

NONLINEAR STEADY-STATE COUPLING OF LH WAVES*

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PFC/RR-81-9

February 1981

* To appear in Proceedings of the 4th Topical Conference on RF Heating of Plasma, University of Texas at Austin, Feb. 1981.

NONLINEAR STEADY-STATE COUPLING OF LH WAVES*

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The coupling of lower hybrid waves at the plasma edge by a two waveguide array with self-consistent density modulation is solved numerically. For a linear density profile, the governing nonlinear Klein-Gordon equation¹ for the electric field can be written as a system of nonlinearly modified Airy equations in Fourier k_z -space. Numerical solutions to the nonlinear system satisfying radiation condition are obtained. Spectral broadening and modifications to resonance cone trajectories are observed with increase of incident power.

The electromagnetic coupling problem of lower hybrid waves with self-consistent density modulation is described by the nonlinear Klein-Gordon equation. In dimensionless space variables (normalized by c/ω), the equation is [1]

$$\frac{\partial^2 E_z}{\partial x^2} + \left(\frac{\partial^2}{\partial z^2} + 1 \right) [K_{\parallel} + (K_{\parallel} - 1) \frac{\delta n}{n_0}] E_z = 0 \quad (1)$$

where $K_{\parallel} = 1 - \frac{\omega_{pe}^2}{\omega^2}$, $\frac{\delta n}{n_0} = \exp(-\beta |E_z|^2) - 1$, $\beta = 1/(4m_e \omega^2 (T_e + T_i))$

Eq.(1) is derived from the steady-state Maxwell's equations in a cold plasma under the assumptions $E_y \sim 0$, $K_{\perp} = 1$, and $K_x = 0$ which are valid near the plasma edge. A slab model is applicable since the coupling is localized in a small region in front of the external source. The distance into the plasma is measured by x , while z is along the static magnetic field. In Eq.(1), $\frac{\delta n}{n_0}$ represents the self-consistent density depression due to ponderomotive effects. These are nonlinear effects generated by large amplitude, high frequency, spatially modulated waves.

The linear coupling to a phased waveguide array is well understood. The nonlinear modification of the waveguide coupling has recently received considerable attention but essentially only the amplitude aspect of the ponderomotive effects was emphasized [2,3]. The numerical treatment here solves the nonlinear waveguide coupling as a two-dimensional problem taking into account the full spatial modulation of the external field [4,5].

For a linear density profile ($\omega_{pe}^2/\omega^2 = \alpha x$), the nonlinear Klein-Gordon equation can be written in Fourier n_z space as a nonlinearly modified Airy equation

$$\frac{d^2 u_j}{d\zeta^2} - \zeta u_j - (\zeta + \epsilon \zeta_c) \mathcal{F} \left[\frac{\delta n}{n_0} u \right]_j = 0 \quad (2)$$

with the transformations $u = \beta^{1/2} E_z$, $\zeta = [1 - n_z^2] \alpha^{1/3} \epsilon (x - x_c)$ where $\epsilon = \text{sgn}(1 - n_z^2)$, $x_c = 1/\alpha$. The coupling to other Fourier modes is given by the convolution \mathcal{F} of the density modulation $\frac{\delta n}{n_0}$ and the field u itself. The problem is then reduced to solving a system of nonlinearly coupled ordinary differential equations subject to appropriate boundary conditions.

* Work supported by U.S. DOE Contract DE-AC02-78ET-51013.

The first boundary condition is the imposed field at the plasma edge. This takes two forms

$$u|_{x=0} = \begin{cases} \pi \text{ phasing,} & 0 \text{ phasing,} \\ U_0 \sin(\frac{\pi z}{b}), & U_0 \cos(\frac{\pi z}{2b}), & |z| \leq b \\ 0, & 0, & |z| > b \end{cases} \quad (3)$$

where b is the waveguide width. The second boundary condition is a radiation condition at the far boundary inside the plasma to satisfy positive power flow requirement. This is achieved by incorporating a temperature profile to eliminate the ponderomotive density change at some distance from the far boundary. Eq.(2) then simplifies to the Airy equation in that region. The radiation condition at the far boundary is then given by the familiar linear solutions

$$\left. \frac{1}{u_j} \frac{du_j}{d\zeta} \right|_{x=\infty} = \begin{cases} \frac{\text{Ai}'(\zeta_\infty)}{\text{Ai}(\zeta_\infty)}, & |n_z| < 1 \\ \frac{\text{Ai}'(-\zeta_\infty) + i \text{Bi}'(-\zeta_\infty)}{\text{Ai}(-\zeta_\infty) + i \text{Bi}(-\zeta_\infty)}, & |n_z| > 1 \end{cases} \quad (4)$$

The prime superscript denotes differentiation with respect to the argument and ζ_∞ is ζ at $x = \infty$. These two boundary conditions, being at different locations, pose a shooting procedure for the numerical integration of the system of equations since only one starting value is available at either boundary. A first guess at the second starting value for the edge boundary can be constructed as follows. The convolution term in Eq.(2) is rewritten in the form

$$\mathcal{F}\left[\frac{\delta n}{n_0} u\right]_j \simeq \left. \frac{\mathcal{F}\left[\frac{\delta n}{n_0} u\right]_j}{u_j} \right|_{x=0} u_j = \sigma_j u_j. \quad (5)$$

