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# Queueing Theory Analysis of Labor & Delivery at a Tertiary Care Center

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## Abstract

Labor and Delivery is a complex clinical service requiring the support of highly trained healthcare professionals from Obstetrics, Anesthesiology, and Neonatology and the access to a finite set of valuable resources. In the United States, the rate of cesarean sections on labor floors is approximately twice as high as considered appropriate for patient care. We analyze one month of data from a Boston-area hospital to assess how well the labor and delivery process can be modelled with tools from queueing theory. We find that the labor and delivery process is highly amenable to analysis under queueing theory models. We also investigate the problem of high cesarean section rates and the potential effects of resource utilization of lowering the rate of cesarean section.

## Introduction

Providing quality to an increasing population with finite resources is a challenging problem bringing together healthcare professionals, lawyers, accountants, and engineers to design better systems (Baum et al., 2014) (Ben-Tal et al., 2011) (Konrad et al., 2013). While the emergency room is the primary hub of patient activity at the hospital, the Labor and Delivery (L&D) floor is a critical component of patient care. The challenges on the L&D floor are unique in that the floor must be staffed and equipped to perform triage, handle emergent surgery, and conduct regular inpatient care for adults *and* neo-natal (i.e., newborn babies) mothers.

One critical concern for care management in Labor and Delivery is the high rate of cesarean section. In 1965, the rate of cesarean section was 4.5% (Taffel et al. 1987). Since 1965, the rate of cesarean section has steadily increased to a national average of 32.8% as recently as 2011 (Hamilton et al. 2012). Some studies have pointed at a "casual" attitude towards caesarian sections (Baicker et al. 2006, Clark et al. 2007). Others have pointed to doctors not properly informing mothers of the value of a vaginal birth (Declercq et al. 2013). Another study points to doctors performing pre-emptive cesarean section as a defense against malpractice lawsuits in the event that a vaginal delivery did not result well (Sakala et al. 2013). Yet, even with knowledge of these factors, the cesarean section rate has not been brought under control. Regardless of the cause of a cesarean section, studies typically agree that cesarean section rates should be somewhere around 15% (Althabe and Belizan 2006).

In this paper, we seek to perform an overall analysis of the process of Labor and Delivery from the lenses of queuing theory. We seek to answer how well processes in Labor and Delivery can be approximated by queueing theory models. Second, we seek to understand whether the workload experienced by doctors vis-á-vis bed utilization has an effect on the method of delivery. Lastly, we draw from queuing theory to understand the effect of decreasing the cesarean section rate to the desired 15% would have of resource utilization of a labor floor.

## Labor and Delivery

The L&D process is complex, consisting of many possible routes through various care centers before giving birth. We first describe the various steps in the overall Labor & Delivery. Next, we discuss how mothers are matched with a set of healthcare professionals for their care.

### *Patient Flow*

A simplified process map is shown in Figure 1. In general, a pregnant woman (hereafter referred to as a mother) receives care. In general, a woman initially arrives at L&D triage for evaluation in response to either an obstetrician's direction or the mother's concern. Mothers are evaluated in triage by a triage nurse and a physician. If the team feels that the patient can follow up with an obstetrician as an outpatient, the mother will be sent home with care instructions. If the mother is found to be in labor (i.e., contractions have begun and the cervix has started to dilate), the mother will be admitted to the L&D floor.

If the healthcare team thinks the mother should be observed due to the likelihood of starting to actively labor or various comorbidities that may require prompt medical intervention, the mother will be admitted to the antepartum floor (or ward). If the mother's or baby's health becomes of concern or the mother has begun to labor, she will be moved to the L&D floor. Otherwise, if the mother's condition has abated and labor is unlikely to begin soon, the mother can be discharged home.

Once a mother has been admitted to the L&D floor, she will be monitored by a healthcare team throughout the labor and delivery process. Approximately two-thirds of babies will be delivered vaginally. For nulliparous mothers (i.e., having never borne a child), this process typically lasts longer than for multiparous (i.e., having borne one or more children), mothers. After a spontaneous vaginal delivery, the mother is moved to the postpartum floor (or ward) for approximately two days before discharge.

If the health of the mother or baby is critical, the healthcare team may decide a cesarean section is in the best interest of the baby and the mother. The rate of cesarean sections can be approximately one-third for tertiary care centers. The mother will be taken to one of the operating rooms on the labor floor, the cesarean section will be performed, and the mother will be taken to a recovery room on the labor floor for a short duration for observation. After sufficient time in the recovery room, the mother and baby will be transferred to the postpartum floor for approximately four days before discharge. In all cases, it is possible that the baby will be transferred to the neonatal intensive care unit if the baby needs acute medical attention but that is beyond the scope of this timeline.

