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# Proposed Method for Evaluating Multiaxial Fatigue in ITER

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**ABSTRACT**

Several of the structural components in the superconducting magnet system for the International Thermonuclear Experimental Reactor (ITER) experience pulsed, multiaxial loading during operation. For components, such as the conductor conduits, with a large population of initial flaws, fatigue crack growth analysis (FCGR) is used to evaluate failure. For other, simpler, unwelded components, such as pins and bolts in the Central Solenoid assembly, in which flaw crack initiation is a dominant factor, S/N data can be used. However, since S/N data is available only for uniaxial load conditions, a reliable design code for multiaxial fatigue is necessary. The problem is that none of the existing design codes gives a complete definition for the equivalent mean stress in multiaxial fatigue. This paper reviews the existing theories and design codes in multiaxial fatigue, and proposes a general definition of equivalent mean stress for multiaxial fatigue for the design of ITER and other pulsed structures with multiaxial loads.

**CONTENTS**

## 1. Introduction

The ITER criteria for magnet design express a preference for using fracture mechanics for fatigue qualification, but allow S-N evaluations in specific cases [1]: e.g. "An S-N fatigue curve evaluation is appropriate to be used in two situations: (a) where the geometry/material microstructure/stress systems are complex and components are small and can be tested as a single unit and in sufficient numbers to obtain a representative sample (bolts or dowels are typical examples), (b) for the evaluation of non-planar flaws in larger components, where there is no possibility of the presence of planar defects and where the characteristic non-planar flaws are much smaller than the specimens being tested. The S-N procedure makes allowance for the initiation of a true crack from the defect and it is essential that the test specimens are representative of the inherent defects in the material and that sufficient tests are made to establish a representative sample."

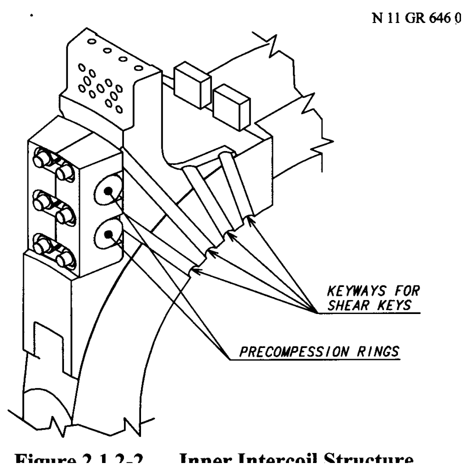



Fig. 1 Pin used in the ITER TF intercoil structure.

Many ITER mechanical parts experience cyclical multiaxial loading during operation, such as pins and bolts in the CS assembly [1-3]. The pins used in ITER are compressed and loaded in reverse shear. Pins support the changing vertical loads on the CS and the alternating out-of-plane loads in the TF. Several other components also see these complex alternating stress states due to the changing Lorentz forces throughout the pulse.

The existing design codes for multiaxial fatigue include ASME, ITER, FIRE, and the German code [1, 4-6]. All of them apply criteria, based on the maximum shear (i.e. Tresca) stress, to describe the alternating stress. As to the mean stress effect, it becomes insignificant if plastic deformation occurs during cyclic loading. The main concern of the abovementioned codes are engineering structures operating in a high stress range, where plastic deformation occurs, i.e., low cycle fatigue. Therefore, none of the above codes provides a complete definition of mean stress for multiaxial fatigue, although the German code gives a definition for the special (nonrotating) case of constant principal direction. Since the ITER parts are designed to operate in the elastic region for a long machine life of up to 30,000 cycles, i.e., high cycle fatigue, the mean stress effect becomes significant and can not be neglected.

This paper reviews the existing theories and design codes in multiaxial fatigue, and proposes a general design code of equivalent mean stress of multiaxial fatigue for ITER.

## 2. Failure mechanism of multiaxial fatigue

Extensive research has been performed for multiaxial fatigue [7-21]. It is found that there is no universal model or parameters that can strongly correlate a multiaxial fatigue process with its life due to the complexity of factors in loading, material and environment. Most of the recent results accept the critical plane damage model [7-9] (see Fig. 1) to describe the mechanism of multiaxial fatigue failure. The critical plane damage model predicts that fatigue cracks nucleate and grow on critical planes in material subjected to cyclically multiaxial loading, and the damage is produced by the shear stress/strain $\tau$ on the critical planes, while the tensile mean stress $\sigma_n$ normal to the critical plane enhances the damage and therefore reduces fatigue life.

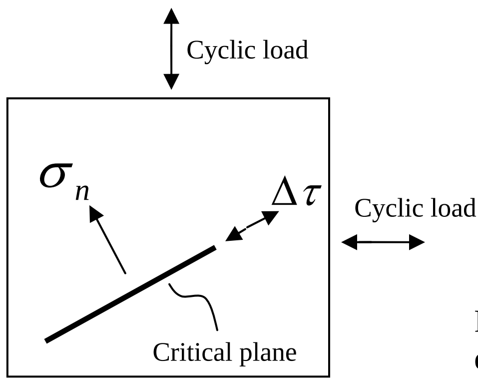



Fig. 2 Critical plane damage model

The mathematical expression of this model as a stress based criterion is summarized in [7-9] as:

$$\Delta\tau + k\sigma_n = f(N), \tag{1}$$

where $\Delta\tau$ is the shear stress range on the critical plane, $k$ is an experimentally determined constant, $\sigma_n$ is the normal stress to the critical plane, and $f(N)$ is a function of fatigue life.

The critical plane has to be determined by a combination of experiments and analysis for each specific material and operation condition. However, the literature shows that, in most cases [7,8,10], the critical plane is the plane either with maximum shear stress or with octahedral shear stress, while the principal stress model [10] only applies in a few cases, such as cast iron, discussed below. Three models of critical planes are shown in Table 1, where the 3 principal stresses are defined as $\sigma_1 > \sigma_2 > \sigma_3$. Table 1 lists, for each critical plane model, the shear stress $\tau$ acting on the critical planes and the tensile stress $\sigma_n$ normal to the critical plane.

