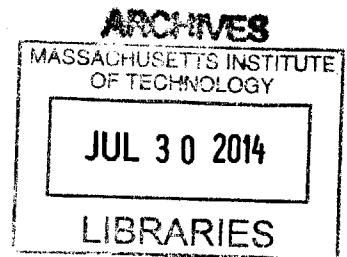# Time-optimal Path Planning for Sea-surface Vehicles Under the Effects of Strong Currents and Winds

by

Benjamin D. Hessels

Submitted to the Department of Mechanical Engineering

In Partial Fulfillment of the Requirements for the Degree of

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Abstract

A path-planning methodology that takes into account sea state fields, specifically
wind forcing, is discussed and exemplified in this thesis. This general methodology
has been explored by the Multidisciplinary Simulation, Estimation, and Assimilation
Systems group (MSEAS) at MIT, however this is the first instance of wind effects
being taken into account. Previous research explored vessels and isotropy, where
the nominal speed of the vessel is uniform in all directions. This thesis explores the
non-isotropic case, where the maximum speed of the vessel varies with direction,
such as a sailboat. Our goal in this work is to predict the time-optimal path between
a set of coordinates, taking into account flow currents and wind speeds. This thesis
reviews the literature on a modified level set method that governs the path in any
continuous flow to minimize travel time. This new level set method, pioneered by
MSEAS, evolves a front from the starting coordinate until any point on that front
reaches the destination. The vehicles optimal path is then gained by solving a
particle back tracking equation. This methodology is general and applicable to any
vehicle, ranging from underwater vessels to aircraft, as it rigorously takes into
account the advection effects due to any type of environmental flow fields such as
time-dependent currents and dynamic wind fields.

Thesis Supervisor: Pierre Lermusiaux
Title: Associate Professor

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1. Introduction

Path planning is a fascinating topic. It is a problem that has been discussed for millennia yet has no universal solution, mainly because it corresponds to numerous definitions of optimality. The ocean makes an especially interesting environment for path planning, as the current and winds are swift, often larger than vehicle speeds, and exceptionally complex. There are many approaches to control guidance at the moment, and there have been many in the past. However there have been few efforts towards rigorous results on optimal path planning for vessels which are heavily affected and/or rely on the wind, such as sail boats. Navigation is the most important task for vessels of this nature.

This thesis aspires to further develop rigorous methodology, so that it may be versatile and applicable not only to submerged vessels, but also to surface vessels that are affected largely by wind speeds. The following chapters illustrate that for a vessel traveling in a time dependent flow field, our methodology will compute the exact fastest path between two locations, accounting for nominal speed that are direction dependent.

The reason why things become significantly more complicated for vessels not operating isotropically is illustrated in Figure 1 below. This is a graphic published by the United States Sailing Association (previously known as the U.S. Yacht Racing Union, or USYRU 1991) that depicts how a boats speed varies as a function of true wind velocity and apparent angle. Plots for a Hobie 33 and an Alchuringa yacht are shown.
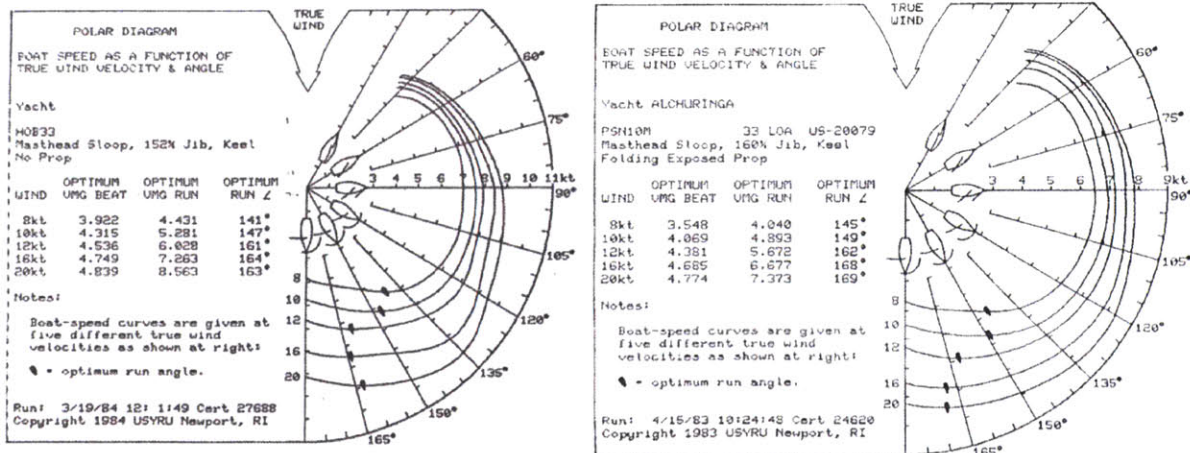
Figure 1. Boat speed as a function of true wind velocity and apparent angle for a Hobie 33 and Alchuringa yacht. [J/Boats, 2007]

The curves seen in Figure 1 are quite typical for vessel speed versus wind velocity and apparent angle. For the first two test cases in this thesis we will be working with a generalized polar diagram case, shown in Figure 2.
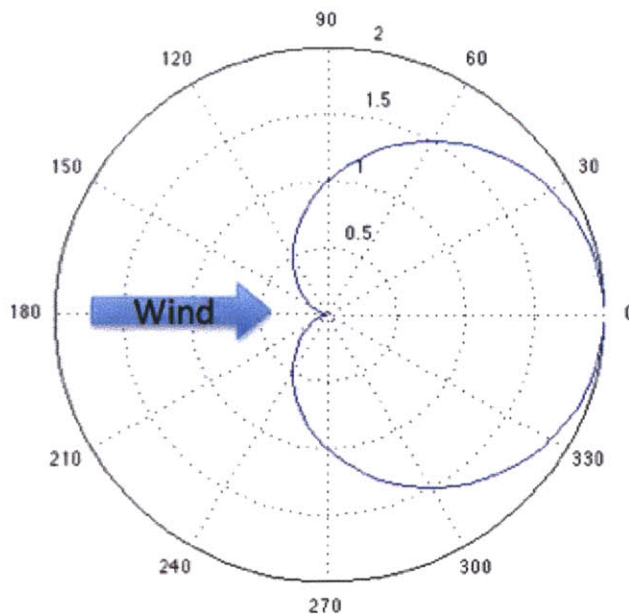
Figure 2. Vessel speed versus wind velocity and apparent angle, with wind blowing at θ = 0. This is the polar diagram function we will use in this thesis.

In the following pages we begin by succinctly reviewing literature pertaining to this project. The problem is then succinctly defined mathematically before a review of the theory used in this thesis is discussed, primarily regarding our usage of the modified level set equations. After this review, the paper addresses a number of test cases on which this methodology has been evaluated and the corresponding results. Finally, a summary and conclusion are given at the end of the paper.

