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Stability of Superconducting Cables with Twisted Stacked YBCO Coated Conductors

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1 Preface

For the design of superconducting applications quench protection is a very important issue. Therefore the limits of stable operation of the superconductors must be well known. As low temperature superconductors are already in use in many applications for decades, there is a lot of experience in the field of stability and the design with this kind of superconductors.

High temperature superconductors like YBCO coated conductors have different material properties and can be operated at much higher temperatures and higher fields. Their quench propagation velocity is in the range of three magnitudes lower than compared to low temperature superconductors. This increases the risk of damage by hot spots. Furthermore the conductivity of the normal conducting layers of YBCO coated conductors is anisotropic and thus a current sharing between multiple tapes in a cable is hindered. Therefore cabling systems have to be developed and the stability of cables with YBCO tapes has to be investigated in order to be able to design stable operating devices.

This work makes a contribution to the development of superconducting high current cables with YBCO coated conductors that can be used for fusion magnets or other power applications. The cable shall meet the following specifications:

- fields < 15 T
- currents < 5 kA
- forces < 75 kN
- current densities > 100 A/mm^2
- operating temperature about 20 K

The assembly of the cable is supposed to be a twisted stack of YBCO coated conductors that are soldered together and are surrounded by a copper support structure. Hence there are multiple parameters in the cable design like the different layer thicknesses of the tapes (YBCO coated conductors), the thickness of the solder layers between the tapes, the number of tapes in a cable and the cross sectional area of the surrounding copper support structure. The appropriate choice of all these parameters is essential for a stable operation of a cable in a fusion magnet or other applications.

In this work the limit of stable operation of such a cable design is investigated. The focus is on electrical stability only. Mechanical strength and manufacturing aspects are not considered in this report.

1 Preface

An electrical circuit model for cables with YBCO coated conductors is introduced. This model is used for the investigation of the current distribution and the joule heating in case of local perturbations on the cable. Based on this cable model a program code was implemented in order to provide tools to apply different established stability criteria on this cable type. This approach allows the variation all geometrical parameters of the cable as well as the temperature and the magnetic field density the cable is exposed to. Therefore the material properties of all materials involved are implemented in fit-functions in the temperature range from 4.2 K to 300 K and the magnetic field density in the range from 0.1 T to 20 T. Additionally the cooling power of different gaseous and liquid coolants is also implemented in fit-functions. In the end the stability criteria are applied on different case examples.

For the implementation of the calculations the commercial numerical computation software Matlab from Mathworks was used. All program code is implemented in *.m files as executable Matlab functions. All code is attached in the appendix of this report.

2 General Cable Assembly and Geometrical Parameters

In this report a simplified cable assembly is assumed by reason that there is no decision on the final cable assembly made yet. The new cable design was already proposed and published in [TMB11], [TCB12]. The simplified cable assembly considered in this report is described in the following.

The YBCO coated conductors (tapes) are supposed to be soldered to a stack that is surrounded by a cupper support structure as shown in Fig. 2.1. The cable is twisted along its axes. The twist has no effect on the stability calculations performed in this report. There is only to mention that if such a cable is used as conductor in a magnet, the magnetic field of the magnet will be perpendicular to the YBCO plane periodically, which causes a periodically reduction of the critical current along the cable as the dependence of the critical current on the magnetic field is anisotropic and the lowest when the critical current is perpendicular to the YBCO plane. But as a conductor in a magnet has to be designed to operate in the highest field range anyway the twist and the resultant variation of the critical current are not considered in the following.

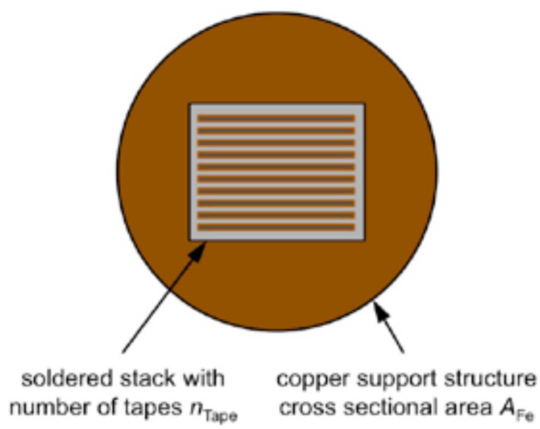


Fig. 2.1: General cross section of cable assembly. A stack of YBCO coated conductors (tapes) is soldered and immersed in a cupper support structure. The cable is twisted along its axis.

The copper support structure is mainly supposed to provide mechanical stabilization and to carry current if all superconductors are normal conducting only. But as there is no decision made on the final design, the copper support structure is not taken into account for the stability calculations in this stage of the work.

Fig. 2.2 shows the relevant dimensions of a part of the cross section of the cable and of a single YBCO coated conductor. The width of the tapes w_{tape} is supposed to be 4 mm. The dimensions given in Fig. 2.2 are the main parameters for the cable stability.

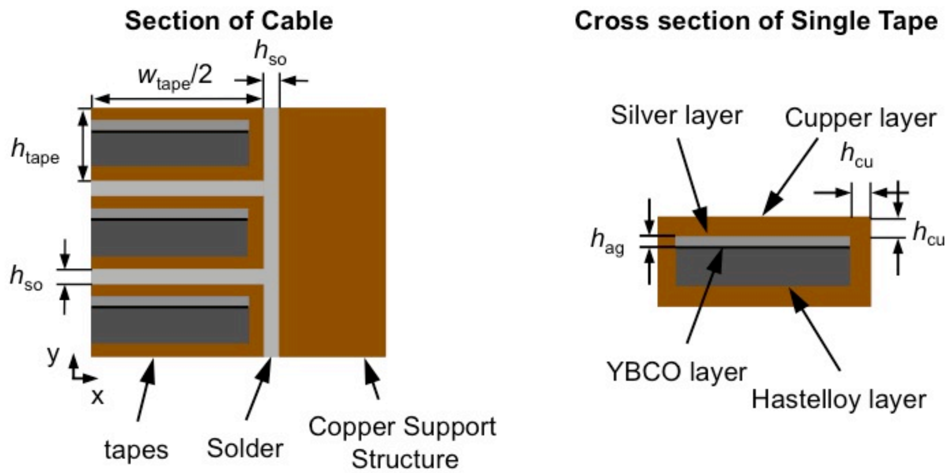


Fig. 2.2: Dimensions of parts of the structure of a cable and of the YBCO Coated conductors.

3 Material Properties

In order to implement stability calculations in a wide range of temperatures and magnetic field densities, the properties of the materials involved have to be known within the designated temperature and field range. Therefore the critical current, the electrical resistivity, the heat capacity and the thermal conductivity are implemented in fit-functions in the temperature range between 4 K and 300 K and magnetic field densities between 0.1 T and 20 T. In the following all fit-functions used for the calculations are listed.

3.1 Critical Current of YBCO Coated Conductors

In order to calculate the critical current I_c of YBCO coated conductors for a wide range of temperature and magnetic field density the equations and parameters given in [Wes11] and [Wes10] were used. These are

$$I_c(B,T) = h_{\mathrm{YBCO}} \cdot w_{\mathrm{tape}} \cdot \frac{A}{B} \cdot B_{\mathrm{irr}}(T)^{\beta} \cdot \left(\frac{B}{B_{\mathrm{irr}}(T)}\right)^{p} \cdot \left(1 - \frac{B}{B_{\mathrm{irr}}(T)}\right)^{q} \tag{3.1}$$

with

$$B_{\mathrm{irr}}(T) = B_{\mathrm{irr}}(0) \cdot \left(1 - \frac{T}{T_c}\right)^{\alpha} \tag{3.2}$$

and the parameters $T_c = 92$ K, $B_{\mathrm{irr}} = 132.5$ T, $A = 4.52 \cdot 10^8$ A, $p = 0.653$, $q = 2.568$, $\alpha = 1.5$, $\beta = 1.789$, the variable temperature T and magnetic field density B, the thickness of the YBCO layer h_{YBCO} and the width of the tapes w_{tape}. This fit-function is valid in the temperature range from 4 K to 300 K and magnetic field densities from 0.1 T to 20 T.

Fig. 3.1 shows the characteristics of the critical current density of a 4 mm wide YBCO tape with a YBCO-layer thickness of $h_{\mathrm{ybco}} = 1$ µm for some selected temperatures depending on the magnetic field density. The values were calculated with fit-function (3.1) and the given parameters.

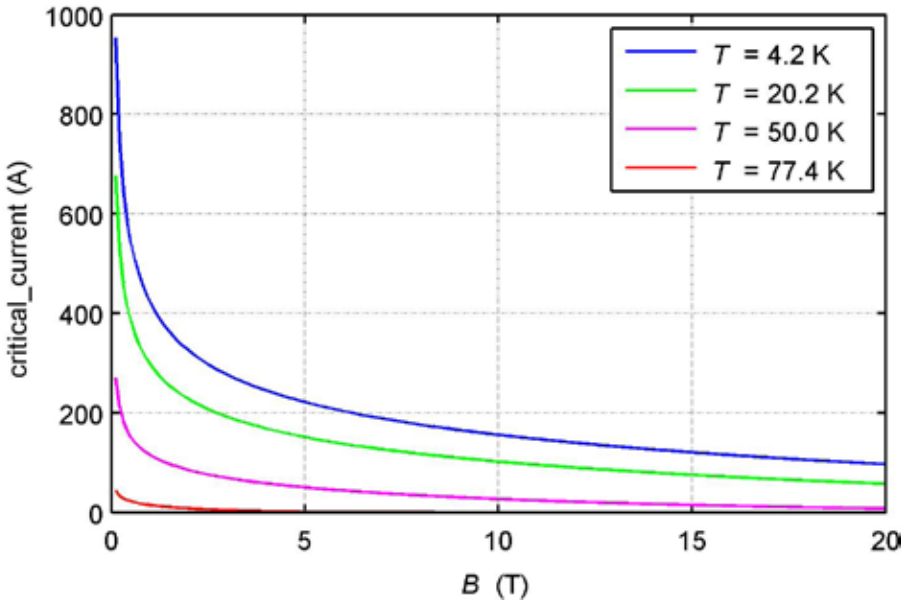


Fig. 3.1: Critical current of 4 mm wide YBCO tape depending on temperature and magnetic field density calculated by the given fit-function. Parameters: h_{YBCO} = 1 µm

In Tab. 3.1 selected values of the critical current for different temperatures and magnetic fields, calculated by the given fit-function are listed.

Tab. 3.1: Critical current of 4 mm wide YBCO-tapes for selected temperatures calculated by fit-function

T (K)	4.2	4.2	20.3	20.3
B (T)	10	15	10	15
I_c (A)	155.7	120.5	95.8	70.0

This function is implemented in Matlab file: *critical_current.m*

3.2 Electrical Resistivity

For the calculation of the electrical resistance load per unit length of single tapes and the conductance load per unit length between two tapes the electrical resistivity of the different materials involved are needed in the temperature range from 4 - 293 K. In the following, several fit-functions for the calculation of these material properties are introduced.

Resistivity of Hastelloy

For the calculation of the temperature depending resistivity of hastelloy, the equation and parameter values given in [CCW70] and [Lu08] are used. The fit-function used in this work is:

$$\rho_{\mathrm{hs}}(T) = a \cdot T + b \begin{cases} a = 0.1469 \cdot 10^{-9}; & b = 1.229 \cdot 10^{-6}\,\mathrm{K} \quad \textit{for} \quad T \leq 12\,\mathrm{K} \\ a = 0.2667 \cdot 10^{-9}; & b = 1.234 \cdot 10^{-6}\,\mathrm{K} \quad \textit{for} \quad T > 12\,\mathrm{K} \end{cases} \tag{3.3}$$

This fit-function is valid in the temperature range from 4 K to 300 K. Fig. 3.2 shows the characteristics of the resistivity of hastelloy depending on the temperature calculated by the given fit-function.

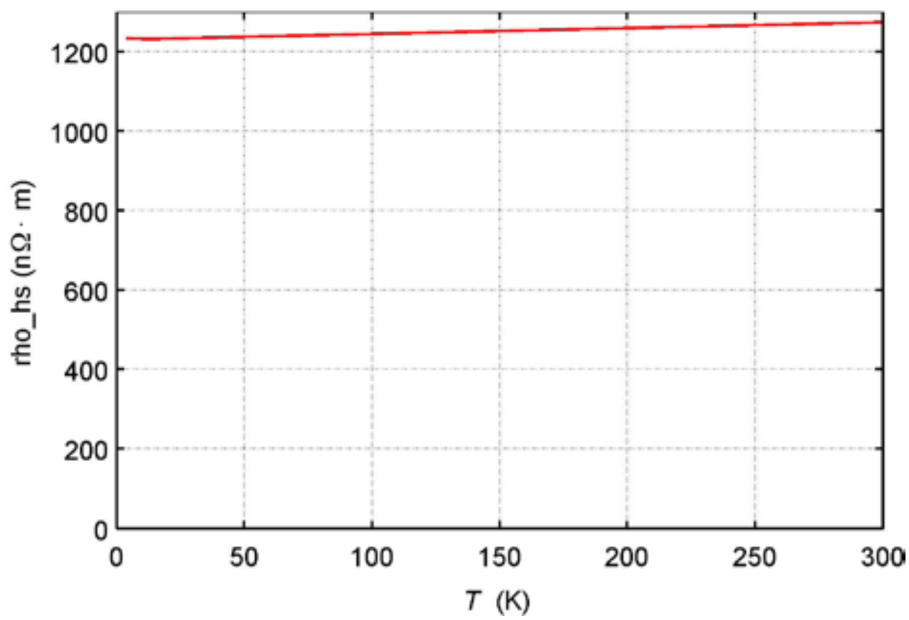


Fig. 3.2: Temperature repentant electrical resistivity of hastelloy calculated by the given fit-function

Tab. 3.2 gives selected values of the electrical resistivity of hastelloy that were also calculated by the given fit-function.

Tab. 3.2: Electrical resistivity of hastelloy calculated by fit-function for selected temperatures

T (K)	4.2	20.3	77.4	293.2
ρ_{hs} (nΩ·m)	1232	1231	1240	1272

This function is implemented in Matlab file: *rho_hs.m*

Resistivity of Silver

For the calculation of the temperature depending resistivity of silver, the equation and parameter values given in [Mat79] are used. The fit-function used in this work is:

$$\rho_{\mathrm{Ag}}(T) = \frac{C}{M \cdot \Theta} \cdot \left(\frac{T}{\Theta}\right)^5 \cdot \int_0^{\frac{\Theta}{T}} \frac{x^5 \cdot e^x}{\left(e^x - 1\right)^2} dz \tag{3.4}$$

With the parameters $C = 0.0011650123$, $M = 107.868$ and $\Theta = 220.9$ K. This fit-function is valid in the temperature range from 4 K to 300 K.

Fig. 3.3 shows the characteristics of the resistivity of silver depending on the temperature, calculated by the given fit-function.

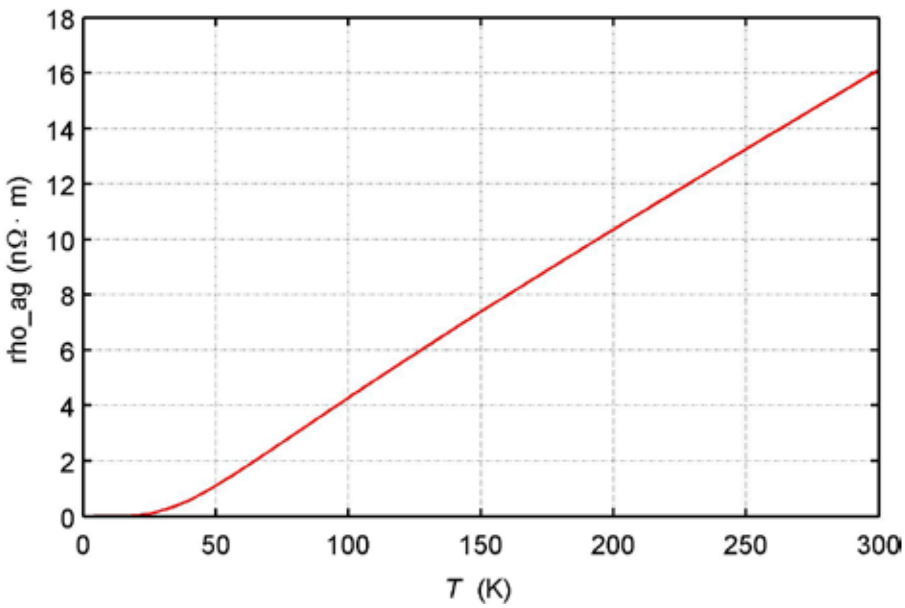


Fig. 3.3: Temperature dependent electrical resistivity of silver calculated by the given fit-function

In Tab. 3.3 selected values of the electrical resistivity of silver calculated by the given fit-function are listed.

Tab. 3.3: Electrical resistivity of silver calculated by fit-function for selected temperatures

T (K)	4.2	20.3	77.4	293.2
ρ_{ag} (nΩ·m)	$1.184 \cdot 10^{-5}$	$3.571 \cdot 10^{-2}$	2.787	14.570

This function is implemented in Matlab file: *rho_ag.m*

Resistivity of Copper

For the calculation of the temperature depending resistivity of copper, the equation and parameter values given in [PK82] are used.

$$\rho_{cu}(T) = \begin{cases} \rho_0 + a \cdot T^2 + b \cdot T^N & for \quad T \leq 12\,\mathrm{K} \\ \rho_0 + \rho_{ss} \cdot T^5 \cdot \dfrac{J_5\left(\dfrac{\Theta}{T}\right)}{124.14} & for \quad T > 12\,\mathrm{K} \end{cases} \tag{3.5}$$

With the parameters $a = 3 \cdot 10^{-15}$ Ωm/K², $b = 5.4 \cdot 10^{-18}$ Ωm/K^N, $N = 4.61$, $\varrho_0 = 7.6 \cdot 10^{-12}$ Ωm and $\Theta = 337$ K and the integral function

$$J_5\left(\frac{\Theta}{T}\right) = \int_0^{\frac{\Theta}{T}} \frac{x^N dx}{\left(e^x - 1\right) \cdot \left(1 - e^{-x}\right)} \tag{3.6}$$

This fit-function is valid in the temperature range from 4 K to 300 K. Fig. 3.4 shows the characteristics of the resistivity of copper depending on the temperature, calculated by the given fit-function.

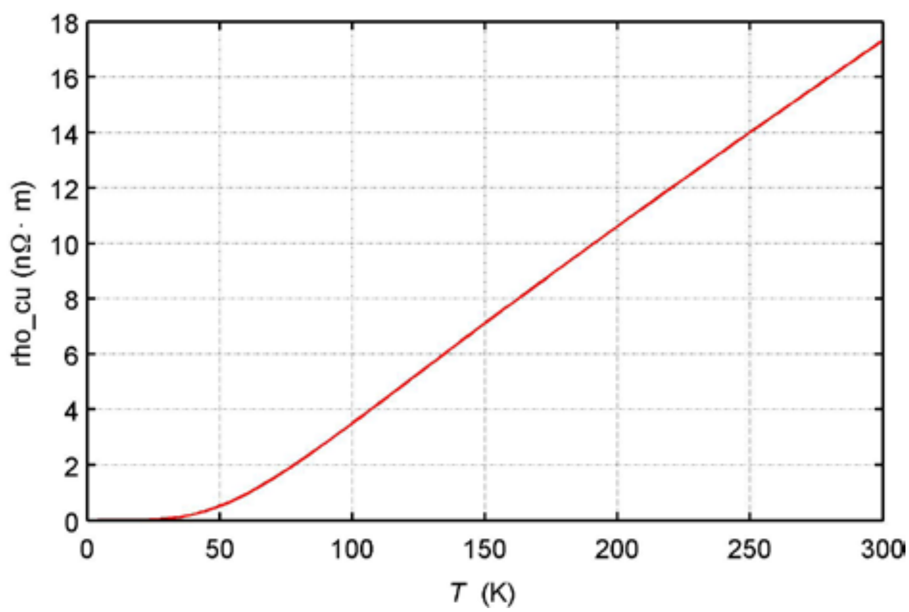


Fig. 3.4: Temperature dependent electrical resistivity of copper calculated by the given fit-function.

In Tab. 3.4 selected values of the electrical resistivity of copper calculated by the given fit-function are listed.

Tab. 3.4: Electrical resistivity of copper calculated by fit-function for selected temperatures

T (K)	4.2	20.3	77.4	293.2
ρ_{cu} (nΩ·m)	$7.651 \cdot 10^{-3}$	$1.523 \cdot 10^{-2}$	1.931	15.533

This function is implemented in Matlab file: *rho_cu.m*

Resistivity of Solder (SnPb)

For the calculation of the temperature depending resistivity of solder (SnPb 6040) the equation and parameter values given in [ES80] are used. This reference allows the calculation of the resistivity with respect to the ratio of components and taking into account the magnetic field dependence. In this work it is calculated by the following fit-function

$$\rho_{so}(T) = r \cdot \rho_{sn}(T) + (1-r) \cdot \rho_{pb}(T) \tag{3.7}$$

Herein the variable r is the volume ratio of the components with $r = V_{sn} / V_{pb}$. The resistivity of the components is calculated as follows. For lead

$$\rho_{pb}(T) = \begin{cases} \frac{10^{-6}}{\Theta} \cdot \frac{T}{\Theta} \cdot \int_0^{\frac{\Theta}{T}} \frac{x^5 \cdot e^x \cdot dx}{\left(e^x - 1\right)^2} & for \quad T \leq \frac{\Theta}{2} \\ \frac{10^{-6} \cdot T}{4 \cdot \Theta^2} \left(1 - \frac{1}{18} \cdot \left(\frac{\Theta}{T}\right)^2 + \frac{1}{480} \cdot \left(\frac{\Theta}{T}\right)^4\right) & for \quad T > \frac{\Theta}{2} \end{cases} \tag{3.8}$$

with the parameter Θ = 83 K, and for tin

$$\rho_{sn}(T) = \begin{cases} \frac{10^{-6}}{\Theta} \cdot \frac{T}{\Theta} \cdot \int_0^{\frac{\Theta}{T}} \frac{x^5 \cdot e^x \cdot dx}{\left(e^x - 1\right)^2} & for \quad T \leq \frac{\Theta}{2} \\ \frac{10^{-6} \cdot T}{4 \cdot \Theta^2} \left(1 - \frac{1}{18} \cdot \left(\frac{\Theta}{T}\right)^2 + \frac{1}{480} \cdot \left(\frac{\Theta}{T}\right)^4\right) & for \quad T > \frac{\Theta}{2} \end{cases} \tag{3.9}$$

with the parameter Θ = 210 K. To estimate the influence of the magnetic field a fit of the given values in [PMS94] is applied by

$$\rho_{so,magn}(T,B) = \rho_{so}(T) \cdot \left(1 + 7.17592 \cdot 10^{-3} \cdot B\right) \tag{3.10}$$

This fit-function is valid in the temperature range from 4 K to 300 K. Fig. 3.5 shows the characteristics of the resistivity of Copper depending on the temperature, calculated by the given fit-function.

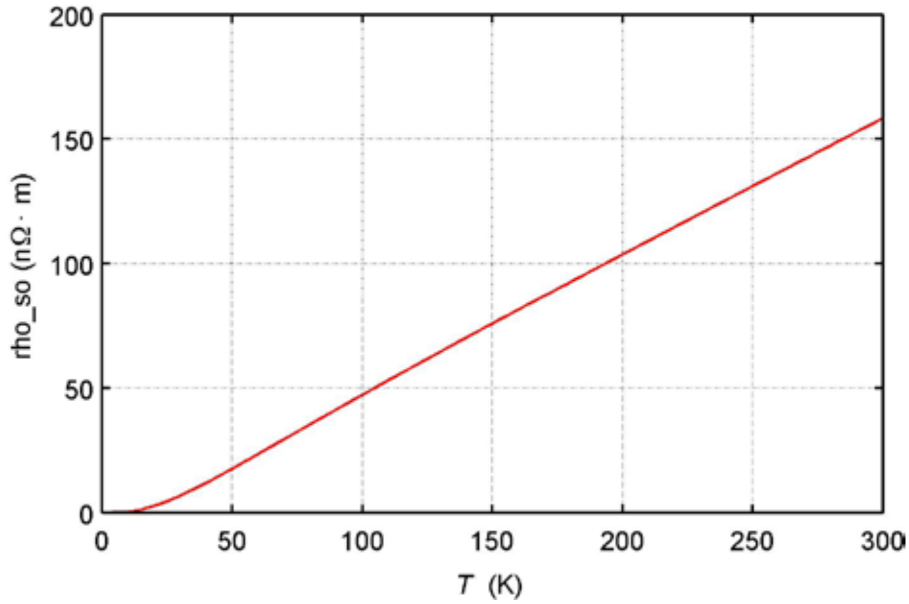


Fig. 3.5: Temperature dependent electrical resistivity of solder (SnPb 6040) calculated by the given fit-function

In Tab. 3.5 selected values of the electrical resistivity of solder for selected temperatures, calculated by the given fit-function are listed.

Tab. 3.5: Electrical resistivity of SnPb 6040 solder calculated by fit-function for selected temperatures

T (K)	4.2	20.3	77.4	293.2
ρ_{so} (nΩ·m)	$2.701 \cdot 10^{-3}$	2.545	33.58	143.6

This function is implemented in Matlab file: *rho_so.m*

3.3 Thermal Conductivity

In the following the fit-functions for the calculation of the heat capacity and thermal conductivity for different materials are given.

Thermal Conductivity of Silver

For the calculation of the thermal conductivity of silver, the equation and parameter values given in [SF95] are used. The fit-function used in this work is:

$$k_{ag}\left(T\right) = a \cdot T^5 + b \cdot T^4 + c \cdot T^3 + d \cdot T^2 + e \cdot T + f \tag{3.11}$$

The values are given in W/m·K. The parameter values for the different temperature ranges are given in Tab. 3.6

Tab. 3.6: Parameters of fit-function of thermal conductivity of silver

T (K)	4 - 33	33 - 100	100 - 300
a (W/m·K^{-5})	-8.085202766065000·10^{-04}	-3.358195377766776·10^{-07}	8.147381572319895·10-^{11}
b (W/m ·K^{-4})	0.084173230577580	1.869961406456805·10^{-04}	-1.042739292996614·10^{-07}
c (W/m ·K^{-3})	-2.898387926986296	-0.041125881979616	5.409027188706845·10^{-05}
d (W/m ·K^{-2})	30.127572196852682	4.473201297735611	-0.013972777872369
e (W/m ·K^{-1})	132.3932096483007	-241.3050714859127	1.569131427472265
f (W/m ·K)	103.1106730600989	5659.032445727915	410.5421708582979

This fit-function is valid in the temperature range from 4 K to 300 K. Fig. 3.6 shows the characteristics of the thermal conductivity of silver depending on the temperature and calculated by the given fit-function.

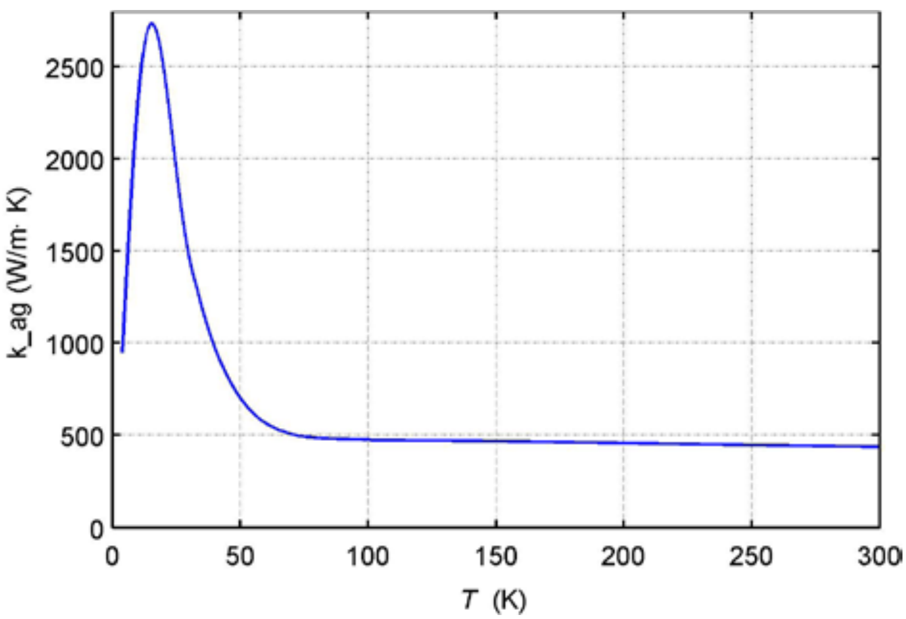


Fig. 3.6: Temperature dependent thermal conductivity of silver calculated by the given fit-function

In Tab. 3.7 selected values of the thermal conductivity of silver calculated by the given fit-function are listed.

Tab. 3.7: Thermal conductivity of silver calculated by the given fit-function for selected temperatures

T (K)	4.2	20.3	77.4	293.2
k_{ag} (W/m·K)	1001.0	2476.3	488.7	438.8

This function is implemented in Matlab file: *k_ag.m*

Thermal Conductivity of Copper

For the calculation of the thermal conductivity of copper (RRR = 100) the equation and parameter values given in [NIS10] are used. The fit-function used in this work is:

$$k_{\mathrm{cu}}(T) = 10^{\frac{a + c\cdot T^{\frac{1}{2}} + e\cdot T + g\cdot T^{\frac{3}{2}} + i\cdot T^{2}}{1 + b\cdot T^{\frac{1}{2}} + d\cdot T + f\cdot T^{\frac{3}{2}} + h\cdot T^{2}}} \tag{3.12}$$

The values are given in W/m·K. The parameters parameter values for the different temperature ranges are given in Tab. 3.8

Tab. 3.8: Parameters of fit-function of thermal conductivity of copper for the temperature range from 4 K to 300 K

a (-)	2.2154	d (-)	0.13871	g (-)	-0.04831
b (-)	-0.47461	e (-)	0.29505	h (-)	0.001281
c (-)	-0.88068	f (-)	-0.02043	i (-)	0.003207

This fit-function is valid in the temperature range from 4 K to 300 K. Fig. 3.7 shows the characteristics of the thermal conductivity of copper depending on the temperature, calculated by the given fit-function.

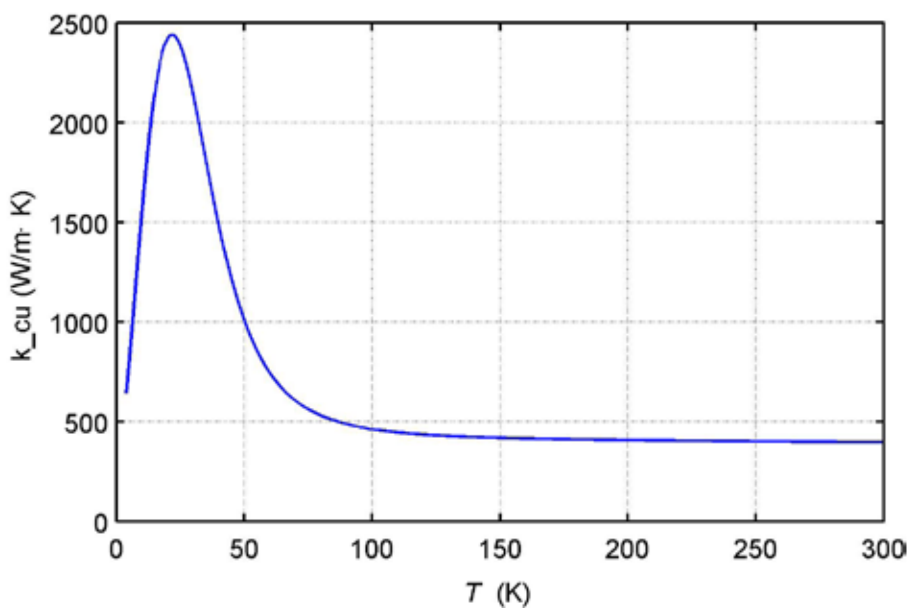


Fig. 3.7: Temperature dependent thermal conductivity of copper calculated by the given fit-function

In Tab. 3.9 selected values of the thermal conductivity of copper calculated by the given fit-function are listed.

Tab. 3.9: Thermal conductivity of copper calculated by the given fit-function for selected temperatures

T (K)	4.2	20.3	77.4	293.2
k_{cu} (W/m·K)	669.7	2441.4	544.6	396.9

This function is implemented in Matlab file: *k_cu.m*

Thermal Conductivity of Hastelloy

For the calculation of the thermal conductivity of hastelloy, the equation and parameter values given in [Lu08], [ZM63] and [GFC11] are used. The fit-function used in this work is:

$$k_{\text{hs}}\left(T\right) = a \cdot T^5 + b \cdot T^4 + c \cdot T^3 + d \cdot T^2 + e \cdot T + f \quad (3.13)$$

The values are given in W/m·K. The parameter values for the different temperature ranges are given in Tab. 3.10.

Tab. 3.10: Parameters of fit-function of thermal conductivity of hastelloy

T (K)	4 - 55	55 - 300
a (W/m·K-5)	$-1.976150097497916 \cdot 10^{-8}$	$3.544514666284345 \cdot 10^{-11}$
b (W/m ·K-4)	$1.692069531515161 \cdot 10^{-6}$	$-3.085569423939295 \cdot 10^{-8}$
c (W/m ·K-3)	$3.428435748706821 \cdot 10^{-5}$	$9.862456406179634 \cdot 10^{-6}$
d (W/m ·K-2)	-0.008289421572708	-0.001448613557764
e (W/m ·K-1)	0.399017559651135	0.120321620954427
f (W/m ·K)	-0.827062339102484	3.632700117022619

This fit-function is valid in the temperature range from 4 K to 300 K. Fig. 3.8 shows the characteristics of the thermal conductivity of hastelloy depending on the temperature, calculated by the given fit-function.

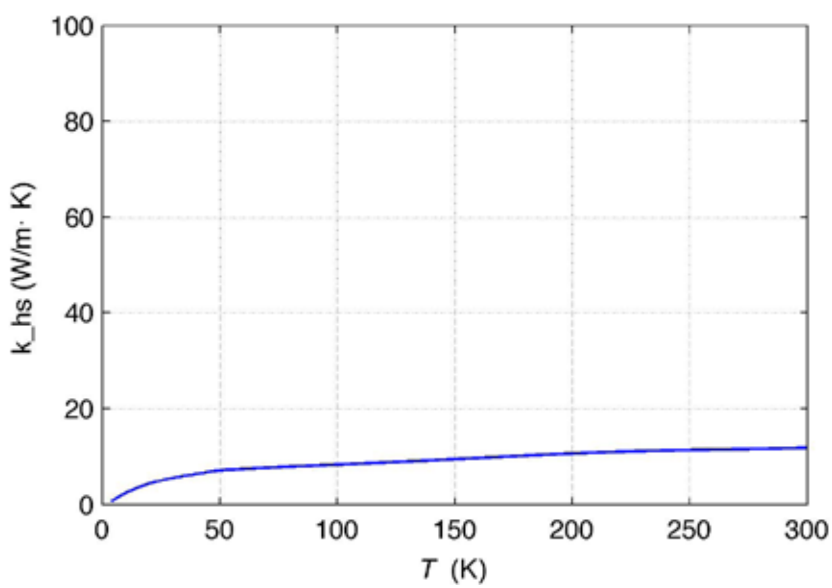


Fig. 3.8: Temperature dependent thermal conductivity of hastelloy calculated by the given fit-function

In Tab. 3.11 selected values of the thermal conductivity of hastelloy calculated by the given fit-function are listed.

Tab. 3.11: Thermal conductivity of hastelloy calculated by the given fit-function for selected temperatures

T (K)	4.2	20.3	77.4	293.2
k_{hs}(W/m·K)	0.706	4.349	7.831	11.74

This function is implemented in Matlab file: *k_hs.m*

Thermal Conductivity of Solder (SnPb)

For the calculation of the thermal conductivity of solder (SnPb 6040), the equation and parameter values given in [CR73], [PB54], [HJR79] are used. The fit-function used in this work is:

$$k_{so}\left(T\right) = a \cdot T^5 + b \cdot T^4 + c \cdot T^3 + d \cdot T^2 + e \cdot T + f \tag{3.14}$$

The values are given in W/m·K. The parameter values for the different temperature ranges are given in Tab. 3.12

Tab. 3.12: Parameters of fit-function of thermal conductivity of solder SnPb 6040

T (K)	4 - 70	70 - 300
a (W/m·K^{-5})	$7.803509145512186 \cdot 10^{-8}$	$-1.546126121833989 \cdot 10^{-10}$
b (W/m ·K^{-4})	$-2.751481293678849 \cdot 10^{-5}$	$1.282710598664956 \cdot 10^{-7}$
c (W/m ·K^{-3})	0.003657305390889	$-3.768920391716192 \cdot 10^{-5}$
d (W/m ·K^{-2})	-0.223437769764658	0.004395839566614
e (W/m ·K^{-1})	6.022184839140108	-0.102794660423839
f (W/m ·K)	-1.192898000639724	46.451188210708640

This fit-function is valid in the temperature range from 4 K to 300 K. Fig. 3.9 shows the characteristics of the thermal conductivity of solder depending on the temperature, calculated by the given fit-function.

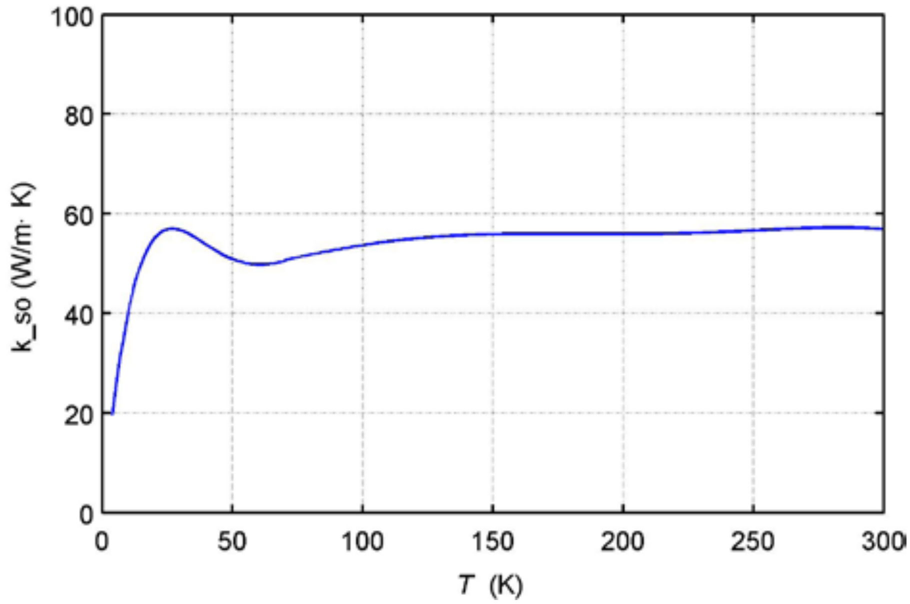


Fig. 3.9: Temperature dependent thermal conductivity of solder calculated by the given fit-function

This fit-function is valid in the temperature range from 4 K to 300 K. In Tab. 3.13 selected values of the thermal conductivity of solder calculated by the given fit-function are listed.

Tab. 3.13: Thermal conductivity of solder calculated by the given fit-function for selected temperatures

T (K)	4.2	20.3	77.4	293.2
k_{so} (W/m·K)	20.42	55.11	51.53	57.17

This function is implemented in Matlab file: *k_so.m*

Thermal Conductivity of YBCO

For the calculation of the thermal conductivity of YBCO the equation and parameter values given in [Wes10] are used. The fit-function used in this work is:

$$k_{\mathrm{ybco}}\left(T\right) = a \cdot T^5 + b \cdot T^4 + c \cdot T^3 + d \cdot T^2 + e \cdot T + f \qquad (3.15)$$

The values are given in W/m·K. The parameter values for the different temperature ranges are given in Tab. 3.14

Tab. 3.14: Parameters of fit-function of thermal conductivity of solder YBCO

T (K)	4 - 70	70 - 300
a (W/m·K^{-5})	-1.266103106492942·10^{-5}	-1.316704893540544·10^{-9}
b (W/m ·K^{-4})	0.002670105477219	2.636151594117440·10^{-6}
c (W/m ·K^{-3})	-0.197035302542769	-0.001601689632073
d (W/m ·K^{-2})	4.933962659604384	0.428760641312538
e (W/m ·K^{-1})	29.651536939501670	-53.306643333287260
f (W/m ·K)	66.578192505447330	3.682252343338599·10^{3}

This fit-function is valid in the temperature range from 4 K to 300 K. Fig. 3.10 shows the characteristics of the thermal conductivity of YBCO depending on the temperature, calculated by the given fit-function.

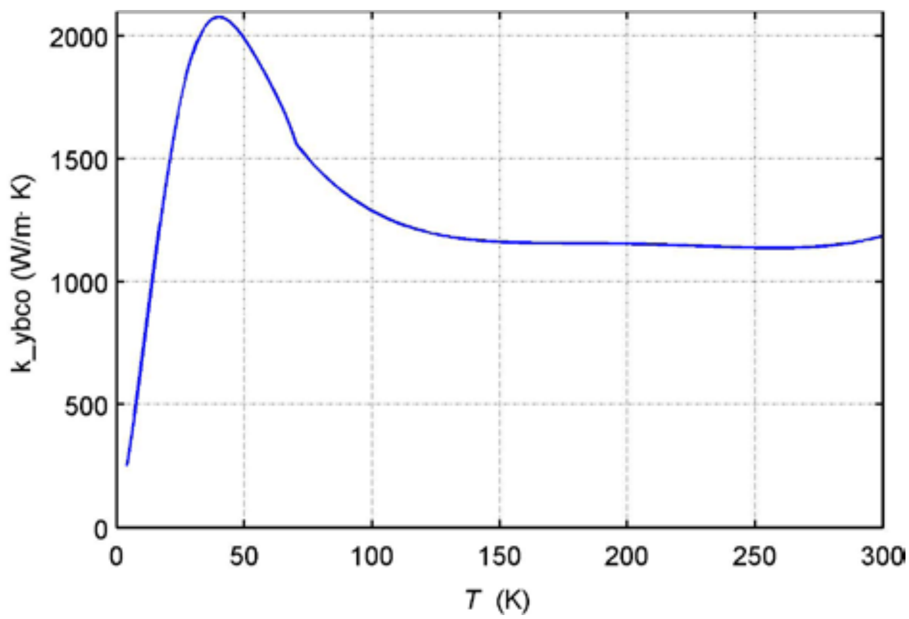


Fig. 3.10: Temperature dependent thermal conductivity of YBCO calculated by the given fit-function

In Tab. 3.15 selected values of the thermal conductivity of YBCO calculated by the given fit-function are listed.

Tab. 3.15: Thermal conductivity of YBCO calculated by the given fit-function for selected temperatures

T (K)	4.2	20.3	77.4	293.2
k_{ybco} (W/m·K)	264.3	1456.7	1473.2	1169.2

This function is implemented in Matlab file: *k_ybco.m*

Thermal Conductivity of Gaseous Helium

The data of the thermal conductivity of gaseous helium are taken from [PHL66]. These data can be fitted by the following function:

$$k_{\mathrm{he}}\left(T\right) = a \cdot T^5 + b \cdot T^4 + c \cdot T^3 + d \cdot T^2 + e \cdot T + f \qquad (3.16)$$

The values are given in W/m·K. The parameter values for the different temperature ranges are given in Tab. 3.16.

