NONLINEAR COUPLING AND PROPAGATION OF LOWER HYBRID WAVES*

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The nonlinear coupling and propagation of lower hybrid waves excited by a waveguide array are examined numerically in a model of an inhomogeneous Tokamak plasma. The nonlinearity is the ponderomotive force which results in self-modulation of the lower hybrid waves. We evolve self-consistently the fields in time to a steady-state. Both travelling and standing wave excitations are considered. The dependence of reflectivity on the nonlinearity is studied and found to exhibit qualitative behavior seen in experiments. The nonlinearity also produces density "holes" in front of the waveguides, and the accompanying fields are strongly focused. In Fourier space, the parallel wave spectrum is broadened and upshifted. In the travelling wave case the spectrum is broadened but remains unidirectional, that is negative k_z are not generated.

I. Introduction : Lower hybrid heating experiments with waveguide arrays have involved large incident power densities, sometimes exceeding 10 kW/cm^2 and have shown strong deviations from linear coupling theory [1,2]. At such power levels ponderomotive effects are expected to be large and might explain this nonlinear behavior. In this paper we investigate numerically coupling and propagation of lower hybrid waves in the presence of self-consistent ponderomotive density modulation.

The numerical work is focused on a number of representative cases: (i) a travelling wave excitation, with a fairly narrow spectrum in k_z space; this might be imposed by a broad waveguide array (at least 4–6 elements) with $\frac{\pi}{2}$ progressive phasing; (ii) a standing wave excitation, with two narrow peaks in the spectrum, obtained by π phasing of the array; and (iii) a two-waveguide array excitation, with 0 or π phasing.

We are principally concerned with two aspects of the coupling, the total power coupled, and the wave spectrum exiting the nonlinear region.

II. Propagation Equations : We assume a slab geometry for edge propagation. The coordinate z points in the toroidal direction, and x is the direction of the equilibrium density gradient. We ignore variation in y and take $E_y = 0$. Because of low temperatures near the edge ($T < 5 - 20$ eV) we neglect thermal dispersion. We consider waves with frequency ω_0, $\Omega_i \ll \omega_0 \ll \Omega_e$, where $\Omega_{i,e}$ are the cyclotron frequencies, and allow for slow time variations by writing: $\mathbf{E}(\mathbf{r},t) = \hat{\mathbf{E}}(\mathbf{r},t)e^{-i\omega_0 t} + c.c.$. We then expand Maxwell's equations, assuming that the elements of the dielectric tensor appear as in the linear case. The density modulation caused by the high frequency fields is $n = n_0(x)\exp\left(-|\hat{E}_z|^2/E_T^2\right)$ where $E_T^2 = 4m_e\omega^2(T_e + T_i)/e^2$. This density appears in the expressions for the elements of the dielectric tensor.

We assume a linear density gradient near the edge. Eliminating $\hat{E}_x$ from the Maxwell equations and neglecting terms in $\partial^2/\partial t^2$, we obtain a single evolution equation for $\hat{E}_z$:

$$i\partial_\tau\left(\partial_z^2(\xi e^{-\beta(\xi)|E|^2}E) + a_0 E\right) + \partial_\xi^2 E - (\partial_\varsigma^2 + 1)\left((\xi e^{-\beta(\xi)|E|^2} - a_0)E\right) = 0 \qquad (1)$$

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with the normalizations: $\xi = a^{1/3}\frac{\omega_0}{c}x$, $\varsigma = \frac{\omega_0}{c}z$, $\tau = \frac{\omega_0}{2}t$, $a_0 = a^{-2/3}$ and $E = E_z/E_T$. The parameter a is proportional to the density gradient, $a = \frac{c}{\omega_0}\frac{d}{dx}\frac{\omega_{peo}^2(x)}{\omega_0^2}\Big|_{x=0}$. In general the parameter a is large, $a \simeq 1000$, so that we can neglect terms in a_0. The function $\beta(\xi)$ allows for temperature inhomogeneity, $\beta(\xi) = (T_{e0} + T_{i0})/(T_e(\xi) + T_i(\xi))$.

