Design of an Individual Vertex Actuator and its use in Understanding the Effect of Constraint Location on Miura-Ori Folding Behavior

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ABSTRACT
The Miura-Ori fold is an origami pattern that can translate from a flat sheet to a compact folded state with a single degree of freedom. Currently little is understood about the relationship between the physical properties of the miura-ori lattice and its folding behavior. The objective of this thesis was to design an actuation setup to quantify the effect of individual vertex actuation and constraint on the adherence of folding behavior to an idealized model. The setup constrains specific vertices to the x/y plane while actuating other vertices along their ideal paths as specified by the idealized geometric model. The minimum number of constraints needed to drive a unit cell miura ori pattern into the folded state was found to be two, adjacent to the center actuator. This rule was also found to be applicable to larger lattice sizes. In addition, the adherence of each node path to the predicted ideal path was found to improve as the number of constraints increased. Several improvements to the setup that would enable further exploration of miura-ori behavior are suggested.

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I would like to acknowledge Professor Martin Culpepper for his advice and support throughout not only this project, but the past year. Professor Culpepper’s advice has inspired me to continue to my engineering studies and pursue a Master’s Degree.

A big thanks to Charlie Wheeler, who created the MATLAB code used to find the ideal folding paths, and Lauren Chai, who constructed the first iteration of the miura-ori lattice in Solidworks.
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1.0 Introduction

The objective of this thesis was to learn whether or not an individual vertex actuation method is able to provide quantitative information about the folding behavior of the miura-ori lattice. To do this, an actuation setup that allows the locations of lattice constraints and actuators to be varied was designed and built. No prior actuation method for individual vertices has been tested, and the setup developed proved the potential of this actuation method as a tool for furthering our understanding of miura-ori behavior. By controlling individual vertices, it is possible to test the response of the miura-ori system to a variety of input conditions. This actuation system was used to characterize the minimum number of constraints needed to induce the desired folding state, and was able to compare the effect of constraints on the lattice motion to the expected movement within 0.3mm. The experimental findings and overall performance of the setup provide a foundation for the future development of a testing platform that will be used to quantitatively determine the relationship between system stiffness and folding behavior. This experimental testing is a crucial step towards creating a model of the miura-ori that can predict the response to constraints and input forces at location depending on the geometry and material of the lattice. Such a model would enable the miura-ori to be quickly optimized for a variety of applications.
Figure 1.1: An example of the actuation setup, without the lattice attached.
2.0 Background

2.1 Miura Ori Fold

In order to design an actuation system, the desired motion of the miura-ori pattern must be understood. The miura-ori is an origami pattern that folds from a flat sheet into a small compact volume. It is comprised of parallelogram ‘panels,’ connected to each other by hinges. During folding, the hinges bend in one of two directions corresponding to the valley and mountain folds of an origami crease pattern. This single degree of freedom movement and the ability to optimize the geometry for varied end configurations makes it a particularly promising candidate for several real world applications [1]. Currently, the pattern is used for solar cell arrays, where it allows the cells to be shipped in compact form and later deployed to their full size.

Much work has been done to model the mathematics behind folding origami patterns. However these models use an ideal pattern with zero thickness and negligible material properties. As a result these models have limited use in the design of structures that move according to these origami patterns. The field of ‘rigid’ origami considers thickness, but makes the assumption that all flexure occurs at the hinges, with none in the panels. While the field of rigid origami has advanced the understanding of thick origami, no work has been done to connect the folding kinematics to the physical properties (e.g. stiffness) of the material. Rigid origami deals with the existence (or not) or a continuous route between final and initial state [2]. The global kinetic behavior of structures and their transformability from one state to another is currently poorly understood, and will be the focus of this thesis project [3].
2.2 Applications of the Miura-Ori

This thesis is the result of an NSF project to explore the potential future application of the Miura-Ori fold to the field of tissue engineering. A bio-compatible miura-ori lattice would allow a single layer of cells to be printed on the lattice in it’s flat state, then a 3D block of tissue could be formed simply by folding the lattice to it’s compact form. One of the current challenges of tissue engineering is the construction of complex vascular structures like blood vessels. This miura-ori based method would not enable the construction of these structures and introduce a speed advantage over current methods.