Tab. 3.16: Parameters of fit-function of thermal conductivity of helium

T (K)	4 - 300
a (W/m·K^{-5})	$2.065416328163720 \cdot 10^{-13}$
b (W/m ·K^{-4})	$-1.829051848815390 \cdot 10^{-10}$
c (W/m ·K^{-3})	$6.100032618364231 \cdot 10^{-8}$
d (W/m ·K^{-2})	$-9.871122362113007 \cdot 10^{-8}$
e (W/m ·K^{-1})	0.001224527275633
f (W/m ·K)	0.004074967617200

This fit-function is valid in the temperature range from 4 K to 300 K. Fig. 3.11 shows the characteristics of the thermal conductivity of helium depending on the temperature, calculated by the given fit-function.

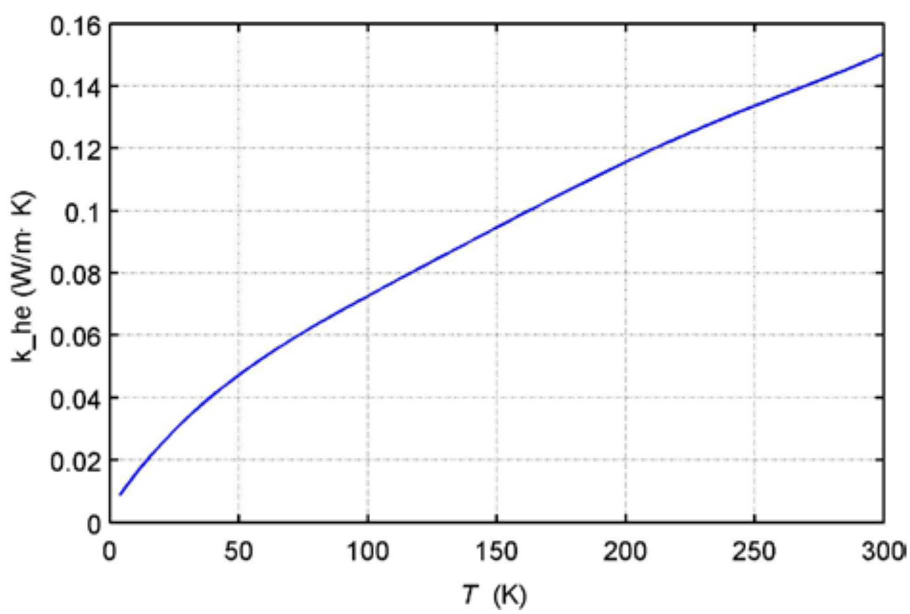


Fig. 3.11: Temperature dependent thermal conductivity of helium calculated by the given fit-function

In Tab. 3.17 selected values of the thermal conductivity of helium calculated by the given fit-function are listed.

Tab. 3.17: Thermal conductivity of helium calculated by the given fit-function for selected temperatures

T (K)	4.2	20.3	77.4	293.2
k_{he} (mW/m·K)	9.0	25.6	62.0	147.8

This function is implemented in Matlab file: *k_he.m*.

Thermal Conductivity of Gaseous Hydrogen

The data of the thermal conductivity of gaseous hydrogen are taken from [WSB48], [AMW86]. These data can be fitted by the following function:

$$k_{h2}(T) = a \cdot T^5 + b \cdot T^4 + c \cdot T^3 + d \cdot T^2 + e \cdot T + f \quad (3.17)$$

The values are given in W/m·K. The parameter values for the different temperature ranges are given in Tab. 3.18.

Tab. 3.18: Parameters of fit-function of thermal conductivity of hydrogen

T (K)	4 - 300
a (W/m·K^{-5})	$1.438979044890086 \cdot 10^{-13}$
b (W/m ·K^{-4})	$-1.179931934921931 \cdot 10^{-10}$
c (W/m ·K^{-3})	$3.406542452329941 \cdot 10^{-8}$
d (W/m ·K^{-2})	$-4.363250395376434 \cdot 10^{-6}$
e (W/m ·K^{-1})	$8.902878936027004 \cdot 10^{-4}$
f (W/m ·K)	-0.001433133271971

Fig. 3.12 shows the characteristics of the thermal conductivity of hydrogen depending on the temperature, calculated by the given fit-function.

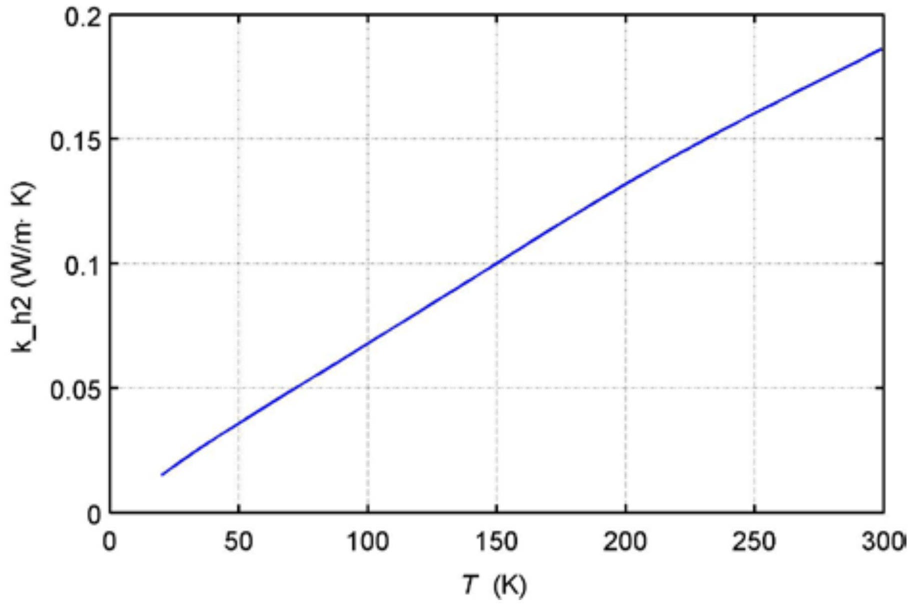


Fig. 3.12: Temperature dependent thermal conductivity of hydrogen calculated by the given fit-function

This fit-function is valid in the temperature range from 4 K to 300 K. In Tab. 3.19 selected values of the thermal conductivity of hydrogen calculated by the given fit-function are listed.

Tab. 3.19: Thermal conductivity of hydrogen calculated by the given fit-function for selected temperatures

T (K)	20.3	77.4	293.2
k_{h2}(mW/m·K)	15.03	53.30	182.9

This function is implemented in Matlab file: *k_h2.m*.

3.4 Heat Capacity

Heat Capacity of Silver

For the calculation of the heat capacity of silver, the equation and parameter values given in [Gro73] are used. The fit-function used in this work is:

$$C_{ag}\left(T\right) = a \cdot T^3 + b \cdot T^2 + c \cdot T + d \qquad (3.18)$$

The values are given in J/kg·K. The parameters parameter values for the different temperature ranges are given in Tab. 3.20.

Tab. 3.20: Parameters of fit-function of heat capacity of silver

T (K)	4 - 30	30 - 125	125 - 300
a (J/kg·K^{-3})	0.00201	$1.1189 \cdot 10^{-14}$	$3.70101 \cdot 10^{-6}$
b (J/kg·K^{-2})	-0.012	-0.04289	-0.00326
c (J/kg·K^{-1})	0.071	6.05068	1.0369
d (J/kg·K)	$-2.66454 \cdot 10^{-14}$	-100.586	116.90736

This fit-function is valid in the temperature range from 4 K to 300 K. Fig. 3.13 shows the characteristics of the heat capacity of silver depending on the temperature, calculated by the given fit-function.

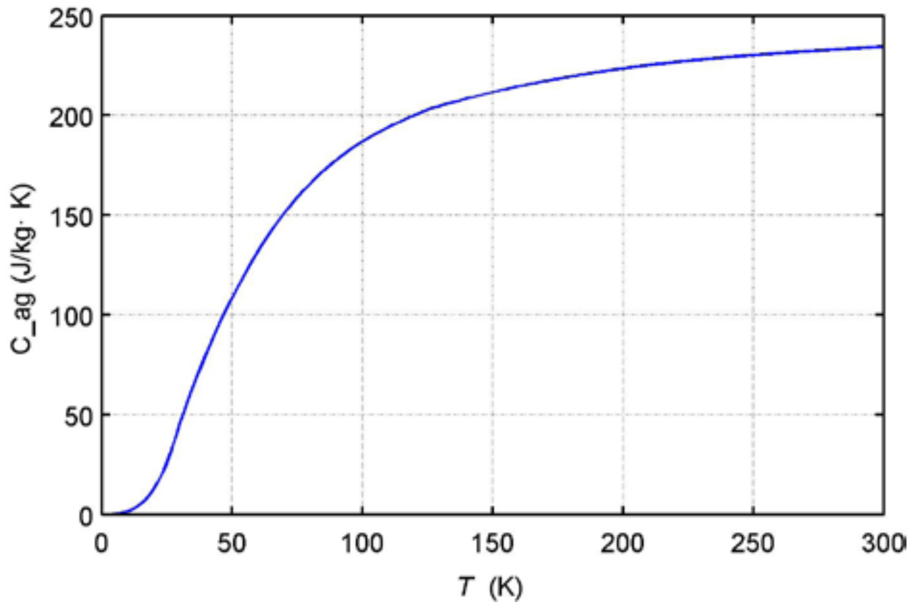


Fig. 3.13: Temperature dependent heat capacity of silver calculated by the given fit-function

In Tab. 3.21 selected values of the heat capacity of silver calculated by the given fit-function are listed.

Tab. 3.21: Heat capacity of silver calculated by the given fit-function for selected temperatures

T (K)	4.2	20.3	77.4	293.2
C_{ag} (J/kg·K)	0.235	13.1	162.3	234.0

This function is implemented in Matlab file: *C_ag.m*

Heat Capacity of Copper

For the calculation of the heat capacity of copper (RRR = 100), the equation and parameter values given in [Wes11] are used. The fit-function used in this work is:

$$C_{cu}\left(T\right)=a\cdot T^{5}+b\cdot T^{4}+c\cdot T^{3}+d\cdot T^{2}+e\cdot T+f \tag{3.19}$$

The values are given in J/kg·K. The parameters parameter values for the different temperature ranges are given in Tab. 3.22.

Tab. 3.22: Parameters of fit-function of heat capacity of copper

T (K)	4 - 30	30 - 125
a (J/kg·K^{-5})	$1.185608689795585\cdot10^{-8}$	$-3.318391520914566\cdot10^{-11}$
b (J/kg·K^{-4})	$-1.754171346610396e\cdot10^{-5}$	$-1.178877722139533\cdot10^{-7}$
c (J/kg·K^{-3})	0.001961392960536	$1.261381319281417\cdot10^{-4}$
d (J/kg·K^{-2})	-0.015485438546988	-0.046922418631570
e (J/kg·K^{-1})	0.088975039227139	8.140457575941973
f (J/kg·K)	1.021961566399155	-204.8755825968253

This fit-function is valid in the temperature range from 4 K to 300 K. Fig. 3.14 shows the characteristics of the heat capacity of copper depending on the temperature, calculated by the given fit-function.

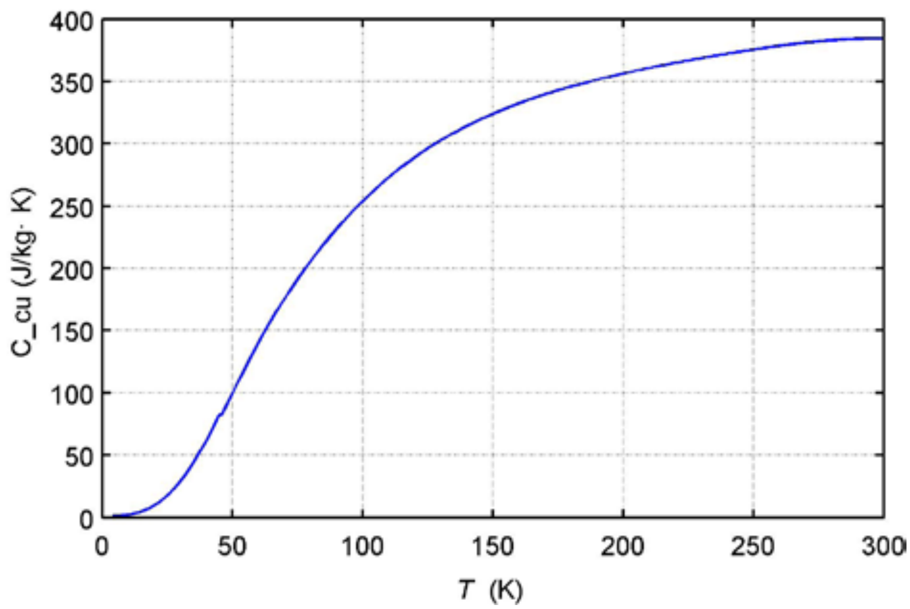


Fig. 3.14: Temperature dependent heat capacity of copper calculated by the given fit-function

This fit-function is valid in the temperature range from 4 K to 300 K. In Tab. 3.23 selected values of the heat capacity of copper calculated by the given fit-function are listed.

Tab. 3.23: Heat capacity of copper calculated by the given fit-function for selected temperatures

T (K)	4.2	20.3	77.4	293.2
C_{cu} (J/kg·K)	1.262	12.63	198.3	384.4

This function is implemented in Matlab file: *C_cu.m*

Heat Capacity of Hastelloy

For the calculation of the heat capacity of hastelloy, the equation and parameter values given in [Lu07] and [Wes11] are used. The fit-function used in this work is:

$$C_{hs}\left(T\right) = a \cdot T^5 + b \cdot T^4 + c \cdot T^3 + d \cdot T^2 + e \cdot T + f \qquad (3.20)$$

The values are given in J/kg·K. The parameters parameter values for the different temperature ranges are given in Tab. 3.24.

Tab. 3.24: Parameters of fit-function of heat capacity of hastelloy

T (K)	4 - 35	35 - 300
a (J/kg·K^{-5})	$-8.659744297212208 \cdot 10^{-7}$	$2.150134339872814 \cdot 10^{-10}$
b (J/kg·K^{-4})	$7.209308577841222 \cdot 10^{-5}$	$-1.458398200879766 \cdot 10^{-7}$
c (J/kg·K^{-3})	-0.001346146407585	$4.376675190884510 \cdot 10^{-5}$
d (J/kg·K^{-2})	0.017992946436959	-0.015104610350977
e (J/kg·K^{-1})	0.084834978274794	4.491152093383801
f (J/kg·K)	0.642508075273653	-110.7126478142060e

This fit-function is valid in the temperature range from 4 K to 300 K. Fig. 3.15 shows the characteristics of the heat capacity of hastelloy depending on the temperature, calculated by the given fit-function.

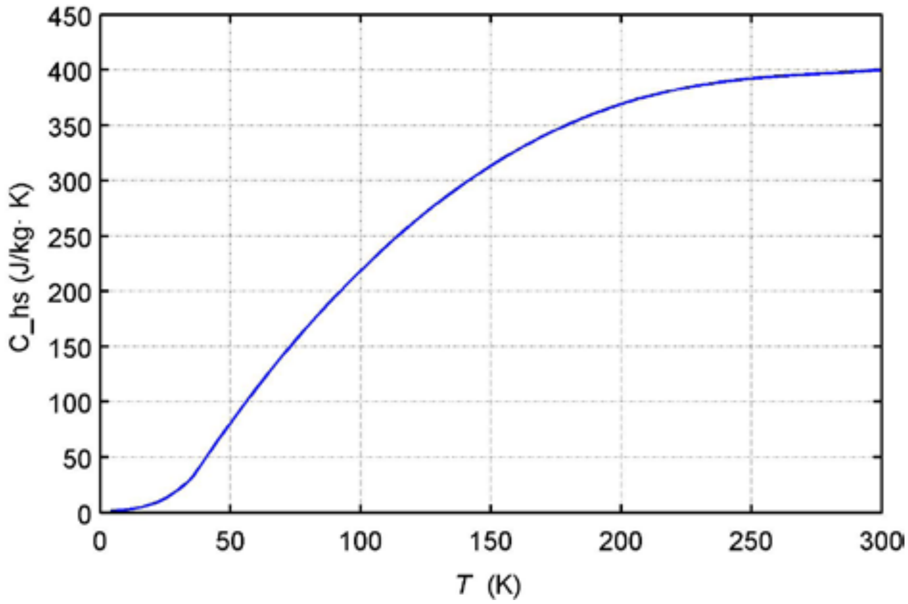


Fig. 3.15: Temperature dependent heat capacity of hastelloy calculated by the given fit-function

In Tab. 3.25 selected values of the heat capacity of hastelloy calculated by the given fit-function are listed.

Tab. 3.25: Heat capacity of hastelloy calculated by the given fit-function for selected temperatures

T (K)	4.2	20.3	77.4	293.2
C_{hs} (J/kg·K)	1.238	7.693	162.1	398.9

This function is implemented in Matlab file: *C_hs.m*

Heat Capacity of Solder (SnPb)

For the calculation of the heat capacity of solder SnPb 5050, the equation and parameter values given in [ZM63] are used. The fit-function used in this work is:

$$C_{so}\left(T\right) = a \cdot T^5 + b \cdot T^4 + c \cdot T^3 + d \cdot T^2 + e \cdot T + f \qquad (3.21)$$

The values are given in J/kg·K. The parameters parameter values for the different temperature ranges are given in Tab. 3.26.

Tab. 3.26: Parameters of fit-function of heat capacity of solder SnPb 5050

T (K)	4 - 300
a (J/kg·K^{-5})	$9.341107159895702 \cdot 10^{-11}$
b (J/kg·K^{-4})	$-1.024659398988616 \cdot 10^{-7}$
c (J/kg·K^{-3})	$4.386446737208523 \cdot 10^{-5}$
d (J/kg·K^{-2})	-0.009191232324487
e (J/kg·K^{-1})	0.972595742967233
f (J/kg·K)	-3.259443448149251

This fit-function is valid in the temperature range from 4 K to 300 K. Fig. 3.16 shows the characteristics of the heat capacity of solder depending on the temperature, calculated by the given fit-function.

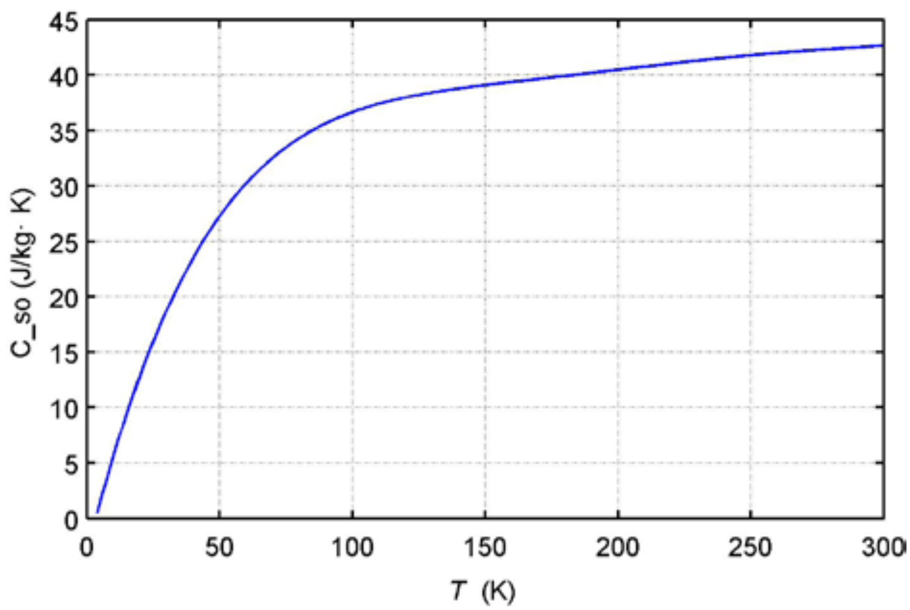


Fig. 3.16: Temperature dependent heat capacity of solder calculated by the given fit-function

In Tab. 3.27 selected values of the heat capacity of solder calculated by the given fit-function are listed. This function is implemented in Matlab file: *C_so.m*

Tab. 3.27: Heat capacity of solder calculated by the given fit-function for selected temperatures

T (K)	4.2	20.3	77.4	293.2
C_{so} (J/kg·K)	0.666	12.98	33.88	42.55

Heat Capacity of YBCO

For the calculation of the heat capacity of YBCO, the equation and parameter values given in [Wes10], [SWB90] are used. The fit-function used in this work is:

$$C_{\text{ybco}}\left(T\right) = a \cdot T^5 + b \cdot T^4 + c \cdot T^3 + d \cdot T^2 + e \cdot T + f \tag{3.22}$$

This fit-function is valid in the temperature range from 4 K to 300 K. The values are given in J/kg·K. The parameters parameter values for the different temperature ranges are given in Tab. 3.30.

Tab. 3.28: Parameters of fit-function of heat capacity of YBCO

T (K)	4 - 300
a (J/kg·K^{-3})	-7.567485538209158 ·10^{-14}
b (J/kg·K^{-2})	6.351452642016898
c (J/kg·K^{-1})	-1.947975786547597
d (J/kg·K)	0.023616673974415
e (J/kg·K)	0.239331954284042
f (J/kg·K)	-1.096191721280114

Fig. 3.18 shows the characteristics of the heat capacity of YBCO depending on the temperature.

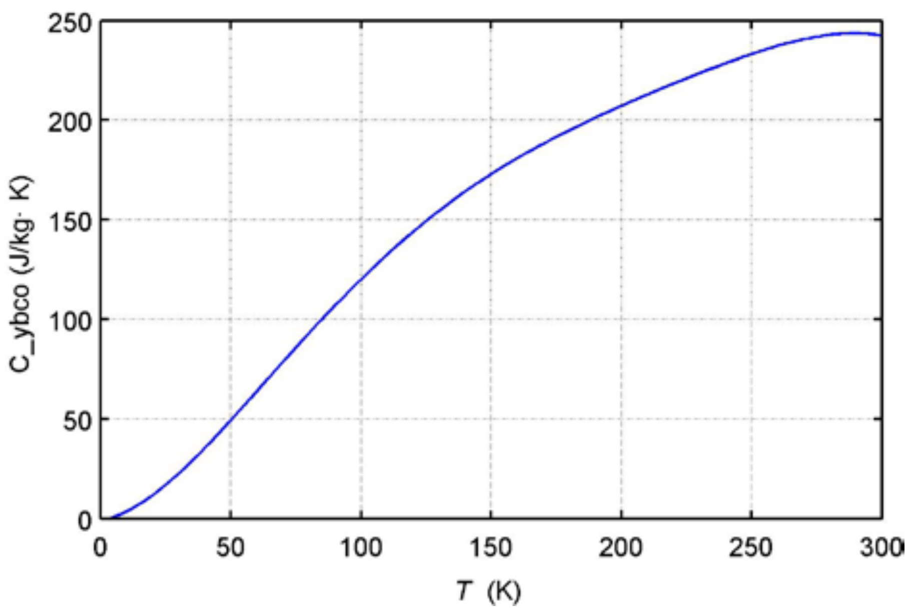


Fig. 3.17: Temperature dependent heat capacity of YBCO calculated by the given fit-function

In Tab. 3.31 selected values of the heat capacity of YBCO calculated by the given fit-function are listed.

Tab. 3.29: Heat capacity of YBCO calculated by the given fit-function for selected temperatures

T (K)	4.2	20.3	77.4	293.2
C_{ybco} (J/kg·K)	0.311	11.87	89.28	243.50

This function is implemented in Matlab file: *C_ybco.m*

3.5 Cooling Properties

In the following the functions to fit the properties of the materials involved are described.

3.5.1 Gaseous Coolants

In this chapter the calculation of the specific cooling power achieved with gaseous streaming coolants is described. This report considers the general cooling properties of not pressurized gaseous Helium and gaseous Hydrogen only. Which means that the temperature of the coolants is assumed to be constant. Hence the cooling power of gases depends on their heat conductivity $k(T)$, the temperature difference between the coolant (gas) T_{cool} and the temperature of the cooled surface T as well as on the geometry. In [RC61] a formula to calculate the heat flux density into the coolant is given

$$\dot{q}(T) = \frac{Nu \cdot k(T) \cdot (T - T_{\mathrm{cool}})}{p_{\mathrm{t}}} \tag{3.23}$$

The influence of the geometry is taken into account by the Nusselt Number Nu and the perimeter of the round tube p_{t} this formula is valid for. In this report the perimeter p_{t} is the perimeter of the superconducting cable and the Nusselt-number is $Nu = 3.66$ in according to the reference. The value is given in W/m^2.

This simple approach can be used as an estimate for the upper limit of the specific cooling power of a streaming gaseous coolant. The effective cooling power of a gas stream along a tube to be cooled is lowered by the heating of the gas along the tube. For a more detailed calculation in a design process, this heating has to be taken into account.

The fit-functions for the thermal conductivity of Helium and Hydrogen were given in the foregoing chapter. Fig. 3.18 shows exemplarily the specific heat transfer in gaseous helium at $T_{\mathrm{cool}} = 10$ K and a perimeter of the tube cooled of $p_{\mathrm{r}} = 4$ mm.

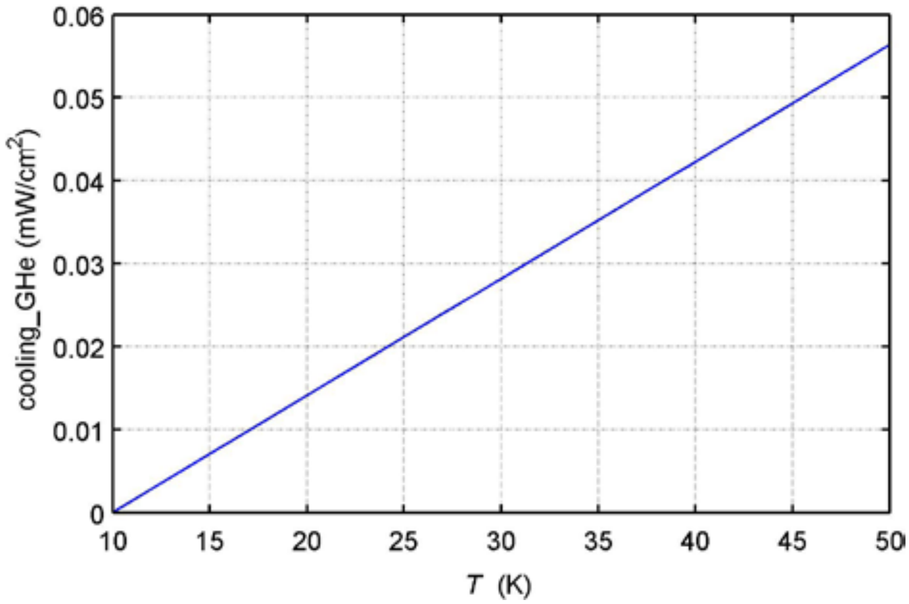


Fig. 3.18: Specific heat transfer in gaseous helium at $T_{cool} = 10$ K

Fig. 3.19 shows exemplarily the specific heat transfer in gaseous hydrogen at $T_{cool} = 30$ K and a perimeter of the tube cooled of $p_r = 4$ mm.

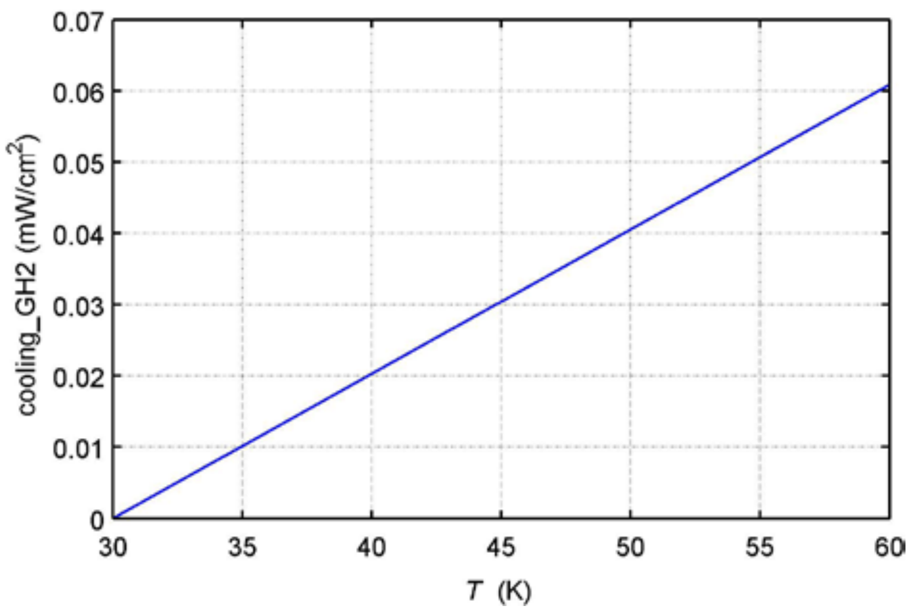


Fig. 3.19: Specific heat transfer in gaseous hydrogen at $T_{cool} = 30$ K

3.5.2 Liquid Coolants

In this chapter the calculation of the specific cooling power achieved with unpressurised liquid coolants at boiling temperature is described. This report considers the general cooling properties of boiling Helium, Hydrogen and Nitrogen depending on the temperature difference between the coolant (liquid) T_{cool} and the temperature of the cooled surface T. The influence of the geometry or forced flow of the liquid is not taken into account because no designated design is specified at this point of the investigations. Hence this simple approach can be used as an estimate of the upper limit of the specific cooling power.

In the following fit-functions for the specific heat transfer in liquid Helium, Hydrogen and Nitrogen at normal pressure and boiling temperature are given.

Boiling Heat Transfer of Liquid Helium

The boiling heat transfer data of liquid helium at $T = 4.2$ K are taken from [BGS10]. These data can be fitted by the following function:

$$\dot{q}_{he}\left(\Delta T\right) = a \cdot \Delta T^5 + b \cdot \Delta T^4 + c \cdot \Delta T^3 + d \cdot \Delta T^2 + e \cdot \Delta T + f \qquad (3.24)$$

The values are given in W/m^2. The parameter values for the different ranges of the temperature differences between coolant and the to be cooled surface are given in Tab. 3.30.

Tab. 3.30: Parameters of fit-function of boiling heat transfer of liquid helium at $T = 4.2$ K

ΔT (K)	0 – 0.335	0.335 – 1.119	1.119 – 1.800	1.800 – 15.00	15.00 - 300
a (-)	$9.7116961 \cdot 10^{-4}$	0.114220579	$-3.5418471 \cdot 10^{-4}$	$-3.4339274 \cdot 10^{-6}$	$3.7053398 \cdot 10^{-12}$
b (-)	-0.431364702	-1.595652097	0.013616234	$2.0883377 \cdot 10^{-4}$	$-2.6797126 \cdot 10^{-9}$
c (-)	8.612750745	6.934406596	-0.165620077	-0.004846221	$4.5516263 \cdot 10^{-7}$
d (-)	2.383663227	-12.42512762	0.86120205	0.054547521	$1.2111368 \cdot 10^{-4}$
e (-)	-0.017527678	8.865159051	-2.009236132	-0.26156897	0.046091218
f (-)	0	-1.227570523	1.934305922	0.600054742	-0.169828008

Fig. 3.12 shows the characteristics of the boiling heat transfer of liquid helium depending on the temperature difference, calculated by the given fit-function.

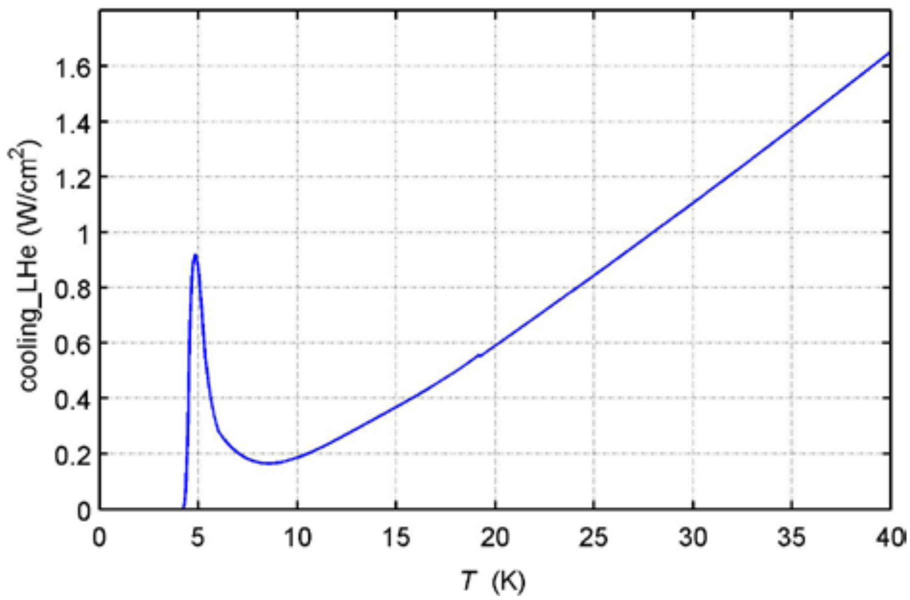


Fig. 3.20: Boiling heat transfer in liquid helium at T = 4.2 K calculated with given fit-function

In Tab. 3.31 the minimum and the maximum values of the boiling heat transfer of liquid helium calculated by the given fit-function are given.

Tab. 3.31: Minimum and maximum value of boiling heat transfer of liquid helium calculated by the given fit-function

T / ΔT (K)	4.84 / 0.64	8.55 / 4.35
q_{he}(W/cm^2)	0.92	0.1649

This function is implemented in Matlab file: *cooling.m*

Boiling Heat Transfer of Liquid Hydrogen

The boiling heat transfer data of liquid hydrogen at T = 20.3 K are taken from [BGS10]. These data can be fitted by the following function:

$$\dot{q}_{h2}\left(\Delta T\right)=\begin{cases} a_1\cdot\Delta T^5+b_1\cdot\Delta T^4+c_1\cdot\Delta T^3+d_1\cdot\Delta T^2+e_1\cdot\Delta T+f_1 & \text{if } \Delta T\leq 7.4\text{K} \\ \dfrac{a_2\cdot\Delta T^3+b_2\cdot\Delta T^2+c_2\cdot\Delta T+d_2}{\Delta T+e_2} & \text{if } \Delta T>7.4\text{K} \end{cases} \quad (3.25)$$

The values are given in W/m^2. The parameter values for the different ranges of the temperature differences between coolant and the to be cooled surface are given in Tab. 3.32.

Tab. 3.32: Parameters of fit-function of boiling heat transfer of liquid hydrogen at T = 20.3 K

ΔT (K)	0 – 1.9	1.9 – 7.4	ΔT (K)	7.4 - 45	45 - 300
a_1 (-)	0.205964281	$3.0364486 \cdot 10^{-4}$	a_2 (-)	$-1.8442529 \cdot 10^{-04}$	$1.8911152 \cdot 10^{-05}$
b_1 (-)	-1.017096695	-0.024040019	b_2 (-)	0.059141049	0.028263491
c_1 (-)	2.124989732	0.584390369	c_2 (-)	-1.321896661	-0.190905407
d_1 (-)	-0.99548425	-5.916830682	d_2 (-)	14.91357646	0.897949095
e_1 (-)	0.311399916	24.34397697	e_2 (-)	-4.121202845	-8.720375198
f_1 (-)	0	-25.12651853			

Fig. 3.21 shows the characteristics of the boiling heat transfer of liquid hydrogen depending on the temperature difference, calculated by the given fit-function.

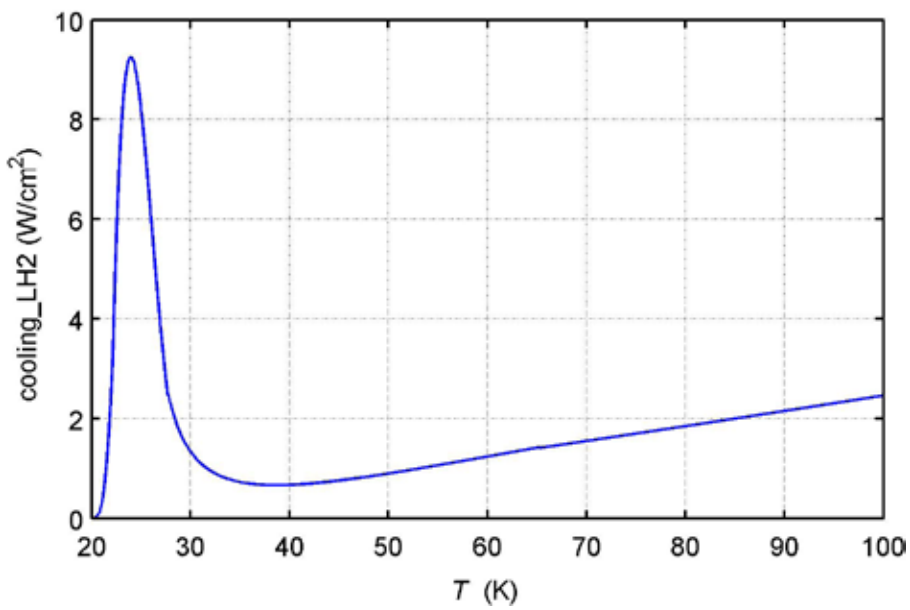


Fig. 3.21: Boiling heat transfer in liquid hydrogen at T = 20.3 K calculated with given fit-function

In Tab. 3.27 the minimum and the maximum values of the boiling heat transfer of liquid hydrogen calculated by the given fit-function are given.

Tab. 3.33: Minimum and maximum value of boiling heat transfer of liquid hydrogen calculated by the given fit-function

T / ΔT (K)	23.97 / 3.77	38.62 / 18.42
q_{h2}(W/cm^2)	9.251	0.6628

This function is implemented in Matlab file: *cooling.m*

Boiling Heat Transfer of Liquid Nitrogen

The boiling heat transfer data of liquid nitrogen at T = 77.4 K are taken from [BGS10]. These data can be fitted by the following function:

$$\dot{q}_{n2}\left(\Delta T\right) = a\cdot\Delta T^5 + b\cdot\Delta T^4 + c\cdot\Delta T^3 + d\cdot\Delta T^2 + e\cdot\Delta T + f \tag{3.26}$$

The values are given in W/m^2. The parameter values for the different ranges of the temperature differences between coolant and the to be cooled surface are given in Tab. 3.34.

Tab. 3.34: Parameters of fit-function of boiling heat transfer of liquid nitrogen at T = 77.4 K

ΔT (K)	0 – 4.6	4.6 – 10.0	10.0 – 18.9
a (-)	0.001680087	0.001523743	0
b (-)	-0.019242022	-0.042849971	0
c (-)	0.087095646	0.443404076	0.030698456
d (-)	-0.074037883	-1.846635116	-1.696351705
e (-)	0.057917877	3.046099292	28.57852002
f (-)	0	0	-133.0831413
ΔT (K)	18.9 – 22.8	22.8 – 89.9	89.9 - 300
a (-)	0	$-7.8404488\cdot10^{-9}$	$-3.37683125\cdot10^{-13}$
b (-)	0	$2.8840356\cdot10^{-6}$	$4.008429818\cdot10^{-10}$
c (-)	-0.019815625	$-0.0004118\cdot10^{-4}$	$-1.60665804\cdot10^{-7}$
d (-)	1.546287856	0.028566069	$3.053044558\cdot10^{-5}$
e (-)	-40.11763931	-0.952535467	0.009349036
f (-)	347.5715056	12.89616643	0.240726932

Fig. 3.22 shows the characteristics of the boiling heat transfer of liquid nitrogen depending on the temperature difference, calculated by the given fit-function.

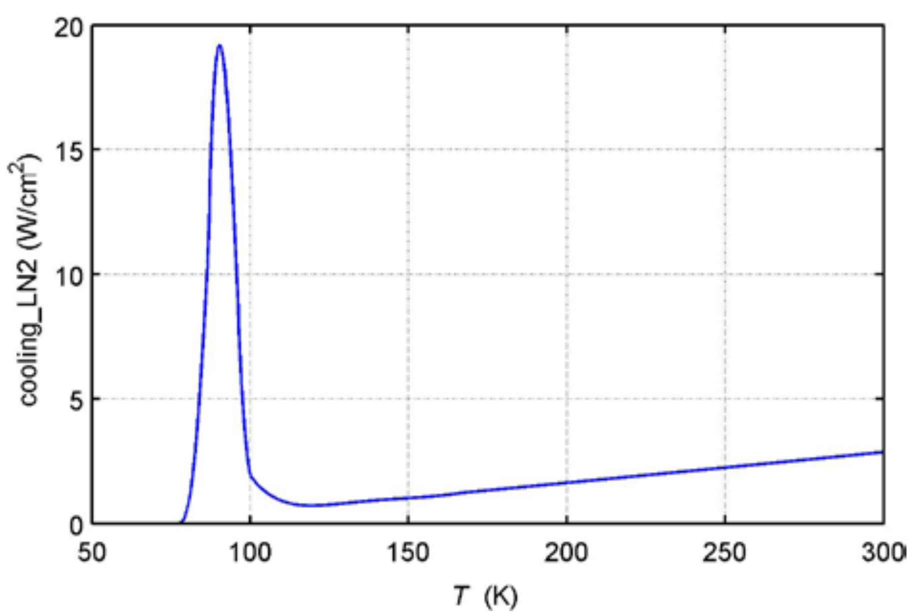


Fig. 3.22: Boiling heat transfer in liquid nitrogen at T = 77.4 K calculated with given fit-function

In Tab. 3.35 the minimum and the maximum values of the boiling heat transfer of liquid nitrogen calculated by the given fit-function are given.

Tab. 3.35: Minimum and maximum value of boiling heat transfer of liquid nitrogen calculated by the given fit-function

T / ΔT (K)	90.39 / 13.03	119.3 / 41.94
q_{he}(W/cm^2)	19.2	0.7165

This function is implemented in Matlab file: *cooling.m*

4 Current Distribution Within Stacks of YBCO coated conductors

In order to make stability calculations in this chapter an electrical circuit diagram of soldered, stacked YBCO coated conductors is introduced. Therefore it is described how the electric circuit elements for the network model can be determined first. In the following the current distribution within such cables in different operating conditions is discussed and a new method for the calculation of the current distribution within superconductors is introduced.

4.1 Calculation of Circuit Elements for Electrical Cable Model

The cable models introduced in chapter 4.2 and 4.3 include the following circuit elements:

- the resistance per unit length R'_{tape}
- the inductance per unit length L_s'
- the capacitance per unit length C'
- and the conductance per unit length G' between the tapes.

The calculation these circuit elements are derived in this chapter. In order to get an understanding of the circuit elements it may be helpful to see the complete circuit diagram in Fig. 4.7 or the cable model used for the stability calculations in Fig. 4.8, Fig. 4.9 and Fig. 4.10.

4.1.1 Resistance per Unit Length

At first the calculation of the resistance per unit length of a single YBCO coated conductor is described. The second part describes the calculation of the resistance load of a soldered stack of multiple YBCO coated conductors.

Resistance per Unit Length of a single YBCO Coated Conductor

For the calculation of the resistance of YBCO tapes the approach published in [SCH09] is used which is explained in the following. The resistivity of superconductors in general strongly depends on the temperature, the transport current and the magnetically field. If the temperature and the magnetic field are constant and below their critical value, the superconductor is in superconducting state. If the transport current is low (below the critical current I_c), the conductors have no resistance. If the current rises in the range of the superconducting – normal conducting transition (increases the critical current I_c) or rises even higher the superconductors show a resistivity. In case of YBCO coated conductors the current starts flowing in the normal conducting layers as well and shares between the different layers with respect to their resistances.