### *Matching of Mothers and Healthcare Professionals*

During pregnancy, women are typically monitored during regular visits to their obstetrician as an outpatient. The obstetrician they select for their care is a member of a team of obstetricians. Team members take turns taking call on the Labor and Delivery floor. While a member of the team is taking call, he or she is directly responsible for managing the care of any women seen by his or her team. Ideally, a woman's obstetrician will deliver her baby; however, because of the randomness in the duration of gestation, a member of the team who is not the woman's primary obstetrician may deliver the baby. The L&D floor in a hospital may support multiple teams who concurrently share the hospital's resources.

It can happen that an obstetrician is delivering a baby in one room, when a second woman under his care enters the second stage of labor (i.e., the cervix is fully dilated) and has started pushing. Different hospitals have different systems models for handling this situation. It is the practice of the L&D floor at the Boston-area hospital for which we perform our analysis that an obstetrician from a different team will assist in delivering a baby if the primary obstetrician is occupied.

## Data Analysis

To understand the process as an engineering system, we analyzed data available from a Boston-area hospital for the month of September 2014. This month was chosen because it is the most recent month for which we were able to access a complete record. The data provided the time a mother was registered in each of the care centers (e.g., L&D triage, L&D floor, and the various inpatient facilities) and the time the mother left each of the care centers.

We note that the data allows one to infer whether a mother was in an antepartum or postpartum ward (i.e., floor), so we aggregate these services as a general inpatient ward.

### *Interarrival Times*

Interarrival times at the hospital are subject to a number of outside factors. Women who are experiencing contractions or other concerning symptoms may first go to their obstetrics clinic to be seen as an outpatient. The obstetrician may then send the mother to L&D triage for evaluation. Obstetrics clinic often operates during normal work hours (e.g., 8am-5pm). Thus, mothers may tend to arrive at L&D Triage in a higher frequency during those hours due to referrals from clinic. This bunching may have a cascading effect to the other care centers. For other reasons (e.g., convenience, circadian rhythm, etc.), arrivals of mothers may tend to cluster around certain intervals.

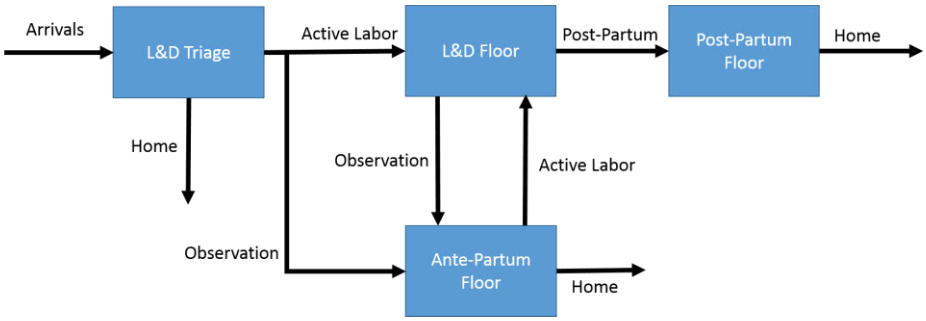


*Figure 1 - Simplified process map for L&D.*

Arrival processes in urban centers (e.g., customers at restaurants, pedestrians at a crosswalk, etc.) can often be approximated as a Poisson arrival process, where interarrival times come from an exponential distribution (Equation 1). This mathematical representation has advantages because it allows for powerful analysis of system performance and can inform system design.

In a Poisson process, the probability of having k arrivals in some interval $[t, t+\tau]$ comes from a Poisson distribution (Equation 2) where $N(t+\tau) - N(t)$ is the number of arrivals in interval $[t, t+\tau]$. In a Poisson arrival process, interarrival times have a mean of $1/\lambda$ and standard deviation of $1/\lambda$. Thus, if the interarrival times at a L&D care center were exponential, we would expect to see that the mean and standard deviation were equal.

$$P_T(t) = \lambda e^{-\lambda t}$$

*Equation 1*

$$P[N(t+\tau) - N(t) = k] = \frac{e^{-\lambda\tau}(\lambda\tau)^k}{k!}$$

*Equation 2*

To determine the overall arrival process, we constructed histograms for the interarrival times for mothers arriving in L&D Triage (Figure 2), the L&D Floor (Figure 3), and the inpatient facilities (Figure 4). The mean, standard deviation, and percent difference of the two measures are shown in Table 1. The data shows that the arrival process that the interarrival times quite nicely approximate an exponential distribution.