This proposed application of the miura-ori pattern includes several unique challenges; the accuracy and repeatability of the folded state has low tolerances, the cells on the panels are unable to withstand significant shear forces, and the scale at which this would occur is on the millimeter level. In order to address these challenges and design a lattice for such an application, the behavior of the miura-ori pattern with respect to its physical properties must be better understood.

2.3 Summary of current actuation methods

One of the obstacles to fully characterizing the behavior of the pattern is the challenge of accurately modeling the geometry of a folded sheet including physical properties and imperfections [4]. Additional challenges lie in the actuation from the flat state to the compact folded state. An ideal folding pattern would prevent significant deformation of the ‘panels,’ require minimal amount of contact with the physical lattice itself, and minimize the input force. The most common method of actuation is currently manual; the lattice is held at opposite corners, which are pushed together, forcing the pattern to fold into the compact state. This method requires the user to change the direction of force
subtly but frequently, and is not successful when the lattice is comprised of materials
stiffer than paper.

Overall, very few methods of actuation have been explored. Schenk explored the use of
cold gas pressure forming as an actuation method on a sheet metal Miura Ori pattern [5].
While this method was successful, it applies force to all surfaces making it impossible to
determine how the system responds to force at specific points. In addition, results
demonstrated that the actual pressure required to fold the sheet was larger than the
theoretical prediction, and there were imperfections in the final stage. This suggests that
there is still work to be done in understanding the response of the miura-ori pattern to
external forces. Self-folding structures that incorporate electromagnetic strips into the
‘hinges’ have proved successful at the nanoscale [6]. This technique shows promise as an
actuation method, but is costly and time-intensive to incorporate into structures. It has yet
to be applied to a miura-ori pattern.

Currently, the relationship between the folding behavior of the miura-ori and its physical
properties is not fully understood, preventing its translation to many real world
applications. The response of the miura-ori to input forces and constraints at individual
vertices is not understood, making it difficult to predict the movement of a randomly
selected point in the lattice in response to a remote input force. To date, no actuation
method has explored the actuation and constraint of individual vertices as a tool for
studying the system behavior.
To date, there are many inadequately explored areas preventing the translation of the miura-ori pattern to real world applications. First, other than Schenk’s cold gas pressure forming, actuation methods have not been explored in detail. Second, no measurement technique has been used to track the movement of each vertex in the panel during the folding process.

2.4 Summary of Design objectives

A model accurately portraying the relationship between input forces and constraints and system behavior with respect to physical properties would allow for optimization of actuation methods and lattice fabrication for specific applications. The goal of this thesis is to design and fabricate a setup that enables specific vertices of the lattice to be either actuated in 3 planes or constrained in the z direction. Such a setup has the potential to:

a) Characterize the force propagation through the system and its relation to system constraints

b) Find the minimum number of actuation points and constraints needed to fold the lattice

c) Characterize the minimum external work required to fold the system as a function of lattice properties.
3.0 Modeling the Miura-Ori folding behavior

3.1 Miura Ori folding geometry

The Miura-Ori fold begins as a flat sheet with hinges that correspond to the “valleys” and “mountains” in origami crease patterns. Via some external force, the system then moves to the final compact state through a single degree of freedom as demonstrated by figure 3.1.1. To reach this final state, each vertex moves along a nonlinear path that is difficult to approximate with polynomial functions. A computer model developed by Charlie Wheeler of the MIT Precision Compliant Systems Lab allows the user to change the geometric parameters of the Miura-Ori lattice and find the resulting paths. This computer model was the source of the actuation paths used to create the actuation setup. Figure 3.1.1 shows the model’s visualization of the miura-ori folding process.
3.2 Folding geometry of a unit cell

Each Miura-Ori fold can be divided into until cells, which comprise four panels and their associated hinges. A unit cell was analyzed to predict the movement and forces at play.

