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Excitation of Quasielectrostatic Modes
in a Magnetized Plasma by a Modulated Hollow E-Beam*

by

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Abstract The power radiated into the modes of an infinite magnetized plasma by a modulated hollow electron beam is calculated for the cases of cold and warm plasmas. The beam is assumed to be sinusoidally density modulated and the induced fluctuating electric field is strong enough to quench any beam plasma interaction. Numerical results are presented for the power deposited into the plasma at frequencies near the lower hybrid frequency for different beam plasma parameters.

1. Introduction

Several experiments were done in the past two decades to investigate the interaction between magnetized plasmas and modulated electron beams⁽¹⁻⁷⁾. Heating of the plasma near the ion cyclotron frequency was observed by Haas and Dandl (1967), Yatsui and Yamamoto (1969), and Hsieh *et al* (1973). It should be mentioned that the plasmas used in these experiments had very small densities. Bhatnagar and Getty (1971), Kovnik and Kornilov (1974), and Zeidlits *et al* (1976) did observe strong heating near the lower hybrid frequency for plasmas that have densities of the order of 10^{12}cm^{-3} .

In this paper we investigate the heating of an infinite magnetized plasma using a hollow thin electron beam as shown in fig. 1. The electron beam is density modulated at a frequency near the lower hybrid resonance, and the beam density is assumed to have a sinusoidal periodicity in the \hat{z} direction with wavelength equal to λ_z .

We neglected the effects of the beam plasma instability near the plasma frequency since this instability is quenched by the induced low frequency field⁽⁶⁻⁷⁾. In section 2 of this paper we found the excited quasiolestatic mode that is excited by the modulated beam in a cold plasma and in section 3 an expression for the power delivered to this mode is written. In section 4 the case of a warm plasma is considered and finally numerical results in section 5 are given for the power delivered to the lower hybrid mode for different beam and plasma parameters.

2. The Case of a Cold Plasma

As shown in fig. 1 the magnetic field is in the \hat{z} direction. Due to the beam modulation an alternating current source will arise which will excite the plasma waves, this current has the following form:

$$\vec{J} = \frac{\hat{z} I_0 \delta(r - a) \exp(ik_{\parallel} z)}{2\pi a} \quad (1)$$

Where

I_0 is the amplitude of the alternating current,

a is the beam radius,

$k_{\parallel} = \omega/v_b$,

and v_b is the beam velocity.

The charge density of the beam is given by

$$\rho_b = \frac{I_0}{2\pi a v_b} \delta(r - a) \exp(ik_{\parallel} z) \quad (2)$$

From the divergence relation we have

$$\nabla(\bar{\epsilon} \nabla \phi) = -\rho_b \quad (3)$$

Hence

$$\epsilon_{\perp} \nabla_{\perp}^2 \phi - \epsilon_{\parallel} k_{\parallel}^2 \phi = -\frac{\rho_b}{\epsilon_0} \quad (4)$$

ϕ is the electrostatic potential.

ϵ_{\perp} the perpendicular plasma relative permittivity.

ϵ_{\parallel} the parallel plasma relative permittivity.

$$\epsilon_{\perp} = 1 - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} \quad (5a)$$

$$\epsilon_{\parallel} = 1 - \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2} \quad (5b)$$

Taking the Hankel transform of eq. 4 we have

$$\epsilon_{\perp} k_{\perp}^2 \Phi + \epsilon_{\parallel} k_{\parallel}^2 \Phi = \hat{\rho}_b \quad (6)$$

Where

$$\Phi(k_{\perp}, z) = \int_0^{\infty} dr 2\pi r \phi(r, z) J_0(k_{\perp} r) \quad (7)$$

$$\hat{\rho}_b(k_{\perp}, z) = \int_0^{\infty} dr 2\pi r \rho_b(r, z) J_0(k_{\perp} r) \quad (8)$$

Hence

$$\hat{\rho}_b(k_{\perp}, z) = \frac{I_o}{v_b} J_0(k_{\perp} a) \exp(ik_{\parallel} z) \quad (9)$$

Hence from (7) (9)

$$\Phi(k_{\perp}, z) = \frac{I_o}{\epsilon_o v_b} \frac{J_0(k_{\perp} a) \exp(ik_{\parallel} z)}{\epsilon_{\perp} k_{\perp}^2 + \epsilon_{\parallel} k_{\parallel}^2} \quad (10)$$

Finding the inverse Hankel transform

$$\phi(r, z) = \frac{I_o}{\epsilon_o v_b} \exp(ik_{\parallel} z) \int_0^{\infty} \frac{dk_{\perp}}{2\pi} \frac{k_{\perp} J_0(k_{\perp} r) J_0(k_{\perp} a)}{\epsilon_{\perp} k_{\perp}^2 + \epsilon_{\parallel} k_{\parallel}^2} \quad (11)$$

We have two regions $r > a$ and $r \leq a$

i) $r > a$

$$\frac{\int_0^{\infty} dk_{\perp} k_{\perp} J_0(k_{\perp} r) J_0(k_{\perp} a)}{\epsilon_{\perp} k_{\perp}^2 + \epsilon_{\parallel} k_{\parallel}^2} = \int_{-\infty}^{\infty} dk_{\perp} \frac{k_{\perp} H_0^{(1)}(k_{\perp} r) J_0(k_{\perp} a)}{2(\epsilon_{\perp} k_{\perp}^2 + \epsilon_{\parallel} k_{\parallel}^2)} \quad (12)$$

From (11) (12)

$$\phi(r, z) = \frac{I_o}{\epsilon_o v_b} \exp(ik_{\parallel} z) \int_{-\infty}^{\infty} \frac{dk_{\perp}}{4\pi} k_{\perp} \frac{J_0(k_{\perp} a) H_0^{(1)}(k_{\perp} r)}{\epsilon_{\perp} k_{\perp}^2 + \epsilon_{\parallel} k_{\parallel}^2} \quad (13)$$

The integrand of eq. (13) has the asymptotic value $k_{\perp} e^{ik_{\perp}(r-a)} / (\epsilon_{\perp} k_{\perp}^2 + \epsilon_{\parallel} k_{\parallel}^2)$ and by Jordan lemma the integral is equal to the residues of the poles lying in the upper half plane.