### *Service Time*

As with the interarrival process of women to the various L&D care centers, the service time (e.g., length of stay) at the care centers is influenced by a

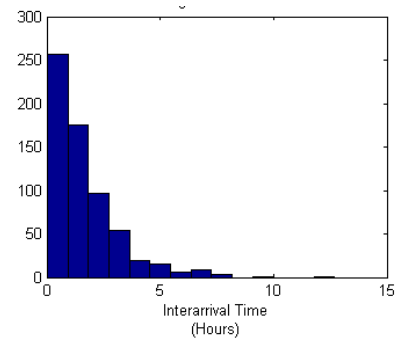


*Figure 2 - Histogram of the interarrival times of mothers arriving at L&D Triage.*

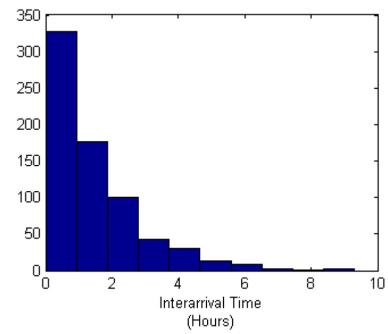


*Figure 3 - Histogram of the interarrival times of mothers being admitted to the L&D floor.*

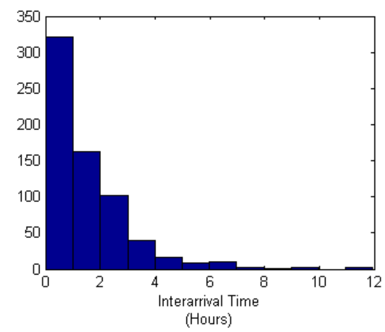


*Figure 4 – Histogram of the interarrival times of mothers being admitted to one of the inpatient floors (e.g., Postpartum, Antepartum, etc.)*

| Interarrival Times | $\mu$ | $\sigma_s$ | % Difference |
|---|---|---|---|
| L&D Triage | 1.62 | 1.56 | 2.21% |
| L&D Floor | 1.44 | 1.47 | -1.03% |
| Inpatient Ward | 1.52 | 1.60 | 2.56% |

*Table 1 – Interarrival times at L&D Triage, the L&D Floor, and on the Inpatient Wards.*

number of factors. In triage, mothers who need to be admitted to the floor may be delayed while the L&D floor prepares to receive them. Mothers in triage who can go home may have an expedited service time due to the reduced acuity of the condition. Women on the antepartum or postpartum floors are typically discharged once or twice per day in batches when doctors round on their mothers.

Women in active labor have three distinct modes of service time. Nulliparous mothers typically experience significantly longer labor than multiparous mothers. Furthermore, doctors may intervene via cesarean section in the normal course of labor if the mother or baby's health becomes of serious concern. The duration of a cesarean section is typically much shorter than the duration of a spontaneous vaginal delivery.

To determine the overall level service time process, we constructed histograms for the service times for mothers in L&D Triage (Figure 5), the L&D Floor (Figure 6), and the inpatient facilities (Figure 7). The mean, standard deviation, and percent difference of the two measures are shown in Table 2. The data shows that the service times on the L&D Floor closely approximates an exponential distribution. The distribution of service times on the inpatient ward reasonably approximate an exponential distribution. Service times in Triage have a long, right-tail with some service times taking much longer than one would expect in an exponential distribution.

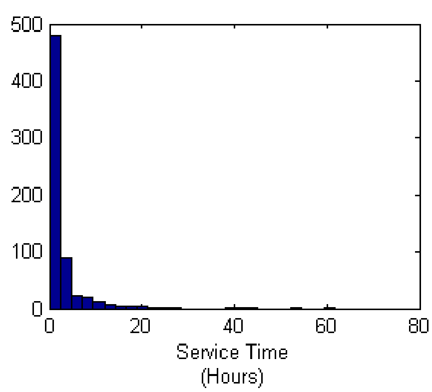


*Figure 5 - Histogram of the service times of mothers arriving at L&D Triage.*

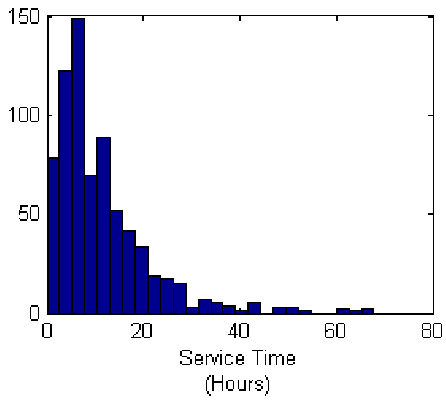


*Figure 6 - Histogram of the service times of mothers arriving on L&D Floor.*

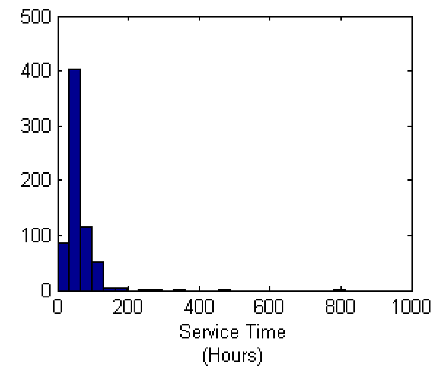


*Figure 7 - Histogram of the service times of mothers arriving on an inpatient ward.*