2. Prior Work

A common problem, which has been the focus of many path planning projects over the past decades, has been dealing with the large number of degrees of freedom of a vessel [Latombe, 1991]. Research specifically pertaining to path planning through unsteady flow-fields has been scarce. A reason that unsteady flow-fields prove difficult for path planning is that the vehicle is actually displaced by the current, thus making the cost of the movement variable in time and anisotropic at different points in space [Isern-Gonzalez et al, 2012]. This causes difficulties even generating feasible tracks for the vessel. Currently, dynamic programming based approaches such as Dijkstra's method and the A* algorithm exist [Rhoads et al, 2010]. For example, Mannarini et al (2013) develop a prototype system for operational ship routing using a modified Dijkstra's algorithm and implementing safety restrictions to avoid surf-riding and parametric rolling. The problem with most of these approaches is that they often output infeasible paths when applied to dynamic flow environments with strong flows, or they only provide an approximate solution especially in applications with non-isotropic drags on the vessel, or, as the environment becomes more complex, their computational cost grows too large.

A fast marching method has been utilized for vessels navigating in isotropic environments [Sethian, 1999]. This method is similar to Dijkstra's algorithm, but

solved in a continuous domain. An Eikonal equation is solved to yield the arrival time function at different points in the domain [Sethian, 1999]. However this method is not easily applied to the non-isotropic case and it is not applicable for flow fields with magnitude larger than the nominal vehicle speed [Lolla et al, 2012]

Techniques that utilize graph searching, such as A*, have been used for underwater path planning, however this technique hinges on the ability to successfully define a good heuristic, which is quite difficult [Carroll et al, 1992, Garau et al, 2009, Lolla, 2012, Rao et al, 2009]. Further, A* requires that the domain be discretized. There is large chance that the optimal path will not always pass through the discretized grid points, and increasing the resolution of the grid significantly increases computational cost. Adaptive grid restricting may be utilized to remedy this, however, this is typically much more effective for simple steady-flow fields and is not successful when applied to complex dynamic flows.

Ge and Li (2013) explored the possibility of using a level set method to design a time-optimal path of a vessel subject to direction-dependent maximum velocities. However, the methodology they pursued did not optimize the level set method at every time step. Ge at al allowed the reachability front to continue to expand outward, normal to itself, at every time step. In contrast, our methodology allows the vessel to go in any direction at every time step from every point on the zero level set, this is key as we can be assured we are finding an exact solution. Further, Lin et al excluded all environmental factors besides wind, the methodology pursued in this thesis takes into account other sea state fields, such as currents and waves.

3. Approach and Theory

The following builds off the progress made by Lolla (2012), Lolla et al (2012, 2013) and of related contributions made by the MSEAS group in the science of autonomy applied to the marine domain (e.g. Lermusiaux et al, 2014). The approach is based on modified level-set equations that govern the evolution of the reachability front.

These are the set of locations that can be reached in shortest time, accounting for environmental forcing including time-dependent currents.

Mathematically, following Lolla et al (2013), let Ω be an open subset of $\mathbb{R}^n$ and $F > 0$. Some vehicle (P) moves in Ω and is influenced by a dynamic flow-field, $\mathbf{V}(\mathbf{x},t) : \Omega \times [0, \infty) \rightarrow \mathbb{R}^n$. The goal is to find a path for P that minimizes its travel time between any given start point ($\mathbf{y_s}$) and a destination ($\mathbf{y_f}$). Let a general continuous trajectory from $\mathbf{y_s}$ to $\mathbf{y_f}$ be denoted as $\mathbf{X}_P(\mathbf{y_s}, t)$, shown in Figure 1. The motion of the vessel is governed by the kinematic relation shown below, which takes into account the vessels own propulsion as well as advection due to the flow-field and wind.

$$\frac{\mathrm{d}\mathbf{X}_P}{\mathrm{d}t} = \mathbf{U}(\mathbf{X}_P(\mathbf{y}_s, t), t) = F_P\left(\hat{h}(t)\right)\hat{h}(t) + \mathbf{V}(\mathbf{X}_P(\mathbf{y}_s, t), t)$$

In the relation above, $F_P\left(\hat{h}(t)\right)$ is the speed of the vessel relative to the flow, where $0 \le F_P(t) \le F$, $\hat{h}(t)$ is the vehicle heading at time t and $\mathbf{U}(\mathbf{X}_P(\mathbf{y}_s, t), t)$ is the total vessel velocity. Let $\widetilde{T}(\mathbf{y}_s) = 0$ denote the 'first arrival time' function, such that it represents the *first time* the vessel reaches any $\mathbf{y}$. The limiting conditions on $\mathbf{X}_P(\mathbf{y_s}, t)$ are detailed below,

$$\mathbf{X}_P(\mathbf{y_s}, 0) = \mathbf{y}_s , \quad \mathbf{X}_P\left(\mathbf{y}_s, \widetilde{T}(\mathbf{y}_f)\right) = \mathbf{y}_f$$

Our goal is to determine the optimal controls for $\hat{h}(t)$ and $F_P(t)$ that minimize $\widetilde{T}(\mathbf{y}_f)$. Let $T^*(\mathbf{y}_f)$ be the optimal travel time to reach $\mathbf{y_f}$, and $\mathbf{X}^*_P(\mathbf{y_s}, t)$ be the optimal trajectory.
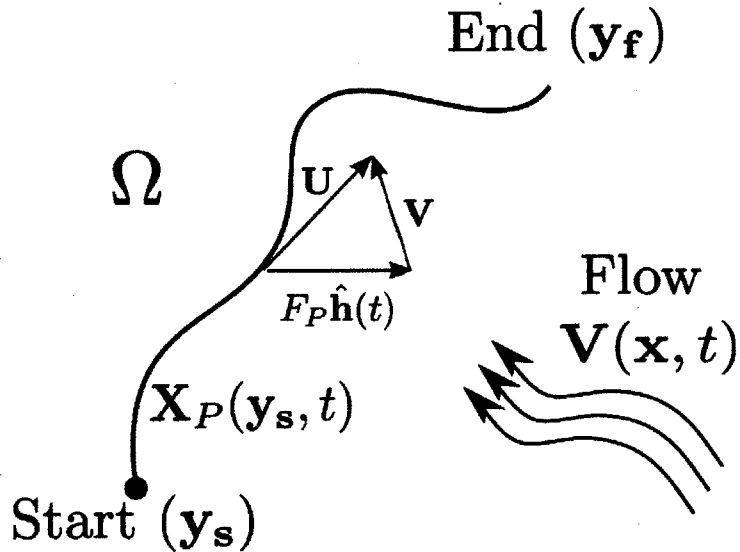
Figure 3. A path $\mathbf{X}_P(\mathbf{y}_s, t)$ between the start point $(\mathbf{y}_s)$ and an end point $(\mathbf{y}_f)$, in flow-field $\mathbf{V}(\mathbf{x}, t)$ (after Lolla et al, 2013).

