A Multi-Scale Model for Piston Ring Dynamics, Lubrication and Oil Transport in Internal Combustion Engines

by

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Abstract

Fuel consumption reduction of more than 20% can be achieved through engine friction reduction. Piston and piston rings contribute approximately half of the total engine friction and are therefore central to friction reduction efforts. The most common method to reduce mechanical losses from piston rings has been to lower ring tension, the normal force providing sealing between the piston ring and the cylinder liner. However tension reduction can result in additional lubricant consumption. The objective of this thesis is to understand and model the physical mechanisms resulting in flow of oil to the combustion chamber in order to achieve optimal designs of piston rings. The optimal design is a compromise between friction reduction and adequate gas and lubricant sealing performance.

To do so a multi-scale curved beam finite element model of piston ring is developed. It is built to couple ring deformation, dynamics and contact with the piston and the cylinder. Oil flow at the interfaces between the ring and the cylinder liner and between the ring and the piston groove can thus be simulated. The piston ring model is used to study the sealing performance of the Oil Control Ring (OCR), whose function is to limit the amount of oil supplied to the ring pack.

The contributions of the three main mechanisms previously identified, to oil flow past the OCR are quantified:

- Deformation of the cylinder under operating conditions can lead to a loss of contact between the ring and the liner.
- Tilting of the piston around its pin can force the OCR to twist and scrape oil from the liner.
- Oil accumulating below the OCR can flow to the groove and leak on the top of the OCR

The OCR is found to be flexible enough to limit the impact of cylinder deformation on oil consumption. Both ring scraping and flow through the OCR groove can contribute to oil consumption in the range of engine running conditions simulated. Reduction of scraping is possible by increasing the ability of both OCR lands to maintain contact with the liner regardless of piston groove tilt. The flow of oil through the OCR groove can be reduced by designing appropriate draining of oil in the groove and an adequate oil reservoir below the OCR. The piston ring oil transport model developed in this thesis will be a valuable tool to optimize ring pack designs to achieve further ring pack friction reduction without increasing oil consumption.
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1 Introduction

1.1 Motivation

1.1.1 Energy efficiency and frictional losses in Internal Combustion Engines

The transportation sector which accounts for approximately a third of all energy use in the United States [1] relies mostly on Internal Combustion Engines (ICE) for powertrains and petroleum as an energy source and is likely to continue doing so in the near future. Consequently transportation and the Internal Combustion Engine will play an important role in the future reduction of emission of pollutants and greenhouse gases. To make the transportation sector fuel efficient and reduce its environmental impact, efficiency of the ICE will have to be improved. A significant portion of energy losses in ICE can be attributed to friction losses (Figure 1.1).

![Energy Flow in an Internal Combustion Engine](image)

Figure 1.1 - Energy Flow in an Internal Combustion Engine – (Adapted from [2])

Figure 1.1 shows that most of the fuel energy is lost through the exhaust or the cooling system. These are heat and thermodynamics losses. Out of the 38% mechanical power outputted by the engine only 21% is used to actually move the vehicle. The rest is mechanical losses either in the engine or in the transmission. Frictional loss reduction has an important role to play in the current effort to increase efficiency of ICEs. An ambitious but yet physically attainable reduction of 50% of engine friction would lead to a gain of 6% of the engine effective output. Since this 6% would
add to the current 21% of fuel energy used to move the vehicle, it corresponds to a decrease of fuel consumption of almost 29%. This highlights the great contribution that engine friction can have on vehicle fuel efficiency. In addition, friction reduction measures are often economical. They correspond to optimization of lubrication systems and incremental design improvements of mechanical components and do not require the addition of costly new systems to the engine.

Engine friction losses is subdivided as follows: 45% for the piston assembly, 30% for bearings, 15% for valve trains and finally 10% for pumping systems ([2]). The power cylinder system, composed of piston and piston rings is thus central to engine friction reduction efforts. This thesis is focusing on simulating the performance of piston rings as they are responsible on average of approximately 50% of the friction losses from the piston assembly ([2]-[3]). The motivation behind this work is that is by improving the understanding of piston ring behavior, improved ring designs can be generated and contribute to increase engine efficiency.

1.1.2 Overview of the Power Cylinder System

The piston assembly which is the focus of this thesis is composed of the piston and the three piston rings (Figure 1.2)
The piston primary function is to transmit to the crankshaft, the mechanical work done by the ignited gas fuel mixture of the combustion chamber. Geometrical features of the piston related to its function are highlighted in Figure 1.3.

Three grooves are machined on the side of the piston to insert the piston rings. In the 3rd groove it is customary to find draining holes to allow flow of gas and release of oil to the crankcase. Below the grooves and 90 degrees on both side of the pin is the piston skirt. It is the surface in contact with the cylinder liner and whose purpose is to sustain the lateral forces and guide the piston in the cylinder.

Each of the piston rings has a specific function. The compression ring or top ring is mostly a gas seal, its function is to prevent leakage of gas from the combustion chamber through the clearance between the piston and the cylinder liner. The 3rd ring or oil control ring (OCR) is mostly responsible for controlling the amount of lubricant supplied to the ring pack as its name indicates. Supply of oil to the piston skirt is large and the OCR is making sure that only a thin oil film is left on the liner to allow proper lubrication of the top two rings while limiting oil consumption. The second ring or scraper ring has a hybrid function, it is both an oil and gas ring. It limits the rise of pressure in the volume above the OCR and stops excessive amount of lubricant that was allowed to pass the OCR.
The nomenclature of the piston ring pack is presented on a section view of piston, cylinder and ring pack (Figure 1.4).

The outer piston surfaces between the rings, in the ring pack are called piston lands. Cylinder-land clearances vary by design to control gas pressure in operation in the various regions of the ring pack. A cut or chamfer can be found at the top of the piston skirt. It has several functions, the first of which is to retain oil to lubricate the piston skirt during the up stroke. Secondly the cut limits the build-up of pressure in the oil below the OCR thus limiting flow to the OCR groove. Several designs of the chamfer exists. It could be a square cut like the one pictured in Figure 1.4 or a diagonal cut.

The three piston rings have different designs tailored to their function. The top ring section is nearly rectangular. Its running face (in contact with the cylinder) has a microscopic parabolic profile to favor generation of hydrodynamic pressure and improve lubrication. Several features can be added such as static twist to expose the upper surface to high pressure gas in order to push
the ring against the lower flank of the top groove and make sure it acts as a proper gas seal. The second ring pictured in Figure 1.4 is a Napier ring which is a common design for second rings. A hook has been cut to act as an oil reservoir to prevent oil flowing on the piston 3rd land from returning to the liner. The 3rd ring or OCR has a distinct design that separates it from the top two rings. The OCR pictured in Figure 1.4 is a two piece OCR, it has two components: a metallic ring and a coiled spring inserted in the groove located at the inner diameter of the metallic ring. This particular design ensures maximum flexibility of the OCR so that it can maintain contact with the cylinder regardless of the cylinder deformation. Top two rings rely on the ring elastic force (generated when closing the ring in the cylinder) to provide a radial contact pressure sealing the ring-liner interface. Since the OCR has a thin section it does not generate enough elastic force when closed which is why a coiled spring is placed behind it. Only two thin portions of the ring outer diameter called the OCR lands are in contact with the liner. Their profile is also flat and parallel to the liner. It increases the contact pressure and limits hydrodynamic pressure generation. This helps control the thickness of the oil film supplied to the top two rings and limits the amount of lubricant consumed.

Several variations of OCR design exist, the two most commonly used are the two piece Oil Control Ring and the three piece Oil Control Ring (Figure 1.5).

![Two Piece Oil Control Ring](image1)

![Three Piece Oil Control Ring](image2)

*Figure 1.5 - Oil Control Ring designs [5]*

The three piece oil control ring design is composed of an expander between two rails. The expander is a split ring pushing the two rails against and the upper and lower flanks of the piston groove and against the cylinder. The three piece OCR design provides better sealing between the ring and the
piston groove but has typically less ability to conform to cylinder deformation. In this thesis, the focus has been placed on two piece oil control rings (most commonly found in engine applications) though the general methodology can be extended to other rings (OCR or top two rings).

1.1.3 The compromise between friction reduction and oil consumption

As stated earlier, the main motivation of this project is to reduce the contribution of piston rings to engine mechanical losses in order to improve fuel efficiency. In the sliding contact between piston ring and cylinder liner, a friction force parallel to, but in the opposite direction of ring motion is generated and results in energy losses (Figure 1.6).

![Figure 1.6 - Illustration of the compromise between friction reduction and oil consumption](image)

By definition this friction force $F_f$ is the product of the normal load of the ring-liner contact and the friction coefficient $f_c$ (1.1).

$$F_f = 2\pi f_c F_t$$

(1.1)

In the case of the ring-liner contact, the normal force is provided by the ring tension $F_t$ and its integration over the ring circumference is equal to $2\pi F_t$. For a ring with standard design, ring tension, which is the tangential force required to close its gap, is statically equivalent to a uniform radial pressure distribution (Figure 1.7).
The relation between ring tension and the radial force per unit length $f_r$ can be calculated by looking at the free body diagram of a half ring (analogous to the effect of surface tension on the difference of pressure across a spherical bubble) (1.2).

$$f_r = \frac{F_t}{R} \quad (1.2)$$

For the top two rings the shape of the free ring is carefully designed to obtain the desired distribution of contact force when inserted in the cylinder. Prescott first investigated the relation between free ring geometry and contact force distribution in the cylinder [6]. He found that to obtain a uniform contact force, the ring free geometry must deviate from a circle and that the geometry was based on the amount of bending the ring undergoes when inserted in the cylinder.

For the OCR, the radial force is provided by the coiled spring. Since the spring can easily bend and adapt to the deformation of the OCR, it is assumed that it provides a radial force that is uniform along the circumference of the ring.

Going back to the definition of the friction force (equation (1.1)), one can see that there are two ways to reduce ring friction. The first is to improve the friction coefficient in order to have less friction force for the same radial load. The second is to reduce ring tension and thus the radial load. Modifying ring tension only requires a change of coiled spring in the case of the OCR, or a change
of free geometry in the case of the top two rings. It is usually easier to achieve than a reduction of friction coefficient. Recent efforts to optimize the ring-liner friction coefficient include design of the liner surface roughness to improve lubrication and tuning of the lubricant properties (viscosity, additive package) to reduce energy losses. Additional research is required to achieve friction coefficient reduction and tension reduction remains a method of choice to directly impact ring pack friction.

A compromise must be made however between tension reduction and sealing performance of the ring. As illustrated in Figure 1.6, ring tension reduction leads to a reduction of the radial force pushing the ring against the cylinder and results in a thicker oil film (low tension situation shown in broken lines). This illustrates how a reduction of tension of the OCR can negatively affect oil consumption by increasing the amount of oil passing through [7]. Though this is a simplified example (for instance the width of the OCR lands can be reduced to maintain the magnitude of the contact pressure) the main argument is valid. Tension reduction reduces the amount of elastic force maintaining the ring in contact with the cylinder and therefore affects its sealing performance. Indeed it is less able to adapt to cylinder deformation or resist dynamic twisting both of which can contribute to oil consumption.

This has been the question central to this work: how far can ring tension be reduced without affecting the ring sealing performance? The follow up question being how can ring design be modified to allow further tension reduction?

1.2 Oil transport in the piston ring pack of Internal Combustion Engines

1.2.1 Modes of oil consumption – contribution from the ring pack

Lubricant stored in the sump (at the bottom of the engine block) can be consumed in different ways. It can be transported through the ring pack and burned in the cylinder. It can also leak from the valves or the turbocharger and be released in the exhaust. This thesis is focusing on the performance of the ring pack and as such only the contribution of the ring pack to oil consumption will be discussed.
Oil is first supplied from the crankcase to the liner via a combination of splashing of the crankshaft and oil jets (depending on the engine architecture). The oil film on the liner lubricates the piston skirt and is scraped by the OCR during down strokes. A fraction of this oil supplied to the liner can make its way to the upper ring pack. Once reaching the upper ring pack several scenarios of oil consumption are possible (Figure 1.8).

![Diagram showing modes of oil consumption](image)

**Figure 1.8 - Modes of oil consumption**

Since combustion occurs in the upper portion of the cylinder, liner temperature is maximum there. When oil reaches the upper region of the cylinder, it is heated and can eventually evaporate. Evaporated oil mixes with gas in the cylinder and can either be burned or leave the cylinder through the exhaust port. To avoid excessive oil consumption, the amount of oil in the upper region of the cylinder must be kept to a minimum. That is why OCR are designed to leave an oil film thickness on the order of the surface roughness which is not optimal for lubrication but enough to protect top rings from excessive wear.

When oil is being scraped from the liner by a ring it can be transported to a piston land. The reciprocating motion of the piston result in an inertia driven flow of oil in the axial direction. If oil accumulates on the top land and forms a large droplet, piston acceleration can drive the oil droplet towards the combustion chamber until it eventually detaches. The oil droplet is then consumed in the combustion chamber either through combustion or evaporation.
Finally under throttled conditions (for spark ignition engines) pressure in the crankcase can exceed the pressure in the combustion chamber during part of the engine cycle. This drives gas to flow from the crankcase towards the combustion chamber. This reverse gas flow can carry small oil droplets and transport them from the ring pack to the combustion chamber thus contributing to oil consumption. Reverse gas flow also occurs outside of throttled conditions, when gas pressure in the second land (below the top ring) exceed the pressure in the combustion chamber.

Oil consumption has several negative effects, it contributes to engine emission of pollutants (particulate matter and hydrocarbons) and poisons after treatment systems \cite{8, 9}. Soot and ashes (due to the minerals present in lubricant additives) are produced when the lubricant is consumed and accumulate on the catalyst and in the particulate filter. It reduces the effectiveness of after treatment systems and increases the pressure drop across the exhaust, thus contributing to energy losses. Though soot can be burned during regeneration, ash must be removed mechanically. Oil consumption therefore shortens the maintenance cycle of after treatment systems and incurs additional cost. It is clear from this description that to limit oil consumption, the volume of oil present in the upper ring pack must be minimized. Ideally it should be limited to a thin oil film on the liner (thickness on the order of liner roughness) to provide protection against wear of the top two rings.

1.2.2 Overview of oil transport mechanisms in the ring pack

Oil transport in the ring pack must be understood to achieve a reduction of piston ring friction without increased oil consumption or degradation of sealing performance. A description of the ring pack oil transport mechanisms identified to date is given here. The purpose of this description is to set the context of this thesis work and understand what is required to simulate the sealing performance of piston rings.

First the transport of oil along the cylinder due to scraping of the OCR is described. In the engine cycle, the piston tilts around its pin axis under the action of contact and friction forces on the piston skirt. Though the magnitude of the tilt motion is small (on the order of $0.1^\circ$), it is sufficient to force the OCR to tilt and affect its ability to control the amount of oil on the liner. Due to piston acceleration, inertia forces the ring against the tilted piston groove (Figure 1.9)
Contact with the piston groove tilts the ring so that it follows the local orientation of the groove. Ring twisting can result in a loss of contact of one of the OCR lands. Since piston tilt varies in the engine cycle it can lead to accumulation of oil between the OCR lands and eventually release on the liner (Figure 1.10).

Figure 1.9 – Equilibrium of the ring in a tilted piston

Figure 1.10 – Sequence of oil control ring scraping
The scraping sequence of Figure 1.10 illustrates how rotation of the ring section can lead to oil transport past the OCR and oil consumption. Film thickness and ring twist have been magnified to allow visualization of the scraping process. In reality ring twist is on the order of 0.1° and film thickness is on the order of 0.1 μm (liner roughness). This magnitude of twist results in a radial displacement of the land of several microns which is significant for oil control. Initially during the down stroke ring twist is such that the OCR scrapes oil from the liner leading to accumulation between the lands. Later in the down stroke the direction of ring twist is reversed and oil is released on the liner. The oil film on the liner is thus much thicker than what the OCR should allow. When ring moves back to this position during the upstroke, oil is scraped from the liner and can flow to the piston. Once on the piston, inertia can move the oil towards the upper ring pack. This shows how this scraping mechanism can contribute to oil consumption and why it must be avoided.

Distortion of the cylinder under thermal and mechanical loads is also a concern for oil consumption (Figure 1.11).

![Distorted bore (magnified x400)](image1)

![Contact force distribution between ring and liner](image2)

*Figure 1.11- Bore distortion and its effect on the contact between the ring and the liner*
Stresses from the engine block assembly and gradients of temperature under running conditions (due to heat transfer from combustion chamber to coolant) deform the cylinder. Though these deformations are small compared to the bore radius (on the order of 1 to 10 µm), they are sufficient to affect the contact between the ring and the liner. As pictured on Figure 1.11, bore distortion creates deviation from the original cylindrical shape resulting in a deformed cylinder with multiple lobes around the circumference. The piston ring might be too stiff and ring tension too small to adapt to these local changes of geometry. Contact pressure is affected and a loss of contact between the ring and the liner can occur. This affects the sealing performance of the ring as gas and oil can leak where ring-liner contact is lost. The difference of ring-bore conformability between the down and up stroke can also generate up-scraping of oil (Figure 1.12)
It can occur for top rings in the intake and compression strokes. Let's consider an engine whose distortion is large enough to cause the second ring to lose contact with the liner in certain locations as pictured on the lower left graph of Figure 1.12. During the intake stroke oil stored in the 2nd ring hook can connect to the liner and a thick oil film is allowed to pass the ring in the non-conforming region. Since the top ring has a similar ability to conform to the distorted bore, it will slide over this thick oil film during the intake stroke. However during the compression stroke, gas pressure is rising in the top ring groove since it is connected to the combustion chamber, and helps the top ring to conform to the bore distortion as pictured in lower right graph of Figure 1.12. The thick oil film that was left on the liner during the intake stoke is scraped and can flow to the top land. Once on the top land, oil is very likely to be consumed either through evaporation or throw off in the combustion chamber. In this case it is not the poor conformability of the top rings that has contributed to oil consumption but the difference of conformability between the intake and the compression stroke.

Scraping and conformability are oil consumption mechanisms that occur at the ring-liner interface. However oil can also flow past a piston ring through the piston groove. A physical description of oil flow to the OCR groove is given (Figure 1.13).
During the down stroke, the OCR scrapes oil that has lubricated the piston skirt, from the liner. This oil accumulates below the OCR and is pressurized due to the relative motion between piston and liner. In the initial portion of the down stroke, inertia and friction are holding the ring against the upper flank of the piston groove. This opens a channel (height of the channel ~50μm) between the ring and the lower flank of the groove for oil to flow. Since oil pressure below the ring is higher than the pressure in the groove, it fills the oil-groove interface. In the second half of the down stroke the direction of inertia is reversed and the ring squeezes the oil film on the lower flank of the groove. A fraction of the oil at the lower ring-groove interface is thus transferred to the groove. This process is repeated every engine cycle and oil starts accumulating in the OCR groove. Once oil accumulation in the groove is large enough, inertia can drive it towards the upper flank of the groove and oil can eventually flow out to the third land through the upper ring-groove interface. Flow through the groove is therefore an additional path for oil to get past the OCR.
As discussed earlier scraping can transfer oil from the liner to the piston. When oil accumulation on the piston lands is large enough, oil can be returned to the liner with the help of inertia. This mechanism is called bridging and has been studied by Fang [10] (Figure 1.14). A succession of scraping and bridging events can cause oil transport through the ring pack and oil consumption.

![Diagram of oil transport through the ring pack](image)

**Figure 1.14- Bridging of oil between ring and liner**

Figure 1.14 describes a bridging event occurring on the second ring at the end of the exhaust stroke. Oil accumulated in the second ring hook is forced out due to the action of inertia and wets the liner. Since the piston is moving up a local thick oil film is left on the liner and scraped by the OCR. The image on the right of Figure 1.14 constitutes experimental evidence supporting this description of bridging. It is an image taken from a single cylinder test engine set up for 2D Laser Induced Fluorescence imaging [11]–[13]. A sapphire window has been installed on the side of the cylinder to allow optical access. The image of Figure 1.14 is taken outside of the engine perpendicular to the radial direction of the cylinder. This LIF technique consists of doping the engine lubricant with fluorescent dye. Once excited with a laser, the dye in the oil emits light that can be filtered. The result is an image where brightness is proportional to the amount of oil. In this particular picture a bridging event can be seen (in the red ellipse). The bright stripe corresponds to

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additional oil that was left on the liner due to bridging from the second ring. This oil is scraped and accumulates on top of the OCR. As can be seen on the LIF image of Figure 1.14, bridging and scraping results in local oil accumulation on top of the OCR. Observations of oil transport with the LIF system have shown that the distribution of oil in the piston ring pack is non-uniform. Hence to simulate oil transport a robust framework to handle local oil accumulation is needed.

Oil present in the ring pack can return to the crank case through the drain holes of the OCR groove (Figure 1.15)

Drain holes in the OCR groove have two functions: to release accumulated oil and to ventilate the ring pack. When the combustion chamber pressure is high, gas can flow through the top ring gaps and make its way to the crankcase through the draining holes in the OCR groove. This flow of gas can entrain oil through the draining holes and return it to the crankcase. This limits the amount of oil accumulation in the OCR groove and hence the flow of oil from the groove to 3rd land.
1.2.3 Challenges in modeling ring pack oil transport and oil consumption

The current objective of ring pack designers is to reduce friction losses while maintaining low levels of oil consumption. To illustrate the scale of oil consumption in engine, the equivalent thickness of oil scraped and consumed per engine cycle \( h_{\text{scraped}} \) is calculated (Figure 1.16).

For passenger cars the oil consumption rate is on the order of 1 to 10 g/h per cylinder. 10g/h is an oil consumption rate already considered large. The thickness of oil film consumed per cycle equivalent to this oil consumption rate is calculated (equation (1.3)).

\[
OC = \rho_{\text{oil}} \times h_{\text{scraped}} \times S \times \pi B \times RPM \times 60
\]  

(1.3)

\( OC \) is the oil consumption rate expressed in g/h, \( \rho_{\text{oil}} \) is the oil density, \( h_{\text{scraped}} \) the thickness of the oil film scraped from the liner, \( B \) and \( S \) the bore and stroke of the engine and \( RPM \) the engine speed in rotation per minute. For an engine with a bore of 80mm and a stroke of 90mm and an oil density of 900kg/m³, an oil consumption rate of 10g/h corresponds to an oil film of approximately 0.01 μm consumed every cycle at 1000 RPM. This shows that current engine oil consumption levels are minimal and oil transport mechanisms at the sub-micron scale can impact significantly oil consumption. It constitutes one of the key challenges of modelling oil transport and oil consumption in engines. One must be able to simulate sub-micron scale oil transport mechanisms over the course of one engine cycle and how changes in ring pack design might affect them.
Oil transport in the piston ring pack is also a multi-physics, multi-scale problem (Figure 1.17)

Gas flowing from the combustion chamber to the crankcase entrains and redistributes oil on the piston lands and in the piston grooves. Ring deformation and dynamics impact the flow of oil through the ring-liner and the ring-groove interfaces. Piston tilting can deform the rings and result in scraping at the ring-liner interface. In summary to address oil transport in the ring pack, ring structural deformation and dynamics, lubrication with groove and liner, oil flow in the groove and on the piston lands must be solved simultaneously. Modeling the ring sealing performance is multi-scale by nature: the ring structural scale (ring radius ~ 40mm) must be coupled to the ring-liner and ring groove lubrication scale (clearance ~0.1μm ~ surface roughness). Oil supply can vary sharply along the circumference of the ring which adds to the difficulty of simulating oil transport.

The multi-physics, multi-scale dimension of oil transport in the piston ring pack as well as the scale of oil consumption explains why predicting oil consumption is a problem still to be solved despite the interest it has generated.
1.3 Existing work on piston ring, oil transport and oil consumption

Early research on piston rings was focused on ring mechanics, in particular on the relation between ring free shape and the contact pressure distribution in the cylinder. Prescott [6] first developed a formula for the ring free shape geometry in order to achieve a uniform pressure distribution. Swift [14] continued studying the relation between geometry and pressure distribution while experimental validation was later provided by Dragoni and Strozzi [15]. When bore distortion was identified as a potential cause of oil consumption [16], [17] [18] several studies of ring conformability were conducted. Mueller [19] developed an analytical criterion to calculate the maximum admissible bore distortion for a given ring configuration. A similar analytical approach and a statistical treatment of bore distortion was used by Dunaevsky to suggest different bore distortion limits [20]–[24]. Tomanik later proposed a new conformability criterion based on experimental measurements of ring conformability limits [25], [26].

Finite element methods have been used to model the structural behavior of the piston ring and its interaction with the cylinder [27]–[30]. Existing ring structural model are based on straight beam finite element models. The straight beam piston ring model has some limitations: discontinuity of physical information such as curvature occur at the nodes of the finite element and a large number of elements are required to reach sufficient accuracy.

Oil transport and oil consumption has been studied both experimentally and theoretically. First order models of oil transport have been developed to understand oil flow in the piston ring pack [11], [31], [32] [33], [34]. These models were derived to characterize oil transport experimental observations. The Laser Induced Fluorescent (LIF) technique has been used with success to visualize oil flow in the piston ring pack [11], [13], [35]–[37]. Oil film thickness and oil consumption measurement have helped support theoretical development.
1.4 Thesis objectives

As discussed in the previous section, piston rings have been studied extensively to be able predict and understand their performance in terms of oil consumption, blow-by, friction and reliability. However because of the complexity of the ring pack system, fundamental questions regarding oil transport and oil consumption are yet to be answered. For instance it is still very difficult to establish what mechanisms contribute to oil consumption for a given engine under given running conditions. Yet that knowledge is essential if ring pack designs are to be optimized in order to achieve minimum friction without excessive oil consumption. Up to now, oil transport mechanisms have been mostly studied or modeled separately. The objective of this thesis work is to build a comprehensive model of piston ring oil transport that can consider relevant oil transport mechanisms simultaneously and help advance ring pack design. The fundamental questions addressed in thesis are: What are the main oil transport mechanisms leading to oil flow past a piston ring? What is their relative contribution to oil transport? Can ring tension be further reduced (to reduce friction losses) without impacting oil consumption?

This thesis work is focusing on the OCR which is the main barrier to oil consumption in ICE and is at the center of the compromise between oil consumption and friction reduction. The methodology developed here can be extended in the future to the full ring pack.