The critical plane model is well established in the academic community, but it is not practical in engineering design because of the difficulties in finding the critical plane for each specific material and operating condition. Therefore, several engineering approaches are proposed as more practical tools for engineering design.

Table 1  Different models of critical plane

| *Critical plane model* | *Shear stress $\tau$ acting on the critical plane* | *Normal stress $\sigma_n$ on the critical plane* |
|---|---|---|
| Maximum shear plane (Tresca) | $\tau_{tresca} = \frac{\sigma_1 - \sigma_3}{2}$ | $\sigma_{tresca} = \frac{\sigma_1 + \sigma_3}{2}$ |
| Octahedral shear plane (Von Mises) | $\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$ | $\sigma_{oct} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$ |
| Principal stress shear plane | $\tau_p = \sigma_1 / 2$ | $\sigma_p = \sigma_1 / 2$ |

## 3.  Engineering approach

A large and adequate test data base is available for uniaxial fatigue. Therefore, the engineering approach to solving the multiaxial fatigue problem is, based on critical plane models, to find the equivalent uniaxial fatigue parameters, i.e. the equivalent alternating stress and the equivalent mean stress. These are then applied to one of three experimental equations - either Gerber's, Goodman's or Soderberg's [13] – in order to estimate the fatigue life.

### *3.1  Equivalent alternating stress*

The equivalent alternating stress is related to the shear stress range $\Delta\tau$ acting on the critical plane. Table 2 lists the equivalent uniaxial alternating stresses derived from the shear stress on the critical plane, where the 3 principal stresses are defined as $\sigma_1 > \sigma_2 > \sigma_3$, and the subscripts "a" and "range" represent stress amplitude and range respectively. The equivalent uniaxial stress is the uniaxial stress under which the shear stress on the critical plane can be obtained.  The literature [10-11,13] shows that, in most cases, the von Mises and Tresca stresses give a good estimation.  Sawert [10] summarizes the biaxial fatigue data for steel, and finds that the von Mises stress criterion gives the best fit, the Tresca stress criterion gives a close and always conservative prediction, and the principal stress criterion is only valid in the first quadrant.  All of the existing design codes [1, 4-6] apply the model of maximum shear (Tresca) to describe the equivalent uniaxial alternating stress.  Note that the maximum shear range on the critical plane is determined by the cyclic stress components, and the static stress components only affect the mean on the critical plane. For example, if the highest and lowest principle stresses are constant with time, they can not be considered simultaneously in the equations used in Table 2 to determine the equivalent alternating stress.

Table 2   Equivalent alternating stress derived from shear stress $\tau$ on critical plane

| Critical plane model | Equivalent uniaxial alternating stress $\sigma_{alt}$ |
|---|---|
| Maximum shear plane (Tresca) | $\sigma_{tresca} = \sigma_1 - \sigma_3$<br>$\sigma_{range} = (\sigma_{tresca})_{\max} - (\sigma_{tresca})_{\min}$<br>$\sigma_{alt} = \sigma_{range} / 2$ |

| | |
|---|---|
| Octahedral shear plane (Von Mises) | $\sigma_{alt} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_{1a}-\sigma_{2a})^2+(\sigma_{2a}-\sigma_{3a})^2+(\sigma_{3a}-\sigma_{1a})^2}$ |
| Principal stress shear plane | $\sigma_{alt} = \sigma_{1a}$ |

*3.2 Equivalent Mean stress*

The test results indicate that the tensile stress normal to the critical plane increases the fatigue damage, while the compressive stress normal to the critical plane decreases the fatigue damage [7, 10]. The equivalent uniaxial mean stress is related to the tensile/compressive stress normal to the critical planes [14-21]. Equivalent mean stress definitions for three different models are listed in Table 3. The equivalent uniaxial mean stress is the uniaxial stress under which the normal stress acting on the critical plane can be obtained. Therefore, the equivalent mean stress is only related to those stresses which control the normal stress acting on the critical plane.

Table 3 Equivalent mean stress derived from normal stress on critical plane

| Critical plane model | Equivalent uniaxial mean stress | Equation No. |
|---|---|---|
| Maximum shear plane (Tresca) | $\sigma_{mean} = (\sigma_1+\sigma_3)_{mean}$ (non-proportional loading)<br>$\sigma_{mean} = \sigma_{1mean}+\sigma_{3mean}$ (proportional loading) | (2) |
| Octahedral shear plane (Von Mises) | $\sigma_{mean} = (\sigma_1+\sigma_2+\sigma_3)_{mean}$ (non-proportional loading)<br>$\sigma_{mean} = \sigma_{1mean}+\sigma_{2mean}+\sigma_{3mean}$ (proportional loading) | (3) |
| Principal stress shear plane | $\sigma_{mean} = \sigma_{1mean}$ | (4) |

Proportional loading means that the ratio of the normal and shear stresses is invariant with time. Nonproportional loading is the more general case that permits arbitrary stress histories. Sines [10] analyzed the test data from 5 different test resources, and suggested using Eq. 3 for the equivalent mean stress. Based on the critical plane model, either Eq. 2 or Eq. 3 can successfully explain the test results listed in Table 4. However, Eq. 3 does not handle the special case well in which hydrostatic loads are significant. Therefore, Eq. 2 will be preferred over Eq. 3 in proposed code statement. Eq. 4, based on the principal stress model is not accepted as a general criterion, because it only applies for very limited cases, such as that of cast iron, discussed below.

Table 4 Summary of Sines' case analysis using either Eq. 2 or Eq. 3

| *Cases* | *Sum of mean* | *Predicted effect on fatigue damage* | *Test data for effect on fatigue damage* |
|---|---|---|---|
| Axial alternating Tensile mean | Positive | Increase | Increase |
| Axial alternating | Negative | Decrease | Decrease |

| | | | |
|---|---|---|---|
| Compressive mean | | | |
| Torsion alternating<br>Torsion mean | Zero | None | None |
| Bending alternating<br>Torsion mean | Zero | None | None |
| Torsion alternating<br>Tension mean | Positive | Increase | Increase |

## 4. The equivalent mean stress of multiaxial fatigue with constant principal direction

Extensive discussion has been under way for establishing the evaluation procedure of the mean stress effect for multiaxial fatigue. For the case of constant principal direction, it is widely accepted to apply the German code,[6] which is based on the Tresca mean stress model. The maximum shear plane is determined by finding the largest shear stress range, and the normal stress acting on the maximum shear plan is obtained from the sum of mean of the 2 principal stresses determining the maximum shear stress.