3.1 Reachability Front Calculation

The infinite number of control choices available to a vessel renders the determination of the reachability front quite complex. At any moment, the vessel may choose from an infinite number of heading directions, and for every time step after that there are further infinite choices, and so on. Due to this, finding all possible paths and choosing the fastest is seemingly impossibly computationally intensive. This has resulted in approximate solutions being explored more than exact solutions. Following the work of Lolla et al. (2012, 2013), this thesis explores an exact solution.

The primary problem with many of these approximate solutions is that they require a set of heuristic control decisions for the vessel to follow. The simplest heuristic steering rule may be to always steer in the direction of your destination [LaValle, 2006]. While these heuristic decision controls typically make for cheap path planning solutions, they are rarely optimal and are not guaranteed to find a feasible path.

Heuristic controls suffer even greater when the flow-field environment is dynamic, as the heuristic control then becomes a function of the velocity field. Again, this method can be executed for every possible control decision, choosing the fastest path once all decision trees have been calculated, but this requires excessive storage and computing power, especially for a method that cannot be guaranteed optimal.

Starting from the origin of the ship, our path planning methodology relies on determining the reachable set and reachability front. The reachable set may be defined as the set of points that the vehicle can reach within a given time frame, where the reachability *front* is the boundary of this set. Keeping track of this front as the time frame is increased allows us to determine the fastest time possible to get from the point of origin to that of the destination. The optimal path is then gained by tracing the point of the reachability front that makes first contact with the point of destination. See Figure 4 [Lolla et al, 2013] below for an illustration this reachability front concept.
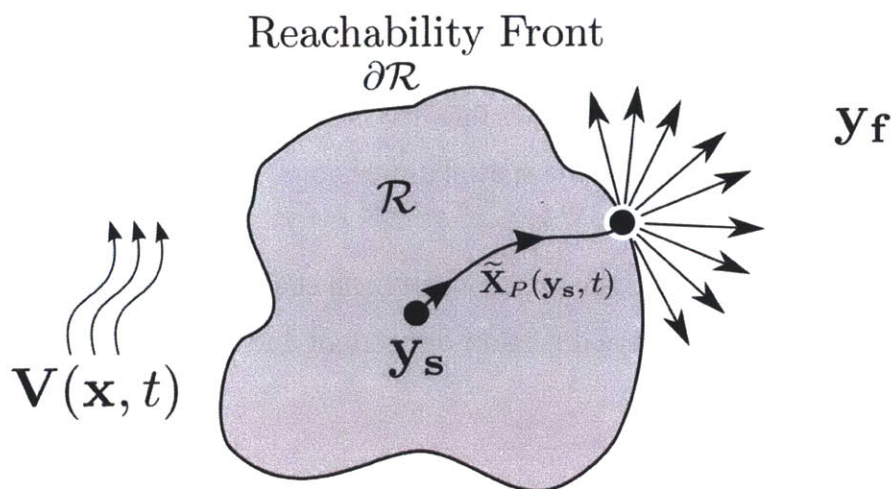

Figure 4. The shaded region R shows the area that can be reached within time t, while ∂R shows the reachability front at time t. [after Lolla et al, 2013]

3.2 New modified Level Set Equation

Level set methods [Sethian, 1999] are proven tools to in tracking the progression of fronts, for example, the interface between two immiscible fluids. Using level set methods is an effective option for modeling the implicit front and capturing the interaction between fluid forcing and the advancement of the front. They may also be applied to problems where the speed of the evolution of the interface is dependent on local or global properties of the environment.

Level set methods track the evolution of an interface by making it a hyper-surface in one higher dimension. For example, if working a 2D problem, the interface is characterized as the zero contour of a 2D scalar field and the advancement of this scalar field determines the front's movement. Thus, in effect it becomes a 3D problem where time is the third dimension. Adding this higher dimension allows the automatic handling of merging, pinching of fronts, and other topological changes. Level sets are an implicit representation of the front rather than explicit.

The new modified level-set equation derived in Lolla et al (2012, 2013) solves the above problem, rigorously providing the solution to the above problem. It evolves the reachability front in time and space. Using this methodology and the corresponding computational schemes, it was shown in [Lolla et al, 2013] that the computational cost of the method grows linearly as the number of vehicles is increased and geometrically with the spatial dimensions.

In what follows, we will extend the approach of Lolla et al. (2012, 2013) to the case of nominal vehicle speed that are direction-dependent due to drag forces by winds (and/or waves and currents).

4. Test Cases

In this section, our path-planning algorithm is checked for two cases that have known solutions, utilizing spatially variable winds. Next, we describe other test cases with obstacles (e.g. islands) and where the winds are time-dependent. Unless otherwise mentioned, the type of vessels we are experimenting with in our test cases correspond to the vessels for which we provided the speed versus direction chart, i.e. the polar diagram, earlier in Fig 2.

4.1 Validation with spatially-dependent winds

The first example is based on a destination which is directly downwind of the vessels origin. This case is one where we know the time optimal path to be a straight line from the origin to the destination (for the type of vessels with the polar diagram shown in Fig 2).

The figure below shows the reachability front for this Case 1 at the moment when it first reaches the destination. The origin is the point, lying at (0.3, 0.5), and the destination is marked by the star, lying at (0.7, 0.5). There is wind in the positive x-direction, hence why the reachability front is warped in that direction, displaying that a vessel may travel from x =0.3 to x=0.7 in less time than traveling to x=0.2.
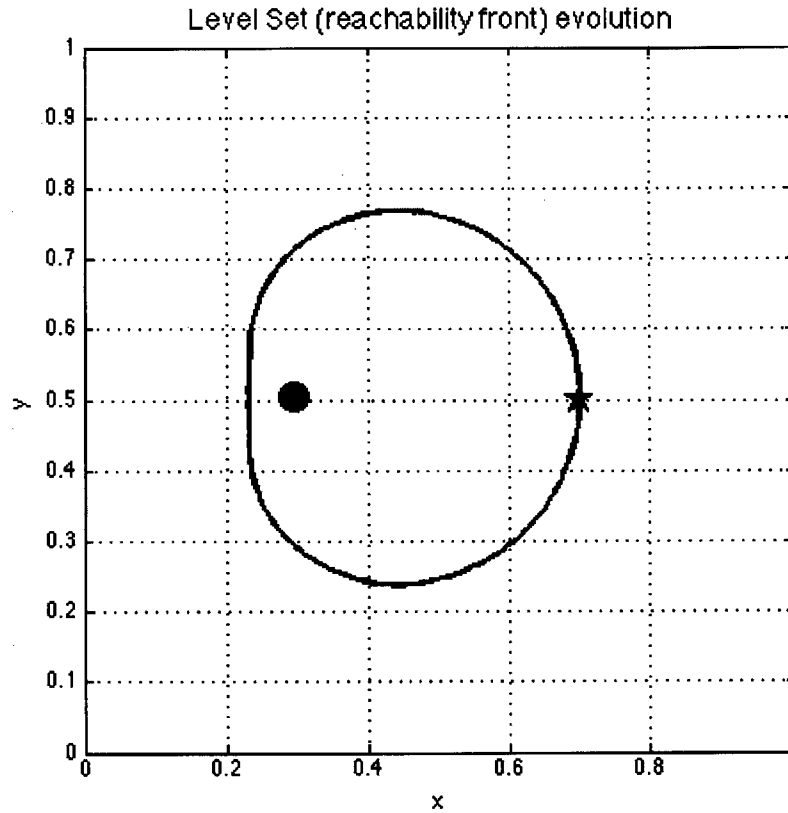
Figure 5. The reachability front at time t^* when the destination point (the star) is first reached.