| Service Times | $\mu$ | $\sigma_s$ | % Difference |
|---|---|---|---|
| L&D Triage | 2.26 | 5.47 | -41.5% |
| L&D Floor | 11.2 | 9.95 | -5.91% |
| Inpatient Ward | 59.5 | 46.3 | 12.5% |

*Table 2 – Service times at L&D Triage, the L&D Floor, and on the Inpatient Wards.*

| Service Times | $\mu$ | $\sigma_s$ | % Difference |
|---|---|---|---|
| Nulliparous Vaginal Delivery | 12.6 | 8.56 | 19.1% |
| Multiparous Vaginal Delivery | 6.91 | 8.06 | -7.68% |
| C-Section | 6.47 | 9.43 | -18.62% |

*Table 3 – Service times at on the L&D Floor for different methods of delivery.*

While service times for the L&D floor closely approximate an exponential distribution overall, the method of delivery strongly influences the duration of the mother's stay on the L&D floor. Histograms reporting the distribution of the length of stay on the

L&D floor for nulliparous vaginal delivery, multiparous vaginal delivery, and cesarean section are shown in Figure 8, Figure 9, and Figure 10, respectively. The mean, standard deviation, and percent difference of the service times are shown in Table 3.

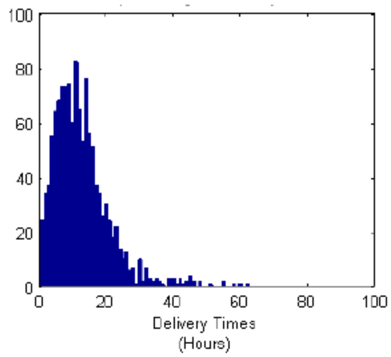


*Figure 8 - Histogram of the service times of nulliparous mothers giving birth vaginally on the L&D floor.*

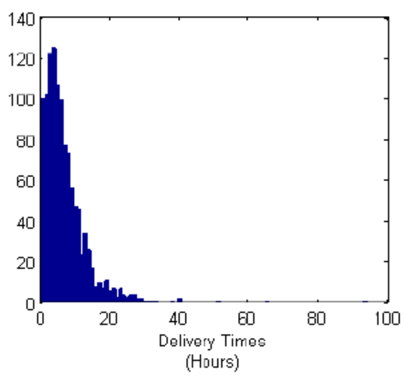


*Figure 9 - Histogram of the service times of multiparous mothers giving birth vaginally on the L&D floor.*

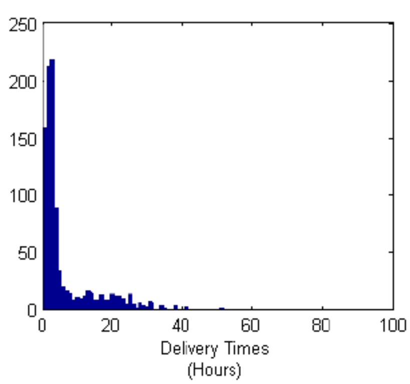


*Figure 10 - Histogram of the service times of mothers giving birth via cesarean section on the L&D floor.*

### *Possible Factors Influencing Service Times*

As we noted, service time in the hospital can be influenced by a number of factors. We sought to determine if the number of beds occupied in a care center (i.e., the number of mothers currently being served) influenced the service time of a given patient. We constructed box plots for the service time of mothers in L&D Triage (Figure 11), the L&D Floor (Figure 12), and the inpatient facilities (Figure 13) as a function of the number of beds occupied at the time of discharge from the care center. We will return to this analysis later.