There are three main oil transport mechanisms affecting the sealing performance of the Oil Control Ring: conformability of the OCR to the distorted bore, scraping of oil on the liner due to OCR twisting and finally oil flow through the OCR groove (Figure 1.18).
To address the contribution of these three mechanisms to oil consumption, ring deformation, dynamics, interaction with the liner and the piston must be studied together. For this purpose a multi-scale curved beam finite element model of the piston ring is developed (Figure 1.19).
The multi-scale model is able to solve structural deformation and dynamics of the piston ring on a coarse mesh and consider the interaction with the piston and the liner on a fine contact grid. This method allows efficient coupling of structural deformation of the ring and lubrication at the ring-liner and ring-groove interfaces which occur at different length scales. As a result contact pressure, friction and oil flow at the ring-liner and ring-groove interfaces can be simulated taking into account relevant factors such as bore distortion, piston tilt and so on.

In chapter 2 of this thesis the development of the dual grid curved beam finite element model of piston rings is presented. Fundamentals of piston ring mechanics are reviewed before formulating the finite element model. The piston ring model is then validated analytically.

The OCR oil transport model is detailed in chapter 3. Contact and lubrication models for the ring-liner and the ring-groove interfaces are presented first. A 2D ring-groove lubrication model was developed in this thesis to study contact and oil flow. The kinematics and secondary motion of the
piston which are key inputs of the OCR model are then reviewed. An overall picture of the OCR oil transport model and its solver is provided.

In chapter 4 the OCR oil transport model is used to calculate oil flow around the OCR for representative engine conditions. First the effect of bore distortion on the oil control function of the OCR is assessed. Then OCR twisting and scraping due to piston dynamic tilting is quantified. Oil from the piston skirt to the OCR groove and from the groove to the piston third land is calculated next. A comparison of the contribution of the three mechanism (bore distortion, OCR scraping and oil flow through the groove) is given at the end of chapter 4.

Finally a summary of the thesis main results can be found in chapter 5. Design modifications to limit oil flow past the OCR either through the groove or through the ring-liner interface are discussed. A list of additions that would best complement this thesis work is given.
2 A dual grid curved beam finite element model of piston rings

2.1 Piston ring mechanics

2.1.1 Beam constitutive equations

Piston rings are thin metallic structures. The cross section dimensions of a ring are small compared to its radius. For instance the radius of a piston ring of a typical passenger car engine is 40 mm for a cross section radial width of 3mm. Since the ratio between cross section dimension and radius of a piston ring is lower than 1/10, beam theory provides an accurate description for the piston ring structural behavior. A review of the elementary beam theory is provided here as it is the foundation of the piston ring model described in this chapter. A complete treatment of the elementary beam theory is available in Timoshenko's strength of materials text [38].

The elementary beam theory, also named Euler-Bernoulli beam theory, is based on the assumption that under the action of bending moments the beam is curved, its cross sections rotate while remaining plane and perpendicular to the beam axis. It has been shown experimentally that this theory provides an accurate model of beam deflection and longitudinal strains. Starting from this assumption the longitudinal strain of the beam can be related to the deformed beam radius of curvature (Figure 2.1)

\[ \varepsilon_x = \frac{y}{R} = \kappa y \]  

\textit{Figure 2.1- Beam deformation under pure bending}
As the curvature of the beam increases under the action of bending moments, the side of the beam closest to the center of curvature is in compression whereas the opposite side is in tension. The plane of zero longitudinal strain is called the neutral plane. Its intersection with the plane of symmetry the ring cross section is the neutral axis. In equation (2.1), $\varepsilon_x$ is the longitudinal strain, $y$ the distance from the neutral axis, $R$ the radius of curvature of the deformed beam and $\kappa$ the curvature (the inverse of the radius of curvature). The stress in the beam cross section can now be calculated using Hooke’s law (equation (2.2)).

$$\sigma_x = E\varepsilon_x = E\kappa y$$  \hspace{1cm} (2.2)

![Figure 2.2: Distribution of axial stresses in the beam cross section](image)

Since the beam is submitted to bending moments only, the resulting axial force on the cross section must equal zero (equation (2.3)).

$$\int \sigma_x dA = E\kappa \int y dA = 0$$  \hspace{1cm} (2.3)

By definition the moment of area of a beam cross section is equal to zero at the centroid. Equation (2.3) shows that for the axial force to be also equal to zero, the beam neutral axis must go through the centroids of cross sections.
The bending moment, $M_z$, is calculated by integrating the contribution of the longitudinal stresses over the cross section (equation (2.4)).

$$M_z = \int \sigma_x y dA = E\kappa \int y^2 dA = EI_z \kappa \tag{2.4}$$

The familiar constitutive equation of beam bending is thus obtained (equation (2.5)).

$$M_z = EI_z \kappa \tag{2.5}$$

The bending moment, $M_z$, is proportional to the curvature, $\kappa$, the coefficient of proportionality being the flexural rigidity, $EI_z$, where $I_z$ is the area moment of inertia of the beam cross section with respect to the axis $z$ (equation (2.6)).

$$I_z = \int y^2 dA \tag{2.6}$$

To satisfy the equilibrium of the beam cross section the moment along the axis $y$, $M_y$, must be equal to zero (equation (2.7)).

$$M_y = \int \sigma_y z dA = E\kappa \int y z dA = E\kappa I_{yz} = 0 \tag{2.7}$$

The quantity $I_{yz}$ is defined as the product of inertia of the cross section for axes $y$ and $z$ (equation (2.8)).

$$I_{yz} = \int yz dA \tag{2.8}$$

The product of inertia is by definition equal to zero if either $y$ or $z$ is an axis of symmetry. Indeed due to symmetry, the contribution of one side of the beam cross section to the product of inertia is cancelled by the contribution from the other side of the axis of symmetry. This means that if the bending moment acts in the plane of symmetry (as it is the case in this example), there are no bending in the perpendicular direction and the beam deforms in the plane of symmetry. However if the bending moments act in a plane where the product of inertia is non-zero then out of plane...
deformations of the beam will occur and the bending moment can no longer be considered proportional to the change of curvature as in equation (2.5).

For any beams of arbitrary cross section, there exists one set of axes $y_p$ and $z_p$ for which the product of inertia is equal to zero. These are called the principal axes of the beam cross section. If the beam cross section has an axis of symmetry then one of the principal axes is the axis of symmetry since it results in a product of inertia equal to zero. For an arbitrary cross section the orientation of the principle axes can be calculated from the product of inertia and the area moment of inertia in the reference frame (Figure 2.3).

![Figure 2.3 - Orientation of the principle axes of a beam cross section](image)

\[
\tan 2\varphi = \frac{2I_{yz}}{I_y - I_z} \tag{2.9}
\]

The angle $\varphi$, gives the orientation of the principle axes in the reference frame of axes $y$ and $z$. $I_{yz}$ and $I_z$ are the product of inertia and the area moment of inertia about $z$ (equations (2.8) and (2.6) respectively). The area moment of inertia about $y$, $I_y$, can be calculated similarly to $I_z$ (equation (2.10)).

\[
I_y = \int z^2 dA \tag{2.10}
\]
The key property of principal axes is that bending moments that are perpendicular to a principal plane, result in deformation of the beam neutral axis in that principle plane. As seen in the pure bending example, it is the only plane where the product of inertia is zero and this is true. Piston rings experience bending both in their plane of curvature and outside of their plane of curvature as a result of interaction with the piston and the cylinder liner. The resulting bending can therefore have any arbitrary orientation with respect to the principle axes of the ring. This bending situation can be solved by working in the principle frame (Figure 2.4).

![Figure 2.4 - Bending outside of the beam principal plane](image)

To calculate the deformation of the beam, the bending moment $M$ which is at an angle $\psi_m$ with $z_p$ is projected on the principle axes $y_p$ and $z_p$ (equation (2.11)).

\[
M_y = M \sin \psi_m \\
M_z = M \cos \psi_m
\]  

(2.11)

Since only small deformations of the beam are considered, the structure response is linear and the principle of superposition can be used. The deformation of the beam under the action of the bending moment $M$ is the superposition of the deformation of the beam under the action of $M_y$ and $M_z$, the projections of $M$ on the principle axes. Since $y_p$ and $z_p$ are principle axes the bending moments $M_y$ and $M_z$ are proportional to the changes of curvature along $y_p$ and $z_p$ (equation (2.12)).
\[ M_y = EI_y \kappa_z \quad M_z = EI_z \kappa_y \] (2.12)

The normal vector \( \mathbf{n} \) to the deformed beam neutral axis is represented on figure 2.4. It is pointing towards the center of curvature of the neutral axis. The deformed beam neutral axis lies in the plane formed by \( \mathbf{n} \) and the tangent to the neutral axis. The orientation of the normal vector can be calculated from the ratio of the curvatures of the neutral axis along \( y_p \) and \( z_p \) (equation (2.13)).

\[
\tan \psi_n = \frac{\kappa_z}{\kappa_y} = \frac{M_y}{EI_y} \frac{M_z}{EI_z} = \frac{l_z}{I_y} \tan \psi_M
\] (2.13)

In the example of figure 4, the beam offers more resistance to bending in the principal plane containing \( y_p \) because of its rectangular shape \( (I_z > I_y) \). As a result the beam neutral axis does not deform in the plane perpendicular to the bending moment but in a plane closer to the principal plane containing \( z_p \) \( (\psi_n > \psi_M) \). Any bending situation can now be solved by working in the principle frame.

The elementary beam theory presented here was derived for a beam whose natural geometry is straight. Piston ring however are naturally curved and are expected to behave differently. Pure bending of curved beam is derived under the same assumptions than straight beams. Under the action of end couples, the beam curvature is modified, its cross section rotate while remaining plane and perpendicular to the neutral axis. Because of the initial curvature of the beam, longitudinal fibers on the concave side (side closest to center of curvature) experience larger strains than the fibers on the convex side. The distribution of stress in the beam cross section is no longer linear but hyperbolic and the neutral axis is moved towards the center of curvature (Figure 2.5).

![Figure 2.5 - Bending of a curved beam](image-url)
As shown by Timoshenko [38], the distribution of stress can be approximated by a linear profile and the neutral axis can be considered coincident with the centroids for slender curved beams. For \( h/R < 10 \) the error on the axial stress from assuming a linear distribution falls below 3%. A constitutive equation similar to the bending of straight beam is thus obtained (equation (2.14))

\[
M = E I_z (\kappa - \kappa_0)
\]  

(2.14)

For naturally curved beam the bending moment is proportional to the change in curvature \( \kappa - \kappa_0 \) where \( \kappa_0 \) is the natural curvature of the ring.

To complete this description of beam mechanics, a review of beam torsion is presented here. The constitutive equation of torsion of a circular shaft is given first. Solutions to the elasticity equations shows that under the action of twisting moment, the circular shaft is in a state of pure shear. Cross sections of the shaft rotate with respect to one another while keeping their diameters and axial positions unchanged (Figure 2.6).

*Figure 2.6 - Torsion of circular shaft*
The lower section of a differential element of the shaft of radius $r$ and length $dx$ is rotated by an angle $d\phi$ with respect to the upper cross section. The shear angle $\gamma$ can thus be calculated (equation (2.15)).

$$\gamma = \frac{d\phi}{dx} \quad (2.15)$$

The twist angle $\theta$ is defined as the rotation of the shaft section per unit length of the beam axis (equation (2.16)).

$$\theta = \frac{d\phi}{dx} \quad (2.16)$$

The equivalent to Hooke’s law for pure shear is used to calculate the shear stress $\tau$ in the shaft cross section (equation (2.17)).

$$\tau = G\gamma = Gr\theta \quad (2.17)$$

The shear modulus, $G$, is a material property that can be expressed as a function of Young’s modulus, $E$, and Poisson’s ratio $\nu$ (equation (2.18)).

$$G = \frac{E}{2(1 + \nu)} \quad (2.18)$$

The shear angle $\gamma$ varies linearly with the distance from the shaft axis $r$ leading to a linear distribution of shear stress in the cross section (Figure 2.7).
The constitutive twisting equation connecting the twisting moment $M_t$ to the twist angle $\theta$ is obtained by integrating the shear stress over the shaft cross section $A$ (equation (2.19)).

$$M_t = \int \tau r dA = G\theta \int r^2 dA = G\theta I_p$$

To simplify the expression of the twisting equation, the quantity $I_p$ is introduced. It corresponds to the polar moment of inertia of the cross section with respect to the shaft axis (equation (2.20)).

$$I_p = \int r^2 dA$$

Twisting of beams with non-circular cross sections (like piston rings) is more complex. The distribution of shear stress is non-linear and the beam cross section experience warping. Warping means that in addition to being rotated, cross sections deform in the direction of the beam axis and are no longer plane.

The constitutive equation between twisting moment and twist angle must be adjusted to account for the particular distribution of shear stress in the beam cross section (equation (2.21)).

$$M_t = G j_f \theta$$

The polar moment of inertia $I_p$ of the torsion equation for circular shaft has been replaced by the torsion constant $j_f$. In its Strength of Materials text Timoshenko gives the following numerical
approximation of the torsion constant for beams of rectangular cross section [38] (equation (2.22)):

\[ J_t = \beta ab^3 \]

Figure 2.8 - Beam rectangular cross section

The dimensions of the rectangular cross section are \( a \) (longest side) and \( b \) (shortest side). The variable \( \beta \) is a non-dimensional constant calculated as function of the ratio \( a/b \). It ranges from 0.14 for \( a/b = 1 \) to 0.33 for \( a/b = \infty \). Numerical values of beta have been computed by Timoshenko [38].

Second ring and oil control ring can have complex cross sections for which no approximations of the torsion constant \( J_t \) are readily available. The torsion constant of arbitrary cross section can be calculated by solving numerically the torsion equation which is a partial differential equation in the plane of the beam cross section.

In this model of piston rings, the effect of shear and axial stresses on the deformation of the ring neutral axis have been neglected. Due to shearing, adjacent cross sections of the beam slide along each other, resulting in a lateral deflection of the neutral axis. For thin beams however this deflection is small compared to the lateral deflection due to bending. This can be shown for a beam laterally loaded in its plane of symmetry (Figure 2.9).
The deflection of this simply supported beam under the uniformly distributed lateral load $q$ has been derived by Timoshenko [38] (equation (2.23)). In this solution the effect of both bending and shearing on the deflection of the beam have been considered.

$$
\delta = \delta_{bending} + \delta_{shearing} = \frac{5}{384} \frac{qL^4}{El_z} + \frac{\alpha L^2}{8GA} q = \frac{5}{384} \frac{qL^4}{El_z} (1 + \frac{48\alpha}{5} \frac{E}{GAL^2}) \quad (2.23)
$$

The deflection at the center of the beam is $\delta$, $A$ is the area of the beam cross section, $q$ is the lateral load per unit length and $\alpha$ is a numerical constant equal to the ratio of shear stress at the centroid and average shear stress. The magnitudes of the shearing and bending contributions to the beam deflection are compared for a beam of rectangular cross section of thickness $h$ (equation (2.24)).

$$
\delta = \frac{5}{384} \frac{qL^4}{El_z} (1 + \frac{48\alpha (1 + \nu)}{5} \frac{h^2}{L^2}) \quad (2.24)
$$

The contribution of the shear force on the deflection of the beam is small for thin beams ($h^2/L^2 \ll 1$). For a rectangular beam ($\alpha = 3/2$) with a Poisson’s ratio of $\nu = 0.3$ (cast iron, steel) and $h/L = 0.1$ shear forces account for only 3% of the deflection. Hence the effect of shearing on the piston ring for which $h/R \sim 0.05$ can be neglected. It can be shown similarly that the contribution of axial stresses to the deflection of the beam is on the same order than the shear contribution and can be neglected.
To summarize this treatment of piston ring mechanics, the constitutive bending and twisting equations of the ring can now be written. They relate changes of ring curvature and twist to the bending and twisting moments. The piston ring is a thin beam whose neutral axis is circular when inserted in the cylinder. The natural geometry of the ring is taken as circular (of radius equal to the bore radius) for the presentation of the constitutive equations. The effect of the ring free shape is discussed in following sections. The ring cross section geometry is assumed symmetrical, thus the ring principle axes are coincident with the radial and axial axes of the cylinder. Figure 2.10 shows the deformation of the ring due to bending in its plane of curvature (plane perpendicular to cylinder axis).

\[
\kappa_{r_0} = \frac{1}{R}
\]

\[
M_z = EI_z (\kappa_r - \kappa_{r_0}) \tag{2.25}
\]

\(\kappa_r\) is the curvature of the deformed ring in its plane of curvature, \(\kappa_{r_0}\) is the original curvature and is equal to the inverse of the bore radius (ring neutral axis reference state is assumed circular).
Similarly the deformation of the ring due to bending outside of its plane of curvature is shown in Figure 2.11.

\[ M_r = E I_r (\kappa_z - \kappa_{z0}) = E I_r \kappa_z \]  

\( \kappa_z \) is the curvature along the cylinder axis. Since initially the ring neutral axis is a plane curve \( \kappa_{z0} = 0 \). The relation between the twisting moment and twist of the ring is more complex than for straight beam due to its curvature. First the natural and principal frames are introduced (Figure 2.12).

The natural frame is the frame attached to the ring neutral axis and composed of the tangent \( e_t \), normal \( e_n \) and binormal \( e_b \) to the neutral axis. The definition and calculation of tangent, normal and binormal vectors of the ring neutral axis is presented in the next section. The principal frame
is composed of the two principal axes $e_2$ and $e_3$ that lie in the plane of the ring cross section and of the tangent $e_t$. The orientation of the principal frame in the cylindrical frame is given by the angle $\alpha$ and the angle $\alpha_r$ marks the orientation of the ring natural frame. In the twisting moment relation, there are two components to the ring twist: $\tau$ the torsion of the ring neutral axis, and the rotation of the ring section per unit length $\frac{d}{ds} (\alpha_r - \alpha)$ (equation (2.27)). A definition of the ring neutral axis torsion is given in the next section, it is a measure of the "twist" of a space curve.

$$M_\theta = GJ_r [\tau + \frac{d}{ds} (\alpha_r - \alpha)] \quad (2.27)$$

Examples are shown to give an interpretation of the two ring twist quantities of equation (2.27). First a circular ring is twisted without deformation of its neutral axis (Figure 2.13)

The red lines in Figure 2.13 show the local orientation of the ring cross section. Under the action of the end twisting moments $M_\theta$ the angle $\alpha$ increases linearly. Since the ring is not deformed torsion of the neutral axis is zero and the normal vector coincide with the cylinder radial vector ($\alpha_r = 0$). This case is thus similar to the torsion of straight beam and the term $\frac{d}{ds} (\alpha_r - \alpha)$ can be interpreted as the twist angle. Let's now assume that the ring sections do not tilt under the action
of the twisting moment but this time the ring neutral axis is deformed in the axial direction (Figure 2.14).

![Figure 2.14 - Ring undergoing torsion with deformation](image)

The circular ring becomes an helix with a constant torsion due to the axial deformation. All along the neutral axis the normal vector and principal direction of the ring section coincide with the radial axis of the cylinder ($\alpha_r = 0$ and $\alpha = 0$). Though the ring section are not tilted, the torsion of the helix places adjacent cross section in a state of shear resulting in a twisting moment.

Equation (2.27) is consistent with the straight beam torsion relation (equation (2.21)): the twisting moment is proportional to the relative rotation of adjacent cross sections. The particular formulation of equation (2.27) accounts for the kinematics of the curved ring.

The ring bending and twisting equations (equation (2.25) to (2.27)) are results of the ordinate approximate theory of naturally curved rods. A thorough treatment of this theory, including derivation and relevant examples is available in Love’s elasticity text [39].
2.1.2 Piston ring equilibrium equations

In addition to the bending and twisting relation, the equilibrium equations of the piston ring are derived. They provide a relation between external forces and stresses in the ring (resulting axial, shear forces and bending moments of the ring cross section) and will be used to study ring conformability for instance. All external forces acting on the ring can be represented by a distribution of radial, axial force and twisting moments per unit length written as \( f_r, f_z \) and \( m_\theta \) respectively. The equilibrium of a differential element of piston ring in its plane of curvature is considered first (Figure 2.15).

![Figure 2.15 - Differential element of piston ring with distributed radial load](image)

The differential element shown in Figure 2.15 is magnified to illustrate the change of radial and tangential direction at both ends of the differential element. The radial force per unit length \( f_r \) results in cross section forces along the tangential and radial directions as well as a bending moment along the axial direction. Since the differential element is infinitesimally small, \( f_r \) can be considered constant. A first equilibrium is obtained by balancing the radial forces acting on the ring differential element (equation (2.28)). Forces at the end of the element are projected on the radial direction at the beginning of the element. Second order quantities are neglected and the prime signs in the equilibrium equation refers to derivation with respect to the polar angle \( \theta \).
The second equilibrium equation is obtained by balancing forces in the tangential direction (equation (2.29)).

\[ F'_{\theta} + F_r = 0 \]  
\[ (2.29) \]

Axial and tangential stresses are coupled due to the curvature of the ring and the rotation of the tangential and radial directions. The bending moments along the axial directions are balanced to obtain the third equilibrium equation (equation (2.30)).

\[ \frac{1}{R} M'_z - F_r = 0 \]  
\[ (2.30) \]

Equations (2.28) to (2.30) can be combined together to obtain a direct relation between the in plane bending moment \( M_z \) and the radial load \( f_r \) (equation (2.31)).

\[ \frac{1}{R^2} (M_z + M_z') = f_r \]  
\[ (2.31) \]

This relation will prove useful when later studying the conformability of piston rings. Distributed axial loads \( f_z \) and twisting moment \( m_{\theta} \) are now considered (Figure 2.16).

Figure 2.16- Differential element of piston ring with distributed axial load and twisting moment
Axial and twisting loads result in twisting and out of plane bending moments as well as axial force in the ring cross section.

By balancing axial forces, bending moments in the radial and tangential directions the three remaining equilibrium equations of the piston ring are obtained (equation (2.32) to (2.34)).

\[
\frac{1}{R} f_z' - f_z = 0 \quad (2.32)
\]
\[
\frac{1}{R} (M_r' - M_\theta) - F_z = 0 \quad (2.33)
\]
\[
\frac{1}{R} (M_\theta' + M_r) + m_t - F_r = 0 \quad (2.34)
\]

Again the curvature of the ring results in a coupling between twisting moment and out of plane bending moment.

2.2 Review of differential geometry

In the previous section covering piston ring mechanics, bending and twisting moments were found to be proportional to changes of curvature and torsion of the ring neutral axis. A review of differential geometry and the calculation of curvature and torsion of a space curve is given in this section. The purpose of this review is to be able to relate deformation of the ring to changes of curvature and torsion and ultimately bending and twisting moments. Since ring deformations are expected to be small compared to the ring radius, curvature and torsion can be approximated and expressed as a simple function of ring deformation. A more complete treatment of differential geometry of space curves is available in Pressley's text [40].
2.2.1 Curvature and torsion of space curves

The neutral axis of a piston ring is a planar curve in its free state. However, out of plane deformation are expected to occur in the engine. Therefore, the three-dimensional shape of the deformed ring has to be taken into account to calculate bending and twisting moments. The geometry of the deformed piston ring axis is most conveniently described in the cylindrical frame of reference. An arbitrary space curve $C$ and its parametric representation with $\theta$, the polar angle, are used to present the derivation of curvature and torsion in cylindrical coordinates (Figure 2.17).

![Figure 2.17 - Space curve in cylindrical coordinates](image)

The unit vectors of the cylindrical coordinate system are $e_r$, $e_\theta$, and $e_z$ in the radial, tangential and axial direction respectively. The spatial curve $C$ admits the following parametric representation (equation (2.35)):

$$C(\theta): \mathbf{r}(\theta) = \rho(\theta)e_r + z(\theta)e_z \quad \theta_1 \leq \theta \leq \theta_2 \quad (2.35)$$
The polar radius \( \rho \) and the axial position \( z \) are the cylindrical coordinates of the position vector \( r \) chosen to represent \( C \). An additional reference frame formed by the unit vectors \( e_t, e_n \) and \( e_b \) and attached to \( C \) is shown on Figure 2.17. It is called the natural frame of curve \( C \).

The curve unit tangent vector \( e_t \) is calculated by taking the derivative of the position vector \( r(\theta) \) with respect to \( \theta \) (equation (2.36)). For the remainder of this section the prime sign is used to refer to a derivation with respect to the polar angle \( \theta \).

\[
e_t(\theta) = \frac{r'(\theta)}{\|r'(\theta)\|} = \frac{\rho'e_r + \rho e_\theta + z'e_z}{\sqrt{\rho^2 + \rho'^2 + z'^2}} \quad (2.36)
\]

The unit normal vector \( e_n \) is calculated by taking the second derivative of the position vector \( r(\theta) \) with respect to \( \theta \) (equation (2.37)).

\[
e_n(\theta) = \frac{r''(\theta)}{\|r''(\theta)\|} = \frac{(\rho'' - \rho)e_r + 2\rho'e_\theta + z''e_z}{\sqrt{(\rho'' - \rho)^2 + 2\rho'^2 + z'^2}} \quad (2.37)
\]

The plane formed by the tangent and normal vector, \( e_t \) and \( e_n \), is called the osculating plane or plane of curvature of curve \( C \). Locally \( C \) can be approximated by a planar curve which is contained in the osculating plane.

The binormal vector \( e_b \) is normal to both \( e_t \) and \( e_n \) and form a right hand coordinate system with them. It can be calculated by taking the cross product of \( e_t \) and \( e_n \) (equation (2.38)).

\[
e_b(\theta) = e_t(\theta) \times e_n(\theta) \quad (2.38)
\]

The coordinates of a space curve can be reconstructed from its curvature \( \kappa \) and torsion \( \tau \). Curvature and torsion are measures of the local radius of curvature and "twist" of the curve. The curvature can be calculated from the differential variation of the unit tangent vector \( e_t \) (equation (2.39)).

\[
\frac{de_t}{ds} = \kappa e_n \quad (2.39)
\]
The variable $s$ is the arc length of curve $C$. The arc length of a different element of $C$ can be calculated from the parametric expression of the position vector using the chain rule (equation (2.40))

$$ds(\theta) = \|r'(\theta)\|d\theta = \sqrt{\rho'^2 + \rho'^2 + z'^2}d\theta$$

(2.40)

Curvature represents the rate at which the tangent vector is rotated towards the normal vector as one moves along curve $C$. It is a measure "how much the curve is curved" locally. A large curvature corresponds to an arc with a small radius of curvature. The concept of osculating circle is helpful in understanding the notion of curvature (Figure 2.18).

Figure 2.18 shows an osculating circle of a plane curve. The following discussion is also valid for a space curve, as locally the space curve is contained in the osculating plane (plane of $e_t$ and $e_n$). At any point along the curve $C$, there exists an osculating circle of radius $R = \frac{1}{\kappa}$ tangent to and
that approximates \( C \). The osculating circle lies in the osculating plane formed by \( e_t \) and \( e_n \) and perpendicular to \( e_b \). The center of the osculating circle is found in the direction of the normal vector \( e_n \). The curvature \( \kappa \) can thus be seen as the inverse of the radius of the circle that approximates \( C \) locally.