In order to calculate the total resistance per unit length of a YBCO tape R'_{tape} the current sharing can be taken into account by a parallel connection of the different layers of the tape as shown in Fig. 4.1.

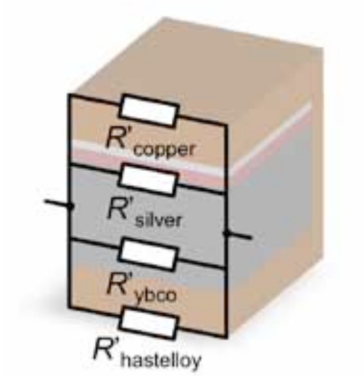


Fig. 4.1: Electrical circuit diagram of a single YBCO coated conductor to calculate the resistance load per unit length R'_{tape}. The resistance of the copper layers on both sides of the tape is combined to one single resistance.

The resistance load of the normal conducting layers R'_{nc} depends on the temperature dependent specific resistivity of the respective materials $\varrho_{cu}(T)$, $\varrho_{ag}(T)$ and $\varrho_{hs}(T)$ and the cross sectional area of each layer that can be calculated by the layer thicknesses h_{cu}, h_{ag} and h_{hs} and the tape width w_{tape} (compare Fig. 2.2). Hence the tape resistance of the normal conducting layers can be calculated by

$$R'_{nc}(T) = \frac{1}{w_{tape}} \cdot \frac{1}{\frac{2h_{cu}}{\rho_{cu}(T)} + \frac{h_{ag}}{\rho_{ag}(T)} + \frac{h_{hs}}{\rho_{hs}(T)}} \tag{4.1}$$

The copper layer thickness h_{cu} is the thickness of one layer on one side of the tape only and the values of the specific resistivity of the materials can be calculated by the fit-functions (3.6), (3.4) and (3.3) as described in chapter 3.2.

The resistance per unit length of the superconducting YBCO layer R'_{sc} depends on the temperature T, the current flowing in this layer I_{sc} and the magnetic field B. To model the superconducting-normal conducting transition of the material, the power law is used for all temperatures below the critical temperature $T < T_c$. Above the critical temperature $T > T_c$ the material is considered to be in normal conducting state and because its resistance is much higher than the resistance of the other normal conducting layers. The resistance load per unit length in normal conducting state is assumed to be constant at 500 nΩ·m [YGK11]. Thus the following equation is used to model the resistance load per unit length of the superconducting YBCO layer R'_{sc}.

$$R'_{sc}\left(B,T,I\right)=\begin{cases}\dfrac{E_c}{I_c\left(B,T\right)}\cdot\left(\dfrac{I_{sc}}{I_c\left(B,T\right)}\right)^{n-1} & for \quad T<T_c\\[2ex] 500\cdot n\Omega\cdot m \quad for \quad T>T_c & \end{cases} \tag{4.2}$$

This function is implemented in Matlab file *R_sc_pul.m*. Here, E_c is the electrical field across the tape at its critical current I_c. The established definition for the critical electrical field is E_c = 1 µV/cm.

The critical current I_c of the YBCO layer depends on the magnetic field B and the temperature of the tape T. It is calculated by the fit-function (3.1) as described in chapter 3.1. This approach allows to model the temperature and field dependence in the range from T = 4 K to 300 K and from B = 0.1 T to 20 T. The value can be scaled by variation of the cross sectional area of the YBCO layer (product of the tape width w_{tape} and layer thickness h_{ybco}). The function to calculate the critical current is implemented in Matlab file *critical_current.m*.

Hence the resistance load per unit length of a single YBCO coated conductor can be calculated by:

$$R'_{tape}\left(B,T,I\right)=\frac{1}{\dfrac{1}{R'_{sc}\left(B,T,I_{sc}\right)}+\dfrac{1}{R'_{nc}\left(T\right)}} \tag{4.3}$$

In order to solve this equation, the current I_{sc} flowing in the superconducting YBCO-layer has to be determined as the current in this layer depends on its resistance and its resistance depends on its current. The total current in a tape I_{tape} shares between the superconducting and the normal conducting layers.

$$I_{tape}=I_{sc}+I_{nc} \tag{4.4}$$

As the voltage across the tape is equal for all layers, the current in the superconducting layer I_{sc} can be calculated numerically by solving the following equation.

$$0 = R'_{sc}(T) \cdot I_{sc} - R'_{nc}(T) \cdot (I_{tape} - I_{sc}) \tag{4.5}$$

Hence the resistance load per unit length of a single tape can be calculated. These functions are implemented in Matlab files: *critical_current.m*, *R_nc_pul.m*, *R_sc_pul.m* and *R_tape_pul.m*.

Resistance per Unit Length of a Stack

In this section the calculation of the resistance per unit length of stacks of multiple YBCO tapes is described. Therefore the method for the calculation of the resistance per unit length of single YBCO coated conductors, described in the foregoing section, was extended in the context of this work. The stack resistance load can be assumed to be the parallel connection of the resistance of n_{tape} tapes in the stack and the resistance of (n_{tape}-1) solder layers between the tapes as shown in Fig. 4.2.

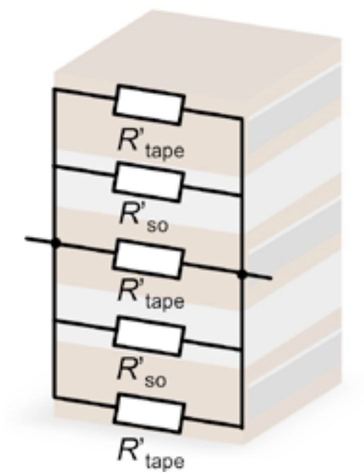


Fig. 4.2: Simplified electrical circuit diagram for the calculation of the resistance load per unit length for a soldered stack the example case of three YBCO coated conductors R'_{stack}.

With this approach the resistance load of the stack can be calculated by

$$R'_{stack} = \frac{1}{\frac{n_{tape}}{R'_{tape}} + \frac{n_{tape} - 1}{R'_{so}}} \tag{4.6}$$

with the resistance of the tapes R'_{tape} and the resistance of the solder slayers R'_{so}. For the calculation of the tape and the solder layer resistance, two assumptions are made:

- The temperature across the whole cross section of the cable is constant
- The current shares equally between the tapes

The resistance load of the solder layer can be calculated by fit-function (3.7) for the temperature dependent resistivity of the solder ϱ_{so} as described in chapter 3.2 and the cross sectional area as the product of the solder layer thickness h_{so} and the tape width w_{tape}.

$$R'_{so} = \frac{\rho_{so}(T)}{h_{so} \cdot w_{tape}} \quad (4.7)$$

The developed calculation for the value of the resistance load per unit length of the tapes R'_{tape} was described in the foregoing section. In order to give the general view, equation (4.3) is shown here again:

$$R'_{tape}(B,T,I) = \frac{1}{\frac{1}{R'_{sc}(B,T,I_{sc})} + \frac{1}{R'_{nc}(T)}} \quad (4.8)$$

Here R'_{nc} is the resistance per unit length of the normal conducting layers that can be calculated by equation (4.1) and R'_{sc} is the resistance per unit length of the superconducting layer that can be calculated by equation (4.2). The resistance of superconducting YBCO layer depends besides the temperature T and the magnetic field density B on the current I_{sc} flowing through it.

In order to calculate the current in the superconducting layer I_{sc}, first the current in each tape I_{tape} has to be calculated. Based on the assumption that the current shares equally between the tapes, the current in each tape can be calculated by

$$I_{tape} = \frac{I_{stack}}{n_{tape}} \quad (4.9)$$

For fusion magnets with very high magnetic fields the self field effect on the critical current of the tapes in the stack can be neglected and so the assumption of homogeneous current distribution can be used as a good estimate. With this assumption the current in the YBCO I_{sc} layer can be calculated by solving equation (4.5) numerically as described in the foregoing chapter. With this approach the current in the solder layer is neglected for the calculation of the tape resistance, but as the resistance of the tapes is much lower than the solder resistance, the error caused by that is small. If there is an error, it appears where the tape resistance and the solder resistance are in the same range.

Based on this, the current in the superconducting layer I_{sc} can be calculated by solving equation (4.5) numerically. With this approach the resistance per unit length of a stack of YBCO tapes R'_{stack} can be calculated. This function is implemented in Matlab file: *R_stack_pul.m*.

4.1.2 Conductance per Unit Length

The conductance per unit length between two soldered YBCO tapes is approximated by an assumed path of the coupling current to flow between the tapes as shown in Fig. 4.3. Hence an analytical equation is obtained, that allows the calculation of the conductance load per unit length for different layer thicknesses and temperatures.

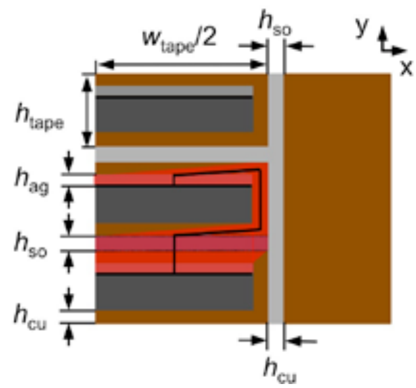


Fig. 4.3: The sketch shows the cut out of the right half side of a stack of three YBCO coated conductors, the solder between the YBCO coated conductors and a segment of the cupper support structure (Compare Fig. 2.1 and Fig. 2.2). The highlighted red area shows the assumed path of the coupling current between two-soldered YBCO coated conductors.

It is assumed, that the coupling current flows in the materials with the least resistivity only. In Fig. 4.3 the path of the coupling current is highlighted red. We have a look on the right half area of the conductor as the same process happens on the other side as well. The current leaves the YBCO layer of the lower coated conductor across its whole cross section and passes through the copper layer first and then through the solder layer into the copper layer of the upper coated conductor. It flows in this copper layer all the way up and distributes on the upper side of the tape to distribute and enter the YBCO layer of the upper tape. Based on this current path and the foundational material equation for conductivity

$$G = \sigma \cdot \frac{A}{\ell} \tag{4.10}$$

that depends on the cross sectional area A the current is flowing through, the effective path length ℓ, and the specific conductivity of the material σ, the following equation is obtained to estimate the conductance load per unit length G' between two coated conductors.

$$G' = 2 \cdot \left(\sigma_{ag} \cdot \frac{\frac{w_{tape}}{2}}{2 \cdot h_{ag}} + \sigma_{so} \cdot \frac{\frac{w_{tape}}{2} + h_{cu}}{h_{so}} + \sigma_{cu} \cdot \left(\frac{\frac{w_{tape}}{2} + \frac{h_{cu}}{2}}{h_{cu}} + \frac{\frac{h_{cu}}{2}}{\frac{w_{tape}}{2} + \frac{h_{cu}}{2}} + \frac{h_{cu}}{h_{tape} + h_{cu}} + \frac{\frac{h_{cu}}{2}}{\frac{w_{tape}}{2} + \frac{h_{cu}}{2}} \right) \right) \quad (4.11)$$

Here σ is the conductivity of the according material and h (layer thicknesses) and w (width) are the dimensions of the tapes and their layers as shown in Fig. 4.3 respectively. As the current can couple on both sides of the superconductors, the total conductivity is doubled. This function is implemented in Matlab file: *G_pul.m*.

4.1.3 Inductance per Unit Length

In order to investigate the current redistribution within stacks of YBCO tapes in case of transient events, the calculation of the self inductance load per unit length of the single tapes within a stack is derived in the following. For this derivation is assumed, that the current flows in the superconducting layers of the YBCO tapes only.

If two parallel straight YBCO tapes carry a current, their magnetic field can be divided into two sections. Because of Maxwell's Law, there is one part of the field, that comprehends both conductors and there is another part of the field around each conductor that comprehends only the conductor itself, as shown in the principal sketch in Fig. 4.4.

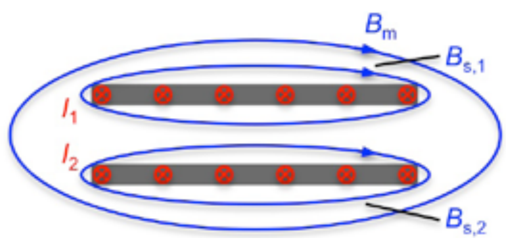


Fig. 4.4: Principal sketch of the magnetic field B in the surrounding of two current carrying parallel, rectangular shaped straight conductors. B_m mutual field, $B_{s,1}$ self field of conductor 1, $B_{s,2}$ self field of conductor 2

The inductances of such an arrangement are calculated due to the magnetic field distribution. There is a mutual inductance L_m that is composed of the magnetic field that encloses both conductors. Besides that, each conductor has a stray inductance L_s that is equal for both in case of symmetrical geometry.

Since both conductors share the mutual inductance, the conductors are magnetically coupled. The electrical circuit diagram of the inductances of a differential element of these parallel inductors of length dx is shown in Fig. 4.5.

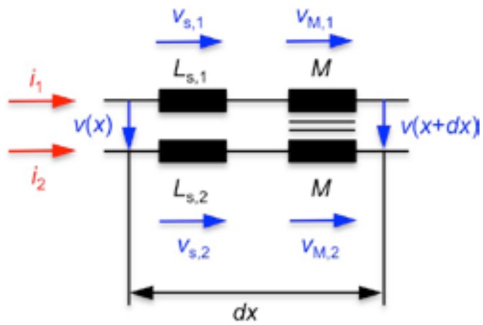


Fig. 4.5: Electrical circuit diagram of the inductance of a differential element of two current carrying parallel conductors of length *dx*.

First we have a look on the voltage between the conductors. The total transport current in the cable is $I_T = I_1 + I_2$. In case of a coil the transport current can be considered to be constant. Hence we assume that if the current in one conductor decreases, it has to increase in the other one and vice versa.

$$\frac{di_1}{dt} = -\frac{di_2}{dt} \quad (4.12)$$

The stray inductance L_s is generally much lower than the mutual inductance M ($L_s << M$). Therefore, we can neglect the stray inductance in a first approximation. The voltage between both conductors at location $x + dx$ is

$$v(x+dx) = v(x) + v_{M,1} - v_{M,2} \quad (4.13)$$

As the mutual inductances are magnetically coupled, the voltage across them is

$$v_{M,1} = v_{M,2} = M \cdot \left(\frac{di_1}{dt} + \frac{di_2}{dt} \right) \quad (4.14)$$

If we insert equation (4.12) in equation (4.14), we get

$$v_{M,1} = v_{M,2} = M \cdot \left(\frac{di_1}{dt} - \frac{di_1}{dt} \right) = 0 \quad (4.15)$$

Hence the voltage across the mutual inductance cancels out and the voltage on both sides of the differential length element dx is equal ($v(x) = v(x + dx)$) because of the magnetic coupling. This means that the mutual inductance has no effect on the redistribution of the current within a superconducting cable of multiple parallel YBCO tapes. The mutual inductance has an effect on chances of the total transport current I_T only. In this investigation the main interests is to get information of the transient current redistribution within a stack in case of local disturbances while the total current is assumed to be constant. Therefore the calculation of the stray inductance L_s is described in the following and the mutual inductance M is not taken into account.

Stray inductance

As the stray inductance of an arrangement of several straight conductors of rectangular shape cannot be calculated analytically, other calculation methods have to be used. One method that allows the calculation of the mutual inductance of two conductors of arbitrary shape is the use of the geometric mean distance (gmd). This method is described in [SF96], [Gro73] and [Ros07] in great detail. The approach of this method is to replace the two conductors (current carrying areas) by two filamentary conductors that are separated by the average distance of the two cross sectional areas. The average distance of all points of both areas is equal to the geometrical mean distance of these areas (not the arithmetical mean distance).

The Mutual inductance of two filamentary conductors with radius a, separated by a distance d can thus be calculated as follows. Based on Maxwell's equation

$$\oint H\,ds = \iint j\,da \quad , \tag{4.16}$$

the magnetic field around a single filamentary conductor carrying the current I is

$$H = \frac{I}{2\pi \cdot r} \quad . \tag{4.17}$$

The stray inductance L_s of this conductor includes the field components that comprehend one conductor only

$$L_s = \frac{\phi}{I} = \frac{\int B\,da}{I} = \int_a^d \frac{\mu \cdot \ell \cdot I}{2\pi \cdot r \cdot I}\,dr \tag{4.18}$$

(compare Fig. 4.4). Here I is the current flowing in the conductor and ℓ is the length of the conductor. For the calculation of the stray inductance per unit length the length and the current cancel out. Hence, its stray inductance per unit length can be calculated by

$$L'_s = \frac{\mu}{2\pi} \cdot \ln\left(\frac{d}{a}\right) \quad for\ a << d \tag{4.19}$$

The stray inductance of conductors with arbitrary shape can be calculated by this equation, if the distance d is the average (or effective) distance between these two areas and if the radius a is the average (or effective) radius of the area, the stray inductance shall be calculated for. Both values are equal to the geometric mean distance that can be calculated as follows. To average the distance between two areas, the areas are segmented in a number of *N* filamentary segments in area *A* and *M* filamentary segments in area *B*. The distance $d_{n,m}$ between a filament *n* of area *A* and a filament *m* of area *B* is

$$d_{n,m} = \sqrt{\left(x_n - x_m\right)^2 + \left(y_n - y_m\right)^2} \quad (4.20)$$

The geometric mean distance of filament *n* (part of area *A*) to the whole area *B* is the *M*-th root of the product of all distances between filament *n* and all *M* filaments in area *B*.

$$gmd_{n,B} = \sqrt[M]{d_{n,1} \cdot d_{n,2} \cdot \quad \cdots \quad \cdot d_{n,M}} \quad (4.21)$$

In the same way, the geometric mean distance between both areas *A* and *B* is calculated by the N^{th} root of the product of the geometric mean distances between all segments in area *A* to all segments in Area *B*

$$gmd_{A,B} = \sqrt[N]{gmd_{1,B} \cdot gmd_{2,B} \cdot \quad \cdots \quad \cdot gmd_{N,B}} \quad (4.22)$$

The average radius of an area (compare radius *a* in equation (4.19)) is the geometric mean distance of an area to itself and can be calculated in the same way as the geometric mean distance between two different areas. The distance between one filament *n* of area *A* and another filament *m* of the same area can be calculated by equation (4.20). The geometric mean distance of a filament *n* of area *A* is then calculated by

$$gmd_{n,A} = \sqrt[N]{d_{n,1} \cdot d_{n,2} \cdot \quad \cdots \quad \cdot d_{n,N}} \quad (4.23)$$

And the geometric mean distance of area *A* to itself is then

$$gmd_{A,A} = \sqrt[N]{gmd_{1,A} \cdot gmd_{2,A} \cdot \quad \cdots \quad \cdot gmd_{N,A}} \quad (4.24)$$

Fig. 4.6 shows a principal sketch of the geometrical mean distance of one conductor to itself and one conductor to another for a geometry that looks like two parallel current carrying YBCO layers.

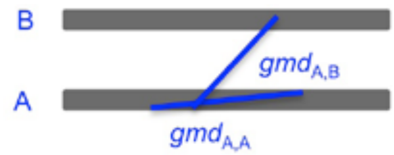


Fig. 4.6: Principal sketch of the geometrical mean distance of one YBCO layer to itself $gmd_{\mathrm{A,A}}$ and one YBCO layer to another $gmd_{\mathrm{A,B}}$.

With these equations the stray inductance per unit length L_s' of a straight conductor A with arbitrary shape, a distance apart from another straight conductor B with also arbitrary shape can be calculated numerically by

$$L_s' = \frac{\mu}{2\pi} \cdot \ln\left(\frac{gmd_{\mathrm{A,B}}}{gmd_{\mathrm{A,A}}}\right) \tag{4.25}$$

with the geometric mean distances $gmd_{\mathrm{A,B}}$ and $gmd_{\mathrm{A,A}}$ of equation (4.22) and (4.24). This formula is based on equation (4.19), of which applicability is limited by the constraint that $a << d$. This constraint is fulfilled, as it refers to the filaments used for the calculation of the *gmd*. Their diameter a is small compared to their separation d.

Hence equation (4.25) can be used to calculate the inductance load per unit length L' of two parallel coated conductors. These functions are implemented in Matlab files: *gmd_11.m*, *gmd_12.m* and *L_pul.m*.

4.1.4 Capacitance per Unit Length

The capacitance load per unit length is proportional to the (longitudinal) cross sectional area of the tapes in parallel and reciprocal to their separation. Hence it can easily be calculated by

$$C' = \varepsilon_0 \cdot \frac{w_{\mathrm{tape}}}{h_{\mathrm{tape}} + h_{\mathrm{so}}} \tag{4.26}$$

with the dielectric constant ε_0, the width of the tape w_{tape}, the height of the tape h_{tape} and the thickness of the solder layer between the tapes h_{so}. This function is implemented in Matlab file: *C_pul.m*.

4.2 Cable Model for Stability Investigations

Superconductors for fusion magnets can reach a length in the km-range. For the investigation of local transient events on the cable it has to be determined whether the cables have to be considered electrically long or if they can be considered electrically short which simplifies the calculation essentially.

This is analyzed by and example calculation for a cable of two parallel YBCO tapes. The run time of an electro-magnetic wave travelling on the cable is determined to see whether this time scale has to be taken into account for the calculation of the heat dissipation in case of a local disturbance. For this case example is assumed, that the cable is 2 km long and that both tapes are connected with joints on both ends. The transport current I_T is constant and shares between both tapes with respect to the resistances of the arrangement. Further more is assumed, that a perturbation in the middle of the cable causes an abrupt resistance rise in one of the tapes only. The total amount of current is not affected by this event only a current redistribution between the tapes will take place.

Fig. 4.7 shows the complete electrical circuit diagram for this case. This model includes all relevant circuit elements for calculations on electrically long cables. It takes into account the joint resistances R_j, the resistance of the faulted tape in the area $R_f = R'_f \cdot \ell_f$ and the resistance of the still superconducting parallel tape $R'_{tape} \cdot \ell_f$ in the faulted area (that is caused by the voltage rise in the range of the critical current).

All other circuit elements of the cable are not considered to be concentrated elements, but distributed over the whole length of the cable. These are the resistance load per unit length of each tape R'_{tape}, the inductance load per unit length L'_s, the capacitance load per unit length C' and the conductance load per unit length G' between the tapes. In this model the two cable segments ℓ_1 and ℓ_2 in front and behind the failed area ℓ_1 are considered to be electrically long.

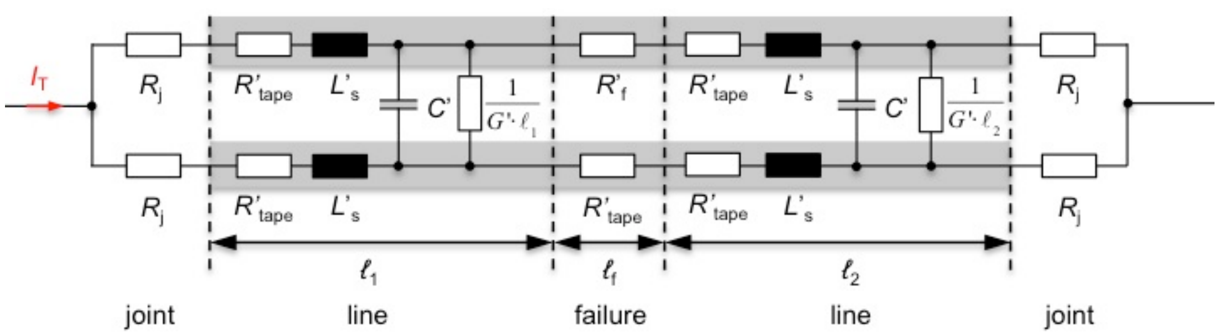


Fig. 4.7: Electrical circuit diagram of electrically long cable of two stacked, parallel YBCO coated conductors and a local failure resistance R_f within the cable. R'_{tape} resistance per unit length of superconducting tape, R_j joint resistances. Grey: electrically long cable with inductance per unit length L_s', capacitance per unit length C', conductance per unit length G'. Total length: $\ell_{tot} = \ell_1 + \ell_f + \ell_2$.

When the failure suddenly occurs in the middle of one tape of the cable, the current can't immediately redistribute from this tape to the other tape because of the capacitances and inductances of the cable. This corresponds to the consideration of the cable to be electrically long. The time scale, an electromagnetically wave needs to travel from the failure location to the end of the cable and back (which is the time the current needs to redistribute within the stack) is considered to be a sub transient process in this report. To get information about the time scale for the sub transient current redistribution, the phase velocity v of the propagating electromagnetic wave travelling on the electrically long cable segments ℓ_1 and ℓ_2 can be calculated by (standard cable model)

$$v' = \frac{1}{\sqrt{L' \cdot C'}} \tag{4.27}$$

Where L' is the inductance per unit length and C' is the capacitance per unit length of the cable. In this case the inductance of the cable L' is the sum of the self inductances in both tapes (compare chapter 4.1.3)

$$L' = L'_{s,1} + L'_{s,2} \,. \tag{4.28}$$

In case of a stack of multiple conductors, the stray inductance of one tape is the inductance of that tape to all the other tapes in the stack. The capacitance load per unit length C' is always the capacity of conductor to the neighbor conductor.

Hence the wave propagation velocity on a cable can be calculated. As the values vary for different geometries, a list for different dimensions of the copper stabilization- and the intermediate solder layer between the tapes is given in Tab. 4.1 for comparison.

Tab. 4.1: Inductance per unit length L', Capacitive per unit length C', Conductive per unit length G' and phase velocity v_0 for a stack of 4 mm wide YBCO-tapes with 50 µm thick Hastelloy, 1.5 µm thick YBCO and varying solder and Copper stabilizer layer thickness.

h_{cu} (µm)	10	20	30	10	20	30	10	20	30
h_{so} (µm)	20	20	20	50	50	50	100	100	100
L' (nV·s/A·m)	247.5	283.6	319.3	301.5	337.0	372.0	389.4	423.9	458.0
C' (pA·s/V·m)	761.6	626.8	532.6	575.8	495.4	434.6	409.4	367	332.6
G' @ 77 K (1/nΩ·m)	106.9	55.1	37.9	105.1	53.3	36.1	104.5	52.7	35.5
G' @ 20 K (1/nΩ·m)	13214	6651	4465	13190	6627	4441	13182	6619	4433
v (m/µs)	72.8	75.0	76.6	75.8	77.4	78.6	79.1	80.1	81.0

For all geometries, the phase velocity is very high (in the range of 1 km / 10 µs). Thus the current redistribution within a cable with a length in the km-range will take place within a time scale of below 1 ms. This time scale can be neglected for the calculation of the heat dissipation within the cable. Thus the general circuit diagram in Fig. 4.7 can be reduced to the resistive circuit elements only for calculations with inhomogeneous current and temperature distribution over the cross section of the stack. Thus for all stability calculations the reduced circuit diagrams introduced in the following chapter are used.

These calculations are implemented in Matlab files: *L_pul.m*, *C_pul.m*, *G_pul.m* and *subtrans.m*.

4.3 Current Sharing Between YBCO Coated Conductors in a Stack

In this chapter the current sharing between the tapes within a stacked cable is discussed. The results of this investigation shall be used for the application of the different stability criteria. Therefore it is assumed in the following that the temperature across the whole cross section of the cable is constant and that the current shares equally between all tapes unless it is explicitly mentioned.

This assumption is useful as it is hard to make assumptions for perturbations of only parts of the cross-section of a stack. The heat dissipation is anyway maximal when the whole cross section of a stack is perturbed. In all other cases (only parts of the cross section are perturbed) the heat dissipation is lower.

Depending on the operational conditions, different circuit elements can be neglected and different network models have to be used. Thus in this chapter a case distinction is done. Three different cases are considered in the following. First the case is low transport current (transport current $I_T <$ critical current I_c), the second case is high transport current ($I_T \geq I_c$) and the third case considers a local perturbation on the cable. The different network models are based on the circuit diagrams shown in Fig. 4.7 and take into account the joint resistances R_j, the tape resistance R_{tape} and the coupling resistances $1/G$ (the coupling resistance 1/G may be divided into different parts) between the tapes. The inductances and the capacitances are not taken into account because they have no effect on the current sharing between the tapes for DC currents as shown in the foregoing chapter.

It is assumed, that the temperature and the magnetic field along the cable is constant. Further more a homogeneous critical current distribution across each tape is assumed in order to simplify the model and to highlight the main effects clearly. The network models show the example case of two parallel YBCO tapes only.

Low Transport Current ($I_T < I_c$)

For low currents (transport current $I_T <$ critical current I_c), the tape resistances R_{tape} across the conductors are much lower than the joint resistances R_j, because both tapes are in superconducting state. Therefore the voltage over the joints is higher than the voltage across the cable and thus the current distribution is mainly affected by the joint resistances and the coupling resistance between the tapes.

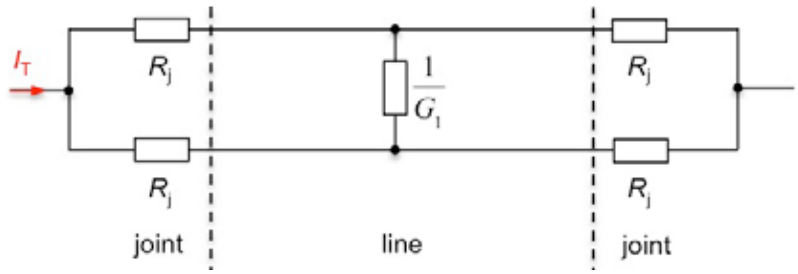


Fig. 4.8: Simplified electrical circuit diagram of two parallel, soldered YBCO coated conductors for low currents ($I_T << I_c$) with joint resistances R_j, conductance load per unit length G.

If the joint resistances are equal, the current shares equally between the tapes. If they differ, the current shares with respect to the relation of the resistances. As long as the current in all tapes stays lower than the critical current, the current distribution doesn't matter and has no effect on the cable stability as there is very little power dissipated along the cable. If there is a difference in the potential of both tapes, the current can couple between both tapes through the coupling resistance along the whole length of the cable.

High Transport Current ($I_T \geq I_c$)

If the current is increased, at some point the voltage across the cable becomes higher than the voltage across the joints and thus the current distribution is mainly affected by the tape resistances. Therefore the cable resistances R_{tape} have to be taken into account. The total voltage across the tapes depends strongly on the length of the cable and therefore the point of equal voltages may even be below the total critical current of the cable (this statement also depends on over which length the critical current is measured). If the cable is exposed to an external magnetic field, the critical current varies periodically because of the twist of the cable.

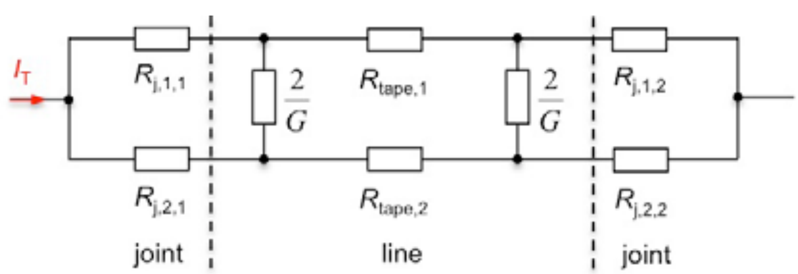


Fig. 4.9: Simplified electrical circuit diagram of two parallel soldered YBCO coated conductors for high currents ($I_T \geq I_c$) with joint resistances R_j, conductance load per unit length G and tape resistances R_{tape}.

The resistance of each tape depends on the relation of the transport current and its critical current. The critical current is affected by the self field of the transport current and external magnetic fields.

There are two effects that have an influence on the effective value of the critical current of the tapes in a stack:

- The self field of the transport current is low in the middle of the stack and increases to its surface. The tapes in the middle of the stack have the magnetic self field to a higher percentage perpendicular to the tape plane. Thus the critical current of the tapes in the middle of the stack may be more reduced than the critical current in the outer parts of the stack. Hence the current in the outer tapes may be higher than of the tapes in the middle of the stack.
- If the cable is penetrated by an external field, the outer tapes may carry shielding currents that prevent the external field from penetrating deep into the stack. These shielding currents may limit the capability to carry transport current and thus may have the effect to reduce the critical current of the outer tapes.

As long as the critical current of the whole stack is not reached locally and the coupling resistance is low so that the current can redistribute between the tapes easily, the current distribution doesn't matter for the cable stability.

Local Perturbation at low current inhomogeneous temperature and current distribution

In this chapter is assumed that the cable is operated with low current in the range of ($I_c/2 < I_T < I_c$), which is the case in most technical applications. Therefore the resistance of the tapes can be neglected because they are in superconducting state. Further more is assumed that there is a local perturbation on one of the tapes (part of the cross section of the cable). The perturbation may be caused by a crack in the superconducting layer where the current has to couple out of the superconducting layer into the normal conducting layers for the length of the perturbation. Thus there appears a resistance over a certain length of the tape (in the range of up to 1 mm). Caused by this resistance the current will couple into the other, still superconducting tape parallel to the disturbed area. If the transport current in the cable is above 50 % critical current, the current in the parallel superconducting tape exceeds the critical current of the tape and thus there is also a resistance in the parallel tape in the area of the perturbation. Because of these local resistances the coupling resistance between both tapes $1/G$ is divided into two parts as shown in Fig. 4.10. Thus the current can couple over the whole length of the cable on both sides of the perturbation.

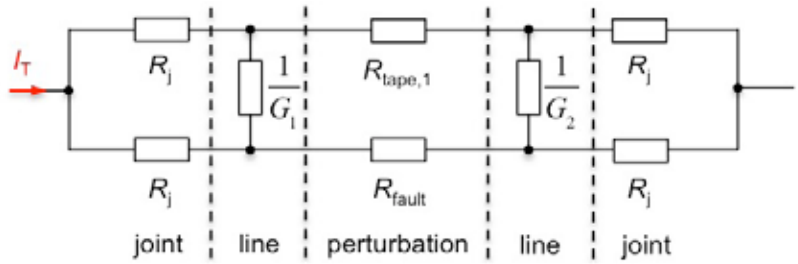


Fig. 4.10: Simplified electrical circuit diagram of two parallel, soldered YBCO coated conductors at low currents ($I_T << I_c$). One conductor is disturbed locally. The electrical network model includes the joint resistances R_j, conductance load per unit length G, the tape resistance $R_{tape,1}$ and the resistance of the local perturbation.

In such a case the current distribution varies along the cable depending on all the resistances and the parameters that have an influence these resistances. Thus the heat dissipation in the perturbed area depends on all these parameters as well as on the relation of the transport current to the critical current of the cable.

This model allows the calculation of the current distribution and the heat dissipation in a perturbed area, when only a part of the cross section of the cable has a reduced critical current. As the amount of the resistance of the superconducting tape depends on the current flowing through it (and vice versa) the current distribution for this case has to be determined by an iterative approximation approach.

In the following chapter, the current distribution within the superconducting layers is discussed and a new method for the calculation of the current distribution within superconductors is introduced.

Local Perturbation at low current homogeneous temperature and current distribution

For the application of the stability criteria it is useful to assume, that the temperature homogeneous over the whole cross section of the cable and that the current shares equally between all tapes. In this case the circuit diagram reduces to the tape resistance in the perturbed area. XX shows the reduced circuit diagram to be used in this case.

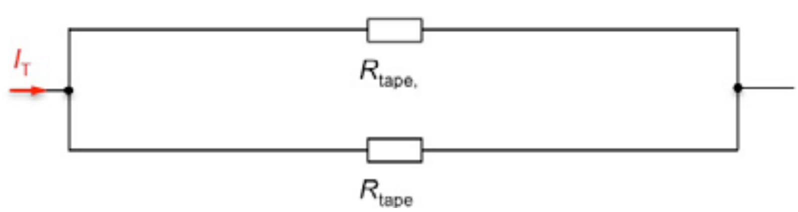


Fig. 4.11: Simplified electrical circuit diagram of two parallel, soldered YBCO coated conductors in case it is assumed that the temperature distribution in homogeneous across the whole cross section of the cable and that the current shares equally between all tapes.

4.4 Numerical Method to Calculate Current Distribution within Superconductors

In the forgoing chapter the different influences on the current sharing between the YBCO tapes in a stack is discussed. In this chapter a new method for the calculation of the current distribution within the superconducting layers is introduced. The method is based on the principle of the calculation of the self inductance load per unit length as described in chapter 4.1.3 and can generally be used for arrangements of superconductors of arbitrary shape. For this calculation is assumed that there is no external magnetic field.

If a superconductor is loaded with current, the current distributes such that the overall energy is minimal. The energy is stored in the magnetic field caused by the current flowing in the superconductor. Thus the current distributes such that the energy stored in the magnetic field is minimized. For the determination of the current distribution the critical state model can be used. This model says that the absolute value of the current density within a superconductor can only be the critical current density j_c or zero. Thus the ratio of the current carrying area A_I to the total cross sectional area of the superconductor A_{YBCO} (superconducting layer only) is the same as the ratio of the transport current I_T in the superconductor to its critical current I_c.

$$\frac{A_I}{A_{YBCO}} = \frac{I_T}{I_c}, \tag{4.29}$$

If a superconductor is cooled below its critical temperature first and a current (lower than the critical current of the superconductor) is applied afterwards, the current will flow in a part of the cross section only and it will distribute in a way that the total energy stored in the magnetic field is minimized. The stored energy can be calculated in dependence of the inductance of the arrangement

$$E = \frac{1}{2} L I^2, \tag{4.30}$$

The inductance can be determined by the following equation as introduced in equation (4.25) in chapter 4.1.3.

$$L_s' = \frac{\mu}{2\pi} \cdot \ln\left(\frac{gmd_{A,B}}{gmd_{A,A}}\right) \tag{4.31}$$

This equation allows the calculation of the self inductance (per unit length) L'_s of a straight conductor A that is parallel to another straight conductor B that carries current in the same direction (compare chapter 4.1.3).

In order to explain the developed method as simple as possible, we have a look at the magnetic field of a straight conductor A that is a distance apart from another parallel straight conductor B. The distance between both conductors is very high. So we can neglect that their magnetic fields effect the current distribution in each other.

The great distance results in the fact that the mutual inductance M' (per unit length) of both conductors is much lower than the self inductance L'_s (per unit length) $M' << L'_s$ of each conductor and thus we can neglect the mutual inductance for the determination of the minimization of the energy. This means for equation (4.31) that the geometrical mean distance between conductor A and B is much higher than the geometrical mean distance of conductor A to itself $gmd_{A,B} >> gmd_{A,A}$, because the distance between both conductors is greater than the diameter of each conductor (compare Fig. 4.6). Hence a variation of the current distribution within conductor A has a major effect on $gmd_{A,A}$, but a minor effect on $gmd_{A,B}$. Thus $gmd_{A,A}$ is the main variable for the determination of the current distribution within conductor A. Equation (4.25) says that the inductance and thus the total energy decreases with increasing geometrical mean distance of the current carrying area A_I to itself ($gmd_{A,A}$). This means, that the current flows in the areas with the highest geometrical mean distance first (in the outer most areas of the conductor).

This correlation can be used for the calculation of the current distribution within superconductors of arbitrary shape. The cross sectional area of the conductor has to be discretized in a number of elements. For each element the geometrical mean distance between itself and all other elements have to be calculated. The elements with the highest geometrical mean distance carry a current. The number of current carrying elements can be determined by equation (4.29). The current carrying area expands from the outermost areas of the conductor to inside the conductor with increasing current. This behavior can be seen in the example calculations shown in Fig. 4.12 where two parallel straight conductors are loaded with different transport currents.

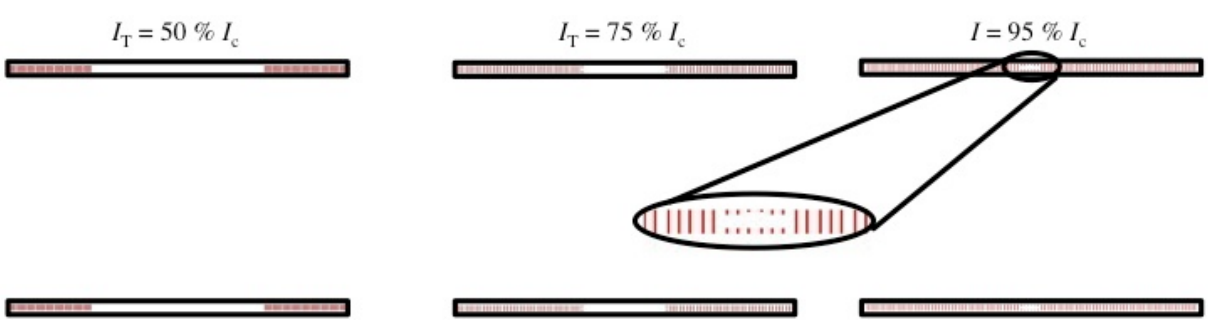


Fig. 4.12: Current distribution in two parallel YBCO layers at 50 % I_c (left) , 75 % I_c (middle) and 95 % I_c (right). It's assumed that the same current flows in each layer. The red lines show the current carrying area.

The approach to calculate the current distribution within superconductors is valid for a superconductor to be cooled below its critical temperature first and loaded with increasing current afterwards only. If the current is decreased at some point the current will start flowing in the opposite direction in the outer most parts of the conductor in order to minimize the total energy. This behavior can be easily taken into account for the calculation of the current distribution of a chronological sequence of increasing and decreasing current within a superconductor.

This method can also be used for the calculation of the current distribution of a number of separate parallel superconductors. In this case the transport current and the critical current of each conductor must be known. The geometrical mean distance has to be calculated between each element and all other elements in all conductors. The number of current carrying elements has to be determined for each conductor separately. The current carrying areas are those with the highest geometrical mean distance. The calculation of the current distribution within the superconducting layers of a stack is implemented in Matlab function *I_distr_stack.m*.

The main advantage of this calculation approach is that the method is simple and can easily be implemented in many Programing languages. The use of a FEM software can be avoided and the detailed knowledge of the current distribution can be used to determine the self inductance per unit length in dependence of the loading of the conductor. This is more accurate because the inductance depends on the current distribution and not on the geometry of a conductor. This method is to be published in “Numerical Method for the Calculation of Current Distribution in Superconductors of Arbitrary Shape” in IOPScience: Superconductor Science and Technology in 2012

5 Stability Calculation Methods

This chapter describes how the standard calculation models for stability can be applied on cables made of stacks of YBCO coded conductors (assembly shown in Fig. 2.1). The stability criteria considered in this report are cryostability, cold end recovery and minimum propagation zone as they are described in literature [Dre95], [Wil83]. Their application is explained in the following chapters and the implementation of the belonging calculations is explained in the following.

In this report the stability of a stack without the support structure is considered only. In order to apply the different stability criteria, it's necessary to calculate the joule heat of a cable (in form of the power load per unit length p') depending on the transport current in the stack I_{stack}. Therefore a method for the calculation of the power load per unit length is developed in this work. Its approach is described in the following.

5.1 Calculation of Electro-Thermal Properties of Soldered Stacks

5.1.1 Heat Dissipation

Homogeneous Current and Temperature Distribution

The power load per unit length (joule heat) of stack of YBCO tapes is calculated by the product of the resistance per unit length of the stack R'_{stack} and the transport current I_T

$$p' = R'_{stack} \cdot I_T^2 \quad . \tag{5.1}$$

The transport current in the stack is given as variable and the resistance load per unit length of the cable R'_{stack} has to be calculated by equation (4.6) as described in chapter 3.2. This approach allows the calculation of the dissipated power per unit length in dependence of all boundary conditions abd cable parameters listed in Tab. 5.1. The material properties needed for this calculation can be calculated by the fit-functions given in chapter 3.