Hence

$$\phi(r, z) = \frac{iI_o}{4\epsilon_{\perp} \epsilon_o v_b} H_0^{(1)}(k_p r) J_0(k_p a) \quad (14)$$

k_p is the solution to the dispersion relation $k_p^2 \epsilon_{\perp} + k_{\parallel}^2 \epsilon_{\parallel} = 0$

ii) For $r \leq a$ we get

$$\phi(r, z) = \frac{iI_o}{4\epsilon_{\perp} \epsilon_o v_b} H_0^{(1)}(k_p a) J_0(k_p r) \quad (15)$$

3. Power Delivered by the Beam

We note from the dispersion relation that we have

$$\epsilon_{\perp} k_p^2 + \epsilon_{\parallel} k_{\parallel}^2 = 0$$

And that k_p can be either positive or negative for propagating waves. One way for choosing the excited mode is to assume that the plasma is lossy by introducing collisions between the particles of the plasma and then pick the mode that has always a positive imaginary part, this mode is a backward wave near the lower hybrid frequency.

Hence the radiated power per unit length is

$$P = -\frac{1}{2} \text{Real} \int dS E_z J_z^* \quad (16)$$

dS is an element of the cross sectional area

$$P = \frac{1}{2} \text{Real} \int_0^{\infty} dr 2\pi r \frac{I_0}{2\pi a} \delta(r-a) \exp(-ik_{\parallel}z) ik_{\parallel} \phi(r,z)$$

Hence

$$P = \text{Real} \left(\frac{I_0^2 k_{\parallel}}{8\epsilon_0 \epsilon_{\perp} v_b} H_0^{(1)}(k_p a) J_0(k_p a) \right) \quad (17)$$

We should note that up to this point we neglected the effect of the radiation on the beam, in other words the beam is taken to be rigid which is not very obvious however if this radiation is only a fraction of the beam dc power when integrated over many wavelength $P\lambda_z \ll I_{dc}V_{dc}$ then our assumption is justified.

4. Modes in a Warm Plasma

The dispersion relation in a warm infinite plasma is given by

$$\mathfrak{D}(\omega, k_{\perp}) = k_{\perp}^2 + k_{\parallel}^2 + \sum_j \omega_{pj}^2 \int_{-\infty}^{\infty} dv_{\parallel} \int_{-\infty}^{\infty} dv_{\perp} 2\pi v_{\perp} \sum_{n=-\infty}^{\infty} J_n^2\left(\frac{k_{\perp} v_{\perp}}{\omega_{cj}}\right) \frac{k_{\parallel} \frac{\partial f_{oj}}{\partial v_{\parallel}} + \frac{n\omega_{cj}}{v_{\perp}} \frac{\partial f_{oj}}{\partial v_{\perp}}}{\omega - n\omega_{cj} - k_{\parallel} v_{\parallel}} \quad (18)$$

Where j refers to the plasma species and

$$\mathfrak{D}(\omega, k_{\perp}) = 0$$

The above equation is derived from the dispersion relation

$$(k_{\perp}^2 + k_{\parallel}^2 + \sum_j q_j \int dv^3 f_{1j}) \Phi(k_{\perp}, z) = \frac{\hat{\rho}_b(k_{\perp}, z)}{\epsilon_0} \quad (19)$$

For zero external sources $\hat{\rho}_b = 0$ we get the dispersion relation of eq. (18). Using eqs. (2) (19) we get

$$\Phi(k_{\perp}, z) = \frac{I_0 J_0(k_{\perp} a) \exp(ik_{\parallel} z)}{\epsilon_0 v_b \mathfrak{D}(\omega, k_{\perp})} \quad (20)$$

Hence

$$\phi(r, z) = \frac{I_0}{\epsilon_0 v_b} e^{ik_{\parallel} z} \int_0^{\infty} \frac{dk_{\perp}}{2\pi} k_{\perp} \frac{J_0(k_{\perp} r) J_0(k_{\perp} a)}{\mathfrak{D}(\omega, k_{\perp})} \quad (21)$$

We know that for $r > a$ the above integral can be written as

$$\phi(r, z) = \frac{I_0}{2\epsilon_0 v_b} \exp(ik_{\parallel} z) \int_{-\infty}^{\infty} \frac{dk_{\perp}}{2\pi} k_{\perp} \frac{H_0^{(1)}(k_{\perp} r) J_0(k_{\perp} a)}{\mathfrak{D}(\omega, k_{\perp})} \quad (22)$$

Taking the residues of the poles in the upper half plane we get

$$\phi(r, z) = \frac{I_0}{2\epsilon_0 v_b} \exp(ik_{\parallel} z) \sum_{p=1}^{\infty} \frac{ik_p H_0^{(1)}(k_p r) J_0(k_p a)}{\partial \mathfrak{D}(\omega, k_{\perp}) / \partial k_{\perp} |_{k_{\perp}=k_p}} \quad (23)$$

And similarly for $r \leq a$ we get

$$\phi(r, z) = \frac{I_0}{2\epsilon_0 v_b} \exp(ik_{\parallel} z) \sum_{p=1}^{\infty} \frac{ik_p H_0^{(1)}(k_p a) J_0(k_p r)}{\partial \mathfrak{D}(\omega, k_{\perp}) / \partial k_{\perp} |_{k_{\perp}=k_p}} \quad (24)$$

Again the power per unit length defined in eq.(16) can be written as follows for a warm plasma

$$P = -\frac{I_o^2 k_{\parallel}}{4v_b \epsilon_o} \sum_{p=1}^{\infty} \text{Real} \left(\frac{k_p H_0^{(1)}(k_p a) J_0(k_p a)}{\partial \mathcal{D}(\omega, k_{\perp}) / \partial k_{\perp} |_{k_{\perp}=k_p}} \right) \quad (25)$$

It can be shown that for $(\omega - k_{\parallel} v_{tj}) / \omega_{cj} \gg 1$ the zeros of the dispersion relation occur in pairs which are the negative of each other complex conjugate when the waves are damped in the \hat{r} direction, consequently the contribution from the poles off the real axis is zero, and the power radiated is provided by those zeros lying on the real k_p axis.