In addition to curvature space curves are “twisted” forcing them to leave the osculating plane. Torsion is a measure of the space curve “twist” and can be calculated from the differential variation of the unit normal vector \( e_b \) (equation (2.41)).

\[
\frac{de_b}{ds} = -\tau e_n
\]  (2.41)

From the expression of equation (2.41), the torsion can be seen as a rotation of normal and binormal vectors around the tangent vector. To illustrate the concept of torsion the example of the helix is considered (Figure 2.19).

Figure 2.19 - Orientation of natural frame of a helix

A helix is a curve with constant curvature and torsion. In Figure 2.19 the natural frame is plotted at two different locations. As the natural frame is displaced along the helix, the normal vector remains flat and points towards the axis of the helix. The tangent and normal vector rotate and tilt.
Without torsion the helix would be transformed in a tilted circle. Torsion forces the curve out of the plane of that tilted circle by rotating normal and binormal vector around the tangent vector and towards the outside of the helix.

By combining equations (2.36), (2.37), (2.39) and (2.40), curvature can be calculated as a function of the cylindrical coordinates of curve $C$ (equation (2.42)).

\[ \kappa = \frac{||r'' \times r'||}{||r'||^3} \]  

(2.42)

Expressions of the position vector and its derivatives are substituted in equation (2.42) to obtain the general expression of curvature in cylindrical coordinates (equation (2.43)).

\[ \kappa = \frac{\sqrt{(\rho z'' - 2\rho' z')^2 + ([\rho'' - \rho] z' - \rho' z'')^2 + (\rho [\rho - \rho'' + 2\rho'^2])}}{(\rho^2 + \rho'^2 + z'^2)^{3/2}} \]  

(2.43)

For planar curve, the axial coordinate $z$ and its derivatives can be removed from the equation and the familiar expression of curvature in polar coordinates is obtained (equation (2.44)).

\[ \kappa = \frac{[(\rho^2 + 2\rho'^2) - \rho \rho'']^2}{(\rho^2 + \rho'^2)^{3/2}} \]  

(2.44)

The definitions of the binormal and normal vectors (equations (2.37), (2.38) and (2.40)) can be used to obtain an expression of the torsion as a function of the position vector and its derivatives (equation (2.45)).

\[ \tau = \frac{(r' \times r'') \cdot r'''}{||r' \times r''||^2} \]  

(2.45)

Taking the derivatives of the position vector (which is somewhat tedious!), the expression of torsion in cylindrical coordinates is obtained (equation (2.46)).

\[ \tau = \frac{(\rho z'' - 2\rho' z')(\rho z'' - 3\rho') + (3\rho'' - \rho)([\rho'' - \rho] z' - \rho' z'') + z''(\rho [\rho - \rho'' + 2\rho'^2])}{(\rho z'' - 2\rho' z')^2 + ([\rho'' - \rho] z' - \rho' z'')^2 + (\rho [\rho - \rho''] + 2\rho'^2)^2} \]  

(2.46)
2.2.2 Small deformation approximation of the curvature and torsion of piston rings

The general expressions of curvature and torsion of space curves in cylindrical coordinates are complex and non-linear (equations (2.43) and (2.46)). To obtain simpler relations between the ring deformation and bending and twisting moments, relevant approximations of curvature and torsion are sought. Piston rings assembled in the engine have a circular neutral axis of radius equal to the radius of the cylinder. Considering the geometry of the piston groove and the magnitude of piston and cylinder deformation, ring deformation can be considered small compared to the neutral axis radius. Indeed for passenger car engines the maximum ring deformation is expected to be in order of $100 \, \mu m$ whereas the ring radius is close to $40 \, mm$.

Under the action of ring tension and inertia (due to the piston acceleration), the piston ring deforms to adapt to the groove and cylinder geometry. The piston ring neutral axis who is circular initially when installed in the engine, deforms in the radial and axial direction (Figure 2.20).

![Figure 2.20 - Magnified deformation of ring neutral axis](image)

The position vector $\mathbf{r}$ of the deformed ring neutral axis is expressed as a function of the nominal ring radius $R$, the radial deformation $y$ and the axial deformation $z$ (equation (2.47)).

$$
\mathbf{r} = (R+y)e_r + z e_z$$

$sizeset[ \frac{y}{R} \ll 1 \quad \text{and} \quad \frac{z}{R} \ll 1 ]$ (2.47)
As stated earlier the radial and axial deformation are expected to be much smaller than the nominal ring radius. Hence the general expression of curvature (equation (2.43)) can be approximated by conserving first order terms only. The following approximation is also made: derivatives of the axial and radial displacements of the ring neutral axis are taken to be of the same order than the axial and radial displacements \((y \sim y' \sim y''\) and \(z \sim z' \sim z''\)). This is valid because ring radial and axial displacement cannot change too rapidly due to the ring stiffness. A simplified expression of curvature is obtained (equation (2.48)).

\[
\kappa \approx \frac{1}{R} - \frac{1}{R^2} (y + y'')
\]  

(2.48)

The curvature approximation becomes a linear function of ring deformation and its derivatives. The axial deformation has disappeared from the simplified curvature expression showing that for small deformations, deformations in the plane of curvature of the ring are decoupled from out of plane deformations. Radial displacements of the neutral axis modify the ring initial curvature \(1/R\).

The first term of equation (2.48), \(-y/R^2\), correspond to a change of curvature when the ring is uniformly expanded. For instance, suppose the ring radius is uniformly increased by the small quantity \(y = dR\). The ring axis radius becomes \(R_n = R + dR\) and the curvature \(\kappa_n = \frac{1}{R_n} = \frac{1}{R + dR}\).

The first order approximation of \(\kappa_n\) shows that an increase of radius of \(y = dR\) has reduced the curvature by \(-y/R^2\) (equation (2.49)).

\[
\kappa_n = \frac{1}{R + dR} = \frac{1}{R} \left( \frac{1}{1 + dR/R} \right) \approx \frac{1}{R} - \frac{dR}{R^2} = \frac{1}{R} \frac{y}{R^2}
\]  

(2.49)

The second term of equation (2.48), \(-y''/R^2\), represents to the local change of curvature separate from the radial displacement of the ring neutral axis.

A small displacement approximation of the torsion expression in cylindrical coordinates (equation (2.46)) is sought. As for the curvature, ring neutral axis displacements and their derivatives are considered small compared to the ring radius. A simplified expression of the ring torsion is obtained (equation (2.50))
\[ \tau \approx \frac{1}{R^2} (z' + z^{(3)}) \quad (2.50) \]

The first order approximation of the torsion of piston rings is a function of the derivatives of the axial displacements only. It shows again that under the small displacement approximation, radial and axial deformations are decoupled.

### 2.2.3 Validation of piston ring curvature and torsion approximation

To verify the validity of the first order approximation, curvature and torsion of a deformed piston rings are calculated. The deformation of the piston ring is chosen to be representative of the maximum deformation that the ring is expected to experience in the engine. The original and approximated expressions of curvature and torsion are compared and the approximation error is quantified. The distribution of radial and axial deformation of the ring axis is calculated with the use of Fourier series (equation (2.51)).

\[ y(\theta) = \sum_{k=1}^{5} A_y k \sin(k\theta + \phi_y k) \quad (2.51) \]

The magnitude of order \( k \) of the Fourier series of the radial deformation is \( A_{yk} \). The phase of order \( k \) is \( \phi_{yk} \). A Fourier series of the same form is used for the calculation of the axial deformation (equation (2.52)).

\[ z(\theta) = \sum_{k=1}^{5} A_z k \sin(k\theta + \phi_z k) \quad (2.52) \]

The magnitudes of radial and axial deformation orders are chosen to be representative of the maximum deformation the piston ring is expected to experience (Table 1).

<table>
<thead>
<tr>
<th>Order of distortion</th>
<th>2(^{nd})</th>
<th>3(^{rd})</th>
<th>4(^{th})</th>
<th>5(^{th})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude ((\mu)m)</td>
<td>100</td>
<td>50</td>
<td>25</td>
<td>5</td>
</tr>
</tbody>
</table>

*Table 1 - Ring displacement magnitude*
The chosen distortion magnitudes are decreasing with increasing order. This is to represent both ring deformation and piston and cylinder distortions. High distortion orders correspond to rapidly varying axial and radial deformations. Because high order deformations require more bending energy, most of the ring deformation corresponds to small orders. This also true for the deformation of the cylinder or the piston groove. The phases of the Fourier series of both radial and axial deformations are chosen randomly. A distribution of radial and axial displacements of the neutral axis around the ring circumference is obtained (Figure 2.21).

![Sample ring deformation for validation of curvature and torsion approximation](image)

*Figure 2.21- Sample ring deformation for validation of curvature and torsion approximation*

With the help of equation (2.43) and (2.48), the small deformation approximation of curvature is compared to the original expression (Figure 2.22)
The first order approximation of curvature gives an accurate description of the deformed ring curvature, so much so that original and approximated curvature cannot be distinguished on Figure 2.22. The relative error to the actual ring curvature (given by equation (2.43)) does not exceed 2%. A similar comparison between original and approximated expression of torsion, given by equations (2.46) and (2.50), is carried out (Figure 2.23).
Once again the first order approximation of torsion is valid for the scale of radial and axial displacements corresponding to the piston ring deformation. The relative error to the actual ring torsion (equation (2.46)) is also found to be below 2%.

Under the small displacement approximation, the tangent vector of the deformed ring can be considered parallel to $e_\theta$. Radial and axial component of the tangent vector are indeed small compared to the tangential component (equation (2.36)). The ring cross section is thus considered perpendicular to the tangential vector. To be able to calculate out of plane bending moment and twisting moment the orientation of the normal vector $\alpha_r$ in the plane of the ring cross section must be known (Figure 2.24).
The angle $\alpha_r$ is calculated by taking the ratio of axial component to radial component of the normal vector (equation (2.37)). As for curvature and torsion, a small displacement approximation of the orientation of the normal is calculated (equations (2.53) and (2.54))

\[
\sin \alpha_r = \frac{z''}{\rho'' - \rho}
\]

(2.53)

\[
\alpha_r \approx -\frac{z''}{R}
\]

(2.54)

2.3 Finite Element formulation of the piston ring model

The constitutive equations of bending and twisting of piston rings have been derived. Bending and twisting of the ring were found to be proportional to changes of curvature and torsion of the ring neutral axis. The parametric expressions of curvature and torsion in cylindrical coordinates were obtained after reviewing the differential geometry theory applicable to space curves. Recognizing that ring deformations are small compared to the ring radius, these expressions were approximated. A set of differential equations relating bending and twisting moments to the ring neutral axis displacements and their derivatives can now be calculated by combining ring bending and twisting equations with the approximated curvature and torsion expressions. In the plane of curvature of the ring, the bending moment is related to the flexural stiffness and change of curvature (equation (2.55)).
\[ M_z = -\frac{EI_z}{R^2} (y + y'') \]  

(2.55)

Bending outside of the ring plane and ring twisting are coupled due to the ring curvature. Twist and axial deformation are present in both the out of plane bending moment and twisting moment expressions (equation (2.56) and (2.57))

\[ M_r = \frac{EJ_r}{R^2} (z'' + R\alpha) \]  

(2.56)

\[ M_\theta = \frac{GJ_c}{R^2} (z' - R\alpha') \]  

(2.57)

The focus of this thesis is to predict the sealing performance of piston rings. To do so dynamics and deformation of the piston ring will have be solved together with piston and cylinder contact interaction. Complex ring loading situations are expected and numerical solution to the ring mechanics equations (equations (2.55) to (2.57)) are required to look at lubricant flow and friction at the ring-liner and ring-piston interfaces. For a structural analysis of this kind, the finite element method is the method of choice. Most finite element analyses focus on determining the strength of a given structure, or in other terms verifying that the maximum stress in the structure remains sufficiently far from the elastic limit. Here however the objective is to couple the deformations of the ring, to the contact interaction with the piston and liner to look at the seal performance. A dual grid finite element method is proposed to provide an efficient solution to this problem. Ring deformations are solved on a coarse structural mesh and contact with the piston and liner is handled on a fine contact grid. In this section the development of this finite element method is detailed.

### 2.3.1 Spline interpolation of the piston ring deformation

The first step in developing the curved beam finite element is to discretize the geometry of the ring neutral axis. The ring neutral axis is a continuous curve which is free to take any shape under deformation. It is discretized in a number of finite elements using spline interpolation. The ring neutral axis geometry is interpolated from the nodal values of displacements and their derivatives by polynomial shape functions. Figure 2.25 shows an example of spline interpolation of the ring radial displacement.
The ring radial deformation (solid line) is approximated by a linear function (dotted line) which matches the actual deformation at node 1 and node 2 (equation (2.58)).

\[ y(\eta) = y_1 + \eta(y_2 - y_1) \tag{2.58} \]

The interpolating function is expressed as a function of the isoparametric variable \( \eta \) (equation (2.59)).

\[ \eta = \frac{\theta}{\theta_e} \tag{2.59} \]

In the above equation \( \theta_e \) is the angular length of a ring element. The isoparametric variable is defined such that \( \eta \) varies linearly from \( \eta = 0 \) at node 1 to \( \eta = 1 \) at node 2. The interpolation function can be rewritten as the sum of two first order polynomials (equation (2.60)).

\[ y(\eta) = (1 - \eta)y_1 + \eta y_2 = N_1 y_1 + N_2 y_2 \tag{2.60} \]

\( N_1 \) and \( N_2 \) are the shape functions of the ring element associated with the nodal displacements \( y_1 \) and \( y_2 \). Equation (2.60) is a first order Hermitian interpolation of the ring radial deformation. The ring can be divided in \( n \) elements with each element having its own interpolation function that are...
assembled together to obtain the deformation of the ring around the entire circumference. By using Hermite polynomials, the continuity of the radial displacement at the nodes is guaranteed. Indeed one can see that in equation (2.60) by definition \( y(0) = y_1 \) and \( y(1) = y_2 \). Higher order Hermite polynomials can be used to guarantee the continuity of the derivatives of the interpolated function. Higher orders lead to higher interpolation accuracy but also more nodal displacements (displacements and derivatives of displacements as opposed to displacement only for the first order Hermite polynomial).

The deformation of the ring to be interpolated can be broken down in three components: the radial and axial displacements of the neutral axis, and the orientation of the cross section (Figure 2.26).

![Figure 2.26 - Displacement of the ring neutral axis](image)

The three components of ring deformation are discretized using Hermitian spline interpolation. As seen in the constitutive bending and twisting equations of the ring (equation (2.55) to (2.57)) bending is related to the variation of curvature in the radial and axial planes. Hence the second derivative of both radial and axial displacements are involved. In this ring finite element model, 5th order Hermite polynomial spline are used to interpolate \( y \) the radial displacement and \( z \) the axial displacement. This will guarantee continuity of the curvature of the ring which is the key variable for bending, thus reaching optimal accuracy per nodal degree of freedom. The twisting moment is proportional to the derivative of the twist angle, therefore 3rd order polynomial interpolation is sufficient.
To carry out the 5th order interpolation of the ring radial deformation, nodal displacements must include the radial displacement, its first and second derivatives (Figure 2.27).

Figure 2.27 - Spline interpolation of ring displacement

\[
y(\eta) = \sum_{k=1}^{6} N_k(\eta) u_{yk} \text{ where } \{u_y\} = \{u_{y1}, \ldots, u_{y6}\}^T = \{y_1, y'_1, y''_1, y_2, y'_2, y''_2\}^T \quad (2.61)
\]

There are six shape functions \(N_k\) in the 5th order polynomial interpolation of the ring radial displacement (equation \(2.61\)). The coefficients of the shape function polynomials can be calculated by using the continuity of the ring radial displacement and its derivative at both nodes (equation \(2.62\)). In this section the prime sign refers to a derivation with respect to \(\theta\), the polar angle.

\[
y(0) = y_1 \quad y'(0) = y'_1 \quad y''(0) = y''_1 \quad y(1) = y_2 \quad y'(1) = y'_2 \quad y''(1) = y''_2 \quad (2.62)
\]

Due to the form of the interpolation function one finds that the first continuity equation \((y(0) = y_1)\) is automatically satisfied for:

\[
N_1(0) = 1 \quad N_2(0) = 0 \quad N_3(0) = 0 \quad N_4(0) = 0 \quad N_5(0) = 0 \quad N_6(0) = 0 \quad (2.63)
\]

Similarly this process can be repeated for the next 5 continuity equations (equations \(2.64\) to \(2.68\)).
\( N_2'(0) = 1 \quad N_k'(0) = 0 \text{ for } k = \{1,3,4,5,6\} \quad (2.64) \)
\( N_3''(0) = 1 \quad N_k''(0) = 0 \text{ for } k = \{1,2,4,5,6\} \quad (2.65) \)
\( N_4(1) = 1 \quad N_k(1) = 0 \text{ for } k = \{1,2,3,5,6\} \quad (2.66) \)
\( N_5'(1) = 1 \quad N_k'(1) = 0 \text{ for } k = \{1,2,3,4,6\} \quad (2.67) \)
\( N_6''(1) = 1 \quad N_k''(1) = 0 \text{ for } k = \{1,2,3,4,5\} \quad (2.68) \)

There are now 6 equations per shape function resulting from continuity conditions. Since the shape functions \( N_k \) are 5th order polynomial, they are uniquely defined by 6 coefficients, that can be calculated from these 6 continuity equations. The continuity equations are solved to obtain the expression of the polynomial shape functions (equation (2.69) to (2.74))

\[
N_1 = 1 - 10\eta^3 + 15\eta^4 - 6\eta^5 
\]
\[
N_2 = \theta_e(\eta - 6\eta^3 + 8\eta^4 - 3\eta^5) 
\]
\[
N_3 = \theta_e^2\left(\frac{\eta^2}{2} - 3\frac{\eta^3}{2} + 3\frac{\eta^4}{2} - \frac{\eta^5}{2}\right) 
\]
\[
N_4 = 10\eta^3 - 15\eta^4 + 6\eta^5 
\]
\[
N_5 = \theta_e(-4\eta^3 + 7\eta^4 - 3\eta^5) 
\]
\[
N_6 = \theta_e^2\left(\frac{\eta^3}{2} - \eta^4 + \frac{\eta^5}{2}\right) 
\]

The 6 shape functions and their first and second derivatives are plotted (Figure 2.28). At each node there is one shape function (or its derivative) that is equal to one and all other are nil. This comes from the form of the Hermite interpolation function (equation (2.61)) that associates one nodal displacement (either displacement or derivative of the displacement) to a given shape function.
The axial deformation of the ring neutral axis is interpolated using the same 5th order Hermite interpolation scheme (equation (2.75))

\[ z(\eta) = \sum_{k=1}^{6} N_k(\eta) u_{zk} \text{ where } \{u_z\} = \{u_{z1}, \ldots, u_{z6}\}^T = \{z_1, z_1', z_1'', z_2, z_2', z_2''\}^T \quad (2.75) \]

The shape functions of the interpolation function of the axial displacement are identical to the shape functions used in the interpolation of the radial displacement. As stated earlier, 3rd order interpolation of the ring twist \( \alpha \) is sufficient (equation (2.76)).
\[
\alpha(\eta) = \sum_{k=1}^{6} N_{ak}(\eta) u_{ak} \text{ where } \{u_{\alpha}\} = \{u_{\alpha_1}, \ldots, u_{\alpha_4}\}^T = \{\alpha_1, \alpha_1', \alpha_2, \alpha_2'\}^T
\]  

(2.76)

The shape functions associated to ring twist \(N_{ak}\) are calculated using equations of continuity at the nodes. It is the same process that was used earlier for the calculation of the 5th order shape functions. The expression of the cubic polynomial shape functions are obtained (equation (2.77) to (2.80))

\[
N_{a1} = 1 - 3\eta^2 + 2\eta^3
\]  

(2.77)

\[
N_{a2} = \theta_\varepsilon (\eta - 2\eta^2 + 8\eta^3)
\]  

(2.78)

\[
N_{a3} = -3\eta^2 - 2\eta^3
\]  

(2.79)

\[
N_{a4} = \theta_\varepsilon (-\eta^2 + \eta^3)
\]  

(2.80)

This discretization of ring deformation leads to 8 degree of freedoms per node: 3 for radial displacements, 3 for axial displacements and 2 for twist. All nodal displacements are assembled in one displacement vector (equation (2.81)).

\[
\{u_k\} = \{y_k, y_k', y_k'', z_k, z_k', z_k', \alpha_k, \alpha_k'\}^T
\]  

(2.81)

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2.3.2 Derivation of stiffness, mass and load finite element matrices

Now that the ring is discretized in elements and its deformation interpolated, the finite element matrices of the ring element can be calculated. The finite element equations of the piston ring model are derived using Hamilton’s principle [41]. It is a variational formulation of the equations of motion based on the Lagrangian of a system (equation (2.82)). It can be seen as an extension of the principle of virtual work that includes the dynamics of the system.

\[ L = T + W - U \]  \hspace{1cm} (2.82)

The Lagrangian \( L \) of the system, combines the kinetic energy \( T \), the strain energy \( U \) and the work of external forces \( W \). Application of Hamilton’s principle to the system leads to the Euler-Lagrange equations (equation (2.83)).

\[ \frac{\partial}{\partial t} \left( \frac{\partial L}{\partial \dot{u}_i} \right) - \frac{\partial L}{\partial u_i} = 0 \text{ for } i = 1, \ldots, n \]  \hspace{1cm} (2.83)

The variable \( u_i \) represents the \( i \)th nodal displacement (degree of freedom) of the discretized ring. The dot sign represents a derivation with respect to time, therefore \( \dot{u}_i \) is the nodal speed. The Euler-Lagrange equation is applied to a one degree of freedom mass spring system to illustrate the equivalence between Hamilton’s principle and the equation of motion (Figure 2.30).

![Figure 2.30 - Spring mass system](image)

First the kinetic energy, work done by \( F \) and the spring energy are calculated as a function of the displacement \( u \) of the mass (equation (2.84)). The system parameters are: \( k \) the spring stiffness and \( m \) the mass attached to the spring.
\[ T = \frac{1}{2} m \dot{u}^2 \quad W = Fu \quad U = \frac{1}{2} ku^2 \] 

The expression of kinetic energy, strain energy and work are substituted in the Euler-Lagrange equation (equation (2.85)).

\[ \frac{\partial}{\partial t} \left\{ \frac{\partial}{\partial \dot{u}} \left( \frac{1}{2} m \dot{u}^2 \right) \right\} - \frac{\partial}{\partial u_i} \left( Fu - \frac{1}{2} ku^2 \right) = 0 \] 

(2.85)

The equation of motion of the spring mass system is found (equation (2.86))

\[ m \ddot{u} = F - ku \] 

(2.86)

One can also note that removing the time derivative term of the Euler-Lagrange equation (equation (2.83)) leads to an expression equivalent to the principle of virtual work. In the present example, it yields the static equation of the spring-mass system.

The Euler-Lagrange equations for one element of the piston ring are now derived. Unlike the spring mass example the ring is a continuous system. First the kinetic energy, strain energy and work of external forces of the ring must be calculated.

The motion of the ring cross section can be split in three parts: the translation in the radial direction, the translation in the axial direction and the rotation around the ring neutral axis. The ring neutral axis displacements (\(y, z\) and \(\alpha\)) are measured in the reference frame of the piston. The zero displacement position corresponds to a ring centered in the piston groove and contacting a perfectly round cylinder. The kinetic energy of the ring element is calculated by integrating the kinetic energy of cross sections along the ring neutral axis (equation (2.87)).

\[ T^{(e)} = \frac{1}{2} \int_0^{L_e} \rho [A(y^2 + \dot{z}^2) + I_p \dot{\alpha}^2] ds \] 

(2.87)

The notation \((e)\) is used to refer to the ring element, for instance \(T^{(e)}\) represents the kinetic energy of the ring element. The integration is carried out over the length of the ring element \(L_e\). The
density of the ring material is $\rho$, $I_p$ is the polar moment of inertia of the cross section (equation (2.20)), and $A$ the cross section area.

Strain energy is stored in the ring structure due to the action of bending and twisting moments. Derivation of the strain energy of a beam due to bending and torsion is available in Timoshenko Strength of Materials text [38]. The strain energy of a beam in pure bending is given by the product of the bending moment $M_b$ with change of curvature $\Delta \kappa$ (equation (2.88)).

$$dU_b = \frac{1}{2} M_b \Delta \kappa ds$$  \hspace{1cm} (2.88)

The contribution of torsion to the beam strain energy is given by the product between the twisting moment $M_t$ and the twist angle $\theta$ (equation (2.89)).

$$dU_t = \frac{1}{2} M_t \theta ds$$  \hspace{1cm} (2.89)

In equations (2.88) and (2.89) $ds$ is the length of a differential ring element. Given the relations between bending moment and curvature change (equation (2.14)) and twisting moment and twist angle (equation (2.21)) the strain energy expression can be rewritten as a function of bending and twisting moments only (equation (2.90)).

$$dU_b = \frac{M_b^2}{2EI} ds \hspace{1cm} dU_t = \frac{M_t^2}{2GJ_t} ds$$  \hspace{1cm} (2.90)

The piston ring is experiencing bending both in and outside of its plane of curvature. The ring elastic strain energy is calculated using the notations of equations (2.55) to (2.57) (equation (2.91)).

$$dU = \frac{1}{2} \left[ \frac{M_b^2}{GJ_t} + \frac{M_t^2}{EI_t} + \frac{M_z^2}{EI_z} \right] ds$$  \hspace{1cm} (2.91)

With the help of equation (2.55) to (2.57), the ring strain energy can be expressed as a function of the neutral axis displacements (equation (2.92)).
The infinitesimal strain energy $dU$ is integrated over the length of the ring element (equation (2.93))

$$U^{(e)} = \frac{1}{2} \int_0^L \left( G_{J_t} \frac{(z' - R\alpha')^2}{R^4} + E I_r \frac{(z'' + R\alpha)^2}{R^4} + E I_z \frac{(y + y'')^2}{R^4} \right) ds$$  (2.93)

The work of external forces must be computed to complete the calculation of the piston ring Lagrangian. During engine operation, the ring deforms and moves within the piston groove due to the action of external forces. Acceleration or deceleration of the piston pushes the ring against the upper or lower flank of the piston groove. Ring tension forces the ring against the cylinder liner. Pressurized piston grooves and lands result in axial and radial forces acting on the ring. Cross section resulting radial force $f_r$, axial force $f_z$ and twisting moment $m_\theta$ can be calculated to take into account all external actions (Figure 2.31). The quantities $f_r$, $f_z$ and $m_\theta$ are forces and moment per unit length respectively.

