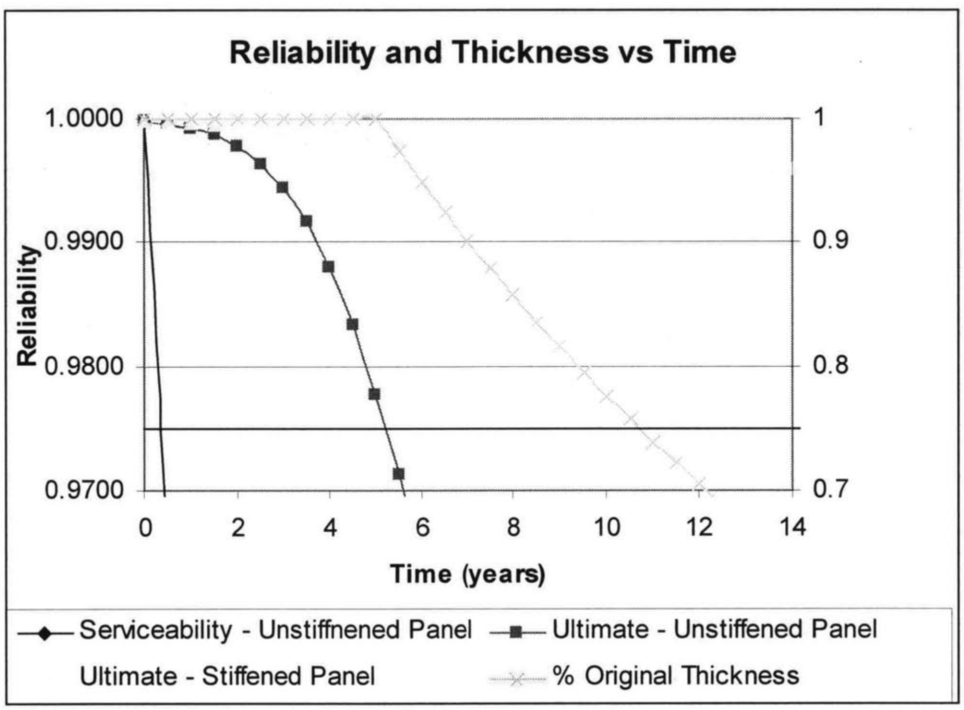

Figure C-18 Test 8 graph.

Test 8

Time, (years)	Serviceability - Unstiffnened Panel		Ultimate - Unstiffened Panel		Ultimate - Stiffened Panel		% Original Thickness
	Reliability	Reliability Index	Reliability	Reliability Index	Reliability	Reliability Index	
0	0.9998	3.5000	0.9998	3.5000	0.9998	3.5000	1
0.5	0.9606	1.7579	0.9996	3.3324	0.9997	3.4232	1
1	0.8948	1.2524	0.9992	3.1637	0.9996	3.3560	1
1.5	0.7969	0.8307	0.9986	2.9986	0.9995	3.2951	1
2	0.6694	0.4384	0.9977	2.8385	0.9994	3.2393	1
2.5	0.5273	0.0684	0.9964	2.6837	0.9993	3.1872	1
3	0.3949	-0.2665	0.9944	2.5344	0.9992	3.1383	1
3.5	0.2932	-0.5442	0.9916	2.3911	0.9990	3.0926	1
4	0.2275	-0.7471	0.9879	2.2548	0.9989	3.0501	1
4.5	0.1906	-0.8758	0.9833	2.1266	0.9987	3.0111	1
5	0.1717	-0.9474	0.9777	2.0081	0.9985	2.9756	1
5.5	0.1578	-1.0037	0.9713	1.9006	0.9984	2.9436	0.974
6	0.1452	-1.0574	0.9645	1.8058	0.9982	2.9152	0.949
6.5	0.1305	-1.1242	0.9577	1.7245	0.9981	2.8900	0.925
7	0.1123	-1.2144	0.9513	1.6571	0.9979	2.8677	0.901
7.5	0.0912	-1.3334	0.9455	1.6028	0.9978	2.8478	0.879
8	0.0698	-1.4773	0.9405	1.5593	0.9977	2.8298	0.857
8.5	0.0506	-1.6392	0.9362	1.5237	0.9975	2.8132	0.835
9	0.0351	-1.8103	0.9322	1.4923	0.9974	2.7975	0.815
9.5	0.0238	-1.9812	0.9281	1.4617	0.9973	2.7824	0.795
10	0.0157	-2.1510	0.9235	1.4292	0.9972	2.7676	0.776
10.5	0.0104	-2.3113	0.9181	1.3923	0.9970	2.7528	0.757
11	0.0068	-2.4691	0.9115	1.3499	0.9969	2.7379	0.739
11.5	0.0046	-2.6038	0.9034	1.3011	0.9968	2.7230	0.722
12	0.0030	-2.7499	0.8936	1.2457	0.9966	2.7077	0.705
12.5	0.0021	-2.8579	0.8818	1.1839	0.9965	2.6924	0.688
13	0.0014	-2.9789	0.8678	1.1162	0.9963	2.6769	0.673
13.5	0.0011	-3.0754	0.8516	1.0432	0.9961	2.6611	0.657
14	0.0008	-3.1736	0.8329	0.9658	0.9959	2.6451	0.643

Figure C-19 Test 8 raw data.