The figure below shows the path found by the methodology for Case 1. We can see that it is a straight line from the origin to the destination. This is encouraging; as it's the path we know to be correct analytically.
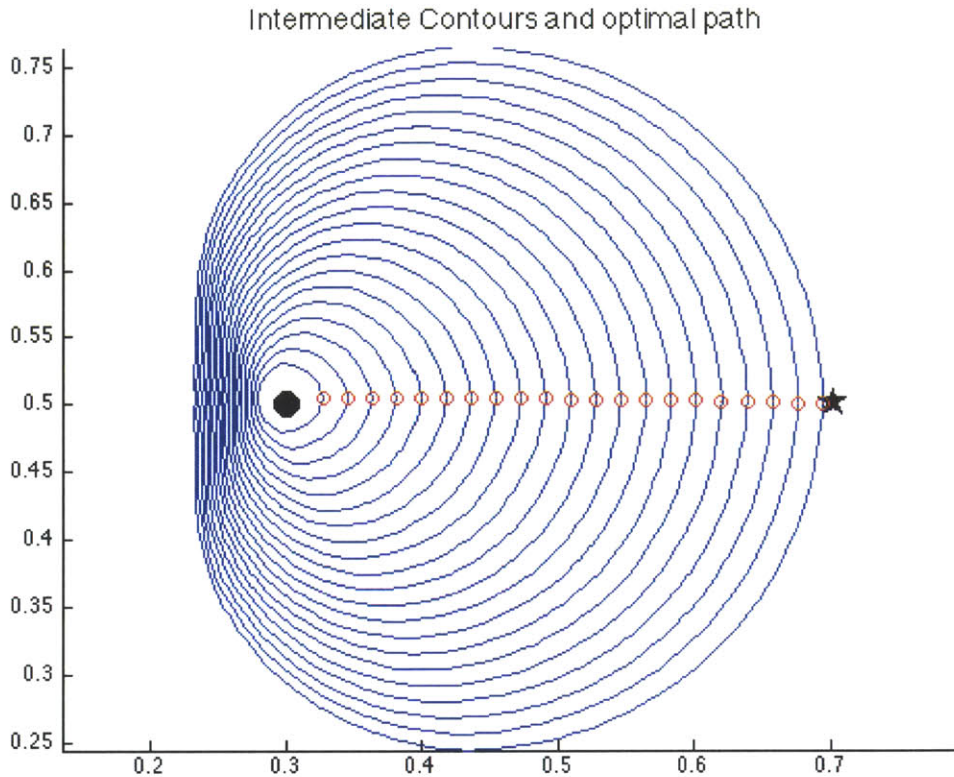
Figure 6. Path found (detailed by red circles) for case 1, where there is wind going in the positive x-direction, with no current flow-fields.

In the second case, we will consider two separate areas of wind. The spatial domain is divided down the middle, split from top to bottom, with wind heading straight down in the left-hand half and wind (of equal magnitude) heading straight up in the right-hand half. The destination and origin for this case each lie on the same horizontal line on the plane, equidistant from the center of the grid's centerline (where the wind changes direction). For this case, we know the analytical solution to be a V-shaped path, where the vessel takes advantage of the wind on either side of the halfway divider.

In summary for this Case 2, we expect thus to see the vessel utilize the wind on both sides on the midway divider, yielding a V-shaped path.
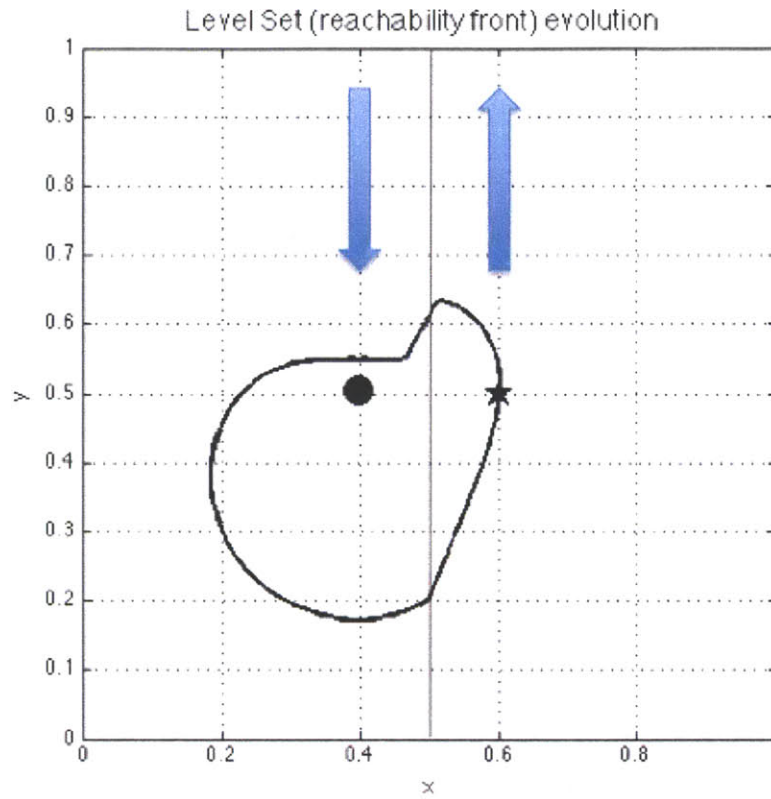
Figure 7. The reachability front at time t^* when the level set first reaches the destination. For case 2, where there are two distinct wind patterns acting oppositely in each half of the plane.

