Multi-modal, Multi-period, Multi-commodity Transportation: Models and Algorithms

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by

Nicholas R. Jernigan

B.S., United States Air Force Academy (2012)

Submitted to the Sloan School of Management
in partial fulfillment of the requirements for the degree of

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Signature redacted

Author ....
Sloan School of Management
May 16, 2014

Signature redacted

Certified by..
Dimitris Bertsimas
Boeing Professor of Operations Research
Sloan School of Management
Co-director, Operations Research Center
Thesis Supervisor

Signature redacted

Accepted by..
Patrick Jaillet
Dugald C. Jackson Professor
Department of Electrical Engineering and Computer Science
Co-director, Operations Research Center

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Abstract

In this paper we present a mixed integer optimization framework for modeling the shipment of goods between origin destination (O-D) pairs by vehicles of different types over a time-space network. The output of the model is an optimal schedule and routing of vehicle movements and assignment of goods to vehicles. Specifically, this framework allows for: multiple vehicles of differing characteristics (including speed, cost of travel, and capacity), transshipment locations where goods can be transferred between vehicles; and availability times for goods at their origins and delivery time windows for goods at their destinations. The model is composed of three stages: In the first, vehicle quantities, by type, and goods are allocated to routes in order to minimize late deliveries and vehicle movement costs. In the second stage, individual vehicles, specified by vehicle identification numbers, are assigned routes, and goods are assigned to those vehicles based on the results of the first stage and a minimization of costs involved with the transfer of goods between vehicles. In the third stage we reallocate the idle time of vehicles in order to satisfy crew rest constraints. Computational results show that provably optimal or near optimal solutions are possible for realistic instance sizes.

Thesis Supervisor: Dimitris Bertsimas
Title: Boeing Professor of Operations Research
Sloan School of Management
Co-director, Operations Research Center

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Chapter 1

Introduction

The use of multiple vehicle types for the shipment of goods represents an important problem for many freight transportation providers. This activity is referred to as multi-modal (or intermodal) transportation [14]. The transfer of goods between vehicles, or transshipment, occurs at an intermodal terminal. Even though both people and freight can be transported in a multi-modal manner, people require additional constraints; for example, people must not be left at hubs for an extended period of time. In this thesis we propose a model for optimally routing vehicles in order to flow goods between O-D pairs. We allow for multiple vehicle types and transshipment.

The practical use for this model is from the perspective of a freight shipper. In particular the shipper owns a finite fleet of vehicles with potentially heterogeneous characteristics. Prior to the planning time horizon of interest, the shipper must construct a schedule to service a set of known demands within specified delivery windows. We provide a modeling framework that allows the shipper to find optimal or near optimal solutions to this problem. Additionally, our model is particularly well suited for shippers who experience some unique and infrequent demands or experience short time windows for shipment. This is because we produce time based schedules, in contrast to many other models which produce service frequencies.

We make the following contributions:

1. Propose a *unique solution methodology* of splitting the problem into sub-problems.

The three sub-problems are solved in a sequential manner, where each sub-problem utilizes the results of the previously solved sup-problem. In this way we increase the tractability of the overall problem by considering only a subset of all decisions in each stage.

2. Introduce a *new formulation* for the multi-modal transportation problem with theoretical underpinnings in the work of Bertsimas et al. 2013 [3].
3. Provide *computational results* that explore the limits of problem size using state of the art hardware and software.

1.1 Model Assumptions and Components

Here we present the assumptions and components of our model.

A time-space network of nodes representing physical locations, arcs representing distance between nodes, and time steps. The distance arcs between nodes are repeated through the time horizon. Consider the network shown in Figure 1-1 that consists of a single arc of distance one, incident on nodes A and B. When extended into the time dimension as in Figure 1-2, the same arc is repeated for each time step and directed through time.

Figure 1-1: Simple network with no time dimension.

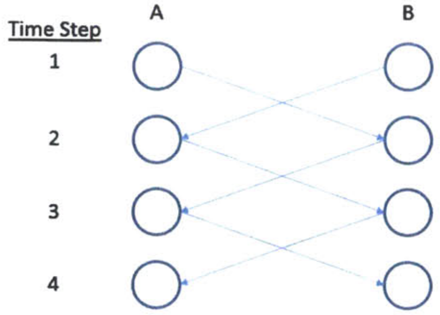

Figure 1-2: Simple network expanded for four time steps.

Multiple vehicle types defined by unique availability dates and locations, speed, cost, and capacity. There may be one or many vehicles of each type. Some of the vehicle types we may include are aircraft, ships, and trucks. Where, for example, aircraft are generally defined by high speed, high cost, and medium capacity. Ships and trucks may be defined with slow speed, low cost, high capacity for ships, and low capacity for trucks. Additionally, not all vehicles may use all network arcs; for example, a truck may not cross an ocean. The values of these characteristics are set by the user.

Multiple commodities with unique availability dates and locations, delivery windows, and quantities. A single commodity is defined as the set of goods with a common destination and delivery window. A single commodity may be split between vehicles. Therefore, a commodity may have multiple origins, where a portion of the commodity becomes available to ship. Additionally, portions of the same commodity may become available to ship at differing time steps. A single commodity may potentially be larger than any one vehicle, as it may be split between several vehicles.

Transfers between vehicles of commodities. The model allows for transshipment at nodes, so that a commodity may arrive at a node on one vehicle and leave on another vehicle. This feature is a key contribution of the model. For example, it allows commodities to be routed from an inland depot to a seaport via truck and then loaded onto a ship. Additionally, there is no requirement for the full amount of a commodity to travel in the same vehicle or along the same route. In this way portions of the same commodity may travel to their destination along unique routes and vehicle type choices. There is no limit to the number of times a commodity may be transferred between vehicles.

Multiple loads and deadheading The number of movements a vehicle makes is only limited by the time horizon and the speed of the vehicle. The number of commodities a vehicle transports is only limited by its capacity. Vehicles may make empty, deadhead, legs for re-positioning purposes. For example, if a node has no vehicles available, some vehicle(s), potentially empty, must be sent to the node to ship any commodities originating at the node.

Time horizon The time horizon is long enough to compare the movements of vehicles with different speeds. For example, an aircraft can traverse many arcs in the same time a ship can traverse one arc. The model can handle enough time steps to account for the discrepancy in speed between the vehicle classes.

1.2 Literature Review

Operations research techniques have greatly contributed to the development of models and strategies to better manage freight transportation networks. These models include three major players: shippers who generate the demand for transportation; carriers who supply the transportation services for moving the demand, and the intermodal network itself composed of multi-modal services and terminals [2]. There are three classical planning levels: strategic, tactical, and operational [15]. At the strategic level, system design decisions address the optimal location of new intermodal terminals that are placed and designed to balance the consolidation and transfer of goods between different modes[4, 12, 30]. Exact and heuristic formulations have been proposed following sequential and parallel computing approaches [19, 20]. Additional strategic decisions involve network design in terms of improving existing infrastructure or establishing new roads, railways, sea links, and freight terminals [33]. Equilibrium conditions are sought for network users, sometimes also using game theoretic approaches [8, 28, 33, 32]. In contrast, the major issue at the tactical planning level concerns the service network design, which includes the selection of vehicle fleet size and mix as well as routes and frequency on which services are offered, and the specification of terminal operations [11, 29]. These questions apply, for instance, to railways, less-than-truckload (LTL) motor carriers, express or parcel services, letter mail services or airlines, where either costs have to be minimized or profit has to be maximized, given constraints on resource availability and level of service [12]. According to [14], "most service network design models proposed in the literature take the form of deterministic, fixed cost, capacitated, multicommodity network design formulations," and may solve very large-scale settings [6, 24]. Finally, at the operational level, planning considers the time factor, where actual arrival and departure times given for each vehicle [15]. Operational decisions involve routing of vehicles, scheduling of services, empty vehicle distribution and repositioning, crew scheduling and allocation of resources [7].