The work of external forces is calculated by multiplying cross sections resultants with corresponding displacements (equation (2.94)) and integrating over the length of the ring element.
\[
W^{(e)} = \int_{0}^{L_e} \left[ f_y y + f_z z + m_\varepsilon \alpha \right] ds \tag{2.94}
\]

Applying the Euler-Lagrange equation (equation (2.83)) on the Lagrangian of the piston ring, leads to the expression of the mass, stiffness and force matrices of this finite element model. To be able to calculate the variation of kinetic, strain energy and work, the ring neutral axis displacements \((y, z\) and \(\alpha\)) are expressed as a function of nodal displacements using the interpolation functions (equations (2.61), (2.75) and (2.76)). The ring element has two nodes and therefore 16 nodal displacements (equation (2.95))

\[
\{u\}^{(e)} = \{u_1, u_2, \ldots, u_{15}, u_{16}\}^T = \{y_1, y_1', y_1'', z_1, z_1', z_1'', \alpha_1, \alpha_1', y_2, y_2', y_2'', z_2, z_2', z_2'', \alpha_2, \alpha_2'\}^T \tag{2.95}
\]

Three sets of indices are introduced to write the nodal expressions of kinetic, strain energy and work (equations (2.96) to (2.98)).

\[
k_y = [1, 2, 3, 9, 10, 11] \tag{2.96}
\]

\[
k_z = [4, 5, 6, 12, 13, 14] \tag{2.97}
\]

\[
k_\alpha = [7, 8, 15, 16] \tag{2.98}
\]

The sets \(k_y\), \(k_z\) and \(k_\alpha\) are designed to select the nodal degrees of freedoms related to radial, axial displacements and twist respectively, from the nodal displacements vector of the ring element \(\{u\}^e\). In addition the mapping function \(m\) is introduced (equation (2.99)).

\[
k \rightarrow m(k) \newline
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16] \rightarrow [1, 2, 3, 1, 2, 3, 1, 2, 4, 5, 6, 4, 5, 6, 3, 4] \tag{2.99}
\]

This mapping function is designed to select the shape function (right hand side) corresponding to a ring element nodal displacement (left hand side). The kinetic energy of the ring is rewritten as a function of the nodal velocities \(\dot{u}_k\) and shape functions \(N_k\) (equation (2.100)).
\[ T^{(e)} = \frac{L_e}{2} \int_0^1 \rho \left[ A \left( \left( \sum_{k=k_y} N_{m(k)} \dot{u}_k \right)^2 + \left( \sum_{k=k_z} N_{m(k)} \dot{u}_k \right)^2 \right) \right. \\
\left. + I_p \left( \sum_{k=k_\alpha} N_{am(k)} \dot{u}_k \right)^2 \right] d\eta \]  

(2.100)

It can be seen that the kinetic energy is a bilinear form of the nodal velocities by expanding the nodal expression of the kinetic energy (equation (2.101)).

\[ T^{(e)} = \frac{L_e}{2} \int_0^1 \rho \left[ A \left( \sum_{k_1=k_y, k_2=k_y} \sum_{k_1=k_z, k_2=k_z} N_{m(k_1)} N_{m(k_2)} \dot{u}_{k_1} \dot{u}_{k_2} \right) \right. \\
\left. + \sum_{k_1=k_\alpha, k_2=k_\alpha} \sum_{k_1=k_x, k_2=k_x} N_{m(k_1)} N_{m(k_2)} \dot{u}_{k_1} \dot{u}_{k_2} \right] d\eta \]  

(2.101)

Hence the nodal expression of the kinetic energy can be written in matrix form. The mass matrix of the ring element \( M^{(e)} \) is introduced to obtain the new expression of the kinetic energy (equation (2.102)).

\[ T^{(e)} = \frac{1}{2} \{ \ddot{u} \}^{(e)} T [M]^{(e)} \{ \ddot{u} \}^{(e)} \]  

(2.102)

The terms of the mass matrix are found by comparing the bilinear form to the matrix form of the kinetic energy (equation (2.103)).

\[ M_{ij}^{(e)} = \begin{cases} 
L_e \int_0^1 \rho A N_{m(i)} N_{m(j)} d\eta & \text{for } \{i, j\} \in k_y \text{ or } k_z \\
L_e \int_0^1 \rho I_p N_{am(i)} N_{am(j)} d\eta & \text{for } \{i, j\} \in k_\alpha \\
0 & \text{for all other } \{i, j\} 
\end{cases} \]  

(2.103)
The same process is repeated for the strain energy of the ring element. Ring deformation is expressed as a function of nodal displacements starting with the contribution of twisting to the ring strain energy (equation (2.104)).

\[ U^{(e)}_{\theta} = \frac{L_e}{2} \int_{0}^{1} GJ_t \left( \frac{(z' - R \alpha')^2}{R^4} \right) d\eta \]

\[ = \frac{L_e}{2R^4} \int_{0}^{1} GJ_t \left\{ \sum_{k=k_{z}} N^{(k)}_{m(k)} u_k - R \sum_{k=k_{a}} N^{(k)}_{am(k)} u_k \right\}^2 d\eta \]  

(2.104)

The in plane bending contribution to strain energy is calculated as a function of nodal displacements (equation (2.105)).

\[ U^{(e)}_z = \frac{L_e}{2} \int_{0}^{1} E I_z \left( \frac{(y + y'')^2}{R^4} \right) d\eta = \frac{L_e}{2R^4} \int_{0}^{1} E I_z \left\{ \sum_{k=k_{y}} (N^{(k)}_m + N^{(k)}_m') u_k \right\}^2 d\eta \]  

(2.105)

Finally the out of plane bending contribution to the ring strain energy is expressed as a function of nodal displacements (equation (2.106))

\[ U^{(e)}_r = \frac{L_e}{2} \int_{0}^{1} E I_r \left( \frac{(z'' + R \alpha)^2}{R^4} \right) d\eta \]

\[ = \frac{L_e}{2R^4} \int_{0}^{1} E I_r \left\{ \sum_{k=k_{z}} N^{(k)}_{m(k)} u_k + R \sum_{k=k_{a}} N^{(k)}_{am(k)} u_k \right\}^2 d\eta \]  

(2.106)

Like the kinetic energy, the strain energy of the ring element is a bilinear form of the nodal displacements and can be written in matrix form. The stiffness matrix \([K]^{(e)}\) of the ring element is introduced and the strain energy is written in matrix form (equation (2.107)).

\[ U^{(e)} = U^{(e)}_r + U^{(e)}_{\theta} + U^{(e)}_z = \frac{1}{2} \{u\}^T [K]^{(e)} \{u\} \]  

(2.107)

The terms of the ring element stiffness matrix are found by comparing the matrix form of the strain energy (equation (2.107)) to the original expression (equation (2.104) to (2.106)).
\[ K_{ij}^{(e)} = \begin{cases} \frac{L_e}{R^4} \int_0^1 EI_z (N_i + N_i')(N_j + N_j') d\eta & \text{for } \{i, j\} \in k_y \\ \frac{L_e}{R^2} \int_0^1 (EI_r N_m(i)N_m(j) + GJ_t N_m(i)N_m(j)) d\eta & \text{for } \{i, j\} \in k_z \\ \frac{L_e}{R^3} \int_0^1 (EI_r N_m(i)N_m(j) - GJ_t N_m(i)N_m(j)) d\eta & \text{for } i \in k_z \text{ and } j \in k_a \\ 0 & \text{for all other } \{i, j\} \end{cases} \quad (2.108) \]

The work of external forces is now written as a function of the nodal displacements (equation (2.109)).

\[ W^e = L_e \int_0^1 \left[ f_r \sum_{k=k_y} N_{m(k)} u_k + f_z \sum_{k=k_z} N_{m(k)} u_k + m_t \sum_{k=k_a} N_{am(k)} u_k \right] d\eta \quad (2.109) \]

The work of external forces on the ring element is a linear form of the nodal displacements. It can be written as a scalar product between two vectors. The load vector \( \{F\}^{(e)} \) is introduced to write this scalar product (equation (2.110)).

\[ W^e = \{u\}^T \{F\}^{(e)} \quad (2.110) \]

The terms of the load vector are the integration of the product between the cross section resulting forces and the shape functions (equation (2.111)).

\[ F_i^{(e)} = \int_0^{L_e} f_i N_i ds \text{ where } f_i = \begin{cases} f_r & \text{if } i \in k_y \\ f_z & \text{if } i \in k_z \text{ and } N_i = \begin{cases} N_{m(i)} & \text{for } i \in k_y \cup k_z \\ N_{am(i)} & \text{for } i \in k_a \end{cases} \\ m_t & \text{if } i \in k_a \end{cases} \quad (2.111) \]

The Lagrangian of the ring element can now be written in matrix form with the help of the mass and stiffness matrices and the load vector (equation (2.112)).

\[ L^{(e)} = T^{(e)} + W^{(e)} - U^{(e)} \]

\[ = \frac{1}{2} \{\dot{u}\}^{(e)} [\dot{M}]^{(e)} \{\dot{u}\}^{(e)} + \{u\}^T \{F\}^{(e)} - \frac{1}{2} \{u\}^T [K]^{(e)} \{u\}^{(e)} \quad (2.112) \]
Applying the Euler-Lagrange equation (equation (2.83)) to the ring element Lagrangian yields the finite element equation of motion (equation (2.113)).

\[
[M]^{(e)}\ddot{u}^{(e)} + [K]^{(e)}u^{(e)} = F^{(e)}
\]  

(2.113)

2.3.3 Assembly of finite element matrices

To obtain the finite element equations for the complete ring, mass and stiffness matrices and load vectors of ring elements are assembled. The same process is used for conventional finite element models. An example of matrix assembly for two ring elements is given (Figure 2.32).

![Figure 2.32- Example of finite element matrix assembly](image)

The ring mesh in this example has two elements and three nodes. Element (1) and element (2) share node 2. The assembly of the element stiffness matrices to obtain the stiffness matrix of the half ring is shown (equation (2.114)).

\[
[K]^{(1)} = \begin{bmatrix} K_{11} & K_{12} \\ K_{12} & K_{22}^{(1)} \end{bmatrix} \quad [K]^{(2)} = \begin{bmatrix} K_{22}^{(2)} & K_{23} \\ K_{23} & K_{33} \end{bmatrix}
\]  

(2.114)

The exponent \((k)\) refers to the element \(k\) and the index \(i\) refers to node \(i\). \(K_{11}\) are the terms of the stiffness matrix related to the displacements of node 1. In the ring finite element model proposed in this thesis there are 8 degrees of freedom per node, hence the element stiffness matrix is of dimension \([16x16]\) and \(K_{11}\) is an \([8x8]\) matrix. The half ring has 3 nodes and 24 nodal
displacements, the global stiffness matrix $[K]$ is obtained by assembling $[K]^{(1)}$ and $[K]^{(2)}$ (equation (2.115)).

$$[K] = \begin{bmatrix} K_{11} & K_{12} & 0 \\ K_{12} & K_{22}^{(1)} + K_{22}^{(2)} & K_{23} \\ 0 & K_{23} & K_{33} \end{bmatrix} \quad (2.115)$$

When assembling the element matrices the contributions of both elements to the stiffness in node 2 are summed. The same process is used to assemble the global mass matrix and the global load vector. The finite element equations of motion can now be written with the global matrices (equation (2.116)).

$$[M]\ddot{\{u\}} + [K]\{u\} = \{F\} \quad (2.116)$$

The global displacement vector $\{u\}$ is the assembly of the nodal displacement vectors (equation (2.117))

$$\{u\} = \{u^{(1)}, u^{(2)}, ..., u^{(i)}, ..., u^{(n-1)}, u^{(n)}\}^T \quad (2.117)$$

2.4 Validation of ring finite element model

In this section the proposed curved beam finite element model of piston rings is validated. First its ability to interpolate the ring deformation is assessed. The deformation of a quarter ring under axial and radial loads is calculated using the ring model and compared to analytical solutions, to ensure that the structural component of the ring model is accurate. Finally a comparison between analytical solution and ring model prediction is made for a gapless ring in a distorted cylinder to validate that the proposed method of separation between contact grid and structural mesh is adequate. The piston ring model is found to provide a numerically efficient solution to the study of ring deformation and its interaction with piston and cylinder. It is able to model ring deformation accurately for a relatively coarse structural mesh (optimal number of ring elements between 10 and 20) while providing a fine description of contact between the ring and liner (contact point every 0.1° of ring circumference) at a low computation cost.
2.4.1 Interpolation of ring deformation

The piston ring interpolation scheme presented in the previous section must provide an accurate description of the ring neutral axis deformation. In the engine, piston rings deform to remain in contact with distorted cylinders and piston grooves. The complexity of the shape of the deformed piston ring is therefore related to the bore or piston distortion. To validate the spline interpolation of the piston ring, a section of a distorted bore is interpolated with the 5th order Hermite polynomial scheme of the proposed finite element model. The assumption is that the maximum complexity of the ring deformation will be close to that of the selected bore distortion.

\[ y_b(\theta) = A_0 + \sum_{k=1}^{n} A_k \sin(k\theta + \phi_k) \] 

(2.118)

Fourier series are commonly used to describe the bore deformation at a given height in the plane perpendicular to the cylinder axis (equation (2.118)). The radial deformation of the cylinder at the circumferential location \( \theta \) is \( y_b(\theta) \) (Figure 2.33). \( A_k \) is the magnitude of the bore distortion for the order \( k \) of the Fourier series, and \( \phi_k \) is the phase of the distortion for order \( k \).

![Distorted bore and Nominal bore](image)

*Figure 2.33 - Magnified section of distorted bore (x300)*
The bore distortion used to validate the ring interpolation scheme is chosen to be representative of the distortion of a passenger car engine (Table 2).

<table>
<thead>
<tr>
<th>$k$: Order of distortion</th>
<th>2$^{nd}$</th>
<th>3$^{rd}$</th>
<th>4$^{th}$</th>
<th>5$^{th}$</th>
<th>6$^{th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_k$: Magnitude (μm)</td>
<td>14.9</td>
<td>4.3</td>
<td>2.9</td>
<td>1.0</td>
<td>0.45</td>
</tr>
<tr>
<td>$\phi_k$: Phase (°)</td>
<td>84</td>
<td>-29</td>
<td>5</td>
<td>-10</td>
<td>29</td>
</tr>
</tbody>
</table>

*Table 2 - Magnitude and Phase of sample bore distortion*

The sample bore distortion is interpolated with increasing number of spline elements (Figure 2.34). The polynomial spline is able to rapidly match the geometry of the distorted bore even its high order features. Indeed 6 elements seem sufficient to interpolate the bore distortion with good accuracy.

*Figure 2.34- Spline interpolation of bore distortion*
The convergence of this interpolation method is studied in more details. The maximum error of interpolation along the circumference of the distorted bore is calculated for increasing number of elements (Figure 2.35).

![Graph showing interpolation error vs number of elements](image)

*Figure 2.35 - Interpolation error*

For the bore distortion used in this validation, the interpolation error reaches sub-micron level at 6 spline elements. The required number of elements for accurate interpolation of the piston ring deformation will depend on the order of the deformation. Since high order deformation translate into large changes of curvature and large bending stresses, the piston ring is not expected to deform significantly for orders higher than 6. The optimal number of ring elements to interpolate the ring deformation is thus between 10 and 20 elements depending on the desired accuracy.

### 2.4.2 Ring structural response

A simple case of a quarter circular ring is considered to validate the structural response of the piston ring finite element model. First the quarter ring is loaded radially and the radial displacement of the ring neutral axis is calculated analytically (Figure 2.36).
Recall the in-plane bending equation of the piston ring (equation (2.119)). Since the ring thickness is considered small compared to its radius, the radial displacement is a result of in-plane bending only and the effect of shear stress is ignored.

\[ M_z = -\frac{EI_z}{R^2} (y + y'') \quad (2.119) \]

The bending moment along the z axis resulting from the radial load is calculated (equation (2.120)).

\[ M_z = -F_r R \cos \theta \quad (2.120) \]

Equations (2.119) and (2.120) are combined together to obtain a differential equation of the radial displacement \( y \) (equation (2.121)).

\[ y + y'' = \frac{F_r R^3}{EI_z} \cos \theta \quad (2.121) \]

The linear differential equation is solved to obtain the radial displacement \( y \) (equation (2.122)). The ring is clamped at \( \theta = 0 \) which gives the two boundary conditions of the differential equation:

\( y(0) = 0 \) and \( y'(0) = 0 \)
The ring radial displacement of equation (2.122) is consistent with the result obtained by Timoshenko [38]. The quarter ring is meshed with both the curved beam model developed in this thesis and a straight beam element previously used to model piston rings [27]. The ring is discretized in two elements and the radial deformation obtained analytically and calculated from both finite element methods are compared (Figure 2.37). For this calculation a ring of rectangular cross section (3x1.2mm) representative of a top ring of a passenger car is chosen. The relevant calculation parameters are given in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$R$</th>
<th>$E$</th>
<th>$I_z$</th>
<th>$F_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>40 mm</td>
<td>210 GPa</td>
<td>3.4 mm$^4$</td>
<td>1 N</td>
</tr>
</tbody>
</table>

*Table 3- Parameters of quarter-ring deformation calculation*

Two curved beam elements are found to be sufficient to calculate the radial deformation with accuracy. On Figure 2.37, the analytical solution and the result of the curved beam finite element developed in this thesis cannot be distinguished. Not only are the position of the ring nodes
accurate but so is the distribution of the radial displacement between two nodes. This is the foundation of the proposed dual grid method: a coarse structural grid is sufficient to provide a fine description of ring deformation and contact with the cylinder. For the straight beam method however, 2 ring elements are insufficient to calculate the radial deformation of the quarter ring. This illustrates why in current piston ring models relying on straight beam finite elements, a large number of elements is required to couple ring deformation and contact with the cylinder.

To evaluate the rapidity of convergence of the straight and curved beam finite elements methods, the radial displacement of the tip of the quarter ring is calculated using both methods and compared to the analytical solution for increasing number of elements (Figure 2.38).

![Figure 2.38- Error of finite element models on the radial displacement of the tip of the quarter ring](image)

A precision of $10^{-4}$ on ring deformation (which corresponds in this example to 0.01 μm, or a 1/10 of the scale of liner roughness) is required to predict ring-liner contact with accuracy. This level of accuracy is reached for one curved beam element or approximately 30 straight beam elements. The ring curved beam model achieves higher accuracy for only one additional degree of freedom per node and is therefore more efficient than straight beam methods currently used.
The out of plane bending and twisting behaviors of the piston ring model are now validated. To do so, the deformation of the quarter ring under axial loading at its tip is calculated (Figure 2.39).

An analytical solution to the ring axial deformation and twist is sought. Recall the out of plane bending and twisting constitutive equations of the piston ring (equations (2.123) and (2.124)).

\[
M_r = \frac{E I_r}{R^2} (z'' + R \alpha) \quad \text{(2.123)}
\]
\[
M_\theta = \frac{G J_t}{R^2} (\alpha' - R \alpha') \quad \text{(2.124)}
\]

Axial deformation and twist of the ring are coupled due to the ring curvature and the out-plane bending and twisting equations will have to be solved together. The distribution of out of plane bending moment \( M_r \) and twisting moment \( M_\theta \) due to the axial load \( F_z \) is calculated (equation (2.125)).

\[
M_r = -F_z R \cos \theta \quad M_\theta = F_z R (1 - \sin \theta) \quad \text{(2.125)}
\]

Combining equations (2.123) and (2.124) together, one can obtain a differential equation in \( z \) only (the axial displacement) (equation (2.126)).
\[ z' + z^{(3)} = \frac{M_r R^2}{EI_r} + \frac{M_\theta R^2}{GJ_t} \]  

(2.126)

Bending and twisting moments are expressed as a function of the axial load in the right hand side of equation (2.126).

\[ z' + z^{(3)} = \frac{F_z R^3}{EI_r} \sin \theta + \frac{F_z R^3}{GJ_t} (1 - \sin \theta) \]  

(2.127)

Clamping of the ring at its base yields the three following boundary conditions (equation (2.128)).

\[ z(0) = 0 \quad z'(0) = 0 \quad \alpha(0) = 0 \]  

(2.128)

The axial deformation \( z \) and the twist \( \alpha \) of the ring neutral axis are obtained by solving the differential equation (equation (2.127)) and using the relation between \( z \) and \( \alpha \) given by equation (2.124). The three constants of the solution are calculated using the boundary conditions.

\[ z(\theta) = \frac{F_z R^3}{GJ_t} (\theta - \sin \theta + \cos \theta - 1) + \frac{F_z R^3}{2} \left( \frac{1}{EI_r} + \frac{1}{GJ_t} \right) \theta \sin \theta \]  

(2.129)

\[ \alpha(\theta) = -\frac{F_z R^2}{GJ_t} \sin \theta + \frac{F_z R^2}{2} \left( \frac{1}{EI_r} + \frac{1}{GJ_t} \right) \theta \sin \theta \]  

(2.130)

The axial deformation and twist found here are consistent with the results obtained by Love in his Theory of Elasticity text [39]. Twisting and out of plane flexural rigidity (\( GJ_t \) and \( EI_r \) respectively) are present in both axial deformation and twist illustrating coupling between bending and twisting. The orientation of sections of the deformed ring under the axial load is represented by the red lines of Figure 2.39. Counter to intuition the twist angle \( \alpha \) is actually negative close to the clamped end and positive close to the free end of the beam. The twist close to the clamped end is due to the twisting moment which is maximum locally. The twist at the free end is however due mostly to the tilting of the ring section when the ring is deformed axially.

The deformation of the quarter ring due to the axial load \( F_z \) is now calculated using straight and curved beam finite elements. The ring is meshed with two elements and the axial deformation
obtained from finite element calculations is compared to the analytical result of equation (2.129) (Figure 2.40).

Once again the axial displacement calculated with the curved beam model cannot be distinguished from the analytical solution. Two straight beam elements are also insufficient to calculate the axial deformation of the ring. The ring tip axial displacement is calculated with both curved and straight beam methods for increasing number of ring elements and compared to the analytical solution (Figure 2.41)
Figure 2.41- Error of finite element models to the axial displacement of the tip of the quarter ring

The curved beam model converges rapidly, the desired accuracy of $10^{-4}$ is reached for two curved elements. However approximately 40 straight beam elements are required for the same accuracy. Twist of the quarter ring under the axial load is calculated with 2 straight beam and 2 curved beam elements and compared to the analytical solution (Figure 2.42)

Figure 2.42- Twist of a quarter ring under axial load
As for radial and axial deformation, the curved beam solution for the ring twist is accurate and cannot be distinguished from the analytical solution on Figure 2.42. The straight beam model is able to give a reasonable value of the tip twist with 2 elements but the distribution is not representative of the analytical solution. The rapidity of convergence of both straight and curved beam models is assessed by calculating the tip twist for increasing number of elements and comparing it to the analytical solution (Figure 2.43).

![Figure 2.43 - Error of finite elements methods to twist of quarter ring](image)

The curved beam model is rapid to converge to the twist analytical solution. The 10^-4 accuracy is reached for two curved beam elements, though approximately 50 straight beam elements are required for the same accuracy.

It can be concluded from this analytical validation of the structural behavior of the curved beam piston ring model, that the method proposed in this thesis is more efficient at calculating deformation of the ring than existing straight beam ring models. It was found that 2 curved beam elements are sufficient to model the deformation of a quarter ring with an accuracy of 10^-4 (relative to analytical solutions). This should mean that 8 curved beam elements are sufficient for the entire ring. One has to keep in mind that in the engine, the deformed shape of the ring is expected to be more complex (quicker variation of ring deformation due for instance to bore distortion). In
practice the optimal number of ring elements is found to be between 10 and 20 depending on the complexity of piston and bore distortion.

2.4.3 Separation of structural mesh and contact grid

So far the proposed curved beam finite element model of the piston ring was found to be efficient at discretizing the ring geometry and calculating the structural deformation of the ring. The focus of this section is to validate the method of separation of structural mesh and contact grid that uses the element shape functions. To do so, a solution of the problem of the conformability of piston rings to distorted cylinders is sought. Indeed in conformability analysis, ring structural deformation and ring-liner contact are coupled making it ideal for testing separation of the structural mesh and contact grid in the proposed ring model.

The piston ring equilibrium equations can be used to derive a relation between the deformation of the ring and the ring-liner contact force (equation (2.131)) [42]. Due to the complexity of the relation between contact force $f_c$ and ring radial deformation $y$, no closed form solution to the conformability of piston rings is readily available.

$$y^{(4)} + 2y'' + y = \frac{R^4}{EI_z} \left( \frac{F_t}{R} - f_c \right)$$  \hspace{1cm} (2.131)

Conventional methods of solving differential equations cannot be used in this case because of the nonlinearity of the contact force $f_c$ with the radial displacement. To obtain an analytical solution the contact between the ring and cylinder liner is linearized (equation (2.132)).

$$f_c = \frac{F_t}{R} + k_c(y - y_b)$$  \hspace{1cm} (2.132)

The radial deformation of the bore is $y_b$ and $k_c$ is the contact stiffness. The simplified contact model is built such that the contact pressure is uniform and equal to $F_t/R$ (ring tension divided by cylinder radius) when there is no bore or ring deformation. The linear conformability equation is obtained by substituting the linear contact model in the general conformability equation (equation (2.133)).

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\[ y^{(4)} + 2y'' + (1 + \lambda)y = \lambda y \quad (2.133) \]

The dimensionless parameter \( \lambda \) represents the ratio between the contact stiffness and the ring stiffness (equation (2.134)).

\[ \lambda = \frac{k_c R^4}{EI_z} \quad (2.134) \]

The linearized conformability equation can be solved to obtain the deformation of the ring in the distorted cylinder at equilibrium [42]. The contact force at equilibrium can be calculated from the ring displacement that solves the linear conformability equation (equation (2.135)).

\[ f_c = \frac{F_t}{R} + \sum_{k=2}^{n} \left\{ k c \left( \frac{1}{1 + \frac{(k^2 - 1)^2}{\lambda}} - 1 \right) A_k \sin(k\theta_k + \phi_k) \right\} \quad (2.135) \]

The general expression of the contact pressure distribution can be simplified by realizing that for piston rings \( \lambda >> (k^2 - 1)^2 \) meaning that ring-liner contact is much stiffer than the ring structure (equation (2.136)).

\[ f_c = \frac{F_t}{R} - \frac{EI_z}{R^4} \sum_{k=2}^{n} ((k^2 - 1)^2 A_k \sin(k\theta_k + \phi_k)) \quad (2.136) \]

The simplified expression of contact distribution is consistent with the result obtained by Muller [19]. Müller obtained this result by making the approximation that ring and distorted cylinder have the same curvature when the ring is conforming to the cylinder. This approximation is equivalent to the approximation made above. For stiff contacts, displacements due to the variation of contact pressure do not affect bending of the ring which is to say that they do not modify the ring curvature. Thus for stiff contacts, one can assume that a conforming ring has the same curvature that the distorted bore. One should note that the contact force expression is only valid if the ring remains in contact with the cylinder. The linearized contact model will not be able to represent a loss of contact between ring and liner.
To validate the method separating structural mesh and contact grid, the conformability of a piston ring in a 4\textsuperscript{th} order distorted bore is studied. The ring is designed to provide a uniform contact pressure along its circumference but the bore distortion affects the contact distribution. Contact pressure increases at the reduced radius locations and decreases at the increased radius locations (Figure 2.44).