### *4.1 Design Code statement*

For any case in which the directions of the principal stresses at the point being considered do not change during a cycle, the steps stipulated in (a) through (d) below shall be taken to determine the mean equivalent stress $\sigma_{mean}$ .

(a) Principal stresses. Calculate the values of the three principal stresses at the key time points of for the complete stress cycle. These are designed as $\sigma_1, \sigma_2, \sigma_3$ for later identification.

(b) Stress differences. Determine the stress differences $S_{12} = \sigma_1 - \sigma_2$, $S_{23} = \sigma_2 - \sigma_3$, and $S_{31} = \sigma_3 - \sigma_1$ at each key time point.

(c) Maximum stress difference range. Determine the maximum stress difference range during one cycle as $S_{ij}^{\max} - S_{ij}^{\min} = \max\left(S_{12}^{\max} - S_{12}^{\min}, S_{23}^{\max} - S_{23}^{\min}, S_{31}^{\max} - S_{31}^{\min}\right)$ . The determination of the maximum stress difference $S_{ij}^{\max} - S_{ij}^{\min}$ is used only for the purpose of determining i and j, the axes to use in (d) below.

(d) Mean equivalent stress is defined [6] as $\sigma_{mean} = \frac{1}{2}\left[(\sigma_i + \sigma_j)^{\max} + (\sigma_i + \sigma_j)^{\min}\right]$. In case of proportional loading, the above expression reduces to $\sigma_{mean} = \sigma_{imean} + \sigma_{jmean}$.

### *4.2 Example of the German code application in the multiaxial fatigue with constant principal direction*

Table 5a Initial stress data

| Stress Component | @ Time Point A (MPa) | @ Time Point B (MPa) |
|---|---|---|
| $\sigma_x$ | 3 | 1 |
| $\sigma_y$ | -20 | -2 |

| | | |
|---|---|---|
| $\sigma_z$ | 40 | 5 |
| $\tau_{xy}$ | 0 | 0 |
| $\tau_{yz}$ | 0 | 0 |
| $\tau_{zx}$ | 0 | 0 |

Table 5b Step "a" to calculate principal stresses

| Principal stresses Stress Component | @ Time Point A (MPa)) | @ Time Point B (MPa)) |
|---|---|---|
| $\sigma_1$ | 40 | 5 |
| $\sigma_2$ | 3 | 1 |
| $\sigma_3$ | -20 | -2 |

Table 5c Step "b" to calculate stress differences

| Principal stresses Stress Component | @ Time Point A (MPa)) | @ Time Point B (MPa)) |
|---|---|---|
| $S_{12}$ | 37 | 4 |
| $S_{23}$ | 23 | 3 |
| $S_{31}$ | -60 | -7 |

Table 5d Step "c" to determine the maximum stress difference range as

| Calculation | Note |
|---|---|
| $S_{12}^{\max} - S_{12}^{\min} = 33MPa$<br>$S_{23}^{\max} - S_{23}^{\min} = 20MPa$<br>$S_{31}^{\max} - S_{31}^{\min} = 53MPa$<br>$Max = S_{31}^{\max} - S_{31}^{\min}$ | This step identifies which pair of principal stresses to be used in the next step; in this case, the 'answer' to step c is: i,j = 1 and 3 |

Table 5e Step "d" to calculate the mean equivalent as

| Calculation | Note |
|---|---|
| $\sigma_{mean} = \sigma_{1mean} + \sigma_{3mean} = 11.5MPa$ | The means are taken from the values in table 5b:<br>$\sigma_{1mean} = (40+5)/2 = 22.5$,<br>$\sigma_{3mean} = (-20-2)/2 = -11$<br>$\sigma_{mean} = \sigma_{1mean} + \sigma_{3mean} = 11.5$ |

**5. The equivalent mean stress of multiaxial fatigue with varying principal direction**

For the case of varying principal direction, there is no commonly accepted procedure to evaluate the mean stress. As the principal direction is rotated during one stress cycles, it is hard to track the rotation path of each principal direction at each moment in time. Therefore, the mean equivalent stress should be determined based on all six stress components. In addition, the shear damage is not concentrated in one single shear plane rather than spreading over many shear planes. A further comment [2] suggested that there must be a continuity between the varying and constant principal direction methods. In this section, a new procedure is developed. It is based on Tresca mean stress model, and can be applied universally for both the varying and constant principal directions.

*5.1 Method 1 vs. method 2 of ASME code*

The ASME code [4] defines 3 steps to calculate the equivalent alternating stress for the case of constant principal stress direction as:

**Method 1: constant principal stress direction**

(a) Calculate the 3 principal stresses $\sigma_1, \sigma_2, \sigma_3$;

(b) Calculate the stress differences between each pair of the principal stresses as $S_{12} = \sigma_1 - \sigma_2$, $S_{23} = \sigma_2 - \sigma_3$, and $S_{31} = \sigma_3 - \sigma_1$;

(c) Calculate the ranges of the stress differences during one stress cycle as $S_{r12} = \Delta S_{12}$, $S_{r23} = \Delta S_{23}$, and $S_{r31} = \Delta S_{31}$, and the largest stress difference range is $S_{rij} = \Delta S_{ij}$ . The equivalent alternating stress is the half of the largest stress difference range $S_{alt} = 0.5 \cdot S_{rij}$ .

The ASME code also defines a second method to calculate the equivalent alternating stress for the case of varying principal stress directions, designated here as **Method 2: varying principal stress direction.** Method 2 begins by calculating the stress range from the original 6 stress components, and then proceeds to calculate the principal stresses for the stress ranges.

We can prove that, for the case of constant principal direction, both methods are equivalent. Therefore, the ASME definition for the case of varying principal direction is an extension of method 2 for the constant principal direction. The method that we will propose for calculating the equivalent mean stress with varying principal direction uses the same logic as the Method 2 used by ASME for defining the equivalent alternating stress with varying principal direction.