In fig. 2 the locii of the propagating modes near the lower hybrid frequency are sketched one of these modes is a backward wave and the other is a forward wave, most of the radiated power is delivered to the backward wave which is usually called the lower hybrid wave.

5. Numerical Results

A computer program was written to calculate the power radiated into the lower hybrid mode in a plasma that has a Maxwellian distribution. The dispersion relation for the infinite plasma can be written as follows:

$$\mathfrak{D}(\omega, k_p) = 0$$

Where

$$\mathfrak{D}(\omega, k_p) = k_p^2 + k_{\parallel}^2 + \sum_{e,i} \omega_{pj}^2 \sum_{n=-\infty}^{\infty} \exp\left(-\left(\frac{k_p v_{tj}}{\omega_{cj}}\right)^2\right) I_n\left(\left(\frac{k_p v_{tj}}{\omega_{cj}}\right)^2\right) \left(1 + \frac{\omega}{k_{\parallel} v_{tj}} Z\left(\frac{\omega - n\omega_{cj}}{k_{\parallel} v_{tj}}\right)\right) \quad (26)$$

Where

v_{te}, v_{ti} are the electron and ion thermal velocities respectively,

ω_{ce}, ω_{ci} are the electron and ion cyclotron frequencies,

I_n is the modified Bessel function of order n of the first kind,

and Z is the plasma dispersion function.

The power radiated increases as the square of the ac current I_o , hence the maximum radiation occurs when the current modulation is 100 % that is $I_o = I_{dc}$. Putting the current equal to this value the ratio η of this maximum power radiation to the dc power transported by the beam is given by

$$\eta = \frac{P_{max}}{I_{dc} V_{dc}} \quad (27)$$

Using eq.(25)

$$\eta = -\frac{I_o k_{\parallel}}{4v_b V_{dc} \epsilon_o} \sum_{p=1}^{\infty} \text{Real}\left(\frac{k_p H_0^{(1)}(k_p a) J_0(k_p a)}{\partial \mathfrak{D}(\omega, k_{\perp}) / \partial k_{\perp} |_{k_{\perp}=k_p}}\right) \quad (28)$$

Putting $I_o = \mathcal{P} V_{dc}^{1.5}$ and $k_{\parallel} = \omega / v_b$ we get

$$\eta = -\frac{\mathcal{P} \omega \sqrt{V_{dc}}}{4v_b^2 \epsilon_o} \sum_{p=1}^{\infty} \text{Real}\left(\frac{k_p H_0^{(1)}(k_p a) J_0(k_p a)}{\partial \mathfrak{D}(\omega, k_{\perp}) / \partial k_{\perp} |_{k_{\perp}=k_p}}\right)$$

Hence

$$\eta = -\frac{\mathcal{P} \omega}{4v_b \epsilon_o} \sqrt{\frac{m_e}{2e}} \sum_{p=1}^{\infty} \text{Real}\left(\frac{k_p H_0^{(1)}(k_p a) J_0(k_p a)}{\partial \mathfrak{D}(\omega, k_{\perp}) / \partial k_{\perp} |_{k_{\perp}=k_p}}\right) \quad (29)$$

Where \mathcal{P} is the beam perveance,

And η is the ratio of the power radiated per unit length to the total power of the beam.

We should note again that when the ratio of power radiated per unit wavelength ($\lambda_z = 2\pi/k_{\parallel}$ is large $2\pi\eta/k_{\parallel} \gg 1$ the above results are not accurate since the beam modulation would be damped in the \hat{z} direction.

In figs.(3-10) η is plotted for a variety of beam plasma parameters. The beam perveance is taken to be 10^{-6} perv and the plasma is a hydrogen plasma with density equal to 10^{12}cm^{-3} . In each of the above figures the real and imaginary parts of k_{\perp} as well as the ratio η are plotted versus frequency for the lower hybrid mode.

Most of the radiated power goes to the lower hybrid mode, so we calculated the contribution of this mode only. As for the other modes more calculations are to be done.

The results show considerable heating near the lower hybrid resonance and by changing the beam velocity then the magnetic field intensity we concluded that the heating goes up as the B field increases and by decreasing the beam velocity. The results where obtained for two temperatures and they show a decrease in heating efficiency as the plasma becomes warmer.

6. Summary

In this paper the quasiolelectrostatic waves that are radiated by a modulated E-Beam in a magnetized plasma are analyzed and analytic formulæ are given for the wave potentials for both cold and warm plasmas at frequencies near the lower hybrid resonance.

It should be emphasized that the high frequency beam plasma interaction was neglected in the analysis and this is justifiable if the beam modulation is large enough such that the high frequency is quenched by the lower hybrid wave.

The numerical results for the power radiated per unit length of the beam in a Maxwellian plasma show that the heating efficiency increases with increasing magnetic field intensity, that it decreases with increasing beam velocity and that the efficiency is lower for warmer plasmas.

