# COUPLED RADIAL AND THERMAL FLUCTUATIONS

OF IGNITED TOKAMAK PLASMAS\*

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## Abstract

The coupling between radial motion and temperature fluctuations of an ignited D-T tokamak plasma are studied using a one-dimensional transport model. The energy balance equations are linearized about the thermal equilibrium (i.e., ignition) point and an eigenvalue analysis is used to determine the growth rate of perturbations. If the density dynamics are relatively slow, it is found that there is at most one growing eigenmode. The growth rate of temperature perturbations and the extent of radial movement depend upon a parameter  $\eta_r$ which describes the elasticity of radial motions ( $\eta_r = T/R dR/dT$ , where R is the major radius and T is the plasma temperature). A simple MHD model is used to determine the dependence of  $\eta_r$  upon the vertical field index ( $\eta = -R/B_v dB_v/dR$ ) and the poloidal beta. The implications of different scaling laws of the electron energy confinement time are examined. Similar values of  $\eta_r$  ( $\eta_r \sim 0.3$ ) can be obtained for representative *ETF/INTOR* and ignition test reactor (*ITR*) parameters. For  $\eta_r \sim 0.3$  the effect of radial motion is to reduce the central ion temperature needed for thermal stability from  $\sim$  50 keV to  $\sim$  30 keV if the electron energy confinement time scales as  $au_e \sim na^2$ ; at lower temperatures (central ion temperatures  $\sim 10-20$  keV), the thermal runaway time is increased by a factor  $\sim 2-3$ . It is pointed out that if the plasma edge is determined by some type of limiter at the midplane, small temperature fluctuations can significantly change the plasma minor radius; active burn control may be needed in some reactor designs to prevent unacceptably large radial movement in addition to providing constant plasma pressure and temperature.

# I. Introduction

An ignited plasma can be unstable against temperature fluctuations [1,2] in a temperature region in which fusion power density is maximized and requirements for adequate energy confinement in tokamaks may be minimum [3]. Most studies of thermal stability have dealt with incompressible plasmas. However, it has been shown that plasma expansion and decompression against a fixed magnetic field can be a strong stabilizing inechanism [4,5].

In this paper a one-dimensional model is used to calculate thermal stability characteristics of ignited D-T tokamak plasmas when plasma radial motion is included. An eigenmode analysis is used to identify temperature fluctuation modes and to determine thermal runaway growth rates. Thermal stability characteristics are determined in terms of  $\eta_r = T/R dR/dT$ , the elasticity of radial expansion with changes in the temperature. The effect of the finite pressure of the decelerating alpha particles is computed. Thermal stability characteristics are calculated for different values of the vertical field index  $\eta = -R/B_v dB_v/dR$  and poloidal beta. The implications of different scaling laws for the electron heat conductivity are determined.

The organization of the paper is as follows: section II describes a simplified model of the physical mechanisms by which radial motion affects the thermal stability characteristics. Section III discusses the onedimensional model and the eigenmode analysis of the plasma stability. In section IV, the effects of various parameters upon  $\eta_r$  are calculated using a simplified model for the *MHD* equilibrium. Section V describes the effects of  $\eta_r$  and different scaling laws upon thermal stability characteristics. A summary is given in section VI.

## II. Simplified Model of Physical Mechanisms

Zero dimensional calculations elucidate the physical mechanism by which the plasma radial motion affects the thermal stability characteristics. The plasma is described using the simplified model

$$\frac{dT}{dt} = nW_{\alpha} - \frac{T}{\tau} - \frac{2}{3}\frac{T}{V}\frac{dV}{dt}$$
(1)

where the last term represents the energy changes in the plasma due to changes in volume; n and T are the central plasma density and temperature, and  $\tau$  is the energy confinement time. It is assumed that the ion and the electron temperatures are equal.  $W_{\alpha}$  is the plasma reactivity,

$$W_{\alpha} = \frac{\overline{n^2 \langle \sigma v \rangle}}{4n_o^2} E_{\alpha} - \frac{\overline{n^2 W_b}}{n_o^2}$$

where  $\overline{n^2 W_b}$  represent the bremsstrahlung losses.  $\overline{n^2 \langle \sigma v \rangle} E_a/4$  is the volume average fusion power density.  $n_o$  is the central plasma density. Impurity radiation and synchrotron radiation have been neglected. Parabolic temperature and density profiles are assumed. The overstrike bars in (2) refer to radial average. V is the plasma volume.