Assume that a stress tensor ($\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{xz}$ ) is applied to a body. The following equation holds for a principal stress $\sigma$ acting on a principal plan:[22]

$$\begin{pmatrix} \sigma - \sigma_x & -\tau_{xy} & -\tau_{xz} \\ -\tau_{xy} & \sigma - \sigma_y & -\tau_{yz} \\ -\tau_{xz} & -\tau_{yz} & \sigma - \sigma_z \end{pmatrix} \begin{pmatrix} l \\ m \\ n \end{pmatrix} = 0 \ , \qquad (5)$$

where: *l, m, n* are the 3 direction cosines of the angles between the principal stress $\sigma$ and the x,y,z axes. σ is a principal stress, $\sigma_1$, $\sigma_2$, or $\sigma_3$, so equation 5 represents 9 equations, used for solving the 9 values of *l*, *m*, and *n* for the three principal stresses.

The direction cosines *l*, *m*, *n* can not be all zero. Therefore, the matrix at left must be zero, which leads to the following equation to evaluate the 3 principal stresses:

$$\begin{aligned}&\sigma^3 - \left(\sigma_x + \sigma_y + \sigma_z\right)\sigma^2 + \left(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_x\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{xz}^2\right)\sigma \\ &- \left(\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{xz} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{xz}^2 - \sigma_z\tau_{xy}^2\right) = 0\end{aligned} . \qquad (6)$$

Since the matrix in Eq. 5 is degenerate, the direction cosines for each principal stress must be obtained by solving Eq.5 with the constraint that:

$$l^2 + m^2 + n^2 = 1 . \qquad (7)$$

When the directions of the principal stresses are known, the principal stresses can be obtained by the simpler equation:

$$\sigma = f(\sigma_{ij}, l, m, n) = \sigma_x l^2 + \sigma_y m^2 + \sigma_z n^2 + 2\tau_{xy} lm + 2\tau_{yz} mn + 2\tau_{zx} nl . \qquad (8)$$

Assume that, according to method 1, the equivalent alternating stress is found to be:

$$S_{alt} = 0.5 S_{r31}, \quad \text{and} \qquad (9)$$

$$S_{r31} = \Delta S_{31} = \Delta\left(\sigma_3 - \sigma_1\right), \quad \text{or} \qquad (10)$$

$$S_{r31} = \Delta\sigma_3 - \Delta\sigma_1 \qquad (11)$$

where:

$$\Delta\sigma_1 = \Delta f(\sigma_{ij}, l_1, m_1, n_1) \quad \text{and} \qquad (12)$$

$$\Delta\sigma_3 = \Delta f(\sigma_{ij}, l_3, m_3, n_3) . \qquad (13)$$

Since the principal stress is a linear function of all 6 stress components according to Eq. 8, for the case of constant principal direction we have

$$\begin{aligned}\Delta\sigma_1 &= f(\Delta\sigma_{ij}, l_1, m_1, n_1) \\ &= \Delta\sigma_x {l_1}^2 + \Delta\sigma_y {m_1}^2 + \Delta\sigma_z {n_1}^2 + 2\Delta\tau_{xy} l_1 m_1 + 2\Delta\tau_{yz} m_1 n_1 + 2\Delta\tau_{zx} n_1 l_1\end{aligned} \quad \text{and} \qquad (14)$$

$$\begin{aligned}\Delta\sigma_3 &= f(\Delta\sigma_{ij}, l_3, m_3, n_3) \\ &= \Delta\sigma_x {l_3}^2 + \Delta\sigma_y {m_3}^2 + \Delta\sigma_z {n_3}^2 + 2\Delta\tau_{xy} l_3 m_3 + 2\Delta\tau_{yz} m_3 n_3 + 2\Delta\tau_{zx} n_3 l_3\end{aligned} \qquad (15)$$

Eqs. 11, 14, and 15 are the theoretical basis for using method 2 to calculate the equivalent alternating stress.

The German code applies method 1 to evaluate the equivalent mean stress for the case of constant principal stress. According to the Tresca model, the equivalent mean stress is

$$S_{mean} = \sigma_{3mean} + \sigma_{1mean} \, . \tag{16}$$

However, again using Eq. 8, we have

$$\sigma_{1mean} = mean\lfloor f(\sigma_{ij}, l_1, m_1, n_1)\rfloor = f(\sigma_{meanij}, l_1, m_1, n_1) \text{, and} \tag{17}$$

$$\sigma_{3mean} = mean\lfloor f(\sigma_{ij}, l_3, m_3, n_3)\rfloor = f(\sigma_{meanij}, l_3, m_3, n_3) \text{, or} \tag{18}$$

$$\sigma_{1mean} = \sigma_{mx} l_1{}^2 + \sigma_{my} m_1{}^2 + \sigma_{mz} n_1{}^2 + 2\tau_{mxy} l_1 m_1 + 2\tau_{myz} m_1 n_1 + 2\tau_{mzx} n_1 l_1 \quad \text{and} \tag{19}$$

$$\sigma_{3mean} = \sigma_{mx} l_3{}^2 + \sigma_{my} m_3{}^2 + \sigma_{mz} n_3{}^2 + 2\tau_{mxy} l_3 m_3 + 2\tau_{myz} m_3 n_3 + 2\tau_{mzx} n_3 l_3 \, . \tag{20}$$

where $\sigma_{mean} = 0.5\ (\sigma_{max} + \sigma_{min})$ for either the Method 1 or 2, and the same rule applies to calculate the means of all stress components: $\sigma_{mx}$, $\sigma_{my}$, $\sigma_{mz}$, $\tau_{mxy}$, $\tau_{myz}$, and $\tau_{mzx}$. Eqs. 19 and 20 are the basis for the Method 2, which can be extended to the case of varying principal direction in line with the ASME treatment for alternating stress.