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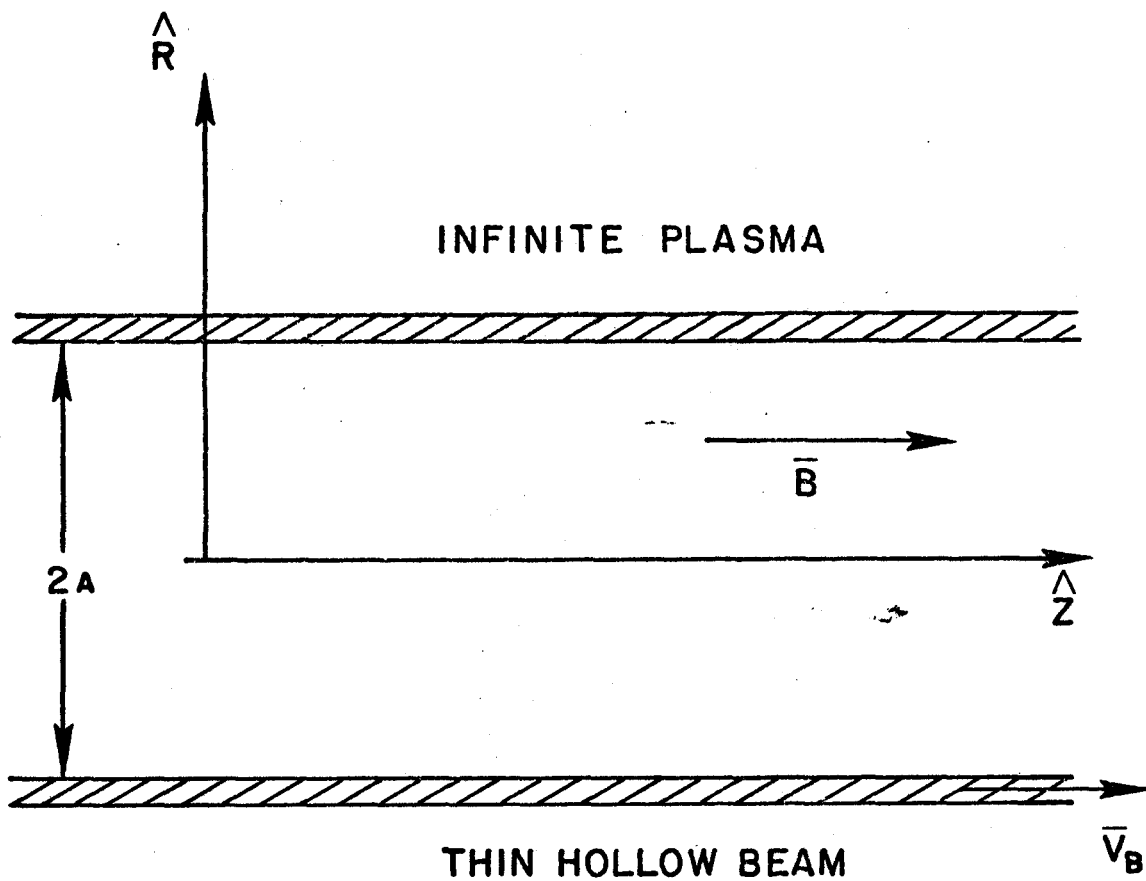


FIGURE 1. CYLINDRICALLY SYMMETRIC HOLLOW E-BEAM

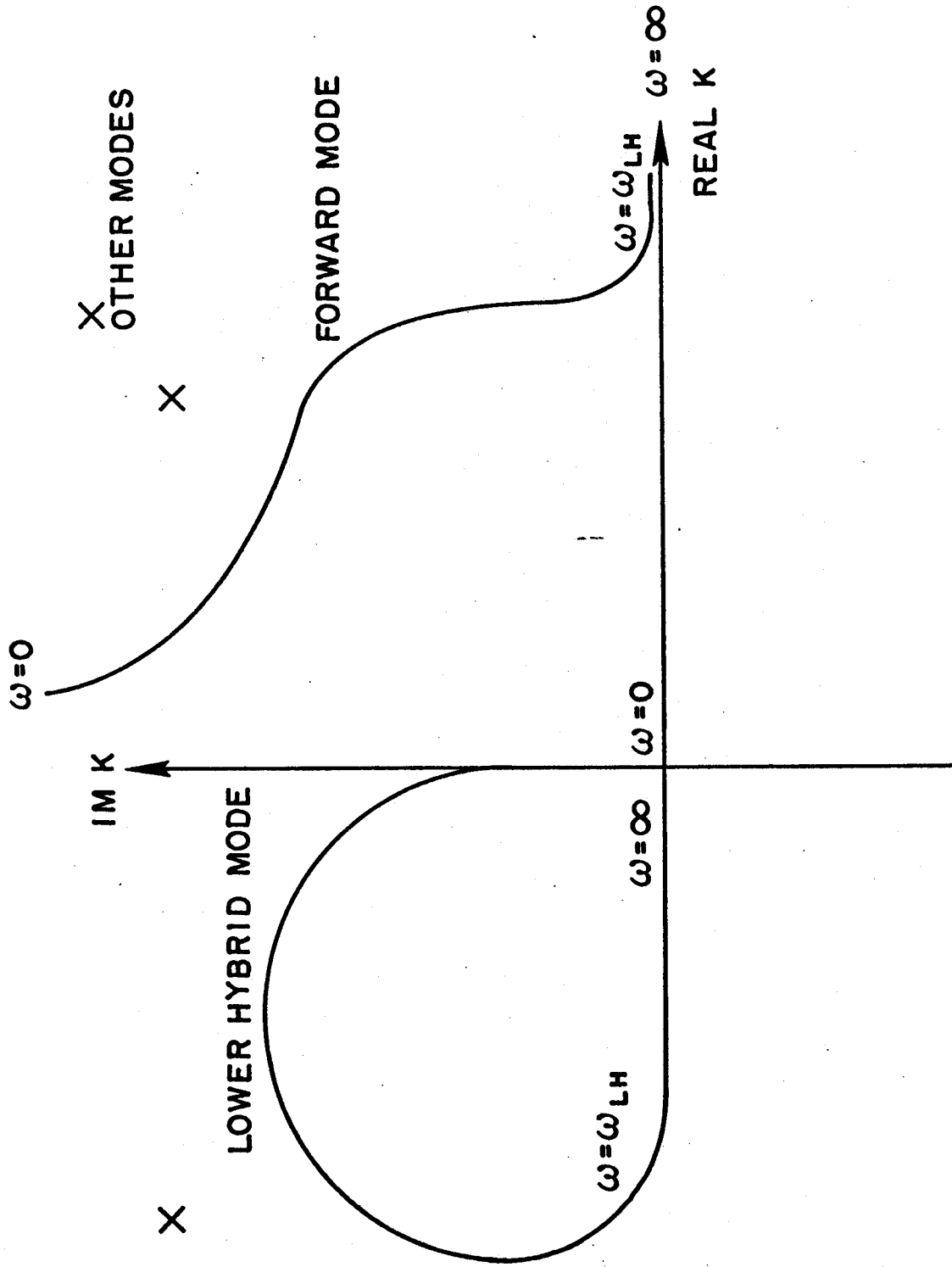


FIGURE 2. ZEROS IN THE COMPLEX K PLANE AS FUNCTION OF ω .