It is assumed throughout this paper that the total number of particles remains constant during a perturbation; the thermal runaway and the energy confinement dynamics generally occur in a time scale that is faster than the density dynamics. Therefore,  $n \sim 1/V$ . It is also assumed that the changes in plasma volume occur in a time scale longer than the ion self-thermalization time. In addition, it is assumed that the radial movement of the plasma is such that no plasma is lost by increased scrape-off. Appropriate positioning of the limiters or feedback control by a divertor could be used to prevent additional scrape-off of the plasma. Furthermore, due to the small mutual inductance between the plasma and the vertical field system, it is assumed that the external magnetic field is constant in time.

For a tokamak plasma, conservation of the plasma toroidal flux implies that the volume varies as  $V \sim Ra^2 \sim R^2$ , where R and a are the major and minor radii of the plasma respectively [7]. (This relation is not appropriate for very low aspect ratios and/or at high plasma pressures [8]). The last term in equation (1) can be rewritten as

$$\frac{2}{3}\frac{T}{V}\frac{dV}{dt} = \frac{4}{3}\left(\frac{T}{R}\frac{dR}{dT}\right)\frac{dT}{dt} = \frac{4}{3}\eta_r\frac{dT}{dt}$$
(2)

The elasticity of the plasma motion,  $\eta_r$  is determined by the *MHD* properties of the system, and will be calculated using a simple model in section IV. With (2), (1) reduces to

$$\frac{dT}{dt} = \frac{nW_a - T/\tau}{1 + 4\eta_r} \tag{3}$$

Thus the energy transfer associated with the last term of (1) reduces the growth or damp rates of temperature perturbations.

Linearizing (3) we let  $T = T_0 + T_1$  where  $T_0$  is the equilibrium value of the plasma temperature and  $T_1$  is a perturbation. Then

$$\frac{dT_1}{dt} = \frac{\partial}{\partial T} \left( \frac{dT}{dt} \right) T_1 + \frac{\partial}{\partial R} \left( \frac{dT}{dt} \right) \frac{dR}{dT} T_1$$

$$= \left( \frac{1}{1 + \frac{1}{3}\eta_r} \right) \left( \frac{\partial}{\partial T} \left( nW_a - \frac{T}{\tau} \right) + \eta_r \frac{\partial}{\partial R} \left( nW_a - \frac{T}{\tau} \right) \right) T_1.$$
(4)

From (4) it can be seen how  $\eta_r$  affects the growth rate of a perturbation around the equilibrium point and how thermal stability can be obtained. Since

$$\frac{\partial}{\partial R} \left( n W_{\alpha} - \frac{T}{\tau} \right) < 0$$

when  $R/\tau d\tau/dR \leq 2$ , the condition for marginal stability

$$\frac{d}{dT_1}\left(\frac{dT_1}{dt}\right) = \mathbf{0}$$

can be obtained at sufficiently large values of  $\eta_r$ . Although this paper deals with coupled radial and thermal fluctuations in the presence of a static value of  $\eta_r$ , it should be noted that  $\eta_r$  can be varied by active control through compression-decompression [6].

# III. One-Dimensional Calculations of Equilibrium and Thermal Stability

In this section, a more exact plasma model will be used to calculate the thermal runaway characteristics of a tokamak plasma.

The ion and electron power balances are given by

$$\frac{3n}{2}\frac{\partial T_i}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}r\chi_i\frac{\partial T_i}{\partial r} - \frac{3n(T_i - T_e)}{2\tau_{ie}} + P_{\alpha,i} - 2\frac{n}{R}\frac{T_i}{dt}\frac{dR}{dt}$$
(5)

and

$$\frac{3n}{2}\frac{\partial T_e}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}r\chi_e\frac{\partial T_e}{\partial r} + \frac{3n(T_i - T_e)}{2\tau_{ie}} + P_{\alpha,e} - P_{br} - 2\frac{n}{R}\frac{T_e}{dt}\frac{dR}{dt}$$
(6)

where *n* is the plasma density,  $T_e$  and  $T_i$  are the electron and ion temperatures,  $\chi_e$  and  $\chi_i$  are the electron and ion thermal conductivities.  $\tau_{ie}$  is the electron-ion equilibration time.  $P_{br}$  is the bremsstrahlung power density. Losses due to neutral particles and synchrotron radiation have been neglected. The ohmic input power has also been ignored.  $P_{\alpha,e}$  and  $P_{\alpha,i}$  are the alpha particle heating power densities of the electrons and ions. It is assumed that the alpha particles deposit their energy in the flux surface where they are born; this approximation is valid for reactor-size tokamaks.