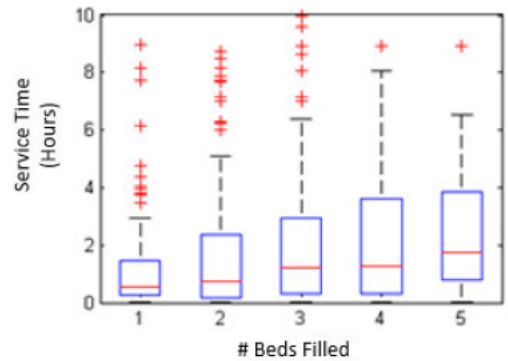


*Figure 11 – Boxplot reporting the distribution of the service times (i.e., time a mother spends in the care center) for mothers in L&D Triage as a function of the number of beds occupied in that center. Data is not reported if the sample size for a given number of occupied beds is less than 30.*

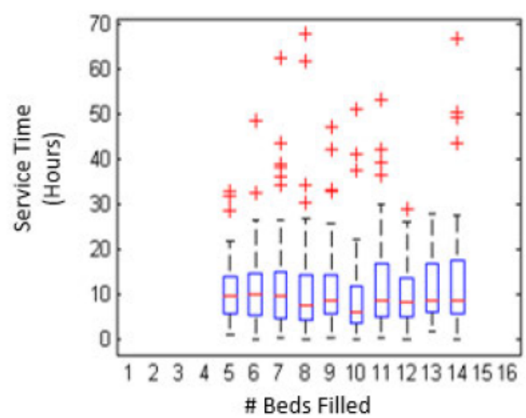


*Figure 12 – Boxplot reporting the distribution of the service times (i.e., time a mother spends in the care center) for mothers on the L&D Floor as a function of the number of beds occupied in that center. Data is not reported if the sample size for a given number of occupied beds is less than 30.*

| Inter-departure Times | $\mu$ | $\sigma_s$ | % Difference |
|---|---|---|---|
| L&D Triage | 1.60 | 1.60 | ~0.00% |
| L&D Floor | 1.48 | 1.54 | -1.98% |
| Inpatient Ward | 1.58 | 3.33 | 35.6% |

*Table 4 – Inter-departure times at L&D Triage, the L&D Floor, and on the Inpatient Wards.*

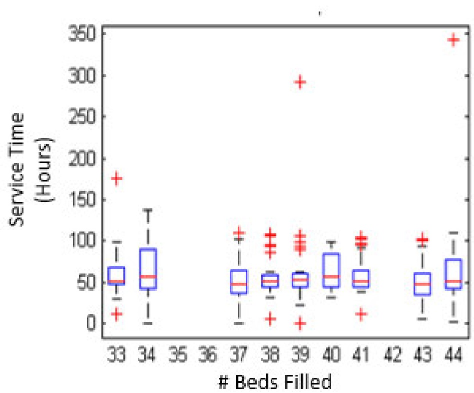


*Figure 13 – Boxplot reporting the distribution of the service times (i.e., time a mother spends in the care center) for mothers in one of the inpatient service floors. Data is not reported if the sample size for a given number of occupied beds is less than 30.*

### Inter-departure Times

We also consider the macro-level inter-departure time process. We constructed histograms for the service times for mothers in L&D Triage (Figure 14), the L&D Floor (Figure 15), and the inpatient facilities (Figure 16). The sample mean, standard deviation, and percent difference of the two measures are shown in Table 4. The data shows that the inter-departure times from L&D triage and the L&D Floor closely approximates an exponential distribution. Service times in the inpatient wards have a long, right-tail with some service times taking much longer than one would expect in an exponential distribution.

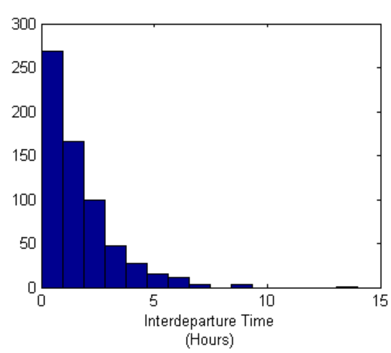


*Figure 14 - Histogram of the inter-departure times of mothers being discharged from L&D triage.*

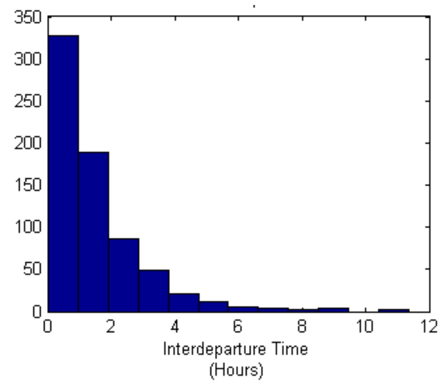


*Figure 15 - Histogram of the inter-departure times of mothers being discharged the L&D floor..*

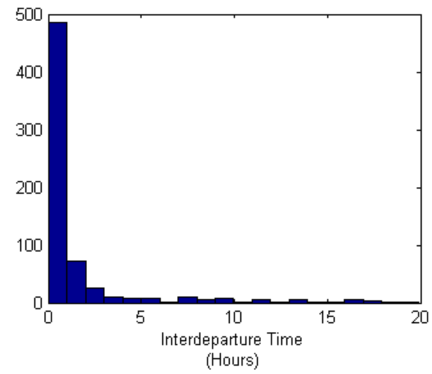


*Figure 16 - Histogram of the inter-departure times of mothers being discharged to one of the inpatient floors (e.g., Postpartum, Antepartum, etc.).*

### Bed Occupancy

One of the most salient measures of system performance is the number of beds occupied in a care center. Hospital beds and the associated resources are one of the primary drivers of the cost of care. Hospitals often employ a systems-level analysis to estimate the correct number of beds to provide to handle the patient population.