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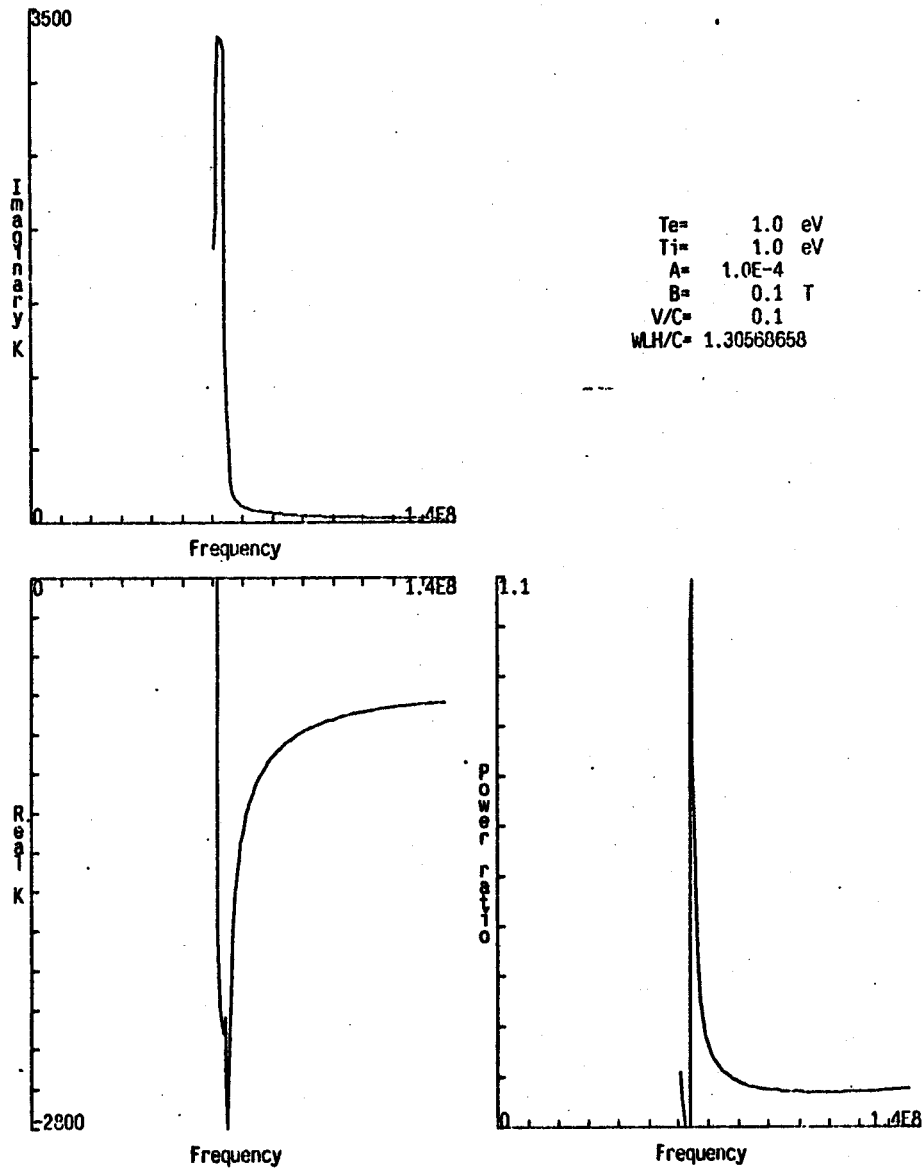


FIGURE 3

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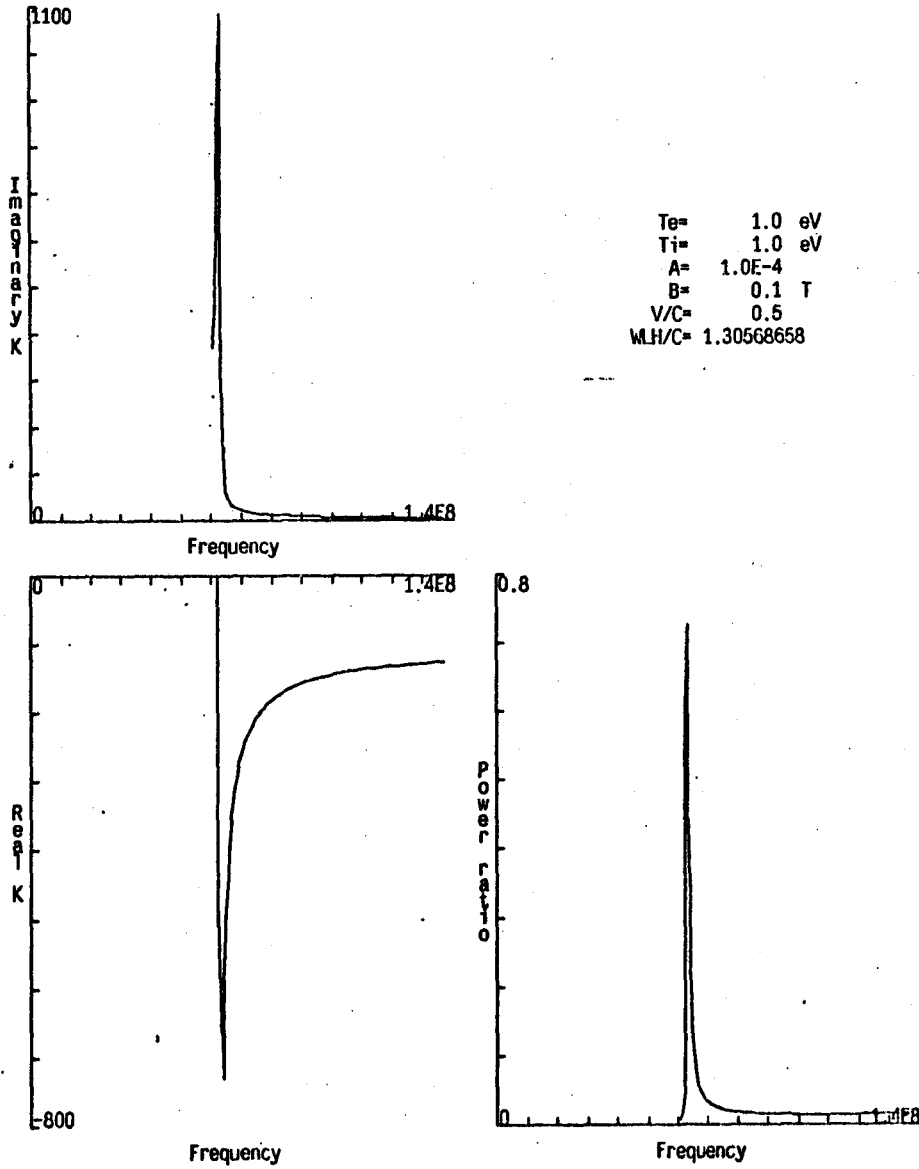