Tab. 5.1: List of input variables used for the calculation of dissipated power (heat) per unit length *p*' of a soldered stack of YBCO coated conductors.

thickness of solder layer between tapes	h_{so}	number of tapes	n_{tape}
thickness of hastelloy layer of tapes	h_{hs}	tape width	w_{tape}
thickness of YBCO layer of tapes	h_{ybco}	Temperature	T
thickness of silver layer of tapes	h_{ag}	Magnetic field density	B
thickness of copper layer of tapes	h_{cu}	Transport current	I

Inhomogeneous Current and Temperature Distribution

In case of local perturbation that causes a inhomogeneous temperature distribution across the cross section of a stack or in case of a defect in one or multiple tapes in a stack, the resistance per unit length of the tapes in a stack may be not equal and a result is a inhomogeneous current and temperature distribution within the cross section of the perturbed area. In the following the calculation of the heat dissipation for this cases is described. Fig. 5.3 shows the circuit diagram used for this calculation (compare Fig. 4.10). The circuit diagram shows the case example of a cable with two parallel tapes and has to be extended in according to the number of tapes in the stack the calculation has to be done for. In the following the determination of the circuit elements is described.

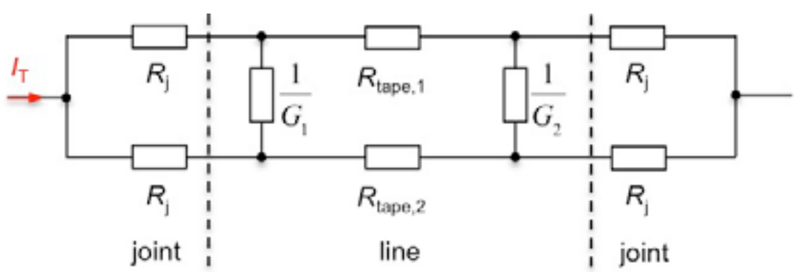


Fig. 5.1: Simplified electrical circuit diagram of two parallel soldered YBCO coated conductors for the calculation of the heat dissipation in case of a inhomogeneous current and temperature distribution in a locally perturbed area. R_j joint resistance, G_1 and G_2 conductance of area, $R_{tape,1}$ resistance of tape 1 in the perturbed area with length ℓ_f, $R_{tape,2}$ resistance of tape 2 in the perturbed area with length ℓ_f.

The joint resistance can be assumed to be in the range of $R_j = 1$ µΩ [TMB11]. The conductance between the tapes in front and behind the faulted area G_1 and G_2 can be calculated by

$$G_1 = G' \cdot l_1 \quad \text{and} \quad G_2 = G' \cdot l_2 \tag{5.2}$$

where *G*' can be determined by equation (4.11). For more detailed calculations a FEM simulation would be useful. The resistances of tape 1 $R_{tape,1}$ represents the resistance of the perturbed tape in the perturbed area ℓ_f and the resistances of tape 2 $R_{tape,2}$ represents the resistance of the still superconducting parallel tape in the perturbed area ℓ_{f}. The tape resistances can be determined by equation (4.3) and

$$R_{\mathrm{tape}} = R'_{\mathrm{tape}}\left(B, I, T\right) \cdot l_{\mathrm{f}} \tag{5.3}$$

This approach allows to assume for example a lower critical current I_{c} or a higher temperature T of the perturbed tape 1 and different values for the not perturbed parallel tape 2.

To calculate the dissipated heat in the perturbed area, the currents in the circuit diagram and the resistances $R_{\mathrm{tape,1}}$ and $R_{\mathrm{tape,1}}$ have to be calculated numerically as they depend on each others. Therefore an equation was derived for each of the currents in circuit diagram by establishing equations for the voltages and currents, inserting them into each others and solving the equations for the current wanted. This was done by using the scientific computing software Maple. The derived formulas for the currents wanted are

$$i_{\mathrm{tape,1}} = \frac{\left(2R_{\mathrm{j}}^2G_2 + 2R_{\mathrm{j}} + 4R_{\mathrm{tape,2}}R_{\mathrm{j}}^2G_1G_2 + 2R_{\mathrm{tape,2}}R_{\mathrm{j}}G_1 + 2R_{\mathrm{tape,2}}R_{j}G_2 + R_{\mathrm{tape,2}} + 2R_{\mathrm{j}}^2G_1\right)i_{\mathrm{T}}}{4R_{\mathrm{j}}^2G_2 + 4R_{\mathrm{j}} + 4R_{\mathrm{tape,1}}R_{\mathrm{j}}^2G_1G_2 + 2R_{\mathrm{tape,1}}R_{\mathrm{j}}G_1 + 2R_{\mathrm{tape,1}}R_{\mathrm{j}}G_2 + R_{\mathrm{tape,1}} + 4R_{\mathrm{j}}^2G_1 + R_{\mathrm{tape,2}}R_{\mathrm{j}}^2G_1G_2 + 2R_{\mathrm{tape,2}}R_{\mathrm{j}}G_1 + 2R_{\mathrm{tape,2}}R_{\mathrm{j}}G_2 + R_{\mathrm{tape,2}}} \tag{5.4}$$

and

$$i_{\mathrm{tape,2}} = \frac{\left(2R_{\mathrm{j}}^2G_2 + 2R_{\mathrm{j}} + 4R_{\mathrm{tape,1}}R_{\mathrm{j}}^2G_1G_2 + 2R_{\mathrm{tape,1}}R_{\mathrm{j}}G_1 + 2R_{\mathrm{tape,1}}R_{j}G_2 + R_{\mathrm{tape,1}} + 2R_{\mathrm{j}}^2G_1\right)i_{\mathrm{T}}}{4R_{\mathrm{j}}^2G_2 + 4R_{\mathrm{j}} + 4R_{\mathrm{tape,1}}R_{\mathrm{j}}^2G_1G_2 + 2R_{\mathrm{tape,1}}R_{\mathrm{j}}G_1 + 2R_{\mathrm{tape,1}}R_{\mathrm{j}}G_2 + R_{\mathrm{tape,1}} + 4R_{\mathrm{j}}^2G_1 + R_{\mathrm{tape,2}}R_{\mathrm{j}}^2G_1G_2 + 2R_{\mathrm{tape,2}}R_{\mathrm{j}}G_1 + 2R_{\mathrm{tape,2}}R_{\mathrm{j}}G_2 + R_{\mathrm{tape,2}}} \tag{5.5}$$

The derivation of these equations as well as the equations to calculate the other currents in the circuit diagram is documented in the Maple files: *Derivation of Equation for Current in Faulted Tape - Total Resistance - with parallel tape resistance.mw* and *Derivation of Equation for Current in Faulted Tape - with parallel tape resistance.mw*.

Once the currents and resistances in the circuit diagram are determined, the total dissipated power can be calculated by

$$P = R_{\mathrm{tape,1}} \cdot I_{\mathrm{tape,1}}^2 + R_{\mathrm{tape,2}} \cdot I_{\mathrm{tape,2}}^2 \tag{5.6}$$

This approach can be used for calculations in case of a inhomogeneous current and temperature distribution across the cross section of the stack. However, to estimate the minimal and maximal dissipated power in a stack, a simple approach is to take into account the superconducting properties of all tapes or to neglect the superconducting properties of all tapes when calculating the resistance per unit length of the tapes.

In Fig. 5.2 an example calculation for the heat dissipation in a stack of 5 parallel conductors with specifications given in Tab. 6.2 in liquid hydrogen at boiling temperature and a magnetic field density of 10 T is shown. The dashed line shows the heat dissipation when superconductivity is taken into account for all YBCO tapes. The dashed-dotted line shows the heat dissipation if only 4 of the 5 tapes show superconducting properties (one of the tapes is always in normal conducting state) and the dotted only line shows the same for the case when 3 of 5 tapes are in superconducting state only. For this calculation is assumed that all tapes are at the same temperature.

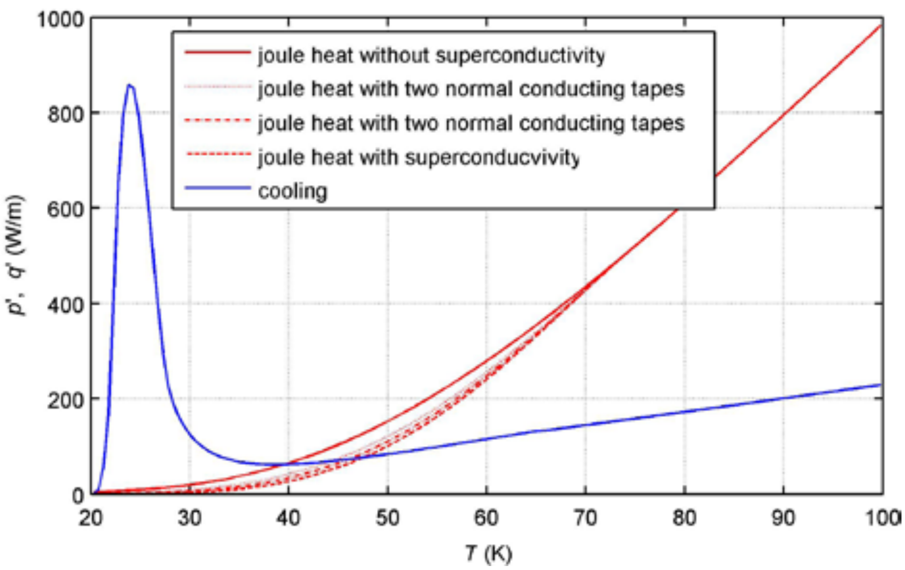


Fig. 5.2: Example calculation for cryostability. Heat dissipation of a stack of 5 parallel tapes with the specifications given in Tab. 6.2 in liquid hydrogen at and a magnetic field density of 10 T.

The example calculation in Fig. 5.2 shows nicely how the heat dissipation is increased when less tapes show superconducting properties. It was assumed that the entire stack had a total length of 1000 m and the disturbance had a length of 1 mm. The joint resistance was assumed to be $R_j = 1$ µΩ and the thickness of the solder layer was assumed to be $h_{so} = 50$ µm. For the calculations where all or no tapes are superconducting were done using circuit diagram.

5.1.2 Heat Capacity

To apply the stability criteria “cold end recovery” and “minimum propagation zone”, the heat capacity of soldered stacks of YBCO coated conductors has to be calculated too. In this work the heat capacity is calculated per unit conductor length by the following equation

$$C'_{\text{tape}}(T) = n_{\text{tape}} \cdot w_{\text{tape}} \cdot \left(\rho_{\text{d,hs}} \cdot h_{\text{hs}} \cdot C_{\text{hs}}(T) + \rho_{\text{d,ybco}} \cdot h_{\text{ybco}} \cdot C_{\text{ybco}}(T) + \rho_{\text{d,ag}} \cdot h_{\text{ag}} \cdot C_{\text{ag}}(T) + \rho_{\text{d,cu}} \cdot h_{\text{cu}} \cdot C_{\text{cu}}(T)\right) + \left(n_{\text{tape}} - 1\right) \cdot w_{\text{tape}} \cdot \left(\rho_{\text{d,so}} \cdot h_{\text{so}} \cdot C_{\text{so}}(T)\right) \tag{5.7}$$

Here w_{tape} is the tape width, n_{tape} is the number of parallel tapes, h is the respective layer thickness, ϱ_{d} the respective material density and C the temperature dependent heat capacity of the respective material calculated by the fit-functions given in chapter 3.4. In Tab. 5.2 the material densities used in this work are listed.

Tab. 5.2: List of density of Hastelloy, YBCO, Silver and Copper at room temperature.

$\rho_{\text{d,hs}}$ (kg/m³)	8890	$\rho_{\text{d,ybco}}$ (kg/m³)	6300
$\rho_{\text{d,ag}}$ (kg/m³)	10490	$\rho_{\text{d,cu}}$ (kg/m³)	8940

This equation is implemented in Matlab file *C_stack.m*.

5.1.3 Thermal Conductivity

To apply the stability criteria "cold end recovery" and "minimum propagation zone", also the thermal conductivity of soldered stacks of YBCO coated conductors has to be calculated. In this work the following equation is used for this calculation

$$k_{\text{stack}}(T) = \frac{n_{\text{tape}} \cdot \left(h_{\text{hs}} \cdot k_{\text{hs}}(T) + h_{\text{ybco}} \cdot k_{\text{ybco}}(T) + h_{\text{ag}} \cdot k_{\text{ag}}(T) + 2 \cdot h_{\text{cu}} \cdot k_{\text{cu}}(T)\right) + \left(n_{\text{tape}} - 1\right) \cdot h_{\text{so}} \cdot k_{\text{so}}(T)}{n_{\text{tape}} \cdot \left(h_{\text{hs}} + h_{\text{ybco}} + h_{\text{ag}} + 2 \cdot h_{\text{cu}}\right) + \left(n_{\text{tape}} - 1\right) \cdot h_{\text{so}}} \tag{5.8}$$

Here n_{tape} is the number of parallel tapes, h is the respective layer thickness and k is the temperature dependent thermal conductivity of the respective material, calculated by the fit-functions given in chapter 3.3. This equation is implemented in Matlab file *k_stack.m*.

5.2 Cryostability

Cryostability is a stability criterion used for designing superconducting magnets. It was established in 1965 and its approach is explained in many books written in this field like [Wil83] and [Dre95]. Cryostable magnets are able to recover after a normalizing perturbation under all circumstances. Cryostability is therefore called unconditioned stability also. This behavior can be achieved by decreasing the resistance per unit length of the cable, so that the joule heat is lower than the cooling. This approach is shown in the principal sketch on the right side of Fig. 5.3. This sketch is usually used to explain cryostability.

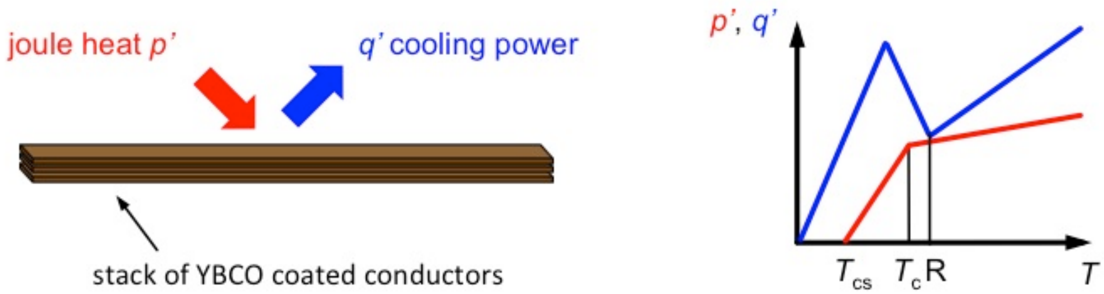


Fig. 5.3: Principal sketch used for the definition of cryostability for low temperature superconductors at helium temperature. Cooling power of liquid helium and dissipated joule heat per unit length of a superconducting cable with a low critical temperature material.

The limit of cryostability is defined at recovery point R which is the minimum of the cooling curve. At this temperature the joule heat (power load per unit length) *p*' must be lower than the cooling *q*'. Thus the relation between both must be lower than 1 that a magnet is cryostable

$$\alpha = \frac{p'}{q'} \leq 1 \quad . \tag{5.9}$$

This equation is called the Stekly criterion. The criterion of cryostability was defined for low temperature superconductors cooled with liquid helium at 4.2 K. The recovery point of unpressurized liquid helium is in the range of 7 K and the critical temperature of the superconducting materials is in the range of 10 K (NbTi) or 18 K (Nb_3Sn).

In Fig. 5.3 an example calculation for the dissipated heat of a single YBCO tape in an external magnetic field of $B = 10$ T and a transport current of $I_T = 100$ A. For comparison the cooling power per unit length of this conductor in liquid hydrogen at boiling temperature (not pressurized without forced flow) is also shown in this diagram. The specifications of the tape are given in Tab. 5.3 and the heat dissipation is calculated as described in chapter 5.1.1. The dashed line shows the dissipated power per unit length when superconductivity is taken into account (minimal dissipated power) and the continued line shows the heat dissipation in the case when the superconducting YBCO layers would not show a superconducting behavior (maximal dissipated power). For this calculation a homogeneous temperature distribution across the cross section of the cable and equal current sharing between the tapes was assumed. Therefore circuit diagram shown in Fig. 4.11 was used for the calculation.

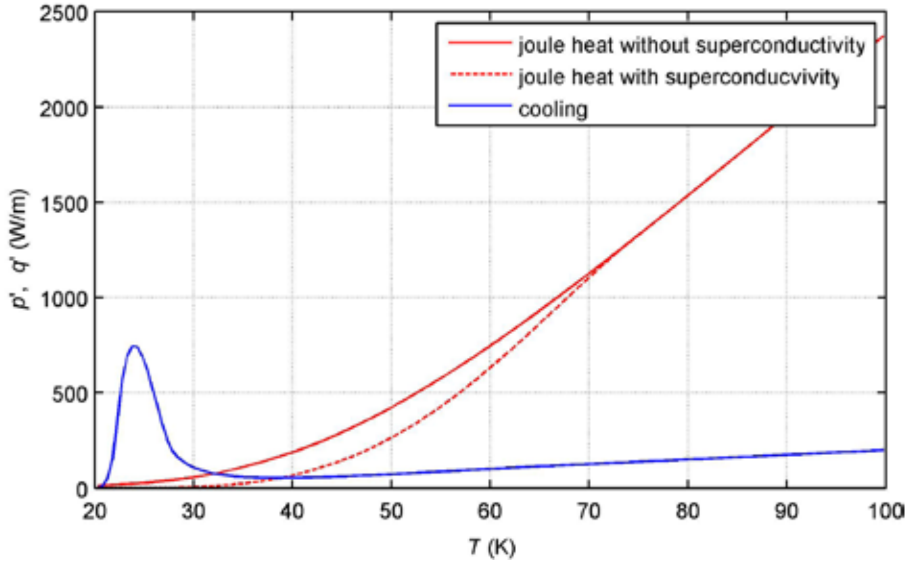


Fig. 5.4: Example calculation for cryostability of a single tape in liquid Hydrogen. The specifications of the tape and the conditions assumed are given in Tab. 5.3. T_{cs} current sharing temperature, T_c critical temperature, R recovery point.

The critical current of this tape in these conditions is I_c = 101.7 A. So the tape is almost operated at its critical current. Its critical temperature is T_c (B, I_T=0) = 75.6 K and the current sharing temperature (critical temperature at transport current) is $T_{cs} = T_c(B, I_T=100\text{ A}) = 20.9$ K so the superconducting-normal conducting transition happens in a wide temperature range. The engineering current density of this conductor in this case is 455 A/mm^2.

The recovery point of liquid hydrogen (minimum of cooling power) is at a temperature of T = 38.8 K. The copper layer thickness for this example is chosen that the Stekly number is just α = 1.0. This means that this conductor is operated at its limit of cryostability as it is operated at critical current and the Stekly number is equal to one at recovery point R. The copper layer thickness is h_{cu} = 1.151 µm. Every increase of the copper layer thickness would cause a reduction of the joule heat and the Stekly number.

The stability criterion of cryostability was defined for low temperature superconductors in liquid hydrogen at 4.2 K. As one can see in the principal sketch on the right in Fig. 5.3 the desired condition for stability is that the joule heat is below the cooling power even in the minimum of the cooling curve and in a certain temperature range above. For YBCO tapes in liquid hydrogen it can only be achieved that the cooling power is higher than the heating up to the minimum of the cooling curve, but not any higher.

Tab. 5.3: Specifications of coated conductor and boundary conditions for example calculation for cryostability

w_{tape} (mm)	4
h_{hs} (µm)	50
h_{ybco} (µm)	1.0
h_{ag} (µm)	1.5
h_{cu} (µm)	1.15
B (T)	10
I_T (A)	100

But the margin for stability is still higher in liquid hydrogen though. the temperature difference between the boiling point and the recovery point of liquid helium is with about 3 K very low and for liquid hydrogen this difference is with about 28 K much higher. Thus the criterion of cryostability of YBCO tapes in liquid hydrogen valid for temperature increases of up to 28 K. Furthermore the heat capacity of the materials involved is very low at liquid helium temperature (4.2 K) and already small perturbations lead to a high temperature increase that may cause a normal zone in the superconductor. At boiling temperature of liquid hydrogen (20.28 K) the heat capacity of YBCO tapes with a copper layer thickness of 20 µm is by a factor of 7.3 higher than at helium temperature (compare Tab. 6.6).

The application of the Stekly criterion on YBCO coated conductors in liquid hydrogen results in very high safety margins. Therefore it should be discussed if this stability criterion is an adequate method to determine stability for this case.

All the foregoing calculations related to the criterion of cryostability are implemented in Matlab file *cryostability.m*. This calculation can be called with defined input parameters by Matlab file *cryostability_function_call.m*.

5.3 Cold End Recovery

Another criterion for stability of superconducting magnets is cold end recovery [Dre95], [Wil83]. This stability criterion allows less copper in the cable structure, which results in higher current densities and a higher cable resistance per unit length. Both leads to a higher heat dissipation in case of perturbations.

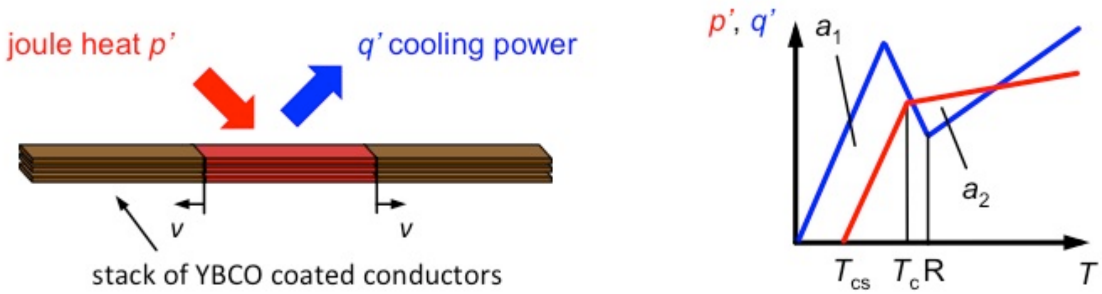


Fig. 5.5: Principal sketch used for the definition of cold end recovery for low temperature superconductor material at helium temperature. T_{cs} current sharing temperature, T_c critical temperature, R recovery point.

In case of a perturbation of a cryostable magnet, the whole normal zone cools down because the heat dissipation of the conductor is lower than the cooling. In case of a magnet that is designed to fulfill cold end recovery, the heat dissipation is higher, so that the normal zone doesn't cool down but stays at a constant temperature and the normal zone shrinks from both ends. In Fig. 5.5 a principal sketch of the heat dissipation of a low temperature superconductor and the cooling properties of liquid helium are shown. The heat dissipation is so high that there are two intersections with the cooling curve (compare with Fig. 5.3). Of these two intersections the second one (the one with the higher temperature) is stable. A perturbation that causes a normal zone leads to a temperature increase up to this temperature. By reason that the length of the perturbation is limited, this normal zone has two ends. As long as area a_1 between the cooling and the heat dissipation curve is greater than area a_2 (see Fig. 5.5), the cooling at both ends is higher than the heating and thus the normal zone will shrink that cable finally recovers. In this case the quench propagation velocity is below zero ($v < 0$ m/s). See Fig. 5.5 on the left for comparison. The limit of cold end recovery where both areas are equal

$$a_1 = a_2 \tag{5.10}$$

This is called the equal area theorem. In Fig. 5.6 an example calculation for cold end recovery is shown. This Figure shows the same diagram as Fig. 5.4. The heat dissipation of a single YBCO tape and the cooling curve of liquid hydrogen. The tape has specifications that exactly meet the conditions for cryostability in liquid hydrogen. The copper layer thickness is $h_{cu} = 1.15$ µm. The external magnetic field is $B = 10$ T and the transport current is $I_T = 100$ A. The complete specifications of this tape are given in Tab. 5.3

This case example shows clearly that there is only a single intersection of the heat dissipation and the cooling curve for a YBCO tape in liquid hydrogen because the critical temperature of YBCO is so high. The intersection is an instable equilibrium. Thus cold end recovery cannot be realized for

YBCO coated conductors in liquid hydrogen. Therefore cold end recovery is not discussed any further in this report.

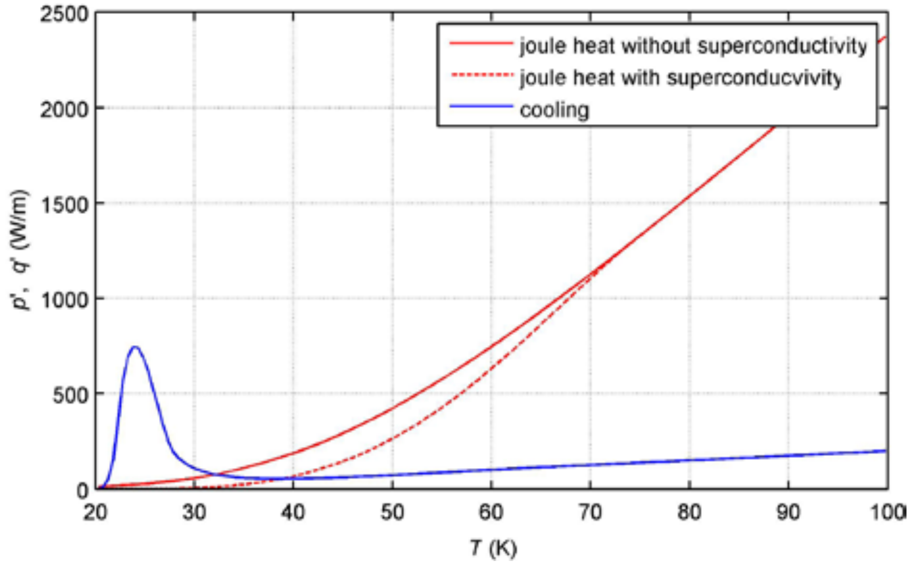


Fig. 5.6: Example calculation for cryostability of a single tape in liquid Helium. The specifications of the tape are given in Tab. 5.3

For a comparison of the heat dissipation of a different stabilized YBCO coated conductor an example calculation for a tape with 20 µm copper layer thickness under the same conditions in shown in Fig. 6.8.

5.4 Minimum Propagation Zone

A third stability criterion is the so-called minimum propagation zone [Dre95], [Wil83]. This criterion is based on the assumption that perturbations of superconducting magnets appear in very small sections of the conductor only and that the amount of energy introduced in a perturbed area is small. A typical example for a perturbation is the local rearrangement of the conductor due to slipping caused by the Lorentz force. These assumptions allow the design of magnets that can be operated with an even lower amount of copper compared to the criteria of cold end recovery and cryostability. Thus the current density and the resistance per unit length are higher which results in a higher heat dissipation in case of disturbances as shown in the sketch on the right of Fig. 5.7.

It is assumed that there is a local no uniform steady state of the temperature distribution if a certain amount of energy is introduced locally. This steady state temperature distribution is called the minimum propagation zone. Its principal characteristic is shown in the sketch in Fig. 5.7 on the left. If a certain amount of energy Q (the formation energy) is introduced in the stack locally, a minimum propagation zone establishes. If the amount of energy introduced is lower, the heat conductivity along the tape and the cooling power is higher than the joule heat and so the perturbed area cools down and recovers. If any higher amount of energy is introduced, a quench starts to propagate from the perturbed area. So the formation energy that establishes a minimum propagation zone is just the limit that is tolerated by the conductor and thus the limit of stable operation with the assumptions mentioned above. Every higher amount of energy leads to a quench of the conductor.

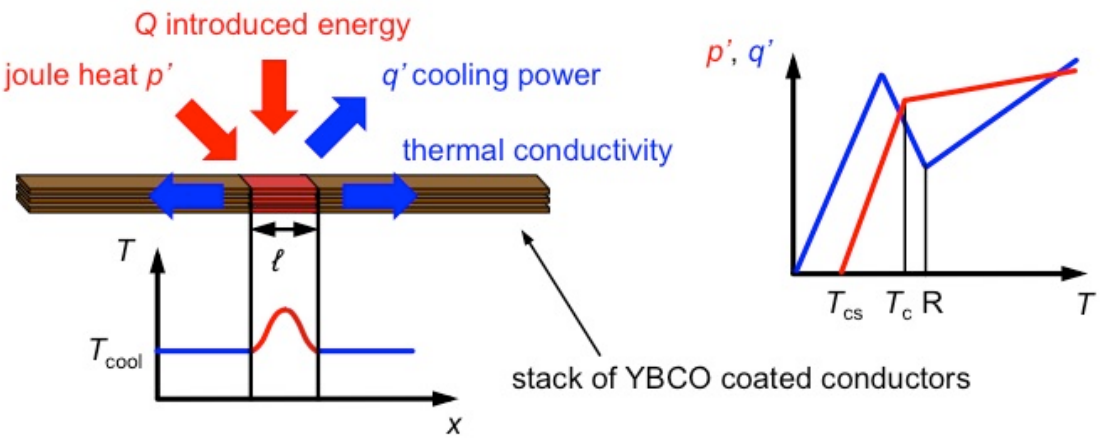


Fig. 5.7: Principal sketch used for the definition of cold end recovery for low temperature superconductor material at helium temperature. T_{cs} current sharing temperature, T_c critical temperature, R recovery point.

The point of interest for the magnet design is the formation energy that leads to the steady state of the minimum propagation zone. In order to determine an estimate of the minimum quench energy, the maximum temperature of the minimum propagation zone as well as the dimension of the minimum propagation zone have to be calculated.

Calculation

The maximum tolerable temperature of the minimum propagation zone T_{max} can be calculated by

$$0 = \int_{T_{cool}}^{T_{max}} k_{stack}(T) \cdot \left(p'_{stack}(T) - q'_{stack}(T)\right) dT \qquad (5.11)$$

[Dre95]. Where $k_{stack}(T)$ is the thermal conductivity of the stack that can be calculated by equation (5.8), $p'_{stack}(T)$ is the power load per unit length that can be calculated by equation (5.1) and $q'_{stack}(T)$ is the cooling per unit length that can be calculated by equation (3.25) for liquid hydrogen. With this equation, the maximum temperature T_{max} can be determined numerically.

In [Wil83] an equation to calculate the dimension of the minimum propagation zone is given. In this approach is assumed, that the tape is heated up to the critical temperature of the tape. As the maximum temperature of the minimum propagation zone is calculated, in this work is assumed that the area is heated up to the maximum temperature T_{max} as calculated by equation (5.11) instead. Thus the dimension of the minimum propagation zone is calculated by the following equation

$$mpz = \sqrt{\frac{2 \cdot k_{stack}\left(T_{max}\right) \cdot \left(T_{max} - T_{cool}\right)}{R'_{stack}\left(T_{max}\right) \cdot \left(\frac{I_T}{A_{stack}}\right)^2}} \quad (5.12)$$

The critical temperature can be calculated by fit-function (3.1). $k_{stack}(T)$ is the thermal conductivity of the stack that can be calculated by equation (5.8), T_{cool} is the temperature of the coolant (LH_2), $R'_{stack}(T)$ is the resistance load of the stack as calculated by equation (4.6) and (4.3) and A_{stack} is the cross sectional area of the stack.

By using equation (5.11) and (5.12) an estimate of the formation energy (minimum quench energy) can be calculated. It is assumed that the minimum quench energy is equally distributed within the minimum propagation zone. Thus the minimum quench energy can be estimated by

$$mqe = mpz \cdot \int_{T_{cool}}^{T_{max}} C'_{stack}\left(T\right) dT \quad (5.13)$$

Simulation

In order to verify that this estimate is in the range of the minimum quench energy, the one dimensional temperature run along the stack after a perturbation is calculated numerically in this work. For this purpose first the minimum propagation zone was calculated by equation (5.12). The YBCO conductors were discretized in infinitesimal length elements *dz,* and as initial condition the temperature of the length elements within the minimum propagation zone was set to the maximum temperature T_{max} calculated by equation (5.11). The temperature of all other length elements was set to the temperature of the coolant T_{cool}. This is equivalent to the introduction of the minimum quench energy within the minimum propagation zone in case of a sudden local perturbation. In the following, the temperature run is calculated numerically for each length element and for infinitesimal time steps *dt*, taking into account the heat dissipation of the stack (equation (5.6)), its heat capacity (equation (5.7)), its thermal conductivity (equation (5.8)) and the cooling power of a liquid coolant (equation (3.24), (3.25) or (3.26)) for each element.

In Fig. 5.8 an example calculation for the temperature run of a minimum propagation zone calculated as described above is shown. This calculation is done for a single YBCO tape in liquid Hydrogen. The tape fulfills the requirements of cryostability in this conditions and has the same specifications as the one in the example calculation for cryostability. Its specifications are given in Tab. 5.4. The magnetic field density is $B = 10$ T and the transport current is $I_T = 100$ A. The critical current of this tape in this field is $I_c = 101.7$ A.

The initial conditions are a maximum temperature of $T_{max} = 65.4$ K over a length of $mpz = 1.291$ mm. The rest of the tape is on the temperature of the coolant $T_{cool} = 20.28$ K. Thus the introduced energy is $mqe = 6.964$ mJ in a volume of 0.283 mm^3. The initial conditions are calculated by equations (5.11) and (5.12). This calculation was done for a total length of $l_{tot} = 10 \cdot mpz = 12.91$ mm. The tape was discretized in length elements of $dz = l_{tot}/200 = 64.55$ µm. The time steps were chosen to $dt = 0.1$ µs for $t < 0.5$ ms and $dt = 1$ µs for $t \geq 0.5$ ms.

Tab. 5.4: Specifications of cryostable YBCO coated conductor for example calculation for minimum propagation zone.

w_{tape} (mm)	4
h_{hs} (µm)	50
h_{ybco} (µm)	1.0
h_{ag} (µm)	1.5
h_{cu} (µm)	1.151
B (T)	10
I_T (A)	100

The blue line in Fig. 5.8 shows the initial temperature run, the red dashed lines show the temperature distribution after 1.67 ms and 3.33 ms after the perturbation and the red continued line shows the temperature distribution 5.00 ms after the perturbation.

The diagram shows the right half side of the perturbation only, because the temperature run is axial symmetric to the point of maximum temperature. There is a clear trend of the maximum temperature to cool down. Thus, a minimum propagation zone is not established and the conductor recovers after the perturbation. This means that the estimates of the minimum propagation zone and the minimum quench energy by equations (5.12) and (5.13) are not accurate and give too low results. The introduced energy needed to establish a minimum propagation zone or a quench is even higher. This is also reported in [IYM05].

A reason for this is that in equations (5.12) and (5.13), the minimum propagation zone is calculated with for the material properties at constant maximum temperature T_{max} and not with a temperature distribution between T_{cool} and T_{max}.

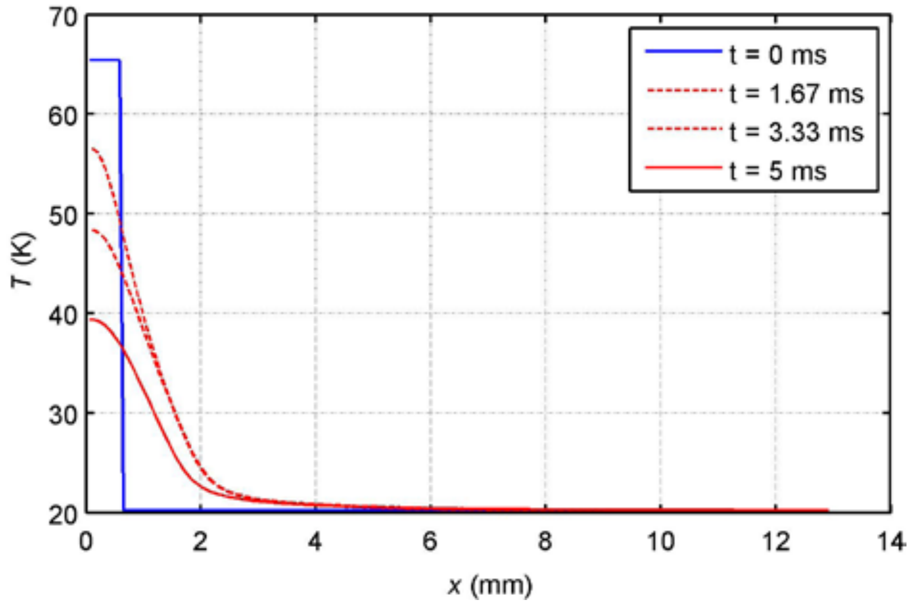
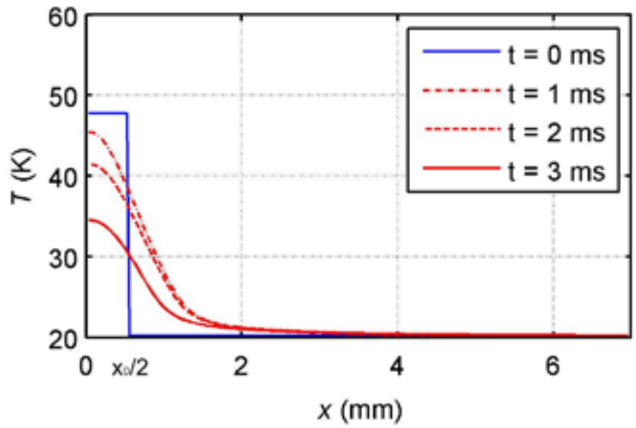


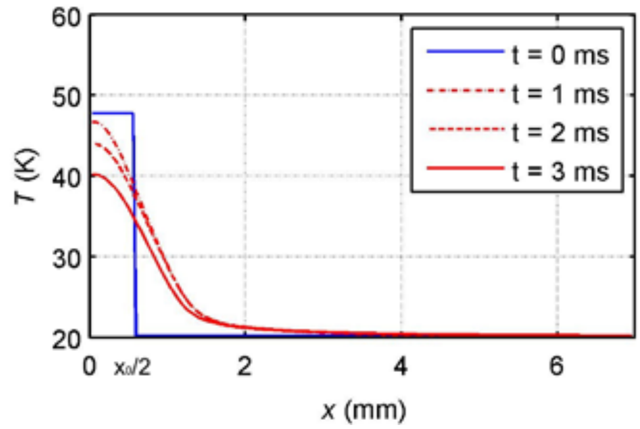
Fig. 5.8: Example calculation for minimum propagation zone of a single tape in liquid Hydrogen. The specifications of the tape are given in Tab. 5.4

If a more accurate value is required, equation (5.13) can be used to get a first estimate. Based on this estimate, the numerical calculation of the temperature run (simulation) can be done to determine a more accurate value of the minimum propagation zone and the minimum quench energy.

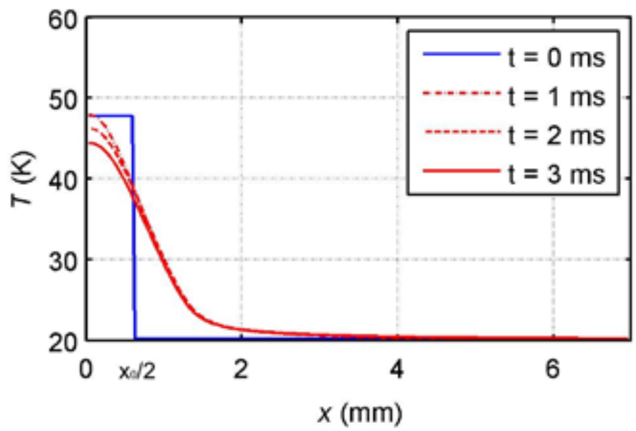
These calculations were done exemplarily for a single tape without copper layer but besides that the specifications given in Tab. 5.4. The result was, that even this tape without copper layer recovers after a perturbation with an energy input as determined by equations (5.12) and (5.13) and a transport current of $I_T = 100$ A in liquid hydrogen. This calculation was repeated for different values of the minimum propagation zone and constant maximum Temperature. The limit, where the tape doesn't recover anymore and a minimum propagation zone establishes, is about 1.7 times the value calculated by equation (5.13) in this case example. In Tab. 5.5 the initial conditions for the calculation are given. The initial conditions of all the calculations were set to the same maximum temperature T_{max} and various lengths of the perturbed area x_0. The value of the minimum propagation zone calculated by equation (5.13) for this tape is $mpz = 0.6958$ mm and the maximum temperature calculated by equation (5.12) is $T_{max} = 47.8$ K.



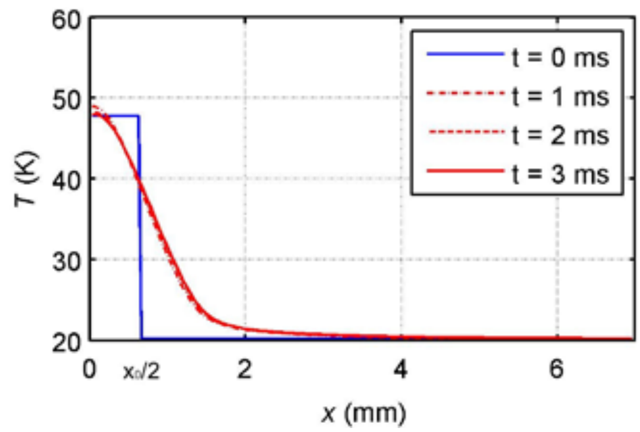
a) $x_0 = 1.6 \cdot \mathrm{mpz}$



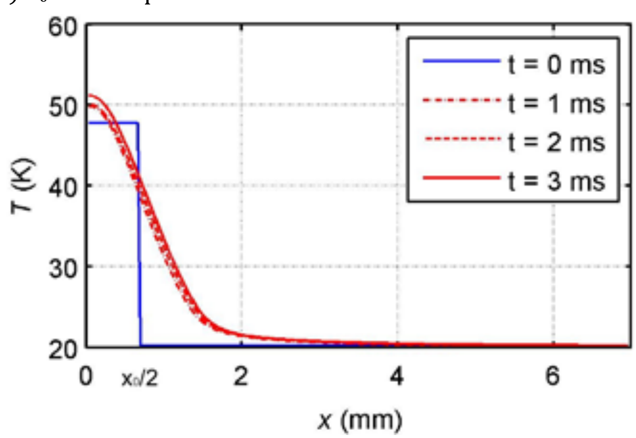
b) $x_0 = 1.7 \cdot \mathrm{mpz}$



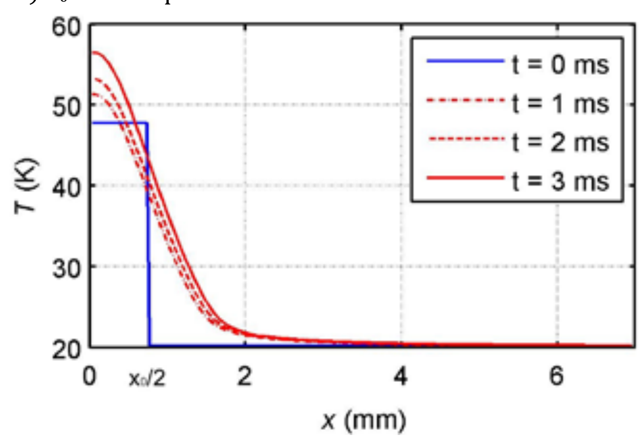
c) $x_0 = 1.8 \cdot \mathrm{mpz}$



d) $x_0 = 1.9 \cdot \mathrm{mpz}$



e) $x_0 = 2.0 \cdot \mathrm{mpz}$



f) $x_0 = 2.1 \cdot \mathrm{mpz}$

Fig. 5.9: Results of numerical calculation of temperature run of minimum propagation zone for a tape with specifications given in Tab. 5.4 but without copper layer.