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1 From Charlie Wheeler's MATLAB code, 2014, author wishes to thank Charlie Wheeler for the use of the code.
during actuation. In order to model the paths of each node, the unit cell was modeled as an unstable truss where each edge (hinge) is a rigid bar [7]. Under this assumption no deformation occurs in these bars, so the movement of the unit cell can be described purely by the geometry. Node 1 is fixed to the plate and therefore has zero degrees of freedom, and points 2, 3, 7, 8, and 9 are all constrained in the z direction such that they never leave the plate. Node 5 is actuated by a pin to a position along its “ideal path,” and the distance travelled by each node can be described as a function of theta and panel geometry. These equations can be found in Appendix A.

![Figure 3.2.1: Geometry of Miura-Ori Unit Cell](image)

3.3 Work of actuation

One aspect of folding behavior is the external work needed to actuate the system. By minimizing the external work of actuation, the ease with which the lattice can be translated from flat to compact states can be increased. Schenk found that the actual
pressure needed to fold the sheet-metal miura-ori lattice using cold gas pressure forming was greater than the theoretical prediction. The external forces acting on the system are shown in figure X. To predict the external work needed to actuate a unit cell of the miura-ori system, the principal of virtual work was utilized.

\[ U = \frac{\phi^2 EI}{2L} \]  

The internal work is equal to the sum of all the displacements of each node multiplied by the force required to move them, added to the strain energy stored in each hinge.
\[-\sum F_{\text{friction}}d_i + \sum F_{\text{pin}}d_s = \sum \frac{\phi^2 EI}{2L} + \sum F_{\text{node}}d_i \]
4.0 Actuation Setup Design

4.1 Functional Requirements for Actuation

Sections 3.1-3.3 show that the motion of each individual vertex is essential to describing the folding behavior. As a result, the actuation setup was designed to address the movement of each individual vertex.

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Value</th>
<th>Implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Each “mountain” vertex actuated along ‘ideal’ path</td>
<td>NA</td>
<td>Slots corresponding to ideal paths</td>
</tr>
<tr>
<td>Allow the position of each vertex to be tracked during folding</td>
<td>NA</td>
<td>Open top to allow for imaging</td>
</tr>
<tr>
<td>Repeatable and precise stage movement in order to image at same z coordinate each time</td>
<td>0.1” +/- .01”</td>
<td>Micrometer for Z axis control</td>
</tr>
<tr>
<td>Pin deviation from ideal point</td>
<td>&lt;= .01”</td>
<td>Pin templates for lining up points,</td>
</tr>
<tr>
<td>Bottom vertices must not detach from bottom plate</td>
<td>Completely constrained in z direction</td>
<td>( F_{\text{magn}} - y &gt; F_{\text{pin}, y} )</td>
</tr>
</tbody>
</table>

One of the challenges in the design is the fact that each vertex moves along a unique path, and at a unique rate, as demonstrated by Figure 3.4.1, which shows the ideal path of every vertex.
Figure 4.1.1: Ideal actuation path for each vertex

Several designs were considered and compared to the cold gas pressure forming method and hand actuation methods.

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2 From Charlie Wheeler's MATLAB code, 2014, author wishes to thank Charlie Wheeler for the use of the code.
Table 4.2 Pugh Chart Comparing Actuation Ideas