FIGURE 4

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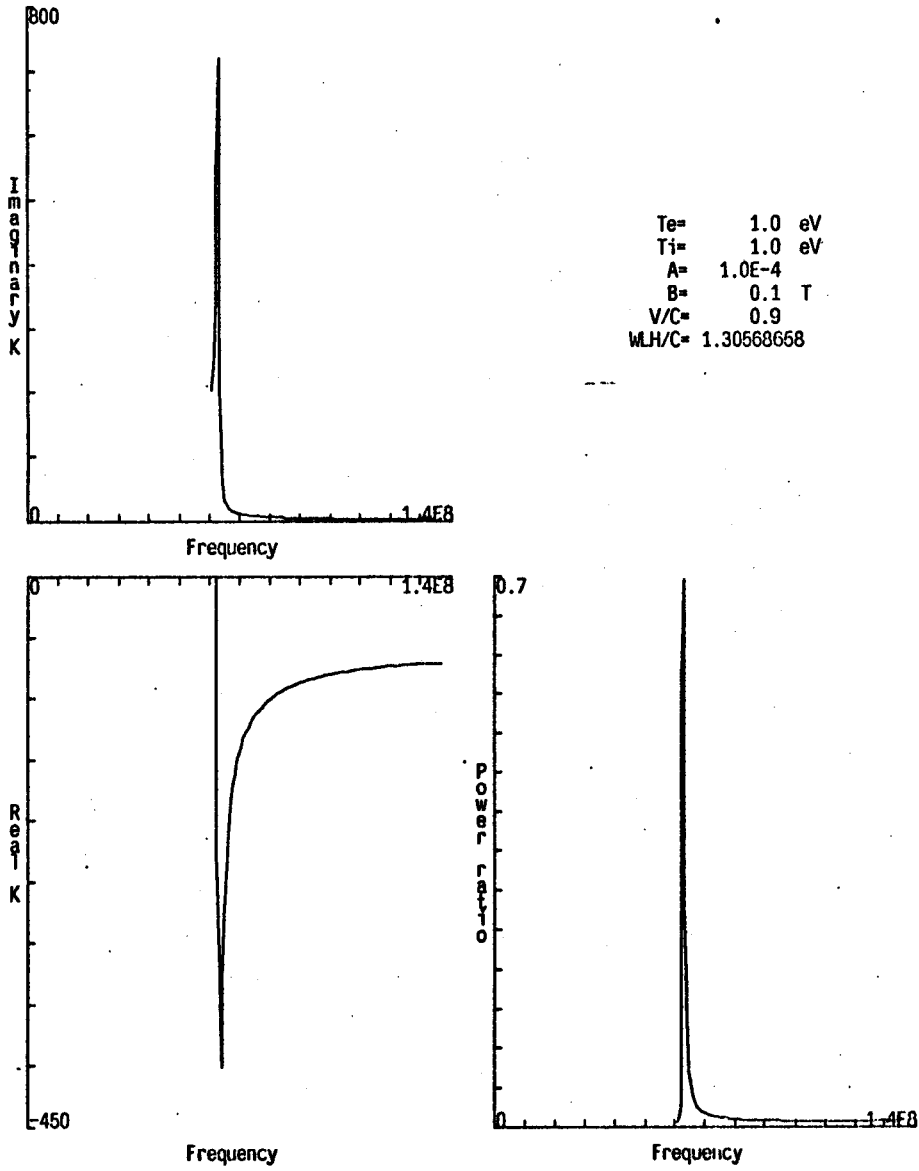