Eq.(1) is not dynamically correct because we took the density modulation to be instantaneous, while in fact it is created at a rate involving the transit of ion acoustic waves. Thus only the steady-state equation is physically correct, and time evolution is principally an artifice for obtaining the steady-state. In the steady-state, $\partial/\partial\tau = 0$, eq.(1) has a conservation law. The total ξ-directed Poynting flux is conserved, $P(\xi) = \int_{-\infty}^{\infty} \mathrm{Re}(-\frac{1}{2}EH^*)\,d\varsigma =$ constant, where H, the normalized H_y, is found by solving: $(\partial_\varsigma^2 + 1)H = i\partial_\xi E$.

Consider, for instance, PETULA-like parameters, with frequency $f = 1.25$ GHz and edge temperatures $T_i \ll T_e \simeq 5$ eV [2] . We assume the coupling region ends at the edge of the limiter shadow, $x_{\max} \simeq 1$ cm, beyond which the temperature rises and reduces nonlinearity. Taking $a \simeq 1000$, we find that $\xi_{\max} = 2.6$. In the experiments, as much as 10 kW/cm^2 were fed into the waveguides, for which we have $E_{\max} = 3.3$. We integrate eq.(1) with these maximum values in mind.

III. Numerical Solutions : Eq.(1) is solved by a finite difference, implicit scheme. The computational region is a rectangle, with, typically $0 < \xi < \xi_m = 3.5$ and $-19.5 = -\varsigma_m < \varsigma < \varsigma_m = 19.5$. A reasonable mesh size is $N_x \times N_z = 27 \times 256$, and this is obtained with $\Delta\xi = 0.135$ and $\Delta\varsigma = 0.15$. The time step used is $\Delta\tau \simeq 0.25$, and roughly $100 - 200$ time steps are necessary to reach the steady-state after the excitation at $\xi = 0$ is turned on. At $\xi = \xi_m$, a numerical radiation boundary condition is implemented. At the boundaries $\varsigma = \pm\varsigma_m$, periodic or zero boundary conditions are applied. The nonlinear region, where $\beta(\xi) = 1$, extends to $\xi_1 = 2.0$. To avoid aliasing problems with the nonlinearity, a digital filter was used. This filter introduces artificial dissipation in $n_{x,z} > n_1$. For $n_{x,z} < n_1$, the dissipation is negligible. The "harm" done to larger Fourier components can be gauged by the total power lost, that is $P(\xi_{\max}) - P(0)$, as waves propagate from the source to the far boundary. In most runs $n_1 \simeq 6$, and power loss was under 10%.

IV. Travelling Wave Excitation : The simplest numerical "experiment" involves a travelling-wave excitation, with a narrow spectrum at the edge. We impose: $E(0,\varsigma) = E_0 e^{i\nu_0\varsigma}\left(1 - \left(\frac{\varsigma}{\varsigma_0}\right)^2\right)^2$, where $\varsigma_0 \gg \pi/\nu_0$. This simulates a long array with $\pi/2$ phasing, with each waveguide of width π/ν_0.

The power coupled for an infinite travelling wave can be found from a one-dimensional model [3,4]. If we refine this model by averaging over the finite envelope, we obtain excellent agreement with the 2D numerical results, even when the "array" is fairly small, containing only 4–6 elements. The main feature is that the power coupled "saturates" at large amplitudes at the edge. When the waveguide impedance is taken into account, there may be regions where the coupling improves because the impedance mismatch between plasma and waveguide decreases. The linear field structure ($E_0 \ll 1$) consists of a single resonance cone, almost symmetrical about its axis. The spectrum changes little in amplitude as the fields penetrate the plasma, each n_z Fourier component picking up the phase of an Airy function. In the nonlinear regime the field structure is changed in two ways. The resonance cone is shifted away from the magnetic field and penetrates faster into the plasma; and a "shock"- like steepening forms on the edge farthest from the magnetic field axis. In Fourier space, this steepening results in a broadening of the spectrum. This broadening is directed toward larger positive n_z's and we see little generation of negative n_z's, which would correspond to nonlinear reflection in the resonance cone. The overall nonlinear effect on the power spectrum is modest. For instance in one run we have $\nu_0 = 2.0$ and $\varsigma_0 = 4.0$. If we define $\bar{\nu}$ as the center of gravity of the power spectrum in $n_z > 0$, then as E_0 increases to 3.0

the shift in $\bar{\nu}$ is only about 0.1.

V. Standing Wave Excitation : The next complication is to introduce a standing wave excitation, of the form $E(0,\varsigma) = E_0 \cos(\nu_0\varsigma)\left(1-\left(\frac{\varsigma}{\varsigma_0}\right)^2\right)^2$. This simulates a large array with alternate zero π phasing.