<table>
<thead>
<tr>
<th>Design Name</th>
<th>Ease of controlling actuation paths</th>
<th>Ability to image</th>
<th>Ease of construction</th>
<th>Allow for removal and addition of constraints</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnets both sides</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>2</td>
</tr>
<tr>
<td>Pulley system</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>Slotted plates with pins</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>3</td>
</tr>
<tr>
<td>Slot car track (electromagnetic driving of points)</td>
<td>0</td>
<td>+</td>
<td>0</td>
<td>+</td>
<td>2</td>
</tr>
<tr>
<td>Cold Gas Pressure Forming (Schenk)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Manual Actuation</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>
The slotted plates with pins approach was chosen over the slot car tracks due to its relative ease of construction. The fabricated setup is shown in Figure 3.4.3.
This setup uses magnets to constrain the “valley” vertices of the lattice in the $z$ direction while allowing them to translate in $x$ and $y$. It also incorporates the ideal paths in which the actuator pins slide, and allows for easy removal and addition of both constraints and pins. As many of the actuation paths overlap, only several were chosen to be included in this stage design in order to ensure that each was properly constrained.

4.2 Machining and Assembly

Twenty equidistant points from each path were selected, drawn in Solidworks and spline fit, then the slots machined using a CNC mill (figure 3.5.1). Both plates (top and bottom) were machined at the same time to minimize offset error.
Each lattice was 3D printed by an Objet printer using Vera White (a rigid plastic) for the panels, and either Tango+ (clear) or TangoBlack (black), both rubber-like materials for the hinges. The magnets (z constraints) and pins (actuators) were attached to the lattice with screws and washers.
3.6 Design Verification

To ensure that the force of the pin pushing up would be great enough to slide the magnet-constrained points, the relationship between the coefficient of friction and theta (the angle between the vertical axis and the panel) was found:

\[ F_{\text{friction}} + F_{\text{gravity}} > F_{\text{mag}} \]

**Figure 4.3.1**: Forces acting on nodes 2, 5, and 8 of a unit cell when an actuator pin is pushing up.

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnets must not become detached from bottom plate</td>
<td>( F_{\text{mag}} + F_g &gt; F_{\text{bar}, y} )</td>
</tr>
<tr>
<td>Magnets must slide horizontally under less force than needed to detach the magnets</td>
<td>( F_{\text{friction}} &lt; F_{\text{bar}, x} )</td>
</tr>
</tbody>
</table>

First, the gravitational force was found to be insignificant in comparison to the magnet force, and was therefore neglected in the force balances.

\[ F_{\text{gravity}} = mg = 9.8mN \]  \( \text{(3)} \)
\[ F_{\text{magnet}} = \frac{B_m^2 A_m}{(8\pi)10^{-7}} = 0.26 \text{lb} = 1.156 \text{N} \] (4)

**Figure 4.3.2:** Forces acting on constrained nodes

\[ \sum F_x = 0 = -F_{\text{friction}} + F_{\text{bar}} \sin \theta \] \hspace{1cm} (5)

\[ \sum F_y = 0 = F_{\text{bar}} \cos \theta - F_{\text{magnet}} \] \hspace{1cm} (6)

**Figure 4.3.3:** Forces acting on actuated nodes

\[ \sum F_y = 0 = F_{\text{pin}} - 2\cos \theta F_{\text{bar}} \] \hspace{1cm} (7)

\[ \sum F_x = 0 = F_{\text{bar}} \sin \theta - F_{\text{bar}} \sin \theta \] \hspace{1cm} (8)

Substituting and solving the above equations, we find that

\[ F_{\text{friction}} < F_{\text{magnet}} \tan \theta \] \hspace{1cm} (9)

Therefore, the inverse tangent of the coefficient of friction must be less than theta for the constrained vertices to slide on the plate. The coefficient of friction between rare earth magnets and steel was cited as 0.5 and theta varies from 0-90 degrees [8]. The inverse
tangent of 0.5 is 25 degrees, so as long as theta is greater than or equal to 25 degrees, the force from the pin will be sufficient to slide the magnets in the x, y plane. As angles of less than 25 degrees only occur at an almost compact folding state, this is not a significant concern (at this stage of folding the pins themselves would interfere with the lattice).