The outputs of our model satisfy many of the questions asked at both the tactical and operational planning levels. As in tactical planning, routes are selected and capacity is allocated. As in operational planning, schedules including a time dimension are created, but unlike many operational models, we do not take routes as an input and generate routes and schedules concurrently. The final outputs of our model are in line with those of an operational model; however they may be easily generalized to answer tactical questions. For example, service frequency– a tactical question– may be derived from vehicle schedules–an operational output.

Some transportation models [16] provide service frequencies and not actual schedules, where a service frequency represents the number of times an arc is traversed for a given period of time. While frequency models are useful for evaluation of the general service network design, they usually do not answer questions regarding the schedule of individual vehicles and commodities and thus are of limited utility to operational planners.

At the operational level a time dimension is added to create actual schedules. Here the literature becomes limited [14] and often more tailored to specific applications such as freight railroads [21]. Most of the proposed models only consider a single mode [17, 21, 24].

A classic problem similar to our problem of interest is the vehicle routing problem (VRP), which was first proposed in 1959 by Dantzig and Ramser [17]. The output of the VRP is a set of vehicle routes to service a set of known customers demands for either pickup or delivery. Each demand is serviced by exactly one of a set of vehicles that are constrained by capacity. The vehicles begin their routes from a single depot at the beginning of the time horizon and must return to the depot before the end of the horizon. Many solution approaches to the VRP have been proposed in the literature (see [10, 26] for a survey). Additionally, there is a generalized VRP with time windows (VRPTW), in which the demands of the customers must be satisfied within a specified time window. There are also many proposals for solution methods to this problem [5, 10].

The problem we examine in this thesis is different from the VRP in several respects. In the VRP each vehicle's route begins and ends at a single location. Demand for a commodity may only be serviced by one vehicle, thus transshipment is prohibited. Only one vehicle type may be considered. The VRP has no time windows, and the VRPTW uses strict time windows, whereas we allow violation of the delivery window at a penalty.

For the operational planner interested in creating a schedule that includes multiple modes, the options become even more limited. Grünert and Sebastian propose a planning model for the large network of the German post office. The problem is broken into sub-problems, but limited solving algorithms are offered to solve the subproblems [22]. Armacost et al. describe a model for express shipment carriers with overnight air operations [1]. One key contribution is the introduction of composite variables, as a combination of vehicle routes and commodity flows into a single class of variables. The formulation is tractable on examples from the United Parcel Service network and allows for the use of multiple vehicle types. However, there are several factors that limit the model. First, all commodities must be transshipped at a hub location. Second, no planning horizons of longer than one night are considered. This is because the decision variables are frequency based and would likely not guarantee feasible flows for all commodities on time horizons longer than one night.

A more complex version of the VRP is known as "rich" VRP allows for multiple modes. These problems include other elements not considered in the standard VRP, such as multiple depots and multiple trips per vehicle. Kytöjoki et al. (2007) propose a heuristic method to solve such a problem efficiently [25]. Their model allows for a heterogeneous fleet of vehicles with potential for different speeds, starting locations, and capacities. While these elements are all common to our proposed method, there are some key differences. The main difference is that each vehicle operates independently of the others. This is possible because each commodity may only be serviced by one vehicle. This requires that there be no transshipment. With the exclusion of transshipment, there is also no transfer of commodities between vehicle types; for example, a truck may not transfer goods to a ship. Additionally, the all commodities must be smaller than the largest vehicle type, whereas in the problem we address, commodities that are larger than the largest vehicle may be split between several vehicles.

Additional multi-modal transportation papers propose methods to solve similar problems to ours. Formulations are proposed to to flow commodities through a network utilizing multiple modes [9, 27]. However, they do not consider the flow of vehicles and rather consider capacities on arcs which correspond with the use of a particular mode. The difference between the two is that [27] does not allow for the splitting of commodities while [9] does. The work of Haghani and Oh is closest to our work [23]. With the major difference being the definition of variables, where they introduce additional variables to represent non-moving vehicles and commodities. Additionally the Haghani and Oh formulation is a single stage and does not consider crew rest.

Our methodology has its intellectual underpinnings in Bertsimas et al. 2013 [3]. Where we take inspiration from their recommendations for modeling dynamic resource allocation problems.

Chapter 2

Modeling

In this Chapter we introduce the formulations for each of the three stages of our model.

2.1 Methodology

Our methodology has three stages. The primary purpose of splitting the problem into stages is to increase tractability. Computationally, the first stage of the model takes the longest to solve; therefore, our modeling effort in this thesis is focused on reducing the number of variables and constraints needed for this stage.

In the first stage, there are integer variables to track the total number of vehicles, by type, on each arc. By using these aggregate integer variables instead of one binary variable per vehicle, we can substantially reduce the number of variables tracking vehicle movements and number of necessary capacity and vehicle constraints. The second stage contains variables that correspond with each individual vehicle. This is not a problem for the tractability because the solution space of the second stage is reduced by only considering solutions that conform with the output of the first stage. In this way the second stage solves very quickly. The third stage adds further detail that would be too complex for the first two stages. In particular, actual travel times and crew rest constraints are introduced.

2.2 First Stage: Assignment of Vehicle Types and Commodities to Routes

In this stage, integer numbers of vehicles are assigned to routes. Commodities are assigned to routes by vehicle type.

Data and Sets

The data for this stage contains a set of commodities $k \in \mathcal{K}$ that must be transported between some nodes $i \in \mathcal{N}$ over a set of arcs $a \in \mathcal{A}$, utilizing vehicles of some types $\ell \in \mathcal{L}$. The vehicle movements and commodity flows take place over a discretized time horizon $t \in \{0, ..., TH\}$. Each commodity becomes available to ship at a set of one or more nodes. The percent of the total amount q^k of commodity k that becomes available to ship at its origin i by time t is $a_i^k(t)$, where for all $k \in \mathcal{K}$:

$$\sum_{i \in \mathcal{N}} a_i^k(TH - 1) = 1 \tag{2.1}$$

In other words, the full amount of each commodity will become available to ship at some point before the end of the time horizon. Each commodity has a delivery time window $[\underline{t}_{d_k}, \bar{t}_{d_k}]$ and penalties for early and late delivery $\underline{\gamma}^k$ and $\overline{\gamma}^k$ respectively. That is, if a delivery is made outside of the delivery window, the penalties $\underline{\gamma}^k$ or $\overline{\gamma}^k$ are charged for each time step outside of the window in proportion to the amount delivered.

Similar to the commodities, vehicles of type ℓ may become available by different time steps and at different nodes specified by $v_i^\ell(t)$.

- $\mathcal{N}$: Set of nodes i
- $\mathcal{A}$: Set of arcs
  - $h(e)$ = Head of arc e
  - $t(e)$ = Tail of arc e
- $\mathcal{K}$: Set of commodities

  - q^k = Total quantity of commodity k to be shipped
  - d_k = Destination of commodity k
  - $\underline{\gamma}^k$ = cost per time step of early delivery for the full amount of commodity k
  - $\overline{\gamma}^k$ = cost per time step of late delivery for the full amount of commodity k
  - $a_i^k(t)$ = quantity of commodity k available to load at node i by time t

- $\mathcal{L}$: Set of vehicle types
  - δ^ℓ = Capacity of vehicle type ℓ
  - c^ℓ = cost of vehicle type ℓ moving for one unit of time
  - t_e^ℓ = travel time on arc e for vehicle of type ℓ
  - $v_i^\ell(t)$ = Number of vehicles of type ℓ with a starting position of node i and available by time t
  - $p_e^\ell(t)$ = The spare capacity of vehicles of type ℓ on arc e and arriving at time t at the head node of arc e. These are vehicle movements which have been scheduled exogenously of our model.

- $\mathcal{T}$: Set of time periods
  - $[\underline{t}_{d_k}, \overline{t}_{d_k}]$ $\equiv$ time window indicating an on time delivery at the destination of commodity k

Decision Variables

The decision variables are:

- $w_e^{k,\ell}(t)$ = Indicates the percent of commodity k moved by vehicle type ℓ on arc e and arriving at time t at the head node of arc e. This implies that each commodity can be parceled.