Eq.(5) assumes that σ_j remains constant for all x with its value determined by the field at the edge boundary. This is, of course, inconsistent with the actual field which disperses and eventually separates into resonance cones. However, at a distance up to where cone separation occurs, Eq.(5) is a reasonable first approximation. Eq.(2) then becomes the Airy equation after renormalization to which solutions similar to (4) can be written:

$$\left. \frac{1}{u_j} \frac{du_j}{d\zeta} \right|_{x=0} = \mu \begin{cases} \frac{\text{Ai}'(-\zeta_c)}{\text{Ai}(-\zeta_c)}, & |n_z| < 1 \text{ or } |n_z| > 1 \text{ and } (1 + \sigma_c) < 0 \\ \frac{\text{Ai}'(\zeta_c) + i \text{Bi}'(\zeta_c)}{\text{Ai}(\zeta_c) + i \text{Bi}(\zeta_c)}, & |n_z| > 1 \end{cases} \quad (6)$$

where

$$\zeta_c = [(1 - n_z^2)|\alpha_p|]^{1/3} x_p, \quad \alpha_p = (1 + \sigma_j)\alpha, \quad x_p = x_c/(1 + \sigma_j), \quad \mu = |1 + \sigma_j|^{1/3}. \quad (7)$$

Although the far field behavior is not strictly consistent, these solutions nevertheless provide a good guess for the second starting value at the edge boundary. Eq.(7) shows two important modifications by ponderomotive effects close to the plasma edge: a change in the ambient density gradient and a shifting of the linear cutoff.

Eq.(2) (a system of equations) is solved on a $250 \times 256 x - n_z$ grid. The shooting procedure for the numerical integration is carried out as follows. Starting at the plasma edge, boundary conditions (3) and (6) are imposed. The field is then evolved nonlinearly to the far boundary to obtain the nonlinear wave spectrum. At the same time, the convolution term in Eq.(2) is stored over the whole integration grid. Since the boundary

condition (6) is only a first guess, an improvement is desired. This is achieved by a backward integration to the edge using the nonlinear spectrum previously generated and imposing the radiation condition (4). The convolution term in this integration step is treated as a source term by using values stored in the forward run. With the corrected second starting value at the edge, the forward integration is repeated. Iteration is performed until convergence is reached.

Figs. 1 and 2 show the spatial fields at the far boundary for the two imposed excitations after one iteration. (Solid lines are nonlinear solutions while the dotted lines are the linear results). In both cases, there is a nonlinear distortion of the resonance cones and a shifting of the cone peaks away from the static magnetic field. The corresponding wave spectra are shown in Figs. 3 and 4 where spectral broadening is observed. It is also seen that the depletion of the $|n_z| < 1$ portion of the spectrum is weaker than in the linear case which indicates a reduction in plasma density. This agrees with Figs. 5 and 6 which show the ponderomotive density depressions for each case.

References

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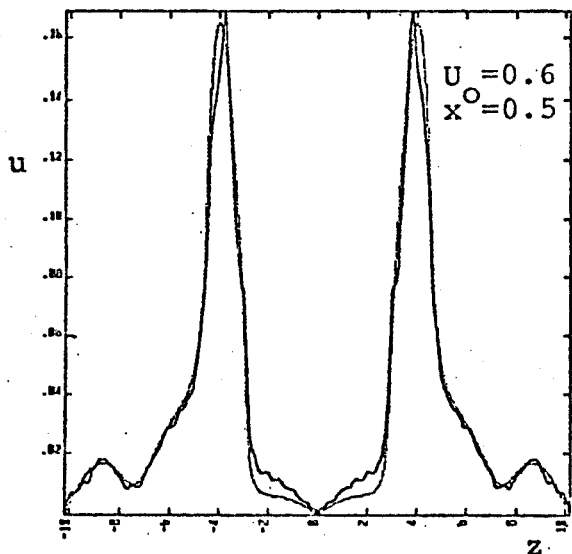


Fig.1 " π " phasing ($b=0.8, \alpha=250$)

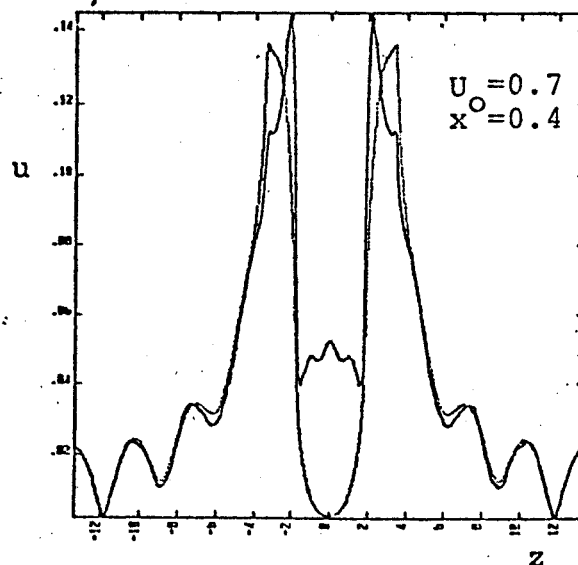


Fig.2 "0" phasing ($b=0.8, \alpha=250$)

