Effect of Impurities and Ripple Upon Power Regulation in Self-Sustained Tokamaks

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Abstract

Tokamak power reactors will likely operate in a self sustained heating mode where additional power losses are introduced to permit higher levels of alpha particle heating (and thus higher levels of total total power) at thermal equilibrium. Illustrative 0-dimensional calculations are made to assess requirements for power regulation of self sustained tokamak plasmas by the use of impurity radiation. Effects of impurities upon allowable fuel density and thermal stability are determined. Requirements are calculated for passive thermal stability control by temperature driven radial motion in the presence of ripple transport losses; it appears that stability might be attained over a relatively wide temperature range with a small amount of ripple transport loss. Requirements for power regulation by the use of ripple transport are also determined.

I. Introduction

The utilization of tokamaks as power reactors will likely require self sustained operation at power levels where intrinsic power loasses are not sufficient to balance the the alpha particle heating; additional power loss must be introduced to achieve thermal equilibrium. The alpha particle heating power (and thus the total fusion power which is five times the alpha particle heating power) is regulated by varying the amount of additional power loss. The use of impurity radiation for power regulation has been suggested [1,2,3]; impurities would be introduced after self sustained operation with intrinsic power loss (*i.e.* ignition) has been achieved. An attractive feature of impurity radiation is that a large fraction of the plasma power loss could be uniformly distributed, reducing the loading on a limiter or divertor [3]. However, the use impurities decreases the allowable fusion power level for fixed plasma β (ratio of plasma pressure to magnetic field pressure) and increases requirements for thermal stability control. In this paper illustrative calculations are made to determine requirements for power regulation in self sustained DT tokamaks by the use of different impurities and to examine tradeoffs between limits on fuel density and thermal stability effects. Requirements for thermal stability control by temperature driven radial motion in the presence of ripple transport loss are computed. The paper also includes a section on the use of ripple transport loss for power regulation.

II. Thermal Equilibrium Control Using Impurities

A 0-dimensional calculation is used to determine thermal equilibrium and stability characteristics. The density and temperature profiles are assumed parabolic and appropiate averages are taken. The electron power balance is given by

$$\frac{1}{2}n_e\frac{dT_e}{dt} = P_\alpha(1-f) - \frac{1}{2}\frac{n_eT_e}{\tau_e} + \frac{3}{5}\frac{n_i(T_i - T_e)}{\tau_{ie}} - P_{br} - P_{rad} - \frac{2}{3}n_e\frac{T_e}{V}\frac{dV}{dt} \qquad (1)$$

where T_i and T_e are the central ion and electron temperatures, n_e and n_i are the central electron and fuel ion densities, P_α is the alpha particle heating power density, f is the fraction of alpha heating power which goes to the ions, P_{br} is the Bremsstrahlung radiation power density, τ_e is the electron energy confinement time, τ_{ie} is the ion-electron equilibration time, and V is the plasma volume. P_{rad} is the line radiation power density which results from the presence of the impurities [4]. It is assumed that the radiation satisfies coronal equilibrium. The last term in (1) represents the power loss due to volume changes resulting from changes in major plasma radius.

For the purpose of illustrative calculations the electron energy confinement time is taken to be given by the *ALCATOR* empirical scaling law

$$\tau_e = 1.9 \times 10^{-21} n_e a^2$$

where a is the minor radius in m and n_e is in m^{-3}.

The power balance equation for the deuterium and tritium ions is

$$\frac{1}{2}n_i\frac{dT_i}{dt} = P_\alpha f - \frac{1}{2}\frac{n_iT_i}{\tau_i} - \frac{3}{5}\frac{n_i(T_i - T_e)}{\tau_{ie}} - \frac{2}{3}n_i\frac{T_i}{V}\frac{dV}{dt} \qquad (2)$$

The deuterium and tritium concentrations are taken to be equal. The relation between deuterium and tritium ion densities, n_i, and the electron density is determined by the impurity concentration, the type of impurity and the electron temperature.

The ion energy confinement time is taken to be given by

$$\frac{1}{\tau_i} = \frac{1}{\tau_{i,neocl}} + \frac{1}{\tau_{i,ripple}}$$

where $\tau_{i,neocl}$ is the neoclassical energy confinement time and $\tau_{i,ripple}$ is the ion energy confinement time due to the ripple losses. In the absence of ripple losses, $\tau_i/\tau_e \gg 1$ in most regimes of interest.

At self sustained thermal equilibrium,

$$\frac{dT_e}{dt} = \frac{dT_i}{dt} = \frac{dV}{dt} = 0$$

The alpha particle heating power just balances all losses. The plasma conditions at thermal equilibrium are found by solving equations (1) and (2) and by using the relation between n_e and n_i at a given impurity concentration level. In determining the effects of impurities upon thermal equilibrium control the simplfying assumption $\tau_i \gg \tau_e$ is made. The values of n_i at equilibrium are found for given values of a, T_i, impurity concentration and atomic number. The fusion power density is then calculated. For electron energy confinement given by $\tau_e \sim n_e a^2$, the products $n_i a$ and $P_\alpha a^2$ are uniquely determined. The values of $P_\alpha a^2$ can then be related in a straight forward way to the total fusion power divided by the major radius, P_T/R.