Test 9

Time, (years)	Serviceability - Unstiffnened Panel		Ultimate - Unstiffened Panel		Ultimate - Stiffened Panel		% Original Thickness
	Reliability	Reliability Index	Reliability	Reliability Index	Reliability	Reliability Index	
0	0.9998	3.5000	0.9998	3.5000	0.9998	3.5000	1
0.5	0.9982	2.9073	0.9997	3.4733	0.9998	3.4936	1
1	0.9950	2.5790	0.9997	3.4354	0.9998	3.4861	1
1.5	0.9894	2.3055	0.9996	3.3859	0.9997	3.4762	1
2	0.9801	2.0551	0.9996	3.3254	0.9997	3.4645	1
2.5	0.9654	1.8174	0.9994	3.2561	0.9997	3.4488	1
3	0.9440	1.5894	0.9993	3.1816	0.9997	3.4313	1
3.5	0.9149	1.3717	0.9990	3.1028	0.9997	3.4110	1
4	0.8786	1.1678	0.9987	3.0224	0.9996	3.3889	1
4.5	0.8373	0.9833	0.9984	2.9427	0.9996	3.3664	1
5	0.7951	0.8244	0.9979	2.8644	0.9996	3.3446	1
5.5	0.7561	0.6937	0.9974	2.7924	0.9996	3.3242	0.979
6	0.7234	0.5930	0.9968	2.7264	0.9995	3.3059	0.959
6.5	0.6979	0.5185	0.9962	2.6681	0.9995	3.2905	0.94
7	0.6783	0.4630	0.9956	2.6181	0.9995	3.2774	0.921
7.5	0.6620	0.4179	0.9950	2.5755	0.9995	3.2672	0.903
8	0.6462	0.3751	0.9945	2.5399	0.9994	3.2588	0.885
8.5	0.6288	0.3287	0.9940	2.5101	0.9994	3.2524	0.868
9	0.6082	0.2747	0.9935	2.4845	0.9994	3.2468	0.852
9.5	0.5838	0.2117	0.9931	2.4614	0.9994	3.2422	0.836
10	0.5557	0.1400	0.9926	2.4385	0.9994	3.2381	0.821
10.5	0.5242	0.0606	0.9921	2.4146	0.9994	3.2340	0.806
11	0.4902	-0.0246	0.9915	2.3878	0.9994	3.2299	0.791
11.5	0.4545	-0.1143	0.9908	2.3568	0.9994	3.2259	0.777
12	0.4179	-0.2073	0.9899	2.3211	0.9994	3.2215	0.764

Figure C-20 Test 9 raw data.

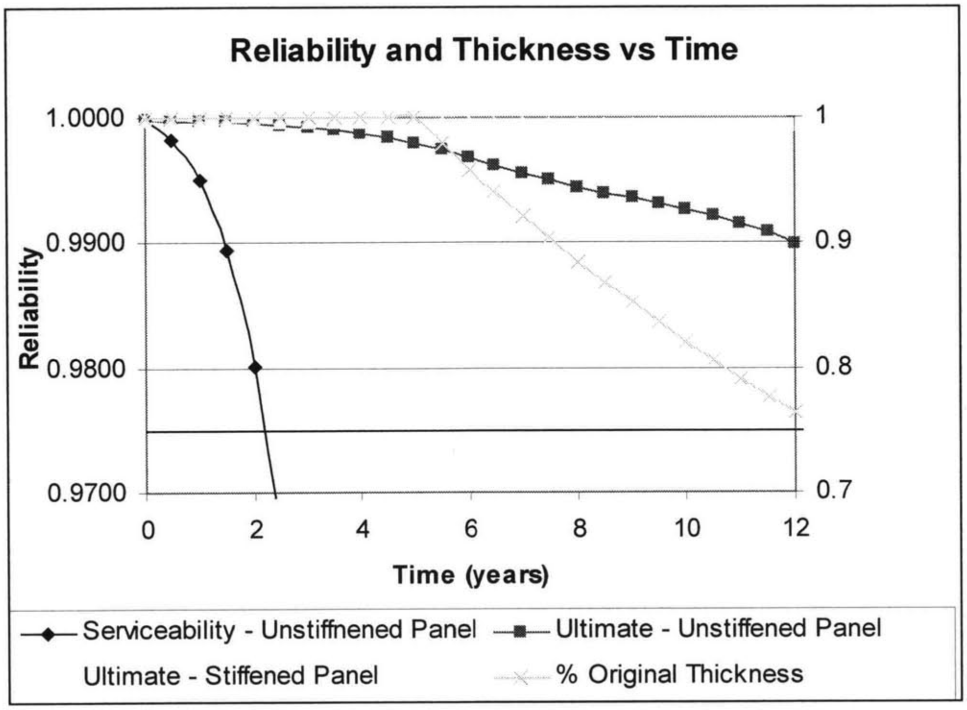

Figure C-21 Test 9 graph.