### *5.2 Proposed procedure and code statement of the mean stress for the varying principal direction*

The proposed procedure starts from the original six stress components. First calculate the stress range for each stress component between time A and time B as:

$$\sigma_{rangei} = \sigma_i(B) - \sigma_i(A) \, . \tag{21}$$

The principals of the stress range can be obtained by inserting Eq. 21 for each stress range component into Eq. 6. The direction cosines for each principal stress range can be calculated by using combined Eqs. 5 and 7.

Take the differences between each pair of the stress ranges to find the maximum difference, which is then divided by 2 to obtain the equivalent alternating stress amplitude.

The next step is to find the mean stresses for each six stress component, as

$$\sigma_{meani} = \left[\sigma_i(B) + \sigma_i(A)\right]/\,2 \, . \tag{22}$$

The principals of the stress means and their direction cosines can be obtained by using the same procedure as discussed above.

The final step is critical.  One has to pick up 2 out of the 3 means to add up for the equivalent mean stress.  *The 2 mean stresses must have the same direction cosines with those which determine the maximum stress difference*.

The mean stress for each principal stress range can also be obtained alternatively by:

$$\sigma_m = \sigma_{mx} l^2 + \sigma_{my} m^2 + \sigma_{mz} n^2 + 2\tau_{mxy} lm + 2\tau_{myz} mn + 2\tau_{mzx} nl \ . \qquad (23)$$

The following is the proposed code statement for the case of varying principal direction.  Since the case of constant principal direction is a special case of varying principal direction, the proposed code can be used universally for either constant or varying principal direction.

For any case in which the directions of the principal stresses at a point change during the stress cycle, the mean equivalent stress $S_{mean}$ shall be calculated based on the six time-dependent stress components as described below in (a) to (h).
(a) Consider the value of the six stress components $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}$ as a function of time for the complete stress cycle.
(b) Find the stress ranges for each stress component during the completed stress cycle, e.g., $\sigma_{rx}, \sigma_{ry}, \sigma_{rz}, \tau_{rxy}, \tau_{ryz}, \tau_{rzx}$.
(c) Calculate the three principal stress ranges $\sigma_{r1}, \sigma_{r2}, \sigma_{r3}$ and the direction cosines *l, m, n* for each principal based on the six stress range components $\sigma_{rx}, \sigma_{ry}, \sigma_{rz}, \tau_{rxy}, \tau_{ryz}, \tau_{rzx}$.
(e)  Take the differences between each pair of principal stress ranges $S_{r12} = \sigma_{r1} - \sigma_{r2}$, $S_{r23} = \sigma_{r2} - \sigma_{r3}$, and $S_{r31} = \sigma_{r3} - \sigma_{r1}$. The largest principal stress range difference is defined as $S_{rij} = \max(S_{r12}, S_{r23}, S_{r31})$ , which have the corresponding direction cosines as *(l,m,n)i and (l,m,n)j*. The equivalent alternating stress is half of the largest stress range difference $S_{alt} = 0.5 \cdot S_{rij}$ .
(f) Find the mean stress for each stress component during the completed stress cycle, e.g., $\sigma_{mx}, \sigma_{my}, \sigma_{mz}, \tau_{mxy}, \tau_{myz}, \tau_{mzx}$.
(g) Calculate the three principal mean stresses $\sigma_{m1}, \sigma_{m2}, \sigma_{m3}$ and the direction cosines *l, m, n* for each principal mean stress based on the six mean stress components $\sigma_{mx}, \sigma_{my}, \sigma_{mz}, \tau_{mxy}, \tau_{myz}, \tau_{mzx}$.

The principal mean stress with given direction cosines can also be calculated alternatively by using

$$\sigma_m = \sigma_{mx} l^2 + \sigma_{my} m^2 + \sigma_{mz} n^2 + 2\tau_{mxy} lm + 2\tau_{myz} mn + 2\tau_{mzx} nl$$

(h) Determine the equivalent mean stress as the sum of the 2 principal means which have the same direction cosines *(l,m,n)i and (l,m,n)j* with those which determine the maximum stress range difference : $S_{mean} = \sigma_{mi} + \sigma_{mj}$ .

### *5.3 Example of application*

An example for the varying principal stress direction is shown below.

Table 6a Step "a" Initial stress component data

| Stress Component | Time Point A (MPa) | Time Point B (MPa) |
|---|---|---|
| $\sigma_x$ | 5 | 0 |
| $\sigma_y$ | -25 | -3 |
| $\sigma_z$ | 50 | -6 |
| $\tau_{xy}$ | 3 | 0 |
| $\tau_{yz}$ | 5 | 0 |
| $\tau_{zx}$ | 2 | 0 |

Table 6b Step "b" To find stress range for each stress component

| Stress Range Component | Stress Range Value (MPa) |
|---|---|
| $\sigma_{ax}$ | -5 |
| $\sigma_{ay}$ | 22 |
| $\sigma_{az}$ | -56 |
| $\tau_{axy}$ | -3 |
| $\tau_{ayz}$ | -5 |
| $\tau_{azx}$ | -2 |

Table 6c Step "c" To calculate principal stresses range

| Principal Stress Range component | Stress Range Value (MPa) | Direction Cosine |
|---|---|---|
| $\sigma_{a1}$ | -56.4 | (0,0,1) |
| $\sigma_{a2}$ | -5.2 | (1,0,0) |
| $\sigma_{a3}$ | 22.6 | (0,1,0) |

Table 6d  Step "d" To calculate stress range differences

| Principal Stress Component | Stress Range Difference (MPa) | Direction Cosine |
|---|---|---|
| $\sigma_{a12}$ | 51.2 | (0,0,1), (1,0,0) |
| $\sigma_{a23}$ | 27.8 | (1,0,0), (0,1,0) |
| $\sigma_{a13}$ | -79 | (0,1,0), (0,0,1), |

Step "e" to find the maximum stress range difference

The maximum stress range difference is $\sigma_{a13} = -79MPa$ with a pair of direction cosines of (0,1,0) and (0,0,1). The equivalent alternating stress amplitude is half the maximum stress range difference: $\sigma_a = \sigma_{a13} / 2 = -39.5MPa$.