Figure 1 shows the dependence of $\overline{n_i}$ (the volume average density) and P_T/R upon impurity concentration n_z/n_i for Krypton (Z =36) for a density average temperature of $\langle T_i \rangle \sim 13$ keV. It is assumed that $a = 1.2$ m. It can be seen that P_T/R can be increased by about a factor of 3 for an impurity to fuel ratio of $n_z/n_i \sim 2 \times 10^{-3}$.

Figure 2 shows the dependence of P_T/R upon impurity concentration for argon (Z = 18), krypton (Z = 36) and xenon (Z = 54). It is assumed that $\langle T_i \rangle = 13$ keV. The minimum value of P_T/R, $(P_T/R)_{min}$ occurs at self sustained operation without the addition of impurities. The impurity concentration required to increase the thermal power level increases significantly with decreasing atomic number. As the fusion power level is increased beyond $2(P_T/R)_{min}$, it becomes a very sensitive function of impurity concentration.

The contribution of impurities to the plasma pressure has been calculated and is taken into account in Figure 1 and 2. The ratio of the impurity pressure to the total plasma pressure, P_Z is given by

$$P_Z = \frac{n_z T_i + n_z T_e Z_{ave}}{(n_i + n_z)T_i + n_e T_e + P_\alpha}$$

where P_α represents the contribution by the alpha particles and Z_{ave} is the average charge of the impurity ions. At small values of P_Z, the fusion power is decreased by $\sim 2P_Z$ through fuel depletion at fixed β. Table I gives the values of P_Z necessary to obtain $P_T/R \sim 3(P_T/R)_{min}$ at $\langle T_i \rangle = 13$ keV for different impurities. The corresponding decrease in the fusion power from fuel depletion is also given.

As mentioned before, it has been assumed that the impurity density, the fuel density and the temperature profiles are parabolic. One dimensional calculations are needed to determine the appropiateness of this approximation. Some qualitative insight as to the effect of the impurities can be obtained by the identification of three regimes: a conduction dominated regime, where impurity radiation has little or no effect on the power balance in most regions of the plasma; a radiation dominated regime, where impurity radiation is much larger than thermal conduction and there is virtually no thermal coupling between the different spatial regions; and an intermediate regime where impurity radiation has a large effect on the overall power loss but where thermal conduction still provides coupling between different spatial regions. In the radiation dominated regime, the temperature profiles will be determined by the radial distribution of the impurities; the temperature may peak off-axis. In the intermediate regime, the temperature profiles tend to be peaked in the center of the plasma. The calculations presented in this paper are representative of the conduction dominated and intermediate regimes.

The temperature profiles can be strongly affected by the atomic number of the impurity as well as by the concentration level and spatial distribution. For low and middle-Z impurities, at temperatures in the keV range, the radiation power loss is relatively flat as a function of temperature. For high-Z impurities in this temperature range the radiation power increases strongly with decreasing temperature. Hence the presence of low and middle-Z impurities could result in flatter temperature profiles and smaller temperature gradients throughout the plasma interior than would the presence of high-Z impurities. The presence of high-Z impurities would lead to steeper temperature profiles with strong conduction in the interior and strong radiation in the outer regions of the plasma.

III. Effect of Impurities on Thermal Stability

Thermal stability characteristics are determined by linearizing the 0-dimensional power balance equations (1) and (2) with respect to ion and electron temperature perturbations from equilibrium. The simultaneous perturbation in major radius for fixed vertical field is also included. The eigenmodes of the linearized equations are determined. It is found that there is only one growing eigenmode. The thermal runaway time, $\tau_{runaway}$ is given by the inverse of the real part of the growth rate of this eigenmode, $\mathrm{Re}(\gamma)$. Marginal thermal stability (where perturbations neither grow nor decay) occurs when $\mathrm{Re}(\gamma) = 0$; in this case, $\tau_{runaway} = \infty$.

Figure 3 shows the dependence of the runaway time $\tau_{runaway}$ on temperature and impurity concentration for self sustained plasma. with krypton impurities. It is assumed that $a = 1.2$ m. Since the vertical field is assumed to be fixed in time, the radial motion which accompanies the temperature excursion reduces the growth rate of the thermal instability by varying the plasma density and the alpha heating power . The effect of radial motion upon thermal stability is determined by the parameter

$$\eta_r = \frac{T_{av}}{R}\frac{dR}{dT_{av}}$$

where R is the major radius of the plasma and T_{av} is the average plasma temperature (including the fast alpha particles) [5,6]. η_r is determined by the MHD characteristics of the plasma; a typical values of $\eta_r = 0.2$ is used. It is assumed in figure 3 that there is no power loss by ripple transport. The thermal stability charateristics for xenon and argon display similar behaviour, if the relative concentrations are adjusted so that the fusion power level is kept constant.