- $y_e^\ell(t) \in Z_+$: Indicates integer number of vehicles of type ℓ traveling on arc e and arriving at time t at the head node of arc e

Objective Function

Transportation costs:

$$\sum_{e\in\mathcal{A},\ell\in\mathcal{L},t\in\mathcal{T}} t_e^\ell \cdot c^\ell \cdot y_e^\ell(t) \tag{2.2}$$

Missed delivery window cost:

$$\sum_{k\in\mathcal{K},e\in\mathcal{A}:h(e)=d_k,\ell\in\mathcal{L},t\leq\underline{t}_{d_k}} \underline{\gamma}^k \cdot (\underline{t}_{d_k} - t) \cdot w_e^{k,\ell}(t) \tag{2.3}$$

$$\sum_{k\in\mathcal{K},e\in\mathcal{A}:h(e)=d_k,\ell\in\mathcal{L},t\geq\bar{t}_{d_k}} \bar{\gamma}^k \cdot (t - \bar{t}_{d_k}) \cdot w_e^{k,\ell}(t) \tag{2.4}$$

The objective function accounts for the transportation costs of the vehicles (2.2) as well as a penalty for deliveries of commodities that arrive at their destination early (2.3) or late (2.4). The transportation costs portion of the objective function (2.2) sums the cost of every vehicle movement. The term $t_e^\ell \cdot c^\ell$ represents the cost of a vehicle of type ℓ traveling the entire length of an arc e. When multiplied by $y_e^\ell(t)$ we obtain the total cost incurred by all vehicles of type ℓ that reach the head of arc e at time t.

Sum (2.4) penalizes late deliveries. The total number of time periods by which the commodity is late $(t - \bar{t}_{d_k})$ is multiplied by a late arrival penalty of $\bar{\gamma}^k$. Similarly sum (2.3) penalizes early deliveries. The total number of time periods by which the commodity is early $(\underline{t}_{d_k} - t)$ is multiplied by an early arrival penalty of $\underline{\gamma}^k$.

Constraints

$$\sum_{\substack{e\in A:t(e)=i,\\ \ell\in\mathcal{L},\\ t'\le t+t_e^\ell}} w_e^{k,\ell}(t') \le \sum_{\substack{e\in A:h(e)=i,\\ \ell\in\mathcal{L},\\ t'\le t}} w_e^{k,\ell}(t') + a_i^k(t) \qquad \forall k\in\mathcal{K}, i\in N, t\in\mathcal{T} \tag{2.5}$$

$$\sum_{e\in A:h(e)=d_k,t\in\mathcal{T},\ell\in\mathcal{L}} w_e^{k,\ell}(t) = 1 \qquad \forall k\in\mathcal{K} \tag{2.6}$$

$$\sum_{k\in\mathcal{K}} q^k \cdot w_e^{k,\ell}(t) \le \delta^\ell y_e^\ell(t) + p_e^\ell(t) \qquad \forall \ell\in\mathcal{L}, e\in A, t\in\mathcal{T} \tag{2.7}$$

$$\sum_{e\in A:t(e)=i,t'\le t+t_e^\ell} y_e^\ell(t') \le \sum_{e\in A:h(e)=i,t'\le t} y_e^\ell(t') + v_i^l(t) \qquad \forall i\in N, \ell\in\mathcal{L}, t\in\mathcal{T} \tag{2.8}$$

$$w_e^{k,\ell}(t) \ge 0 \qquad \forall k\in\mathcal{K}, e\in A, t\in\mathcal{T} \tag{2.9}$$

Constraints (2.5) represent flow balance for the commodities. The sum on the left hand side represents commodities that have left node i at all times less than or equal to t. The sum on the right hand side represents commodities arriving at node i at all times less than or equal to t and commodities that have become available for shipment by time t. Thus, the sum of commodities that have left node i must be less than or equal to the sum of commodities that have arrived at the node.

Constraints (2.6) ensure that all commodities reach their destination by the end of the time horizon.

Constraints (2.7) are a capacity constraints for the vehicles. They state that the total amount of commodities carried by vehicles of type ℓ that reach the head of arc e at time t must be less than or equal to the total capacity of those same vehicles.

Constraints (2.8) represent flow balance for the vehicles and are similar to constraints (2.5). The sum on left hand side represents vehicles that have left node i at all times less than or equal to t. The sum on the right hand side represents vehicles arriving at node i at all times less than or equal to t and the vehicles which become available for use at node i by time t. Thus, the sum of commodities that have left node i must be less than or equal to the sum of commodities that have arrived at the node.

Of some note, this formulation is the combination of two network flow problems coupled by a capacity constraint. Constraints (2.5) and (2.6) represent the network flow problem for the commodities while constraints (2.8) represent the network flow problem for the vehicles. The two sub-problems are dynamically coupled by the capacity constraints (2.7). The interaction between the two sub-problems is what makes this stage non-trivial to solve. For example, with the removal of constraints (2.7), no vehicles would be routed and all commodities would be set to flow on arbitrary paths on the time-space network to arrive at their destination within their specified time windows.

2.3 Stage Two: Assignment of Individual Vehicles to routes and Commodities to Vehicles

In this stage we assign individual vehicles to routes, by their vehicle identification numbers. We also assign commodities to individual vehicles. As an input, stage two takes the optimal values of the decision variables from the first stage. The routes of the individual vehicles are consistent with the aggregate routes from the first stage.

Data and Sets

In addition to the data and sets presented in Section 2.3, we now introduce:

- $\tilde{w}_e^{k,\ell}(t)$= The optimal value of $w_e^{k,\ell}(t)$ obtained from the first stage.
- $\tilde{y}_e^{\ell}(t)$= The optimal value of $y_e^{\ell}(t)$ obtained from the first stage.
- $R(\ell)$ = The set of vehicles r of vehicle type ℓ.
- $\underline{\rho_\ell^k}$= Cost of unloading commodity k from a vehicle of type ℓ.
- $\overline{\rho_\ell^k}$= Cost of loading commodity k onto a vehicle of type ℓ.

Decision Variables

We need to decide how to route individual vehicles and decide upon which vehicles to place the commodities in order to satisfy all the problem constraints. To accomplish this result, we define the following decision variables.

- $w_e^{k,r}(t)$ = Indicates the percent commodity k moved by vehicle r on arc e and arriving at time t at node i the head node of arc e.

- $y_e^r(t) \in \{0,1\}$ = Indicates whether vehicle r travels on arc e and arriving at time t at node i the head node of arc e.

Objective Function

The vehicle routing and assignment of commodities to vehicles is handled minimizing the cost of transferring commodities between vehicles. We may consider different costs for unloading, $\underline{\rho_\ell^k}$, and loading commodities, $\overline{\rho_\ell^k}$.