4.4 Error Model

To model the error in vertex accuracy, the bending of the pins must be taken into account. They are modeled as a simply supported beam on one side, with forces \( F_{bar} \) acting on the tip. In a worst case scenario, \( F_{bar} \) would be zero on one side and maximum (theta = \( \pi/2 \)) on the other.

![Figure 4.4.1: Forces acting on actuator pins](image)

\[
\delta = \frac{FL^3}{3EI} \tag{10}
\]

\[
\delta_{\text{max}} = 0.0016 \text{nm} \tag{11}
\]

The model used to describe the motion of the miura-ori in sections 3.1-3.3 has made several assumptions about the system: First, it assumed the hinges do not deform along their length, but only in the bending mode along their width. Second, that the connections
between the magnets (z constraints), pins (actuation points), and the lattice are perfect.

The predicted error from the real case of the above assumptions is included in the system error model (Table 4.4)

<table>
<thead>
<tr>
<th>Source</th>
<th>Worst case value, Direction</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Misalignment of plates</td>
<td>+/-0.005&quot;X, +/-0.005&quot;Y</td>
<td>Diameter tolerance of alignment rods</td>
</tr>
<tr>
<td>Accuracy of machined paths in plates</td>
<td>+/-0.005&quot;X, +/-0.05&quot;Y</td>
<td>CNC machining tolerance</td>
</tr>
<tr>
<td>Pin hole misalignment when attached to lattice</td>
<td>+/-0.01&quot;</td>
<td>Sliding space between screw and 3D printed hole in lattice</td>
</tr>
<tr>
<td>Bending of pins</td>
<td>6.29e-11&quot;X, 6.29e-11&quot;Y</td>
<td>Worst case bending due to force</td>
</tr>
<tr>
<td>Pin height</td>
<td>+/-0.005&quot;Z</td>
<td>Machining tolerance</td>
</tr>
<tr>
<td>Levelness of surface</td>
<td>+/- 0.006 Z</td>
<td>Stock tolerance</td>
</tr>
<tr>
<td>Variation in pin length</td>
<td>+/-0.01&quot;</td>
<td>Machining tolerance</td>
</tr>
</tbody>
</table>

If each of the tolerances were to stack up, the worst case error in each direction from the expected position:

X: +/- 0.015"
Y: +/- 0.015"
Z: +/- 0.021"
5.0 Results

5.1 Experimental Design

Using the setup, two experiments were performed. In the first, the effect of the number and location of constraint points on the folding pattern of a unit cell was studied. Each cell was actuated at node 5, while the constrained nodes were varied. The lattice was imaged at a minimum of 3 points during the folding process (until a folding angle of 30 degrees was achieved). Using the images, the distance travelled by each node was found and summed, then compared to the expected distance (equations can be found in Appendix A). The error associated with these points corresponds to the image pixel size, and is +/- 0.3mm.

In the second experiment the size of the lattice and the number and location of actuation and constraint points was varied. The materials used to make the lattice resulted in large forces resisting folding, which made it difficult to keep 'actuated' nodes in the correct position, so the findings will be discussed qualitatively.

5.2 Unit Cell Results

For the Unit Cell test, the system was expected to move as outlined in Section 3.2. The folding progression for different numbers of constraints is shown below in figures 5.2.1 through 5.2.4.
The minimum number of constraints needed to ‘pop’ the unit cell into the correct folding pattern such that each node remained in the correct plane was two. These points were located at nodes 2 and 8 (shown in Figure 5.2.5), in a vertical line with the actuated node.
Figure 5.2.3: Unit Cell with minimum number of constraints needed to enter ideal folding state.

When only 1 constraint was used, one side of the unit cell was raised to the same level as the actuated node. Without an additional force or constraint, the unit cell cannot be folded to its compact position.