In the absence of impurities, thermal stability ($\tau_{runaway} \sim \infty$) occurs for $\langle T_i \rangle \sim 20$ keV. For $n_Z/n_i \sim 2.3 \times 10^{-3}$ thermal stability occurs at $\langle T_i \rangle \sim 28$ keV.

Figure 3 also shows contours of constant levels of P_T/R. For a given temperature the thermal runaway times become shorter as P_T/R is increased by adding impurities. It can be seen that the impurity levels which correspond to significant increases in P_T/R result in the presence of thermal instability over a very wide temperature range.

The 0-dimensional calculations are valid if the profiles of the temperature perturbations have the same shape as the equilibrium temperature profiles. 1-dimensional calculations made for plasmas without the presence of the impurities indicate that these profiles are approximately the same; there is only one growing eigenmode [5]. However, the presence of a sufficiently large amount of impurity radiation may provide enough decoupling between the radial zones of the plasma to allow additional modes to grow. The stabilizing effect of radial motion is reduced for these modes; this reduction occurs since the average temperature, which is responsible for the plasma motion, can be relatively unaffected by oppositely directed changes in temperature at different radial locations.

IV. Passive Thermal Stability by Ripple Transport and Radial Motion

Thermal stability control is needed to allow the flexibility to operate at whatever temperatures optimze plasma performance characteristics, such as maximization of the fusion power density and minimization of adverse plasma-wall interactions. It has been shown that in the absence of impurities passive thermal stability can be obtained over a very wide temperature range by the combination of a small amount of fixed ripple and the presence of radial motion driven by temperature fluctuations [7]. This approach is now applied with the additional effect of the impurities. The minimum amount of ripple transport loss power ($P_{i,ripple} = 0.5 n_i T_i / \tau_{i,ripple}$) required for thermal stability is determined by the requirement of marginal stability. $\tau_{i,ripple}$ is written as

$$\tau_{i,ripple} \sim C T_i^{-\epsilon}$$

where C is an adjustable parameter and $\epsilon = 7/2$ for the case of ripple trapping loss [8] or $\epsilon = 3/2$ for ripple plateau loss [9]. C is determined from the requirement that the system of equations (1) and (2) be marginally stable.

The presence of the ripple transport loss affects the linearized power balance equations through the presence of a power loss which increases with increasing plasma temperature. Furthermore, because the ripple varies in space (it generally increases with increasing major radius) as the plasma moves it sees a varying ripple. The effect of plasma movement upon $\tau_{i,ripple}$ can be characterized by the parameter

$$\gamma_i = \frac{R}{n\tau_i}\frac{dn\tau_i}{dR}$$

For ripple trapping transport, typical values of γ are $\gamma = -40 - 50$.

Figure 4 shows values of $P_{i,ripple}/P_\alpha$ required for marginal stability as a function of the plasma temperature and the impurity concentration for krypton; $\epsilon = 7/2$ and $\gamma_i = -40$. Also shown are contours of constant values of P_T/R. It can be seen that over a relatively wide range in temperature and power production level thermal stability can be achieved with a relatively small amount of ripple trapping power loss. Ripple trapping loss can thus play a major role in providing for thermal stability while at the same time having a rather small effect on thermal equilibrium characteristics.

V. Use of Ripple for Thermal Equilibrium Control

It has been suggested that ripple transport loss can be used to provide power regulation by variation of density at fixed ripple [10] or by varying both the density and the amount of ripple [11]. Equations (1) and (2) have been used to determine the density and ripple dependence of temperature and fusion power at thermal equilibrium It is assumed that there are no impurities. Figure 5 shows P_T/R as a function of the average plasma density for $\epsilon = 7/2$. Also shown are contours of constant $\langle T_i \rangle$. Contours of constant value of K correspond to contours of constant ripple. The ratio of ripple transport loss to electron transport loss described by the *ALCATOR* empirical scaling is represented by $\tau_e/\tau_{i,ripple} = K(\langle T_i \rangle/10)^{7/2}$ where K depends on the ripple and $\langle T_i \rangle$ is in keV. It can be seen that for $K = 0$ (no ripple losses), there is a relatively narrow range of power variation wich is accompanied by a rather large variation in temperature. The presence of a small amount of fixed ripple allows a large power variation with small variation of the plasma temperature. The capability to vary the ripple is necessary for independent power level and temperature control.

Figure 6 shows results for $\epsilon = 3/2$. Larger temperature changes are required for a given increase in fusion power than that required for $\epsilon = 7/2$. This difference is due to the weaker temperature dependence of the ripple losses.

Conclusions

In the case of power regulation by ripple transport, the ripple requirements for passive thermal stabilization are easily satisfied. The possible difficulties associated with the use of ripple for power level control are large increase in thermal conduction (which could result in increases in non-uniform heating of the first wall, limiter or divertor); adverse effects on the confinement of energetic particles, and the addition of the technological complexity of variable ripple for independent power level and temperature control.