Transferring costs (unloading costs):

$$\sum_{k\in\mathcal{K},\ell\in\mathcal{L}} \left(\underline{\rho_\ell^k} \cdot \sum_{i\in N,t\in T,r\in R(\ell)} \max \left\{0; \sum_{e\in A:h(e)=i} w_e^{k,r}(t) - \sum_{e'\in A:t(e)=i} w_e^{k,r}(t+t_e^r)\right\} \right) \tag{2.10}$$

Transferring costs (loading costs):

$$\sum_{k\in\mathcal{K},\ell\in\mathcal{L}} \left(\overline{\rho_\ell^k} \cdot \sum_{i\in N,t\in T,r\in R(\ell)} \max \left\{0; \sum_{e'\in A:t(e)=i} w_e^{k,r}(t+t_e^r) - \sum_{e\in A:h(e)=i} w_e^{k,r}(t)\right\} \right) \tag{2.11}$$

Constraints

$$\sum_{r\in R(\ell)} w_e^{k,r}(t) = \tilde{w}_e^{k,\ell}(t) \qquad \forall e \in A, k \in \mathcal{K}, \ell \in \mathcal{L}, t \in \mathcal{T}. \tag{2.12}$$

$$\sum_{r\in R(\ell)} y_e^{r}(t) = \tilde{y}_e^{\ell}(t) \qquad \forall e \in A, \ell \in \mathcal{L}, t \in \mathcal{T}. \tag{2.13}$$

$$\sum_{\substack{e\in A:t(e)=i,\\ \ell\in\mathcal{L}:r\in R(\ell),\\ t'\leq t+t_e^{\ell}}} w_e^{k,r}(t') \leq \sum_{\substack{e\in A:h(e)=i,\\ \ell\in\mathcal{L}:r\in R(\ell),\\ t'\leq t}} w_e^{k,r}(t') + a_i^k(t) \qquad \forall k \in \mathcal{K}, i \in N, t \in \mathcal{T} \tag{2.14}$$

$$\sum_{e\in A:h(e)=d_k,t\in\mathcal{T},r\in R(\ell)} w_e^{k,r}(t) = 1 \qquad \forall k \in \mathcal{K} \tag{2.15}$$

$$\sum_{k\in\mathcal{K}} a_{o_k}^k(t) \cdot w_e^{k,r}(t) \leq \sum_{r\in R(\ell)} \delta^{\ell} y_e^{r}(t) + p_e^{\ell}(t) \qquad \forall \ell \in \mathcal{L}, e \in A, t \in \mathcal{T} \tag{2.16}$$

$$\sum_{e\in A:t(e)=i,t'\leq t+t_e^{\ell}} y_e^{r}(t') \leq \sum_{e\in A:h(e)=i,t'\leq t} y_e^{r}(t') + v_i^{\ell}(t) \quad \forall \ell \in \mathcal{L} : r \in R(\ell), i \in N, t \in \mathcal{T} \tag{2.17}$$

$$w_e^{k,\ell}(t) \geq 0 \qquad \forall k \in \mathcal{K}, e \in A, t \in \mathcal{T} \tag{2.18}$$

Constraints (2.12) and (2.13) set the total number of vehicles and commodities traveling on each arc equal to the value prescribed in the first stage. Constraints (2.14) through (2.17) are of the same form as the constraints contained in the first stage,but they are for the individual vehicle level, not the aggregate vehicle type level.

2.4 Stage Three: Creation of an “Exact Time" Schedule

In the previous two stages the model utilizes a time horizon of discrete time steps, where each input or decision related to time is in increments of the time steps. In Figure 2-1 we show a time horizon that is split into six hour time steps. The choice of the size of the time step is a molding decision. In the first two stages the travel times of vehicles must be mapped to some number of time steps. In order to guarantee real world feasibility, the mapping of travel times to time steps must always round up. For example, in the context of the six hour time steps of Figure 2-1, we represent a travel time of ten hours as two time steps. Rounding up between actual travel time and time steps necessarily introduces slack into the schedule. Consider the example presented in Figure 2-2. The model states that the movement will arrive at its destination at a time of two time steps (12 hours); however the actual travel time is ten hours, so the vehicle will arrive early. The vehicle will remained parked at the port for at least two hours, this is the slack time. The modeler may control the amount of slack time introduced into the model by reducing the size of the time steps; however, this comes at the cost of a larger problem size.

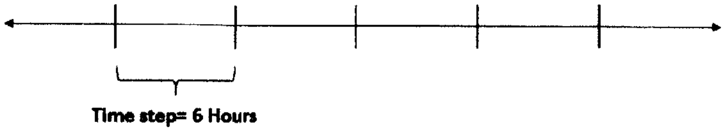

Figure 2-1: Time horizon with discretized time steps of six hours.

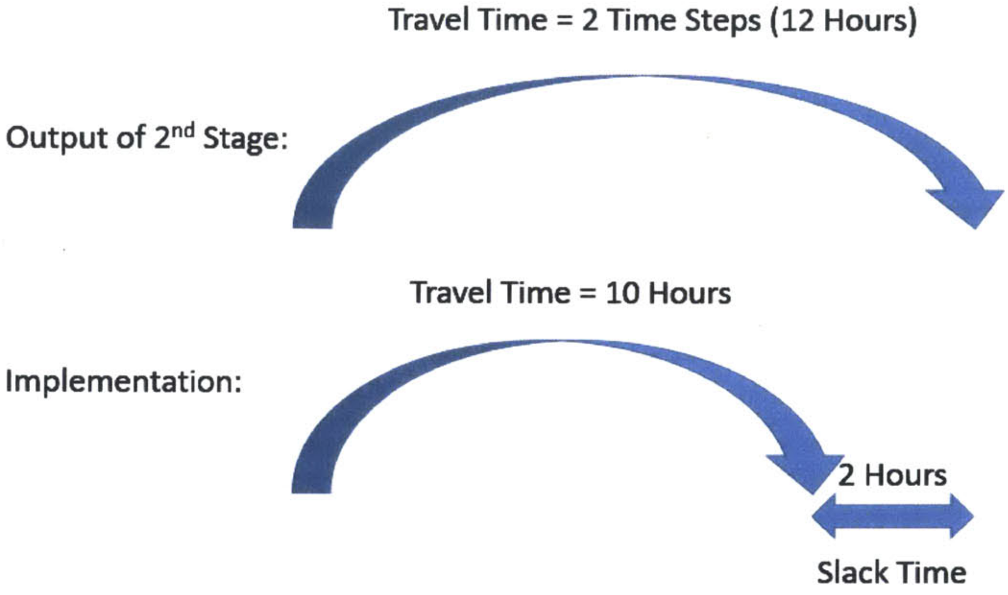
Travel Time = 2 Time Steps (12 Hours)
Output of 2nd Stage:
Travel Time = 10 Hours
Implementation:
2 Hours
Slack Time

Figure 2-2: Comparison of output of the second stage to schedule implementation. The second stage must schedule the movement for some number of time steps. The difference between actual travel time and its mapping to time steps introduces slack into the schedule.

The previous stages may schedule a vehicle to be parked at a node. This is different from the slack time that is introduced by the use of discrete time steps. The sum of slack time and planned parked time represents the total amount of idle time for each vehicle in implementation.

The purpose of the third stage is to reallocate the idle time for each vehicle in order to:

1. Provide adequate transfer time of commodities between vehicles at the point of transshipment.
2. Minimize the number of replacement crews needed to satisfy crew rest requirements for vehicles.

We refer to one leg of travel by a given vehicle on a single arc as a *movement* of the vehicle. Note that if vehicle r has a movement on arc e arriving at time t, then $y_e^r(t) = 1$, so that the number of movements of vehicle r is:

$$n_r = \sum_{e \in A, t \in T} y_e^r(t) \tag{2.19}$$

In the third stage the ordering of vehicle movements is preserved. Additionally, all transshipment remains feasible and all deliveries are occur on time. Outside of these constraints, the timing of movements may not be preserved.

Decision Variables

There are three sets of decision variables. They represent the amount of time that each vehicle will stay parked at each port it visits, the arrival time of each vehicle at each port it visits, and missed transfers.

- p_q^r: Indicates the continuous amount of time vehicle r is parked before its qth movement, where $q \in \{1, ..., n_r\} = Q_r$.
- v_q^r: Continuous variable indicating the actual arrival time of the qth movement of vehicle r.
- p_f^r: the amount of time vehicle r is parked at the final node it visits after movement n_r.
- $\phi_q^r = \begin{cases} 1, & \text{A replacement vehicle ships the commodities that missed a transfer}, \\ 0, & \text{otherwise.} \end{cases}$

Parameters and Ordering Information

- TH= the total amount of time being modeled, the end of the time horizon.
- $TT_r^{r'}$ = the amount of time needed to transfer a commodity from vehicle r to r'.
- t_q^r = the actual travel time (not the binned) that maps to movement q of vehicle r.