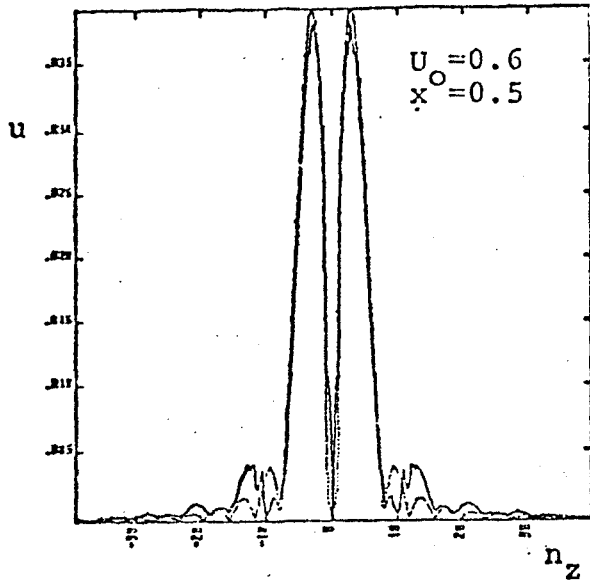


Fig.3 " π " phasing

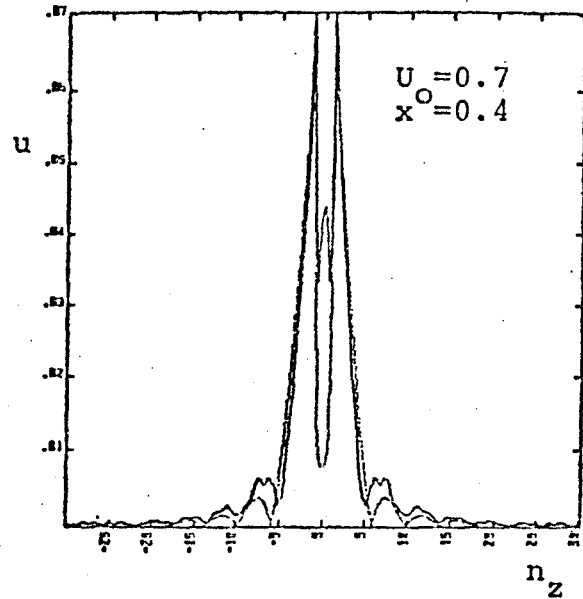


Fig.4 "0" phasing

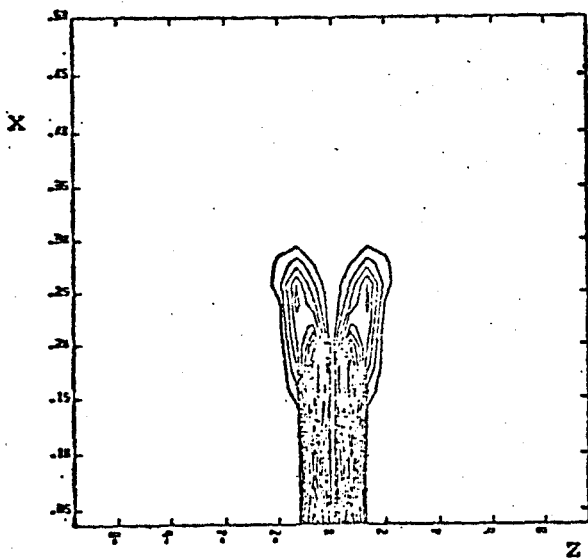


Fig.5. $\delta n/n_0$ for " π " phasing

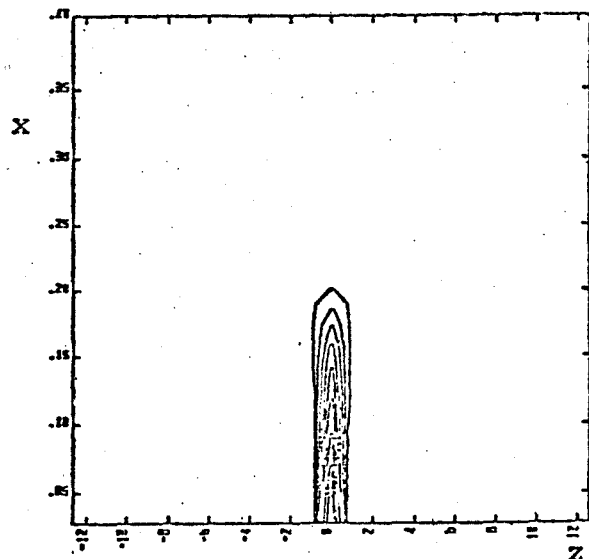


Fig.6 $\delta n/n_0$ for "0" phasing