![Deformed ring and contact pressure](image)

*Figure 2.44 - Distribution of contact pressure between ring and 4\textsuperscript{th} order bore distortion (ring deformation magnified 1000 times)*

The equilibrium of the ring in the distorted bore is calculated with the curved beam finite element method developed in this thesis and compared to the solution to the linear conformability equation. The same ring configuration than for structural validation is used to remain representative of a top ring of passenger car. The bore distortion is a 4\textsuperscript{th} order only of magnitude 4\textmu m. Calculation parameters are summarized in table 4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$R$</th>
<th>$E$</th>
<th>$I_z$</th>
<th>$A_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>40 mm</td>
<td>210 GPa</td>
<td>3.4 mm$^4$</td>
<td>4\textmu m</td>
</tr>
</tbody>
</table>

*Table 4 - Parameters of ring conformability calculation*
First the impact of changing the mesh size on the accuracy of the dual grid curved beam model is studied. To isolate the effect of the mesh size, a fine contact grid with one contact point every 0.1° is chosen for all mesh sizes. The ring equilibrium is calculated for increasing number of ring elements (Figure 2.45).

![Diagram showing contact pressure distribution](image)

**Figure 2.45- Distribution of contact pressure between ring and distorted bore for increased number of ring elements**

With only 6 elements the dual grid ring model is able to predict the pressure distribution with good accuracy. The shape functions of the ring elements are able to describe the ring-liner interaction between the nodes. The maximum error along the ring circumference between the ring model and the analytical solution is calculated for increasing number of ring elements (Figure 2.46).
Figure 2.46 - Variation of the conformability calculation error with number of elements

A relative error lower than 1% is reached for 12 elements. The error has a local minimum for 8 elements which can be explained by the fact that for 8 elements the ring nodes are in phase with the distorted bore and the contact pressure distribution which is ideal numerically. Now the impact of the size of the contact grid on the accuracy of the dual grid ring model is studied. A fine structural mesh of 24 elements is used to guarantee accurate calculation of the deformation of the ring at the nodes and the number of contact points is gradually increased (Figure 2.47).
Only a quarter of the ring circumference is displayed on Figure 2.47 to highlight the differences between the analytical solution and the model calculation. Since the pressure distribution is periodic the three remaining quarters of the ring will behave similarly. Even for one contact point per node the calculated contact pressure at the nodes is accurate. However the contact grid is too coarse to look at the distribution of contact force along the ring circumference. A good approximation of the contact force is obtained by going to a total of 72 contact points for the entire ring. The maximum error at the contact points between the analytical solution and the ring model is calculated for increasingly finer contact grids (Figure 2.48).
Even with only one contact point per node, the relative error of the ring model is low (on the order of 0.1%). It decreases further when the contact grid is refined but then reaches a minimum. The residual error is due to the fixed size of the structural mesh (24 elements in this case). The size of the contact grid does not affect the accuracy of contact pressure at the grid points strongly, it depends rather on the structural mesh size. However the number of contact points is chosen based on how finely the distribution of contact pressure along the ring circumference needs to be modeled. The optimal number of contact points is a function of how quickly the contact pressure varies along the circumference. For the 4th order bore distortion of this example, 48 points are enough to describe the distribution of contact force with sufficient accuracy. That corresponds to 12 points per period of the pressure distribution. Hence one can expect the optimal contact grid size to be 24 for a 2nd order contact force distribution, 36 for a 3rd order and so on. When a ring is inserted in a distorted bore the contact pressure tends to vary rapidly around the ring gap. This would set in practice the optimal contact grid size.
2.5 Conclusion

A dual grid curved beam finite element model has been developed in this thesis for the purpose of simulating piston ring sealing performance. This finite element model is tailored to the mechanics of piston rings which are thin curved structure experiencing small deformations in the engine. The key issue with modeling the sealing performance of piston rings is the necessity to couple ring deformation to contact interaction with piston and liner and the fact that structural deformation, and contact are on different length scales. The method of separation of structural mesh and contact grid using the element shape functions has proved to be efficient at addressing this problem. Ring structural deformations are solved with sufficient accuracy on a coarse structural mesh and contact with the liner and the piston is handled on a fine contact grid. Moving to a curved beam formulation has solved issues of the straight beam method which has so far been widely used to model piston rings. Discontinuities of quantities such as bending moment and curvature are removed with the curved beam formulation. The singular behavior of the ring gap can also be studied with a higher degree of confidence.

The dual grid model of piston rings provides a numerical framework for the study of ring performance. Due to the efficiency of this method engine cycle simulations can be computed in a reasonable amount of time. Flow of lubricant between ring and liner, and ring and piston can be calculated to model oil transport in the ring pack. Friction between the ring and the liner and contact with liner and piston can be studied to look at power loss and wear. In the following chapters the model is used to simulate the Oil Control Ring (OCR) sealing performance. The objectives of this simulation is to understand how various mechanism such as lack of conformability, OCR scraping and oil flow through the OCR groove can contribute to oil transport past the OCR. The motivation of this analysis is to find ways to reduce OCR tension (and hence friction) without causing penalties in oil consumption.

Finally one should keep mind that certain approximations were made to study specifically the performance of OCR and several adjustments are required to make the method more general. The current method is valid only for ring with symmetrical cross sections undergoing small deformation. For ring free shape analysis and calculation of ring static twist, the small
displacement approximation no longer holds and the general expression of the curvature must be used. In addition top rings sections can have chamfers or hooks making them asymmetrical. The section principle axes are no longer coincident with the cylinder frame of reference and ring deformation must be solved in the reference frame of the principal axes.
3 OCR oil transport model

3.1 Interaction between the OCR and the cylinder liner

3.1.1 Lubrication of sliding contacts

The contact interaction between the piston ring and the cylinder liner is characteristic of sliding contacts. Stribeck curves are commonly used to describe the behavior of sliding contacts (Figure 3.1)

On the x-axis of the Stribeck curve is a dimensionless parameter combining the oil viscosity $\mu$, the sliding speed $V$ and the normal contact load $F$. There are three separate regimes of lubrication: boundary lubrication at high load and low sliding speed, hydrodynamic lubrication at low load and high sliding speed and mixed lubrication at the transition between the last two. Both liner and ring surfaces are rough. In boundary lubrication, the oil film is too thin to separate both surfaces and contact between metal asperity occurs. In hydrodynamic lubrication, pressure in the oil film is
sufficient to have full separation of the sliding surfaces. In the boundary friction regime, the friction coefficient of the sliding contact is close to the non-lubricated friction coefficient. With increased sliding speed, the oil film helps reduce contact between surface asperities and hence friction. In the hydrodynamic regime, the friction coefficient is seen to increase again due to the shearing of the oil film.

The oil control ring model presented in this thesis combines an asperity contact model with a ring-liner lubrication model to be able to simulate the ring-liner interaction across the three regimes of lubrication.

3.1.2 Asperity contact interaction

The asperity contact model used in this thesis is derived from the work Greenwood and Tripp on the contact of rough surfaces [43]. The simplified formulation derived by Hu is adopted [44] (equation (3.1)).

\[
P_c = \begin{cases} 
0 & \frac{h_l}{\sigma_l} \geq 4 \\
\frac{1}{k_c} \left(4 - \frac{h_l}{\sigma_l}\right)^z & \frac{h_l}{\sigma_l} < 4 
\end{cases}
\]  \hspace{1cm} (3.1)

\(P_c\) is the asperity contact pressure, \(h_l\) the ring-liner clearance and \(\sigma_l\) the standard deviation of the liner roughness. The correlation constant \(k_c\) is a function of the properties of the ring and liner material (equation (3.2)). In this analysis ring and liner are assumed to be made of the same material.

\[
k_c = C \frac{E}{1 - v^2}
\]  \hspace{1cm} (3.2)

\(C\) and \(z\) are correlation constant calculated by Hu [44] (equation (3.3))

\[
C = 5.279 \times 10^{-9} \text{ and } z = 6.804
\]  \hspace{1cm} (3.3)

Figure 3.2 is a plot of the asperity contact pressure for ring-liner clearance ranging between one and 4 times the standard deviation of the liner roughness.
The contact force per unit length acting on the ring can be calculated by multiplying the asperity contact pressure to the OCR land width (equation (3.4)).

\[ f_c = 2lwP_c \]  

(3.4)

3.1.3 Lubrication model for the ring-liner interface

The sliding surface of the OCR is parallel to the liner, thus conventional lubrication models (solutions to the Reynolds equation) would predict no generation of hydrodynamic pressure in the oil film between the ring and the liner. OCR lands are parallel to the liner by design to control the thickness of the oil film on the liner down to the level of surface roughness. The roughness geometry can no longer be neglected and is the main contributor to hydrodynamic pressure generation in the oil film. An example of cylinder liner roughness geometry is shown on Figure 3.3.
This kind of surface micro-geometry is obtained from the cross hatch honing process. Tilted groove of depth ranging from 1 to 3μm are formed on the liner. Plateaus are found between the grooves and have a roughness on the order of 0.1μm. The radial load of the ring liner contact is supported by the plateaus. Figure 3.4 is a plot of the cylinder roughness along the sliding direction that highlights roughness of the plateau seen by the OCR as it slides along the liner.
A deterministic model solving the Reynolds equation of the sliding contact between the OCR and the cylinder liner at the scale of the liner surface asperities has been developed by Li and Chen [45]–[48]. It considers the effect of the liner roughness on hydrodynamic pressure generation in the oil film. Cavitation behind surface asperities is modelled. In this model the OCR surface is considered smooth which is to say its roughness is considered much smaller than the liner roughness.

Steady state simulations results for a range OCR-liner clearance are used to build a correlation between the ring-liner clearance ($h_l$) and the average hydrodynamic pressure in the oil film ($p_h$) (equation (3.5)). In these simulations, oil supply is assumed to be sufficient to have fully-flooded lubrication over the entire range of ring-liner clearance.

$$p_h = p_o \frac{\mu V}{(\mu V)_0} \left(\frac{h_l}{\sigma_l}\right)^{-a}$$  \hspace{1cm} (3.5)

The correlation coefficients are $p_o$, $(\mu V)_0$ and $a$. One set of correlation coefficients is calculated per set of OCR – liner geometry. The oil viscosity is $\mu$, $V$ is the sliding speed, $h_l$ is the OCR-liner clearance.
clearance and $\sigma_l$ is the standard deviation of the liner roughness. A second correlation is built to describe the variation of hydrodynamic shear stress with sliding speed and clearance (equation (3.6)).

$$\tau_h = \frac{\mu V}{h} (c_1 + c_2 e^{-c_3})$$ (3.6)

Figure 3.5 shows the variation of average hydrodynamic pressure in the oil film between the OCR and the liner with clearance and sliding speed.

In this thesis, interaction between the OCR and cylinder liner is simulated assuming fully-flooded lubrication. However during the up-stroke, oil supply to the OCR is limited to the oil film left on the liner during the down-stroke and might not be sufficient. The effect of oil starvation on the lubrication of piston rings has been studied by Chen [49].
3.1.4 Summary

The interaction between the OCR and the liner is a combination of asperity contact and hydrodynamic forces. The total normal force of the ring liner contact is calculated by combining both contributions. (equation (3.7))

\[
f_i = lw(PR \times P_c + P_h) = lw \left[ PR \times k_c \left( 4 - \frac{h_l}{\sigma_l} \right)^2 + p_o \mu V \frac{h_l}{\sigma_l} \right] \tag{3.7}
\]

\(f_i\) is the normal force acting on one land of the OCR, \(lw\) the land with and \(PR\) the plateau ratio. The plateau ratio is the ratio between the surface of the plateaus and surface of the grooves. In the normal force formulation it is assumed that asperity contact can only occur on the plateau. Since the OCR land is parallel to the liner contact and hydrodynamic pressure are considered constant over the land width.

Asperity friction and hydrodynamic shear are combined to obtain the total friction force (equation (3.8))

\[
f_f = lw(PR \times f_c P_c - \tau_h) = lw \left[ PR \times f_c k_c \left( 4 - \frac{h_l}{\sigma_l} \right)^2 - \mu V \frac{h}{h} (c_1 + c_2 e^{-c_3}) \right] \tag{3.8}
\]

The boundary friction coefficient is \(f_c\). The minus sign in front of the shear stress is to account for the fact that the direction of the shear stress is opposite to the direction of the ring velocity.

3.2 Interaction between the OCR and the piston groove

3.2.1 Geometry of ring-groove interface

The OCR lower or upper flank interacts with the piston groove between the inner diameter of the ring and the outer diameter of the piston (Figure 3.6).
The width of the ring-groove interaction is close to the radial width of the OCR (in the baseline geometry considered in this thesis it is approximately 2mm). Due to the twisting of the ring and the tilting of the piston, variation of the ring-groove clearance along the radial direction must be considered. Since the ring and the piston groove can deform in the direction of the cylinder axis, variation of the ring-groove clearance along the circumferential direction must also be considered.

As for the ring-liner contact, the ring-groove interaction has two components: a contact pressure generated by the interaction between the ring and the groove asperities and a hydrodynamic pressure generated by squeezing the oil film between the ring and the groove.

The friction interaction between the piston and the ring is considered negligible compared to the action of ring tension and its impact on the radial motion of the ring is neglected.

3.2.2 Asperity contact interaction

The simplified Greenwood-Tripp asperity contact model is again used to describe the contact interaction between the ring and the piston (equation (3.9)).
\[
P_c = \begin{cases} 
0 & \frac{h_g}{\sigma_g} \geq 4 \\
k_c \left(4 - \frac{h_g}{\sigma_g}\right)^z & \frac{h_g}{\sigma_g} < 4
\end{cases} \quad (3.9)
\]

\(h_g\) and \(\sigma_g\) are the ring-groove clearance and standard deviation of groove roughness. As mentioned earlier, the ring-groove clearance vary along the radial direction (equation (3.10)).

\[
h_g = h_{go} + \delta y \quad (3.10)
\]

\(h_{go}\) is the ring-groove clearance at the inner diameter of the ring, \(\delta\) is the relative angle between the ring and the groove and \(y\) is radial distance from the ring inner diameter.

### 3.2.3 Lubrication model for the ring-groove interface

A 2D lubrication model is built to simulate hydrodynamic forces and oil flow at the ring-groove interface. Since the ring-groove clearance (h~40μm) is much smaller than the length of the ring-groove interface (L~2mm) the lubrication approximations are valid and can be used to build this model (equation (3.11)).

\[
\left(\frac{h}{L}\right)^2 \ll 1 \quad Re_L\left(\frac{h}{L}\right)^2 \ll 1 \quad (3.11)
\]

In this model the ring-groove interface can be partially filled with oil (for instance when the OCR is moving away from the oil film). The oil volume ratio \(\phi\) is defined to track the presence of oil in the ring-groove interface (equation (3.12)).

\[
\phi = \frac{V_{oil}}{V_{oil} + V_{gas}} \quad (3.12)
\]

In the expression of the oil volume ratio \(V_{oil}\) is the oil volume in a given cell of the lubrication grid and \(V_{gas}\) is the gas volume. The ring-groove lubrication equations are derived from differential form of the mass conservation equation (equation (3.13)).
\[ \nabla \cdot \vec{m} = \frac{\partial}{\partial t}(\rho h) \]  

(3.13)

The density of the oil-gas mixture is \( \rho \), \( h \) is the ring-groove clearance and \( \vec{m} \) is the mass flow rate of the oil-gas mixture (in vector form). The mass flow rate of the mixture is the product of density of the mixture and volumetric flow rate \( q \) (equation (3.14)).

\[ \vec{m} = \rho q \]  

(3.14)

The density of the mixture can be calculated from the oil volume ratio and the oil and gas densities (equation (3.15))

\[ \rho = \phi \rho_{oil} + (1 - \phi) \rho_{gas} \approx \phi \rho_{oil} \]  

(3.15)

Since the oil density is three orders of magnitude larger than the density of gas the density of the mixture can be approximated by the product of oil volume ratio and oil density. The radial motion between the ring and the groove is neglected, therefore the flow at the ring-groove interface can be model as a viscous pressure flow (Poiseuille flow) (equation (3.16)).

\[ q = \frac{h^3}{12\mu} \nabla p \]  

(3.16)

The oil viscosity is \( \mu \) and \( \nabla p \) is the pressure gradient. The flow equation is inserted in the mass conservation equation to obtain a simplified form of the Reynolds equation (equation (3.17)).

\[ \nabla \cdot (\phi \rho_{oil} \frac{h^3}{12\mu} \nabla p) = \frac{\partial}{\partial t}(\phi \rho_{oil} h) \]  

(3.17)

Since the density of the oil can be assumed constant in the flow domain and in time, the equation can be further simplified (equation (3.18))

\[ \nabla \cdot (\phi \frac{h^3}{12\mu} \nabla p) = \frac{\partial}{\partial t}(\phi h) \]  

(3.18)
The simplified Reynolds equation for the oil-gas mixture is now expressed in Cartesian coordinates (equation (3.19))

\[
\frac{\partial}{\partial x} \left( \phi \frac{h^3}{12\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \phi \frac{h^3}{12\mu} \frac{\partial p}{\partial y} \right) = \frac{\partial}{\partial t} (\phi h) \tag{3.19}
\]

Since oil supply conditions and the shape of the ring-groove interface can vary, closed form solutions to equation (3.19) cannot be found and a numerical solution is sought. The ring-groove contact domains is discretized in trapezoidal cells and a finite volume solver of the Reynolds equation is developed (Figure 3.7).

Figure 3.7 - Lubrication grid of ring-groove interface (radial direction has been magnified)
Since the difference between inner and outer radius is small (2mm over an average radius of 40mm), the Cartesian form of the mass conservation equation can be used and the discretization error of the trapezoidal grid is small. For each cell the finite volume form of the mass conservation equation is applied (Figure 3.8).

\[ \Delta x_N \]
\[ \Delta y \]
\[ E \]
\[ W \]
\[ S \]
\[ h, p, \Phi \]
\[ h_N, p_N, \Phi_N \]

\[ q_N \Delta x_N + q_w \Delta y + q_s \Delta x_s + q_E \Delta y = \frac{[(\phi h)_t + \Delta t] - (\phi h)_t}{\Delta t} A_{trpz} \] \hspace{1cm} (3.20)

\( \Delta t \) is the length of a calculation time step, \( A_{trpz} \) is the area of a trapezoidal cell. \( q_N \) is the flow from the cell to the northern neighbor, \( \Delta x \) and \( \Delta y \) the length of the cell edges along the x and y direction. The trapezoid area is calculated from the length of its sides (equation (3.21)).

\[ A_{trpz} = \frac{\Delta x_N + \Delta x_S}{2} \sqrt{\Delta y^2 - \left(\frac{\Delta x_s - \Delta x_N}{2}\right)^2} \] \hspace{1cm} (3.21)

The volumetric flow rate to the neighboring cells is expressed as a function of clearance, pressure difference and oil volume ratio (equations (3.22) to (3.25)).
\[
q_N = \frac{\phi_N h^3}{12\mu} \frac{p - p_N}{\Delta y}
\]
\[
q_W = \frac{\phi_W h^3}{12\mu} \frac{p - p_W}{\Delta x}
\]
\[
q_S = \frac{\phi_S h^3}{12\mu} \frac{p - p_S}{\Delta y}
\]
\[
q_E = \frac{\phi_E h^3}{12\mu} \frac{p - p_E}{\Delta x}
\]

\(\Delta x\) is the average cell width along the x direction (equation (3.26)).

\[
\Delta x = \frac{\Delta x_N + \Delta x_S}{2}
\]

Oil volume ratios are averaged between the cell and its neighbors to consider both flow from the cell or flow to the cell (equation (3.27)).

\[
\overline{\phi}_N = \frac{\phi + \phi_N}{2}, \quad \overline{\phi}_W = \frac{\phi + \phi_W}{2}, \quad \overline{\phi}_S = \frac{\phi + \phi_S}{2}, \quad \overline{\phi}_E = \frac{\phi + \phi_E}{2}
\]

Cavitation is considered in the 2D ring-groove lubrication model. Oil cells can be in two states full (oil fills the entire volume) or partially filled (volume filled by a mixture of gas and oil). Cells can go from a partially filled state to a full state by transport of oil due to pressure gradient or squeezing. Likewise cells can go from full to partially filled in the event of cavitation. Cavitation can occur due to separation of ring and groove surface (reversal of axial force).

If a cell is full then the oil volume ration is known, \(\Phi = 1\), and the mass conservation equation is solved to obtain the pressure in the cell. If the cell is partially filled then the pressure in the cell is equal to the cavitation pressure, \(p = p_c\), and the mass conservation is solved to obtain the oil filling ratio in the cell. Thus oil transport and pressure distribution in the oil film at the ring-groove interface can be solved for.
In the lubrication solver the state of a cell (full or partial film) is detected to know whether to solve for $p$ or for $\Phi$. The switch variable $S$ is used for this purpose (equations (3.28) and (3.29)).

$$S = \phi \frac{p}{p_c} \quad (3.28)$$

$$S \geq 1 : \text{full film cell}$$
$$S < 1 : \text{partial film cell} \quad (3.29)$$

The switch variable is a way to handle the numerical discontinuity at the transition between full and partial film. When the pressure of a full film cell drops below the cavitation pressure then the solver will detect cavitation and allow the filling ratio to decrease below 1. In the simulations presented in this thesis the cavitation pressure was set to be the atmospheric pressure.

Using the finite volume form of the mass conservation equation, distribution of oil in the calculation domain and pressure at the current time step can be calculated from the knowledge of clearance at the current time step and clearance and oil film distribution at the previous time step. Boundary conditions (in pressure and oil volume ratio) at both inlet and outlet of the ring groove interface must be known. Local oil supply conditions can be considered in this model since the boundary condition is also discretized by the lubrication grid. Though the mass conservation equation is linear in pressure and oil volume ratio, the lubrication problem is non-linear due to the transition between full and partial film cells. Pressure and oil volume ratio are solved implicitly using the Newton-Raphson technique.

In the future the ring groove lubrication model could be extended to take into account the effect of the boundary gas pressure and gas flow. Due to the presence of the draining holes, the pressure in the oil control ring groove can be assumed atmospheric. However during the expansion stroke the 3rd land pressure can rise above atmospheric. In addition the purpose of this thesis is to build a numerical framework for a complete ring pack oil transport model. Hence when the oil transfer model built for the oil control ring will be adapted for the top two rings, the effect of gas pressure
in piston groove or lands will become of importance. The multiphase lubrication scheme developed by Li can be used to address this problem [50].

3.3 Piston kinematics

3.3.1 Piston primary motion

Piston position, speed and acceleration are needed to simulate the ring behavior in an engine cycle. The piston slides up and down in the cylinder. Its reciprocating motion is translated in rotation of the crank through the connecting rod (Figure 3.9).

In figure 3.9, $z_p$ is the axial position of the piston, $CA$ the crank angle, $R_C$ the crank radius and $L_{CR}$ the connecting rod length. The engine stroke is equal to two times the crank radius (equation (3.30)).
Using the geometry of the crank rod mechanism, a relation between the piston position and the crank angle is obtained (equation (3.31)).

\[ z_p = R_c \cos(CA) + \sqrt{L_{CR}^2 - R_c^2 \sin^2(CA)} \] (3.31)

The speed and acceleration of the piston in steady state operation can now be calculated. First the angular velocity of the crank (constant) is defined (equation (3.32)).

\[ \omega = \frac{dCA}{dt} \] (3.32)

It can be calculated from the engine speed expressed in rotation of the crank per minute (equation (3.33)).

\[ \omega = \frac{2\pi \text{RPM}}{60} \] (3.33)

The expression of the piston velocity is obtained (equation (3.34)).

\[ V_p = \omega \left\{-R_c \sin(CA) - \frac{R_c^2 \sin(CA) \cos(CA)}{\sqrt{L_{CR}^2 - R_c^2 \sin^2(CA)}} \right\} \] (3.34)

The piston acceleration is calculated by taking the time derivative of the piston velocity (equation (3.35)).

\[ A_p = \omega^2 \left\{-R_c \cos(CA) - \frac{R_c^2 \left[\cos^2(CA) - \sin^2(CA)\right]}{\sqrt{L_{CR}^2 - R_c^2 \sin^2(CA)}} \right. \]
\[ \left. - \frac{R_c^4 \sin^2(CA) \cos^2(CA)}{(\sqrt{L_{CR}^2 - R_c^2 \sin^2(CA)})^3} \right\} \] (3.35)
For given architecture (stroke and connecting rod length) the position, speed and acceleration of the piston over one engine revolution are calculated and shown on Figure 3.10. Piston position and acceleration have been divided by their maximum value over the engine cycle.

*Figure 3.10 - Piston position, speed and acceleration over one engine revolution*

It can be seen that the variation of piston speed in the cycle is not sinusoidal which leads to a asymmetric piston acceleration. Due to the variation of piston speed, the ring experiences the three regimes of lubrication (boundary, mixed and hydrodynamic) over the course of a stroke. When the engine speed is modified, the magnitude of piston speed and acceleration is modified without changing the shape of their distribution in the cycle. The maximum piston speed and acceleration over a crank rotation are calculated for engine speeds ranging from 0 to 4000 RPM (Figure 3.11).
As expected piston speed varies linearly with engine speed whereas piston acceleration has a quadratic dependence with engine speed. The fast reciprocating motion of the piston generates large accelerations at high RPM (on the order of $1000g$) which impact the axial dynamics of the ring.

3.3.2 Piston secondary motion

The piston is free to rotate around the pin connecting the piston to the connecting rod. Radial clearance between the piston skirt and the cylinder liner allow lateral movement (in the radial direction) of the piston. Tilt and lateral displacement constitute the piston secondary motion. Piston tilt cannot be neglected when modeling piston rings as tilted piston grooves force the rings to tilt and twist. Tilt and lateral displacement of the piston are shown on Figure 3.12.
The piston tilt is $\beta_p$ and $y_p$ is the lateral displacement of the piston. A positive piston tilt corresponds to an upward movement of the thrust side and a downward movement of the anti-thrust side. Positive lateral motion means that the piston is moving towards the thrust side in the convention adopted in this thesis.