Ordering information determined from output of stage 2:

- $\mathcal{K}_q^r \subseteq \mathcal{K}$: the set of commodities k that are delivered to their final destination on the qth movement of vehicle r.
- $d_q^r = \min_{k \in \mathcal{K}_q^r} \max\{t_{d_k}, \bar{t}_{d_k}\}$: where t_{d_k}= the arrival time of commodity k at its final destination as prescribed in the previous stages. Recall, $\bar{t}_{d_k}$ is the latest time for an on time delivery of k.
- $\tilde{Q}_r \subseteq Q_r$: the set of movements q of vehicle r that deliver any amount of any commodity to its final destination.
- $(\underline{v_q^r}, \overline{v_{q'}^{r'}})$: a tuple that represents a transfer of commodities between vehicles.

Remarks. We use the parameter d_q^r to represent the latest acceptable completion time for movement q of vehicle r. Recall that $\bar{t}_{d_k}$ is a user set parameter from the first stage which represents the latest time for an on time delivery of commodity k at its destination. Additionally, the first stage may choose to deliver k at some time after $\bar{t}_{d_k}$ for a penalty, we call the actual delivery time of k, as specified in the first stage, t_{d_k}. Thus $\min_{k \in \mathcal{K}_q^r} \max\{t_{d_k}, \bar{t}_{d_k}\} = d_q^r$ represents the earliest of all the latest acceptable delivery times for the commodities k being delivered to their final destination on movement q of vehicle r. By defining the parameter in this way, the earliest possible required delivery time in the third stage corresponds to $\bar{t}_{d_k}$ of some commodity being delivered by r; however, we allow deliveries after $\bar{t}_{d_k}$ if this aligns with the output of the first stage.

The tuple $(\underline{v_q^r}, \overline{v_{q'}^{r'}})$ is defined for each time a commodity is transshipped in the second stage, that is, each time a commodity leaves a node on a different vehicle than that which delivered the commodity. An example in the context of our notation is: if $w_{e:h(e)=i}^{k=1,r=1}(t) = 1$ and $w_{e:t(e)=i}^{k=1,r=2}(t + t_{e:t(e)=i}^{\ell}) = 1$ then we define the tuple $(\underline{v_q^{r=1}}, \overline{v_{q'}^{r'=2}})$, where q corresponds to the movement of vehicle $r = 1$ and q' corresponds to the movement of vehicle $r = 2$.

Objective

$$\min \sum_{r \in R : q \in Q_r} \phi_q^r \tag{2.20}$$

Here we minimize the sum of replacement vehicles. It is reasonable to assume that the objective will often be zero. This occurs when the required transfer time of commodities is small relative to the aggregate slack time of the vehicles.

Constraints

$$\max\nolimits_{g_q^r \in \mathcal{U}} v_{q-1}^r + p_q^r + t_q^r + g_q^r \leq v_q^r \qquad \forall r \in R : q \in Q_r \tag{2.21}$$

$$v_{n_r}^r + p_f^r = TH \qquad \forall r \in R : n_r \in Q_r \tag{2.22}$$

$$\overline{v_{q'}^{r'}} - \underline{v_q^r} \geq TT_r^{r'} - M\phi_q^r \qquad \forall(\underline{v_q^r}, \overline{v_{q'}^{r'}}) \tag{2.23}$$

$$v_q^r \leq d_q^r \qquad \forall r \in R : q \in \tilde{Q}_r \tag{2.24}$$

$$p_q^r \geq 0 \qquad \forall r \in R : q \in Q_r \tag{2.25}$$

Constraints (2.21) state that the arrival time of a vehicle is equal to the sum of that vehicle's arrival time at its previous node with the amount of time parked at that node and the travel time to the node. Constraints (2.22) state that the sum of every vehicle's arrival time at its final node and the time parked at that node must equal the time horizon. Constraints (2.23) ensure there is a sufficient amount of time to transfer commodities between vehicles. In cases of insufficient transfer time a contract carrier may complete the delivery. This event is represented by ϕ_q^r. Constraints (2.24) ensures that all commodities will arrive at their final destination at or before the time specified by the first two stages.

Crew Rest Constraints

For some vehicle types it could be important to give the crews time to rest before a movement. For example, the US Air Force requires aircrew to have 12 consecutive hours of rest before any 16 hour duty period [18]. Crew rest constraints of this type may be added with the addition of binary variables. These additional constraints will not cause the model to become infeasible.

For each vehicle r that we would like to impose a crew rest constraints and for each movement of that vehicle $q \in Q_r$, we find the set of all prior resting periods that could possibly satisfy the crew rest requirements. Call this set:

$$\Omega_q^r = \{\omega \mid \sum_{i=\omega}^{q} t_i^r \leq 16, \omega \leq q\} \tag{2.26}$$

Additional Decision Variables

We introduce the new binary decision variable β_ω^r to represent the use of a replacement crew. We introduce $\epsilon_\omega^{r,q}$ to indicate the periods which satisfy crew rest for movement q of vehicle r.

- $\beta_\omega^r = \begin{cases} 1, & \text{a replacement crew is obtained prior to movement } \omega \text{ of vehicle } r, \\ 0, & \text{otherwise.} \end{cases}$
- $\epsilon_\omega^{r,q} = \begin{cases} 0, & \text{crew rest for movement } q \text{ of vehicle } r \text{ is fulfilled by } p_\omega^r \text{ or } \beta_\omega^r, \\ 1, & \text{otherwise.} \end{cases}$

Additional Constraints

For every movement q of each vehicle r that requires crew rest, we add the following constraints and variables to constraints (2.21) through (2.24):

$$p_\omega^r + 12\beta_\omega^r \geq 12 - 12\epsilon_\omega^{r,q} \quad \forall \omega \in \Omega_q^r, \omega > 1 \tag{2.27}$$

$$\sum_{\omega \in \Omega_q^r} \epsilon_\omega^{r,q} \leq |\Omega_q^r| - 1 \tag{2.28}$$

$$\sum_{i=\omega+1}^{q} p_i^r + \sum_{i=\omega}^{q} t_i^r \leq 16(1 - \epsilon_\omega^{r,q}) + M\epsilon_\omega^{r,q} \quad \forall \omega \in \Omega_q^r, \omega < q \tag{2.29}$$

Constraint (2.27) together with (2.28) state that one or more potential resting periods must provide at least 12 hours of rest or introduce a replacement crew $\beta_\omega^r = 1$. Note that we assume that crews are fully rested at the beginning of their first movement, therefore, we do not add constraint (2.27) for the case $\omega = 0$. Constraint

(2.29) ensures that the conclusion of the active resting period occurs within 16 hours of the ending of movement q of vehicle r.

Objective

In addition to (2.20), we now also minimize the need for replacement crews:

$$\min \sum_{\omega \in \Omega_q, r \in R} \beta_{\omega}^{r} \tag{2.30}$$

Chapter 3

Computational Results

The stage with the largest relative computational burden is the first stage. Thus we provide computational testing on the first stage model. Testing is conducted on a linux computer with 16 core Intel Xeon E5-2687W (3.1 GHz) processor and 128 GB of RAM, the amount RAM of is much greater than required for problems tested. We use Gurobi version 5.6 as the solver.

We test instances on two networks, the smaller of the two consists of 10 nodes, 34 directed arcs, and 20 time steps. The larger network contains 20 nodes, 112 arcs, and 20 time steps. We generate various instances of randomly placed vehicles and commodities, with each commodity having a random availability time and delivery window. Each instance includes two vehicle types. The vehicles are evenly split between types. In Table 3.1 we list the number of decision variables for the instances before presolve.

As is common in applications of mixed integer programming, not all instances solve to provable optimality. In Figure 3-1 we show an instance which solves to provable optimality. In Figure 3-2 we show an instance which fails to reach provable optimality before a preselected cutoff time of three hours. Note that the MIP gap represents the maximum possible percentage from optimality of the current best feasible solution. The actual distance from optimality is often less than the MIP gap. For example, in Figure 3-1 we find the optimal solution at 169 seconds but must wait for the lower bound to improve to this value at 3,578 seconds in order to prove optimality. In the same way, it is possible that the final best feasible solution shown in figure 3-2 is optimal, but we do not know for sure at the cutoff time.