Table 6e  Step "f" To find mean stress for each stress component

| Mean Stress Component | Mean Stress Value (MPa) |
|---|---|
| $\sigma_{mx}$ | 2.5 |
| $\sigma_{my}$ | -14 |
| $\sigma_{mz}$ | 22 |
| $\tau_{mxy}$ | 1.5 |
| $\tau_{myz}$ | 2.5 |
| $\tau_{mzx}$ | 1 |

Table 6f  Step "g" To calculate principal mean stresses

| Principal Mean Stress Component | Mean Stress Value (MPa) | Direction Cosines |
|---|---|---|
| $\sigma_{m1}$ | 22.2 | (0,0,1) |
| $\sigma_{m2}$ | 2.56 | (1,0,0) |
| $\sigma_{m3}$ | -14.3 | (0,1,0) |

Step "h" to find the equivalent mean stress

The equivalent mean stress is the sum of the 2 mean stresses with the same direction cosines in the step e: $\sigma_{mean} = \sigma_{m1} + \sigma_{m3} = 7.9MPa$ with direction cosines (0,0,1), (0,1,0).

Table 6f'  Step "g'" Principal mean stresses from Eq. 23

| Direction Cosines | Principal Mean Stress |
|---|---|
| (0,0,1) | $\sigma_{m1}$=22.23 |
| (0,1,0) | $\sigma_{m3}$=-14.29 |

Eq. 23 can be applied alternatively to obtain the required principal mean stresses:

$$\sigma_m = \sigma_{mx} l^2 + \sigma_{my} m^2 + \sigma_{mz} n^2 + 2\tau_{mxy} lm + 2\tau_{myz} mn + 2\tau_{mzx} nl \quad . \tag{23}$$

The results are listed in Table 6f', and the exactly same result with that in Step "h" is obtained: $\sigma_{mean} = \sigma_{m1} + \sigma_{m3} = 7.9 MPa$

However, in some cases, if the direction cosines in the step "g" (principal means) are not consistent with those in step "e" (maximum stress range difference), Eq. 23 must be used to obtain required principal mean stresses with the same direction cosines against those of maximum stress range differences.

*5.4 Comparison check*

As a universally applicable procedure, it must be in agreement with those of constant principal directions. Again use the same stress values listed in Table 6a, but assume constant principal direction.

Table 7a Initial stress data

| Stress Component | Time Point A (MPa) | Time Point B (MPa) |
|---|---|---|
| $\sigma_x$ | 5 | 0 |
| $\sigma_y$ | -25 | -3 |
| $\sigma_z$ | 50 | -6 |
| $\tau_{xy}$ | 3 | 0 |
| $\tau_{yz}$ | 5 | 0 |
| $\tau_{zx}$ | 2 | 0 |

Table 7b Calculate principal stresses for each time moment

| Principal Stress | Time point A (MPa) | Time point B (MPa) |
|---|---|---|
| $\sigma_1$ | -25.6 (0,1,0) | -3 (0,1,0) |
| $\sigma_2$ | 5.15 (1,0,0) | 0 (1,0,0) |
| $\sigma_3$ | 50.47 (0,0,1) | -6 (0,0,1) |

Note: the direction cosines must be compared to find the right match of each principal stress at 2 time moments

Table 7c Calculate principal stresses ranges and means

| Principal Stress | Range (MPa) | Mean (MPa) | Direction Cosines |
|---|---|---|---|
| $\sigma_1$ | 22.6 | -14.3 | (0,1,0) |
| $\sigma_2$ | -5.15 | 2.57 | (1,0,0) |
| $\sigma_3$ | -56.47 | 22.23 | (0,0,1) |

The pair with maximum stress difference is $\sigma_{a13} = \sigma_3 - \sigma_1 = -79MPa$ with direction cosines of (0,1,0), (0,0,1). Therefore the equivalent stress amplitude is half of the maximum stress difference $\sigma_a = \sigma_{a13} / 2 = -39.5MPa$

The equivalent mean stress is the sum of the 2 means with the same direction cosines of the pair of stresses to determine the maximum stress difference $\sigma_{mean} = \sigma_{m1} + \sigma_{m3} = 7.9MPa$ with direction cosines (0,0,1), (0,1,0). Note that the results, one using the code for the constant principal and another one using the code for varying principal direction, are identical.

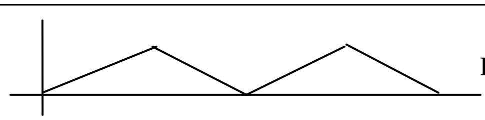
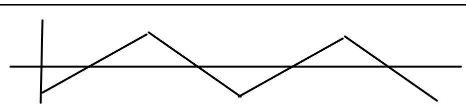
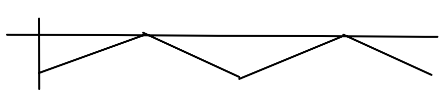
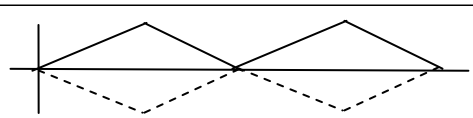
5.5 *Exceptions to the proposed mean stress definition[10]*

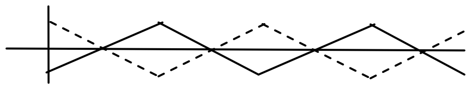
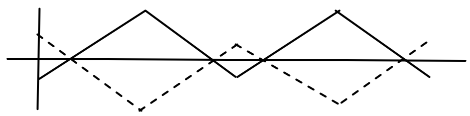
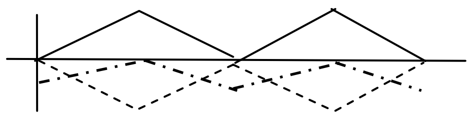
The above code statements are based on Eqs. 2, the Tresca mean stress model. Literature also indicates that Eq. 3, the Von Mises mean stress model can also be used. However, there are some exceptions, where only the model of maximum principal stress is applicable, e.g., the multiaxial fatigue on a cast iron specimen. It is found that the test data of multiaxial fatigue on the cast iron specimens fall nearer to the maximum principal stress criterion than to the maximum shear criterion. The mechanism is explained by Sines. Many flakes of graphite exist in the microstructure of the cast iron. Since the flakes of graphite carry little stress, they behave like porosity in the structure. Similar to crack growth behavior in most structural materials, the crack initiation caused by each flake and their combination is mostly governed by the maximum principal stress.