Middle and high-Z impurities can be used for power level regulation of self sustained tokamaks with a small effect on the β limits on fuel density. The power level and the temperature at thermal equilibrium can be varied independently. However, the equilibrium temperature profiles depend in a complicated way upon the impurity transport and the atomic number of the impurity. The use of impurities makes the plasma less thermally stable; some type of passive or active thermal stability control will be required to allow for stable operation over a wide temperature range. Passive control could be achieved by allowing for radial motion in the presence of a small amount of fixed ripple transport loss if there only one growing mode. However, 1-dimensional effects on thermal stability could introduce significant differences and must be calculated.

	Argon	Krypton	Xenon
n_z/n_i	1×10^{-2}	2.2×10^{-3}	6×10^{-4}
P_Z	9×10^{-2}	3×10^{-2}	1.3×10^{-2}
$\Delta P_f/P_f$	1.8×10^{-1}	6×10^{-2}	2.6×10^{-2}

Table 1. Ratio of the impurity pressure to the plasma pressure, P_Z for $P_T/R \sim 3(P_T/R)_{min}$. $\langle T_i \rangle = 13$ keV. Also shown are ther required impurity concentrations n_Z/n_i and the relative decrease in fusion power resulting from fuel depletion for β-limited plasmas.

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List of Figures

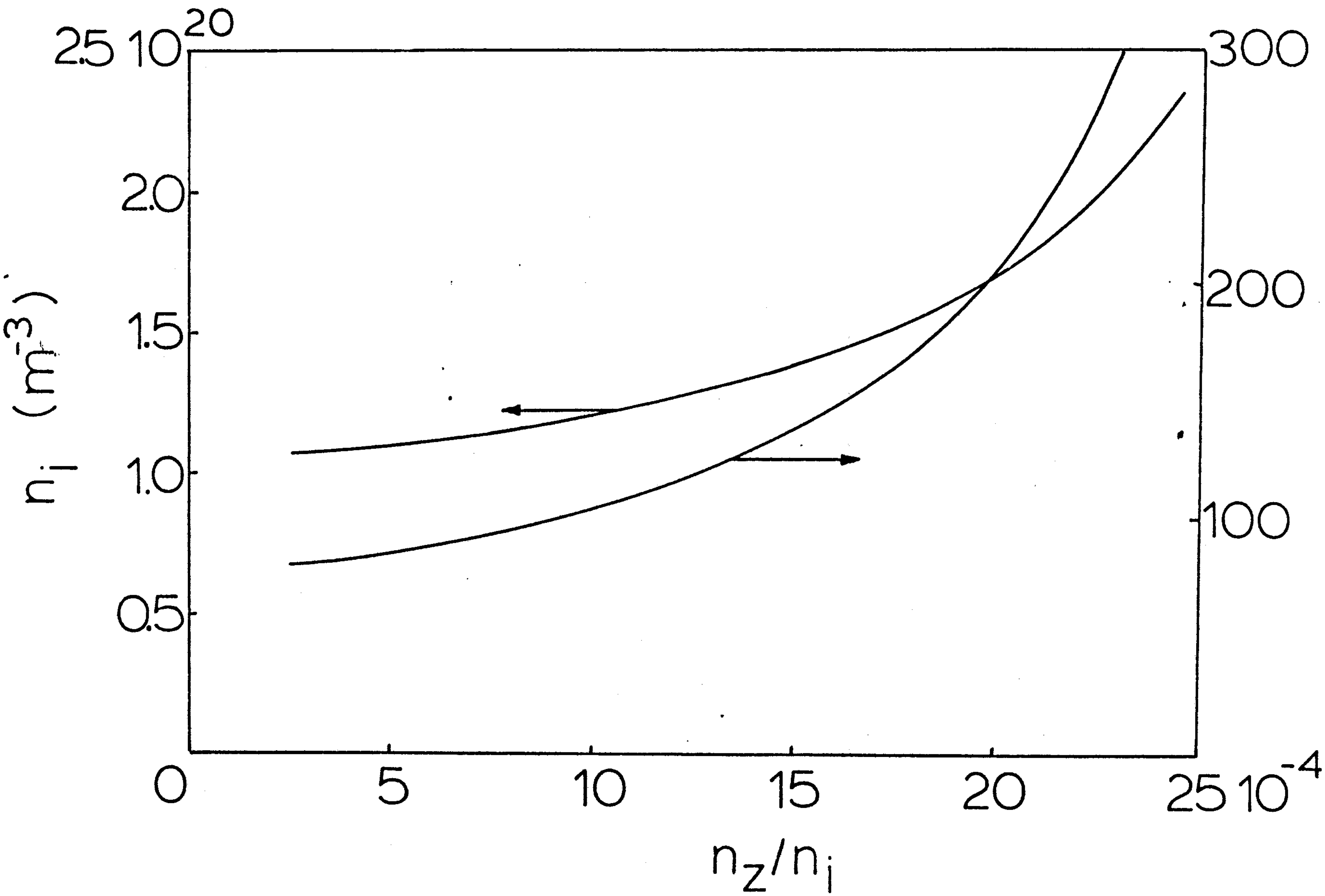

2.5 10^20
2.0
1.5
1.0
0.5
n_i (m^{-3})
0
5
10
15
20
25 10^-4
n_Z/n_i
300
200
100
P_T/R (MW/m)