Table 3.1: Number of decision variables before presolve for various instance sizes

	25 Commodities	40 Commodities	55 Commodities	70 Commodities	85 Commodities
Network Size	10 Vehicles	20 Vehicles	30 Vehicles	40 Vehicles	50 Vehicles
10 Nodes, 32 Arcs	39,520	62,320	85,120	107,920	130,720
20 Nodes, 112 Arcs	116,480	183,680	250,880	318,080	385,280

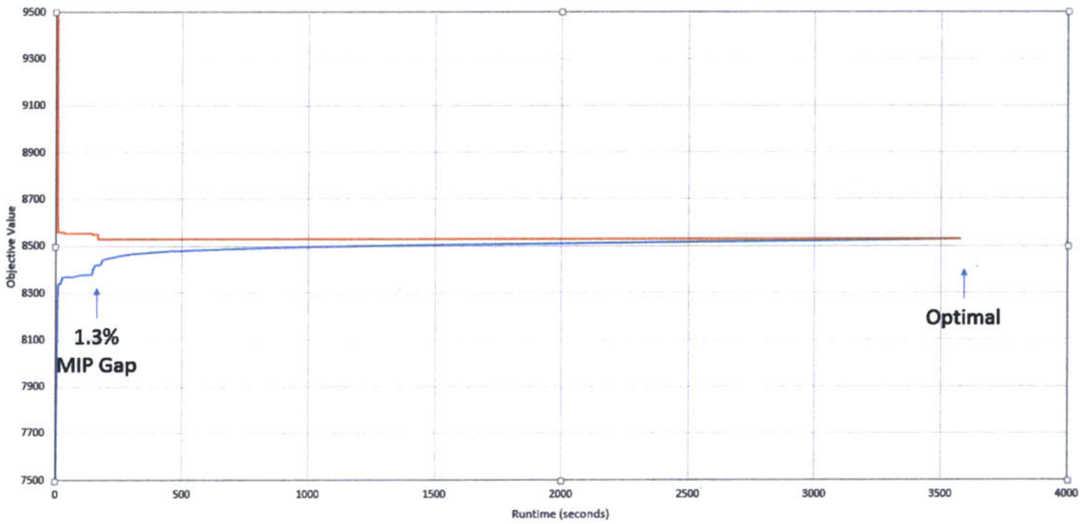

Figure 3-1: Comparison of lower and upper bounds for an instance that solves to optimality. This particular instance includes: 10 nodes, 34 directed arcs, 20 time steps, 85 commodities, and 50 vehicles.

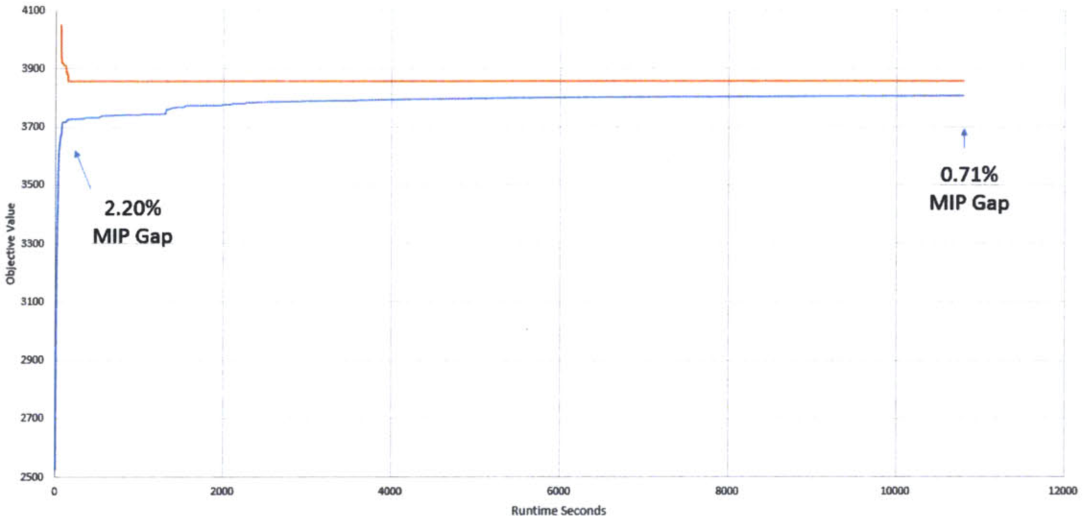

Figure 3-2: Comparison of lower and upper bounds for an instance that does not solve to optimality before the cutoff time of three hours. This particular instance includes: 10 nodes, 34 directed arcs, 20 time steps, 40 commodities, and 20 vehicles.

We also test the effects of increasing the scale of the instance as well as differing initial conditions for the vehicles and commodities. In figures 3-3 through 3-6 we show the MIP gaps for five runs of various instances of the same size. Figures 3-3 and 3-4 show MIP gaps for instances on the small network. Where figure 3-3 shows instances of 25 commodities and 10 vehicles and Figure 3-4 shows instances of 85 commodities and 50 vehicles.

In Figures 3-5 through 3-6 we show the MIP gaps for instances on the large network. Where Figure 3-5 shows instances of 25 commodities and 10 vehicles and Figure 3-6 shows instances of 85 commodities and 50 vehicles.

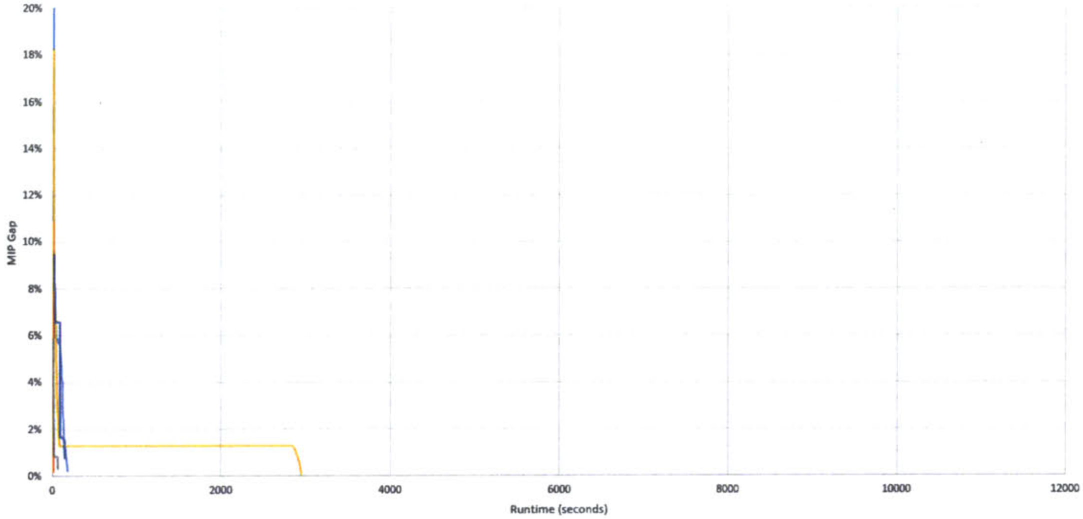

Figure 3-3: MIP gap over time for five randomly generated instances of size: 10 nodes, 34 directed arcs, 20 time steps, 25 commodities, and 10 vehicles.

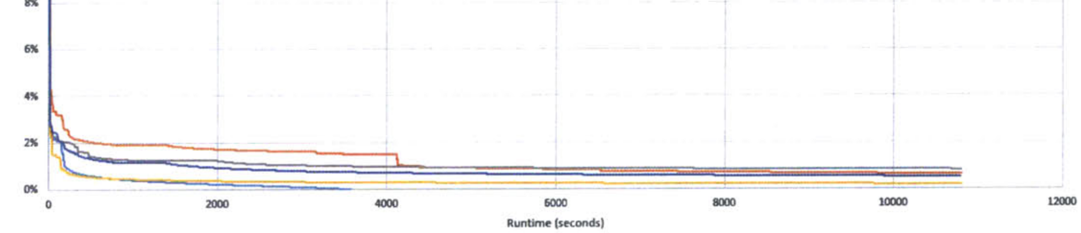

Figure 3-4: MIP gap over time for five randomly generated instances of size: 10 nodes, 34 directed arcs, 20 time steps, 85 commodities, and 50 vehicles.