An example of linear field structure is shown in fig.(1). In this run, $\nu_0 = 1.7$ and $\varsigma_0 = 6.0$. The excitation results in two smooth resonance cones exiting the coupling region. The nonlinear structure is shown in fig.(2), with $E_0 = 3.0$. The resonance cones are now split into several parallel channels. In Fourier space, we see coupling to odd harmonics of ν_0, with subsidiary peaks of the spectrum forming at $n_z \simeq 5.1, 8.5, \ldots$, and it is the beating of these harmonics which results in filamentation of each resonance cone. Such an effect can be predicted qualitatively from the equation for an infinite cosine excitation. The higher harmonics do not carry very much power. For instance, with $E_0 = 3.0$, we find that $\Delta\bar{\nu} \simeq 0.15$, a small shift in the center of the positive power spectrum. The nonlinear effect on the total power coupled is large however, and we have found a one-dimensional model to predict it. Because higher harmonics are small in the immediate vicinity of the excitation, we neglect them in doing a Fourier expansion of the fields in harmonics of $e^{i\nu_0\varsigma}$. The coupling equation for the amplitude of $e^{i\nu_0\varsigma}$ is: $\frac{d^2E_1}{d\xi^2} + (\nu_0^2 - 1)\xi Q(\frac{1}{2}|E_1|^2)E_1 = 0$ where $Q(y) = e^{-y}(I_0(y) - I_1(y))$, $E_1(0) = E_0$ and radiation is required at large ξ. This equation ressembles the one for the travelling wave excitation, and the same power saturation occurs. Of course, the harmonic generation and filamentation in propagation are not described by this simplified equation.

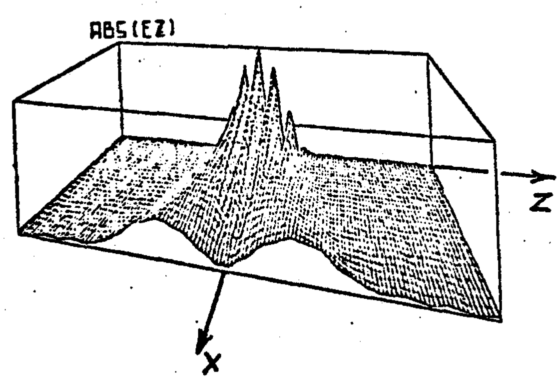

fig.(I)

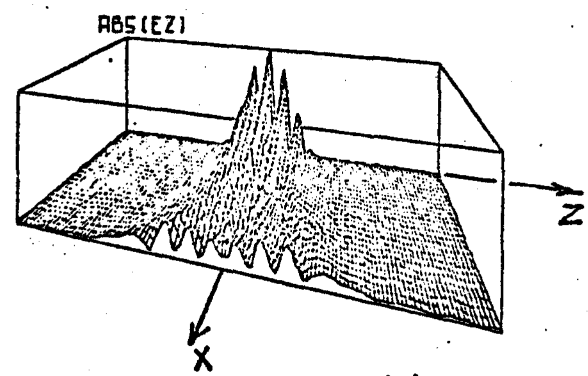

fig.(2)

VI. Waveguide Coupling : The sources considered above are idealized versions of large arrays .We now consider a more finite excitation, that of two phased waveguides.

Our computations are done with only the dominant modes in the waveguides. In each run, we measure the plasma admittance Y_P, and then calculate the reflection coefficient according to: $R^2 = \left|\frac{1-a^{1/3}Y_P}{1+a^{1/3}Y_P}\right|^2$. Some results are shown in fig.(3) where R^2 is plotted against the self-consistent incident power. We considered a PETULA like array, with waveguide width $b = 1.0$ in normalized units. Both $\phi = 0$ and $\phi = \pi$ phasings are considered, for density gradients of $a = 540$ and $a = 64$. Power densities are obtained assuming an edge temperature $T_e = 5$ eV. When the normalized amplitude is $E = 1$, the power density is 0.93 kW/cm^2.

For both density gradients there is convergence and crossing of the reflection coefficients with increasing power. This is the result of a steady decrease in the plasma admittance Y_P. The field structure is also changed considerably. We see the formation of narrow channels in front of the waveguide mouths where the density is strongly depressed. At the end of this channel the fields seperate into resonance cones which themselves are filamented as in fig.(2).