As in the last section, the density dynamics are ignored. The density profile is assumed to be parabolic. Equal D and T densities are assumed. No impurities are included in the calculations. The alpha particle dynamics are also ignored; the alpha slowing down time is significantly shorter than the runaway time and the alpha population is close to equilibrium. The partition of the alpha fusion energy between the ions and the electrons is calculated using the results of Sigmar and Joyce [9].

The last two terms in (5) and (6) represent the changes in temperature due to changes in major radius. For the purpose of this paper, the radial dynamics can be described by

$$\frac{dR}{dt} = \frac{R_{MIID} - R}{\tau_d} \tag{7}$$

where  $R_{MHD}$  is the radius determined by the *MHD* equilibrium (depedent upon the electron and ion temperatures) and  $\tau_d$  is the characteristic time for changes in the plasma position.  $\tau_d$  is determined by the decay time of the induced currents in the material surrounding the plasma. Under most circumstances,  $\tau_d \ll \tau_{runaway}$  (where  $\tau_{runaway}$  is the characteristic time for changes of the plasma temperature). This case is assumed in the calculations in this paper. (7) reduces to

$$\frac{dR}{dt} = \eta_r \frac{R}{T_{ave}} \frac{dT_{ave}}{dt}$$
(7a)

where

$$\eta_r = \frac{T_{ave}}{R_{MIID}} \frac{dR_{MIID}}{dT_{ave}}.$$
(8)

Here  $T_{ave}$  is defined by

$$T_{ave} = \frac{1}{\pi a^2 n_o} \int_0^a (n T_i + n T_e) 2\pi r dr.$$

where a is the plasma minor radius.

In the case when  $\tau_d \gg \tau_{runaway}$ , the stability of the plasma is not affected by the *MHD* equilibrium. The case  $\tau_d \gg \tau_{runaway}$  can represent either the case where the decay time of external currents opposing the plasma motion is very long, or where the vertical field is actively adjusted to maintain the plasma position. Previous calculations of the thermal stability characteristics of the plasma should be considered in this context [1,2,3,10].

It is assumed that the ion thermal conductivity is described by the neoclassical theory [11]. The assumed electron thermal conductivity is given by

$$\chi_e = \frac{6.5 \ 10^{19}}{F(T_e)} \ /\mathrm{m \ s} \tag{9}$$

With  $F(T_e) = 1$  and parabolic density profile, the electron energy confinement time given by (9) is in agreement with the *ALCATOR* empirical scaling law [12]

$$\tau_e = \frac{1}{2} \tau_E = 1.9 \, 10^{-21} \, n_o \, a^2 \tag{10}$$

where  $n_o$  is in m<sup>-3</sup>, a is in m and  $\tau_e$  is in s.  $F(T_e)$  indicates the temperature dependence of the electron thermal conductivity. It is assumed that the electron energy confinement time does not depend on the ion temperature. As the electron and ion temperatures vary together for the mode with the largest growth rate [10], for the

stability calculations it is immaterial whether the electron conductivity varies with the ion or with the electron temperatures. In this paper the assumed dependence of  $F(T_e)$  is given by

$$F(T_e) \sim \begin{cases} 1, & \text{if } T_e < 3; \\ T_e^{\epsilon}, & \text{if } T_e > 3, \end{cases}$$
(11)

where  $T_e$  is in keV. Two values of  $\epsilon$  will be considered:  $\epsilon = 0$  (ALCATOR scaling [12]) and  $\epsilon = 1$  [13].

The system of equations (5)-(7) is solved by a procedure similar to the one used in reference [10]. The equilibrium is solved by requiring that

$$\frac{\partial}{\partial t} = 0.$$
 (12)

At equilibrium (*i.e.*, ignition), (5) and (6) are reduced to a set of ordinary nonlinear differential equations. These equations are solved by standard methods. The profiles obtained are significantly more peaked than parabolic, specially in the case when the electron energy confinement time improves with temperature ( $\epsilon > 0$ ).