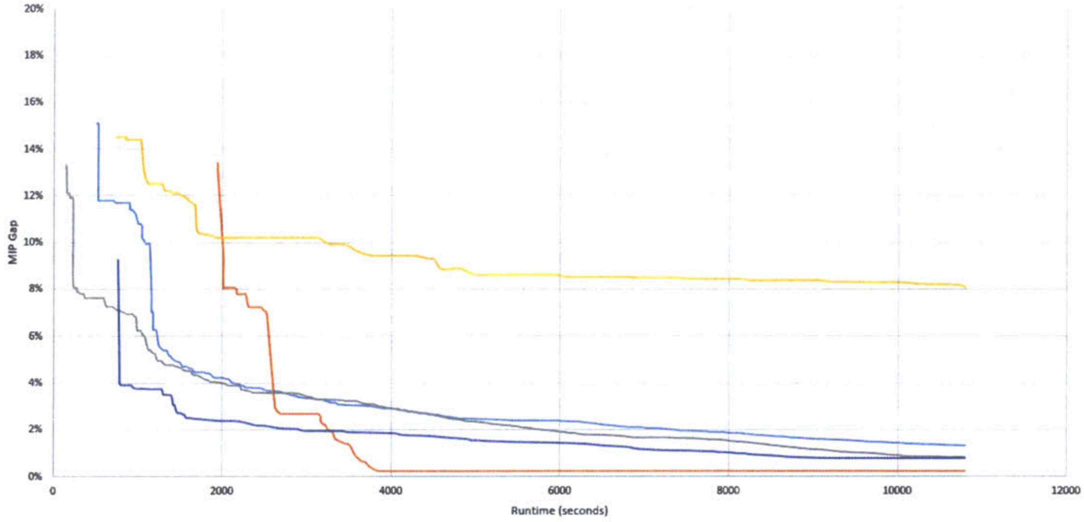

Figure 3-5: MIP gap over time for five randomly generated instances of size: 20 nodes, 112 directed arcs, 20 time steps, 25 commodities, and 10 vehicles.

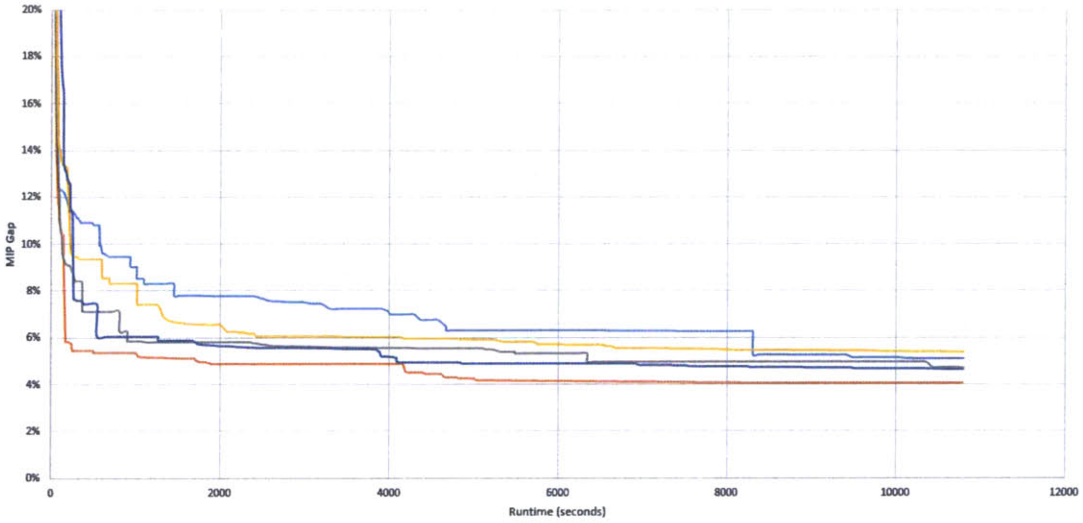

Figure 3-6: MIP gap over time for five randomly generated instances of size: 20 nodes, 112 directed arcs, 20 time steps, 85 commodities, and 50 vehicles.

We are also test the solving characteristics as the instance size grows very large. In Table 3.2 we show the MIP gaps achieved at one and three hours for instances of increasing size. While the gap size increases as problem size grows, we still achieve reasonable feasible solutions, even for the largest instance of 550 commodities and

330 vehicles.

Table 3.2: MIP gaps achieved for instances of increasing size. All instances shown in this table are on the 20 node network with 20 time steps.

(Comms, Nodes)	1 hour gap	3 hour gap
100, 60	3.70%	3.27%
	3.59%	3.34%
150, 90	4.98%	4.17%
	4.65%	4.05%
200, 120	4.05%	3.94%
	3.13%	2.54%
250, 150	4.22%	3.19%
	5.93%	5.14%
300, 180	4.58%	4.35%
	6.20%	4.12%
350, 210	5.78%	5.25%
	11.20%	6.41%
400, 240	4.54%	3.41%
	infeas	infeas
450, 270	10.80%	6.71%
	8.00%	3.43%
500, 300	13.20%	8.23%
	10.60%	7.50%
550, 330	9.45%	8.38%
	8.75%	7.79%

Chapter 4

Conclusion

In this thesis we present a multi-stage mixed integer program to optimally route vehicles and goods. Our model allows for a fleet of heterogeneous vehicles, transshipment and time windows for delivery. By splitting the optimization into multiple stages we are able to increase its tractability. We provide computational results that show the model produces provably optimal or near optimal solutions for realistic instance sizes.

4.1 Future Research

Multi-modal transportation is a very large area of study and there are many additional avenues to explore. In the context of our work, one potential avenue of exploration is to continue to work on increasing the tractability of the model. In this thesis we explore networks of up to 20 time steps and 112 arcs. In the future it would be nice to be able to achieve quick results on instances with arcs in the thousands.

In our modeling we account for many constraints, including crew rest; however, there are many other constraints which could be important for a freight shipper. These include vehicle capacity at nodes, node operating hours, cargo incompatibility, and many more. The exploration of the addition of these constraints is an important area of research.

We assume that the input data is known with certainty; however, this is not always the case in reality. Incorporating known uncertainty into the model, perhaps with robust optimization, could prove very promising.

Appendix A

Mathematical Model with By Variables

A.1 Motivation

An additional formulation we have tested for the first stage we call the 'by variable' formulation.

By changing the definition of the decision variables we are able to make the constraint matrix much more sparse for the flow balance constraints. This is because many of the sums no longer need to include limits over all time t. However, this comes at the expense of a more dense set of capacity constraints. Testing of the by variable formulation does not show any gains over the formulations with at variables.

A.2 Data and Sets

All data and sets remain the same as in the formulation with at variables.

A.3 Decision Variables

The decision variables are:

- $w_e^{k,\ell}(t)$ = Indicates the percent of commodity k moved by vehicle type ℓ on arc e and arriving by time t at the head of arc e

- $y_e^{\ell}(t) \in Z_+$: Indicates integer number of vehicles of type ℓ traveling on arc e and arriving BY time t at the head node of arc e

These are strictly increasing monotone variables with respect to time. They are a cumulative measure of vehicle and commodity movement over time.