The hospital upon which my analysis is based has three L&D Triage beds and three Gynecology Triage beds that can be used as overflow. The L&D floor consists of thirteen L&D floor beds, three OR beds, and six beds in a recovery room, which can be used as overflow.

To determine the level of bed occupancy, we constructed histograms for the number of beds filled in L&D Triage (Figure 17), the L&D Floor (Figure 18Figure 7), and the inpatient facilities (Figure 19).

The mean, standard deviation, and 99th percentile are shown in Table 5.

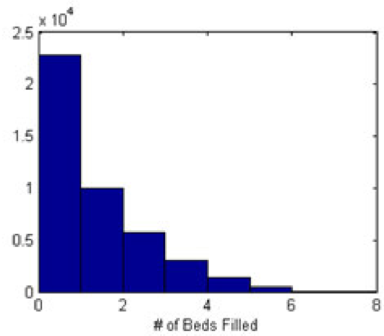


*Figure 17 - Histogram of the number of beds occupied in L&D Triage. The occupancy is sampled 1000 times per day.*

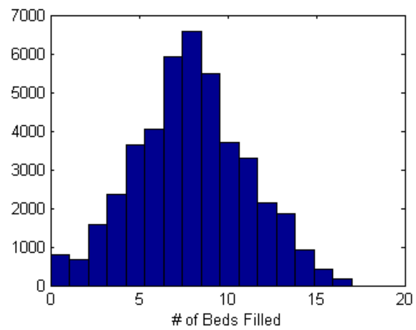


*Figure 18 - Histogram of the number of beds occupied on the L&D Floor (right). The occupancy is sampled 1000 times per day.*

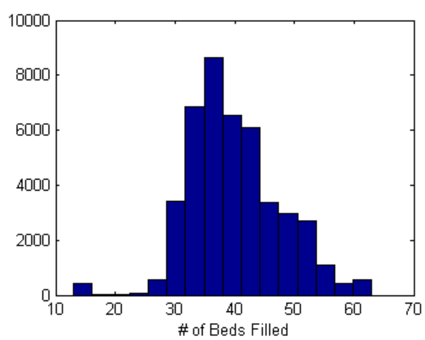


*Figure 19 - Histogram of the total number of beds occupied of on the inpatient floors (e.g., Postpartum, Antepartum, etc.).*

| Bed Occupancy | $\mu$ | $\sigma_s$ | 99th Percentile |
|---|---|---|---|
| L&D Triage | 1.65 | 1.46 | 6 |
| L&D Floor | 7.96 | 3.01 | 15 |
| Inpatient Ward | 40.1 | 7.75 | 61 |

*Table 5 – Bed occupancy at L&D Triage, the L&D Floor, and on the Inpatient Wards.*

# Queueing Theory Analysis

In this section, we draw from queuing theory to perform a theoretical analysis of the L&D system. First, we consider how well a queueing theory model predicts the number of beds occupied in each of the care centers. Second, we predict how likely a patient will have their baby delivered by their own team's obstetrician, known as the intra-response frequency

## *Bed Occupancy*

To determine the expected number of mothers in the system (i.e., bed occupancy), we assume that no mother will be turned away because of a lack of a bed. The nurses will use resources from other centers as overflow. We assume that interarrival times are exponentially distributed with rate $\lambda$ and mothers are processed with an average service time $1/\mu$.

Given these assumptions, an M/M/∞ queuing model is most amenable to analysis. Figure 20 shows the transition diagram for the M/M/∞ queue, and Equation 3 yields the probability of having n beds occupied at steady-state. We can find the expected number of beds occupied in steady state in Equation 4.

Table 6 shows the average, expected (Equation 4), and percent difference of the average and expected number of beds occupied in each care center. Despite the factors that can affect the process of patient care, the M/M/∞ is able to predict the average bed occupancy for all three care centers within 3%, which is an impressive result.