The stability of the system of equations (5)-(7) is solved by a procedure similar to the one used in reference [10]. The stability analysis is done in the following manner: equations (5) and (6) are replaced by a set of difference equations; this is the physical equivalent of dividing the plasma into several radial zones. These difference equations are linearized about the equilibrium electron and ion temperatures. The linearized system of equations depends on  $R_{MIID}$  through  $\eta_r$ . Therefore, as in the simplified model of the previous section, the stability analysis can be done parametrically on  $\eta_r$ . ( $\eta_r$  will be calculated using a simple model in the next section).

The alpha particle dynamics have been neglected (previously it has been shown that the alpha dynamics do not significantly affect the calculation of the thermal runaway time [10]).

The eigenvalues of the resulting set of linear equations are obtained. The number of eigenvalues is equal to two times plus one the number of zones. If the real part of the largest eigenvalue,  $\text{Re}(\gamma_{max})$ , is positive then the system is unstable. The runaway time,  $\tau_{runaway}$ , is defined by

$$\tau_{runaway} = \frac{1}{\operatorname{Re}(\gamma_{max})} \tag{13}$$

The effect of the number of zones on the largest eigenvalues is discussed in reference [10]. As the number of zones is increased, the number of eigenvalues increases. The new eigenvalues are strongly damped; however,

the largest eigenvalues remain approximately constant as the number of zones is increased.

# **IV. Effect of MHD Equilibrium Requirements**

The *MIID* equilibrium determines the value of  $T_{ave}/R dR/dT_{ave}$ . In a tokamak plasma, as the plasma pressure increases, the plasma expands in major radius against the vertical magnetic field. In this paper the externally imposed vertical magnetic field assumed to be fixed in time, although it may vary in space. The assumed vertical field is given by

$$B_{ver} = B_{ver,o} \left(\frac{R_o}{R}\right)^{\eta} \tag{14}$$

where  $R_o$  is the equilibrium position of the plasma and  $B_{vcr,o}$  is the value of the vertical magnetic field at equilibrium.  $\eta$  is sometimes called the index of the vertical field.

For a circular plasma with a large aspect ratio (A = R/a > 1) the radial position is determined from the relation

$$B_{ver} = \frac{\mu_0 I}{4\pi R} \left( \log(8A) + \beta_p - 1.5 + \frac{l_i}{2} \right)$$
(15)

where I is the plasma current and  $l_i/2$  is related to the poloidal field energy inside the plasma

$$\frac{l_i}{2} = \frac{4\pi}{\mu_o I_p^2} \int_0^a \frac{B_p^2}{2\mu_o} 2\pi r dr$$
(16)

Here  $B_p$  is the poloidal magnetic field and the integral is over the plasma cross-section. The poloidal beta is defined as

$$\beta_p = \frac{\langle n_i k T_i + n_c k T_c + \overline{P_\alpha} \rangle}{B_p^2 / 2\mu_o} \tag{17}$$

where  $\overline{P_{\alpha}}$  represents the contribution to the plasma pressure by the suprathermal alpha particles.  $\overline{P_{\alpha}}/n_o$  at ignition depends only on the electron and ion temperatures.

If the plasma is flux conserving and has an aspect ratio  $A \gg 1$ , then the plasma current scales as [7]

$$I(R) = I(R_o) \frac{R_o}{R}$$
(18)

This scaling law breaks down for small aspect ratios and/or large  $\beta_p[8]$ .

In this paper it will be assumed that the plasma shifts position without the generation of skin currents. This implies a feedback system such that the *OII* field is adjusted to cancel any induced skin currents. This case is considered because the dynamics of current penetration are not well understood. A simplified model including skin currents indicates that the skin currents result in larger values of  $\eta_r$ , and therefore in more stable plasmas. It is further assumed that the relative profile of the plasma current does not vary during volumetric changes; in this case,  $l_i/2$  is constant.