FIGURE 5

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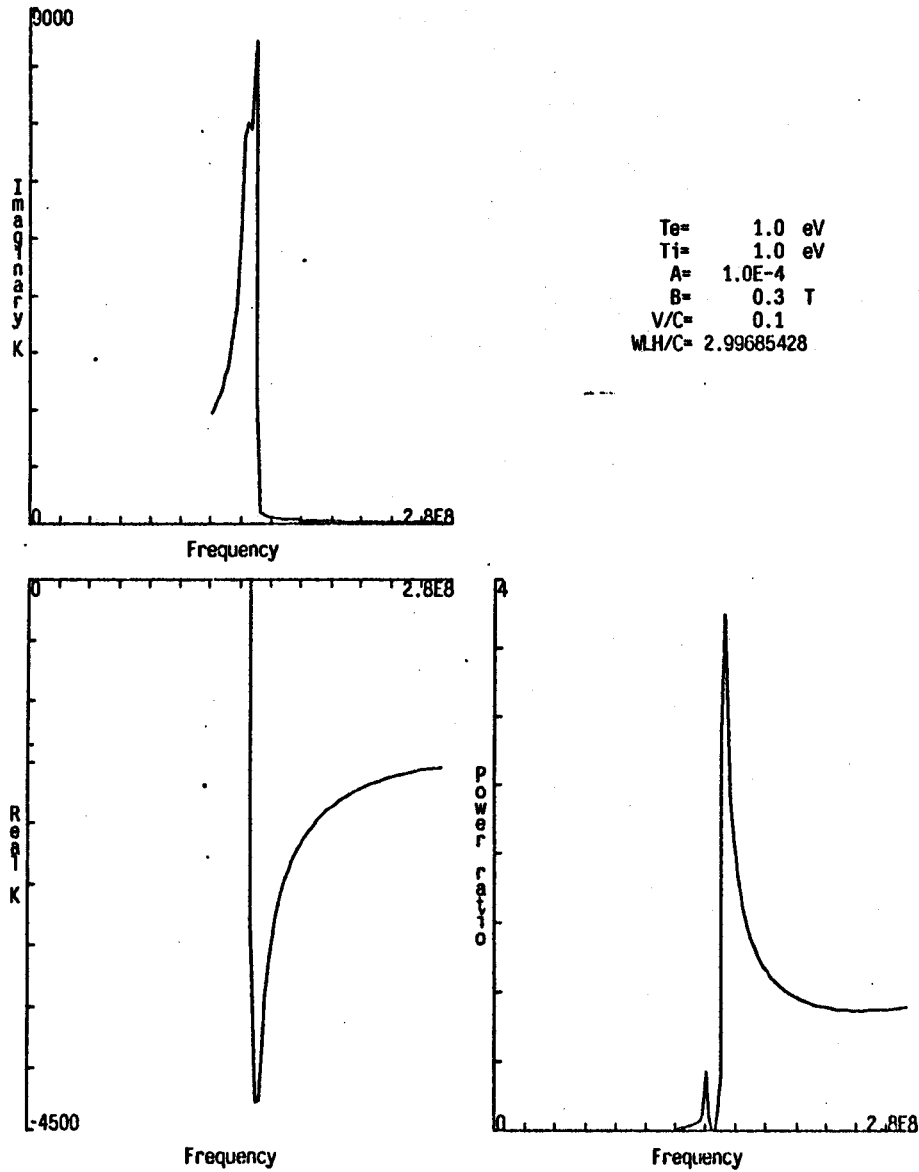


FIGURE 6

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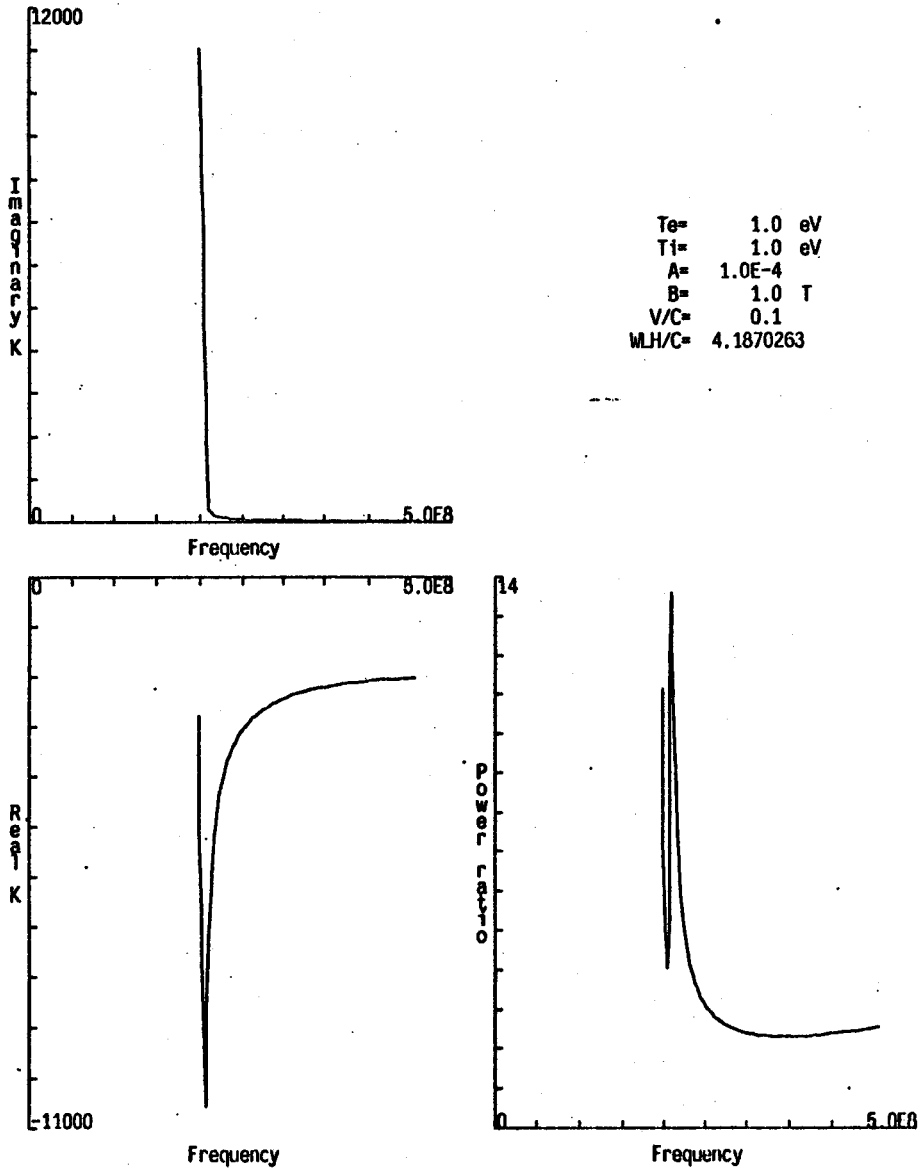