Figure 5.2.4: (a) Unit Cell with 1 constraint, (b): the cell in an actuated position corresponding to a 30-degree fold angle

Table 5.2 shows the relationship between the number of constraints, the percentage of the expected distance travelled by all of the points, and the average variance between the predicted position and the actual position. As the number of constraints increased, each node’s adherence to the ideal path improved (demonstrated by the variance), as seen in figure 5.2.5. The percentage of total expected distance travelled is a measure of the work done on the system in comparison to the expected work.
Table 5.2: Comparison of experimental and predicted results

<table>
<thead>
<tr>
<th>Number of Constraints</th>
<th>Percentage of total expected distance travelled (+/-2%)</th>
<th>Average variance (mm) (+/-0.3mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15%</td>
<td>4.20</td>
</tr>
<tr>
<td>2</td>
<td>130%</td>
<td>1.51</td>
</tr>
<tr>
<td>3</td>
<td>87%</td>
<td>0.44</td>
</tr>
<tr>
<td>6</td>
<td>110%</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Figure 5.2.5: Node variance (+/-0.3mm) from theoretical position as a function of number of constraints (c).

5.3 Multiple Cell Lattice Results

Multiple Unit Cells were combined to make larger lattices of sizes ranging from 6x7 panels to 3x3 panels. As the unit cell results demonstrated that constraints directly adjacent to an actuation point were the minimum necessary, this trend was tested for a larger size lattice. As demonstrated in Figure 5.3.1, constraining every bottom node did
not offer any advantages, as the same behavior was observed when only adjacent nodes were constrained.

Figure 5.3.1: (a): only adjacent nodes constrained. (b): all interior nodes constrained

When non-adjacent nodes were the ones constrained as in Figure 5.3.2, the unconstrained bottom nodes strayed from their plane (Figure 5.3.3). Without any bottom constraints, the lattice exhibited no adherence to ideal folding behavior, offering great resistance to the actuator pins as it attempted to remain flat, and also buckling in different directions when actuated.

Figure 5.3.2: translation away from bottom plane of unconstrained adjacent nodes.
Figure 5.3.3: No bottom constraints. The entire lattice translates upwards as one unit and the hinges resist buckling such that the lattice never enters the ‘folding path.’

It should be noted that none of the 3D printed lattices used for testing contained hinges biased in a certain direction. Pre-stressing or biasing these hinges may change the results, as the least energy state for the affected hinges may be in a ‘folded’ state.

5.4 Performance of actuation setup

Due to the internal stiffness of the materials used to create the miura-ori lattices, accurate position data could not be gathered for lattice sizes larger than a unit cell. In these cases the actuation pins were not able to maintain their position, and bent and slid out of place to find the least energy state.

While this actuation setup was adequate for unit cell tests and demonstrated potential to be used with more compliant lattices, several changes are proposed that would not only offer greater accuracy and control, but allow the user to collect data corresponding to system stiffness. First, adding drivers that slide the actuator pins along the paths (instead of allowing them to move independently) would prevent the internal resistance of the lattice to folding from moving the actuated points to a least energy state and enable more
accurate positioning at each actuation stage. This driven actuation system is non-trivial to fabricate as each vertex moves at a different rate along a unique path in the x and y plane. Several ideas, including an electromagnetic slot car-like track and a marionette pulley system were proposed during the ideation phase, but each requires a great deal of room and as a result it may be easier to fabricate this system on a larger scale. Second, reinforcing the holes in the lattice where constraints and actuators attach would prevent ripping of lattice material and the associated position error. Third, the friction between the magnets and the bottom plate, and the actuator pins and slots was significant. Minimizing this, perhaps using a Teflon coating or different materials, may allow for smoother actuation. It becomes doubly important to support the actuated vertices at each stage, as less friction also makes it easier for the lattice to seek out the flat, least energy state. Finally, using a measurement device such as a CMM to find the location of each vertex during the folding process would provide more accurate data, allowing conclusions to be drawn with greater confidence.
6.0 Conclusions

The objective of this thesis was to design an actuation setup to quantify the effect of individual vertex actuation and constraint on the adherence of folding behavior to an idealized model. This information can be used to better understand the relationship between physical characteristics of the miura-ori and its folding behavior, as well as direct the design of future actuators. Improving our understanding of the miura-ori pattern increases its potential for use in a variety of applications. Using an individual vertex approach, the minimum number of points needed to actuate a miura-ori unit cell in the proper folding pattern was found to be three: one actuation point and two constraints. The constraints needed to be directly adjacent to the actuated point, and this rule carried through to larger size lattices as well. It was also discovered that increasing the number of constraints improved the adherence of each node’s movement with respect to the theoretical, ideal path.