In Fig. 5.9 the results of the simulations are shown. The pictures show the temperature run of one half side of the perturbed area. The blue lines show the initial condition, and the red lines show the temperature run after one, two and three ms respectively. In these calculations the initial length of the perturbed area x_0 is varied between 1.6 and 2.1 times the calculated value of the minimum propagation zone. Picture d) shows the temperature run for x_0/mpz = 1.9, which remains almost constant between 1 and 3 ms after the perturbation. Pictures a), b) and c) show simulations for shorter perturbed areas. It can be clearly seen, that the conductors cools back in these cases. Pictures e) and f) show simulations for longer perturbed areas and it's obvious that the conductor heats up further after the perturbation, which leads to a local quench of the conductor. These results are summarized in table Tab. 5.5.

Tab. 5.5: Results of simulation of temperature run of minimum propagation zone for a tape with specifications given in Tab. 5.4 but without copper layer. The length x_0 is given relative, referred to the calculated value of the *mpz*.

Initial conditions							
x_0/mpz (-)	1.0	1.6	1.7	1.8	1.9	2.0	2.1
mqe (mJ)	1.24	1.984	2.108	2.232	2.356	2.48	2.604
Temperature trend after 3 ms	recovery	recovery	recovery	recovery	stable	quench	quench

This comparison shows, that the minimum quench energy is approximately by a factor of 1.9 higher than the minimum quench energy as calculated by the equations given above in this case example. For the design of superconducting magnets, this difference can be neglected. In the design process, a reasonable value for the minimum quench energy has to be chosen that the magnet has to tolerate without a quench. As this value is just an estimate for the energy possibly introduce by a conductor movement, a difference of the calculated value of the minimum propagation zone or the minimum quench energy just doesn't matter.

Another interesting result of this calculation is, that the minimum quench energy of this single tape without an additional copper layer is about 1.24 mJ when operated at critical current $I_T = I_c$ = 100 A at liquid hydrogen temperature T_{cool} = 20 K in a magnetic field of B = 10 T. Typical values of the minimum quench energy of a NbTi wire with a diameter of 0.8 mm in a magnetic field of 8 T in liquid helium are in the range of a few to a few tens of μJ [Dre95] for comparison. This means that the margin of the minimum quench energy for a single YBCO tape in liquid hydrogen is already more than a magnitude higher than for a NbTi superconductor in liquid helium. Thus it seems to be very likely that a conductor movement would probably not cause a quench at this temperature, because the maximum introduced energy doesn't change but the heat capacity as well as the critical current of YBCO tapes is much higher at an operating temperature of 20 K in comparison to a NbTi wire in liquid helium.

All the foregoing calculations related to the criterion of minimum propagation zone are implemented in Matlab file *minimum_propagation_zone.m*. This calculation can be called with defined input parameters by Matlab file *minimum_propagation_zone_function_call.m*.

6 Example Calculations for Cable Stability

In this chapter example calculations for cable stability are shown. In order to get a general overview of the properties of YBCO coated conductors in a wide temperature range in chapter 6.2 the properties of a single YBCO coated conductor are shown in the following.

6.1 Proof of Calculation of Resistivity and Critical Current

In order to prove that the developed code gives results in a reasonable range, the calculation of the resistance load per unit length and of the critical current in self field at 77 K was compared with measurement results For this comparison the measurements published in [BNK11] were used. In this paper, the resistance of different YBCO tapes was measured in the temperature range between 77 K and almost 300 K without external magnetic field. Three of the given example measurements were chosen for a comparison. In Tab. 6.1 the width and the thickness of the different layers of these tapes are given. The sample with tape ID A4-40 is a 4 mm wide tape with 50 µm substrate thickness and additional copper stabilization. Sample A12-0 is a 12 mm wide tape with also 50 µm substrate thickness. This tape has no copper stabilization, but a quite thick silver layer. Sample A12-40 also has a 50 µm thick substrate layer and an additional surrounding copper stabilization layer.

As in the publication only the total thickness of the tapes is given, the layer thicknesses given in the table were determined by fitting the resistance curves. The resulting layer thicknesses are in a reasonable range. The buffer layer of the tapes is not considered in this comparison. The calculation of the resistance of the tapes was performed with the Matlab files *R_tape_pul*.

Tab. 6.1: Width and layer thicknesses of YBCO coated conductors for comparison of calculation and measurement of tape resistivity and critical current. Measurement results published in [BNK11], Buffer layer neglected.

Sample tape	**A4-40**	**A12-0**	**A12-40**
w_{tape} (mm)	4.1	12.1	12.0
h_{hs} (µm)	50.0	52.0	52.0
h_{ybco} (µm)	1.0	1.0	1.0
h_{ag} (µm)	2.0	3.7	1.0
h_{cu} (µm)	23.0	-	21.5
Total thickness	80.1	56.7	75.5

In Fig. 6.1, Fig. 6.2 and Fig. 6.3 the comparison of the calculation and the measurement of the tapes resistances is shown. The maximum of the deviation between calculation and measurement is at a temperature of 92 K 7.85 % for sample A4-40, 5.11 % for sample A12-0 and 8.25% for sample A12-40. The deviation is much lower at higher temperatures. The deviation may be caused by a poor fit of the layer thicknesses, a differing RRR value of the copper or the silver used for the fit-functions of the resistivity and also by an inexact measurement. These results prove that the calculation of the resistance is adequate for the use in stability calculations.

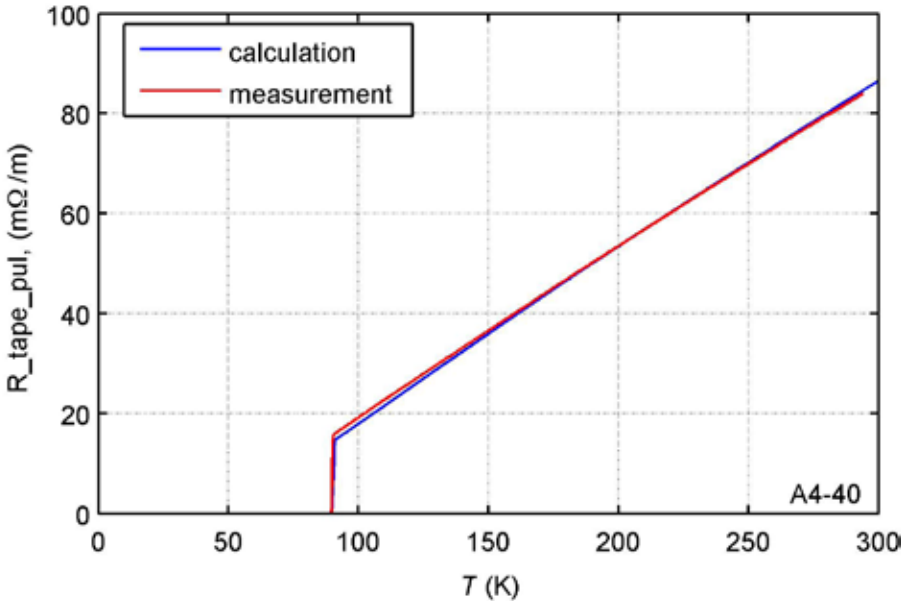


Fig. 6.1: Comparison of measurement and calculation of temperature dependent resistance load per unit length *R*’ of 4 mm wide, cupper stabilized sample tape A4-40 (see Tab. 6.1)

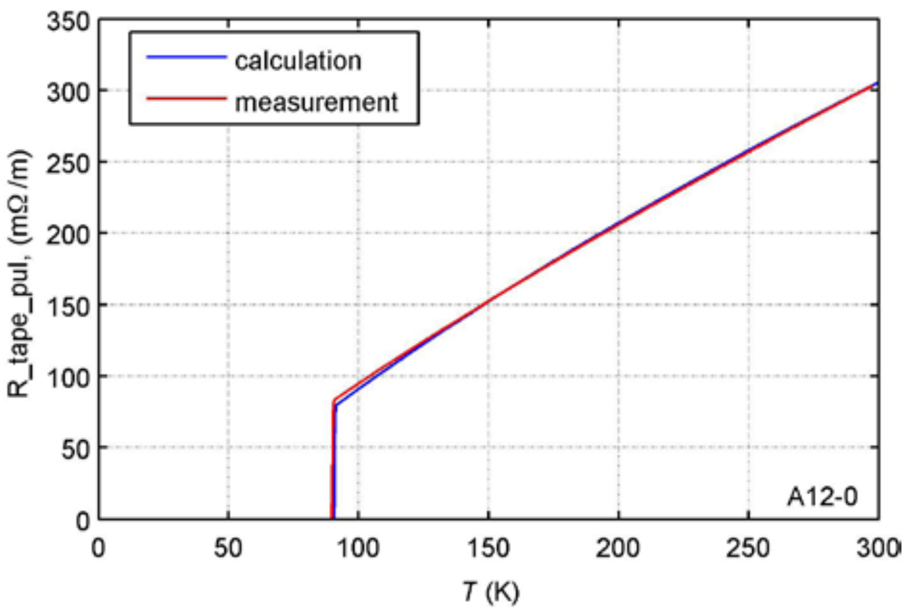


Fig. 6.2: Comparison of measurement and calculation of temperature dependent resistance load per unit length *R*’ of 12 mm wide, not copper stabilized sample tape A12-0 (see Tab. 6.1).

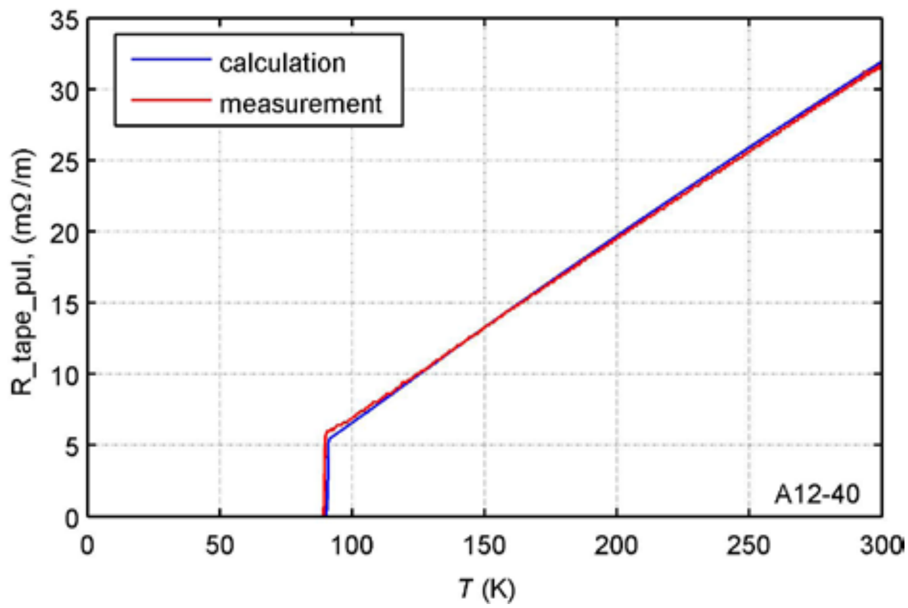


Fig. 6.3: Comparison of measurement and calculation of temperature dependent resistance load per unit length *R*' of 12 mm wide, copper stabilized sample tape A12-40 (see Tab. 6.1).

As the resistance of the tapes were measured between liquid nitrogen temperature (T_{LN2} = 77.4 K) and room temperature, the diagrams show also the transition between superconducting and normal conducting state in self field. As the fit-function used for these calculations is limited to a magnetic filed density in the range of 0.1 – 20 T, a critical field density of 100 mT was set as boundary condition for these calculations. Without using a criterion, it can be seen in the diagrams, that also the calculation of the critical temperature / critical current fits quite good with the measurement results. Hence it can be assumed that the calculations give good results in the whole temperature and magnetic field range and can be used for stability calculations.

6.2 Electro-Thermal Properties of a Single YBCO Coated Conductor

In this chapter the material properties of a single 4 mm, copper laminated wide YBCO coated conductor are given. The critical current, resistance load per unit length, heat capacity and thermal conductivity are calculated as described in the chapters 3.1, 3.2 5.1.2 and 5.1.3. In Tab. 6.2 the specifications of the tape are listed.

Tab. 6.2: Specifications of YBCO coated conductors for example calculations

w_{tape} (mm)	4
h_{hs} (µm)	50
h_{ybco} (µm)	1.0
h_{ag} (µm)	1.5
h_{cu} (µm)	20

Critical Current

Fig. 6.4 shows the critical current of the YBCO tape as calculated by fit-function (3.1) in chapter 3.1. In Tab. 6.3 selected values of the critical current for selected temperatures and magnetic fields are listed. The specifications of the tape are given in Tab. 6.2.

Tab. 6.3: Critical current of a single YBCO coated conductor with the specifications given in Tab. 6.2 for selected values of temperature T and magnetic field density B calculated by fit-function (3.1)

T (K)	4.2	4.2	20.3	20.3
B (T)	10	15	10	15
I_c (A)	155.7	120.5	101.7	70.0

Because of limitations of the fit-function, the critical current can be calculated for magnetic field densities of $0.1\ \mathrm{T} < B < 20\ \mathrm{T}$ only. The calculated critical current of the tape at $T = 77$ K and $B = 0.1$ T is $I_c = 45.7$ A for comparison.

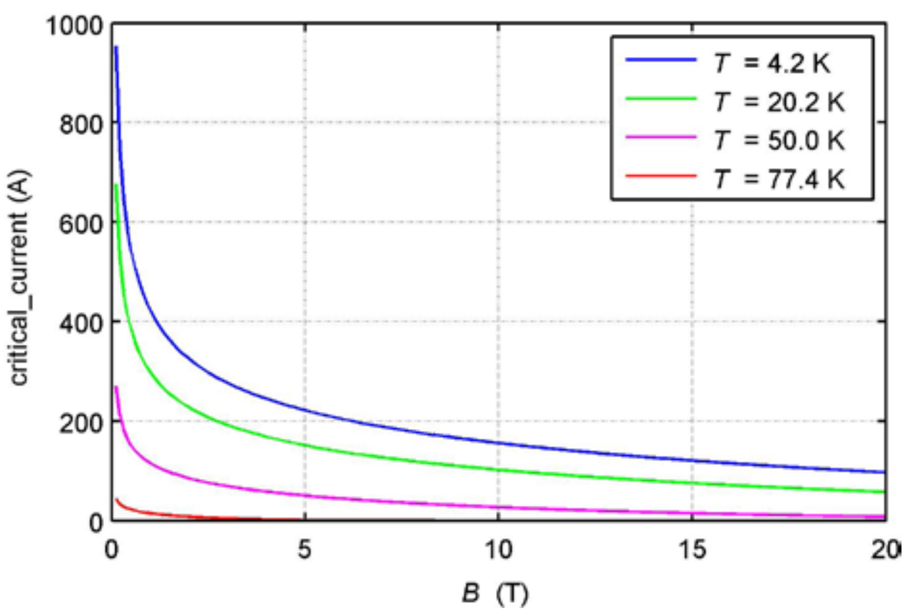


Fig. 6.4: Critical current of a 4 mm wide YBCO coated conductor with the specifications given in Tab. 6.2, calculated by fit-function (3.1).

Resistance per Unit Length

In Fig. 6.5 the resistance per unit length of the single YBCO tape is shown. The values are calculated by equations (4.1) and (4.2) as described in chapter 4.1.1. The resistance drop at the critical temperature $T_c = 92$ K can nicely be seen. The specifications of the tape are given in Tab. 6.2.

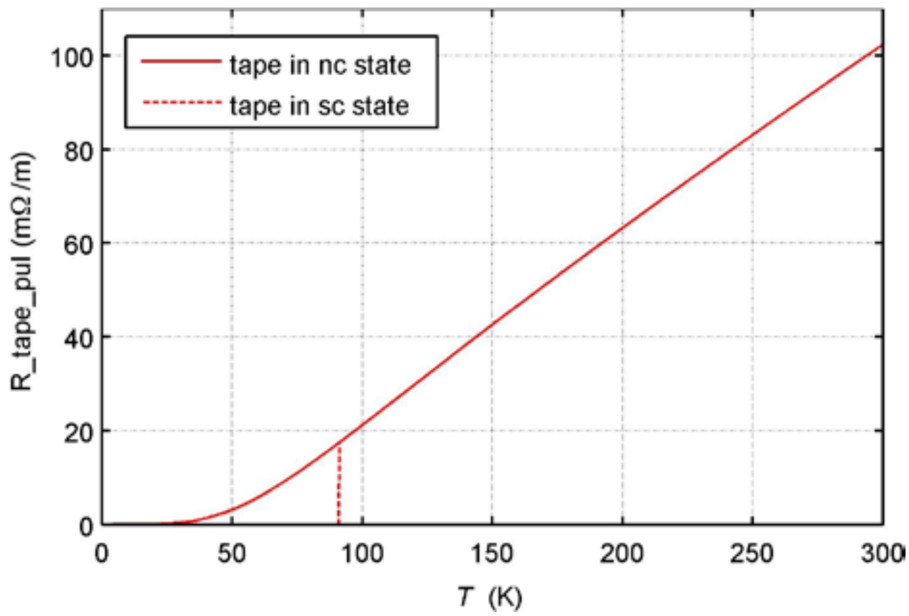


Fig. 6.5: Resistance per unit length of a single YBCO coated conductor with specification given in Tab. 6.2, calculated by equations (4.1) and (4.2). The magnetic field density chosen for this calculation is $B = 0.1$ T.

Selected values of the resistance per unit length of the example calculation shown in Fig. 4.3 are listed in Tab. 6.4.

Tab. 6.4: Selected values of the resistance per unit length of a single YBCO coated conductor with specification given in Tab. 6.2 calculated by equations (4.1) and (4.2). The magnetic field density chosen for this calculation is $B = 0.1$ T.

T (K)	4.2	20.3	77.4	293.2
R'_{nc}	2.393 μΩ/m	97.34 μΩ/m	11.39 mΩ/m	99.75 mΩ/m

Thermal Conductivity

Fig. 6.6 shows the thermal conductivity of the YBCO coated conductor. The values are calculated by equation (5.8) as described in chapter 5.1. The specifications of the tape are listed in Tab. 6.2. The peak of the thermal conductivity at about 20 K is mainly affected by the copper layer. The silver and YBCO layers contribute a miner part because of their low layer thicknesses. The contribution of the hastelloy layer is very low because of its material properties.

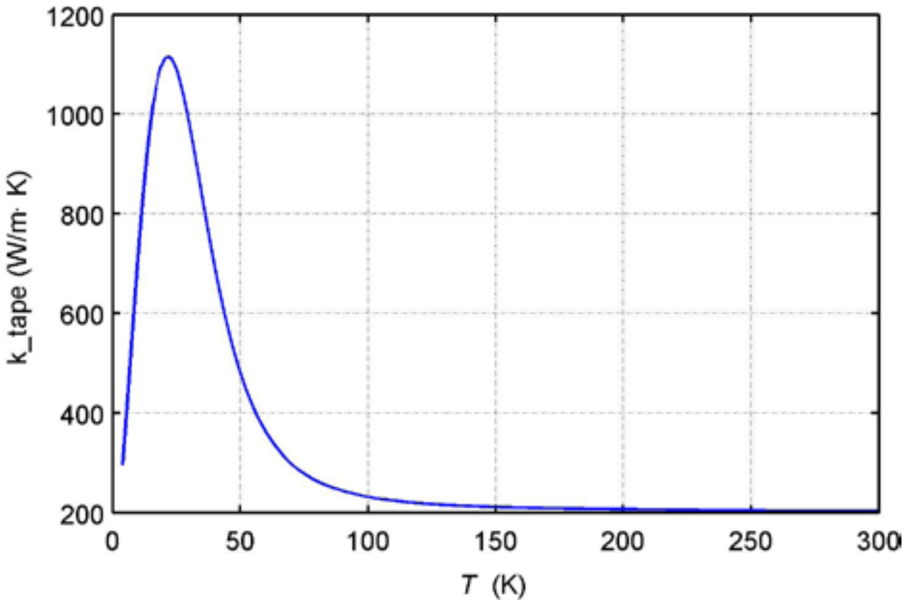


Fig. 6.6: Calculated thermal conductivity of a single YBCO coated conductor with specification given in Tab. 6.2 calculated by equation (5.8).

Selected values of the thermal conductivity of the example tape shown in Fig. 6.6 are listed in Tab. 6.5.

Tab. 6.5: Selected values of thermal conductivity of a single YBCO coated conductor with specification given in Tab. 6.2 calculated by equation (5.8).

T (K)	4.2	20.3	77.4	293.2
k_{tape} (W/m·K)	308.8	1110.3	270.1	202.9

Heat Capacity

Fig. 6.7 shows the heat capacity of a single YBCO coated conductor with specification given in Tab. 6.2 calculated by equation (5.7) as described in chapter 5.1. The heat capacity of YBCO tapes is very low at low temperatures.

In Tab. 6.6 selected values of the heat capacity of the example calculation shown in Fig. 6.7 are listed.

Tab. 6.6: Calculated heat capacity of a single YBCO coated conductor with specification given in Tab. 6.2 calculated by equation (5.7).

T (K)	4.2	20.3	77.4	293.2
C_{tape} (mJ/m·K)	4.03	29.30	585.3	1282.9

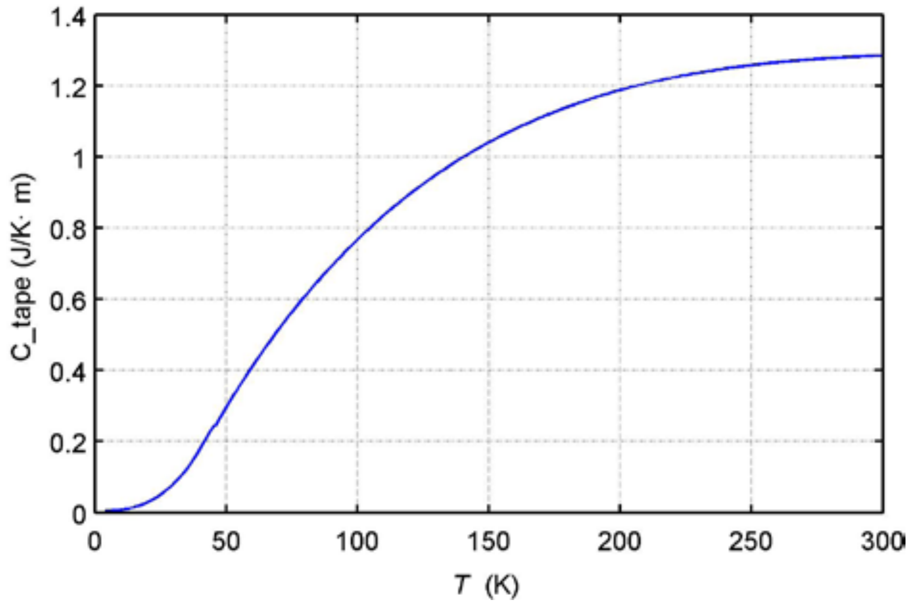


Fig. 6.7: Calculated heat capacity of a single YBCO coated conductor with specification given in Tab. 6.2 calculated by equation (5.7).

Cryostability

Fig. 6.8 example calculation for cryostability of a single YBCO tape with specifications given in Tab. 6.2 in liquid hydrogen (LH_2) and a magnetic field density of $B = 10$ T. Transport current $I_T = 100$ A, critical current $I_c = 101.7$ A

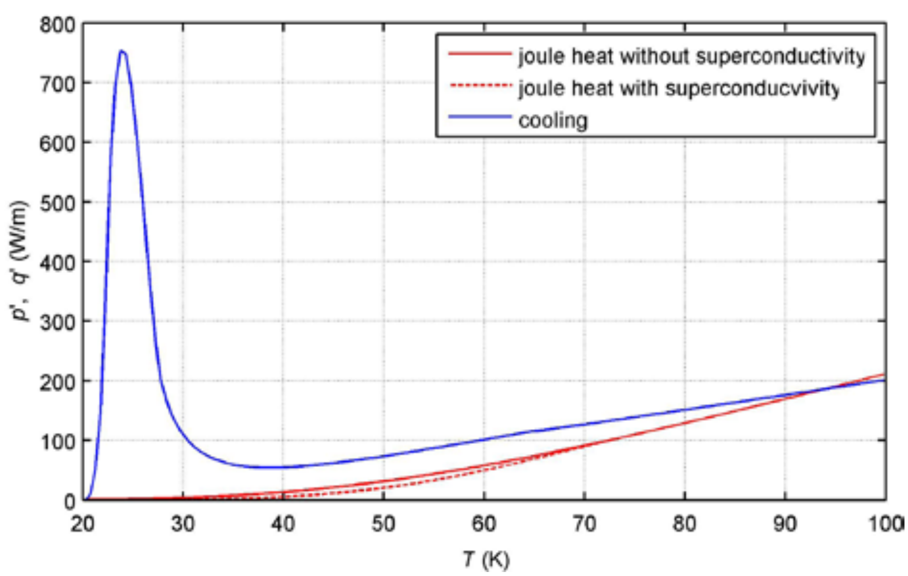


Fig. 6.8: Example calculation for cryostability of a single tape with specificatons given in Tab. 6.2 in liquid Hydrogen in a magnetic field density of $B = 10$ T and a transport current of $I_T = 100$ A.

For this case the Stekly criterion is $\alpha = 0.081$ at recovery point ($T = 38.8$ K). The current sharing Temperature $T_{cs} = 20.9$ K and the critical temperature is $T_c = 75.6$ K.

Minimum Propagation Zone

Also the minimum propagation zone was calculated for the given tape cooled by liquid hydrogen at 20.3 K and operated with a transport current of a transport current of $I_T = 100$ A ($I_c(T = 20.3\text{ K}) = 101.7$ A). The minimum propagation zone was calculated analytically and the results were used for a simulation to prove the analytical calculation.

The minimum propagation zone was calculated by equation (5.12). It is $mpz = 6.34$ mm and the maximum temperature calculated by equation (5.11) is $T_{max} = 281.4$ K. Taking into account the cross sectional area of the tape of $A_{tape} = 0.37\text{ mm}^2$, the estimate for the minimum quench energy is thus $mqe = 1486$ mJ.

The calculated values of the minimum propagation zone and of the maximum temperature were used as initial conditions for a simulation. The temperature run for these initial conditions is shown in Fig. 6.9.

The picture shows, that the tape cools down and recovers after the perturbation. This means, that the minimum quench energy that establishes a minimum propagation zone is even higher than the amount of energy introduced.

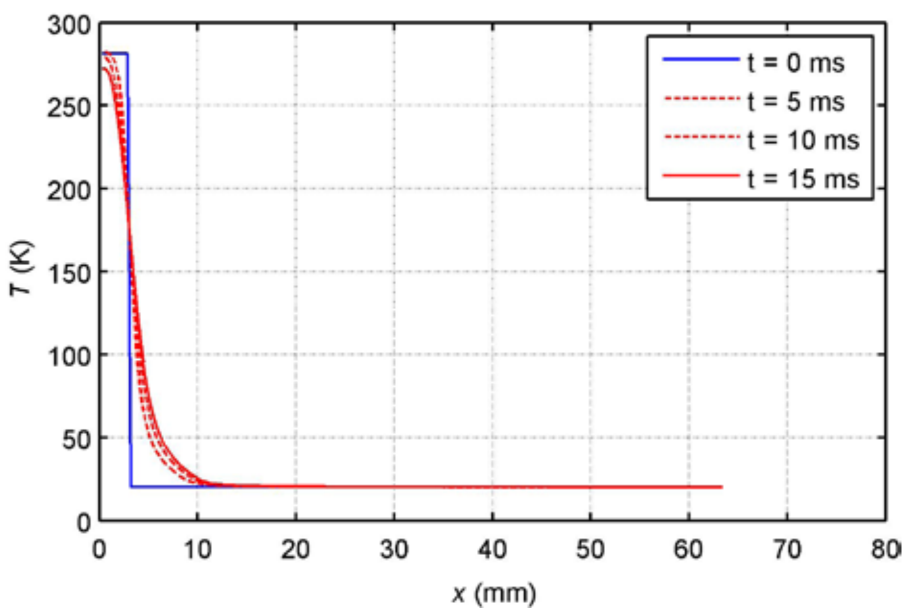


Fig. 6.9: Example calculation for minimum propagation zone of a single YBCO tape with specifications given in Tab. 6.2 in liquid Hydrogen. Blue line: Assumed initial temperature distribution. Red lines: Calculated temperature distribution after five, ten and fifteen ms respectively.

6.3 Parameter Analysis for Heat Dissipation

In this chapter a parameter analysis is described that gives information on the influence of different variables on the dissipated heat in a locally perturbed area of a stack of two parallel soldered YBCO tapes. An inhomogeneous temperature distribution across the cross section of the tape is assumed and therefore the circuit diagram shown in Fig. 5.3 and the equation (5.7) was used for the calculations.

In order to evaluate the influence of multiple parameters on the dissipated power in the faulted area, a parameter analysis was done. Therefore the dissipated energy is calculated for a set of the different parameters. The influence of each parameter is rated by correlation coefficients for the varied parameters and the total power dissipated in the faulted area. In Tab. 6.7 all values of the parameters used for the parameter analysis are given.

Tab. 6.7: Chosen parameters for calculation of dissipated energy in faulted area. The dissipated power is calculated for all combination of these parameters

w_{tape} (mm)	4							
h_{hs} (µm)	50							
h_{ag} (µm)	1.0							
R_j (µΩ)	1							
I_c (A)	100							
h_{so} (µm)	20	50	100					
h_{cu} (µm)	10	20	30					
i_T (A)	100	120	140	160	170	180	190	200
ℓ_1 (m)	10	333	500					
$T_{tape,1}$ (K)	100							
ℓ_f (mm)	0.01	0.1	1	10				
T_{cool} (K)	20	77						

The total length of the cable was assumed to be 1 km and therefore ℓ_1 represents the distance from one end of the cable. The temperature of the perturbed tape was assumed to be 100 K, the temperature of the other parallel tape was assumed to be the temperature of the coolant. The total power P_{tot} dissipated in the faulted area ℓ_f is calculated for every combination of these values.

In Tab. 6.8, the maximum and minimum values of all calculations are listed. It is remarkable, that the dissipated power in the still superconducting tape R_{tape} can be higher than in the faulted tape R_f, even tough it has a lower resistance. But because of its lower resistance the current in this tape is higher and the dissipated power rises with the current squared. For the calculation of these values the Matlab file *calc_current_dist.m* was used.

Tab. 6.8: Maximal and minimal values of total dissipated power P_{tot} in faulted area ℓ_f for set of parameters given in Tab. 6.7. Left: Values and belonging parameters for T_{cool} = 20 K. Right: Values and belonging parameters for T_{cool} = 77 K.

	max(P_{tot})	min(P_{tot})
T_{cool} (K)	**20**	**20**
w_{tape} (mm)	4	4
h_{hs} (µm)	50	50
h_{ag} (µm)	1.5	1.5
R_j (µΩ)	32	32
I_c (A)	100	100
h_{so} (µm)	50	50
h_{cu} (µm)	30	10
i_T (A)	100	200
ℓ_1 (m)	200	10
$T_{tape,1}$ (K)	100	100
ℓ_f (mm)	10	0.1
$R_{tape,1}$ (µΩ)	142.7	0.4
$R_{tape,2}$ (µΩ)	0.007	0.001
$I_{tape,1}$ (A)	0.0	0.4
$I_{tape,2}$ (A)	100.0	199.6
$P_{tape,1}$ (W/m)	0.0	0.0
$P_{tape,2}$ (W/m)	0.0	3.0
P_{tot} (W/m)	0.0	3.0

	max(P_{tot})	min(P_{tot})
T_{cool} (K)	**77**	**77**
w_{tape} (mm)	4	4
h_{hs} (µm)	50	50
h_{ag} (µm)	1.5	1.5
R_j (µΩ)	32	32
I_c (A)	100	100
h_{so} (µm)	20	100
h_{cu} (µm)	30	10
i_T (A)	100	200
ℓ_1 (m)	200	10
$T_{tape,1}$ (K)	100	100
ℓ_f (mm)	1	0.1
$R_{tape,1}$ (µΩ)	14.3	0.4
$R_{tape,2}$ (µΩ)	4.966	0.181
$I_{tape,1}$ (A)	25.8	61.3
$I_{tape,2}$ (A)	74.2	138.7
$P_{tape,1}$ (W/m)	9.5	153.8
$P_{tape,2}$ (W/m)	27.3	348.2
P_{tot} (W/m)	36.8	502.0

There is a wide spread of values of the dissipated power for the different sets of parameters between P_{tot} = 0.0 W and 502 W. To valuate the influence of the different parameters on the rise of dissipated total power, the parameters are rated by their correlation coefficient.

The correlation coefficient generally rates the linear dependency of two variables. Its values are in the range between -1 and +1. Where -1 and +1 means that there is a fully linear (positive or negative) correlation between these two variables. The variables are absolutely not correlated, if the correlation coefficient is 0.

Tab. 6.9: Correlation coefficient for total dissipated power and different parameters at T_{cool} (K) = 20 K (left) and T_{cool} (K) = 77 K (right)

T_{cool} (K) = 20	correlation coefficient
i_T	0.785
ℓ_1	0.000
h_{so}	0.000
h_{cu}	-0.470

T_{cool} (K) = 77 K	correlation coefficient
i_T	0.652
ℓ_1	0.000
h_{so}	0.000
h_{cu}	-0.671

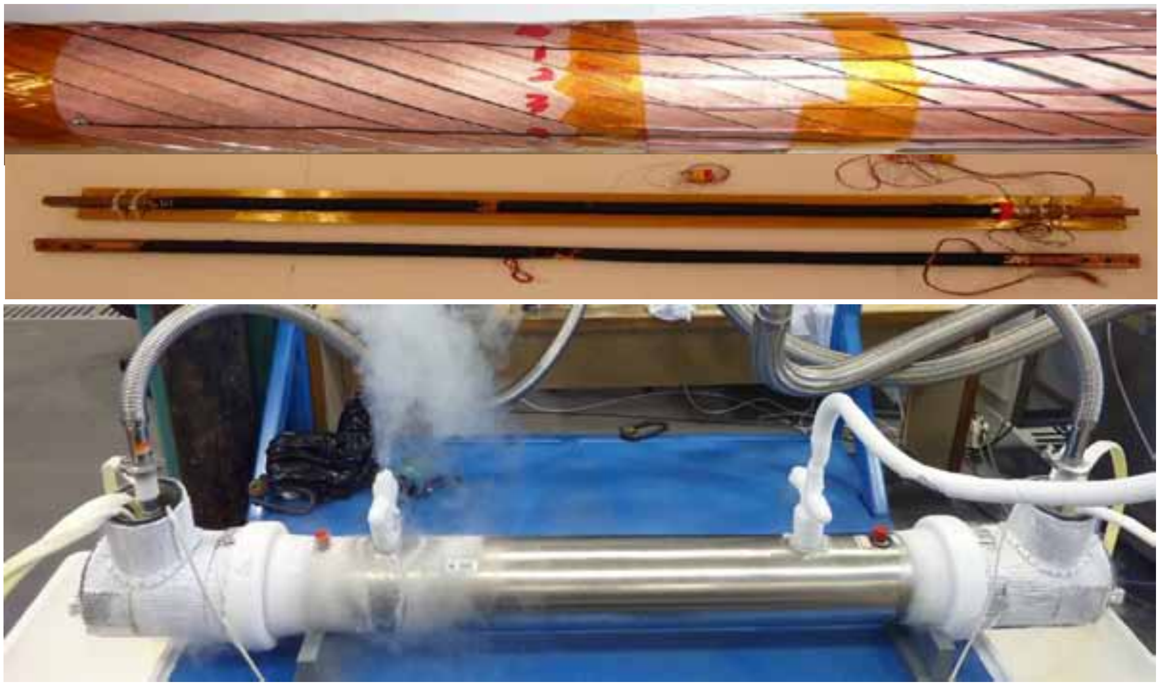
The main result of this analysis is that the transport current i_T and the copper layer thickness of the YBCO coated conductors have the greatest influence on the dissipated power in the faulted area. This is plausible because the voltage across the resistance of the still superconducting tape R_{tape} rises very fast in the range of the critical current of the tape. And the thickness of the copper layer has the main effect on the resistance of both tapes R_f as well as R_{tape} and hence on the dissipated power in the faulted area. The correlation coefficient is negative in this case, because the dissipated power decreases with increasing copper layer thickness. The location as well as the thickness of the solder between the YBCO coated conductors is not correlated with the dissipated power in the faulted area. For these calculations the Matlab file *correlation_coefficient.m* was used the full list of input and output variables is given in Excel file *Correlation Data.xlsx*.

6.4 Stability of a Wound Cable with 12 YBCO Coated Conductors

On June 14th 2011 Sastry Pamidi from Florida State University, Center of Advanced Power Systems, Tallahassee, Florida published measurement results of a high current cable with YBCO coated conductors on the CEC/ICMC in Spokane, Washington. The title of his talk was "*Progress on the Helium Gas Cooled Superconducting DC Cable Project*" (Presentation #: C1OrF-04) [PKK11]. The assembled cable used another cabling concept as proposed from our group. Their cable was measured with DC current in liquid nitrogen and gaseous helium between 65 K and 78 K. Under these conditions the cable was loaded with various currents. The measurements showed that the cable could be operated stable for longer than one minute at up to 1.25 times its critical current in liquid nitrogen at 77 K. It's remarkable that the cable could not be operated stable at critical current in gaseous hydrogen at the same temperature. Here, the cable quenched within one minute.

This observation is a very interesting case example for stability calculations. Therefore this case was theoretically investigated in this report. The dissipated heat and the cooling was calculated and compared for gaseous and liquid cooling using the developed tools. All sample pictures and measurement results related with these experiments are taken from the presentation of S. Pamidi in June 2011.

In Fig. 6.14 the 1m long test samples and the cryostat used for the measurements of Pamidi are shown. The YBCO tapes are wound on a copper support tube. Voltage taps and thermo couples are soldered on the tapes for measurement. The voltage was measured across each tape over a distance of 75.5 cm.



a)

b)

c)

Fig. 6.10: a) Wound cable with voltage taps (left side) and thermo couples (middle).
b) Entire cables (two samples) with joints and measuring lines
c) Cryostat for test samples
(Source: [PKK11])

In Fig. 6.11 an overcurrent measurement of a sample cooled with liquid nitrogen at 77 K is shown. The diagram shows the current and the temperature run for a measurement of 3 minutes. After about 30 s ramping, the current is kept constant for one minute at critical current (890 A). Then the current was ramped further to 1.25 times its critical current (1218 A) and kept constant for another minute. Afterwards the current was switched off. During this measurement there was not voltage or temperature rise observed. The cable operated stable.

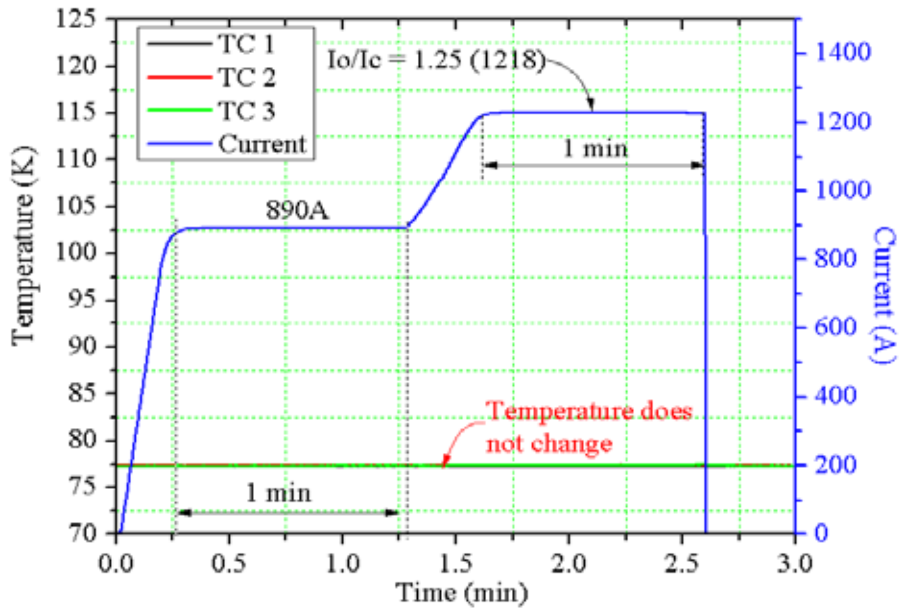


Fig. 6.11: Run of total current and temperature during overcurrent measurement of a sample in liquid nitrogen at 77 K. (Source: [PKK11])

Fig. 6.12 shows the same measurement procedure with the same sample, but cooled with gaseous helium at the same temperature (78 K). In this measurement, the current was ramped up to almost 700 A within about 10 s. Within the next minute the current was increased slower to critical current (850 A) and afterwards the current was kept constant. The temperature measurement shows that the temperature starts to increase already about 15 s after begin of the measurement. The current was switched off after 1.5 min, because of a quench on the cable.

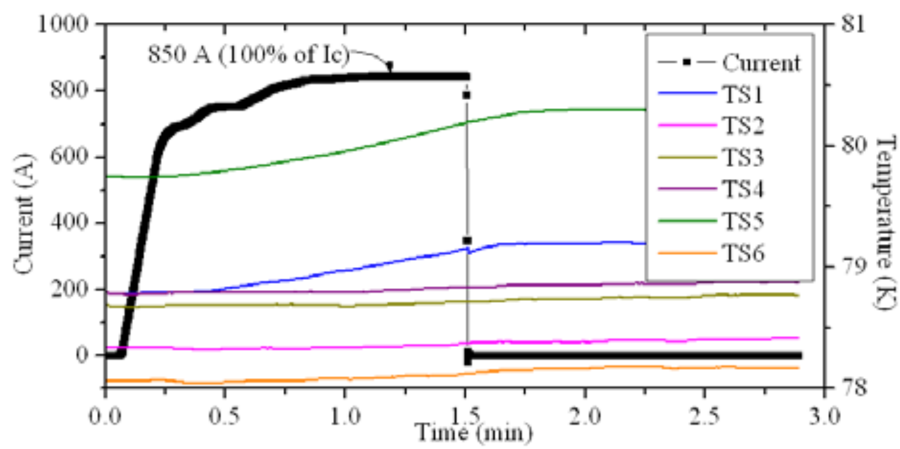


Fig. 6.12: Run of total current and temperature during overcurrent measurement of a sample in liquid nitrogen at 77 K. (Source: [PKK11])

The quench can be seen in the voltage rise in some of the tapes as shown in the voltage measurement in Fig. 6.13. It is interesting that the voltage decreases in some tapes and increases in others. The reason for this is most likely a current redistribution between the tapes caused by a quench outside of the voltage taps closer to the joints. The resistance increases in these section in some tapes and causes a current redistribution to the other tapes. The current decreases in the tapes with decreasing voltage and increases in the tapes with increasing voltage. With increasing current in the other tapes there appears another quench between the voltage tapes that can causes the voltage rise. So there are two hot spots on the cable. One outside the voltage taps on the tapes with decreasing voltage in the end and another one between the voltage taps in the tapes with increasing voltage in the end.