From (14)-(18), the value of  $\eta_r$  is given by

$$\eta_r = \frac{T_{ave}}{R} \frac{dR}{dT_{ave}} = \frac{T_{ave}}{T_{ave} + \overline{P_a}/n_o} \frac{d}{dT_{ave}} \left( T_{ave} + \frac{\overline{P_a}}{n_o} \right) \left( \frac{R}{\beta_{po}} \frac{d\beta_{po}}{dR} - 1 \right)^{-1}$$
(19)

where

$$\frac{\beta_{po}}{R} \frac{dR}{d\beta_{po}} = \frac{\beta_{po}}{(2-\eta)(\ln(8A) + \beta_{po} + li/2 - 1.5) - 1/2}$$
(20)

 $\beta_{po}$  is the value of  $\beta_p$  at equilibrium (*i.e.*, ignition).  $\eta_r$  would be independent of temperature if the alpha particles were not prelent. The presence of the alpha particles increases the value of  $\eta_r$  by  $\sim 20\%$  over a wide range of temperatures. This is because of the dependence of  $\overline{P_{\alpha}}$  on the plasma temperature. The value of  $\overline{P_{\alpha}}/n_o$  is determined solely by  $T_e$  and  $T_i$  [3].

 $\eta_r$  is shown in Figure 1 as a function of the index of the field  $\eta$  for several values of  $\beta_{po}$ . This figure is calculated for central temperatures  $T_{io} \sim T_{co} \sim 15$  keV. As the index of the field  $\eta$  and/or as the plasma  $\beta_{po}$  increase, the radial excursion for a given excursion in temperature increases. For low value of  $\beta_{po}$ ,  $\eta_r$  is small; this is the situation in present day experiments. For  $\eta \sim 1.5$ ,  $\eta_r$  is very large; this corresponds to the usual limit of  $\eta$  for stability of the plasma against horizontal motions [7]. Similar values of  $\eta_r$  ( $\eta_r \sim 0.3$ ) can be obtained for representative *ETF/INTOR* parameters at ignition ( $a \sim 1.2$  m,  $R \sim 5.5$  m,  $\beta_{po} \sim 3$ ,  $\eta \sim -1$ ) [14,15] and for representative ignition test reactor (*ITR*) parameters ( $a \sim 0.5$  m,  $R \sim 1.5$  m,  $\beta_{po} \sim 1.5$ ,  $\eta \sim 0.2$ ) [16]. The lower value of  $\eta$  in the reactor size machine ( $\eta \sim -1$  as compared to  $\eta \sim 0.2$  for the ignition test reactor) is balanced by the larger value of  $\beta_p$  in the reactor size machine. The characteristic values of  $\eta_r$  will be altered in high  $\beta_p$ , flux conserving devices [17,18] because of the lack of applicability of simple models of *MHD* equilibrium.

The radial motion due to temperature fluctuations results in amplified changes in the plasma minor radius if the plasma edge is determined at the midplane; the resulting changes in the plasma minor radius is

$$\frac{\delta a}{a} = A\eta_r \frac{\delta T}{T}$$

For temperature perturbations  $\sim 10\%$ ,  $\delta a/a \sim 15\%$  for  $A \sim 5$  and  $\eta_r \sim 0.3$ . Such large changes in minor radius are probably unacceptable; if thermally driven plasma motion is allowed, it would probably be necessary to use limiters on the top and bottom of the vacuum chamber and to allow sufficient space for radial movement. Active plasma control may be needed in some device designs to prevent unacceptably large radial movement as well as to provide constant plasma pressure and temperature during burn.

#### V. Implications for Thermal Instability Growth Rates and Thermal Stability Requirements

The runaway time  $\tau_{runaway}$  of the thermal instability at ignition is shown as a function of the central ion temperature for different values of  $\eta_r$  in Figures 2 and 3. It is assumed that  $a \sim 1.2$  m. The curves in Figure 2 are calculated assuming  $\epsilon = 0$ . The curves in Figure 3 are calculated for  $\epsilon = 1$ . As  $\eta_r$  and the plasma temperature increase,  $\tau_{runaway}$  increases (*i.e.*, the plasma is more stable against temperature perturbations). The plasma is marginally stable when  $\text{Re}(\gamma_{max}) = 0$  (that is,  $\tau_{runaway}$  is infinite). For larger values of the temperature, the plasma becomes stable, and perturbations decay away. The mode with the largest growth rate has a spatial dependence approximately equal to the spatial profile of the temperature.

The negative values of  $\tau_{runaway}$  in Figures 2 and 3 represent the inverse of the damping rate of a perturbation with a structure similar to the eigenmode corresponding to the eigenvalue  $\gamma_{inax}$ ; these values provide a measure of how stable the system is.

Allowing for radial plasma motions significantly decreases the temperature required for marginal stability and significantly increases the thermal runaway time. For  $\epsilon = 0$  the required central ion temperature for marginal stability when  $\eta_r = 0$  is  $T_{io} \ge 50$  keV. Allowing for the radial motion reduces the temperature needed for stability to  $T_{io} \le 30$  keV for representative *ETF/INTOR* and ignition test reactor parameters characterized by  $\eta_r \sim 0.3$ .