The significance of the experimental findings is that not every hinge or vertex needs to be actuated in order to force the system into the folding pattern. Future exploration of miura-ori actuation methods may consider systems in which only certain hinges need to be pre-stressed or buckled, potentially simplifying fabrication processes or allowing the lattice to be designed such that not every panel is essential.

The actuation setup worked well enough to enable the findings outlined above and demonstrated the promise of the individual vertex approach as a tool for furthering the understanding of miura-ori system behavior. The setup was difficult to control and less
accurate than desired, suggesting that an automated system with drivers for each vertex would allow the user greater control and accuracy over the folding process. Such a setup may also enable the use of force sensors, allowing the stiffness of the lattice and external work needed at different points in the folding process to be found. Future work is needed to create a model that represents the folding behavior as a function of physical properties, input force and location, and constraints. Such a tool would be invaluable in designing and optimizing the miura-ori fold for various applications.
References

Appendix A: Distance travelled by each node of the unit cell

These equations correlate to the dimensions shown in Figure 3.2.1, and the distance $d_i$ corresponds to the distance travelled by the node $i$.

$$W = 2b \frac{\cos \theta \tan \gamma}{\sqrt{1 + \cos^2 \theta \tan^2 \gamma}}$$

$$B = 2a \sqrt{1 - \sin^2 \theta \sin^2 \gamma}$$

$$h = a \sin \theta \sin \gamma$$

$$d_1 = 0$$

$$d_2 = \sqrt{\left(\frac{1}{2}W - b \sin \gamma\right)^2 + \left(\sqrt{\frac{1}{4}W^2 - b \cos \gamma}\right)^2}$$

$$d_3 = 2b \sin \gamma - W$$

$$d_4 = \sqrt{(W - 2b \sin \gamma)^2 + (\sqrt{a^2 - h^2 - a^2})^2 + h^2}$$

$$d_5 = \sqrt{\left(\frac{1}{2}W - b \sin \gamma\right)^2 + \left(\sqrt{\frac{1}{4}W^2 + \sqrt{a^2 - h^2 - b \cos \gamma} - a}\right)^2 + h^2}$$

$$d_6 = \sqrt{(\sqrt{a^2 - h^2 - a^2})^2 + h^2}$$

$$d_7 = B - 2a$$

$$d_8 = \sqrt{\left(\frac{1}{2}W - b \sin \gamma\right)^2 + \left(\sqrt{\frac{1}{4}W^2 + 2\sqrt{a^2 - h^2 - b \cos \gamma - 2a}}\right)^2}$$

$$d_9 = \sqrt{(W - 2b \sin \gamma)^2 + (B - 2a)^2}$$
### Appendix B: Pugh Chart comparing actuation ideas

<table>
<thead>
<tr>
<th>Design Name</th>
<th>Ease of controlling actuation paths</th>
<th>Ability to image</th>
<th>Ease of construction</th>
<th>Allow for removal and addition of constraints</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnets both sides</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>2</td>
</tr>
<tr>
<td>Pulley system</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>Slotted plates with pins</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>3</td>
</tr>
<tr>
<td>Slot car track</td>
<td>0</td>
<td>+</td>
<td>0</td>
<td>+</td>
<td>2</td>
</tr>
<tr>
<td>(electromagnetic driving of points)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cold Gas Pressure Forming (Schenk)</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Manual Actuation</td>
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<td>+</td>
<td>+</td>
<td>-</td>
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