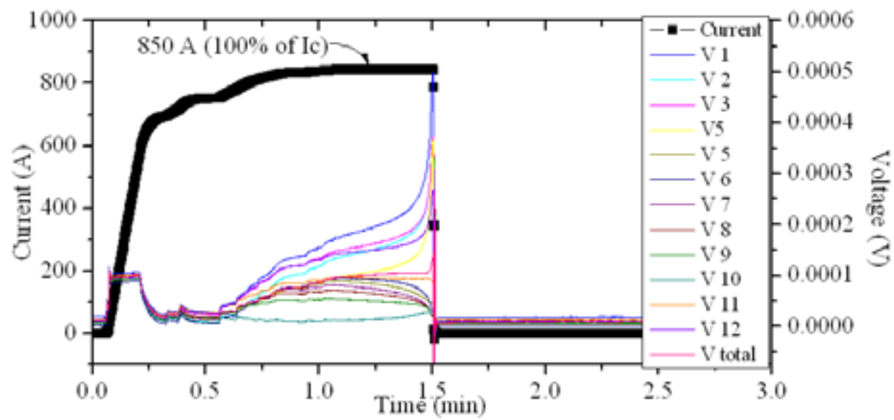


Fig. 6.13: Run of total current and tape voltages during overcurrent measurement of a sample in gaseous helium at 78 K. The voltage of each tape was measured over a length of 75.5 cm. The cable is cooled with gaseous helium. (Source: [PKK11])

These measurements show that this YBCO cable can be operated at currents above critical current in liquid nitrogen, but not even at critical current in gaseous helium at the same temperature. In the following this behavior is investigated theoretically. Therefore the heat dissipation and the cooling were calculated for both cases, gaseous and liquid cooling.

In Fig. 6.14 the cross section of the cable, assumed for the stability calculations is shown. The 12 YBCO tapes are wound on a copper tube. This cable assembly is immersed in a round cryostat. For the calculations is assumed, that the conductors are cooled on the outer surface of the cable only. In order to use the developed Matlab-functions on this cable design, the calculations were done exemplarily for a single YBCO tape only. For the calculations was assumed, that the current shares equally between the tapes.

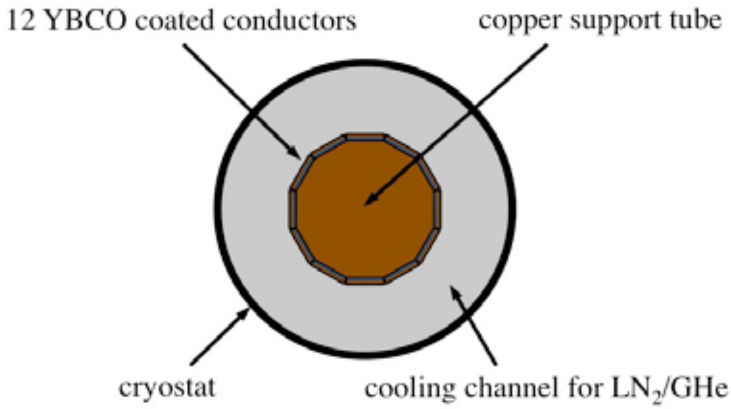


Fig. 6.14: Cross section of assembled cable in cryostat. 12 4 mm wide YBCO coated conductors are wound on a cupper support tube. The cable is immersed in a cryostat and is cooled by gaseous helium at 50 K, unpressurized with forced flow.

The specifications of the YBCO tapes used for the calculations are given in Tab. 6.10. The tapes are 4 mm wide and have a 50 µm thick Hastelloy substrate and an additional 20 µm thick cupper stabilization layer on each side. The average critical current of the tapes (determined during the measurements) was 70.8 A/tape at a temperature of 78 K. It was mentioned in the presentation, that the critical current of the different tapes varied in the range of +/- 10 % of the average value.

Tab. 6.10: Width and layer thicknesses of YBCO coated conductors for calculation of heat dissipation.

w_{tape} (mm)	4.1
h_{hs} (µm)	50.0
h_{ybco} (µm)	1.706
h_{ag} (µm)	1.0
h_{cu} (µm)	20.0
I_c(B = 0.1 T, 78 K) (A/tape)	70.8

In Fig. 6.15 and Fig. 6.16 the calculated heat dissipation (per tape and per unit length) for a constant current of 70.8 A/tape and the cooling (per tape and per unit length) is shown for cooling with gaseous helium at 78 K and liquid nitrogen at 77.4 K. As the cable is operated at critical current, the current sharing temperature is (in both cases) T_{cs} = 78 K and the critical Temperature is T_c = 91 K.

Gas Cooling

In case of gas cooling, the equilibrium of heat dissipation and cooling is at about 78.8 K. The cooling power of the gas is in the range of 205 mW/m·K very low and above the equilibrium temperature the heat dissipation rises strongly while the cooling rises only very little. Thus the temperature margin for cryostability is with below 1 K very low.

The minimum propagation zone is 2.3 mm and the minimum quench energy is 2.8 mJ per tape (with a maximum Temperature of T_{max} = 79.6 K). The minimum quench energy of the tapes is very high in comparison to magnet applications with low temperature superconductors.

During the measurements, one of the tapes showed a resistive part. The current leads seem to be quite short and the cryostat seems to be poorly insulated as both ends are icy. Thus it is very likely that there is heat introduced in the cable by all these effects (see the openings on the top side of the tube). This explains the fast quench of the conductor. The margin for cryostability is very low and there is heat introduced al the time.

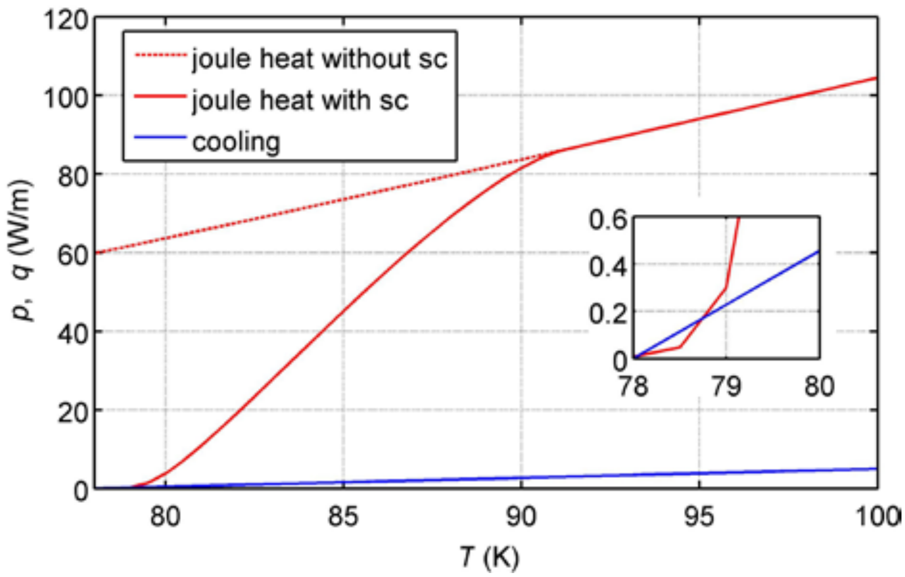


Fig. 6.15: Calculation of heat dissipation and cooling for a wound cable with specifications given in Fig. 6.14 and Tab. 6.10. Values given per tape and per unit length, one side of the tapes is considered to be cooled only. Coolant is gaseous helium at 78 K and transport current is critical current with $I = I_c$ = 70.8 A/tape. Current sharing temperature T_{cs} = 78 K and critical temperature T_c = 91 K.

On top of this it was mentioned above that the critical current of 70.8 A/tape at a temperature of 78 K was just the average critical current of the tapes. Their critical current varied by about +/- 10 %. For the calculation of the heat dissipation the critical current was assumed to be 78.8 A/tape. In this case the heat dissipation of a tape is 7.08 mW/m per tape. In the sections of the tapes where the critical current is reduced by 5 %, the heat dissipation is 33 mW/m and in the sections where the critical current is reduced by 10 %, the heat dissipation is already 155 mW/m at the same transport current. The cooling power is in the range of 228 mW/m·K. As both the calculation of the cooling as well as the calculation of the heating are estimates only it can be said that both are in the same range under these conditions. Every further heat introduction (as for example by heat flow through the current leads) would lead to a quench.

Liquid Cooling

In case of liquid cooling, the equilibrium of heat dissipation and cooling is at 97.9 K. Even though the current sharing temperature is $T_{cs} = 78$ K and the critical temperature is $T_c = 91$ K, the cooling power is still higher than the dissipated heat in this temperature range and thus the temperature margin for cryostability is with $\Delta T = 20.5$ K very high.

The minimum propagation zone is 8.1 mm and the minimum quench energy is 1314 J per tape in this case (with a maximum Temperature of $T_{max} = 232$ K).

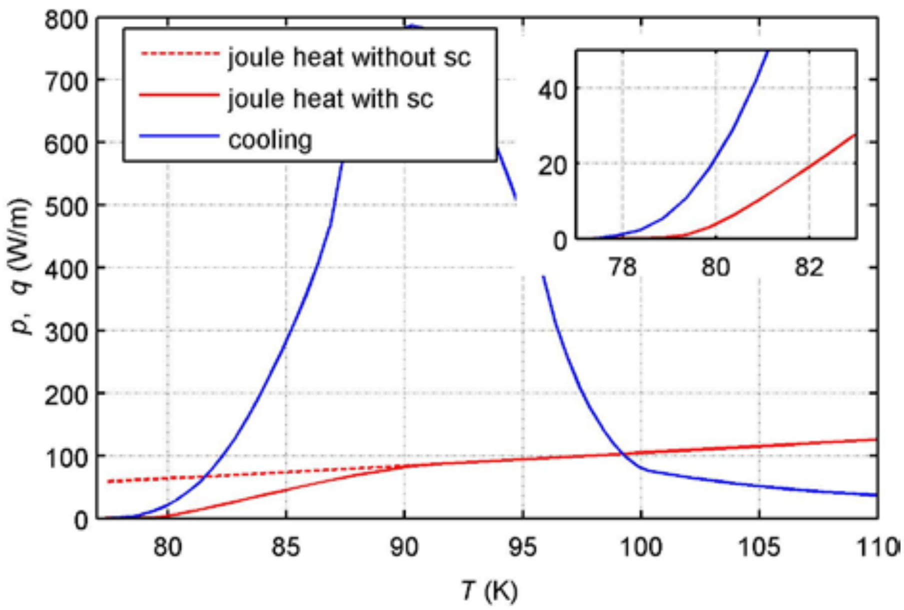


Fig. 6.16: Calculation of heat dissipation and cooling for a wound cable with specifications given in Fig. 6.14 and Tab. 6.10. Values given per tape and per unit length, one side of the tapes is considered to be cooled only. Coolant is liquid nitrogen at 77.36 K and transport current is critical current with $I = I_c = 70.8$ A/tape. Current sharing temperature $T_{cs} = 78$ K and critical temperature $T_c = 91$ K.

As mentioned shown Fig. 6.11, the cable was also operated stable with a higher transport current of 1.25 times critical current in liquid nitrogen (1218 A). This case example was also calculated. The results are shown in Fig. 6.17. As the transport current in the cable is above critical current, there is heat dissipated already at cooling temperature $T = 77.4$ K. The equilibrium between cooling and heating is at a temperature of 81.3 K. This equilibrium is stable as the cooling rises above the heating when the temperature rises and vise versa. Thus the temperature remains stable at this point and the heat dissipation is in the range of 58 W/m.

Above this equilibrium condition, the cooling is much higher than the heating up to a temperature of about 97.5 K. So if the cooling power is not limited for some reason, there is still a stability margin of 16.2 K.

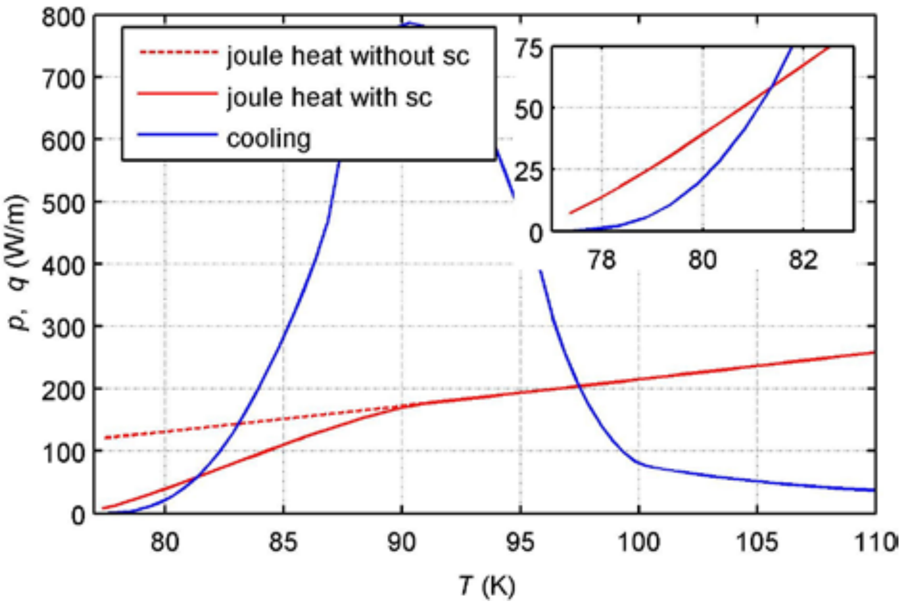


Fig. 6.17: Calculation of heat dissipation and cooling for a wound cable with specifications given in Fig. 6.14 and Tab. 6.10. Values given per tape and per unit length, one side of the tapes is considered to be cooled only. Coolant is liquid nitrogen at 77.36 K and transport current is 1.25 times critical current with $I = 101.5$ A/tape and $I_c = 70.8$ A/tape. Critical temperature is $T_c = 91$ K.

Discussion

The calculations proof that the quench of the cable when it was operated in gaseous helium was caused by a very low stability margin and insufficient cooling. This is a very important information for the cable design. YBCO coated conductors need very high stability margins when they are operated with gaseous coolants. The measurements show that even the mounting on a copper support tube and a copper stabilization layer of 20 µm doesn't prevent a quench at critical current. The cooling power of gas is just too low.

For stable operation in gaseous coolants must be ensured that the transport current is much lower than the critical current $I_T << I_c$ and that there is no heat introduced through the current leads or any other potential heat source. This includes that the gas cooling must be effective on the whole cable surface to avoid weak spots.

In case of liquid cooling (in this case example with liquid nitrogen) the stability margin may be much lower as the cooling power is so high that the cable can even be operated stable at 1.25 times its critical current even though there is heat dissipation all the time.

6.5 Stability of a Stack with 32 YBCO Coated Conductors

Our group proposed the new cabling concept for high current cables with twisted stacked YBCO coated conductors and published already first successful measurement results in [TCB12], [TMB11]. In the context of this report, the stability of the measured example cable with 32-stacked conductors operated in different boundary conditions is investigated. Example calculations are made for the cable to be operated as transmission line and as conductor in a 10-T magnet cooled with different coolants under different conditions as follows:

- Transmission line: Operated in self field and cooled with LN_2 at 77.4 K, I_T = 70 % $I_c(T)$
- Magnet: Operated in external field of 10-T and cooled with GHe at 50.0 K, I_T = 70 % $I_c(T)$
- Magnet: Operated in external field of 10-T and cooled with LH_2 at 20.3 K, I_T = 70 % $I_c(T)$
- Magnet: Operated in external field of 10-T and cooled with LHe at 4.2 K, I_T = 70 % $I_c(T)$

This example calculation takes into account the soldered stack only. As mentioned in the introduction of this report, a copper support structure is not taken into account (see Fig. 2.1). In Tab. 6.11 the specifications of the cable and the conductors used are given.

Tab. 6.11: Specifications of cable and conductors used for stability calculations and published measurements [TMB11].

w_{tape} (mm)	4.1	number of tapes	32
h_{hs} (µm)	50.0	solder between tapes	SnPb6040
h_{ybco} (µm)	1.067 (*0 T*) / 0.7195 (*10 T*)	h_{so} (µm)	20
h_{ag} (µm)	1.0	no copper support structure	
h_{cu} (µm)	20.0		
$I_c(B$ = 0 T, 77 K) (A/tape)	85		

At first, the critical current of the cable was calculated for the different operating conditions (with Matlab function *critical-current.m*). Two different approaches were used to calibrate the calculated value of the critical current to the measurement results. The used fit-function is valid for the calculation of the critical current of a single conductor in a homogeneous external field between 0.1 T and 20 T only. Thus, the influence of the stacking of the tapes on the critical current of each tape cannot be calculated by this fit-function.

In order to get a rough estimate of the magnetic self field of the stack at critical current, the stack can be assumed to be a round conductor with homogeneous current density across its cross section. With a radius of the stack of 2 mm, the maximum magnetic flux density at the surface of the conductor is in the range of B = 153 mT (calculated by Ampere's law).

To estimate the temperature dependence of the stack's critical current in its **self field at 77 K**, the magnetic field density was set to 0.1 T for the calculations, and the YBCO-layer thickness was fit to meet the measured critical current of the 32-tapes stack in self field at 77 K $I_c(T = 77\text{ K}, B = 0\text{ T}) = 1530\text{ A}$ as mentioned in the publication. Thus the YBCO-layer thickness was set to $h_{ybco} = 1.067\ \mu\text{m}$ for this calculation.

For the calculation of the critical current of the stack in a **magnetic field of 10 T at various temperatures**, the self field of the stack can just be neglected, as it is more than one magnitude lower than the external field. In this case, the critical current was calibrated in the calculation to the value of a single tape of $I_c(T = 77\text{ K}, B = 0\text{ T}) = 85$ A/tape as given in the specification of the tapes. Thus the YBCO-layer thickness was set to $h_{ybco} = 0.7195\ \mu\text{m}$ for these calculation. In Fig. 6.18, the critical current of the 32-tapes stack is shown in dependence of the temperature (left) and magnetic field (right).

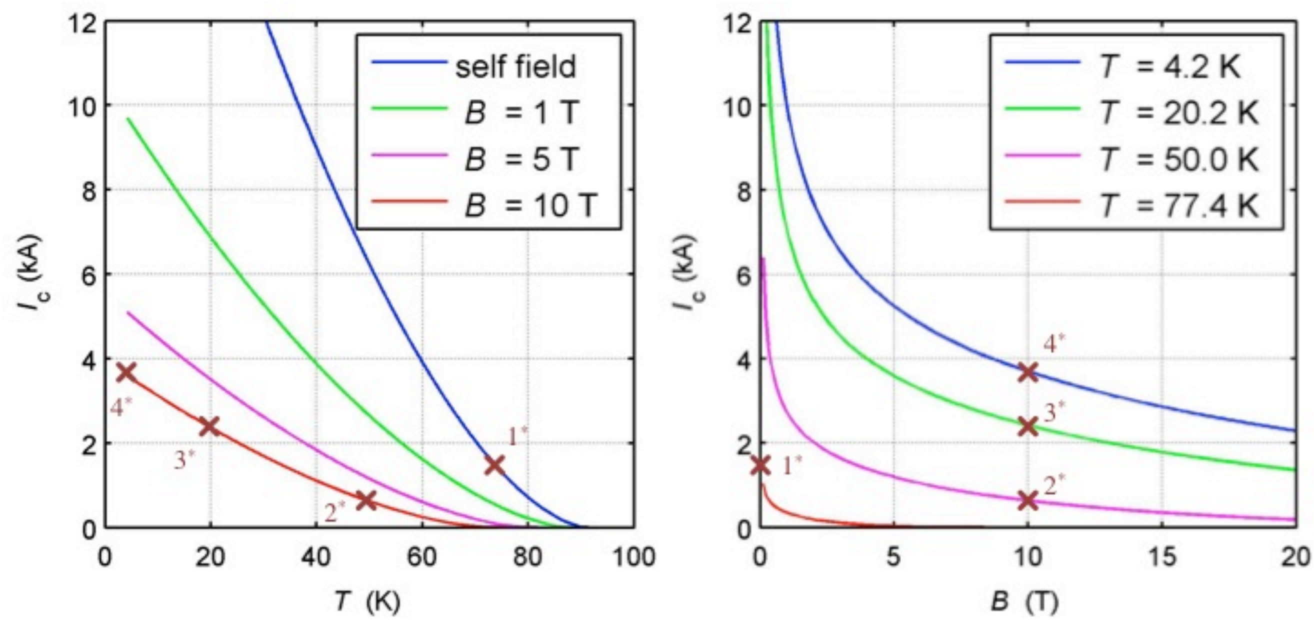


Fig. 6.18: Critical Current of the 32-tapes stack with specifications as given in Tab. 6.11 in dependence of the temperature (left) and magnetic field (right). Example calculations made for the highlighted operation conditions: *1** high current cable cooled with liquid nitrogen; *2**, *3** and *4** 10-T Magnet cooled with gaseous helium, liquid hydrogen and liquid helium respectively.

These diagrams show the potential of the cable to be operated superconducting in different conditions. The conditions, the stability calculations are made for are highlighted. At liquid nitrogen temperature, the magnetic field tolerance is very low, so that it makes sense to use the cable in self field or in application with very low magnetic fields only. At lower temperatures the current carrying capability is generally very high and remains quite high even in very high fields up to 20 T.

In Fig. 6.19 to Fig. 6.22 the calculated joule heat of the cable and the cooling power operated at different temperatures with different coolants are given for a wide temperature range.

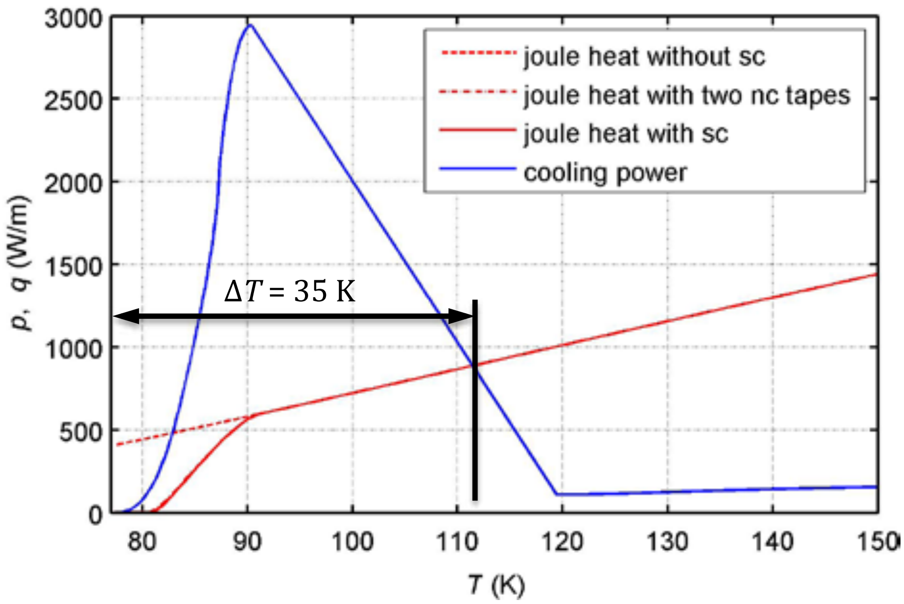


Fig. 6.19: Joule heat and the cooling power of the 32-tapes cable with specifications given in Tab. 6.11 when operated in LN_2 at 77.4 K. The margin for cryostability is approximately $\Delta T = 35$ K, and transport current $I_T = 70\ \% \cdot I_c(T) = 1.53$ kA

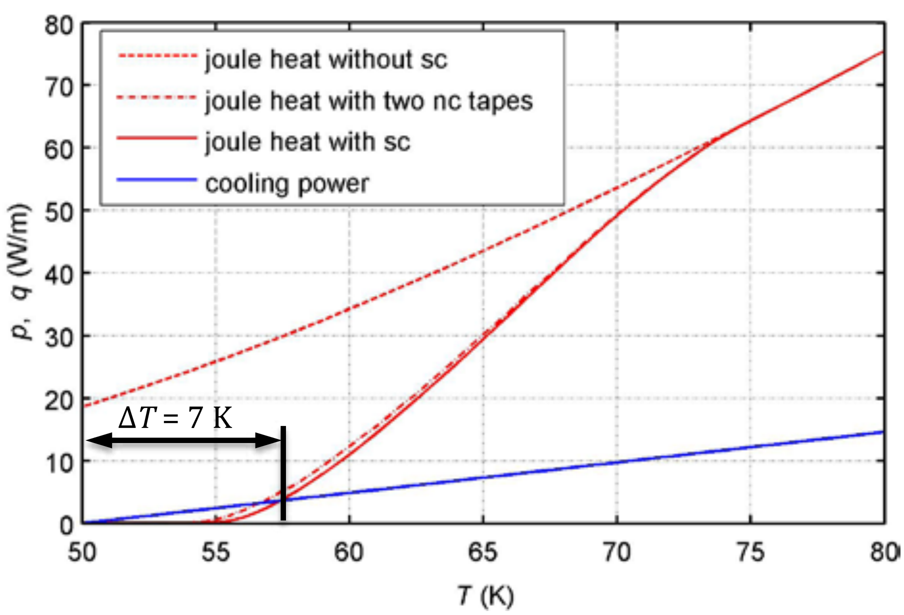


Fig. 6.20: Joule heat and the cooling power of the 32-tapes cable with specifications given in Tab. 6.11 when operated in GHe at 50 K. The margin for cryostability is $\Delta T = 7$ K and transport current $I_T = 70\ \% \cdot I_c(T) = 0.663$ kA

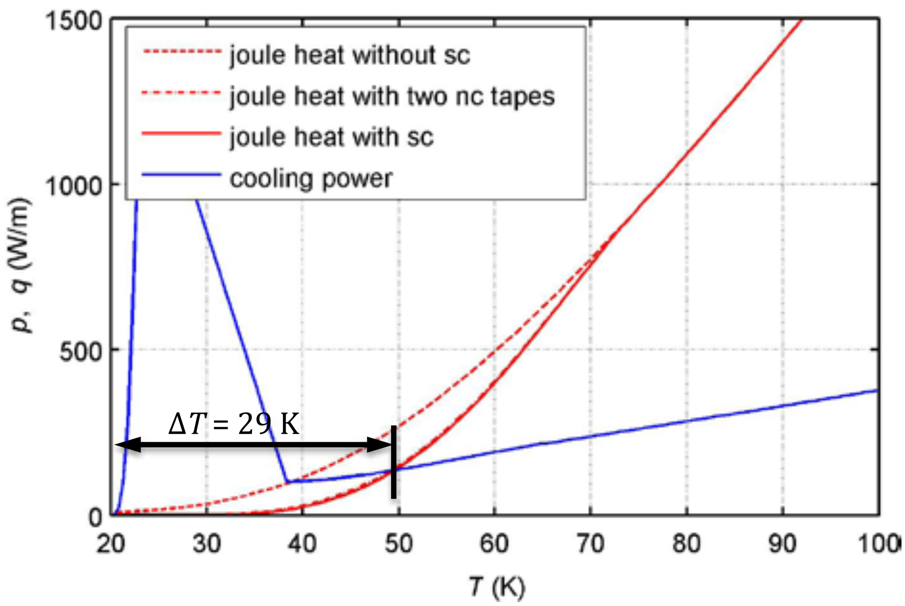


Fig. 6.21: Joule heat and the cooling power of the 32-tapes cable with specifications given in Tab. 6.11 when operated in LH_2 at 22.3 K. The margin for cryostability is $\Delta T = 29$ K and transport current $I_T = 70\ \% \cdot I_c(T) = 2.40$ kA kA

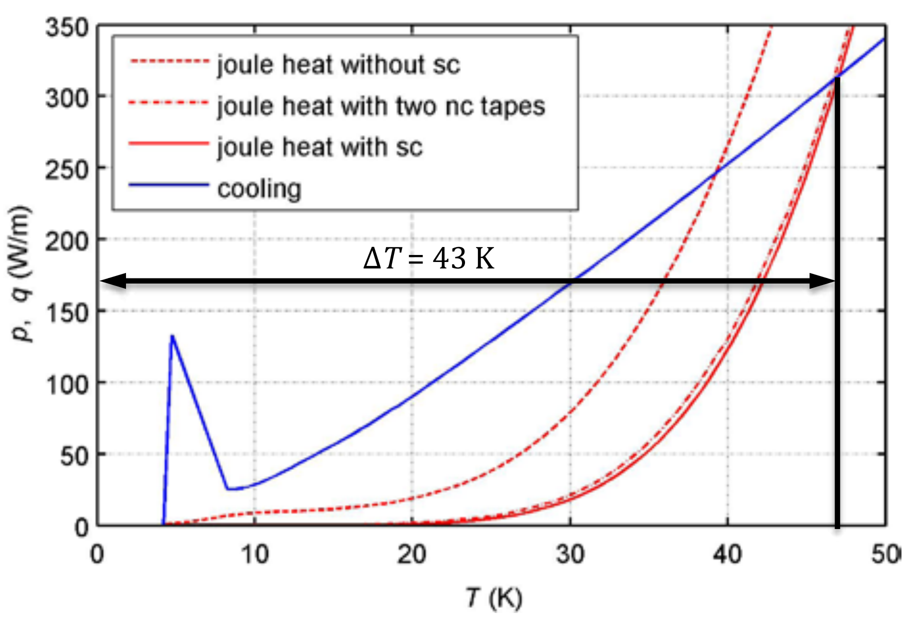


Fig. 6.22: Joule heat and the cooling power of the 32-tapes cable with specifications given in Tab. 6.11 when operated in LHe 4.2 K. The margin for cryostability is $\Delta T = 43$ K and Transport current $I_T = 70\ \% \cdot I_c(T) = 3.67$ kA

The blue lines show the cooling power, the red dashed lines show the joule heat in case there would be no superconductivity (maximum dissipated power), the red continued lines show the joule heat taking into account superconductivity in all tapes (minimum dissipated power) and the dotted-dashed lines show the joule heat in case that two out of the 32 tapes are not superconducting and the other 30 tapes are superconducting. In each diagram, the temperature margin for cryostability ΔT is highlighted. Thus the heat dissipation and cooling can be seen in great detail depending on the temperature. The most interesting values are listed for comparison in Tab. 6.12.

Tab. 6.12: Summary of operating conditions and corresponding values for cable stability.

	High Current Cable	**10-T Magnet**	**10-T Magnet**	**10-T Magnet**
Cooling	LN_2 @ 77.4 K	GHe @ 50.0 K	LH_2 @ 20.3 K	LHe @ 4.2 K
Critical current I_c	1.53 kA (48 A/tape)	0.663 kA (18 A/tape)	2.40 kA (75 A/tape)	3.67 kA / (115 A/tape)
Transport current I_T (70 % of I_c)	1071 A	432 A	1680 A	2571 A
Current density j	89 A/mm^2	36 A/mm^2	140 A/mm^2	214 A/mm^2
Temperature margin for cryostability ΔT	~35 K	7.5 K	29 K	43 K
Corresponding energy introduction ΔE	~54 mJ/mm^3	6.4 mJ/mm^3	10.1 mJ/mm^3	9.0 mJ/mm^3
Temp. margin to current sharing temperature ΔT_{cs}	2.7 K	4.6 K	10.5 K	13.8 K
Corresponding energy introduction ΔE	4.1 mJ/mm^3	3.8 mJ/mm^3	1.6 mJ/mm^3	0.45 mJ/mm^3

High Current cable

The cable has with I_c = 1.07 kA and with an engineering current density of 89 A/mm^2 (referred to the cross sectional area of the stack) a high current carrying capability with a low weight and volume. At a transport current of 70 % of critical current, the safety margin to the current sharing temperature is with 2.7 K quite low, but this temperature difference corresponds to a heat introduction of 4.1 mJ/mm^3, which is a sufficient value for stable operation. The temperature margin for cryostability is with about 35 K very high in comparison to applications of low temperature superconductors. In case that two out of the 32 tapes are defect and completely normal conducting in the same section, the critical current is reduced by 6.25 %. But the heat dissipation is only slightly higher in this case. The joule heat of two normal conducting tapes is almost the same as when all tapes were superconducting (compare Fig. 6.19).

10-T Magnet

The cable shows a potential for an application in a 10-T magnet with coolants in a temperature range from 4.2 K to 50 K. The current carrying capability (at 70 % of critical current) ranges from 432 A at 50 K over 1.68 kA at liquid hydrogen temperature to 2.57 kA at liquid helium temperature.

Thus the engineering current density is with 36 A/mm^2 (referred to the cross sectional area of the stack) at 50 K relatively low for superconducting magnets but increases with decreasing cooling temperature to 140 A/mm^2 at 20 K and 214 A/mm^2 at 4.2 K. In all cases the safety margin from operating temperature to current sharing temperature is with 4.6 K to 13.8 K quite high and the safety margin for cryostability is with 4.6 K to 43 K also quite high. In case that two out of the 32 tapes are defect and completely normal conducting in the same section, the critical current is reduced by 6.25 %. But the heat dissipation is only slightly higher compared to the case that all tapes are superconducting. The joule heat of two normal conducting tapes is almost the same as when all tapes were superconducting (compare Fig. 6.20 to Fig. 6.22).

The proposed cable shows a very high potential for its use as superconducting high current cable operated with liquid nitrogen and as conductor in a 10 T magnet with a varying current carrying capability depending on the cooling temperature. At liquid Helium temperature low temperature superconductors are most likely a cheaper solution and therefore a better choice for applications in this field range. But here the cable shows a potential for applications in higher magnetic fields. Most promising in a field range of 10 T is an application of this cable in a temperature range between 20 and 50 K. Here, gaseous helium offers flexibility in the cooling temperature up to 50 K, but very low cooling power. Generally it can be said that Liquid Hydrogen offers very good cooling properties for superconducting applications with YBCO coated conductors.

This example calculation is to be published as “Potential and Stability of a High Current Cable with 32 Twisted Stacked YBCO Coated Conductors” in IOPScience: Superconductor Science and Technology in 2012.

7 Manual for Developed Computation Tools

This chapter contains a manual for the use of the different Matlab m-files that were developed in the context of this work. The used equations and calculation strategies used in these files are described in the foregoing chapters.

All files have the same structure. Their function head of contains a short description of what this function does, a list of all input and output variables including their symbol, name and unit and if the function uses data or an equation published elsewhere, the references are listed as well. The function head is followed by the actual calculation. The functions can be called in the Matlab command window with their function name (filename without the ".m" in the end) followed by their input variables in brackets and separated by commas "," like "R_tape_pul(I_tape, w_tape, h_hs, h_ag, h_cu, T, I_c)" for example. The variables can be inserted as number alternatively. The functions respond with the specified output variables. Some functions open a figure window and/or save the results in a file in a specified folder.

7.1 Calculation of Material Properties

Heat Capacity, Thermal Conductivity and Electrical Resistivity

All implemented fit-functions for the calculation of the different material properties are listed in Tab. 7.1 with their function name. All functions have the same structure and have only the temperature as input variable and the according material property as output variable. Thus the function can be called with a single temperature value and responds with the corresponding single value of the material property as output. The units of the input variable (temperature) and output variable (material property) are given in the function head.

Tab. 7.1: List of Matlab m-file function names for the calculation of material properties.

	Heat capacity	Thermal Conductivity	Electrical Resistivity
Silver RRR = 100	*C_ag.m*	*k_ag.m*	*rho_ag.m*
Copper RRR = 100	*C_cu.m*	*k_cu.m*	*rho_cu.m*
Hastelloy C96	*C_hs.m*	*k_hs.m*	*rho_hs.m*
Solder SnPb6040	*C_so.m*	*k_so.m*	*rho_so.m*
YBCO	*C_ybco.m*	-	-
Helium	-	*k_he.m*	-
Hydrogen	-	*k_h2.m*	-

All fit-functions are valid in the temperature range between 4.2 K and 300 K only. The fit-functions used for the calculation as well as diagrams of the calculated material properties and values at typical temperatures are given in 3.2 to 3.4. The program code is given in Appendix A.2 to A.4.

Critical Current

The calculation of the critical current is implemented in Matlab function *critical_current.m*. The function can be called for a set of input variables which are the magnetic field density *B* (field density perpendicular to the tape plane), the temperature of the YBCO *T*, the layer thickness of the YBCO *h_ybco* and the tape width *w_tape*. Output is a single value of the critical current of a YBCO layer in the given boundary conditions. As this value is determined by a fit-function, the YBCO layer thickness is not necessarily the thickness of the YBCO layer, but has to be varied in order to calibrate the calculation to the specified value of the tape the calculation has to be done for.

The calculation is valid in a temperature range from 4.2 to 92 K and in a magnetic field range from 0.1 to 20 T. It is important that these limitations are taken into account for the calibration.

Example for calibration of calculation: The critical current of a 4 mm wide tape shall be calculated in a magnetic field of 10 T at a temperature of 20 K. The critical current of the tape is 100 A at a temperature of 77 K in self field. In order to calibrate the calculation to the given values, it can be assumed that the critical current in a 100 mT field is 38 % of its value in self field at 77 K (use measurement results of the given tape). Thus in this case example the thickness of the YBCO layer has to be chosen so that the calculated critical current at 77 K and 0.1 T fits 38 A. (The exact value of the reduction of the critical current in a magnetic field can be determined by a comparison of available measurement results).

Cooling Power

All implemented fit-functions for the calculation of the cooling properties of different coolants are implemented in Matlab function *cooling.m*. All implemented coolants are listed in Tab. 7.2 with their

Symbol and their given temperature (liquid coolants at boiling temperature) or valid temperature range (gaseous coolants).

Tab. 7.2: List of fit-functions to calculate the cooling properties of different coolants implemented in *cooling.m*.

Liquid Coolants	Symbol	Temperature	Gaseous Coolants	Symbol	Temperature
Boiling Helium	LHe	4.2 K	Helium	GHe	4.2 – 300 K
Boiling Hydrogen	LH2	20.3 K	Hydrogen	GH2	4.2 – 300 K
Boiling Nitrogen	LN2	77.4 K			

The function can be called for a set of input variables. The einput variables are the temperature of the cooled surface *T*, the temperature of the coolant *T_cool* (the specified value has an effect for gaseous coolants only), the symbol of the coolant (for example 'LHe') and the cooled perimeter. The variable cooled_perimeter has an effect for gaseous coolants only. It is the perimeter of the to be cooled tube and has an influence on the heat transfer per unit surface (compare calculation of gas cooling in chapter 3.5.1). The output variable is the cooling power per unit surface. The lower limit of the valid range for calculations is the cooling temperature (boiling temperature for liquid coolants) and the upper limit is generally 300 K, as all fit-functions for material properties are limited to room temperature.

7.2 Calculation of Properties of a Single YBCO-Coated Conductors and Stacks

Properties of Single YBCO Coated Conductors

There are three files that allow the calculation of the properties of a single YBCO coated conductor. These are the calculation of the heat capacity of a tape by Matlab function *C_tape.m*, the calculation of the thermal conductivity of a single tape by Matlab function *k_tape.m* and the calculation of the resistance load per unit length of a single tape *R_tape_pul.m*. All three functions can be called with a set of input variables that can be found in the function head and give a single value per unit length back as output variable.

The approach for the calculation of the tape resistance load is described in chapter 4.1.1, the approach for the calculation of the heat capacity is described in chapter 5.1.2 and the approach for the calculation of the thermal conductivity is described in chapter 5.1.3. Each function calls other functions listed in chapter 7.1 to calculate the material properties. Therefore all functions for the calculation of the material properties must be in the same folder.

Properties of Stacks of YBCO Coated Conductors

Comparable to the calculation of the properties of single tapes of YBCO coated conductors, there are also three files that allow the calculation of the properties of stacks of multiple soldered YBCO coated conductors. These are the calculation of the heat capacity of a tape by Matlab function *C_stack.m*, the calculation of the thermal conductivity of a single tape by Matlab function *k_stack.m* and the calculation of the resistance load per unit length of a single tape *R_stack_pul.m*, the capacitance load per unit length between two tapes in a stack *C_pul.m*, the inductance load per unit length between two tapes in a stack *L_pul.m* and the conductance load per unit length *G_pul.m*.

These files have the same input variables as the files for the single tapes plus the number of tapes and the solder layer thickness. Each file has a single output variable that contains a single output value as described in the function head.

The files for the calculation of the properties of stacks basically call the functions for the calculation of the values of a single conductor and use these values to calculate the value of a stack. Therefore all functions for the calculation of the basic material properties as well as for the properties of a single tape must be in the same folder as the function in use. The file *L_pul.m* uses two additional functions to calculate the geometrical mean distance that are implemented in *gmd_11.m* and *gmd_12.m*. These files have also to be included in the current folder in order to execute the function *L_pul.m*.

Calculation of Current Distribution within Superconductors

The calculation of the current distribution within superconductors is implemented in file *I_dist_stack.m*. The method used in this function is explained in Chapter 4.4. This function is implemented for the calculation of the current distribution within the YBCO layers of the parallel tapes in a stack that has a structure of the proposed cable design (compare Fig. 2.1) and uses the dimensions and variables as shown in Fig. 2.2. The current distribution within the YBCO layers can be calculated for different load currents under the assumption that there is no external magnetic field, that the superconductor was cooled down first and transport current was applied afterwards and that the current shares equally between all tapes. A description of the input variables is given in the function head. The output variable is a matrix that contains the coordinates of the points of the grid, the identification number of the tape the point belongs to, the identification number of the point itself and the information if there is a current flowing at this position.

The results can be visualized in a figure window by calling function *plot_current_dist.m*. The output-matrix of function *I_dist_stack.m* must be used as input variable.

7.3 Calculation of Stability Criteria

The calculation of the stability criteria is more complex than the functions mentioned in the forgoing chapters and therefore they have a different structure. These more complex functions call some of the other functions mentioned in the foregoing chapters and therefore all functions must be included in the same folder.

7.3.1 Cryostability

The calculation of cryostability is separated in two functions. One function has no input variables but contains the definition of all variables used in the calculation and calls the other function that contains all the calculations. After the calculation is performed, a figure window opens that contains the results. This figure window as well as another file that contains all input parameters and calculated values is stored in a folder, specified in the calling function. The file names are generated within the calculation.

Comparable to all other functions the function head contains a description of the input and output variables. The parameters limits are the same as mentioned in the sections where the material parameters are defined.

7.3.2 Minimum Propagation Zone

The calculation of the minimum propagation zone is separated in two functions. One function has no input variables but contains the definition of all variables used in the calculation and calls the other function that contains all the calculations. When the calculation is started, a figure window opens that shows the current calculation results. This calculation is quite time consuming and can last several hours depending on the specifications. In order to get good results, the time and length discretization have to be varied and fit to the calculation. This can be done by try and error. The parameters for the discretization have to be changed if the calculation doesn't run stable.

Once the calculation is done, the figure window as well as another file that contains all input parameters and calculated output variables is stored in a folder specified in the calling function. The file names are generated within the calculation.

Comparable to all other functions, the function head contains a description of the input and output variables. The parameters limits are the same as mentioned in the sections where the material parameters are defined.

8 Summary

This work makes a contribution to the development of high current cables with a new cabling concept for twisted stacked YBCO coated conductors that can be used for fusion magnets or other power applications. The first step of this work was the development and implementation of fit-functions that allow the calculation of different material properties of all the materials involved in a soldered stacked cable with YBCO coated conductors in a wide range of boundary conditions. These fit-functions were used for the development of a numerical method to calculate the critical current, resistivity, heat capacity and thermal conductivity of a cable depending on the different layer thicknesses, magnetic field density and temperature. This approach allows the calculation of the joule heat of the cable depending on all relevant design variables and operating conditions. In order to apply different stability criteria fit-functions for the calculation of the cooling power of different coolants were implemented as well. The developed calculation methods and software tools were used to perform some example calculations in order to get an understanding of current sharing processes within a cable including the current distribution within the superconducting layers, as well as the limit of the stable operation of cables with YBCO coated conductors. The calculations are compared with measurement results and thus the developed calculation methods are proven and the potential of this kind of cable is shown. The developed tools can be used for further investigations and cable designs.