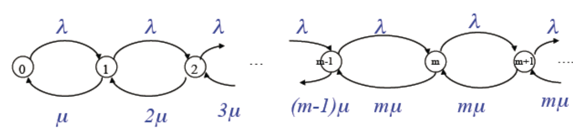


*Figure 20 - M/M/∞ transition diagram (Courtesy: MIT Course 16.76J Logistical and Transportation Planning Methods, Queueing Systems: Lecture 3).*

$$P_n = \frac{\left(\frac{\lambda}{\mu}\right)^n e^{-\left(\frac{\lambda}{\mu}\right)}}{n!}$$

*Equation 3*

$$E[L] = \frac{\lambda}{\mu}$$

*Equation 4*

| Bed Occupancy | $\bar{L}$ | $E[L]$ | % Difference |
|---|---|---|---|
| L&D Triage | 1.65 | 1.62 | -1.87% |
| L&D Floor | 7.96 | 7.80 | -2.08% |
| Inpatient Ward | 40.1 | 39.0 | -2.62% |

*Table 6 – Actual, predicted, and percent difference of the bed occupancy at L&D Triage, the L&D Floor, and on the Inpatient Wards.*

### Inter/Intra-Team Deliveries

It is natural for a mother to prefer having her baby delivered by a physician with whom she is more familiar. A mother would first prefer her primary obstetrician to deliver her baby. If the primary obstetrician is unavailable, then a mother would prefer that an obstetrician from the same team would deliver the baby. Lastly, the mother would prefer an obstetrician from a different team deliver the baby over a resident. We assume that there are a sufficient quantity of residents to deliver babies if all obstetricians are occupied.

We assume that mothers starting the second stage of labor arrive according to an exponential distribution with rate $\lambda$ and mothers are processed with an average service time $1/\mu$. Here, the duration of service is equal to the duration of the second stage of labor, when the mother is actively pushing. It is during this stage that we seek to understand how likely it is that an obstetrician from the desired team is present.

At the Boston-area hospital we analyzed, there are two main obstetric teams as well as a set of other are provider teams that serve a subset of the patient populations. We assume that 1/3 of the mothers arriving have a primary obstetrician in Team 1, 1/3 of the mothers arriving have a primary obstetrician in Team 2. We assume that the remaining 1/3 of mothers are distributed across the other obstetric teams on the labor floor.

While we cannot estimate $\mu$ directly from our data, we can use data from prior studies (Rouse, et al., 2001). The data provided from this study was from a cohort of 4,126 mothers. The duration of the second stage of labor was between 0-1 hours for 1,901 mothers, 1-2 hours for 1,251 mothers, 2-3 hours for 217 mothers, 4-5 hours for 97 mothers, and greater than 5 hours for 46 mothers. If we assume that average duration of the second stage of labor is equal to the weighted sum of the middle of the range of the bin (e.g., 0.5 hours for the 0-1 hour bin) and 5.5 hours for the >5 hours bin, then the average duration of the second stage of labor is 1.41 hours +/- 1.10 hours. This distribution is approximately exponentially distributed. Thus, we estimate the service rate $\mu = 0.709$ mothers per hour.

We model the 1/3 of mothers who are seen primarily by an obstetrician not in the first two teams as random erasures. Thus, we estimate from the data in our analysis that the arrival rate of mothers to the first two teams is $\lambda = \left(\frac{2}{3}\right)0.694 = 0.462$.

The hypercube (Larson, 1974) queueing model is amenable to analysis of the likelihood that a mother will receive care from her own team or from a different team. We assume for simplicity that there are two main teams on the L&D floor. Mothers who enter the second stage of labor when the obstetrician from their primary team is serving another mother are served by the obstetrician from the secondary team. Mothers who enter the second stage of labor when the on-call doctors from both teams are already serving mothers are assumed to be served by an obstetrics resident. If an obstetrician becomes available while one of his or her team's mothers is in the second stage of labor, that obstetrician would intervene to deliver the baby.

To understand how likely it is that mother will receive care from her team's obstetrician throughout the entire duration of the second stage of labor, we can approximate the system with a two-server hypercube model, as shown in Figure 21. In this figure, states are denoted as nodes, and transition probabilities are shown as weighted arcs connecting nodes. States $S_{0,0}$ represents a state where no mother is in the second stage of labor. $S_{1,0}$ represents the state where the obstetrician from Team 1 is attending to a mother in the second stage of labor and there are no other mothers in the second stage of labor (vice versa for $S_{1,0}$). $S_{1,1}$ refers to the state where both obstetricians from Team 1 and Team 2 are attending mothers in the second stage of labor and there are no other mothers in the second stage of labor.