The figure below details the time-optimal path found. This path is just as expected.
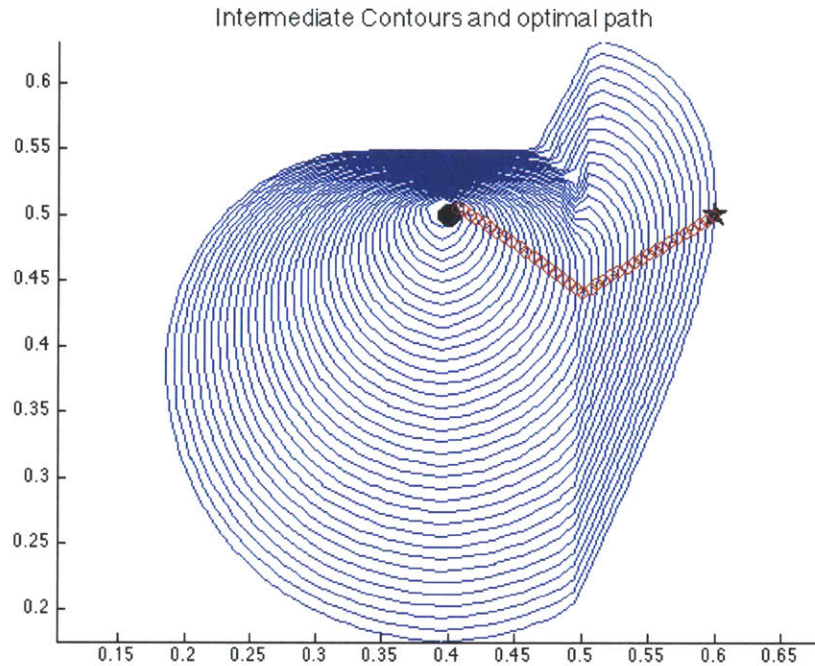
Figure 8. The time-optimal path for case 2, shown by red circles, found by solving the particle tracking equation backwards in time.

4.2 Flow around an island

Now we will move on to a more interesting scenario, flow around a circular island in the ocean. We will begin merely by treating the island as an obstacle, with the only flow-field being that of wind coming off the coast. From this, we will next add a realistic time varying current field about the island, including eddies, etc., to illustrate how a ship might realistically navigate this scenario. And lastly, we will make the wind time varying in direction and magnitude to illustrate the potential of this methodology.

4.2.1 Flow around an island: fixed winds, no currents

For the case where there is only simple wind, and no currents, we expect the vessel to navigate around the top-side of the island, as the wind is coming off the coast at a 26.6° angle as shown in Figure 9 below.
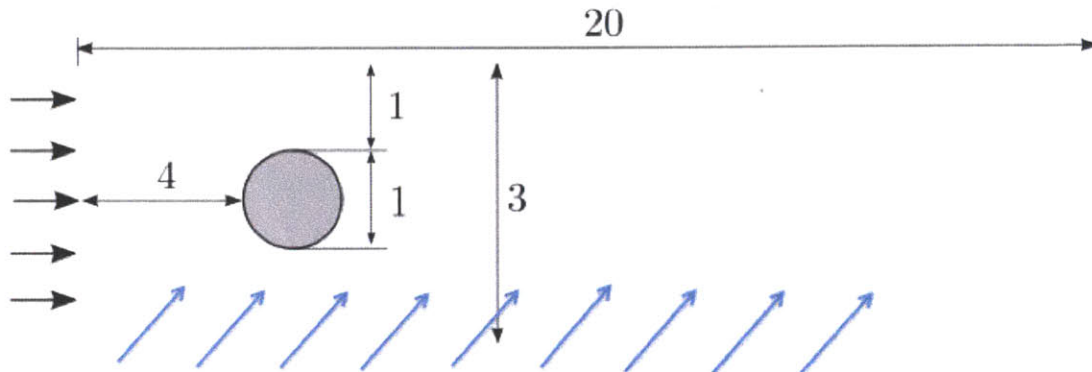
Figure 9. The space in which the case will be executed. A rectangle 20 units long and 3 units tall, where the island is located 4 units in with a diameter of 1 unit. The blue arrows on the bottom show the wind blowing off the coast at a 26.6° angle.

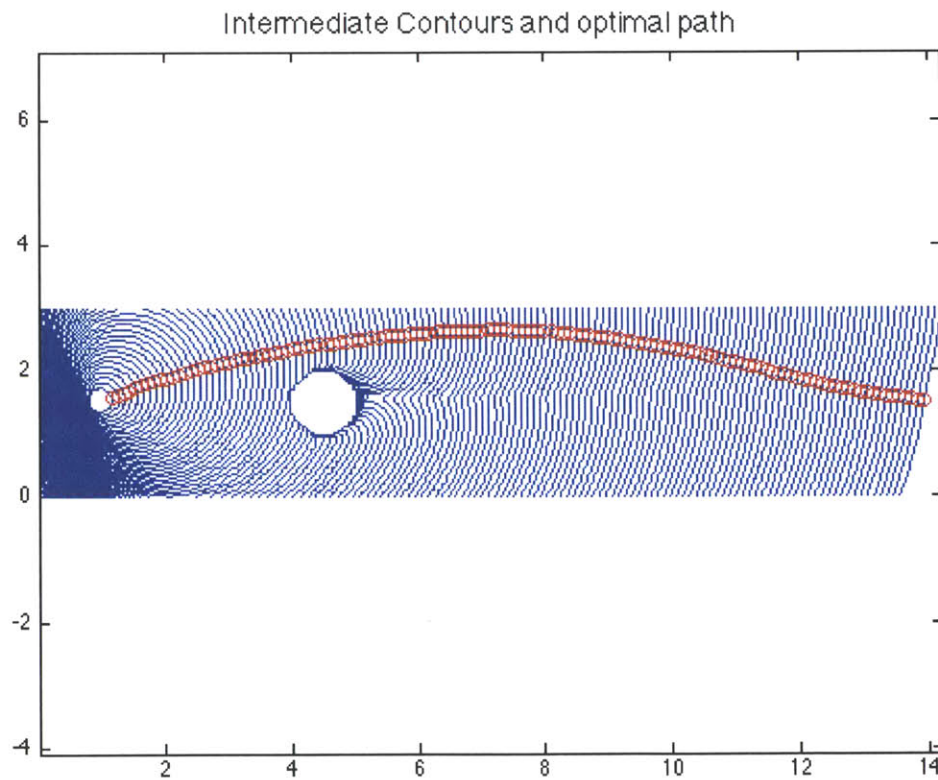The path found using our methodology is shown below in Figure 10.

Intermediate Contours and optimal path

Figure 10. Path found around island for case when there is wind coming off the shoreline and no current flow-field.

4.2.2 Flow around an island: fixed winds, but with currents

The current flow-field is shown in the four plots below, illustrating how the flow-field varies in time over the course of a single trial.