It is possible to obtain W AT variables from the W BY variables with the following transformations:

BY Variables: $(w_e^{k,\ell}(t) - w_e^{k,\ell}(t-1)) \rightarrow$

AT Variable: $w_e^{k,\ell}(t)$

The same is true for the Y variables:

BY Variables: $(y_e^{\ell}(t) - y_e^{\ell}(t-1)) \rightarrow$

AT Variable: $y_e^{\ell}(t)$

A.4 Objective Function

Transportation costs:

$$\sum_{e\in\mathcal{A},\ell\in\mathcal{L}} d(e)\cdot c^{\ell}\cdot y_e^{\ell}(t_{final}) \tag{A.1}$$

Missed deliver window cost:

$$\sum_{k\in\mathcal{K},e\in\mathcal{A}:h(e)=d_k,\ell\in\mathcal{L},t\geq\bar{t}_{d_k}} \bar{\gamma}^k\cdot(t-\bar{t}_{d_k})\cdot a_i^k(t)\cdot(w_e^{k,\ell}(t)-w_e^{k,\ell}(t-1)) \tag{A.2}$$

$$\sum_{k\in\mathcal{K},e\in\mathcal{A}:h(e)=d_k,\ell\in\mathcal{L},t\leq\underline{t}_{d_k}} \underline{\gamma}^k\cdot(\underline{t}_{d_k}-t)\cdot a_i^k(t)\cdot(w_e^{k,\ell}(t)-w_e^{k,\ell}(t-1)) \tag{A.3}$$

The objective function accounts for the transportation costs of the vehicles as well as a penalty for deliveries of commodities which fall outside of their specified delivery time window. The transportation costs portion of the objective function sums the cost of every vehicle movement. The term $d(e)\cdot c^{\ell}$ represents the cost of a vehicle of type ℓ traveling the entire length of an arc e. When multiplied by $y_e^\ell(t_{final})$ we obtain the total cost incurred by all vehicles of type ℓ which have traversed arc e.

Sum A.2 penalizes for deliveries that occur after the $\overline{t}_{d_k}$. The total number of time periods for which the commodity is late $(t - \overline{t}_{d_k})$ is multiplied by a late arrival penalty of $\overline{\gamma}^k$. Similarly sum A.3 penalizes for deliveries that occur before the $\underline{t}_{d_k}$. The total number of time periods for which the commodity is early $(\underline{t}_{d_k} - t)$ is multiplied by an early arrival penalty of $\underline{\gamma}^k$.

A.5 Constraints

$$\sum_{e\in A:t(e)=i,\ell\in\mathcal{L}} w_e^{k,\ell}(t+t_e^\ell) \le \sum_{e\in A:h(e)=i,\ell\in\mathcal{L}} w_e^{k,\ell}(t) + a_i^k(t) \qquad \forall k\in\mathcal{K}, i\in N, t\in\mathcal{T} \tag{A.4}$$

$$\sum_{e\in A:h(e)=d_k,\ell\in\mathcal{L}} w_e^{k,\ell}(t_{final}) = 1 \qquad \forall k\in\mathcal{K} \tag{A.5}$$

$$\sum_{k\in\mathcal{K}} a_{o_k}^k(t)\cdot(w_e^{k,\ell}(t) - w_e^{k,\ell}(t-1)) \le \delta^\ell\cdot(y_e^\ell(t) - y_e^\ell(t-1)) \qquad \forall \ell\in\mathcal{L}, e\in A, t\in\mathcal{T} \tag{A.6}$$

$$\sum_{e\in A:t(e)=i} y_e^\ell(t+t_e^\ell) \le \sum_{e\in A:h(e)=i} y_e^\ell(t) + v_i^l(t) \qquad \forall i\in N, \ell\in\mathcal{L}, t\in\mathcal{T} \tag{A.7}$$

$$y_e^\ell(t-1) \le y_e^\ell(t) \qquad \forall \ell\in\mathcal{L}, e\in A, t\in\mathcal{T} \tag{A.8}$$

$$w_e^{k,\ell}(t-1) \le w_e^{k,\ell}(t) \qquad \forall k\in\mathcal{K}\ell\in\mathcal{L}, e\in A, t\in\mathcal{T} \tag{A.9}$$

Constraints A.4 represent flow balance for the commodities. The sum on left hand side represents commodities that have left node i by time t. The sum on the right hand side represents commodities that have arrived at node i by time t. $a_i^k(t)$ represents commodities which arrive at node i that have become available for shipment by time

t. $a_i^k(t)$ is a parameter of the model. Thus, the sum of commodities which have left node i must be less than or equal to the sum of commodities which have arrived at the node.

Constraints A.5 ensure that all commodities reach their destination by the end of the time horizon.

Constraints A.6 are capacity constraints for the vehicles. They state that the total amount of commodities carried by vehicles of type ℓ which reach the head of arc e at time t must be less than or equal to the total capacity of those same vehicles.

Constraints A.7 represent flow balance for the vehicles and is a similar to constraints A.4. The sum on left hand side represents vehicles that have left node i by time t. The sum on the right hand side represents vehicles that have arrived at node i by time t. $v_i^l(t)$ represents the vehicles which become available for use at node i by time t. $v_i^l(t)$ is a parameter of the model. Thus, the sum of commodities which have left node i must be less than or equal to the sum of commodities which have arrived at the node.

Constraints A.8 and A.9 ensure that $y_e^\ell(t)$ and $w_e^{k,\ell}(t)$ are monotone.

Appendix B

First Stage with Additional Variables

We present here our original formulation. In this formulation we include variables that represent the inventory level of vehicles and commodities at the nodes.

B.1 Decision Variables

The decision variables are:

- $w_e^{k,\ell}(t)$ = Indicates the quantity of commodity k moved by vehicle type ℓ on arc e and arriving AT time t at node j (head node of arc e)
- $x_i^k(t)$ = Indicates the inventory level of commodity k at node i AT time t.
- $y_e^\ell(t) \in Z_+$: Indicates integer number of vehicles of type ℓ traveling on arc e and arriving AT time t at node j (head node of arc e)
- $z_i^\ell(t)$ = Indicates the number of vehicles of type ℓ parked at node i AT time t.

B.2 Objective Function

Transportation costs:

$$\sum_{e \in \mathcal{A}, \ell \in \ell} \sum_{t \in [\underline{t}_e^k, \bar{t}_e^k]} c_e^\ell \cdot y_e^\ell(t) \tag{B.1}$$

(B.2)

Missed deliver window cost:

$$\sum_{k\in\mathcal{K},e\in\mathcal{A}:h(e)=d_k,\ell\in\mathcal{L},t\in T} \max\{0;\ \overline{\gamma}^k\cdot(t-\bar{t}_{d_k});\ \underline{\gamma}^k\cdot(\underline{t}_{d_k}-t)\}\cdot w_e^{k,\ell}(t) \qquad \text{(B.3)}$$

$$x_i^k(t+1)+\sum_{\substack{e\in A:t(e)=i,\\ \ell\in\mathcal{L}}} w_e^{k,\ell}(t+t_e^\ell)=x_i^k(t)+\sum_{\substack{e\in A:h(e)=i,\\ \ell\in\mathcal{L}}} w_e^{k,\ell}(t)+a_i^k(t)\quad \forall k\in\mathcal{K},i\in N,t\in\mathcal{T}$$

(B.4)

$$\sum_{e\in A:t(e)=d_k,t\in[\underline{t}_{d_k},\bar{t}_{d_k}],\ell\in\mathcal{L}} w_e^{k,\ell}(t)=q^k \qquad \forall k\in\mathcal{K}$$

(B.5)

$$\sum_{k\in\mathcal{K}} w_e^{k,\ell}(t)\le\delta^\ell y_e^\ell(t) \qquad \forall\ell\in\mathcal{L},e\in A,t\in\mathcal{T}$$

(B.6)

$$z_i^\ell(t+1)+\sum_{e\in A:t(e)=i} y_e^\ell(t+t_e^\ell)=z_i^\ell(t)+\sum_{e\in A:h(e)=i} y_e^\ell(t) \qquad \forall i\in N,\ell\in\mathcal{L},t\in\mathcal{T}$$

(B.7)

$$z_i^\ell(0)+\sum_{e\in\ A:t(e)=i,t=0} y_e^\ell(t+t_e^\ell)=v_i^\ell \qquad \forall i\in\mathcal{N},\ell\in\mathcal{L}$$

(B.8)

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