Figure 1

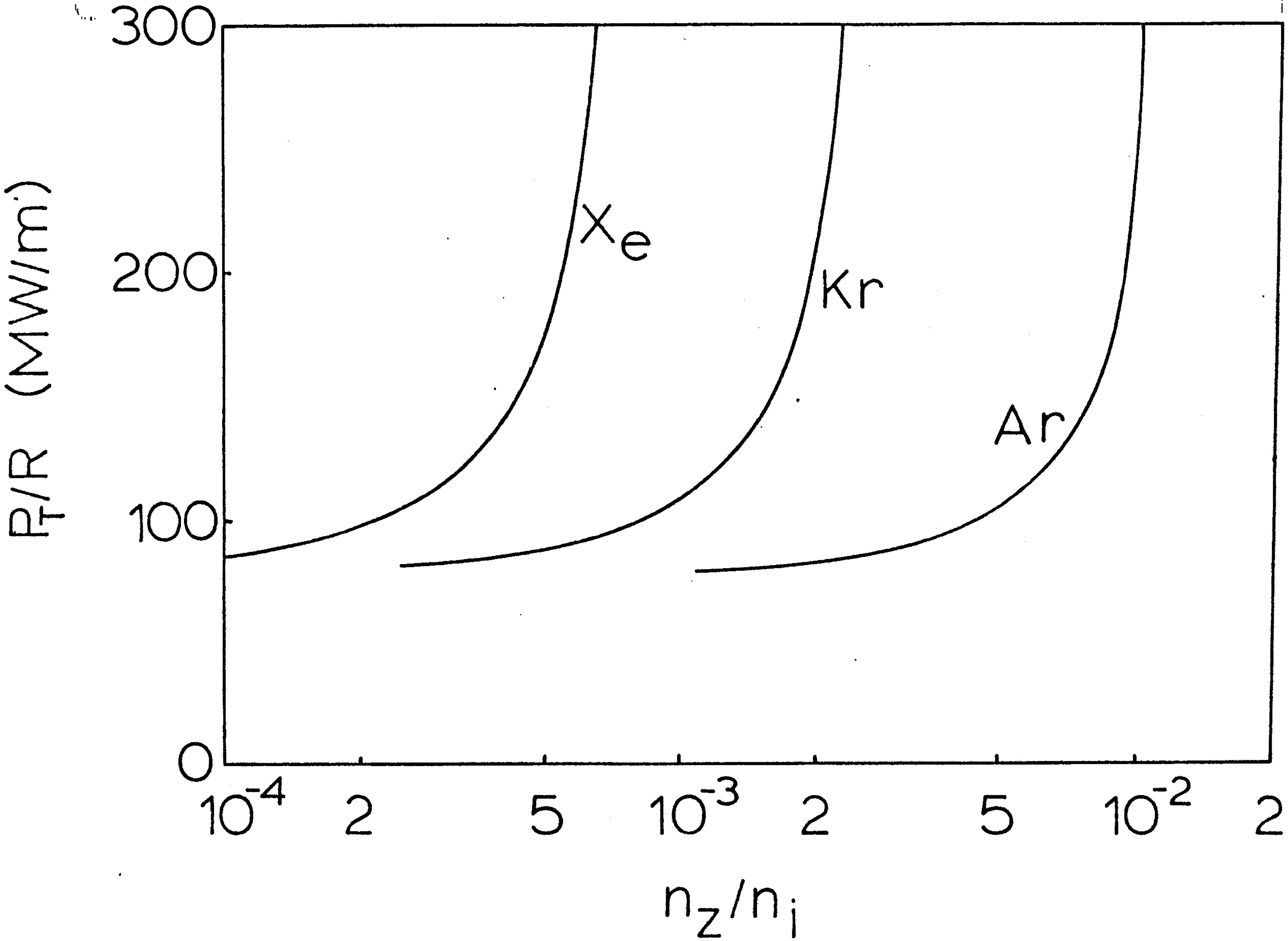