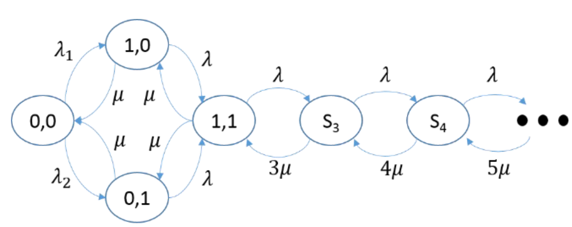


*Figure 21 – Infinite-server hypercube state space representation with two primary servers.*

We can use Equation 4 to calculate the steady-state probabilities that the system is in each state $S_i$ (i.e., a state where i beds are occupied), which are shown in Table 7. Here, State Q (i.e., queueing state) refers to all of the states in which mothers are in the second stage of labor but are not being seen by an obstetrician.

| | $S_0$ | $S_1$ | $S_2$ | $S_Q$ |
|---|---|---|---|---|
| $P(S_i)$ | 0.521 | 0.340 | 0.111 | 0.028 |

*Table 7 – Steady-state probabilities of being in state $S_i$ for the M/M/∞ queuing model shown in Figure 20.*

With these steady-state probabilities, we can determine the probability of being in the sub-states $S_{0,0}$, $S_{1,0}$, $S_{0,1}$, and $S_{1,1}$ in Equation 5-Equation 8.

$$P(S_{1,0}) = \frac{\lambda_1 P_{0,0} + \mu P_{1,1}}{(\lambda + \mu)} = 0.170$$

*Equation 5*

$$P(S_{0,1}) = \frac{\lambda_2 P_{0,0} + \mu P_{1,1}}{(\lambda + \mu)} = 0.170$$

*Equation 6*

$$P(S_{0,0}) = P(S_0) = 0.521$$

*Equation 7*

$$P(S_{1,1}) = P(S_2) = 0.111$$

*Equation 8*

We next want to determine the fraction of all dispatches (i.e., an obstetrician responding to a mother starting the second stage of labor) that send the primary team's obstetrician to the mother and incur no queue delay, which is shown in Equation 9.

$$f_{1,1} = f_{2,2}$$

$$= \frac{\lambda_1}{\lambda}(P_{0,0} + P_{0,1})$$

$$= \frac{1}{2}(0.521 + 0.170) = 0.346$$

*Equation 9*

The fraction of total dispatches where the primary obstetrician is present for the entire duration of the second stage of labor is shown in Equation 10.

$$F_I = f_{1,1} + f_{2,2} = 0.346 + 0.346 = 0.692$$

*Equation 10*

It is important to note that this intra-response frequency is less than what this author has anecdotally witnessed, and there is likely an error in the assumptions made to support theoretical analysis of the system.

## Obstetrician Workload and Method of Delivery

We want to determine if obstetrician workload might play a factor in whether a labor is allowed to take its course (i.e., spontaneous vaginal delivery). From a queuing theory perspective, a cesarean section is advantageous because the average service time is much less than the average duration of a spontaneous vaginal delivery. However, cesarean sections have a negative effect on bed occupancy in the postpartum ward. Women receiving a cesarean section must stay twice as long (i.e., four days as opposed to two days) in a post-partum ward before discharge.

We performed a Wilcoxon Rank-Sum Test, which showed that there was not a statistically significant difference for the number of beds filled when a cesarean versus a vaginal delivery occurred ($z = 0.0541$, $p = 0.957$). In future work, we will further investigate whether there are other workload-based drivers that my affect the likelihood of a particular delivery method.

However, we can draw from queuing theory to evaluate how lowering the cesarean section rate from 35% to 15% as recommended may influence labor floors. Returning to our M/M/∞, we can approximate the effect of decreasing the cesarean section rate by randomly erasing 25 out of every 35 samples and re-computing the average service time for patients in the reduced sample. We run a Monte-Carlo simulation to approximate that the average duration of labor on the floor would increase from 8.72 hours to 9.24 hours (~5.7%). We note that the average stay duration on the floor, including visits where mothers do not deliver before being discharged, is actually 11.2 hours. If we do a first-order approximation that the average duration of a visit (i.e., 11.2 hours) likewise increases by ~5.7%, the M/M/∞ queueing model predicts that the average number of occupied beds would increase from 7.8 beds to 8.2 beds, and the $99^{th}$ percentile occupancy would increase from 15 beds to 16 beds.

Each additional bed requires a large investment and is critical in the design of the labor floor.

# Conclusions

Labor & Delivery is a complex clinical service requiring the orchestration of a diverse set of critical resource to properly care for mothers and their babies. We analyzed data from the Department of Obstetrics and Gynecology at a Boston-area hospital. We found that the performance of the L&D floor can be well-approximated by an M/M/∞ queuing model. This model is able to accurately predict the expected number of beds occupied in the various care centers associated with Labor and Delivery. We also investigated whether obstetrician workload vis-á-vis bed occupancy was correlated with the type of delivery (cesarean versus vaginal), but we did not find a significant effect. Lastly, we investigated the potential effects of lowering the cesarean section rate on resource utilization on the labor floor.

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