The piston moves laterally due to the reaction of the tilted connecting rod to both the action of inertia and gas pressure on the piston. Axial motion of the center of the skirt contact and variation of the skirt friction cause the piston to tilt. The piston secondary motion has been calculated by Bai [51], [52]. It is an output of his piston skirt lubrication model which considers the primary forces acting on the piston. Figure 3.13 is an example of piston lateral motion calculated from Bai skirt lubrication model. It corresponds to a medium load and speed case for a passenger car engine (3500 RPM, 700 mbar).
The piston is seen to shift several times between the thrust and anti-thrust side in the engine cycle. The timing of these lateral displacement events correspond to changes in the connecting rod orientation and the magnitude of the vertical force resulting from the effect of inertia and gas pressure. The total magnitude of displacement between thrust side and anti-thrust side is consistent with piston skirt clearance which is close to 50 microns in this simulation.

The variation of piston dynamic tilting in the engine cycle shown next, is taken from the same simulation case (Figure 3.14).
For piston dynamic tilting, it is more difficult to distinguish a clear pattern. The piston tilting moment result from the offset between skirt contact and pin axis which varies due to lubrication condition. Changes in piston skirt friction also contribute to the variability of the piston tilt. The piston dynamic tilting obtained from these simulations will be used as an input in the OCR simulations presented in this thesis. The primary objective is to understand the impact of a given magnitude of piston tilt on the twisting and scraping behavior of the OCR.

3.4 Calculating forces acting on the OCR

Forces at the ring-groove and ring-liner interfaces can be calculated from the knowledge of the position of the ring relative to the piston groove and the cylinder liner. The position of the cross section of the deformed ring is given by the radial displacement $y_r$, the axial displacement $z_r$ and the rotation of the section $\alpha_r$ (Figure 3.15).
In the reference position, the ring is centered axially in the piston groove and its radial position is such that it is in contact with the non-deformed cylinder. The piston groove moves axially and is tilted due to piston dynamic tilting (Figure 3.16).

Figure 3.15 - Displacement of ring neutral axis

Figure 3.16 - Displacement of the groove of a tilted piston
Since the magnitude of piston tilt is small ($\beta_p \approx 0.1^\circ$), the rotation of the groove located at the polar angle $\theta$ can be approximated by the expression of equation (3.36).

$$\beta \approx \cos \theta \beta_p$$

Likewise a small displacement approximation of the axial displacement of the groove, $z_g$ is calculated (equation (3.37)).

$$z_g \approx (R \cos \theta - off) \beta_p$$

In equation (3.37), $off$ corresponds to the piston offset (positive when the pin axis is moved towards the thrust side) and $R$ is the ring radius. In addition to piston tilt, the thermal deformation of the piston and bore distortion are considered in this model (Figure 3.17).

![Figure 3.17 - Deformation of the cylinder and of the piston groove](image)

The radial displacement of the cylinder from its nominal position due to bore distortion is $y_b$. The axial displacements of the upper and lower flanks of the groove due to thermal distortion are $z_{gu}$.
and \( z_{gl} \). They are measured at the radial location of the ring cross section centroid \( C \). \( \beta_{gu} \) and \( \beta_{gt} \) are the groove tilt angles due to thermal distortion.

The clearances between the ring and the flanks of the piston groove and between the ring lands and the cylinder can now be calculated (Figure 3.18)

![Figure 3.18 - Ring-groove and ring-liner clearances](image)

The clearance between the OCR upper and the liner, \( h_{tu} \), is a function of bore distortion \( y_b \), ring radial displacement \( y_r \), rotation of the OCR section \( \alpha_r \), and the axial distance between the OCR upper land and the OCR centroid \( h_{fu} \) (equation (3.38)).

\[
h_{tu} = y_b - y_r + \alpha_r h_{fu} \tag{3.38}
\]

The clearance between the OCR lower land and the liner can be calculated similarly. In equation (3.39), \( h_{fl} \) refers to the axial distance between the OCR lower land and the OCR centroid.

\[
h_{tl} = y_b - y_r - \alpha_r h_{fl} \tag{3.39}
\]
The clearance between the ring and the groove upper flank at the radial location $y_{gu}$ is calculated by taking into account ring displacement, piston tilt and piston thermal deformation. In equation (3.40), $h_g$ is the axial clearance between the OCR and the OCR groove, $a_r$ is ring radial thickness and $y_c$ is the radial distance between the section centroid and the cylinder.

$$h_{gu}(y_{gu}) = \frac{h_g}{2} + z_g + z_{gu} - z_r + (y_{gu} + a_r - y_c)(\beta + \beta_{gu} - \alpha_r) \quad (3.40)$$

Similarly the clearance between the ring and the groove lower flank is calculated (equation (3.41)).

$$h_{gl}(y_{gl}) = \frac{h_g}{2} - z_r - z_g - z_{gl} + (y_{gl} + a_r - y_c)(\alpha_r - \beta + \beta_{gl}) \quad (3.41)$$

With the expressions of the ring-liner and ring-groove clearances, forces resulting from the interaction between the ring and the piston and the ring and the liner can be calculated. The forces acting on the cross section of the OCR are shown on Figure 3.19.

Figure 3.19 - Forces acting on the OCR

Ring tension is modeled as an external force. The spring behind the OCR metallic section is considered flexible enough to provide a radial force uniformly distributed along the circumference.
of the ring. The expression of the radial force per unit length \( f_t \) due to the ring tension \( F_t \) is given in equation (3.42).

\[
f_t = \frac{F_t}{R}
\]  

(3.42)

In the model, the ring natural shape is considered circular with a radius equal to the nominal bore radius. The effect of free shape is simulated by applying a distribution of radial force statically equivalent to the bending of the ring in the cylinder. For the OCR, the effect of free shape is secondary and most of the radial force is provided by the coiled spring. For the top two rings however, the radial force distribution is a reaction to the bending of the ring from its free shape to its circular shape in the cylinder. The inertia force is calculated from the piston acceleration and the mass of the OCR and spring assembly (equation (3.43)).

\[
f_i = -\frac{(m_r + m_s)A_p}{2\pi R}
\]  

(3.43)

The mass of the ring is \( m_r \) and \( m_s \) is the mass of the spring. The inertia force per unit length, \( f_i \) is uniformly distributed along the ring circumference. The external forces are summed to obtain the resulting radial force per unit length \( f_r \), the resulting axial force per unit length \( f_z \) and the resulting moment per unit length \( m_\theta \) (equations (3.44) to (3.46)).

\[
f_r = f_t - f_{iu} - f_{il}
\]  

(3.44)

\[
f_z = f_i + f_{gl} - f_{gu} + f_{fl} + f_{fu}
\]  

(3.45)

\[
m_\theta = f_{iu}h_{fu} - f_{il}h_{ft} - f_{gu}A_{gu} - f_{gi}A_{gi} + (f_{fu} + f_{fl})y_c + f_i(y_m - y_c)
\]  

(3.46)

In equation (3.46), \( y_m \) is the radial distance between the liner and the center of mass. As can be seen on Figure 3.19, the center of mass of the ring and spring assembly does not coincide with the centroid of the ring cross section.
3.5 OCR oil transport model solver

By combining the dual grid curved beam finite element model of the OCR with the ring-liner and ring-groove interaction models, comprehensive simulations of OCR behavior and oil transport around the OCR can be conducted (Figure 3.20).

To solve for oil transport around the OCR, the engine cycle is discretized. At each time step, inputs of the structural model such as piston position, speed, acceleration, and bore distortion are updated based on the position in the cycle and on the location of the ring on the cylinder. The ring finite element dynamics equation is solved iteratively to find the position and deformation of the ring at the given time step (globally convergent Newton-Raphson solver for system of non-linear equations [53]). The dynamics solver is implicit, as ring position and contact forces must be solved simultaneously. An additional iteration loop is required to solve the lubrication at the ring-groove
interface. For each guess of ring position and deformation made when solving the dynamics equation, lubrication at the ring-groove interface is solved with the corresponding distribution of ring groove clearance. Figure 3.21 gives an overview of the solver of the OCR oil transport model.

Figure 3.21 - Overview of OCR oil transport model
4 Oil Transport around the OCR

A comprehensive model of the piston ring taking into account the relevant physics (ring deformation and dynamics, interaction with cylinder and piston) has been developed in this thesis to simulate ring sealing performance. It is now used to simulate the behavior of the oil control ring (two piece design) in the engine cycle to understand how oil can flow past the OCR and the potential impacts on oil consumption. In the introduction three key mechanisms contributing to oil flow past the OCR were identified (Figure 4.1). In this chapter their relative contributions to oil transport to the upper ring pack are calculated.

![Figure 4.1 - Mechanisms contributing to oil flow past the OCR](image)

One baseline configuration is used for all the simulation results presented in this chapter. Unless specified otherwise, all simulation inputs are consistent with the engine data presented in this introduction to chapter 4. The baseline configuration is representative of a passenger car light duty...
gasoline engine. The nomenclature adopted for ring and piston groove geometry is presented on Figure 4.2 and in table 5.

![Diagram of OCR and OCR groove nomenclature](image)

**Figure 4.2 - OCR and OCR groove nomenclature**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_r$</td>
<td>Ring radial wall thickness</td>
</tr>
<tr>
<td>$w_r$</td>
<td>Ring axial width</td>
</tr>
<tr>
<td>$l_w$</td>
<td>OCR land width</td>
</tr>
<tr>
<td>$h_{fl}$</td>
<td>Axial location of contact between OCR land and liner</td>
</tr>
<tr>
<td>$C$</td>
<td>Centroid of OCR cross section</td>
</tr>
<tr>
<td>$M$</td>
<td>Center of mass of OCR and coiled spring assembly</td>
</tr>
<tr>
<td>$y_c$</td>
<td>Radial position of OCR centroid</td>
</tr>
<tr>
<td>$y_m$</td>
<td>Radial position of center of mass</td>
</tr>
<tr>
<td>$D_b$</td>
<td>Bore diameter</td>
</tr>
<tr>
<td>$D_s$</td>
<td>Diameter of piston skirt below OCR ring</td>
</tr>
<tr>
<td>$D_g$</td>
<td>Piston OCR groove diameter</td>
</tr>
</tbody>
</table>
The ring-groove clearance $h_g$ is calculated from the axial width of both ring and groove (equation (4.1)).

$$h_g = w_g - w_r$$  

The piston skirt chamfer clearance $h_s$ is calculated from the bore and skirt chamfer diameters (piston is considered centered in the cylinder) (equation (4.2))

$$h_s = \frac{D_b - D_s}{2}$$  

The parameters of the baseline geometry are listed in table 6.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$a_r$</th>
<th>$w_r$</th>
<th>$l_w$</th>
<th>$h_{fl}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>2 mm</td>
<td>2 mm</td>
<td>0.2 mm</td>
<td>0.65 mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$y_c$</th>
<th>$y_m$</th>
<th>$D_b$</th>
<th>$D_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.85 mm</td>
<td>1.2 mm</td>
<td>80 mm</td>
<td>73 mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$F_t$</th>
<th>$h_s$</th>
<th>$h_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>15 N</td>
<td>250 μm</td>
<td>40 μm</td>
</tr>
</tbody>
</table>
4.1 Contribution of bore distortion to oil flow past OCR

In this section the effect of bore distortion on the oil control performance of the OCR is simulated. Here the focus is to understand how the deformation of the cylinder affects the ability of the OCR to control the thickness of lubricant left on the liner.

4.1.1 Overview of Bore distortion

Bore distortion was briefly presented in the introduction to this thesis. During engine operation, cylinders of the engine block deform due to mechanical and thermal stresses. Mechanical stresses are a result of the fastening of the engine block whereas constraints and temperature gradients around the cylinder cause thermal-mechanical stresses. Temperature gradients are due to the flow of heat from the combustion chamber to the coolant. The distribution of thermal and mechanical stresses can vary significantly depending on the architecture of the engine block (location of bolts for block assembly, position of cooling chamber, ...) and power output. As a result bore distortion is engine specific and changes with running condition (speed and load). Though the magnitude of the cylinder distortion is small, it is sufficient to affect the contact between the ring and the cylinder liner (Figure 4.3)
In order to maintain contact with the distorted bore the ring must deform and adapt to its geometry. If the ring is too stiff and ring tension (the elastic force pushing the ring against the cylinder) is too small then contact with the liner can be lost locally. This negatively impacts the sealing performance of the ring since gas and lubricant can flow where the ring is no longer touching the cylinder. It is customary to describe the geometry of a distorted bore with a discrete Fourier series (Figure 4.4 and equation (4.3)). The Fourier series is used to interpolate the deformation of sections of the cylinder perpendicular to the cylinder axis.
The radial displacement of the distorted cylinder from the nominal cylinder (without out of roundness) is $y_b$, $k$ is the order of distortion, $A_k$ and $\Phi_k$ are the magnitude and phase of distortion of order $k$. A cylinder distortion of order 0 corresponds to a change of radius that is uniform along the cylinder circumference. The first order distortion gives eccentricity to the cylinder (displacement of its axis). The second order quantifies the ovality of the distorted cylinder. A third order distortion has three lobes of deformation equally spaced around the circumference, the fourth order has four and so on. The Fourier series provides a convenient way of separating various contributions to the shape of the distorted bore.

Bore distortion information can be obtained from thermal-mechanical finite element simulations of the engine block. Boundary conditions of these simulations take into account the fastening of the engine block as well as the heat input from combustion and cooling of the block. An example of bore distortion obtained from simulation is shown in Figure 4.5. It corresponds to the deformation of the cylinder of a light-duty diesel engine with an open-deck design under high load high speed conditions (most severe conditions for bore distortion).
Figure 4.5 - Example of a light-duty diesel engine bore distortion

Though there are some high order features, the cylinder has mostly expanded and has become oval under engine operation. Table 7 summarizes the typical range of bore distortion seen in passenger car engines.

<table>
<thead>
<tr>
<th>Distortion order</th>
<th>0</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude (µm)</td>
<td>0 - 200</td>
<td>0 - 50</td>
<td>0 - 15</td>
<td>0 - 10</td>
<td>0 - 1</td>
</tr>
</tbody>
</table>

*Table 7 - Typical range of bore distortion for passenger car engines*

Most of the cylinder deformation occurs at orders of distortion between 0 and 4. Distortion for higher orders gets closer to the scale of the liner roughness and the limit of precision of the finite element simulation is reached. 0th order distortion corresponds to the expansion of the cylinder when the engine block is heated under operation. For engines with 4 in-line cylinders the outside
of the block is cooler than the center resulting in 2\textsuperscript{nd} and 3\textsuperscript{rd} order thermal distortion consistent with the temperature distribution. 4\textsuperscript{th} order distortion is often a result of mechanical deformation when the 4 bolts surrounding the cylinder are tightened to assemble the engine block. In addition machining of the cylinder can cause high order distortion (3\textsuperscript{rd} order and up). The machining error can be due to the variation of stiffness of the engine block along the circumference of the cylinder and to mechanical deformation when the block is fastened for the machining operation.

### 4.1.2 Sample results of ring equilibrium in distorted bore

A slice of the distorted cylinder of the light duty diesel engine presented in the previous section is selected to calculate the impact of distortion on the interaction between the ring and the cylinder. The slice is selected to be representative of severe bore distortion (Table 8).

<table>
<thead>
<tr>
<th>Distortion order</th>
<th>0</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude ((\mu\text{m}))</td>
<td>160.0</td>
<td>14.9</td>
<td>15.9</td>
<td>8.9</td>
</tr>
<tr>
<td>Phase ((^\circ))</td>
<td>-88.3</td>
<td>58.4</td>
<td>-43</td>
<td></td>
</tr>
</tbody>
</table>

*Table 8 - Magnitude and phase of selected bore distortion*

The curved beam model of piston ring is used to calculate the equilibrium of the OCR in the distorted cylinder. The baseline OCR with 15N tension and 0.2mm land width is used for this calculation. The ring sliding velocity is set at 5m/s (quasi-static calculation) and a liner roughness of 0.2\(\mu\text{m}\) is chosen. The effects of bore distortion on the distribution of ring-liner contact force and clearance are discussed. First a 2D representation of the ring equilibrium in the distorted bore in the plane perpendicular to the cylinder axis is given (Figure 4.6)
The outer blue line is a magnified representation of bore distortion (radial displacement has been magnified 200 times). The black line inside represents the deformed OCR neutral axis at equilibrium. The black circles mark the position of the nodes of the finite element model simulating ring deformation. The space between the bore distortion and ring lines has been magnified to visualize the variation of ring-liner clearance around the circumference. For this case the OCR is flexible enough to maintain contact with the cylinder and the ring-liner clearance is uniform except in the ring gap region. The nodes corresponding to the ring tips are colored in red to show the position of the gap (in this figure they are located at 180°). The inner-most red curve is a plot of the contact force distribution. Contact force increases towards the center of the figure and the red dotted lines show the scale of the contact force (from 0 to 1 N/mm). The ring is found to conform to the distorted bore, the contact force distribution is slighted affected but contact is maintained all around the circumference of the ring.
The distribution of radial position, clearance and contact force are plotted on separate graphs to be more conveniently analyzed (Figure 4.7, Figure 4.8, Figure 4.9).

![Figure 4.7 - Ring and bore radial position](image)

On Figure 4.7, the blue line shows the radial position of the bore and the red line with the circles the radial position of the ring. The OCR is flexible enough and has enough tension to follow the geometry of the distorted cylinder. The distribution of contact force along the ring circumference is shown on Figure 4.8. The contact force between the OCR and the liner includes the contribution of both contact between asperities and hydrodynamic pressure in the oil film.
On Figure 4.8 the solid line shows the contact force obtained from the ring model, the broken line is the contact force given by the approximate solution to conformability. The analytical expression of the contact force \( f_c \) for a ring at equilibrium with the cylinder was derived by assuming that ring and bore curvature are identical for a conforming ring (equation (4.4)). For this calculation the ring is also assumed gapless.

\[
f_c = \frac{F_t}{R} - \frac{E I_z}{R^4} \sum_{k=2}^{n} \{(k^2 - 1)^2 A_k \sin(k\theta_k + \phi_k)\}
\]

(4.4)

In the present case, the ring conforms to the liner and equation (4.4) provides a good approximation of the contact force as can be seen on Figure 4.8. The only significant deviation occurs at the ring gap (circumferential position of 0° and 360°). The OCR must bend to follow the distorted bore geometry and the contact force is modified as a result of this bending. Sharp deformation (high order deformation) translate into large variation of ring curvature and bending which explains why the contact force depends strongly on the order of distortion (\( k^4 \) dependence as seen in equation (4.4)). The modification of ring-liner contact force impact the distribution of clearance between the ring and the liner (Figure 4.9).
Ring-liner clearance is increased in regions where the contact force is decreased by the bore distortion. The opposite is also true (increased contact force results in decreased ring-liner clearance). For this particular ring bore configuration, bore distortion does not lead to significant increase of the ring-liner clearance since the ring conforms well. The variation around the average clearance falls below the cylinder roughness (0.2 μm). The ring tip located at 0° is close from losing contact with the cylinder, though the affected region is narrow. Behavior of the ring gap region will be discussed in more details in the next section. The average clearance between the ring and the liner is a good indicator of the oil control performance of the OCR. It is directly related to the thickness of the oil film left on the liner after the passage of the OCR (oil film thickness is half of the ring-liner clearance, Couette flow).

In the baseline configuration (15N OCR tension), the OCR is able to conform to the severe bore distortion used in this sample calculation. The impact of reducing OCR tension on the OCR sealing performance is assessed. The objective of this study to understand the extent to which OCR friction can be reduced without seeing an increase in oil consumption due to bore distortion.
The contact pressure (and not contact force as shown previously) is plotted for reduced OCR tension on Figure 4.10, bore distortion is kept constant (table 8). For these calculations OCR land width is reduced proportionally to the tension to keep the average contact pressure constant. This is what is done in practice to allow a reduction of OCR tension without changing the nominal oil film thickness. The ring-liner contact pressure is increasingly affected when ring tension is reduced so much that a loss of contact is near for certain locations around the ring. When ring tension is reduced so is the amount of elastic force available to deform the ring and make it conform to the liner. The variation of the contact force (and not pressure) due to bore distortion is the same for the different ring tensions. However the average contact force decreases with reduced tension until contact between the ring and the liner is lost (Figure 4.11).
The increased variation of contact pressure with reduced tension affects the OCR ability to control the oil film thickness left on the liner (Figure 4.12)

Figure 4.11 – Effect of ring tension on ring-liner contact force

Figure 4.12 - Effect of tension reduction on ring-liner clearance
With decreased tension, the oil film left on the liner becomes less uniform. The average oil film thickness is not affected at first as can be seen on Figure 4.12 when tension is divided by two from 15N to 7.5N. When the tension is further decreased, the ring is close to losing contact with the liner at certain locations. Locally the oil film thickness is significantly increased (for instance at $\theta = 150^\circ$ on Figure 4.12).

It can been inferred from the calculation with reduced OCR tension that for the present bore distortion (which belongs to the upper range of bore distortion for passenger car engines) there is potential to reduce OCR tension (hence friction) without negatively impacting oil control.

To further study the effect of bore distortion on sealing performance of the OCR, the oil film thickness left on the liner by the OCR during a down stroke is calculated. It is assumed that oil supply below the OCR during the down stroke is sufficient to have fully flooded lubrication between the OCR lands and the liner. This assumption is reasonable since the oil film left by the piston skirt-cylinder contact usually exceed the oil supply required by the OCR. The oil film thickness distribution on the liner is calculated for an engine speed of 2000 RPM (Figure 4.13). Engine speed affects the average oil film thickness but not significantly the structural behavior of the ring in the distorted bore. The oil film thickness displayed corresponds to half of the ring – liner clearance (Couette flow between the ring and the liner).
The colored region of the distorted bore corresponds to the portion of the cylinder swept by the OCR between its top dead center and bottom dead center. The thick dotted line marks the position of the thrust-side of the cylinder, the thin dotted line locates the anti-thrust side. The liner is colored according to the oil film thickness. For the baseline configuration with an OCR tension of 15N, bore distortion hardly affects the oil control performance of the OCR. The oil film thickness only increases in locations where bore distortion is sharp and stays within reasonable level (3 times the surface roughness). Even when OCR tension is reduced by a factor of 5 (down to 3N), the ring is able to maintain control of the oil film thickness in most locations. However thick oil film regions appear and seem to correspond to a portion of the bore where the 4th order distortion is high. This problem could be addressed by reviewing the design of the engine block and limiting the effect of fastening on bore distortion.
4.1.3 Behavior of the ring gap region

In the bore distortion cases calculated so far, the region near the gap of the ring was found to behave singularly. To gain a better understanding of the region near the ring gap, the OCR is rotated in a distorted bore and equilibrium of the ring for each position is calculated. Piston rings have been observed to rotate during engine operation, meaning that any ring gap position is likely to occur. A single 4th order distortion of 5μm magnitude is chosen. The equilibrium of the ring (baseline configuration) with the distorted bore for five successive gap positions is shown on Figure 4.16.
Figure 4.14 - Rotation of a ring in a distorted bore
The boxes at the bottom of each graph indicate the pressure at the ring tips. The rest of the graph is identical to Figure 4.6. When the ring is rotated in the distorted bore, the region near the ring gap is the only one to experience a change in contact pressure and clearance. The region affected by the ring gap is contained within a 30° arc. Ring tips experience large contact pressure at the location of maximum bore radius and a loss of contact at the location of minimum bore radius. Experimental observations of wear at the ring tips are consistent with the large contact pressures observed in simulations in the ring gap region.

The behavior of the ring gap region can be explained by the fact that the end sections of the ring are free of stresses, since they are open sections due to the ring gap cut. With the absence of bending stresses, the curvature and shape of the ring close to the gap cannot be modified significantly. Therefore in the ring gap region, the ring cannot adapt to geometry of the distorted bore and severe contact or loss of contact occur. The portion of the ring close to the gap can be seen as a rigid curved beam being pushed against the liner by the rest of the ring.

It can be argued that since the region affected by the ring gap behavior is sufficiently narrow it is of secondary importance for sealing performance of OCR. A ring tip not in contact would not affect significantly the average oil film thickness the OCR leaves on the liner. Since the behavior of the ring tip is based on the mismatch of geometry between distorted bore and ring, lower orders of distortion with higher magnitudes of deformation lead to higher clearances and contact pressures. The affected ring gap region also becomes wider.

Though the ring gap region does not affect the oil control of the OCR in an average sense, it can enable larger oil supply locally that can lead to scraping of the top two rings. The piston ring model developed in this thesis can be used to evaluate the behavior of the ring gap region and assess its impact on oil supply to the upper ring pack.

4.1.4 Sensitivity Analysis

It has been showed that ring tension reduction eventually deteriorates the oil control performance of the OCR, allowing thicker oil films to be left on the liner when bore distortion is severe. In this section conformability limits of ring-bore configurations are sought. To evaluate the effect of
bore distortion on the oil control function of the OCR, the average oil film thickness around the ring circumference is calculated. It is used as an indicator whether bore distortion is allowing thick oil film through the OCR. First the average oil film thickness is calculated for a single order bore distortion for a range of ring tension from 1N to 25N (Figure 4.15). The OCR land width is adjusted when ring tension is modified to keep the average contact pressure constant.