At first fit-functions for different material properties of all materials involved in a cable were determined and implemented for a wide temperature range from 4.2 K to room temperature. This includes the calculation of the thermal conductivity, heat capacity, electrical resistivity and the determination of the critical current in the temperature range from 4.2 to 92 K and a magnetic field range from 0.1 T to 20 T. These fit-functions for material properties were used for the development of equivalent circuit elements like the resistance per unit length of a cable and the conductance per unit length between conductors in a cable. This allows calculations in a wide spread of dimensions and operation conditions like various layer thicknesses, temperatures, magnetic field densities and transport currents in the cable

In the following different electrical circuit diagrams for twisted stacked cables with YBCO coated conductors were introduced in order to calculate the current distribution along the cable in different operation conditions. In this context a new method for the numerical calculation of the current distribution within superconductors was introduced and also implemented in a mathematic computation software. All these developed and implemented functions can be used for the calculation of the joule heat of cables depending on the multiple cable dimensions and operating conditions.

The developed tools were used for the calculation of the wave propagation velocity on cables with different dimensions taking into account the inductance and capacitance per unit length of the cable. This investigation shows that the cable can be considered to be electrically short for stability calculations that take into account current redistribution processes caused by sudden local events. Furthermore the resistance per unit length was calculated for different example tapes in a temperature range from 77 K to room temperature and the results were compared with measurement results. Thus the calculation method and the implemented fit-functions were proven.

Besides the developed and implemented method of the calculation of the joule heat dissipated in the cable in case of perturbations also different fit-functions for the calculation of the cooling power of different liquid and gaseous coolants were determined and implemented. This includes the calculation of the cooling power of liquid Helium, Hydrogen and Nitrogen unpressurized at boiling temperature and forced flowed, unpressurized gaseous Helium and Hydrogen above their boiling temperature. This allows the application of different stability criteria on the cable. The implemented stability criteria are cryostability and minimum propagation zone. It was shown that the stability criteria of cold end recovery cannot be realized with YBCO coated conductors in liquid Helium and Hydrogen because of its high critical temperature. The implementation of cryostability allows the calculation of the Stekly number and the plot of a diagram of the heat dissipation and the cooling depending on the temperature. The implementation of the minimum propagation zone allows the analytical calculation of the values minimum quench energy as well as a numerical simulation of the heat distribution and dissipation after a local perturbation.

In order to proof the calculation of the minimum propagation zone, an example calculation for a single conductor in liquid hydrogen was performed. The results of the analytical calculation were compared with the simulation results. This comparison showed that the analytically calculated value of the minimum quench energy is by a factor of 1.9 higher than the result achieved by the simulation in this example case. The main reason for this is the variation of the material properties over the wide temperature range. The material properties are assumed to be constant in case of the analytical calculation but they are taken into account in dependence of the temperature in case of the simulation.

In order to proof the implemented calculation of cryostability an interesting case example was chosen. The case example was a measurement of a cable with YBCO coated conductors recently published by Sastry Pamidi from Florida State University. His group measured the stability of a cable operated at critical current in liquid hydrogen and gaseous helium at 77 K. The measurements showed that the cable could be operated at 1.25 times the critical current in liquid hydrogen, but it quenched already at critical current when it was cooled with gaseous helium at the same temperature. The calculation showed that there is a high cooling power and temperature margin for cryostability in case of liquid cooling but a very low cooling power and almost no temperature margin in case of gas cooling. The belonging calculations explained the observed behavior and also proved the implemented calculation.

Furthermore the heat capacity, thermal conductivity, resistance and critical current depending on at different magnetic fields and a wide temperature range from 4.2 K to 300 K was calculated exemplarily for a 4 mm wide YBCO coated conductor with a 20 µm thick cupper stabilization layer. The given diagrams for this typical conductor allow the fast access to the material properties in a wide temperature range to make estimates in a design process or to double check calculations.

In following workings the developed and implemented fit-functions for the calculation of the material properties can be used for all kinds of calculations where these data is needed. The implemented methods for the calculation of heat dissipation and cooling of cables allow the application of the different stability criteria in design processes of YBCO cables. In order to proof the calculation of minimum propagation zone a comparison of the results of the numerical simulation with quench experiments would be useful.

A Appendix

In the following the implemented executable in Matlab code for the stability calculations described in this report is provided. Each calculation is included in a single file that can be called from other functions with the specified input and output parameters.

A.1 Program Code for Calculation of Critical Current

```
function [I_c] = critical_current (B, T, h_ybco, w_tape)

%Calculation of critical current depending on magnetical field density and temperature
%applicable in the parameter range of T = 4 K - 92 K and B = 0.1 T - 20 T

%Input: Symbol / Description / Unit
% B / Magnetical Fild densitie / T
% T / Temperature / K
% I_c_77_0 / critical current of single tape at T = 77K and B = 0T / A

%Output: Symbol / Description / Unit
% I_c / Critical Current / A

T_c = 92;

A = 4.52*10^8;
B_irr_0 = 132.5;
p = 0.653;
q = 2.568;
alpha = 1.5;
beta = 1.789;
B_irr = B_irr_0 * (1-(T/T_c))^alpha;
if (T/T_c) < 1 && (B/B_irr) < 1
    j_c = (A/B) * B_irr^beta * (B/B_irr)^p * (1-(B/B_irr))^q;
else
    j_c = 0;
end
I_c = h_ybco * w_tape * j_c;
```

A.2 Program Code for Calculation of Electrical Resistivity

Resistivity of Copper

```
function [rho] = rho_cu (T)

%Calculation of specific resistivity of copper depending on temperature

%Input: Symbol / Description / Unit
% T / Temperature / K

%Output: Symbol / Description / Unit
% rho / Specific Resistivity / Ohm*m
```

```
%Source: D. B. Poker, C.E. Klabunde; Temperature dependence of electrical
%resistivity of Vanadium, Platinum and copper; Physical Review B, Vol 26,
%No 12; 1982

e = 2.718281828459;
rho_ee = 0; %15.5*10^-9;
Theta = 337;
lowlim = 0;
uplim = Theta/T;

rho_sd = 0;
fun = @(x) (x.^3) ./ ( (e.^x- 1) .* (1-e.^-x));
J_3 = quadl( fun, lowlim, uplim);

rho_ss = 2.38*10^-18;
fun = @(x) (x.^5) ./ ( (e.^x- 1) .* (1-e.^-x));
J_5 = quadl( fun, lowlim, uplim);

rho_0 = 7.6 * 10^-12;   % Ohm * Meter
a = 3 * 10^-15;
b = 5.4 * 10^-18;
N = 4.61;

if T < 12 ;
    rho = rho_0 + a*T^2 + b*T^N;
else
    rho = rho_0 + rho_ee*T^2 + rho_sd*T^3*(J_3/7.212) + rho_ss*T^5*(J_5/124.14);
end
%rho = rho * 10^6; %conversion from Ohm*m to Ohm*mm^2/m
```

Resistivity of Silver

```
function [rho] = rho_ag (T)

%Calculation of specific resistivity of Silver depending on temperature

%Input: Symbol / Description / Unit
% T / Temperature / K

%Output: Symbol / Description / Unit
% rho / Specific Resistivity / Ohm*m

%Source: Electrical Resistivity of Copper, Gold, Palladium and Silver, R.A.
%Matula, 1979

e = 2.718281828459;
M = 107.868;
Theta = 220.9;
C = 0.0011650123;
lowlim = 0;
uplim = Theta/T;

fun = @(x) (x.^5) .* e.^x ./ (e.^x- 1).^2 ;
J_5 = quadl( fun, lowlim, uplim);

rho = (C/(M*Theta)) * (T/Theta)^5 * J_5;
```

Resistivity of Hastelloy

```
function [rho] = rho_hs (T)

%Calculation of specific resistivity of Hastelloy depending on temperature
```

```
%Input: Symbol / Description / Unit
% T / Temperature / K

%Output: Symbol / Description / Unit
% rho / Specific Resistivity / Ohm*m

%Source:
% 1. Electrical resistivity of some engineering alloys at low temperatures, A.F. Clark,G.
E. Childs, G. H. Wallace, 1970
% 2. Physical properties of Hastelloy C-276TM at cryogenic temperatures, J. Lu, E. S. Choi,
H. D. Zhou, 2008


%Simple linear curvefit because of low temperature dependance
if T > 12
    p1 = 0.0001469;
    p2 = 1.229;
    rho = (p1 * T + p2) * 10^-6;
else
    p1 = -0.0002667;
    p2 = 1.234;
    rho = (p1 * T + p2) * 10^-6;
end

%rho = rho * 10^6; %conversion from Ohm*m to Ohm*mm^2/m
```

Resistivity of Solder

```
function rho_sn_pb = rho_so (T)

%Calculation of specific resistivity of SnPb 60/40 solder depending on temperature

%Input: Symbol / Description / Unit
% Fraction of Sn in given in [%]
% B / Magnetic Flux Density/ T (optional)
% T / Temperature / K

%Output: Symbol / Description / Unit
% rho / Specific Resistivity / Ohm*m

%Source:
% 1. Pressure and temperature dependence of electrical resistivity of Pb and Sn
%from 1-300K and 0-10 GPa-use as continuous resistive pressure monitor accurate over
%wide temperature range; superconductivity under pressure in Pb, Sn and In,
%A. Eiling, J.S. Schilling, 1980
% 2. Progress in the Manufacture of the Cable-in-Conduit Nb3Sn Outsert Coils
%for the 45 Tesla Hybrid Magnet, T.A. Painter,J.R. Miller, L.T. Summers,
%1994

ratio = 0.6; % Gives volume ratio between Sn and Pb => ratio = volume_Sn / volume_Pb
e = 2.718281828459;

%Calculation of specific resistivity of Sn
K = 7440 * 1/100;
Theta = 210;
if T > Theta/2
    rho_sn = (K*T/(4*Theta^2)) * (1 - (1/18)*(Theta/T)^2 + (1/480)*(Theta/T)^4);
else
    lowlim = 0;
    uplim = Theta/T;
    fun = @(x) (x.^5) .* e.^x ./ ((e.^x)- 1).^2;
    J_5 = quadl(fun, lowlim, uplim);
    rho_sn = (K/Theta) * (T/Theta)^5 * J_5;
end
```

```
%Calculation of specific resistivity of Pb
K = 2091 * 1/100;
Theta = 86;
if T > Theta/2
    rho_pb = (K*T/(4*(Theta)^2)) * (1 - (1/18)*(Theta/T)^2 + (1/480)*(Theta/T)^4);
else
    lowlim = 0;
    uplim = Theta/T;
    fun = @(x) (x.^5) .* e.^x ./ ((e.^x)- 1).^2;
    J_5 = quadl(fun, lowlim, uplim);
    rho_pb = (K/Theta) * (T/Theta)^5 *J_5;
end

%Calculation of specific resistivity Sn/Pb
rho_sn_pb = 10^-6 * (rho_sn * ratio + rho_pb * (1-ratio));

%Influence of magnetic field
%grad = 0,00717592;
%rho_sn_pb = rho_sn_pb * (1 + grad * B);
```

A.3 Program Code for Calculation of Thermal Conductivity

Thermal Conductivity of Copper

```
function [thermal_conductivity_cu] = k_cu (T)

%Calculation of specific heat conductivity of copper depending on temperature
%in the temperature range between 4K and 300K

%Input: Symbol / Description / Unit
% T / Temperature / K

%Output: Symbol / Description / Unit
% thermal_conductivity_cu / Specific heat conductivity/ W/(m*K)

%Source: http://cryogenics.nist.gov/MPropsMAY/OFHC%20Copper/OFHC_Copper_rev1.htm

%RRR = 50 => n = 1
%RRR = 100 => n = 2
%RRR = 150 => n = 3
%RRR = 300 => n = 4
%RRR = 500 => n = 5

n = 2; %Standard value n = 2

a = [1.8743, 2.2154, 2.3797, 1.357, 2.8075];
b = [-0.41538, -0.47461, -0.4918, 0.3981, -0.54074];
c = [-0.6018, -0.88068, -0.98615, 2.669, -1.2777];
d = [0.13294, 0.13871, 0.13942, -0.1346, 0.15362];
e = [0.26426, 0.29505, 0.30475, -0.6683, 0.36444];
f = [-0.0219, -0.02043, -0.019713, 0.01342, -0.02105];
g = [-0.051276, -0.04831, -0.046897, 0.05773, -0.051727];
h = [0.0014871, 0.001281, 0.0011969, 0.0002147, 0.0012226];
i = [0.003723, 0.003207, 0.0029988, 0, 0.0030964];


thermal_conductivity_cu = 10^((a(n) + c(n)*T^0.5 + e(n)*T + g(n)*T^1.5 + i(n)*T^2) / (1 +
b(n)*T^0.5 + d(n)*T + f(n)*T^1.5 + h(n)*T^2));
```

Thermal Conductivity of Silver

```
function [thermal_conductivity_ag] = k_ag (T)

%Calculation of specific heat conductivity of silver depending on temperature
%in the temperature range between 4K and 300K

%Input: Symbol / Description / Unit
% T / Temperature / K

%Output: Symbol / Description / Unit
% thermal_conductivity_ag / Specific heat conductivity/ W/(m*K)

%Source: R. Smith, F. R. Fickett; Low-Temperature Properties of Silver; J. Res. Natl. Inst.
Stand. Technol. 100, 119 (1995)

a = [-8.085202766065000e-04, -3.358195377766776e-07, 8.147381572319895e-11];
b = [0.084173230577580, 1.869961406456805e-04, -1.042739292996614e-07];
c = [-2.898387926986296, -0.041125881979616, 5.409027188706845e-05];
d = [30.127572196852682, 4.473201297735611, -0.013972777872369];
e = [1.323932096483007e+02, -2.413050714859127e+02, 1.569131427472265];
f = [1.031106730600989e+02, 5.659032445727915e+03, 4.105421708582979e+02];

if T <= 33
    n = 1;
else
    if T <= 100
        n = 2;
    else
        n = 3;
    end
end

thermal_conductivity_ag = a(n)*T^5 + b(n)*T^4 + c(n)*T^3 + d(n)*T^2 + e(n)*T + f(n);
```

Thermal Conductivity of Hastelloy

```
function [thermal_conductivity_hs] = k_hs (T)

%Calculation of specific heat conductivity of Hastelloy depending on temperature
%in the temperature range between 4K and 300K

%Input: Symbol / Description / Unit
% T / Temperature / K

%Output: Symbol / Description / Unit
% thermal_conductivity_hs / Specific heat conductivity/ W/(m*K)

%Source:
%1) Lu, J., et al., Physical Properties of HastelloyÆ C-276TM at Cryogenic Temperatures, J.
Appl. Phys., 101, 123710 (2007).
%2) Wesche HTS Conductors for Fusion Thermal Stability and Quench Karlsruhe 2011
%3) http://www.goodfellow.com/E/Hastelloy-C276-Heat-Resisting-Alloy.html


a = [-1.976150097497916e-08, 3.544514666284345e-11];
b = [1.692069531515161e-06, -3.085569423939295e-08];
c = [3.428435748706821e-05, 9.862456406179634e-06];
d = [-0.008289421572708, -0.001448613557764];
e = [0.399017559651135, 0.120321620954427];
f = [-0.827062339102484, 3.632700117022619];

if T <= 55
```

```
    n = 1;
else
    n = 2;
end

thermal_conductivity_hs = a(n)*T^5 + b(n)*T^4 + c(n)*T^3 + d(n)*T^2 + e(n)*T + f(n);
```

Thermal Conductivity of YBCO

```
function [thermal_conductivity_ybco] = k_ybco (T)

%Calculation of specific heat conductivity of YBCO depending on temperature
%in the temperature range between 4K and 300K

%Input: Symbol / Description / Unit
% T / Temperature / K

%Output: Symbol / Description / Unit
% heat_cap / Specific heat conductivity/ W/(m*K)

%Source:
%1)

a = [-1.266103106492942e-05, -1.316704893540544e-09];
b = [0.002670105477219, 2.636151594117440e-06];
c = [-0.197035302542769, -0.001601689632073];
d = [4.933962659604384, 0.428760641312538];
e = [29.651536939501670, -53.306643333287260];
f = [66.578192505447330, 3.682252343338599e+03];

if T <= 70
    n = 1;
else
    n = 2;
end

thermal_conductivity_ybco = a(n)*T^5 + b(n)*T^4 + c(n)*T^3 + d(n)*T^2 + e(n)*T + f(n);
```

Thermal Conductivity of a coated conductor

```
function [thermal_conductivity_tape] = k_tape (h_hs, h_ybco, h_ag, h_cu, w_tape, T)

%Calculation of specific heat capacity of YBCO tapes depending on temperature

%Input: Symbol / Description / Unit
% T / Temperature / K

%Output: Symbol / Description / Unit
% thermal_conductivity_tape / Specific heat conductivity/ W/m*K

%Source: (material densities) PSFC/RR-10-7 - Jun Feng - Thermohydraulic-Quenching
Simulation for
%Superconducing Magnets Made of YBCO HTS Tape - 2010

thermal_conductivity_tape = w_tape * ( h_hs * k_hs(T) + h_ybco * k_ybco(T) + h_ag * k_ag(T)
+ 2*h_cu * k_cu(T) ) / (  w_tape * ( h_hs + h_ybco + h_ag + 2*h_cu ));
```

Thermal Conductivity of a stack

```
function [thermal_conductivity_h2] = k_h2 (T)

%Calculation of specific heat conductivity of gaseous Hydrogen depending on temperature
%in the range between 0.5 K and 5000K
```

```
%Input: Symbol / Description / Unit
% T / Temperature / K

%Output: Symbol / Description / Unit
% thermal_conductivity_he / Specific heat conductivity / W/(m*K)

%Source: Woolley, Scott, Brickwedde. Compilation of Thermal Properties of
%Hydrogen in Its Various Isotopic and Ortho-Para Modifications. National
%Buroeau of Standards. Research Paper RP1932. Volume 41, 1948.
%and
%Assael, Mixafendi, Wakeham. The Viscosity and Thermal Conductivity of
%Normal Hydrogen in the Limit of Zero Density. J. Phys. Chern. Ref. Data,
%Vol. 15, No.4, 1986

a = [1.438979044890086e-13];
b = [-1.179931934921931e-10];
c = [3.406542452329941e-08];
d = [-4.363250395376434e-06];
e = [8.902878936027004e-04];
f = [-0.001433133271971];

n = 1;

thermal_conductivity_h2 = a(n)*T^5 + b(n)*T^4 + c(n)*T^3 + d(n)*T^2 + e(n)*T + f(n);
```

Thermal Conductivity of Solder

```
function [thermal_conductivity_so] = k_so (T)

%Calculation of specific heat conductivity of Solder SnPb 6040 depending on temperature
%in the temperature range between 4K and 300K
%It is not sure if the values below 70K a correct. Different values were
%found in literature

%Input: Symbol / Description / Unit
% T / Temperature / K

%Output: Symbol / Description / Unit
% thermal_conductivity_so / Specific heat conductivity/ W/(m*K)

%Source: Chuan, Ratnalingam, Thermal conductivity of soft solder from 90 K
%to 300 K, CRYOGENICS. MARCH 1973

%Material data for Pb60/Sn40

a = [7.803509145512186e-08, -1.546126121833989e-10];
b = [-2.751481293678849e-05, 1.282710598664956e-07];
c = [0.003657305390889, -3.768920391716192e-05];
d = [-0.223437769764658, 0.004395839566614];
e = [6.022184839140108, -0.102794660423839];
f = [-1.192898000639724, 46.451188210708640];

if T <= 70
    n = 1;
else
    n = 2;
end

thermal_conductivity_so = a(n)*T^5 + b(n)*T^4 + c(n)*T^3 + d(n)*T^2 + e(n)*T + f(n);
```

Thermal Conductivity of Gaseous Hydrogen

```
function [thermal_conductivity_h2] = k_h2 (T)
```

```
%Calculation of specific heat conductivity of gaseous Hydrogen depending on temperature
%in the range between 0.5 K and 5000K

%Input: Symbol / Description / Unit
% T / Temperature / K

%Output: Symbol / Description / Unit
% thermal_conductivity_he / Specific heat conductivity / W/(m*K)

%Source: Woolley, Scott, Brickwedde. Compilation of Thermal Properties of
%Hydrogen in Its Various Isotopic and Ortho-Para Modifications. National
%Buroeau of Standards. Research Paper RP1932. Volume 41, 1948.
%and
%Assael, Mixafendi, Wakeham. The Viscosity and Thermal Conductivity of
%Normal Hydrogen in the Limit of Zero Density. J. Phys. Chern. Ref. Data,
%Vol. 15, No.4, 1986

a = [1.438979044890086e-13];
b = [-1.179931934921931e-10];
c = [3.406542452329941e-08];
d = [-4.363250395376434e-06];
e = [8.902878936027004e-04];
f = [-0.001433133271971];

n = 1;

thermal_conductivity_h2 = a(n)*T^5 + b(n)*T^4 + c(n)*T^3 + d(n)*T^2 + e(n)*T + f(n);
```

Thermal Conductivity of Gaseous Helium

```
function [thermal_conductivity_he] = k_he (T)

%Calculation of specific heat conductivity of gaseous Helium depending on temperature
%in the range between 0.5 K and 5000K

%Input: Symbol / Description / Unit
% T / Temperature / K

%Output: Symbol / Description / Unit
% thermal_conductivity_he / Specific heat conductivity / W/(m*K)

%Source: R. Powell, C. Ho, P. Liley, Thermal Conductivity of Selected Materials, 1966

a = [2.065416328163720e-13];
b = [-1.829051848815390e-10];
c = [6.100032618364231e-08];
d = [-9.871122362113007e-06];
e = [0.001224527275633];
f = [0.004074967617200];

%if T <= 33
    n = 1;
% else
%     if T <= 100
%         n = 2;
%     else
%         n = 3;
%     end
% end

thermal_conductivity_he = a(n)*T^5 + b(n)*T^4 + c(n)*T^3 + d(n)*T^2 + e(n)*T + f(n);
```

A.4 Program Code for Calculation of Heat Capacity

Heat Capacity of Copper

```
function [heat_capacity_cu] = C_cu (T)

%Calculation of specific heat capacity of copper depending on temperature
%in the temperature range between 4K and 300K

%Input: Symbol / Description / Unit
% T / Temperature / K

%Output: Symbol / Description / Unit
% heat_cap / Specific heat capaciy/ J/(kg*K)

%Source: R. Wesche, HTS Conductors for Fusion Thermal Stability and Quench. Karlsruhe 2011

a = [1.185608689795585e-08, -3.318391520914566e-11];
b = [-1.754171346610396e-05, -1.178877722139533e-07];
c = [0.001961392960536, 1.261381319281417e-04];
d = [-0.015485438546988, -0.046922418631570];
e = [0.088975039227139, 8.140457575941973];
f = [1.021961566399155, -2.048755825968253e+02];

if T <= 45
    n = 1;
else
    n = 2;
end

heat_capacity_cu = a(n)*T^5 + b(n)*T^4 + c(n)*T^3 + d(n)*T^2 + e(n)*T + f(n);
```

Heat Capacity of Silver

```
function [heat_capacity_ag] = C_ag (T)

%Calculation of specific heat capacity of Silver depending on temperature
%in the temperature range between 4K and 300K

%Input: Symbol / Description / Unit
% T / Temperature / K

%Output: Symbol / Description / Unit
% heat_cap / Specific heat capaciy/ J/(kg*K)

%Source: Jun Feng, PSFC/RR-10-7 Thermohydraulic Quenching Simulation for
%Superconducting Magnets made of YBCO HTS Tape 2010

a = [0.00201, 1.11189*10^-4, 3.70101*10^-6];
b = [-0.012, -0.04289, -0.00326];
c = [0.071, 6.05068, 1.0369];
d = [-2.66454*10^-14, -100.586, 116.90736];

if T <= 30
    n = 1;
else
    if T <= 125
        n = 2;
    else
        n = 3;
    end
end
```

```
heat_capacity_ag = a(n)*T^3 + b(n)*T^2 + c(n)*T + d(n);
```

Heat Capacity of Hastelloy

```
function [heat_capacity_hs] = C_hs (T)

%Calculation of specific heat capacity of Hastelloy depending on temperature
%in the temperature range between 4K and 300K

%Input: Symbol / Description / Unit
% T / Temperature / K

%Output: Symbol / Description / Unit
% heat_cap / Specific heat capaciy/ J/(kg*K)

%Source:
%1) Lu, J., et al., Physical Properties of HastelloyÆ C-276TM at Cryogenic Temperatures, J.
Appl. Phys., 101, 123710 (2007).
%2) Wesche HTS Conductors for Fusion Thermal Stability and Quench Karlsruhe 2011

a = [-8.659744297212208e-07, 2.150134339872814e-10];
b = [7.209308577841222e-05, -1.458398200879766e-07];
c = [-0.001346146407585, 4.376675190884510e-05];
d = [0.017992946436959, -0.015104610350977];
e = [0.084834978274794, 4.491152093383801];
f = [0.642508075273653, -1.107126478142060e+02];

if T <= 35
    n = 1;
else
    n = 2;
end

heat_capacity_hs = a(n)*T^5 + b(n)*T^4 + c(n)*T^3 + d(n)*T^2 + e(n)*T + f(n);
```

Heat Capacity of Solder

```
function [heat_capacity_so] = C_so (T)

%Calculation of specific heat capacity of solder SnPb5050 depending on temperature
%in the temperature range between 4K and 300K

%Input: Symbol / Description / Unit
% T / Temperature / K

%Output: Symbol / Description / Unit
% heat_cap / Specific heat capaciy/ J/(kg*K)

%Source: W. T. ZIEGLER and J. C. MULLINS, SPECIFIC HEAT OF 50-50 PER CENT
%(WT) LEAD-TIN SOLDER FROM 20 TO 300∞K, Cryogenics, 1964


a = [9.341107159895702e-11];
b = [-1.024659398988616e-07];
c = [4.386446737208523e-05];
d = [-0.009191232324487];
e = [0.972595742967233];
f = [-3.259443448149251];

n = 1;

heat_capacity_so = a(n)*T^5 + b(n)*T^4 + c(n)*T^3 + d(n)*T^2 + e(n)*T + f(n);
```

Heat Capacity of YBCO

```
function [heat_capacity_ybco] = C_ybco (T)

%Calculation of specific heat capacity of YBCO depending on temperature
%in the temperature range between 4K and 300K

%Input: Symbol / Description / Unit
% T / Temperature / K

%Output: Symbol / Description / Unit
% heat_cap / Specific heat capaciy/ J/(kg*K)

%Source:
%1) Rainer Wesche, HTS Conductors for Fusion Thermal Stability and Quench Workshop
Karlsruhe 2011
%2) Schviv, Westurum. The heat capacity and derived thermophysical properties of the high
Tc superconductor YBa2Cu307_6 from 5.3 to 350 K, 1989


a = -7.567485538209158e-10;
b = 6.351452642016898e-07;
c = -1.947975786547597e-04;
d = 0.023616673974415;
e = 0.239331954284042;
f = -1.096191721280114;

heat_capacity_ybco = T^5*a + T^4*b + T^3*c + T^2*d + T*e + f;
```

Heat Capacity of Stack

```
function [heat_cap_stack] = C_stack (n_tape, h_hs, h_ybco, h_ag, h_cu, w_tape, h_so, T)

%Calculation of specific heat capacity of a stack depending on temperature

%Input: Symbol / Description / Unit
% T / Temperature / K

%Output: Symbol / Description / Unit
% heat_cap_times_mass_stack / Heat capaciy per meter of a YBCO tape / J/(K*m)

%Source: -

%Material diensities in kg/m^3
rho_hs = 8.89*10^3;
rho_ybco = 6.3*10^3;
rho_ag = 10.49*10^3;
rho_cu = 8.94*10^3;
rho_so = 8.67*10^3;

heat_cap_stack = w_tape * ( n_tape * rho_hs * h_hs * C_hs(T) + n_tape * rho_ybco * h_ybco *
C_ybco(T) + n_tape * rho_ag * h_ag * C_ag(T) + n_tape * rho_cu * 2*h_cu * C_cu(T) +
(n_tape-1) * h_so * rho_so * C_so(T));
```

A.5 Program Code for Calculation of Resistances

Coupling Resistance Between Tapes

```
function cond = G_pul (w_tape, h_tape, h_ag, h_cu, h_so, T)
```

```
%Calculation of Conductivity per unit length between  two soldered
%Conductors (compare documentation for Failure Case 1)

%Input: Symbol / Description / Unit
% w_tape / width of entire tape / m
% h_tape / thickness of entire tape / m
% h_cu / thickness of copper stabilization layer / m
% h_so / tickness of solder between tapes / m
% T / Temperature / K

%Output: Symbol / Description / Unit
% cond / temp. dependent conductivity per unit length depending / (Ohm*m)^-1

cond = ( (1/rho_ag(T)) * (w_tape)/(2*h_ag) + (1/rho_cu(T)) * ((w_tape+h_cu)/(2*h_cu) +
(2*h_cu)/(w_tape+h_cu) + (h_cu)/(h_tape+h_cu)) + (1/rho_so(T)) * (((w_tape/2+h_cu))/h_so)
);
```

Resistance Load per Unit Length for Normal Conducting Layers

```
function res = R_nc_pul (w_tape, h_hs, h_ag, h_cu, T)

%Calculation of resistance load per unit length in x direction for failure case 2

%Input: Symbol / Description / Unit
% w_tape / width of entire tape / m
% h_hs / thickness of hastelloy layer / m
% h_ag / thickness of silver layer / m
% h_cu / thickness of copper layer / m
% T / Temperature / K

%Output: Symbol / Description / Unit
% res / resistance load per unit length in of normal conducting layers at temperature T /
(Ohm*m)

res = (1/w_tape) * (1/( (2*h_cu/rho_cu(T)) + (h_hs/rho_hs(T)) + (h_ag/rho_ag(T) ) ));
```

Resistance Load per Unit Length for Superconducting Layers

```
function res = R_sc_pul (I_c, I_sc, T)

%Calculation of resistance load per unit length in x direction for failure case 2

%Input: Symbol / Description / Unit
% I_c / critical current at 77 K / A
% I_sc / current in superconducting layer / A
% T / Temperature / K

%Output: Symbol / Description / Unit
% res / resistance load per unit length of superconducting layer at temperature T and
current I_sc/ (Ohm*m)

E_c = 1 * 10^-6 / 0.01;
n = 30;

if I_c > 0
    res = (E_c/I_c) * (I_sc/I_c)^(n-1);
else
    res = 1*10^9;
end
```

Resistance Load per Unit Length of Entire Tape

```
function [res, I_sc, I_nc] = R_tape_pul (I_tape, w_tape, h_hs, h_ag, h_cu, T, I_c)
```

```
%Calculation of resistance load per unit length in along a tape depending on current and
Temperature

%Input: Symbol / Description / Unit
% T_m / Temperature the critical current is given for / K
% I_tape / total current in tape / A
% w_tape / width of entire tape / m
% h_hs / thickness of hastelloy layer / m
% h_cu / thickness of silver layer / m
% B / Magnetical Fild densitie / T
% T / Temperature / K
% I_c_77_0 / critical current of single tape at T = 77K and B = 0T / A

%Output: Symbol / Description / Unit
% res / resistance load per unit length of tape / (Ohm*m)

%Calculation of critical current depending on temperature
epsilon = 0.0000001; %break off criterion

%Calibration on critical current Ic
R_nc = R_nc_pul(w_tape, h_hs, h_ag, h_cu, T);
I_nc_c = 100*10^-6 / R_nc;
I_c_cal = I_c - I_nc_c;

%Calculation of current sharing between sc and nc layers
if I_tape > 0.2 * I_c_cal
    n = 1;
    I_sc(n,1) = 0.1*I_c_cal;
    delta_u(n,1) = R_sc_pul(I_c_cal, I_sc(n,1), T) * I_sc(n,1) - R_nc * (I_tape-I_sc(n,1));

    n = 2;
    I_sc(n,1) = 0.2*I_c_cal;
    delta_u(n,1) = R_sc_pul(I_c_cal, I_sc(n,1), T) * I_sc(n,1) - R_nc * (I_tape-I_sc(n,1));

    if I_c_cal > 0.001
        while (abs(delta_u(n,1)) > epsilon)
            n = n + 1;
            if delta_u(n-1,1) < 0
               if (delta_u(n-1,1) * delta_u(n-1,1)) > 0
                    I_sc(n,1) =  I_sc(n-1,1) + abs( (I_sc(n-2,1) - I_sc(n-1,1)) );
                else
                    I_sc(n,1) =  I_sc(n-1,1) + abs( (I_sc(n-2,1) - I_sc(n-1,1))/2 );
                end
            else
                if (delta_u(n-1,1) * delta_u(n-1,1)) <= 0
                    I_sc(n,1) =  I_sc(n-1,1) - abs( (I_sc(n-2,1) - I_sc(n-1,1)) );
                else
                    I_sc(n,1) =  I_sc(n-1,1) - abs( (I_sc(n-2,1) - I_sc(n-1,1))/2 );
                end
            end
            delta_u(n,1) = R_sc_pul(I_c_cal, I_sc(n,1), T) * I_sc(n,1) - R_nc * (I_tape-
I_sc(n,1));
        end
    else
        I_sc(n,1) = 0.001;
    end
else
    n = 1;
    I_sc(n,1) = I_tape;
end

%Calculation of resistivity per unit length of tape
I_sc = I_sc(n,1);
I_nc = I_tape - I_sc;
```

```
res = 1/( (1/R_nc) + (1/R_sc_pul(I_c_cal, I_sc, T)) );


%Display of intermediate results
%delta_u(n,1)
%I_nc = I_tape - I_sc;
%Matrix = [ u , I_sc , I_nc ]
```

A.6 Program Code for Calculation Cooling

```
function [cooling_power] = cooling (T, T_cool, coolant, cooled_perimeter)

%Calculation of cooling power liquid Hydrogen

%Input: Symbol / Description / Unit
% T / Temperature of cable / K
% T_cool / Temperature of coolant / K
% coolant / LHe = liquid Helium, LH2 = liquid Hydrogen, LN2 = liquid
% Nitrogen, GHe = gaseous Helium, GH2 = gaseous Hydrogen / [string]

%Output: Symbol / Description / Unit
% cooling_power  / Cooling Power  / W/m^2


%Cooling with Liquid boiling Helium
%Source: E. Brentari, P. Giarratano, R. Smith. Boiling Heat Transfer for Oxygen,
%Nitrogen, Hydrogen and Helium. U.S. Department of Commerce National Bureau
%of Standards. Technical Note No. 317. Boulder 1965
if coolant == ['LHe']

    T_cool = 4.22;
    Delta_T = T-T_cool;

    a1 = [9.711696132610116*10^-04, 0.114220579269682, -3.541847140082002*10^-04, -
3.433927475440869*10^-06, 3.705339803033855e-12];
    b1 = [-0.431364701542670, -1.595652097149223, 0.013616233955146, 2.088337768149688*10^-
04, -2.679712663940987e-09];
    c1 = [8.612750744768492, 6.934406595529862, -0.165620076532440, -0.004846220961957,
4.551626340671213e-07];
    d1 = [2.383663227152308, -12.425127622894069, 0.861202050283444, 0.054547520847638,
1.211136844405880e-04];
    e1 = [-0.017527677806502, 8.865159051459754, -2.009236131890110, -0.261568969600508,
0.046091217901146];
    f1 = [0, -1.227570523284329, 1.934305922352881, 0.600054741974201, -0.169828008064109];

    if Delta_T < 0
        cooling_power = 0;
    else
        if Delta_T < 0.335
            n = 1;
        else
            if Delta_T < 1.119
                n = 2;
            else
                if Delta_T < 1.8
                n = 3;
                else
                    if Delta_T < 15
                        n = 4;
                    else
```

```
                        n = 5;
                    end
                end
            end
        end
        cooling_power = 10000 * (Delta_T^5*a1(n) + Delta_T^4*b1(n) + Delta_T^3*c1(n) +
Delta_T^2*d1(n) + Delta_T*e1(n) + f1(n));
    end
end

%Cooling with Liquid boiling Hydrogen
%Source: E. Brentari, P. Giarratano, R. Smith. Boiling Heat Trnasfer for Oxigen,
%Nitrogen, Hydrogen and Helium. U.S. Department of Commerce. National
%Bureau of Standards. Technical Note No. 317. Boulder. 1965

if coolant == ['LH2']

    T_cool = 20.28;
    Delta_T = T-T_cool;

    a1 = [0.205964280664305, 3.036448670791589*10^-04];
    b1 = [-1.017096695192534, -0.024040018611891];
    c1 = [2.124989731552975, 0.584390369028840];
    d1 = [-0.995484250403823, -5.916830682011566];
    e1 = [0.311399915562726, 24.343976965176020];
    f1 = [0, -25.126518534838360]; %(Field 1 originally was 0.001209250648352)

    a2 = [-1.844252911076747e-04, 1.891115260810556*10^-05];
    b2 = [0.059141048795790, 0.028263491011630];
    c2 = [-1.321896661322377, -0.190905407424569];
    d2 = [14.913576459912234, 0.897949095224492];
    e2 = [-4.121202844838504, -8.720375197991760];

    if Delta_T < 0
        cooling_power = 0;
    else
        if Delta_T < 1.9
            n = 1;
            cooling_power = 10000 * (Delta_T^5*a1(n) + Delta_T^4*b1(n) + Delta_T^3*c1(n) +
Delta_T^2*d1(n) + Delta_T*e1(n) + f1(n));
        else
            if Delta_T < 7.4
                n = 2;
                cooling_power = 10000 * (Delta_T^5*a1(n) + Delta_T^4*b1(n) +
Delta_T^3*c1(n) + Delta_T^2*d1(n) + Delta_T*e1(n) + f1(n));
            else
                if Delta_T < 45
                    n = 1;
                    cooling_power = 10000 * ((a2(n)*Delta_T^3 + b2(n)*Delta_T^2 +
c2(n)*Delta_T + d2(n)) / (Delta_T + e2(n)));
                else
                    n = 2;
                    cooling_power = 10000 * ((a2(n)*Delta_T^3 + b2(n)*Delta_T^2 +
c2(n)*Delta_T + d2(n)) / (Delta_T + e2(n)));
                end
            end
        end
    end
end


%Cooling with Liquid boiling Nitrogen
%Source: Brentari. Giarratano. Smith. Boiling Heat Trnasfer for Oxigen.
%Nitrogen. Hydrogen and Helium. U.S. Department of Commerce. National
```

```
%Bureau of Standards. Technical Note No. 317. Boulder. 1965
if coolant == ['LN2']

    T_cool = 77.36;
    Delta_T = T-T_cool;

    a1 = [0.001680087317345, 0.001523742594545, 0, 0, -7.840448867640097e-09, -
3.376831251299163e-13];
    b1 = [-0.019242022013695, -0.042849971468586, 0, 0,  2.884035691766796e-06,
4.008429818030891e-10];
    c1 = [0.087095646378018, 0.443404075725262, 0.030698455830402, -0.019815624626693, -
4.118523712519108e-04, -1.606658041297706e-07];
    d1 = [-0.074037882770234, -1.846635115903137, -1.696351704757089, 1.546287855732148,
0.028566068632537, 3.053044558646029e-05];
    e1 = [0.057917877207038, 3.046099292101531, 28.578520020396105, -40.117639308958790, -
0.952535467253945, 0.009349035738659];
    f1 = [0, 0, -1.330831413182242e+02, 3.475715056624032e+02, 12.896166432201340,
0.240726932374015];


    if Delta_T < 0
        cooling_power = 0;
    else
        if Delta_T < 4.6
            n = 1;
        else
            if Delta_T < 10
                n = 2;
            else
                if Delta_T < 18.9
                    n = 3;
                else
                    if Delta_T < 22.8
                    n = 4;
                    else
                        if Delta_T < 89.9
                            n = 5;
                        else
                            n = 6;
                        end
                    end
                end
            end
        end
        cooling_power = 10000 * (Delta_T^5*a1(n) + Delta_T^4*b1(n) + Delta_T^3*c1(n) +
Delta_T^2*d1(n) + Delta_T*e1(n) + f1(n));
    end
end


%Cooling with gaseous Helium
%Source: Warren M. Rohsenow, Harry, Choi. Heat, Mass, and Momentum Transfer, 1961, page 141
if coolant == ['GHe']

    %Nusselt Number
    Nu = 3.66;

    cooling_power = Nu * k_he(T_cool) * (T - T_cool)/cooled_perimeter ;

end


%Cooling with gaseous Hydrogeon
%Source: Warren M. Rohsenow, Harry, Choi. Heat, Mass, and Momentum Transfer, 1961, page 141
```

```
if coolant == ['GH2']

    %Nusselt Number
    Nu = 3.66;

    cooling_power = Nu * k_h2(T_cool) * (T - T_cool)/cooled_perimeter ;

end
```

A.7 Program Code for Calculation of Stability Criterion

Cryostability Function Call

```
function [output] = cryostability_function_call

I_c_soll_at_77K_0_1T = 85*0.38

%number of tapes in stack / -
n_tape = 32;

%total current in stack / A
I_stack = 0.6 * 32 * 32.3;

%tape width / m
w_tape = 4.1*10^-3;

%thickness of hastelloy layer / m
h_hs = 50*10^-6;

%thickness of YBCO layer / m
h_ybco = 0.7195*10^-6;

%thickness of silver layer / m
h_ag = 1.0*10^-6;

%thickness of copper layer / m
h_cu = 20*10^-6;

%thickness of solder layer / m
h_so = 20*10^-6;

% Length 1 (0 - failure) / m
l_1 = 3.5;

%Length 2 (failure - cable end) / m
l_2 = 3.5;

%Joint resistance of single tape / Ohm
R_j = 1*10^-6;

%length of failure / m
l_f = 1*10^-3;

%Magnetical field density / T
B = 40;

%Temperature of coolant / K
%LHe = liquid Helium
%LH2 = liquid Hydrogen
%LN2 = liquid Nitrogen
```

```
%GHe = gaseous Helium
%GH2 = gaseous Hydrogen
coolant = ['LHe'];
T_cool = 50; %if coolant is liquid, T_cool must be defined, but has no effect

%%%%%%%%%%%%%%%%%%%%%%%Calculation%%%%%%%%%%%%%%%%%%%%%%%
if coolant == ['LHe']
    T_cool = 4.22;
end
if coolant == ['LH2']
    T_cool = 20.28;
end
if coolant == ['LN2']
    T_cool = 77.36;
end

for w = 1:length(n_tape)
%    for x = 1:length(calculation_duration)

            %Calculate and save results
            name = ['Cryostability ' num2str(n_tape(w)) 'tapes ' num2str(I_stack(w)) 'A '
num2str(B) 'T '  coolant ' ' num2str(T_cool) 'K'];
            directory= ['/Users/Andre/Desktop/Results/' name '/'];
%['C:\Users\adberger\Desktop\Results\' name '\']

            if exist (directory) == 7
            else
                mkdir (directory);
            end


            [stackley, I_c, I_c_stack, T_c_i_stack, j] = cryostability (I_stack, n_tape,
w_tape, h_hs, h_ybco, h_ag, h_cu, h_so, l_1, l_2, R_j, l_f, B, T_cool, coolant, directory,
name);

%    end
end
```