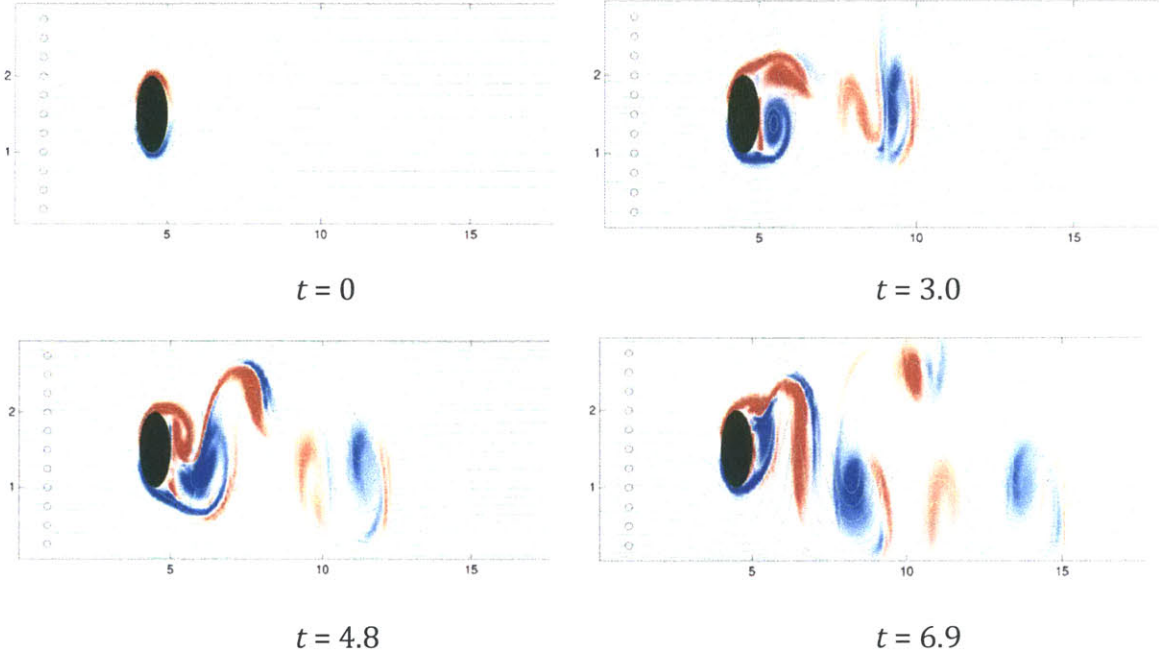$t = 0$

$t = 3.0$

$t = 4.8$

$t = 6.9$

Figure 11. The flow-field shown at four different instances in time over the course of a trial.

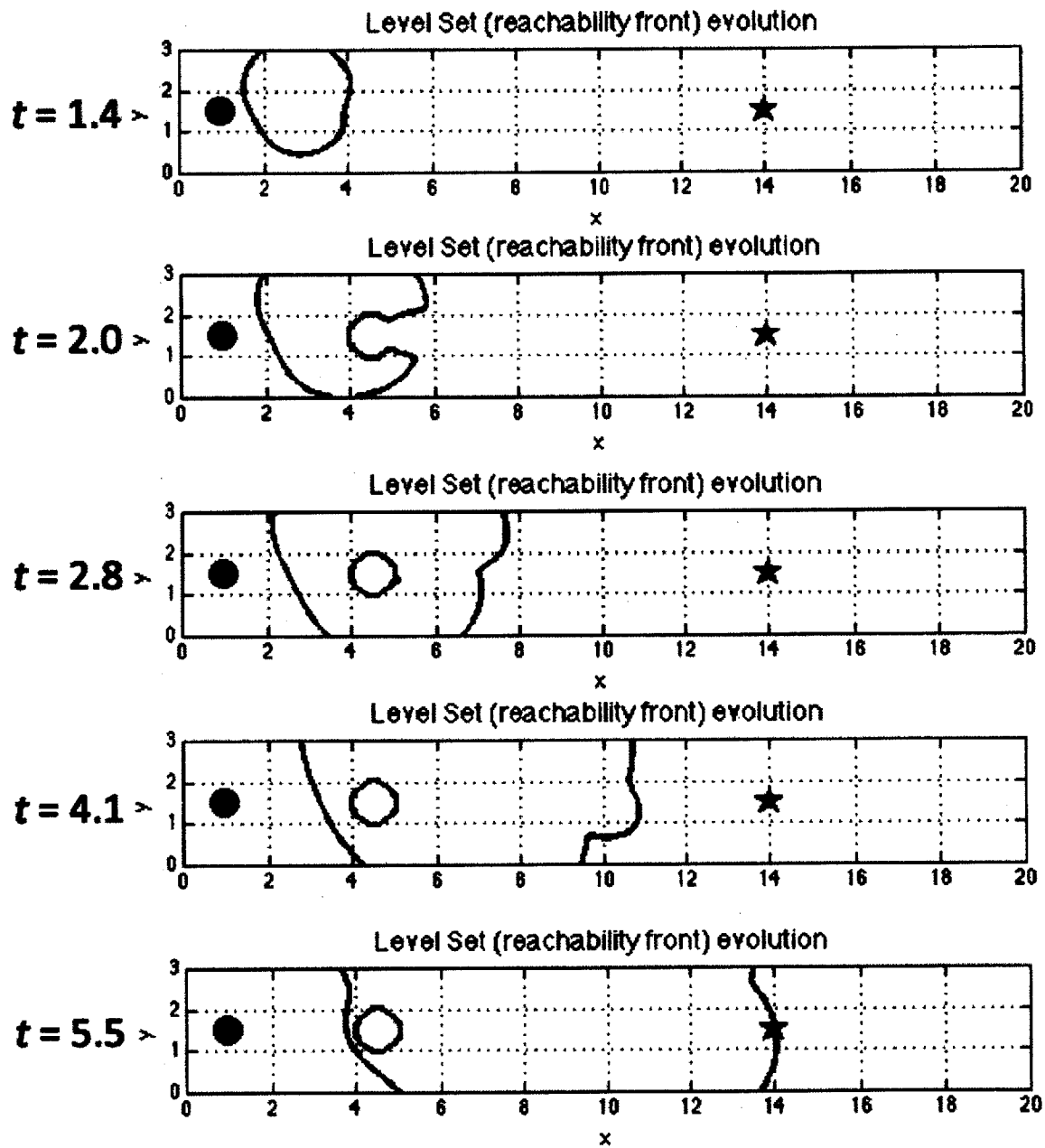The reachability front is shown below in Figure 12 at five different times.

Figure 12. The reachability front at five different times during the evolution of the flow-field and the vessels path around the island.