Although similar thermal stability characteristics are obtained for representative *ETF/INTOR* and *ITR* parameters, the time-scales of the thermal instability are different. As long as  $\chi_e \gg \chi_i$  (which occurs for  $\chi_i$  given by neoclassical theory and  $\chi_e$  given by (9) with  $\epsilon \leq 1$ ) the results in Figures 2 and 3 can be scaled to other machine sizes by the scaling

### $au_{runaway} \sim a$

Therefore, the runaway time of a representative *ETF/INTOR* design is a factor of  $\sim 2$  longer than the runaway time of a representative ignition test reactor design.

For fixed  $\eta_r$  and  $T_{io}$  the runaway time of the instability decreases with increasing  $\epsilon$ . The system is more unstable, because of the destabilizing effect of an energy confinement time that increases with temperature. For  $\epsilon \sim 1$ , the plasma conduction losses are independent of the plasma temperature.

Figure 4 shows the value of  $\eta_r$  required for marginal stability  $\eta_{r,marg}$  as a function of the central ion temperature for the cases  $\epsilon = 0$  and  $\epsilon = 1$ . As the temperature increases, the value of  $\eta_r$  required for stability

decreases. This is because the plasma is inherently more stable. The value of  $\eta_r$  required for stability could in principle be provided by active position control of the plasma by compression-decompression [6].

## VI. Conclusions

A one dimensional model has been used to perform an eigenmode analysis of coupled radial and temperature fluctuations of an ignited plasma. The growth rates and the structure of the eigenmodes have been determined. For the electron heat conductivities given by  $\chi_e \sim T_e^{-1}$  or  $\chi_e \sim$  constant, the system has at most one growing mode, with a radial structure similar to the equilibrium profiles. The growth rate of temperature fluctuations and the extent of radial movement depend upon the parameter  $\eta_r = T/R dR/dT$  which represents the elasticity of radial expansion. A simplified model for the *MHD* equilibrium have been used to determine  $\eta_r$ . It is found that similar values of  $\eta_r$  ( $\eta_r \sim 0.3$ ) can be obtained for representative *ETF/INTOR* parameters and representative ignition test reactor (*ITR*) parameters; hence the stability characteristics for these representative design parameters can be similar. Allowing for radial motion of the plasma during thermal excursions results in a large stabilizing force. If the electron energy confinement scales as  $\tau_e \sim na^2$ , an *ETF/INTOR* plasma at ignition (with  $\beta_p \sim 3$  and  $\eta \sim -1$ ) could be stable at central ion temperatures  $\sim 30$  keV; in contrast, if no radial excursions are allowed, central ion temperatures  $\sim 50$  kev would be required for thermal stability. At lower temperatures ( $T_{io} \sim 10 - 20$  keV) the radial motion has the effect of increasing the runaway time by a factor  $\sim 2 - 3$ . If the energy confinement time increases with temperature ( $\chi_e \sim T_e^{-1}$ ), the effect of the radial motion is not sufficient for stability for central ion temperatures  $T_{io} < 50$  keV.

Because  $\delta a/a \sim A\eta_r \delta T/T$  if the plasma edge is determined by some type of limiter at the midplane, small temperature fluctuations can significantly change the minor radius; active burn control may be needed in some reactor designs to prevent unacceptably large radial movement in addition to providing constant plasma pressure and temperature.

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Figure 1. Dependence of  $\eta_r = T_{ave}/R dR/dT_{ave}$  upon  $\eta = -R/B_v dB_v/dR$  for several values of  $\beta_p$ . This figure is calculated using a simplified *MHD* equilibrium model.  $\Lambda = 4$  and  $T_{eo} \sim T_{io} \sim 15$  keV.

Figure 2. Value of  $\tau_{runaway}$  as a function of the central ion temperature for an ignited device with a = 1.2 m.

 $\epsilon = 0$  is assumed. Negative values of  $\tau_{runaway}$  correspond to the damping times.

Figure 3. Same as Figure 3, but for  $\epsilon = 1$ .

Figure 4. Value of  $\eta_{r,marg}$  for stability as a function of the central ion temperature for  $\epsilon = 0$  and  $\epsilon = 1$ .









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