![Graph](attachment:image.png)

**Figure 4.15 - Variation of average and standard deviation of oil film thickness with tension**

The solid lines in Figure 4.15 correspond to average oil film thickness around the circumference of the ring and the broken lines represent the standard deviation of the oil film thickness. Two sets of simulation were run: the red curves correspond to calculations with a 3rd order bore distortion of magnitude 35μm and the blue correspond to calculation with a 4th order distortion of magnitude 10 μm. For high OCR tension the oil film is uniform and equal for both sets of calculation. Oil film thickness remains relatively constant for tension as small as 5N, though the standard deviation starts to increase meaning that the oil film becomes less uniform. A large increase in oil film thickness is seen when tension is further decreased signaling that the conformability limit of the ring has been reached.
Equation (4.4) can be used to define a conformability criterion and a dimensionless conformability parameter. A criterion for conformability can be that contact must be maintained all around the ring circumference (equation (4.5))

\[ f_c > 0 \] (4.5)

Substituting for the expression of the contact force in equation (4.5), and simplifying the bore distortion to a simple order yields:

\[ \frac{F_t}{R} > \frac{E l_z}{R^4} (n^2 - 1)^2 A_n \] (4.6)

A conformability criteria connecting ring tension, ring stiffness, distortion magnitude \( A_n \) and order \( n \) is thus obtained (equation (4.7)).

\[ CF > 1 \text{ with } CF = \frac{F_t R^3}{(n^2 - 1)^2 A_n E l_z} \] (4.7)

\( CF \) is the dimensionless conformability factor. Piston rings are conforming (no loss of contact pressure around the circumference) for conformability factors larger than one. In the calculation of Figure 4.15, the magnitudes of the 3rd and 4th order distortion were chosen to have the same effect on ring conformability (equation (4.8)).

\[ (3^2 - 1)^2 A_3 = (4^2 - 1)^2 A_4 \] (4.8)

To verify the validity of the conformability criteria, the average oil film thickness as a function of the conformability factor is shown on Figure 4.16. The average oil film thickness for varying ring stiffness \( l_z \) and the same 4th order distortion than before is calculated and added to Figure 4.16.
The average oil film thickness curve of the 3\textsuperscript{rd} and 4\textsuperscript{th} order distortions remain close except for low conformability factors (lower than 1.5). At low conformability factors contact between the ring and the liner is lost at certain locations. Consequently the contact force expression of equation (4.4) no longer holds meaning that the dimensionless parameter is no longer representative. It is a region of non-conformability and therefore does not need to be modeled as it must be avoided. Stiffness and tension variation result in two curves that cannot be distinguished on Figure 4.16. This shows that the effect of stiffness and ring tension is modelled correctly in the conformability factor.

At $CF = 1$, the increase of average oil film thickness on the liner for both 3\textsuperscript{rd} and 4\textsuperscript{th} order distortion is low enough to consider that it has not affected the oil control performance of the OCR. Therefore the conformability criteria (equation (4.6)) can be used to determine the tension reduction limit for a given ring – distorted bore configuration.
The baseline configuration (15N ring tension) with a 4th order bore distortion of 10μm has a conformability factor of approximately 4. Thus for this configuration the tension can be reduced by a factor 3 without affecting significantly the oil control performance of the OCR.

4.1.5 Summary

The interaction between the OCR and distorted bores has been studied using the curved beam piston ring model developed in this thesis. It has been found that in current engine configurations, oil control rings were sufficiently flexible and had enough tension to conform to even severe bore distortion. In most distortion cases, there is potential to reduce OCR tension without affecting the ability of the OCR to control the amount of oil left on the liner for the top two rings.

The ring model also helped investigate the singular behavior of the ring gap region. Being absent of bending stresses due to the proximity of free ends, the ring gap region is unable to deform significantly to adapt to the local bore distortion. As a result high contact pressure or loss of contact is seen at the ring tips which is problematic for both wear and oil leakage. The region affected by the ring gap remain narrow and does not affect the oil control function of the OCR in an average sense but can increase oil locally the oil supply to the top two rings.

Finally it has been shown that the approximate solution to ring conformability can be used to design a conformability criteria. By forcing contact pressure to remain positive around the circumference of the ring, a relation between ring properties (tension, stiffness) and bore distortion magnitude is obtained. The conformability criteria can be used to estimate the tension reduction limit for a given ring – bore distortion configuration.
4.2 Contribution of OCR twisting and scraping to oil flow past OCR

The impact of OCR scraping on oil transport and oil consumption was introduced in chapter 1. Twist of the OCR can lead to accumulation of oil between the ring lands and release on the liner. Oil released on the liner can later be scraped and supply the piston 3rd land, eventually contributing to oil consumption (Figure 4.17)

![Diagram showing scraping and release sequence for a twisted OCR]

1: Accumulation between OCR lands  
2: Release on the liner  
3: Up-scraping

*Figure 4.17 - Scraping and release sequence for a twisted OCR*

In addition to scraping oil on the liner, twist of the OCR affects the oil control function of the upper land. 2D LIF observations have shown that in steady state engine operation, oil accumulates between the lands of the OCR. This accumulated oil can bridge with the liner and if the OCR section is rotated in a way that increases the upper land clearance, thick oil films can be left on the cylinder.

The piston ring model presented in chapter 2 is used to simulate the twisting and scraping of the OCR. Its contribution to oil flow past the OCR is quantified and compared to the other oil transport mechanisms: conformability and oil flow through the groove.
4.2.1 Physics of OCR twisting and scraping

A description of the physics of OCR scraping is given prior to presenting the simulation results. The neutral plane of the oil control ring follows the piston dynamic tilt. When the inertia force is strong enough, the orientation of the ring cross section follows the local orientation of the groove. Figure 4.18 shows a 3D view of an OCR in the groove of a tilted piston. The colored plane helps visualize the orientation of the OCR section. It is a plane that goes through the section centroid and is perpendicular to the cylinder axis when the OCR is not twisted or tilted. As a result of piston tilt, OCR sections are tilted with respect to the liner which can lead one of the lands to lose contact. Ring tension provides resistance against the rotation of the OCR sections and the orientation of the sections can be seen to be different from the tilt plane.

Figure 4.18 - 3D view of ring tilt and ring twist in reaction to piston dynamic tilting
The magnitude of rotation of the OCR sections in the reference frame of the cylinder is maximum at the thrust and anti-thrust side of the piston. Since piston tilt does not rotate the OCR groove on the pin side it also does not affect the OCR.

The contributions to OCR twisting in the case of a tilted piston are described, starting with the contact on the piston groove (Figure 4.19). The force balance situations on the thrust and anti-thrust side of the piston where groove tilt is maximum are pictured.

Let’s assume the piston is approaching the bottom dead center and it is tilted such that rotation around the pin has risen the thrust side and lowered the anti-thrust side. Since the piston is decelerating, inertia is forcing the OCR against the lower flank of the tilted groove. Because of the relative angle between the piston groove and the ring section, the center of the ring-groove contact is moved towards the ring outer diameter on the thrust side and the inner diameter on the anti-thrust side. There is therefore an offset between the groove contact and the OCR section centroid, resulting in a twisting moment. The twisting moment $M_t$ can be expressed as a function of the offset $\Delta$ and the groove contact force $F_g$ (equation (4.9)).
Since the centroid is closer to the outer diameter of the piston groove $\Delta_{ID} > \Delta_{OD}$ and the twisting moment is larger when contact occurs close to the inner diameter of the OCR. The magnitude of the groove contact force is related to the magnitude of inertia. Hence maximum twisting moments are expected to occur at high engine speed close to the dead centers.

Ring-liner friction, ring tension, inertia also contribute to twisting of the OCR (Figure 4.20).

Since the center of mass of the spring and ring assembly does not coincide with the centroid of the OCR, inertia contributes to twisting of the ring (equation (4.10))

$$M_t = F_i(y_m - y_c) \quad (4.10)$$

In the expression of the contribution of inertia to the twisting moment, $y_m$ is the radial position of the center of mass and $y_c$ the radial position of the centroid. Similarly friction between the ring and the liner contributes to twisting of the OCR (equation (4.11)).

$$M_t = F_f y_c \quad (4.11)$$

The contact force on the liner $F_f$ balances the ring tension $F_t$. When the OCR is not twisted and its lands are aligned with the liner, the contact forces on both lands are equal and no twisting.
moment is generated. When the ring section is rotated, the contact force of the land moving away from the liner is reduced, while the contact force on the other land is increased. This generates a twisting moment bringing both lands back in contact with the liner. Therefore for the two piece oil control ring design, ring tension helps resist rotation of the OCR section with respect to the liner. Contribution of the liner contact to twisting can be expressed as a function of $\Delta F_i$, the difference of contact force between the lands of the OCR, and $H$ the axial distance between the land contact and the section centroid (equation (4.12)).

$$M_t = \Delta F_i H$$  \hfill (4.12)

$\Delta F_i$ is a function of the rotation of the OCR with respect to the liner. When contact at one of the land is lost then the maximum twisting resistance from ring tension is obtained (equation (4.13))

$$M_t = F_i H$$  \hfill (4.13)

In addition to piston tilt, thermal and mechanical distortion of the piston can cause the OCR to twist. Figure 4.21 is a section view of a piston finite element model simulating thermal distortion of the piston. Deformation of the piston has been magnified to show the rotation and displacement of piston groove.

![Axial deformation and tilt of piston OCR groove](image)

*Figure 4.21 - Thermal deformation of the piston*
Piston thermal distortion can cause the groove to rotate or move vertically. The magnitude of the groove thermal tilt is on the order of $0.1^\circ$ for light duty engines pistons, and is thus on the same order as piston tilt. Axial thermal deformation can reach 20 $\mu$m. Figure 4.22 shows twisting of an OCR forced against a piston groove with uniform thermal down tilt. In the tilted piston situation, contact with the groove is forcing the OCR sections to remain parallel to the tilted plane of curvature of the ring and ring tension is twisting the ring outside of its plane of curvature. In the uniformly tilted groove situation, groove contact is twisting the OCR sections outside of the plane of curvature of the ring and ring tension helps keeping the OCR sections aligned with the plane of curvature.

*Figure 4.22 - Twisting of OCR for a piston with thermal down tilt*

To prevent scraping of oil on the liner, the OCR lands must be kept aligned with the liner. Stiffness of the ring has an opposite effect on maintaining both lands of the OCR on the liner for piston dynamic tilting and groove thermal distortion. For piston dynamic tilting it is preferable to have a flexible ring to allow twisting whereas a stiff ring would perform better in the case of groove thermal distortion.
The axial deformation of the piston groove can be an additional source of OCR twisting. Because of its curvature, sections of the piston ring rotate when the ring is deformed outside of its plane of curvature. Indeed twisting and bending outside of the plane of curvature of the ring are coupled. Figure 4.23 shows the rotation of the sections of an OCR when it is deformed axially due to the thermal distortion of the groove.

In the engine the OCR is subject to a combination of piston distortion and piston tilting. Piston tilt, groove thermal tilting and axial deformation all contribute to twisting the OCR when inertia forces are strong enough. In all these situations, ring tension helps maintaining both lands of the OCR in contact with the liner and ensure adequate oil control at the ring liner interface. It can be inferred that reducing ring tension can trigger excessive scraping and result in oil flow past the OCR.
4.2.2 OCR scraping algorithm

In the simulations presented in this section the effect of piston dynamic tilting only was considered and thermal distortions of the piston were neglected. The variation of piston tilt over the engine cycle was obtained from the lubrication model of the piston skirt taking into account piston secondary motion developed by Bai [51]. In particular tilt calculated for a medium load and speed operating point (3500 RPM, 700 mbar intake pressure) was used for all simulations (Figure 4.24).

![Figure 4.24 - Piston tilt obtained from piston skirt lubrication simulation](image)

In Figure 4.24, positive piston tilt corresponds to the piston moving up on the thrust-side and down on the anti-thrust side. Since the effect of gas flow or gas pressure on the OCR is not considered in current model, conditions in the first or second half of the engine cycle are the same for the OCR. OCR scraping simulations are conducted over half of an engine cycle and the tilt data from the intake and compression stroke was selected.

In the following simulation results, the oil film thickness on the liner is calculated from the clearance between the OCR lands and the liner (Figure 4.25).
Oil flow between the OCR land and the liner can be approximated by a Couette flow. By conservation of mass the oil film thickness on the liner must equal half of the ring liner clearance.

To calculate the oil film thickness on the liner and the scraping from the OCR, several assumptions are made. During a down stroke oil supply to the lower land of the OCR is assumed to be sufficient to have fully flooded lubrication. This assumption is reasonable considering that the oil film left on the liner by the piston skirt is typically larger than what is required for lubrication of the OCR. During the down stroke two scraping scenarios are possible (Figure 4.26).
When OCR twist is such that the upper land clearance is smaller than lower land clearance, oil is scraped from the liner and accumulates between the OCR lands (equation (4.14)).

\[
\Delta V_{OCR} = \left(\frac{h_{il} - h_{lu}}{2}\right) U \Delta t = q_{sc} \Delta t \quad \text{if} \quad h_{lu} < h_{ll}
\]  

\(\Delta V_{OCR}\) is the variation of the volume of oil accumulated between the OCR lands in the time \(\Delta t\) (per unit length in the circumferential direction), \(U\) is the ring sliding velocity and \(q_{sc}\) is the scraping flow rate (volumetric flow rate per unit length). If OCR twists in the opposite direction, the lower land scrapes oil from the liner to the top of the piston skirt. The thickness of the oil film left on the liner after the passage of the OCR is set by the land with the lowest clearance. During an up stroke, the oil supply to the OCR is given by the oil film thickness that was left on the liner during the down stroke. Scraping can occur both between the lands of the OCR or on top of the OCR if the oil film on the liner is thick enough (Figure 4.27)
The amount of oil up-scraped by the OCR ($V_{3^{rd}}$) can be calculated from the ring clearance and the oil film left on the liner (equation (4.15)).

$$\Delta V_{3^{rd}} = \frac{(h_{oil} - h_{tu})}{2} U \Delta t = q_{upsc} \Delta t \quad \text{if} \quad h_{tu} < h_{oil} \quad (4.15)$$

Similarly scraping between the lands during the down stroke can be calculated (equation (4.16)).

$$\Delta V_{OCR} = \frac{(h_{oil} - h_{il})}{2} U \Delta t = q_{sc} \Delta t \quad \text{if} \quad h_{il} < h_{oil} \quad (4.16)$$

In the OCR scraping simulations presented in this section oil is assumed to stay with the ring and the piston once scraped. In reality oil can remain attached to the liner and a fraction of the scraped volume can return to the liner when ring twist is reversed. The effect of starvation on the OCR land normal force is also not considered in the current model. This scraping algorithm could be revised in the future, the current assumptions are used to obtain a first set of results.
4.2.3 Sample OCR scraping results

The oil film thickness left on the liner by the OCR during the intake and compression strokes is calculated using the piston ring model. The piston dynamic tilting data of Figure 4.24 is used as input and the engine speed is set to 4000 RPM. A high speed case was chosen to observe the effect of high inertia forces and piston dynamic tilting on the oil control function of the OCR. The baseline ring configuration with 15N ring tension is used. The oil film thickness on liner at BDC of the intake stroke and TDC of the compression stroke is shown on Figure 4.28.

![Figure 4.28 - Effect of piston dynamic tilting and OCR twisting on liner oil film thickness](image)

This 3D view of the oil film thickness on the liner is similar to Figure 4.13 of the ring conformability section. The colored region of the distorted bore corresponds to the portion of the cylinder swept by the OCR between its top dead center and bottom dead center. The thick dotted line marks the position of the thrust-side of the cylinder, the thin dotted line locates the anti-thrust side. The liner is colored according to the oil film thickness. Two curves were added inside the cylinder: the red curve shows the variation of piston dynamic tilt during the down-stroke and the blue curve corresponds to the up-stroke piston tilt. The two curves lie in the plane of the piston.
pin. When the tilt curve is on the right side of the cylinder axis it indicates positive piston tilt values (thrust side of the piston going up, anti-thrust side going down). When it is on the left side of the cylinder axis, piston tilt is negative (Figure 4.29). The tilt curves are positioned to show the value of piston tilt at each position along the stroke.

![Piston tilt sign convention](image)

*Figure 4.29 – Piston tilt sign convention*

The oil film was found to be rather uniform on a distorted bore for the baseline ring configuration. Piston dynamic tilting and OCR twisting however lead to a non-uniform oil film distribution on the liner. Control of the oil film thickness is maintained as the maximum thickness observed is between two and three times the liner roughness (0.2 μm). Oil film thickness is found to be minimum in regions where the piston tilt is maximum. When the piston tilt is large, rotation of the OCR section is expected to be large, meaning that one land of the OCR can lose contact and the other land support ring tension alone. Since the pressure on the land in contact with the liner is larger than for a non-twisted OCR it reduces the oil film thickness allowed through the OCR. The equilibrium of the OCR in the groove of a tilted piston is shown on Figure 4.30.
Forces acting on the cross section of the OCR located at thrust side of the piston are shown on the left of Figure 4.30. The cross section of the OCR located at the anti-thrust side is shown on the right. The green arrow represents the inertia force, ring-groove and ring-liner contact forces are shown in red and the friction forces in black. The length of each arrow is proportional to the magnitude of the force. In the frame pictured the OCR is approaching TDC of the up-stroke of the simulation case presented in this section. The OCR is rotated due to piston and the normal force is supported by only one land. Consequently the clearance at the land in contact is smaller than if both lands were in contact.

On the pin side, OCR twist is small and one can see that the oil film is thicker and more uniform. On the thrust and anti-thrust side the oil film thickens or is reduced depending on piston tilt and OCR twisting.

The same pattern of oil film distribution can be observed when the OCR has reached TDC. Part of the oil that was left on the liner during the down-stroke has been scraped by the OCR during the up stroke. This is due to the asymmetry of piston tilt and OCR twisting. If piston tilt and OCR twisting were the same during the down and up stroke then no net scraping would be observed.

The same simulation is repeated but this time no piston dynamic tilting is considered. This is done to validate that piston dynamic tilting is the major cause of OCR twisting and scraping of oil on the liner. The oil film thickness distribution on the liner for this case is shown on Figure 4.31.
With the absence of piston dynamic tilting, the oil film thickness on the liner is uniform and again in the order of two to three times the liner roughness. Moreover there is hardly any change of oil film thickness between the down and up stroke meaning that OCR scraping is minimum. Two thin bands of lower oil film thickness can be seen on Figure 4.31. They correspond to the position in the down and up stroke when the ring shifts from one flank of the piston groove to the other. Dynamics can cause the ring to twist in such events, leading to thinner oil films on the liner.

As discussed earlier, there are two scraping situations in the current scraping algorithm. Oil can be scraped from the liner to move to the volume between the two OCR lands or be scraped from the liner to move to the top of the OCR and eventually the 3rd land. First the amount of oil scraped from the liner to the volume between the OCR lands is shown (Figure 4.32).
One should note that the down-stroke and up-stroke pictures have different scales. As expected most of the scraping occurs on the thrust and anti-thrust side and during the down stroke when the oil supply at the leading edge of the oil control ring is large. Up to one micron of oil is picked up from the liner which results in significant oil flow to the volume between the OCR lands. During the down stroke scraping occurs on the anti-thrust side when the piston tilt is positive and on the thrust side when the tilt is negative. This is consistent with the orientation of the piston groove and the corresponding rotation of the OCR section. The amount of oil scraped during the up stroke is much smaller. Indeed in the up stroke oil supply to the OCR is determined by the oil film left on the liner during the down-stroke. There is thus less oil available to be scraped than during the down-stroke.
The amount of oil scraped between the OCR lands for the simulation with no piston dynamic tilting is shown on Figure 4.33.

![Diagram of oil scraped between OCR lands](image)

**Figure 4.33 - Oil scraped between OCR lands when no piston dynamic tilting is considered**

The initial observation based on the oil film thickness distribution is confirmed. Without piston dynamic tilting, OCR twisting is limited and does not generate any significant amount of scraping between the OCR lands. In this case, ring tension is able to maintain both lands of the OCR in contact with the liner.

Now the second scraping situation, up scraping to the top of the OCR and the piston 3\textsuperscript{rd} land, can be studied. The amount of oil up-scraped on the liner is shown on Figure 4.34 (up scraping only occurs during the upstroke).
Results from three separate simulations are shown: two with piston dynamic tilting and one without. The upper left picture shows the result of a low engine speed case (1000 RPM), the upper right is a high engine speed case (4000 RPM). The oil film thickness scale is the same for the three pictures to allow comparison. Without dynamic tilting of the piston the amount of oil up scraped remains small. The one up-scraping event visible on Figure 4.34 occurs when the OCR moves from the lower flank of the piston groove to the upper flank towards the end of the up-stroke. Even without piston tilt, friction can contribute to up-scraping. The direction of the friction force
reverses between the down-stroke and the up-stroke thus creating an asymmetry in OCR twist. The magnitude of friction up-scraping is smaller than the contribution of piston dynamic tilting in most cases.

For the simulations using the piston tilt input shown in Figure 4.24, most of the up-scraping occurs on the anti-thrust side, when the piston tilt is positive. The thickness of oil up-scraped can reach 0.1 μm. The low and high speed cases have similar scraping patterns. The amount of oil up-scraped per stroke increases with speed as inertia becomes stronger and forces the OCR to follow piston tilt more closely.

4.2.4 Sensitivity analysis

OCR scraping is quantified for varying engine speed (between 1000 RPM and 4000 RPM) and varying ring tension (5 to 50 N). These simulations are conducted to understand how ring tension impacts the OCR scraping mechanism over the operating range of the engine. The volume of oil scraped by the OCR over the intake and compression stroke is normalized. To allow comparison with oil flow through the OCR control ring groove the normalization of equation (4.17) is adopted.

\[
V^* = \frac{V}{V_0} \quad \text{with} \quad V_0 = 2\pi R h_0 L_g
\]  

\( V^* \) is the normalized oil volume, \( V \) the absolute oil volume and \( V_0 \) the reference oil volume. The reference volume corresponds to the volume of an oil film of thickness \( h_0 \) covering the whole ring-groove interface whose width is \( L_g \) and radial position is \( R \). A film thickness value of \( h_0 = 1 \, \mu m \) is chosen. \( V^* = 40 \) corresponds approximately to a volume of oil transferred equal to the entire volume of the ring-groove interface (since the ring-groove clearance is close to 40 μm). The normalized volume of oil transfer can be converted in oil mass flow rate (equation (4.18)).

\[
\dot{m}_{oil} = \frac{V^* V_0 \rho_{oil}}{t_{rotation}} = V^* V_0 \rho_{oil} \frac{RPM}{60}
\]  

\( 175 \)
The oil density is $\rho_{oil}$ and $t_{\text{rotation}}$ is the duration of a crank rotation (half of the engine cycle). The normalized volume of oil scraped during the down and up stroke for varying ring tension and engine speed is shown on Figure 4.35. OCR land width is adjusted to keep the contact pressure with the liner constant when ring tension is changed.

![Figure 4.35 - Scraping between OCR lands for varying tension and engine speed](image)

Ring tension has a strong effect on the scraping behavior of the OCR. As OCR tension is decreased, so is the corresponding twisting moment forcing both lands of the OCR to remain in contact with the liner. The OCR is forced to follow piston tilt and the OCR sections are tilted with respect to the liner resulting in scraping of oil from the liner. At high ring tension (50N), the twisting moment from ring tension is large enough to maintain both lands of the OCR in contact with the liner for the entire engine operating range (1000 to 4000 RPM). For low OCR tension values, the amount of oil scraped increases with engine speed. Indeed the inertia force becomes larger and forces the OCR to follow piston tilt more closely. At low tension high engine speed, the OCR follows the tilted piston and scraping is maximum whereas at high tension low speed the OCR stays flat on the liner and scraping is minimum.
The normalized volume of oil up-scraped to the top of the OCR is shown on Figure 4.36. The volume of oil up-scraped is an order of magnitude lower than the volume scraped between the OCR lands. This is expected since the amount oil supplying the OCR during the up-stroke is limited to the oil film left on the liner by the OCR during the down-stroke.

![Figure 4.36 - OCR up-scraping for varying tensions and engine speeds](image)

Since the magnitude of up-scraping is smaller than the scraping between the lands, secondary effects not discussed in this section are likely to play a role. For instance the effect of ring tension on scraping is non-trivial. In these simulations increasing ring tension leads to an increase of up-scraping. Though ring tension helps maintain both lands of the OCR in contact with the cylinder, it also increases the friction force that contributes to negative OCR twist in the up-stroke and up-scraping. The inertia force increases with engine speed which forces the OCR to follow piston tilt more closely and leads to an increase OCR up-scraping.

The volume of oil scraped between the OCR lands and up-scraped over a crank rotation is converted in mass flow rate (Figure 4.37 and Figure 4.38).
Figure 4.37 - Mass flow rate of oil scraped between OCR lands

Figure 4.38 - Mass flow rate of oil up scraped
Because of the linear relation between the mass flow rate and the engine speed, the maximum oil scraping flow rate occurs at high speed. This is assuming that oil supply to the OCR during the down stroke remains sufficient to have fully-flooded conditions when engine speed is increased. With the current scraping algorithm, the mass flow rate of oil for scraping between the OCR lands can exceed 700 g/h. For reference the scale of oil consumption for a passenger car engine is 1 to 10 g/h per cylinder. Fortunately the oil accumulation between the OCR lands can flow to the groove through the draining slots that are machined in the OCR section. Once in the groove, oil can be released in the crank case through the OCR groove draining holes. This limits the impact of oil accumulation between the lands of the OCR on oil consumption. Oil can also be released at the trailing edge of the OCR during the up-stroke if bridging occurs. The up-scraping mass flow rate can reach 30 g/h in the current model which is lower than scraping between the lands but still exceeds the scale of acceptable oil consumption. Oil up-scraped can also be returned to the liner if bridging occurs, therefore it is not trivial to estimate the fraction of up-scraping that contributes directly to oil consumption.

4.2.5 Summary

Tilting of the OCR due to piston dynamic tilting was found to negatively affect the oil control performance of the OCR by causing scraping of oil from the liner. Mass flow rates of up to 700 g/h for scraping between the lands of the OCR and up to 30 g/h for up-scraping were obtained from the OCR model developed in this thesis. Part of the oil scraped is released on the liner or in the groove through the OCR draining slots and a fraction of the oil scraped contribute to oil consumption. OCR tension is the main factor limiting scraping at the ring liner interface. The mechanics of OCR twisting must be considered to achieve tension reduction without seeing an increase in ring twist and thus a penalty in oil consumption.

4.3 Flow through ring-groove lower interface

4.3.1 Description of oil flow through the ring-groove lower interface

To finish characterizing oil transport around the OCR, the flow of oil through the OCR groove is studied starting with oil flow at the lower ring-groove interface. The sequence of events leading to
flow of oil to the OCR groove is presented on Figure 4.39. In this scenario, inertia is stronger than ring friction and dominates the axial motion of the ring.

During the down stroke, the OCR scrapes oil from the liner. This oil accumulates below the OCR and is pressurized due to the relative motion between piston and liner. In the initial portion of the down stroke, inertia is holding the ring against the upper flank of the piston groove leaving the ring-groove interface open for oil to flow. Pressure in the oil accumulation below the OCR drives flow towards the groove. In the second half of the down stroke the direction of inertia is reversed and the ring squeezes the oil film on the lower flank of the groove. A fraction of this oil film is transferred to the groove.

Results from the ring-groove lubrication model coupled with ring dynamics are used to validate this description of oil flow to the groove. In the results presented next, oil was supplied at the top of the piston skirt (and not on the pin side) at a constant pressure of 10 kPa during the down-stroke
and the engine speed was set at 4000 RPM. Figure 4.40 explains the different components of the ring-groove plot that will be used to illustrate oil flow at the ring-groove interface.