Calculation of Cryostability

```
function [Stekly, I_c, I_c_stack, T_c_i_stack, j] = cryostability (I_stack, n_tape, w_tape,
h_hs, h_ybco, h_ag, h_cu, h_so, l_1, l_2, R_j, l_f, B, T_cool, coolant, directory, name)

%Calculation of Stekly criterion

%Input: Symbol / Description / Unit
% I_stack / total current in stack / A
% n_tape / Number of tapes in stack / -
% w_tape / width of tapes / m
% h_hs / thickness of hastelloy layer / m
% h_ybco / thickness of YBCO layer / m
% h_ag / thickness of silver layer / m
% h_cu / thickness of copper layer / m
% h_so / thickness of solder layer / m
% l_1 / Length 1 (0 - failure) / m
% l_2 / Length 2 (failure - cable end) / m
% R_j / Joint resistance of single tape / Ohm
% l_f / length of failure / m
% B / Magnetical field density / T
% T_cool / Temperature of coolant / K

%Output: Symbol / Description / Unit
% Stekly  / Value of Strackley Criterion for stack @T_c(B,I_stack) / -
% Plot of heating and cooling power over Temperature (all tapes in normal conducting state
```

```
only)

%Source: L. Dresner, Stability of Superconductors, 1995

dT = 0.5; %step size of Temperature

%Critical current I_c(B,T_cool)
I_c = critical_current(B, T_cool, h_ybco, w_tape);
['I_c(B,T_cool) = ' num2str(I_c) ' A/tape']

%Calculation of critical temperature Tc(B,I=0)
for N = 4:0.1:92
    T_c = N;
    if critical_current(B, N, h_ybco, w_tape) == 0
        break
    end
end
['T_c(B,I=0) = ' num2str(T_c) ' K']

I_tape = I_stack/n_tape;
for T_c_i_stack = 4:0.1:92
    if critical_current(B, T_c_i_stack, h_ybco, w_tape) <= I_tape
        break
    end
end
['T_c(B, I=I_stack) = ' num2str(T_c_i_stack) ' K']


%Plot of introduced and dissipated power per unit length of the cable
%for fix Cool Temperature and varying cable temperature
h_tape = h_hs + h_ybco + h_ag + 2*h_cu;
h_stack = n_tape * h_tape + (n_tape-1) * h_so;
cooled_perimeter = sqrt(h_stack^2 + w_tape^2);
%w = waitbar(0,'power');
n = 1;
if coolant == ['LN2']
    uplim = 200;
else
    uplim = 100;
end
for T = T_cool:dT:uplim
    diagram(n,5) = 0;
    diagram(n,1) = T_cool;
    diagram(n,2) = T;
    R_stack_so = (rho_so(T)/(w_tape * h_so)) / (n_tape-1);
    I_c_T = critical_current(B, T, h_ybco, w_tape);

    % entire cable in superconducting state (no disturbance)
    R_tapes_stack_pul_sc = R_tape_pul (I_stack, w_tape, n_tape*h_hs, n_tape*h_ag,
n_tape*h_cu, T, n_tape*I_c_T);
    R_stack_pul_sc = 1/( (1/R_tapes_stack_pul_sc) + (1/R_stack_so) );
    diagram(n,3) = R_stack_pul_sc * I_stack^2;

    % one single tape in normal conducting state
    if n_tape >= 2
        diagram(n,7) = P_partially_nc_stack (w_tape, h_hs, h_ybco, h_ag, h_cu, h_so, T,
l_1, l_2, R_j, l_f, I_stack, B, I_c_T,  n_tape, 1);
    end

    % two single tapes in normal conducting state
    if n_tape >= 3
        diagram(n,8) = P_partially_nc_stack (w_tape, h_hs, h_ybco, h_ag, h_cu, h_so, T,
l_1, l_2, R_j, l_f, I_stack, B, I_c_T,  n_tape, 2);
    end
```

```
    % entire cable in normal conducting state
    R_tapes_stack_pul_nc = R_nc_pul(w_tape, h_hs, h_ag, h_cu, T)/n_tape;
    R_stack_pul_nc = 1/( (1/R_tapes_stack_pul_nc) + (1/R_stack_so) );
    diagram(n,4) = R_stack_pul_nc * I_stack^2;

    %Cooling
    diagram(n,5) = cooling(T, T_cool, coolant, cooled_perimeter) * (2*h_stack+2*w_tape);

    n = n+1;
    %waitbar((T-T_cool)/(uplim-T_cool));
end
%close(w);
j = I_stack / (w_tape * h_stack);
I_c_stack = n_tape * I_c;

diagram(1,3) %dissipated power at T_cool
% cooling(T_cool+1, T_cool, coolant, cooled_perimeter) * w_tape
% cooling(T_cool+1, T_cool, coolant, cooled_perimeter) / 10000

%Calculation of Stekly Criterion - for liquid cooling only at T=min(p_cooling)
if strcmp(coolant, 'GHe') == 1 || strcmp(coolant, 'GH2') == 1
    q_specific = cooling(T_cool+1, T_cool, coolant, cooled_perimeter);
    Stekly = 0;
end
if strcmp(coolant, 'LN2') == 1 || strcmp(coolant, 'LH2') == 1 || strcmp(coolant, 'LHe') ==
1
    for N = 1:uplim
        if diagram(N+1,5) < diagram(N,5) && diagram(N+2,5) < diagram(N,5)
            T_q_max = diagram(N,2);
            q_max = diagram(N,5);
            break
        end
    end
    for N = N:uplim
        if diagram(N+1,5) > diagram(N,5) && diagram(N+2,5) > diagram(N,5)
            T_q_min = diagram(N,2);
            q_min = diagram(N,5);
            break
        end
    end
    I_c_R = critical_current(B,T_q_min, h_ybco, w_tape);
    R_tapes_stack_pul_sc = R_tape_pul (I_stack, w_tape, n_tape*h_hs, n_tape*h_ag,
n_tape*h_cu, T_q_min, n_tape*I_c_R);
    R_stack_so = ( (rho_so(T_q_min))/(w_tape * h_so)) / (n_tape-1);
    R_stack_pul = 1/( (1/R_tapes_stack_pul_sc) + (1/R_stack_so) );
    P_intro_pul = R_stack_pul * I_stack^2;
    q_diss_pul = cooling(T_q_min, T_cool, coolant, cooled_perimeter)*(2*h_stack+2*w_tape);
    Stekly = P_intro_pul / q_diss_pul;
    ['Stekly = ' num2str(Stekly)]
end

%Point of equilibrium of cooling and joule heat
%Calculation of Temperature
for n = 2:length(diagram(:,5))
    if diagram(n,5) <= diagram(n,3)
        T_max_p_q = diagram(n,2);
        break
    end
end
if n == length(diagram(:,5))
    T_max_p_q = 0;
end
h_stack = n_tape * (h_hs + h_ybco + h_ag + 2*h_cu) + (n_tape-1) * h_so;
```

```
A_stack = h_stack * w_tape;

%Maximum temperature for minimum propagation zone => Dresner Page 66
T_max_mpz = 0;
dT = 0.1;
Integral = 0;
for T = T_cool+dT:dT:92
    I_c = critical_current (B, T, h_ybco, w_tape);
    R_tapes_stack_pul_sc = R_tape_pul (I_stack, w_tape, n_tape*h_hs, n_tape*h_ag,
n_tape*h_cu, T, n_tape * I_c);
    R_stack_so_pul = ( (rho_so(T))/(w_tape * h_so)) / (n_tape-1);
    R_stack_pul = 1/( (1/R_tapes_stack_pul_sc) + (1/R_stack_so_pul) );
    p_stack_pul = R_stack_pul * I_stack^2;
    cooled_perimeter = sqrt(h_stack^2 + w_tape^2);
    q_stack_pul = cooling(T, T_cool, coolant, cooled_perimeter)*(2*h_stack+2*w_tape);

    dIntegral = k_stack(n_tape, w_tape, h_hs, h_ybco, h_ag, h_cu, h_so, T) * (p_stack_pul -
q_stack_pul)/cooled_perimeter * dT;
    Integral = Integral + dIntegral;
    T_max_mpz = T;
    if Integral >= 0
        break
    end
end
['T_max_mpz = ' num2str(T_max_mpz)]

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%Calculation of energy introduction%%%%%%%%%%%%%%%%%%%%%%%%%%
%Calculation of energy input for q = p
E_mpz_specific = 0;
dT = 0.5;
for T = T_cool:dT:T_max_p_q
    dE_mpz = C_stack(n_tape, h_hs, h_ybco, h_ag, h_cu, w_tape, h_so, T) * dT / A_stack;
    E_mpz_specific = E_mpz_specific + dE_mpz;
end
mqe_p_q = E_mpz_specific ;
['mqe_p_q = ' num2str(mqe_p_q)]

%Calculation of energy input for Tc
E_mpz_specific = 0;
dT = 0.5;
for T = T_cool:dT:T_c
    dE_mpz = C_stack(n_tape, h_hs, h_ybco, h_ag, h_cu, w_tape, h_so, T) * dT / A_stack;
    E_mpz_specific = E_mpz_specific + dE_mpz;
end
mqe_T_c = E_mpz_specific ;
['mqe_T_c = ' num2str(mqe_T_c)]

%Calculation of energy input for Tcs
E_mpz_specific = 0;
dT = 0.5;
for T = T_cool:dT:T_c_i_stack
    dE_mpz = C_stack(n_tape, h_hs, h_ybco, h_ag, h_cu, w_tape, h_so, T) * dT / A_stack;
    E_mpz_specific = E_mpz_specific + dE_mpz;
end
mqe_T_c_i_stack = E_mpz_specific ;
['mqe_T_c_i_stack = ' num2str(mqe_T_c_i_stack)]

%Calculation of energy input for T_max_mpz
E_mpz_specific = 0;
dT = 0.5;
for T = T_cool:dT:T_max_mpz
    dE_mpz = C_stack(n_tape, h_hs, h_ybco, h_ag, h_cu, w_tape, h_so, T) * dT / A_stack;
    E_mpz_specific = E_mpz_specific + dE_mpz;
end
```

```
mqe_T_max_mpz = E_mpz_specific ;
['mqe_T_max_mpz = ' num2str(mqe_T_max_mpz)]

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%Plot results%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(1)
plot(diagram(:,2), diagram(:,4), '-r'); hold on;
if n_tape >=3
    plot(diagram(:,2), diagram(:,8), '--r'); hold on;
end
if n_tape >= 2
    plot(diagram(:,2), diagram(:,7), '-.r'); hold on;
end
plot(diagram(:,2), diagram(:,3), ':r'); hold on;
plot(diagram(:,2), diagram(:,5), '-b'); hold on;
xlabel('\it T\rm (K)');
ylabel('\it p\rm, \it q\rm (W/m)');
hold off
if n_tape >=3
    legend('joule heat without superconductivity', 'joule heat with two normal conducting
tapes', 'joule heat with two normal conducting tapes', 'joule heat with superconducvivity',
'cooling');
end
if n_tape == 2
    legend('joule heat without superconductivity', 'joule heat with two normal conducting
tapes', 'joule heat with superconducvivity', 'cooling');
end
if n_tape == 1
    legend('joule heat without superconductivity', 'joule heat with superconducvivity',
'cooling');
end

x=xlim;
y=ylim;
Text_2 = cell(4,1);
Text_2{1,1} = ['n\_tape = ' num2str(n_tape) ];
Text_2{2,1} = ['B = ' num2str(B) ' T'];
Text_2{3,1} = ['T\_cool = ' num2str(T_cool) ' K (' coolant ') '];
Text_2{4,1} = ['T\_cs = ' num2str(T_c_i_stack) ' K'];
Text_2{5,1} = ['T\_c(B,I=0) = ' num2str(T_c) ' K'];
Text_2{6,1} = ['I\_stack = ' num2str(I_stack) 'A (' num2str(I_stack/n_tape) ' A/tape)'];
Text_2{7,1} = ['I\_c (B, T\_cool) = ' num2str(round(10*I_c_stack)/10) ' A ('
num2str(round(10*I_c_stack/n_tape)/10) ' A/tape)'];
Text_2{8,1} = ['j = ' num2str(round(10*j/10^6)/10) ' A/mm^2'];
Text_2{9,1} = ['Distance to T\_c(B,I=0): \DeltaT = ' num2str(round(10*(T_c-T_cool))/10) '
K, equivalent energy input = ' num2str(mqe_T_c/10^6)     ' J/cm\^3'];
Text_2{10,1} = ['Distance to T\_cs = T\_c(B,I\_stack): \DeltaT = '
num2str(round(10*(T_c_i_stack-T_cool))/10) ' K, equivalent energy input = '
num2str(mqe_T_c_i_stack/10^6)     ' J/cm\^3'];
Text_2{11,1} = ['Distance to p = q: \DeltaT = ' num2str(round(10*(T_max_p_q-T_cool))/10) '
K, equivalent energy input = ' num2str(mqe_p_q/10^6) ' J/cm\^3'];
Text_2{12,1} = ['Distance to T\_max\_mpz: \DeltaT = ' num2str(round(10*(T_max_mpz-
T_cool))/10) ' K, equivalent energy input = ' num2str(mqe_T_max_mpz/10^6)     ' J/cm\^3'];

if strcmp(coolant, 'GHe') == 1 || strcmp(coolant, 'GH2') == 1
    Text_2{13,1} = ['q = ' num2str(q_specific*2*(h_stack+w_tape)*1000) ' mW/m*K ('
num2str(round(10*q_specific*1000/10000)/10) ' mW/cm^2*K)'];
    text(x(1)+0.05*(x(2)-x(1)), y(2)-0.25*(y(2)-y(1)), {Text_2{1,1}, Text_2{2,1},
Text_2{3,1}, Text_2{4,1}, Text_2{5,1}, Text_2{6,1}, Text_2{7,1}, Text_2{8,1}, Text_2{9,1},
Text_2{10,1}, Text_2{11,1}, Text_2{12,1}, Text_2{13,1}});
end
if strcmp(coolant, 'LN2') == 1 || strcmp(coolant, 'LH2') == 1 || strcmp(coolant, 'LHe') ==
1
    Text_2{13,1} = ['\alpha = ' num2str(Stekly) ' @min(q)']; % = ' num2str(T_q_min) 'K '];
    Text_2{14,1} = ['min(q) = ' num2str(round(10*q_min*1000/10000)/10) ' mW/(cm^2) @ T = '
```

```
num2str(round(10*T_q_min)/10) ' K'];
    text(x(1)+0.05*(x(2)-x(1)), y(2)-0.25*(y(2)-y(1)), {Text_2{1,1}, Text_2{2,1},
Text_2{3,1}, Text_2{4,1}, Text_2{5,1}, Text_2{6,1}, Text_2{7,1}, Text_2{8,1}, Text_2{9,1},
Text_2{10,1}, Text_2{11,1}, Text_2{12,1}, Text_2{13,1}, Text_2{14,1}});
end
grid on
figure(1)
set(gcf,'PaperUnits','centimeters')
xSize = 8; ySize = 12; xLeft = 0; yTop = 0;
set(gcf,'PaperSize',[12 8]);
set(gcf,'PaperPosition',[xLeft yTop xSize ySize])

%%%%%%%%%%%%%%%%%%%%%%%%%%%% plot tape resistance %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% n = 0;
%
% for T = 4.2:0.2:300
%     n = n+1;
%     I_c = critical_current(B, T, h_ybco, w_tape);
%     resistance_pul_tapes(n,2) = R_tape_pul (I_stack/n_tape, w_tape, h_hs, h_ag, h_cu, T,
I_c)*1000;
%     resistance_pul_tapes(n,1) = T;
% end
%
% figure(2)
% plot(resistance_pul_tapes(:,1), resistance_pul_tapes(:,2))
% xlabel('\it T\rm (K)');
% ylabel('R\_tape\_pul, (m\Omega/m)');
% grid on

% ylimits = ylim;
% y = [ylimits(1),((ylimits(2)-ylimits(1))*0.3)];
% line([T_q_min,T_q_min],y ,'Color','k');
% text(T_q_min*0.9,y(2)*1.15, 'T(B,I=I\_c)');

%hold off

%%%%%%%%%%%%%%%%%%%%%%%%%%%% save results to file %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

h=gcf;
figfilename = [ name '.fig'];
if exist ([ directory  figfilename ], 'file') == 2
    n = 1;
    figfilename = [ name '_' num2str(n) '.fig' ];
    while exist ([ directory  figfilename ], 'file') == 2
        n = n + 1;
        figfilename = [ name '_' num2str(n) '.fig' ];
    end
end
saveas(h, [directory figfilename], 'fig');

%create struckt
file.parameters.I_stack = I_stack;
file.parameters.n_tape = n_tape;
file.parameters.w_tape = w_tape;
file.parameters.h_hs = h_hs;
file.parameters.h_ybco = h_ybco;
file.parameters.h_ag = h_ag;
file.parameters.h_cu = h_cu;
file.parameters.h_so = h_so;
file.parameters.l_1 = l_1;
file.parameters.l_2 = l_2;
file.parameters.R_j = R_j;
file.parameters.l_f = l_f;
file.parameters.B = B;
```

```
file.parameters.T_cool = T_cool;
file.parameters.coolant = coolant;

file.results.Stekly = Stekly;
file.results.I_c = I_c;
if strcmp(coolant, 'LN2') == 1 || strcmp(coolant, 'LH2') == 1 || strcmp(coolant, 'LHe') ==
1
    file.results.T_q_min = T_q_min;
    file.results.q_min = q_min;
    ['T_q_min = ' num2str(T_q_min) ' K']
end
if strcmp(coolant, 'GHe') == 1 || strcmp(coolant, 'GH2') == 1
    file.results.q_specific = q_specific;
end
file.results.j = j;
file.results.T_max_p_q = T_max_p_q;

file.data = diagram;

parameterfilename = figfilename(1:length(figfilename)-4);
filename = [ directory parameterfilename ' parameters.mat'];
save( filename,  'file' );
```

Minimum Propagation Zone Function Call

```
function [location, T, T_start] = minimum_propagation_zone_function_call

%number of tapes in stack / -
n_tape = [32];

%total current in stack / A
I_stack = 827;

%tape width / mm
w_tape = 0.0041;

%thickness of hastelloy layer / m
h_hs = 50*10^-6;

%thickness of YBCO layer / m
h_ybco = 0.7195*10^-6;

%thickness of silver layer / m
h_ag = 1.0*10^-6;

%thickness of copper layer / m
h_cu = 20*10^-6; %%%%%%%%%1.151

%thickness of solder layer / m
h_so = 50*10^-6;

%Magnetical field density / T
B = 40; %0.01; @75K

%Temperature of coolant / K
%LHe = liquid Helium
%LH2 = liquid Hydrogen
%LN2 = liquid Nitrogen
%GHe = gaseous Helium
%GH2 = gaseous Hydrogen
coolant = ['LHe'];
T_cool = 50.0; %if coolant is liquid, T_cool must be defined, but has no effect

%number of differential length elements / -
```

```
n_x = 100; %250;

%Total duration of Calculation / s
calculation_duration = [0.01*10^-3];

%length of time steps
dt_1 = 0.1*10^-6;
t_1 = 5*10^-3;
dt_2 = 0.5*10^-6;
t_2 = 1*10^-3;
dt_3 = 1*10^-6;


%Resolution of calculation (factor to scale time steps dt)
%standard values: 200 faster, but higher accuracy / 500 slower but higher accuracy
%resolution = 500; %not in use in current version

%Scaling of maximum Temperature (T_max = factor * T_mpz_max)
factor_T = [1];

%Scaling of mpz (length(T_cool) = factor_mpz * mpz)
factor_mpz = [1.0];

%calculation area (calculated area = calculation_area * mpz)
calculation_area = 10; %standard value is 10

for w = 1:length(n_tape)
    for x = 1:length(calculation_duration)
        for y = 1:length(factor_T)
            for z = 1:length(factor_mpz)

                s_time = clock;
                fprintf(2,['start time of calculation = ' num2str(s_time(3)) '.'
num2str(s_time(2)) '.' num2str(s_time(1)) ',  ' num2str(s_time(4)) ':' num2str(s_time(5))
':', num2str(round(s_time(6))) ]);


                %Calculation and save results
                name = [num2str(n_tape(w)) 'Tapes ' num2str(I_stack(w)) 'Istack '
num2str((1000*calculation_duration(x))) 'ms ' num2str(factor_T(y)) 'Tmax '
num2str(factor_mpz(z)) 'mpz' ];
                if exist('C:\Users\') == 7
                    directory = ['C:\Users\adberger\Desktop\Results\' name ];
                    if exist (directory) == 7
                        n = 1;
                        directory = ['C:\Users\adberger\Desktop\Results\' name  '_'
num2str(n) '\'];
                        while exist (directory) == 7
                            n = n + 1;
                            directory = ['C:\Users\adberger\Desktop\Results\' name  '_'
num2str(n) '\'];
                        end
                    else
                        directory = [directory '\'];
                    end
                    mkdir (directory);
                else
                    directory = ['/Users/Andre/Desktop/Results/' name ];
                    exist (directory)
                    if exist (directory) == 7
                        n = 1;
                        directory = ['/Users/Andre/Desktop/Results/' name '_' num2str(n)
'/'];
                        while exist (directory) == 7
```

```
                            n = n + 1;
                            directory = ['/Users/Andre/Desktop/Results/' name   '_'
num2str(n) '/'];
                        end
                    else
                        directory = [directory '/'];
                    end
                    mkdir (directory);
                end

                [mpz, mqe, T_max_mpz, location, T_start, T, dT_dx, dT_dt, total_time,
Integral, I_c, propagation_location , dE_dt, dt, T_c_i_stack, T_c] =
minimum_propagation_zone (I_stack(w), n_tape(w), w_tape, h_hs, h_ybco, h_ag, h_cu, h_so, B,
T_cool, n_x, calculation_duration(x), calculation_area, directory, factor_T(y),
factor_mpz(z), name, coolant, dt_1, t_1, dt_2, t_2, dt_3);

                name_parameters = [ directory name ' parameters.mat'];
                save( name_parameters , 'I_stack', 'w_tape', 'h_hs', 'h_ybco', 'h_ag',
'h_cu', 'h_so', 'B', 'T_cool', 'n_x', 'calculation_duration', 'calculation_area', 'dt',
'coolant' );

                name_data = [ directory name ' data.mat' ];
                save( name_data , 'location', 'T_start', 'T', 'dT_dx', 'dT_dt');

                name_results = [ directory name ' results.mat'];
                save( name_results , 'total_time', 'mpz', 'mqe', 'T_max_mpz', 'Integral',
'I_c', 'propagation_location' , 'dE_dt', 'T_c_i_stack', 'T_c');
            end
        end
    end
end
```

Minimum Propagation Zone Calculation

```
function [mpz, mqe, T_max_mpz, location, T_start, T, dT_dx, dT_dt, total_time, Integral,
I_c, propagation_location , dE_dt, dt, T_c_i_stack, T_c] = minimum_propagation_zone
(I_stack, n_tape, w_tape, h_hs, h_ybco, h_ag, h_cu, h_so, B, T_cool, n_x,
calculation_duration, calculation_area, directory, factor_T, factor_mpz, name, coolant,
dt_1, t_1, dt_2, t_2, dt_3)

%Calculation of stackely criterion

%Input: Symbol / Description / Unit
% A_stack / Cross sectional area of stack / m^2
% I_stack / total current in stack / A
% n_tape / Number of tapes in stack / -
% w_tape / width of tapes / m
% h_hs / thickness of hastelloy layer / m
% h_ybco / thickness of YBCO layer / m
% h_ag / thickness of silver layer / m
% h_cu / thickness of copper layer / m
% h_so / thickness of solder layer / m
% l_1 / Length 1 (0 - failure) / m
% l_2 / Length 2 (failure - cable end) / m
% R_j / Joint resistance of single tape / Ohm
% l_f / length of failure / m
% B / Magnetical field density / T
% T_cool / Temperature of coolant / K
% T_i_c / Temperature of critical current in magnetic field T(B,I_c) / K

%Output: Symbol / Description / Unit
% mpz_classic / Minimum propagation zone / m

%Source: M. Wilson, Superconducting Magnets, 1983, pp76
```

```
Plot_on = 1; %1 = creates a plot every time step otherwise every 10th time step
clear global

if coolant == 'LHe'
    T_cool = 4.2;
end
if coolant == 'LH2'
    T_cool = 20.2;
end
if coolant == 'LN2'
    T_cool = 77.36;
end

global T location T_start dT_dx
I_c = critical_current(B, T_cool, h_ybco, w_tape);
['I_c = ' num2str(I_c)]

%Calculation of T_c(B,I_stack)
for T = 4:0.1:92
    T_c_i_stack = T;
    if critical_current(B, T, h_ybco, w_tape) <= I_stack/n_tape;
        break
    end
end
['T_c_i_stack = ' num2str(T_c_i_stack)]

%Calculation of T_c(B,I = 0) (=T_cs)
for T = 4:0.1:92
    T_c = T;
    if critical_current(B, T, h_ybco, w_tape) <= 0;
        break
    end
end
['T_c = ' num2str(T_c)]

I_tape = I_stack/n_tape;
h_stack = n_tape * (h_hs + h_ybco + h_ag + 2*h_cu) + (n_tape-1) * h_so;
A_stack = h_stack * w_tape

%Calculation of maximum temperature of minimum propagation zone => Dresner Page 66
T_max_mpz = 0;
dT = 0.1;
Integral = 0;
for T = T_cool+dT:dT:300
    I_c = critical_current (B, T, h_ybco, w_tape);
    R_tapes_stack_pul_sc = R_tape_pul (I_stack, w_tape, n_tape*h_hs, n_tape*h_ag,
n_tape*h_cu, T, n_tape * I_c);
    R_stack_so_pul = ( (rho_so(T))/(w_tape * h_so)) / (n_tape-1);
    R_stack_pul = 1/( (1/R_tapes_stack_pul_sc) + (1/R_stack_so_pul) );
    p_stack_pul = R_stack_pul * I_stack^2;
    cooled_perimeter = sqrt(h_stack^2 + w_tape^2);
    q_stack_pul = cooling(T, T_cool, coolant, cooled_perimeter)*(2*h_stack+2*w_tape);

    dIntegral = k_stack(n_tape, w_tape, h_hs, h_ybco, h_ag, h_cu, h_so, T) * ((p_stack_pul
- q_stack_pul)/cooled_perimeter) * dT;
    Integral = Integral + dIntegral;
    T_max_mpz = T;
    if Integral >= 0
        break
    end
end
['T_max_mpz = ' num2str(T_max_mpz)]
```

```
%Calculation of Minimum propagation zone => Wilson Page 76
R_tapes_stack_pul_nc = R_nc_pul(w_tape, h_hs, h_ag, h_cu, T_max_mpz)/n_tape;
R_stack_so_pul = ( (rho_so(T_max_mpz))/(w_tape * h_so)) / (n_tape-1);
R_stack_pul = 1/( (1/R_tapes_stack_pul_nc) + (1/R_stack_so_pul) );
k = k_stack(n_tape, w_tape, h_hs, h_ybco, h_ag, h_cu, h_so, T_max_mpz);
mpz = sqrt( 2 * k * (T_max_mpz - T_cool) / ((R_stack_pul*A_stack) * (I_stack/A_stack)^2 ));
['mpz = ' num2str(mpz)]

%Calculation of Minimum propagation zone => Wilson Page 76 extended by integration over T
and upper Integration limit T_max_mpz
mpz_extended = 0;
dT = 0.5;
for T = T_cool:dT:T_max_mpz
    R_tapes_stack_pul_nc = R_nc_pul(w_tape, h_hs, h_ag, h_cu, T)/n_tape;
    R_stack_so_pul = ( (rho_so(T))/(w_tape * h_so)) / (n_tape-1);
    R_stack_pul = 1/( (1/R_tapes_stack_pul_nc) + (1/R_stack_so_pul) );
    k = k_stack(n_tape, w_tape, h_hs, h_ybco, h_ag, h_cu, h_so, T);
    d_mpz = sqrt( 2 * k * dT / ((R_stack_pul*A_stack) * (I_stack/A_stack)^2 ));
    mpz_extended = mpz_extended + d_mpz;
end
['mpz_extended = ' num2str(mpz_extended)]
dT = 0;

%Calculation of minimum quench energy
E_mpz_specific = 0;
dT = 0.5;
for T = T_cool : dT : T_max_mpz
    dE_mpz = C_stack(n_tape, h_hs, h_ybco, h_ag, h_cu, w_tape, h_so, T) * dT / A_stack;
    E_mpz_specific = dE_mpz + E_mpz_specific;
end
mqe = E_mpz_specific * mpz * A_stack;
['mqe = ' num2str(mqe)]
mqe_extended = E_mpz_specific * mpz_extended * A_stack;
['mqe_extended = ' num2str(mqe_extended)]

%Calculation of energy of minimum propagation zone by numerically solving differential
equation
%Declaration of start values
X = calculation_area*mpz; %%%%
extended = 0; %%%%
dx = X / n_x;
location(n_x) = 0;
for x = 1:n_x
    location(x) = X * x/n_x;
end
for x=1:n_x
    if x < ((factor_mpz*mpz/2)/X)*n_x %%%%
        T(x,1) = T_max_mpz*factor_T;
    else
        T(x,1) = T_cool;
    end
end
T_start = T;
['T_max = ' num2str(T_max_mpz*factor_T)]

figure(1);
plot(location(:), T(:),'-b');
xlabel('x (m)');
ylabel('T (K)');
Text_1 = ['Minimum Propagation Zone'];
title( Text_1 )
legend('t = 0 ms');
%hold on
```

```
%Calculation of Temperature distribution T(x,t)
w = waitbar(0,'progress');
time(1) = 0;
t = 1;

%Calculation of Temperature run
i = 0;
j = 0;
marker_plot_1_3 = 0;
marker_plot_2_3 = 0;
marker_plot_runtime = 0;
dT_dt(1,n_x) = 0;
Energy = 0;
propagation = 1;
tic %start time measurement
correction = 0;
T_last = T;
while time(t) < calculation_duration
    if time(t) < t_1 || correction == 1
        dt = dt_1;
        correction = 0;
    else
        if time(t) < t_2
            dt = dt_2;
        else
            dt = dt_3;
        end
    end
    dT_dx(1:n_x-1,1) = diff(T)./dx;
    dT_dx(n_x,1) = dT_dx(n_x-1,1);
    for x = 1:n_x
        if x > 1 && T(x) == T(x-1) && dT_dx(x) == dT_dx(x-1) && t > 2
            k(x) = k(x-1);
            C(x) = C(x-1);
            p_stack_pul(x) = p_stack_pul(x-1);
            q_stack_pul(x) = q_stack_pul(x-1);
        else
            k(x) = dT_dx(x) * k_stack(n_tape, w_tape, h_hs, h_ybco, h_ag, h_cu, h_so,
T(x));
            C(x) = C_stack(n_tape, h_hs, h_ybco, h_ag, h_cu, w_tape, h_so, T(x) ) /
A_stack;
            I_c(x) = critical_current (B, T(x), h_ybco, w_tape);
            R_tapes_stack_pul_sc(x) = R_tape_pul (I_stack, w_tape, n_tape*h_hs,
n_tape*h_ag, n_tape*h_cu, T(x), n_tape * I_c(x));
            R_stack_so_pul(x) = ( (rho_so(T(x)))/(w_tape * h_so)) / (n_tape-1);
            R_stack_pul(x) = 1./( (1./R_tapes_stack_pul_sc(x)) + (1./R_stack_so_pul(x)) );
            p_stack_pul(x) = R_stack_pul(x) .* I_stack^2 ./ A_stack;
            q_stack_pul(x) = cooling(T(x), T_cool, coolant,
cooled_perimeter)*(2*h_stack+2*w_tape) / A_stack;
        end
    end
    dk_dx(2:n_x) = diff(k(:)) ./ dx;
    dk_dx(1) = (k(2)-k(1))/dx;
    dT = ((dk_dx + p_stack_pul - q_stack_pul) ./ C) .* dt;
    T_last_last = T_last;
    T_last = T;
    T = T + dT';
    [a,b] = size(T);
    if a == 1
        T = T';
    end
    dT_dx = diff(T(:)) ./ dx;
    dT_dx(n_x) = dT_dx(n_x-1);
```

```
%%%%%%%%%%%%%%%%%%%%%%Check plausibility and correction%%%%%%%%%%%%%%%%%
    if t > 2
        %Checking dT_dx
        a = find(dT_dx > 0);

        %Checking T < T_cool and T > 300
        b = find(T < T_cool);
        c = find(T >= 300);

        %Checking dT_dt
        dT_dt = (T-T_last)/dt;
        Sign = diff(sign(dT_dt));
        Sign(2:n_x-1) = Sign(1:n_x-2).*Sign(2:n_x-1);
        Sign(n_x) = 0;
        d = find(Sign < 0);
        if length(d) < 3
            d = [];
        else
            for n=1:length(d)-3
                if d(n) ~= d(n+1)+1 && d(n) ~= d(n+2)+2
                    d(n) = 0;
                end
            end
        end

        %Delete doubled detected elements
        j = unique([a',b',c',d'])';
        if isempty(j) == 0
            e = find(j < 4);
            f = find(j > n_x-4);
            g = find(j == 0);
            if isempty([e,f]) == 0
                j([e,f,g]) = [];
            end
        end

        %Correction of detected elements
        if isempty(j) == 0

            correction = 1;
            if ( isempty(a) == 0 || isempty(b) == 0 || isempty(c) == 0 )
                T = T_last;
                T_last = T_last_last;
                t = t-1;
            end
% check(:,1) = T;
            j = j';
            ['Correction: t = ' num2str(t) ', time = ' num2str(time(t)) ' s, j = '
num2str(j) ]%', location = ' num2str(location(j)) ' m ' ]
            T_temp = T;
            T_temp_last = T_temp;
            dt_temp = dt/10;
            for n = 1:10
                dT_dx_temp(1:n_x-1,1) = diff(T_temp)./dx;
                dT_dx_temp(n_x,1) = dT_dx_temp(n_x-1,1);
                for x = 2 : n_x
                    if T_temp(x) == T_temp(x-1) && dT_dx_temp(x) == dT_dx_temp(x-1)
                        k_temp(x) = k(x);
                        C_temp(x) = C(x);
                    else
                        k_temp(x) = dT_dx_temp(x) * k_stack(n_tape, w_tape, h_hs, h_ybco,
h_ag, h_cu, h_so, T_temp(x));
                        C_temp(x) = C_stack(n_tape, h_hs, h_ybco, h_ag, h_cu, w_tape, h_so,
T_temp(x) ) / A_stack;
```

```
                    end
                end
                dk_dx_temp(2:n_x) = diff(k_temp(:)) ./ dx;
                dk_dx_temp(1) = 0;
                T_temp_last = T_temp;
                dT_temp = (dk_dx_temp ./ C_temp) .* dt_temp;
                for m = 1:length(j)
                    T_temp((j(m)-3):(j(m)+3)) = T_temp((j(m)-3):(j(m)+3)) + dT_temp((j(m)-
3):(j(m)+3))';
                end
            end
            [a,b] = size(T_temp);
            if a == 1
                T = T_temp';
            else
                T = T_temp;
            end
%   check(:,2) = T;
%   check(:,3) = (1:n_x)
%   j
%   pause
        end
    end

    %Plot runtime results
    waitbar(time(t)/calculation_duration);
    if rem(t+9,10) == 0 || Plot_on == 1
        marker_plot_runtime = 1;
    end

    %Calculation of total energy
    Energy_last = Energy;
    Energy = 0;
    for x = 1 : propagation
        dEnergy= C_stack(n_tape, h_hs, h_ybco, h_ag, h_cu, w_tape, h_so, T(x)) * T(x) * dx;
        Energy = dEnergy + Energy;
    end
    dE_dt = (Energy - Energy_last)/dt;

    if marker_plot_runtime == 1
        propagation = max(find(dT_dx(:) ~= 0));
        marker_plot_runtime = 0;
        figure(1)
        plot(location(:)*1000,T(:),'--r');
        xlimit=xlim;
        ylimit=ylim;
        legend(['t = ' num2str(time(t)*1000) ' ms ']);
        Text_2{1,1} = ['dt = ' num2str(dt*1000000) ' \mus '];
        Text_2{2,1} = ['propagation = '
num2str(round(1000*location(propagation)*1000)/1000) ' mm '];
        text(xlimit(1)+0.7*(xlimit(2)-xlimit(1)), ylimit(2)-0.2*(ylimit(2)-
ylimit(1)),{Text_2{1,1}, Text_2{2,1}});
        xlabel('x (mm)');
        ylabel('T (K)');

        figure(2)
        plot(location(:)*1000,dT_dt(:)/1000,'-b');
        xlabel('x (mm)');
        ylabel('dT/dt (K/ms)');
        legend(['t = ' num2str(time(t)*1000) ' ms']);
        Integral = sum(dT_dt(1:propagation).*dx)/location(propagation);
        Text_3{1,1} = ['avg(dT/dt) = ' num2str(Integral/1000) ' K/ms '];
        Text_3{2,1} = ['dE/dt = ' num2str(dE_dt*10^3) ' \muJ/ms '];
        xlimit=xlim;
```

```
        ylimit=ylim;
        text(xlimit(1)+0.68*(xlimit(2)-xlimit(1)), ylimit(2)-0.2*(ylimit(2)-
ylimit(1)),{Text_3{1,1}, Text_3{2,1}});
    end

    if time(t) >= calculation_duration*1/3 && marker_plot_1_3 == 0
        t_1_3 = time(t);
        T_1_3 = T;
        marker_plot_1_3 = 1;
    end
    if time(t) >= calculation_duration*2/3 && marker_plot_2_3 == 0
        t_2_3 = time(t);
        T_2_3 = T;
        marker_plot_2_3 = 1;
    end

    time(t+1) = time(t) + dt;
    t = t+1;
end
location = location';
duration = toc; %finish time measurement
close(w)


%Command window output
total_time = time(t);
['t = ' num2str(total_time) ' s']
Integral = sum(dT_dt(1:propagation).*dx)/location(propagation);
['integral over dT_dt from x = 0 to propagation = ' num2str(Integral/1000) ' K/ms ']
seconds = mod(duration,60);
minutes = mod((duration-seconds)/60,60);
houres = ((duration-seconds)/60-minutes)/60;
['duration for calculation is = ' num2str(houres) ' h ' num2str(minutes) ' min '
num2str(round(seconds)) ' s ']
propagation_location = location(propagation);
['propagation = ' num2str(propagation_location/1000) ' mm ']


%Plot results
figure(1);
plot(location(:)*1000,T_start(:),'-b');
xlabel('\it x\rm (mm)');
ylabel('\itT\rm (K)');
% Text_1 = ['Minimum Propagation Zone'];
% title( Text_1 )
hold on;
if marker_plot_1_3 == 1 || marker_plot_2_3 == 1
    if marker_plot_1_3 == 1 && marker_plot_2_3 == 0
        plot(location(:)*1000, T_1_3(:), '--r')
        plot(location(:)/1000, T(:),'-r')
        legend('t = 0 ms', ['t = ' num2str(round(100*t_1_3*1000)/100) ' ms'], ['t = '
num2str(round(100*time(t)*1000)/100) ' ms']);
    end
    if marker_plot_2_3 == 1 && marker_plot_1_3 == 0
        plot(location(:)*1000, T_2_3(:), '--r')
        plot(location(:)*1000, T(:),'-r')
        legend('t = 0 ms', ['t = ' num2str(round(100*t_2_3*1000)/100) ' ms'], ['t = '
num2str(round(100*time(t)*1000)/100) ' ms']);
    end
    if marker_plot_2_3 == 1 && marker_plot_1_3 == 1
        plot(location(:)*1000, T_1_3(:), '--r')
        plot(location(:)*1000, T_2_3(:), '--r')
        plot(location(:)*1000, T(:),'-r')
        legend('t = 0 ms', ['t = ' num2str(round(100*t_1_3*1000)/100) ' ms'], ['t = '
```

```
num2str(round(100*t_2_3*1000)/100) ' ms'], ['t = ' num2str(round(100*time(t)*1000)/100) '
ms']);
    end
else
    plot(location(:)/1000, T(:),'-r')
    legend('t = 0 ms', ['t = ' num2str(time(t)*1000) ' ms']);
end
I_c = critical_current (B, T_cool, h_ybco, w_tape);
Text_4{1,1} = ['T\_cool = ' num2str(T_cool) ' K '];
Text_4{2,1} = ['B = ' num2str(B) ' T '];
Text_4{3,1} = ['I\_c,stack = ' num2str(round(10*I_c*n_tape)/10) ' A '];
Text_4{4,1} = ['I\_stack = ' num2str(I_stack) ' A '];
Text_4{5,1} = ['n\_tape = ' num2str(n_tape)];
Text_4{6,1} = ['mpz = ' num2str(round(10*mpz*1000000)/10) ' \mum '];
if extended == 1
    Text_4{7,1} = ['mpz\_extended = ' num2str(round(10*mpz_extended*1000000)/10) ' \mum
x'];
else
    Text_4{7,1} = ['mpz\_extended = ' num2str(round(10*mpz_extended*1000000)/10) ' \mum '];
end
Text_4{8,1} = ['mqe = ' num2str(round(100*mqe*1000)/100) ' mJ '];
Text_4{9,1} = ['mqe\_extended = ' num2str(round(100*mqe_extended*1000)/100) ' mJ '];
Text_4{10,1} = ['T\_max,mpz ' num2str(round(100*T_max_mpz)/100) ' K '];
Text_4{11,1} = ['Propagation ' num2str(round(1000*location(propagation)*1000)/1000) ' mm
'];
Text_4{12,1} = ['dE/dt = ' num2str(dE_dt*10^3) ' \muJ/ms '];
Text_4{13,1} = ['avg(dT/dt) = ' num2str(Integral/1000) ' K/ms '];
xlimit=xlim;
ylimit=ylim;
text(xlimit(1)+0.68*(xlimit(2)-xlimit(1)), ylimit(2)-0.4*(ylimit(2)-
ylimit(1)),{Text_4{1,1},Text_4{2,1}, Text_4{3,1}, Text_4{4,1}, Text_4{5,1}, Text_4{6,1},
Text_4{7,1}, Text_4{8,1}, Text_4{9,1}, Text_4{10,1}, Text_4{11,1}, Text_4{12,1},
Text_4{13,1}});
hold off

h=gcf;
%name = [num2str((1000*calculation_duration)) 'ms ' num2str(factor_T) ' Tmax '];
figfilename = [  directory  name 'T_x.fig'];
saveas(h,figfilename, 'fig');

%Plot Time derivative
figure(2)
dT_dt = 1000*((T(:)-T_last(:)))/dt;
dT_dt(n_x) = 0;
plot(location(:)*1000,dT_dt(:)/1000,'-b');
xlabel('x (mm)');
ylabel('dT/dt (K/ms)');
% Text_1 = ['change rate'];
% title( Text_1 )
legend(['t = ' num2str(round(100*time(t)*1000)/100) ' ms']);
xlimit=xlim;
ylimit=ylim;
text(xlimit(1)+0.68*(xlimit(2)-xlimit(1)), ylimit(2)-0.28*(ylimit(2)-
ylimit(1)),{Text_4{1,1},Text_4{2,1}, Text_4{3,1}, Text_4{4,1}, Text_4{5,1}, Text_4{6,1},
Text_4{7,1}, Text_4{8,1}, Text_4{9,1}, Text_4{10,1}});
h=gcf;
figfilename = [  directory name ' dT_dx.fig'];
saveas(h,figfilename, 'fig');
```

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