## 6. Case study of the proposed mean stress definition

The proposed mean stress definition is applied in different cases listed in Table 8, and produces reasonable results for all these cases.

Table 8 Case applications of the proposed mean stress definition

| Multiaxial fatigue case<br>(solid line $\sigma_1$, dot-dash line $\sigma_2$, dash line $\sigma_3$ ) | Mean defined by<br>$\sigma_m = \sigma_{1m} + \sigma_{3m}$ | Logically or tested true mean |
|---|---|---|
|  Pure tension | Positive | Positive |
|  Reversal | Zero | Zero |
|  Pure compression | Negative | Negative |
|  Pure shear | Zero | Zero |

| | | |
|---|---|---|
|  Symmetric shear | Zero | Zero |
|  Asymmetric shear | Zero | Zero |
|  Triaxial (nonproportional) loading | Zero | Zero |

Note that, for the last case, if the fatigue failure is controlled by the maximum shear model, the result is zero since the normal stress acting on the maximum shear plane is zero although a small compression is acting on a non-critical shear plane.

## 7. Fatigue life evaluation for a pin subjected to rotating loads

As an application example, consider a pin made of stainless steel 316LN in the ITER CS assembly. The pin is part of a proposed vertical support system that utilizes 9 toroidally oriented shear pins that are clamped by preloaded bolts. The pin is subjected to rotating shear and compression during operation. Let us determine the equivalent alternating stress according to ASME B&PV, and the equivalent mean stress according to this proposal.

Figure 3 gives 3 stress components (Sy, Sz, Syz) at the pin as a function of load step, in which A and B represent 2 extreme time points during one cycle. For this case, x is the radial direction and y is the vertical direction. The original stresses are listed in Table 9a:

Table 9a Initial stress components of the pin

| Stress Component | @ Time Point A (MPa) | @ Time Point B (MPa) |
|---|---|---|
| $\sigma_x$ | 0 | 0 |
| $\sigma_y$ | -23 | -16 |
| $\sigma_z$ | -91 | -96 |
| $\tau_{xy}$ | 0 | 0 |
| $\tau_{yz}$ | -55.4 | 44 |
| $\tau_{zx}$ | 0 | 0 |

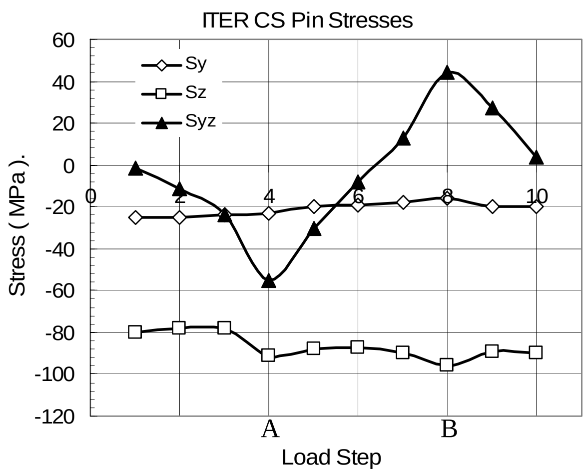



Fig. 3 Stresses at ITER CS gravity support pin, well preloaded, as a function of load step, A and B represent two extreme time points. $S_{yz}$ is the same as $\tau_{yz}$ in Table 9a

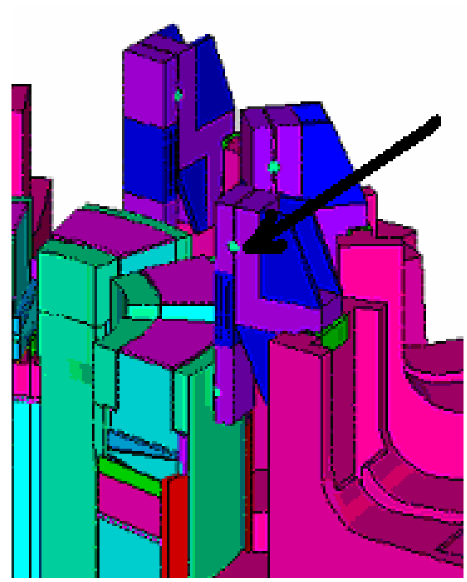


Fig. 4 Location of a Pinned Joint Utilized in One of the Proposed CS Structural Support Concepts

The stress ranges in Table 9b for each stress component are obtained by taking the difference of each stress component between the 2 extreme time points A and B, as listed in Table 9a. The principal stress ranges in Table 9c are then calculated from the stress range of 6 stress components, using the techniques discussed above.

Table 9b: The range for each stress component

| Stress Range Component | Stress Range Value (MPa) |
|---|---|
| $\sigma_{ax}$ | 0 |
| $\sigma_{ay}$ | 7 |
| $\sigma_{az}$ | -5 |
| $\tau_{axy}$ | 0 |
| $\tau_{ayz}$ | 99.4 |
| $\tau_{azx}$ | 0 |

Table 9c: Principals of stress range

| Principal Stress Range component | Stress Range Value (MPa) | Direction Cosine |
|---|---|---|
| $\sigma_{a1}$ | 100.58 | (0, 0.728, 0.685) |
| $\sigma_{a2}$ | 0. | (1, 0, 0) |
| $\sigma_{a3}$ | -98.58 | (0, 0.685, -0.728) |

The maximum stress range difference is $\sigma_{a13} = 199.16MPa$ with a pair of direction cosines of (0, 0.728, 0.685) and (0, 0.685, -0.728). The equivalent stress amplitude is half the maximum stress range difference. $\sigma_a = \sigma_{a13} / 2 = 99.58MPa$.

Then, let us discuss the mean stress evaluation. Table 9d lists the mean stress for each stress components in Table 9a.