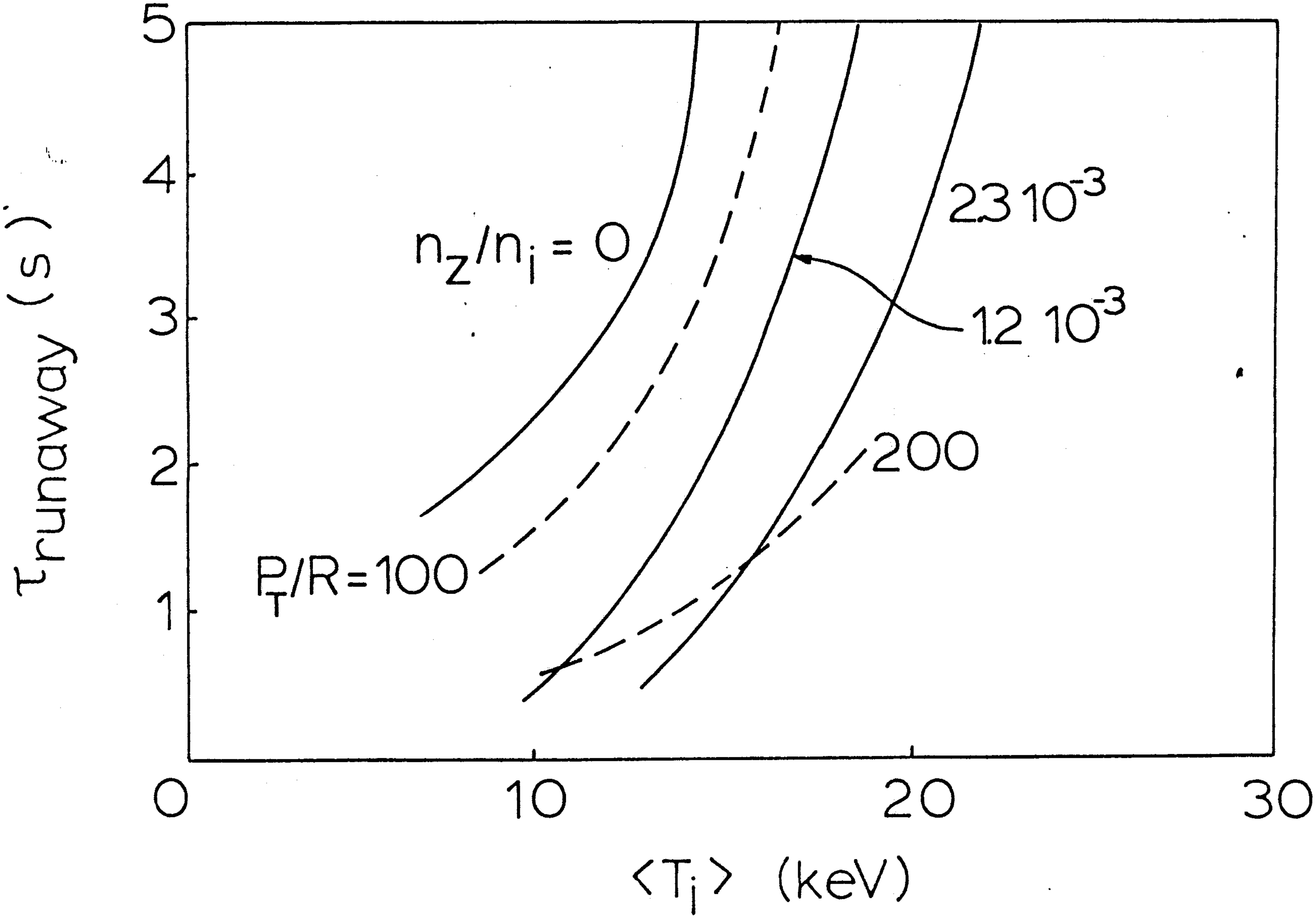
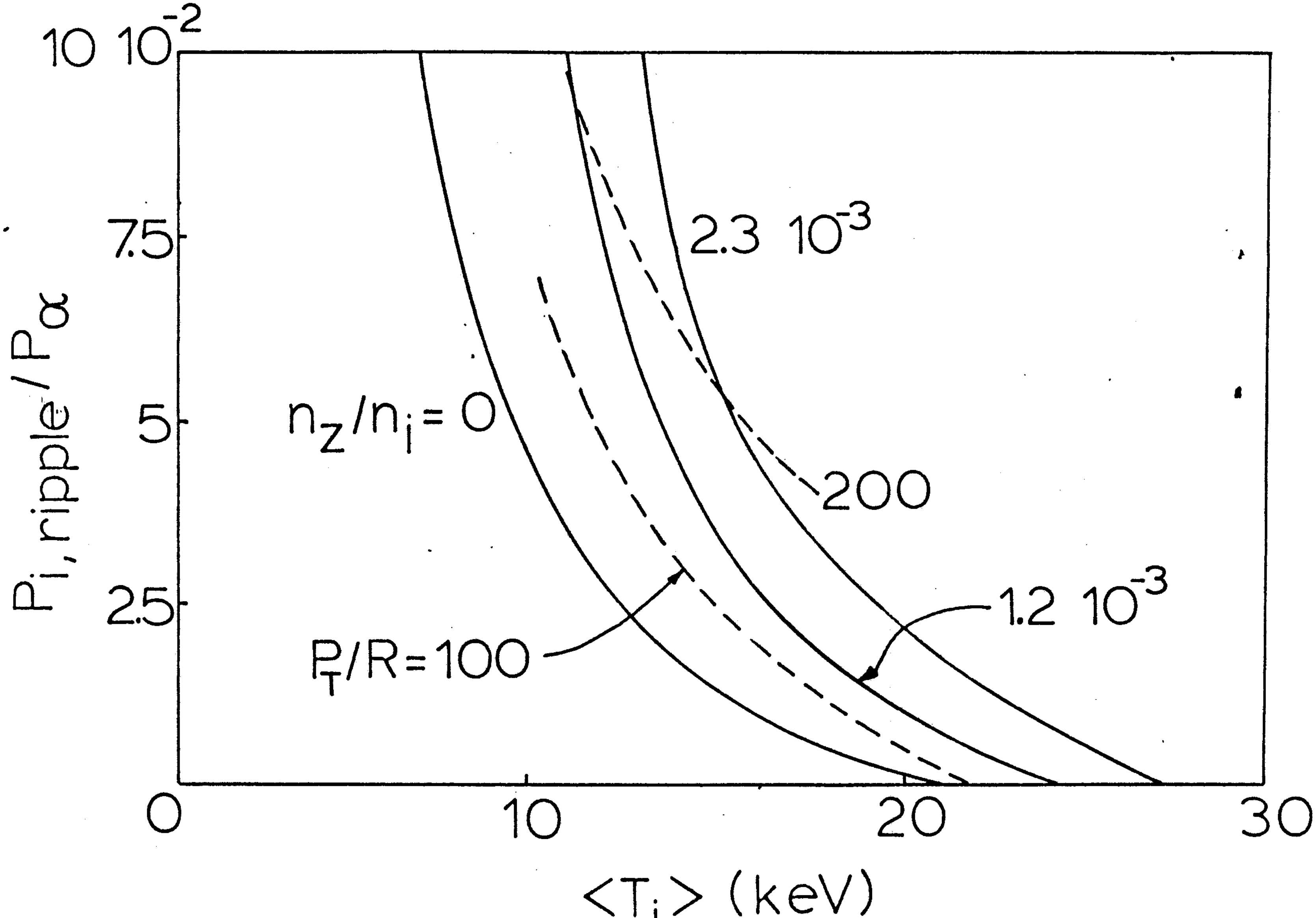