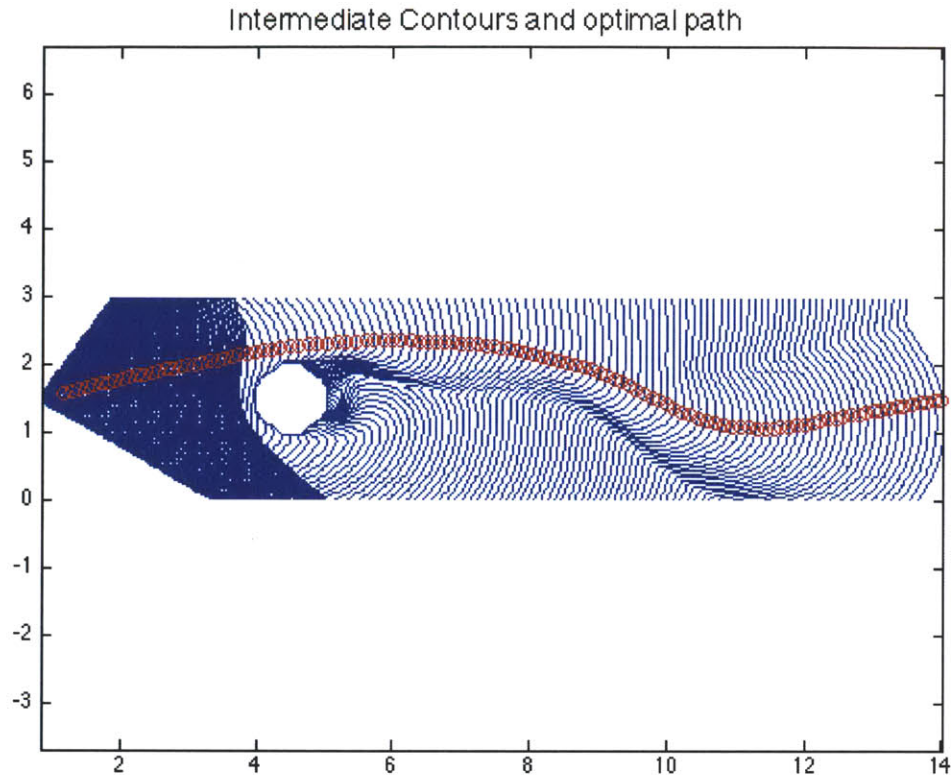Shown below is the optimal path found for this case.

Figure 13. Time-optimal path for case with constant wind coming off the shore and flow-field about the island.

4.2.3 Flow around an island: time-dependent winds and currents

In this last case we will look at the same current flow-field as above, however the wind will be time-dependent, varying in direction and magnitude. The x-component of the wind will vary in magnitude according to a *sine* function, however it will always remain positive, between 0 and 1. The y-component of the wind will also oscillate according to a *sine* function, however it will vary between -1 and 1. The reachability front is shown below at five different times.
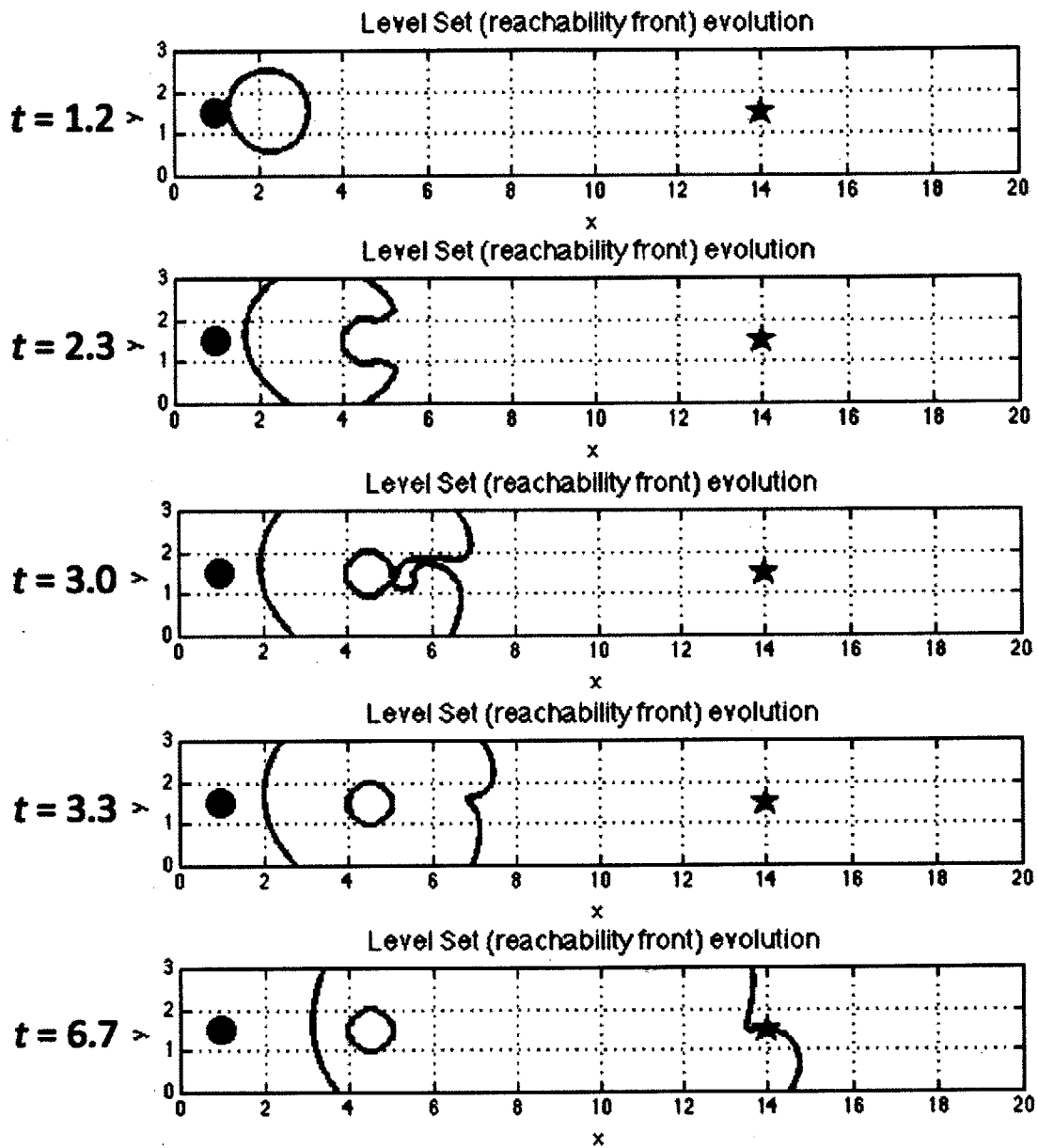
Figure 14. The reachability front at five different times during the evolution of the flow-field and the vessels path around the island. For case with time-dependent wind and current flow-field.