Table 9d: Mean stress for each stress component

| Mean Stress Component | Mean stress value (MPa) |
|---|---|
| $\sigma_{mx}$ | 0 |
| $\sigma_{my}$ | -19.5 |
| $\sigma_{mz}$ | -93.5 |
| $\tau_{mxy}$ | 0 |
| $\tau_{myz}$ | -5.7 |
| $\tau_{mzx}$ | 0 |

For this example, it is found that the direction cosines of principal means are not consistent with those of maximum stress range difference in Table 9c. Therefore, Eq. 23 is applied to evaluate the required principal means. The results are listed in Table 9e.

Table 9e Principal mean stresses from Eq. 23

| Direction Cosines | Principal Mean Stress |
|---|---|
| (0, 0.728, 0.685) | $\sigma_{m1}$ = -59.96 |
| (0, 0.685, -0.728) | $\sigma_{m3}$ = -53.04 |

We now have the equivalent mean stress according to the suggested code statement:

$$\sigma_{mean} = \sigma_{m1} + \sigma_{m3} = -113MPa$$

The Soderberg relation (Eq. 24) [13] is then applied to account for the combined alternating and mean stress effects on the equivalent fatigue stress $\sigma_{fs}$:

$$\sigma_{fs} = \frac{\sigma_a}{\left(1 - \frac{\sigma_m}{\sigma_y}\right)} \tag{24}$$

where $\sigma_a$, $\sigma_y$ and $\sigma_m$ are the equivalent stress amplitude, yield stress, and mean stress respectively (MPa). $\sigma_{fs}$ is the equivalent fatigue stress at load ratio R=-1 to be used with an S/N curve for calculation of lifetime.

For the pin made of stainless steel 316LN, $\sigma_y \approx 1235MPa$. Substituting $\sigma_a = 99.58MPa$ and $\sigma_m = -113MPa$ into Eq. 24 gives the equivalent R=-1 stress or $\sigma_{fs} = 91.23MPa$. Apparently, the compressive mean stress reduces the effective applied stress amplitude. Substituting $\sigma_{fs} = 91.23MPa$ into S-N curve of this alloy at 4K gives fatigue life no less that $10^6$ cycles. For ITER, the life requirement is for 30000 of these cycles. ITER criteria requires a margin of the more conservative of a factor 2 on stress or 20 on life. This is the same as for ASME. Thus we would enter a R= -1 SN curve (shown below) for the appropriate temperature, at an alternating stress of 2 * 91.23 = 182.5 MPa , and the pin design satisfies the criterion.

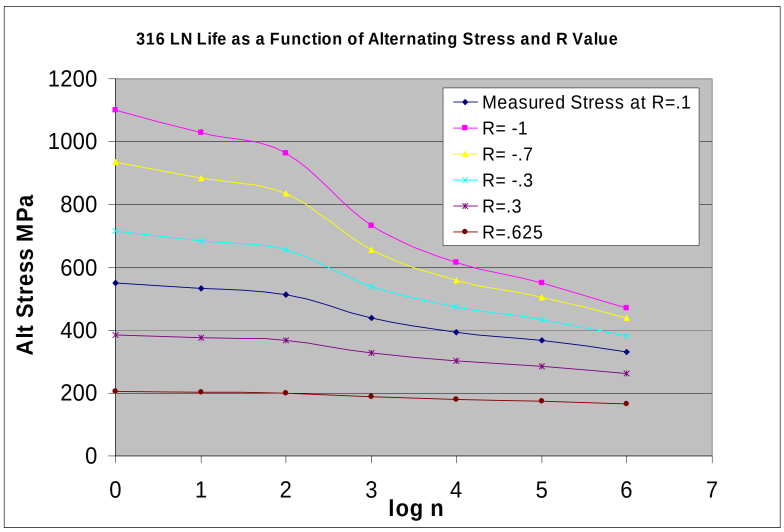



Fig. 5 R=-1 4K 316 LN Fatigue Data, Developed from Measure R=.1 Measured data, and the Solderberg Relation (5)

Finally, let us check the rotation feature of the stresses in Table 9a. Tables 9f and 9g give the 3 principal stresses and their direction cosines at time point A and B respectively. Comparing Table 9f to 9g indicates that the principal directions rotate from the time point A to the time point B. Since each principal stress rotation path is not tracked during typical database, it is hard to apply the German code for further calculation of the equivalent alternating and mean stresses.

Table 9f principal stresses at time point A of the pin

| Stress Component | @ Time Point A (MPa) | Direction Cosines |
|---|---|---|
| $\sigma_1$ | -122 | (0, 0.488, 0.873) |
| $\sigma_2$ | 0. | (1, 0, 0) |
| $\sigma_3$ | 8 | (0, 0.873, -0.488) |

Table 9g principal stresses at time point B of the pin

| Stress Component | @ Time Point B (MPa) | Direction Cosines |
|---|---|---|
| $\sigma_1$ | -115 | (0, 0.405, -0.915) |
| $\sigma_2$ | 0. | (1, 0, 0) |
| $\sigma_3$ | 3.46 | (0, 0.915, 0.405) |

## 8. Conclusions

Existing theories and design codes have been reviewed. The mean stress effect can not be neglected when a material is operated in the elastic range, e.g., in ITER structure design.

A German structural standards code that has been gaining international acceptance is adopted, using a maximum shear critical plane model to define the equivalent mean stress of multiaxial fatigue with a constant principal direction. However, some example shows that it is hard to apply directly for the case of varying principal direction.

For the more general case of varying principal stress direction, the equivalent mean stress is determined by a more complex method. The mean stresses of all 6 stress and shear components are calculated, before calculating the principal mean stresses. A universally applicable procedure to evaluate the equivalent mean stress is developed, based on a Tresca mean stress model. The key is that the user must determine the direction cosines for each principal stress, stress range, maximum stress difference, and mean stress. The sum of the 2 mean stresses that are added to find the equivalent mean stress must be the normal stress acting on the maximum shear plane, i.e., they must have the same direction cosines as the two stress ranges that determine the maximum stress difference.

The proposed procedure generates identical results with German procedures for cases with constant principal directions. It is also found that the proposed procedure can be applied very well to most special cases, including uniaxial, biaxial fatigues, and static combined cyclic loading. One exception comes when the fatigue failure mechanism is not controlled by the maximum shear plane model, as with cast iron.

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