Figure 2



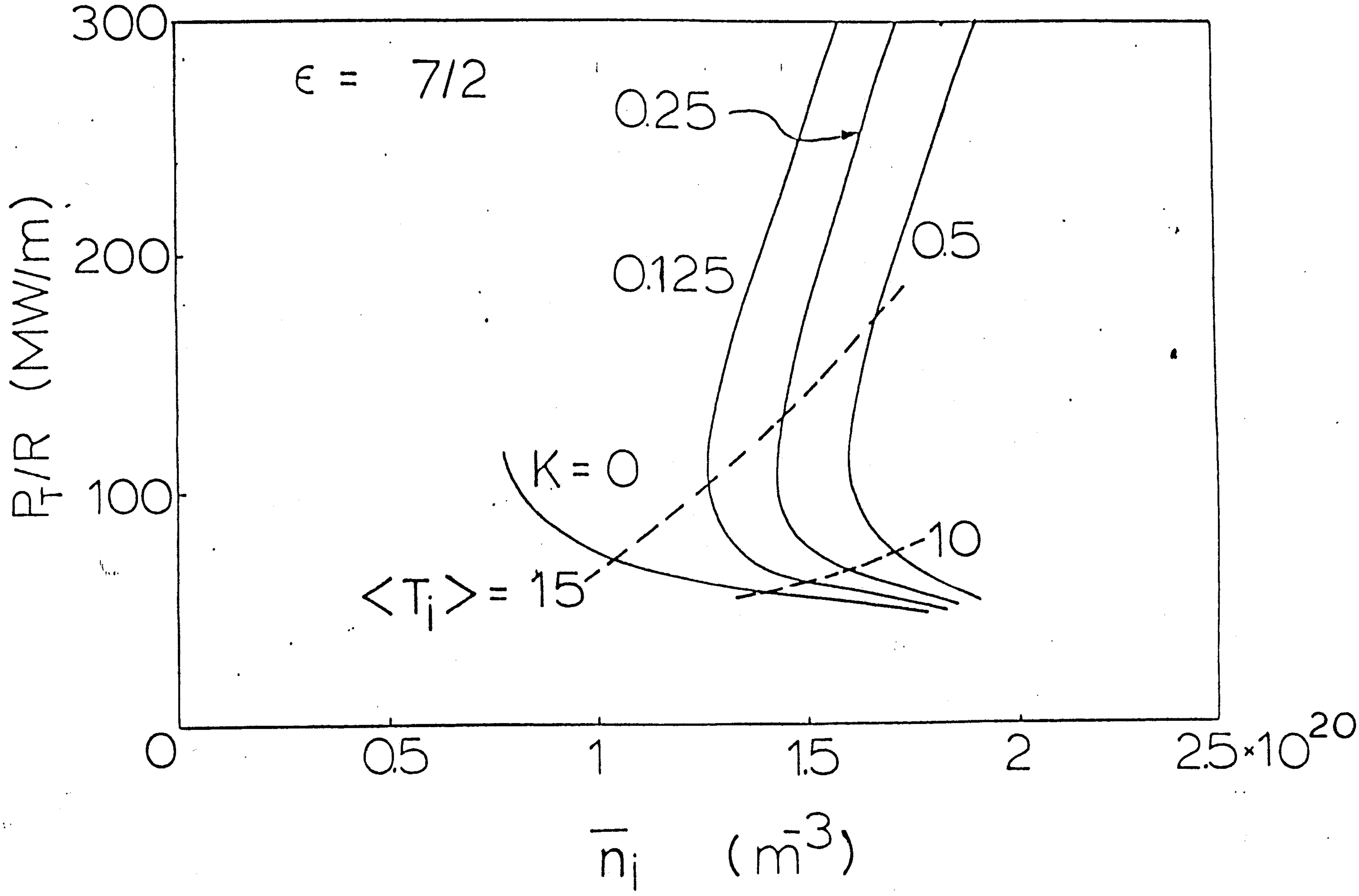
5
4
3
2
1
0
10
20
30
τrunaway (s)
nZ/ni = 0
PT/R=100
2.3 10-3
1.2 10-3
200
<Ti> (keV)