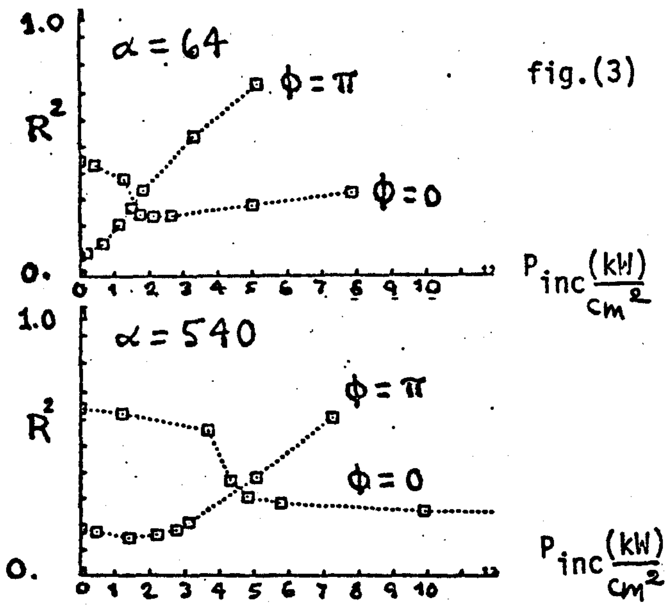

fig.(3)

In Fourier space, the spectrum of the excitation is shifted and broadened by the nonlinear effects. In particular, we see growth of Fourier components about $\nu = 2\pi/b$ for both phasings. The shift of the center of gravity of the power spectrum is again modest. For instance, with $b = 1.5$, we found a shift $\Delta\bar{\nu} = 0.2$ for $\phi = \pi$ and $\Delta\bar{\nu} = 0.5$ for $\phi = 0$, as E_0 increases to 3.0 . On the other hand the broadening of the spectrum may suffice to shift a considerable amount of power above accessibility. This is particularly true for $\phi = 0$ phasing. The linear spectrum has less than 10% power above $\nu = 2.0$, and this is increased to over 50% when $E_0 = 2.0$.

VII. Conclusions : We analyzed the nonlinear excitation of lower hybrid waves with models of waveguide arrays. Our focus was on predicting the total power coupled into the plasma and how it is distributed in the nonlinear power spectrum.

We found that power coupled from large arrays (at least 4-6 waveguides), for travelling or standing wave excitations, can be found by solving one-dimensional equations. The nonlinear effects on the power coupled are strong, with saturation at large powers. . However, when matching to the waveguide impedance is taken into account, the coupling may improve in some range of incident power. The spectrum generated inside the plasma is not drastically changed, in part because of the finite length in x of the nonlinear region. With the standing wave, Fourier modes at odd spatial harmonics are generated but do not carry very much power. In real space, the nonlinear effects are more striking and result in the distortion or filamentation of the resonance cones.

We also considered a two-waveguide excitation. Again, the nonlinear effects on the coupling are strong, with the reflection coefficients for 0 and π phasing changing considerably. In real space, straight narrow channels form in front of the waveguides. In Fourier space the spectrum is broadened. When accessibility effects are taken into account, the nonlinear modification of power which reaches the plasma interior may be considerable. In particular, these results suggest that a linearly "poor" coupler (i.e. a single waveguide) may be improved by nonlinear effects, both because its matching to the plasma is improved and because a large amount of its spectrum is pushed above accessibility.

References

[1] Nagashima, T., Fujisawa, N., Lower Hybrid Heating Experiment in JFT-2 Tokamak, Proc. of Joint Varenna-Grenoble Symposium on Heating in Toroidal Plasmas, Grenoble, 1978.

[2] Briffod, G., Review of Lower Hybrid Results in the WEGA and PETULA Tokamaks, IAEA Metting, Nagoya Plasma Physics Conf., April 1980.

[3] Chan, V., Chiu, S., Phys. of Fluids, **22**, 1724 (1979)

[4] Fukuyama, A., Morishita, T., Furutani, Y., Plasma Phys., **22**, 565 (1980).

[5] Krapchev, V. B., Theilhaber, K., Ko., K. C., Bers, A., MIT Report PFC/JA-80-20, Jan. 1980.