Figure 4.40 - Introduction to ring-groove interface flow visualization
There are three main components to the ring-groove interface graph: a view of the ring position and the oil film (top), a top view of distribution of hydrodynamic pressure in the ring-groove interface (middle) and a plot of the axial force balance (bottom). In Figure 4.40 the radial and axial scale have been magnified in order to observe the movement of the ring and of the oil film front. For the baseline configuration the radial width of the ring-groove interface is close to 2mm and the ring-groove axial clearance is approximately 40µm. Figure 4.41 is the same graph but this time with the real radial scale.

![Ring Groove Lubrication Model - CA=74.5/569](image)

*Figure 4.41 – Ring-groove interface flow visualization with real radial scale*

At the start of the intake stroke the OCR sits at the top of the groove and the lower ring-groove interface is open for oil to flow (Figure 4.42). Initial conditions with a uniform oil film of 5µm on the lower flank of the groove were chosen. As shown later in this section the OCR does not squeeze the oil film on the groove much farther than 5µm.
Pressure at the top of the skirt chamfer drives the flow of oil towards the OCR groove. Oil gradually fills the ring-groove interface until the net axial force points down and the ring starts moving towards the lower flank of the groove (Figure 4.43). It needs to be reminded that the circumference of the ring-groove interface is much larger than the radial width. Flow of oil appears one dimensional but the circumferential and radial motion of oil are similar.
After the first half of the intake stroke, the net axial force is moving the ring towards the lower flank of the groove. The oil film at the ring-groove interface is squeezed and continues flowing towards the groove (Figure 4.44). Since the oil film only supports the ring on the skirt side, the OCR deforms axially and make contact on the pin side. Part of the axial load is supported there which reduces the oil film squeezing force.
Oil eventually reaches the groove with further ring squeezing. As the oil film thickness reduces, the ring-groove contact becomes stiffer and ring squeezing is made harder. The end of squeezing occurs around the middle of the up stroke at which point the oil film has been squeezed down to a thickness of approximately 5 μm (Figure 4.45). The hydrodynamic pressure in the oil film exceeds 100 kPa during the squeezing phase which is an order of magnitude higher than the difference of pressure between the skirt and the groove. Considering also that the ring-groove clearance reduces during the squeezing phase, oil flow due to the difference of pressure between the skirt and groove becomes negligible compared to oil flow due to ring squeezing. During the squeezing phase a fraction of the oil that filled the ring-groove interface due to pressure flow is transferred to the groove. If pressure flow was able to fill the entire interface and if ring flank and groove flank are parallel, then half of the ring-groove interface volume would be squeezed to the groove the other half returned to the piston.
Late in the up-stroke the net axial force is moving the ring towards the upper flank of the groove (Figure 4.46). Cavitation occurs in the oil film between the ring and the groove. For simplicity it was assumed in the ring-groove lubrication model that all the oil remains attached to the groove surface. Since in the current version gas flow is not considered the attachment assumption has no influence on oil flow at the interface. In the simulations of oil flow at the ring-groove interface, cavitation pressure was set to 1 bar.
Oil flow to the groove is normalized using the same reference volume as in the OCR scraping section (equation (4.17)). It is divided by the volume of an oil film 1μm thick uniformly distributed in the ring-groove interface. \( V^* \) refers to the normalized flow of oil to the groove in the graphs to follow. Oil flow and flow rate to the groove is calculated for the example shown on Figure 4.42 to Figure 4.46 (Figure 4.47). Flow to the groove is integrated all around the ring circumference.
Shortly after squeezing starts (around CA=100°) the oil film at the ring liner interface reaches the groove. Flow rate to the groove increases initially due to squeezing until the oil film becomes thinner and the contact stiffer. A second increase in flow rate is seen at BDC when the direction of the friction force reverses which adds to the squeezing force. After a crank rotation, a total volume of oil of $V^* = 3$ is transferred to the groove. This means that an oil volume of 3/40 of the ring-groove interface volume is transferred to the groove per engine rotation, which corresponds to a mass flow rate of approximately 40 g/h at 4000RPM. Therefore under this skirt pressure boundary condition only a small fraction of the oil flowing through the ring groove interface makes it to the groove.

4.3.2 Modeling oil pressure in the piston skirt chamfer

The oil pressure boundary condition at the OCR-skirt interface depends on the nature of oil flow below the OCR and is influenced by the design of the skirt chamfer, skirt lubrication and piston secondary motion. Determining the oil pressure boundary condition at the OCR-skirt interface is critical to predict flow of oil to the OCR groove. While the oil flow below the OCR is complex,
here a simplified model is provided. During the down stroke, the oil scarped from the liner together with the oil flowing up along the piston accumulate in the piston chamfer (Figure 4.48).

The oil accumulated in the skirt chamfer is assumed to fill the clearance between the liner and the skirt and that the flow in that region is assumed to be fully viscous. The cut at the top of the piston skirt is parallel to the liner in this case.

In this first order model, two mechanisms contribute to pressurization of the oil at the inlet of the OCR groove, namely, the hydrostatic effect from the inertia force and the resistance and the resistant to the oil flow caused by piston sliding. The former is shown in Figure 4.49.
Inertia generates a linear pressure gradient in the oil accumulation below the OCR (equation (4.19)).

\[ p_i = -\rho_{oil} A_p h_{ch} \]  

(4.19)

The gage pressure at the inlet of the groove due to inertia is \( p_i \), \( \rho_{oil} \) is the oil density, \( A_p \) the piston acceleration and \( h_{ch} \) the height of the oil accumulation below the OCR. In addition to the contribution from inertia, the relative motion between the piston and the liner coupled to the OCR scraping generates additional pressure in the oil accumulation. The pressure contribution of OCR scraping can be calculated using mass conservation on the oil accumulation in the chamfer (Figure 4.50).
The oil accumulated below the OCR must move in average at the speed of the piston. Therefore the shear flow and pressure flow components of the oil flow in the chamfer must add up to be equivalent to a uniform velocity distribution (equation (4.20)). The pressure distribution in the oil accumulation is once again linear since the wall of the piston and the wall of the liner are parallel.

\[
q_{chamfer} = h_{ch} \frac{V_p}{2} + \frac{h_{ch}^3}{12\mu h_{acc}} p_{visc} = h_{ch}V_p
\]  

(4.20)

The mass conservation equation can be rewritten to express the contribution of scraping to the pressure at the inlet of the groove (equation (4.21)).

\[
p_{visc} = 6\mu V_p \frac{h_{acc}}{h_{ch}^2}
\]

(4.21)

To calculate the pressure contributed by both inertia and scraping, one must know the amount of oil accumulated in the chamfer (height of the oil column below the OCR). This information is obtained from simulations conducted with the piston skirt lubrication model developed by Bai and Totaro [51], [54]. Oil accumulation information used in the simulations in this section is shown on Figure 4.51.
The oil accumulation data corresponds to a case with a large supply of oil to the liner and a skirt chamfer clearance of 250µm. The piston chamfer fills up at the end of the intake stroke which is consistent with experimental observations that have been made with Laser Induced Fluorescence (LIF). First, the position of the rings and the skirt chamfer are shown on Figure 4.52, on an image obtained from a test engine set up with a LIF system.
The LIF image of Figure 4.52 shows the portion of the piston and piston ring pack visible through the sapphire window on the thrust side of the engine block. The three rings and the piston skirt chamfer are visible. The brightness of the image is correlated to the amount of oil in the ring pack with light regions corresponding to regions of oil accumulation. A sequence of LIF images taken during the intake stroke are shown in Figure 4.53 to illustrate the accumulation of oil below the OCR during the down stroke. In this example the engine was running at a speed of 2500 RPM with an intake pressure of 700 mbar.
In Figure 4.53 CA=0 corresponds to the top dead center of the intake stroke and CA=180 to the bottom dead center of the intake stroke. It can be seen that oil accumulates below the OCR as the piston slides down in the intake stroke. When the piston nears bottom dead center of the intake stroke, the skirt chamfer is almost entirely full. It can also be observed that the distribution of oil and the nature of the flow below the OCR is not trivial.

Oil pressure in the piston skirt chamfer calculated from the first order model presented in this section is shown on Figure 4.54. Oil pressure was calculated for an engine speed of 4000 RPM and a chamfer clearance of 250µm.
CA=0 corresponds to the beginning of the down-stroke. During the up-stroke, oil that accumulated in the skirt chamfer supplies the skirt contact and the pressure at the inlet of the groove is assumed to be zero. The contribution of scraping to the oil pressure below the OCR (shown in black in Figure 4.54) is found to be more important than the contribution of inertia. The oil pressure in at the inlet of the ring-groove interface would therefore vary approximately linearly with the engine speed.

Figure 4.54 - Oil pressure in accumulation below the OCR (4000 RPM – chamfer clearance 250μm)
4.3.3 Sensitivity analysis

The OCR oil transport model is used to simulate the effect of ring tension, engine speed, piston skirt chamfer clearance, and piston dynamic tilting on the flow of oil to the groove. First the effect of ring tension and engine speed on the flow of oil to the OCR is shown on Figure 4.55.

![Figure 4.55 - Normalized oil flow to the groove (No piston dynamic tilt, skirt clearance 250μm)](image)

For this case, no piston dynamic tilting was considered and the skirt chamfer clearance was set at 250μm. Contrary to intuition the maximum oil flow to the groove occurs at low speed and high ring tension. The physics of flow to the groove are reviewed to explain why an increase of ring tension and a decrease of engine speed contribute to flow to the groove. Ring tension and engine speed mainly affect the axial force balance of the ring and the time during which the OCR sits on the upper flank and allow oil to flow at the lower ring-groove interface (Figure 4.56).
Two situations are pictured in Figure 4.56: a friction and an inertia dominated axial dynamics. Friction at the ring-liner interface and inertia are the two forces moving the ring in the axial direction. The friction force is related to the ring tension by the friction coefficient. If ring tension is large enough, the friction force can remain larger than the inertia force over the entire down-stroke. In this case, the axial motion of the ring is dictated by friction and the ring moves to the lower flank of the groove when BDC of the down-stroke is reached. Now if the magnitude of the inertia force exceeds the magnitude of the friction force, then the shift to the lower flank of the groove occurs earlier in the down-stroke. It can occur as early as the middle of the down-stroke when the direction of the inertia force is changed. Hence the ring-groove interface remains open longer for oil to flow to the groove when the axial dynamics of the ring is dominated by friction. The switch between friction dominated behavior and inertia dominated behavior is further illustrated by looking at the competition between friction and inertia in the down stroke (Figure 4.57).
In Figure 4.57 the green, blue and broken black lines correspond to the inertia, friction and net axial force respectively. Ring tension is kept constant and engine speed is increased from 1000 RPM to 4000 RPM to go from a situation where friction dominates to a situation where inertia dominates. At 1000 RPM, the inertia force is small and the friction force maintains the ring on the upper flank for the entire down-stroke. When engine speed goes up to 4000 RPM hydrodynamic friction is increased but not enough to prevent inertia in the second part of the down-stroke to move the ring to the lower flank of the groove. The effect of engine speed and ring tension on the duration when the ring-groove channel is open is showed conceptually in Figure 4.58.
Entire stroke

Time spent by ring on upper flank during intake stroke

Until mid stroke

Figure 4.58 - Competition between friction and inertia

For high ratios of ring tension over engine speed the axial dynamics of the ring is dominated by friction. The ring stays on the upper flank for the entire intake stroke. At low tension over engine speed ratios the ring moves to the lower around mid-stroke when the direction of inertia is switched.

As discussed earlier the oil transferred to the groove over a rotation is a fraction of oil that filled the ring-groove interface due to pressure flow (equation (4.22)).

\[ V_{oil} = \alpha q_{Ap} \Delta t - \alpha q_{Ap} \Delta t \]  \hspace{1cm} (4.22)

In equation (4.22), \( V_{oil} \) is the volume of oil transferred to the groove over one rotation, \( q_{Ap} \) is the flow rate of the pressure flow between the skirt and the groove and \( \Delta t \) is the time during which the OCR is sitting on the upper flank of the groove. The fraction \( \alpha \), which represents the fraction of the oil at the ring-groove interface going the groove during squeezing, will depend on how full the ring-groove interface is, on the squeezing force and on the relative orientation of ring and groove flank. In the pressure boundary condition model, the pressure below the OCR varies mostly
linearly with engine RPM since the scraping contribution dominates. Cycle time however decreases linearly with RPM which counteract the pressure effect. Engine speed and tension impact \( \Delta t \) by modifying the time the OCR spends on the upper flank during the intake stroke. As seen on Figure 4.55, ring tension increase the volume of oil flow the groove by keeping the ring on the upper flank while an increase in engine speed reduces the flow the groove by starting the ring squeezing motion earlier. Since the inertia force depends more strongly on engine speed than friction varies with tension, engine speed has a stronger influence on flow to the groove.

The volume of oil transferred to the groove per rotation is now converted in mass flow rate (Figure 4.59).

![Figure 4.59 - Mass flow rate to OCR groove (No piston dynamics tilt, skirt clearance 250um)](image)

Significant mass flow rates to the OCR groove ranging from 100 to 400 g/h are obtained. This highlights the importance of draining oil in the OCR groove to avoid oil consumption issues. The same simulations cases were ran this time considering piston dynamic tilting (Figure 4.60)
Piston dynamic tilting is found to slightly reduce the amount of oil transferred to the groove. Since the sign of piston tilt varies frequently in the cycle, the tilted groove merely adds to the viscous resistance to flow. The effect of piston dynamic tilting is secondary compared to the effect of the other parameters discussed in this section (engine speed, tension and most importantly skirt chamfer clearance).

Now the clearance of the piston skirt chamfer is reduced to 150 μm and the effect on flow to the groove is calculated and compared to the previous simulation (Figure 4.61).
For the skirt chamfer clearance of 150μm, almost the entire ring-groove interface volume is transferred to the groove every crank rotation. In the viscous model of oil flow between the skirt and liner, the pressure below the oil control ring varies quadratically with the skirt clearance. The same quadratic relation is found between the amount of oil transferred to the groove for the chamfer clearance of 150μm and for the chamfer clearance of 250μm. In these simulations the height of oil accumulated below the OCR was kept constant when the chamfer clearance was changed. In reality if the volume of oil supplied to the skirt remains constant when the chamfer clearance is made smaller, then the height of the oil accumulation would increase. In this case flow of oil to the groove is expected to have a cubic dependence to the chamfer clearance. This shows that the skirt chamfer must be designed properly to be able to hold oil to lubricate the piston skirt during up-strokes without pressuring oil below the OCR.
4.3.4 Summary

Simulations of oil flow from the skirt chamfer to the piston groove have shown that flow to the groove is primarily addressed with piston design. Indeed the skirt chamfer was found to have a stronger effect on flow to the groove than ring tension or engine speed. Without an adequate skirt chamfer design oil pressure below the OCR can rise, leading to significant oil flow to the groove. The volume of the chamfer must be designed to hold the quantity of oil scraped from the liner in the down stroke without getting full and becoming pressurized. Considering that under certain conditions oil can flow to the groove, draining in the OCR groove is important to limit oil accumulation and potential leakage to the 3rd land.

4.4 Flow through ring-groove upper interface

In this section flow of oil to the 3rd land through the upper ring-groove interface is quantified using the OCR oil transport model. Its contribution to the supply of oil to the upper ring pack and its impact on oil consumption is assessed.

4.4.1 Modeling oil pressure in the OCR groove

Oil is supplied to the OCR groove through the lower ring groove interface and the OCR draining slots. Once enough oil accumulates in the groove, pressure can rise due to the action of inertia and oil can leak to the 3rd land through the ring-groove upper flank. A first order model is used to calculate the oil pressure at the inlet of the upper ring-groove interface as a function of inertia and engine speed (Figure 4.62).
Under the action of inertia, oil is assumed to be distributed uniformly around the piston groove and inertia generates a linear pressure gradient in the oil accumulation (equation (4.23))

\[ p_i = -\rho_{oil}h_{groove}GFA_p \]  

(4.23)

The pressure at the inlet of the ring groove upper flank is \( p_i \), \( \rho_{oil} \) is the oil density, \( GF \) the groove filling ratio and \( A_p \) the piston acceleration. Variation of the groove oil pressure over the compression and exhaust stroke is calculated according to this model (Figure 4.63).
Pressure in the oil is maximum at TDC of the compression stroke when the inertia force is maximum. The maximum oil pressure reaches almost 20 kPa for a full groove and 10 kPa for a groove half full. Those figures of oil pressure are sufficient to cause leakage of oil to the piston 3\textsuperscript{rd} land. It needs to be reminded that large accumulation of oil in the groove is unlikely in the engine because of the release through the OCR groove draining holes. Therefore the simulations conducted here are representative of a worst case scenario. Figure 4.64 shows the evolution of the maximum oil groove pressure with engine speed. The solid line corresponds to a full groove and the broken line to a groove half full.
4.4.2 Sensitivity analysis

Oil flow to the piston 3\textsuperscript{rd} land, through the upper ring groove interface is calculated using the pressure boundary condition model of equation (4.23). OCR tension values of 5, 10, 25N and engine speeds ranging between 1000 RPM and 4000 RPM are chosen. The normalized oil transfer and oil mass flow rates for a full groove and a groove 25\% full are shown on Figure 4.65. The engine speed was set to 2000 RPM for these simulations.
Flow of oil to the 3rd land is significantly lower than flow from the skirt to the OCR groove. The maximum normalized oil transfer observed is lower than 3 as opposed to 13 for flow to the groove (in the baseline configuration). Ring tension helps maintain the upper ring-groove interface open and hence increased tension results in increased oil transfer to the 3rd land. The effect of engine speed is not trivial, it increases the pressure in the groove but also reduces the time for oil to flow to the 3rd land. When inertia dominates over ring friction, pressure rise in the groove and ring axial motion are in phase. The upper ring-groove interface closes when the oil pressure in the groove rises thus limiting the flow to the 3rd land.

The volume of oil transferred to the 3rd land per rotation is converted in mass flow rate and shown for the full range of engine speed (Figure 4.66).
4.4.3 Summary

The rate of leakage of oil from the OCR groove to the 3\textsuperscript{rd} land ranges from 20g/h to 120g/h. As for OCR scraping it exceeds the range of oil consumption which is 1 to 10 g/h per cylinder. This shows that the OCR must be properly drained to avoid leakage to the 3\textsuperscript{rd} land and contribution to oil consumption.

Certain running conditions not considered here can promote leakage from the OCR groove. For spark ignition engines, throttled conditions can drive oil to flow from the OCR to the 3\textsuperscript{rd} land. Indeed reverse gas flow and favorable pressure gradient between the groove and the 3\textsuperscript{rd} land can generate leakage even if the oil accumulation in the groove is not large.

Under adequate draining conditions in the OCR groove, the analysis conducted in this thesis shows that it is easier for the oil to flow to the groove than to leak to the 3\textsuperscript{rd} land. The relative motion between piston and liner pressurizes the oil at the inlet of the lower ring groove interface. In the groove however, inertia is the only contribution to oil pressure and a full groove is required.
to obtain the same pressure build up than below the OCR. In addition gas flowing from the 3rd land to the OCR groove draining holes helps prevent leakage of oil to the 3rd land.

4.5 Summary of oil transport around the oil control ring

The multi-scale curved beam finite element model of piston ring developed in this thesis was used to quantify the impact of conformability, scraping and flow to the groove on oil flow past the OCR (Figure 4.67).

Figure 4.67 - Mechanisms contributing to oil flow past the OCR

Table 9 is a summary of the range of oil mass flow rate at the interfaces around the OCR obtained from the OCR oil transport model developed in this thesis. These results were obtained for baseline ring and piston configuration (OCR tension 15N, skirt chamfer clearance 250μm, OCR groove full oil) and an engine speed ranging from 1000 to 4000 RPM.
<table>
<thead>
<tr>
<th>Oil transport mode</th>
<th>Range of mass flow rate (g/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil consumption per cylinder</td>
<td>1-10 g/h</td>
</tr>
<tr>
<td>Scraping between the OCR lands</td>
<td>5-300 g/h</td>
</tr>
<tr>
<td>OCR up-scraping</td>
<td>5-30 g/h</td>
</tr>
<tr>
<td>Flow from piston skirt to OCR groove</td>
<td>150-350 g/h</td>
</tr>
<tr>
<td>Flow from groove to piston 3(^{rd}) land</td>
<td>40-100 g/h</td>
</tr>
</tbody>
</table>

*Table 9 – Range of oil mass flow rates at the interfaces around the OCR*

Up-scraping and flow to the piston 3\(^{rd}\) lands are the two sources of supply to the upper ring pack and thus the two potential sources of oil consumption. Their range exceed measured engine oil consumption and flow to the 3\(^{rd}\) land seems to be the biggest contributor. One has to keep in mind that the worst case scenario of a groove full of oil was studied for leakage to the 3\(^{rd}\) land and it is expected to be much smaller with efficient groove draining. The scraping prediction depends on the piston tilt input and the assumptions of the scraping algorithm. However this analysis has shown the impact of OCR twisting on oil transport and why it must be considered when ring tension is reduced.

For representative bore distortion and ring configuration, the OCR was found to be flexible enough to conform to distorted cylinders. Bore distortion started affecting the oil film thickness left on the liner by the OCR for ring tension as low as 5N.
5 Conclusions

5.1 Conclusions

Oil transport in the piston ring pack involves oil flows on the piston, liner, and the transfer of oil between piston and the liner. To flow through the ring pack, oil has to pass the barriers set by the piston rings. The interactions between the piston rings and their mating parts, namely, piston grooves and the liner, are multi-scale ranging from the scale of the ring structure to the scale of an asperity of the liner roughness. These interactions together with local oil availability and pressure conditions determine the oil transfer rate through the ring-groove and ring-liner interfaces. Therefore, establishing a multi-scale model of piston rings connecting the structural response of the rings to the local lubrication and contact conditions of the ring-groove and ring-liner interfaces is essential to advance the predictability of the oil consumption. While this work mainly covers the oil control ring, the established framework is a foundation that can be extended in the future to the top two rings by considering the effect of gas pressure and gas flows.

The main motivation for this project is to understand the mechanisms that contribute to oil consumption to be able to reduce OCR tension and hence friction without a penalty in oil consumption. The simulations and analysis conducted in this project has shown that OCR twisting and oil flow through the OCR groove could contribute to oil consumption under certain engine operating conditions. Due to the flexibility of the OCR, bore distortion was found to have little impact on the ability of the OCR to control the oil film thickness on the liner. In most engine OCR tension reduction is most likely to have a larger impact on the ability of the OCR to maintain both lands in contact with the cylinder rather than conformability.

Oil flow through the groove is mostly addressed with the piston design and does not depend strongly on OCR tension. Indeed designing the piston skirt chamfer to hold oil scraped from the liner without building pressure and designing draining in the OCR groove to limit oil accumulation can be sufficient to remove most of the leakage of oil from the groove to the piston 3rd land.

Therefore the emphasis must be placed on OCR twisting and scraping when trying to reduce the ring tension. OCR designs can be modified to limit twisting due to piston dynamic tilting and
groove thermal tilting. This would allow ring pack friction reduction through OCR tension reduction without affecting the sealing performance of the OCR.

5.2 Design recommendations

In this section, design guidelines to reduce the contribution of OCR scraping and flow through the OCR groove are given. Simulations conducted in this thesis have shown that piston tilt affects the oil control function of the OCR. The neutral plane of the OCR follows the groove of the tilted piston. The OCR section is rotated with respect to the liner and ring tension is trying to twist the ring so that both lands of the OCR can maintain contact with the cylinder (Figure 5.1).

![Diagram showing tension, inertia, and groove contact]

Figure 5.1 Design recommendations to reduce OCR scraping

A healthy design of the two piece OCR should allow both lands to maintain contact with the cylinder even when the piston is tilted. This helps reduce scraping of oil from the liner and improves the oil control function of the OCR. In the current context of friction reduction, the ability of the OCR to remain parallel to the liner should be increased without relying on ring tension. This can be done by reducing the out of plane and twisting stiffness of the ring. It would help ring tension to twist the ring outside of its plane of curvature to maintain oil control. The contact with
the groove provides resistance to twisting of the OCR by the ring tension. The geometry of the ring-groove interface can be modified (for instance by adding a parabolic profile as suggested on Figure 5.1) to move the center of the groove contact closer to the center of stiffness of the OCR. The twisting resistance of the groove would thus be reduced. Reducing the offset between the center of the groove contact and the ring section centroid is also helping reduce scraping for a groove with thermal tilt.

Reducing oil pressure below the OCR and oil accumulation in the OCR groove is an effective strategy to limit oil flow through the groove. This can be done by optimizing the design of the OCR groove draining holes and of the piston skirt chamfer (Figure 5.2).

The volume of the skirt chamfer must be designed based on the amount of oil supplied to the liner and expected to accumulate below the OCR. The oil accumulating in the skirt chamfer during the down-strokes is essential to lubricate the piston skirt in the up-strokes and limit friction and wear. However the chamfer must be big enough so that it does not get full during the down-stroke and oil pressure remains acceptable. A compromise must be made between the size of the skirt chamfer and the stability of the OCR in the OCR groove. Twisting instability of the OCR is likely to occur if the ring cross section centroid is outside of the outer diameter of the piston groove. Other
geometries of the oil reservoir below the OCR can be thought of. For instance the oil reservoir can be moved down to make sure that diameter of the lower flank of the OCR groove is large enough to control the dynamics of the OCR.

5.3 Future work

Oil transport and oil consumption in the ring pack of internal combustion engines is an ongoing topic of research. This thesis work is providing a numerical framework to overcome one the main difficulties of simulating oil transport: coupling the dynamics and deformation of the OCR with the piston and liner contact interaction. However additional research will be required to obtain a comprehensive ring pack oil transport model which is needed if ring pack designs are to be optimized.

First the OCR oil transport model predictions can be improved by iterating on the current model of oil flow in the groove, in the piston skirt chamfer and on the piston third land. Oil transport prediction depend strongly on the oil supply and pressure boundary conditions. It will also be important to take into account the effect of gas flow on oil transport at the ring-groove interface. For this, an approach similar to the multiphase model developed by Li [50] can be adopted.

The OCR scraping results can vary significantly with the attachment assumptions. In the current model scraped oil is assumed to go the ring. It will be useful to review other scraping algorithm that consider attachment of scraped oil to the liner and their impact on scraping predictions.

Finally in the longer term the OCR oil transport model can be extended to the top two rings to build a complete ring pack oil model. A stronger influence of gas pressure and gas flow is expected for the top two rings. Reliable oil flow models on the piston land and in the piston groove will be required to be able to predict oil transport from the OCR to the top of the compression ring.
References