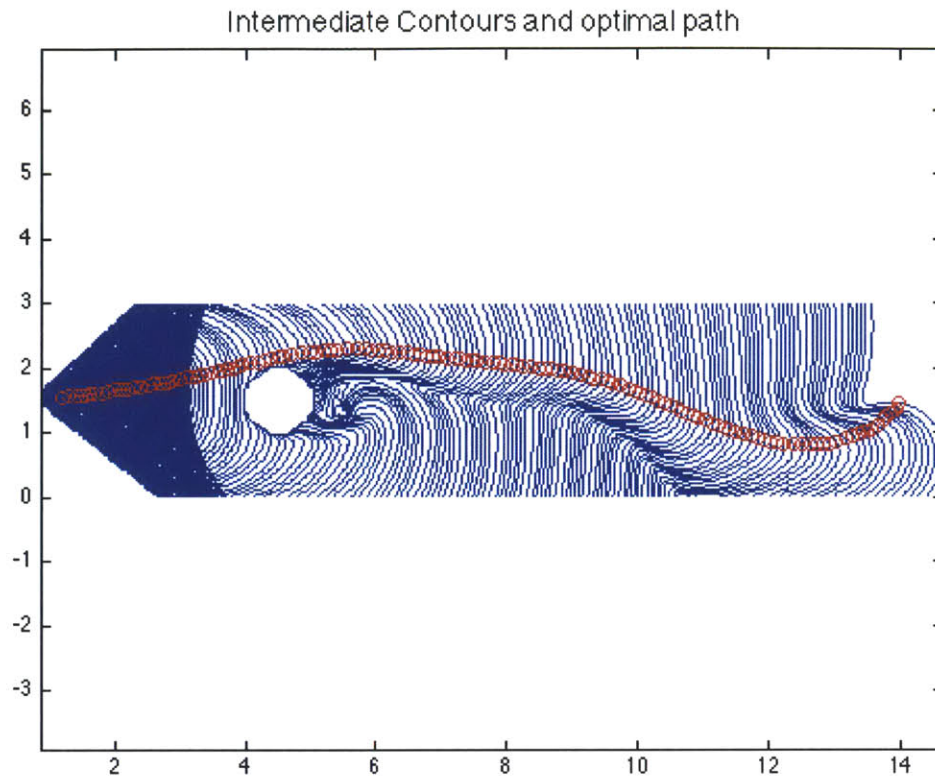The optimal path is shown below in Figure 15.

Figure 15. Time-optimal path for case with time-dependent wind and current flow-field.

5. Summary and Conclusion

Building on the results of Lolla et al (2012, 2013), this thesis proposed a methodology for determining the exact fastest path between two locations, for a vessel traveling in a time dependent flow field and with a nominal engine speed that is direction-dependent. As in Lolla et al (2012, 2013), the computational cost that grows linearly with the number of vehicles and geometrically with the spatial dimensions. A modified level set equation was utilized to track the progression of the 'reachability front' of a vessel in a time varying flow field, accounting for the direction-dependent drag due to winds along the path. To represent this direction dependent constraint, a polar diagram is employed. This front was evolved, taking into account the vessels steering power and advection from the environment, until it reached the destination, at this point the optimal time had been found. Then the

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time-optimal path was deduced from the progression of the reachability front by solving a particle backtracking equation. We illustrated the methodology in varied conditions, including simple fixed winds, spatially variable winds, time-dependent winds, with and without currents and with and without obstacles. To illustrate the effect of vessel shapes and capabilities, different polar diagrams were employed.

6. References

[1] Carroll K, McClaren S, Nelson E, Barnett D, Friesen D, William G (1992) AUV path planning: An A* approach to path planning with consideration of variable vehicle speeds and multiple, overlapping, time-dependent exclusion zones. In: Proceedings of the Symposium on Autonomous Underwater Vehicle Technology, pp 79-84

[2] Garau B, Bonet M, Alvarez A, Ruiz S, Pascual A (2009) Path planning for autonomous underwater vehicles in realistic oceanic current fields: Application to gliders in the Western Mediterranean Sea. Journal of Maritime Research 6(2):5-22

[3] Isern-Gonzlez J, Hernndez-Sosa D, Fernndez-Perdomo E, Cabrera-Gmez J, Domnguez-Brito AC, Prieto-Maran V (2012) Obstacle avoidance in underwater glider path planning. Journal of Physical Agents 6(1):11-20

[4] J/Boats. (2007) J/30 IMS Velocity Prediction. [Online].
http://www.jboats.com/j30/j30polar.htm

[5] Latombe JC (1991) Robot Motion Planning. Kluwer Academic Publishers, Boston, MA

[6] LaValle SM (2006) Planning Algorithms. Cambridge University Press, Cambridge, U.K.

[7] Lermusiaux P.F.J, T. Lolla, P.J. Haley. Jr., K. Yigit, M.P. Ueckermann, T. Sondergaard and W.G. Leslie, 2014. *Science of Autonomy: Time-Optimal Path Planning and Adaptive Sampling for Swarms of Ocean Vehicles.* Chapter 11, Springer Handbook of Ocean Engineering: Autonomous Ocean Vehicles, Subsystems and Control, Tom Curtin (Ed.). In press

[8] Lolla T, Lermusiaux PFJ, Ueckermann MP. Haley PJ (2013) Time-optimal path planning in dynamic flows using level set equations: theory and schemes. Department of Mechanical Engineering, Massachusetts Institute of Technology

[9] Mannarini G, Coppini G, Oddo P, Pinardi N (2013) A Prototype of Ship Routing Decision Support System for an Operational Oceanographic Service. International Journal on Marine Navigation and Safety of Sea Transportation. Volume 7, Number 1. DOI: 10.12716/1001.07.01.06

[10] Ge L, Li Y (2013) Non-Isotropic Planar Motion Planning for Sailboat Navigation. Master's Thesis in Embedded and Intelligent Systems 120 ECTS, Halmstad University. 91pp.

[11] Lolla T (2012) Path planning in time dependant flows using level set methods. Master's thesis, Department of Mechanical Engineering, Massachusetts Institute of Technology

[12] Lolla, T.; Ueckermann, M.P.; Yigit, K.; Haley, P.J.; Lermusiaux, P.F.J., 2012, *Path planning in time dependent flow fields using level set methods,* 2012 IEEE International Conference on Robotics and Automation (ICRA), 166-173, 14-18 May 2012, doi: 10.1109/ICRA.2012.6225364.

[13] Rao D, Williams SB (2009) Large-scale path planning for underwater gliders in ocean currents. In: Proceedings of Australasian Conference on Robotics and Automation

[14] Rhoads B, Mezic I, Poje A (2010) Minimum time feedback control of autonomous underwater vehicles. In: Decision and Control (CDC), 2010 49th IEEE Conference on, pp 5828 – 5834

[15] Sethian JA (1999) Fast marching methods. SIAM Rev 41(2):199-235

[16] Sethian JA (1999) Level Set Methods and Fast Marching Methods: Evolving Interfaces in Computational Geometry, Fluid Mechanics, Computer Vision, and Materials Science. Cambridge University Press: Cambridge, U.K.