Figure 3



10 10-2
7.5
5
2.5
0
10
20
30
Pi, ripple / Pα
<Ti> (keV)
nZ/ni = 0
2.3 10-3
200
1.2 10-3
PT/R = 100

Figure 4



ε = 7/2
0.25
0.125
0.5
K = 0
10
<Tᵢ> = 15
300
200
100
P_T/R (MW/m)
0
0.5
1
1.5
2
2.5×10^20
n̄ᵢ (m^-3)

Figure 5

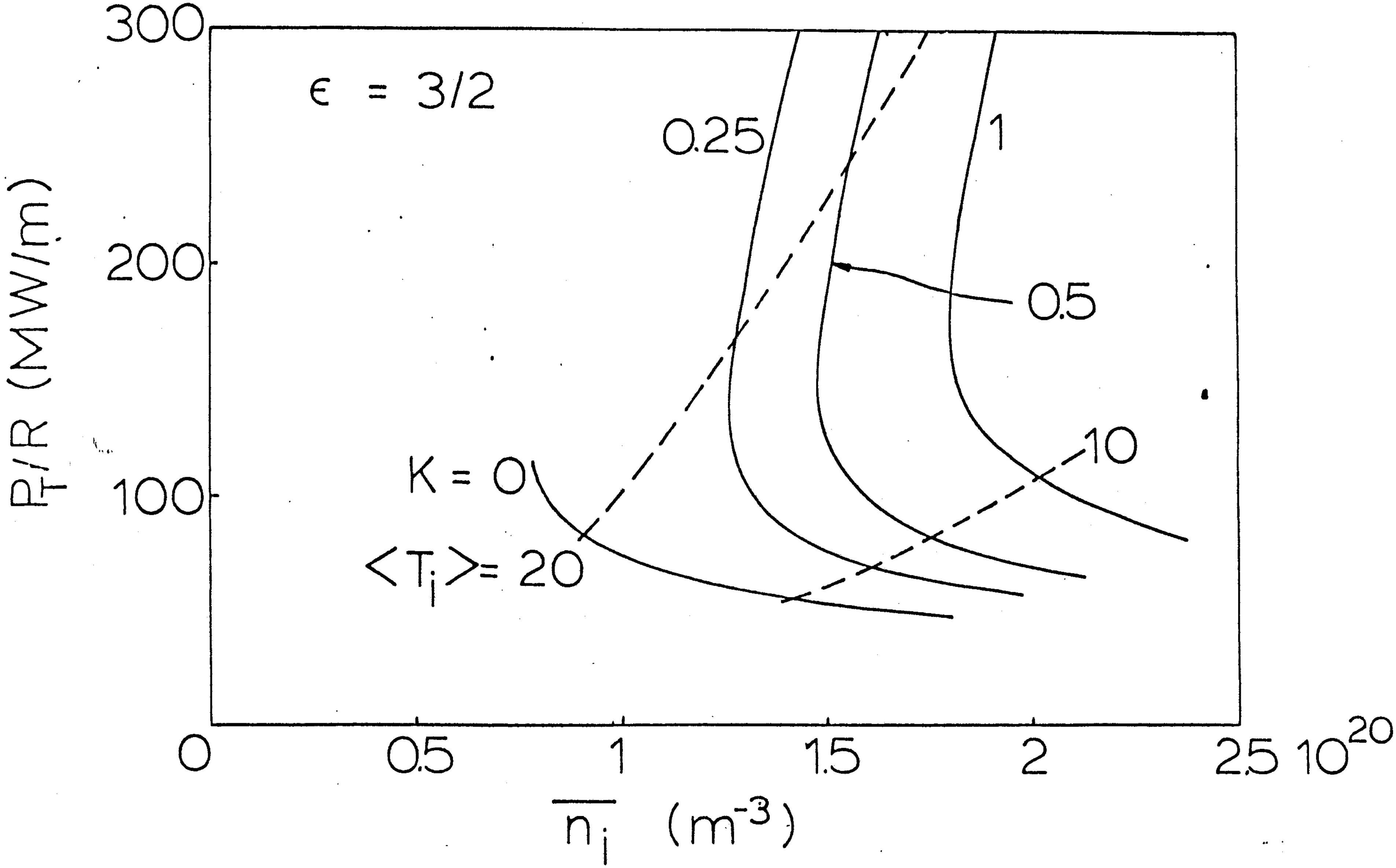


Figure 6