14.02

Bank Runs

Fall 2009
1 Bank runs

- Diamond and Dybvig (1983)
- Three periods $T = 0, 1, 2$
- Continuum of agents
- Preferences

$$u(c_1 + \eta c_2)$$

where $\eta$ idiosyncratic shock

$$\eta = \begin{cases} 
0 & \text{with probability } \tau \quad (\text{“early consumer”}) \\
1 & \text{with probability } 1 - \tau \quad (\text{“late consumer”})
\end{cases}$$
• Agents have an endowment normalized to 1

• At time 0, each agent invests without knowing his shock \( \eta \)

• No aggregate uncertainty: exactly \( \tau \) fraction of agents will have \( \eta = 0 \)

• Technology: if an agent invests 1 at time 0, he can get:
  
  1. \( x \) if he chooses to liquidate a fraction \( x \) at time 1
  2. \( R (1 - x) \geq (1 - x) \) if he liquidates \( 1 - x \) at time 2
Autarky:

- single agent will choose how much to consume in two periods

\[
\max_{x, c_1, c_2} \quad E \left[ u \left( c_1 (\eta) + \eta c_2 (\eta) \right) \right]
\]
\[
c_1 (\eta) \leq x
\]
\[
c_2 (\eta) \leq R (1 - x)
\]

- optimal to choose

\[
c_1 (1) = 0, c_2 (1) = R
\]
\[
c_1 (0) = 1, c_2 (0) = 0
\]
Banks:

- risk sharing arrangement

\[
\max \quad E \left[ u \left( c_1(\eta) + \eta c_2(\eta) \right) \right] \\
E \left[ c_1(\eta) \right] \leq x \\
E \left[ c_2(\eta) \right] \leq R (1 - x)
\]

- optimal

\[
c_1(1) = 0, \quad c_2(0) = 0
\]

and

\[
u'(c_1(0)) = Ru'(c_2(1))
\]
Assumptions

- Assumption 1: high coefficient of relative risk aversion
- Assumption 2: avoiding liquidation is more profitable

\[ R > 1 \]
Optimal liquidation policy

- use $C_1$ for $c_1(0)$ and $C_2$ for $c_2(1)$

- resource constraint at time 2 imposes

$$ (1 - \tau C_1) R = (1 - \tau) C_2 $$

- find $C_1$ s.t.

$$ u'(C_1) = R u' \left( R \frac{1 - \tau C_1}{1 - \tau} \right) $$

- Result: under $A1$ and $A2$

$$ 1 < C_1 < C_2 < R $$
• Some, but not complete, insurance for the early consumers

• → autarky is suboptimal

• check that A2 is needed: with \( u(c) = \log c \), then \( C_1 = 1 \) and \( C_2 = R \) works

\[
C_1^{-1} = R \left( R \frac{1 - \tau C_1}{1 - \tau} \right)^{-1}
\]

\[
1 = R \left( R \frac{1 - \tau}{1 - \tau} \right)^{-1}
\]

• but not with CRA > 1, e.g. \( u(c) = c^{1-\gamma} / (1 - \gamma) \), then

\[
1^{-\gamma} > R \left( R \frac{1 - \tau}{1 - \tau} \right)^{-\gamma} \text{ if } \gamma > 1
\]
How do we implement a banking allocation?

- Offer all consumers the option to withdraw $C_1$ in the first period: *demand deposit* contract

- incentive compatibility:
  
  1. for the early consumers is trivial:
      \[ u(C_1) \geq u(0 + 0C_2) = 0 \]

  2. for the late consumers:
      \[ u(C_1) \geq u(C_2) \]
      because $C_2 > C_1$ (from result above)
Unique implementation?

- Given that the bank offers to all consumers the possibility to withdraw there exists an equilibrium where only early consumers withdraw (IC ensures that)

- but is that the only equilibrium?

- with a demand deposit contract NO
Bad equilibrium

- All consumers apply for $C_1$ in first period
- The bank only has 1 unit of asset to liquidate
- Some consumers are rationed (the ones last in line)
• Why late consumers do not wait?

• If you do not apply for $C_1$ you get

$$\tilde{C}_2 = R \frac{\max \{1 - \tilde{\tau} C_1, 0\}}{1 - \tilde{\tau}}$$

where $\tilde{\tau}$ is the number of consumers who apply for $C_1$

• So if $\tilde{\tau} = 1$ then $\tilde{C}_2 = 0$

• If you expect everyone to run, running is a best response
Suspension of convertibility

- The bank announces: I'll give $C_1$ to the first $\tau$ people that show up in period 1, 0 to the rest of them

- Now it is optimal to wait for a late consumer

- Equilibrium is unique
• But now introduce some aggregate uncertainty about $\tau$: $\tau$ is a random variable with CDF $F(\tau)$

• sometimes there is more early consumers, sometimes less

• Now optimum has

$$u'(C_1(\tau)) = Ru'\left(\frac{R\left(1 - \tau C_1(\tau)\right)}{1 - \tau}\right)$$

• This optimum is incentive compatible but it cannot be implemented if the bank is facing a *sequential service constraint*: You can only assign consumption to consumers who show up in period 1 on the basis of their position in the line
Simple alternative: Demand deposits + deposit insurance

- Historically this combination has proved very successful

- Now is the government that takes care of making $C_1$ state contingent: if too many people show up, everyone is taxed so that they get paid and the late consumers are protected

- The government effectively has a way of intervening after $\tau$ is realized

- In this way the bad equilibrium is ruled out
• Important principle: the gov’t does not actually intervene in equilibrium

• Just announcing intervention off-the-equilibrium path, eliminates the bad equilibrium

• These are very desirable policy interventions: no actual intervention (no tax levied, no distortion created), very big effects (sometimes too good to be true?)
Repo market: reinterpreting a bank run

- The bank borrows short term from the consumers to invest in long run project and at the same time sells equity shares

- Promises to repay rate of return $C_1/1 > 1$ with expectation that the loan will be rolled over

- If consumers decide to roll over they will get $C_2/1 > C_1/1$

- If loan not rolled over the bank won’t be able to offer positive return to consumers $C_2/1$
• Bad equilibrium: banks refuse to roll over → “run” followed by bankruptcy

• The model can be reinvented to better match the competitive determination of interest rates in repo markets (and the role of collateral)

• But the underlying logic is there

• See letter of Cox (SEC Chairman) to the Basel Committee
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