FIGURE 7

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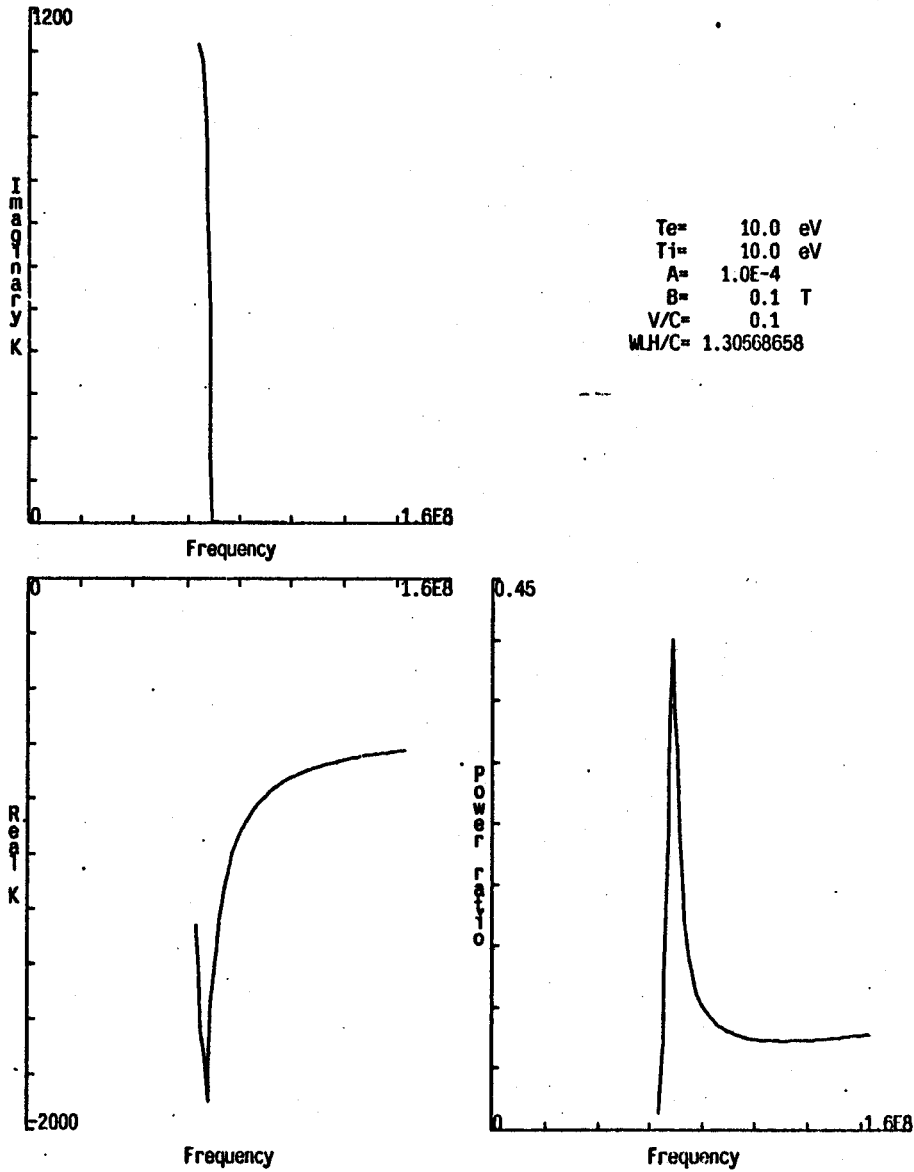


FIGURE 8

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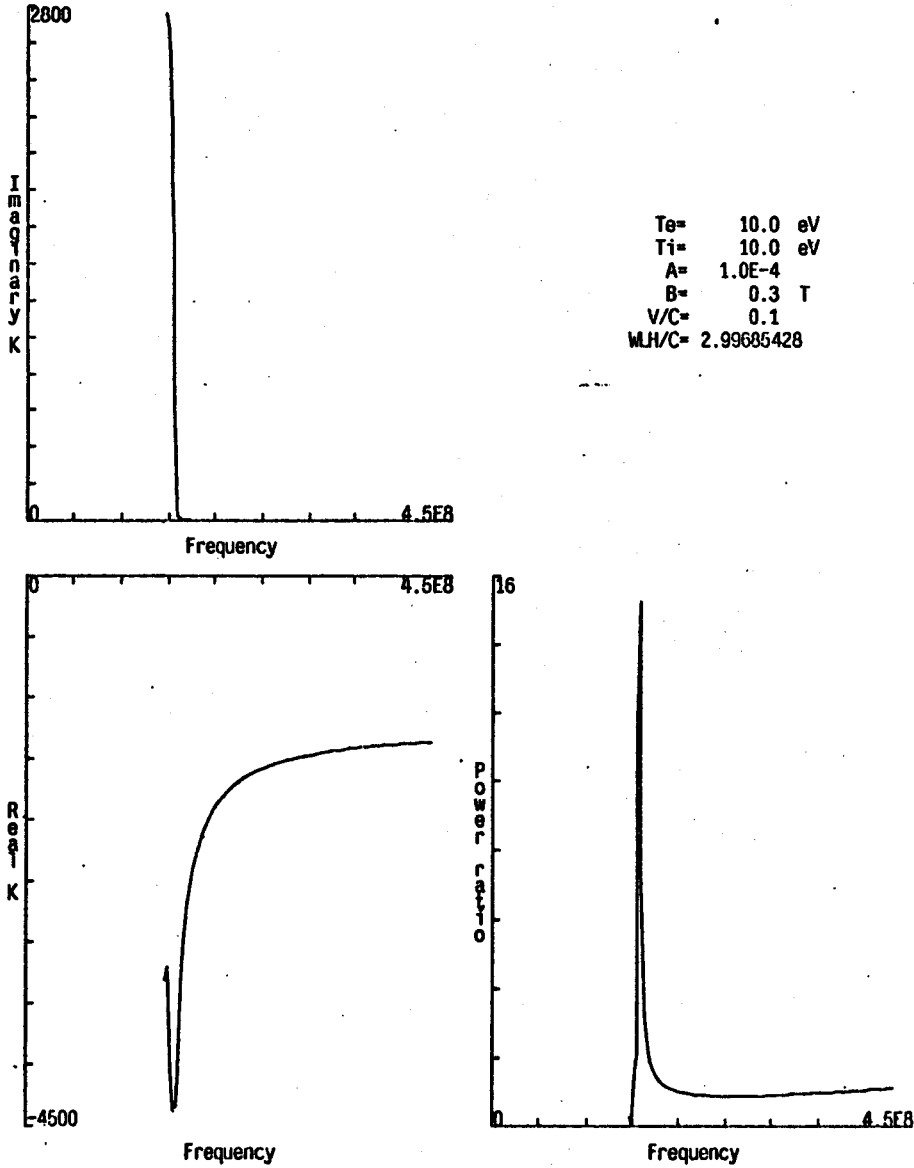


FIGURE 9

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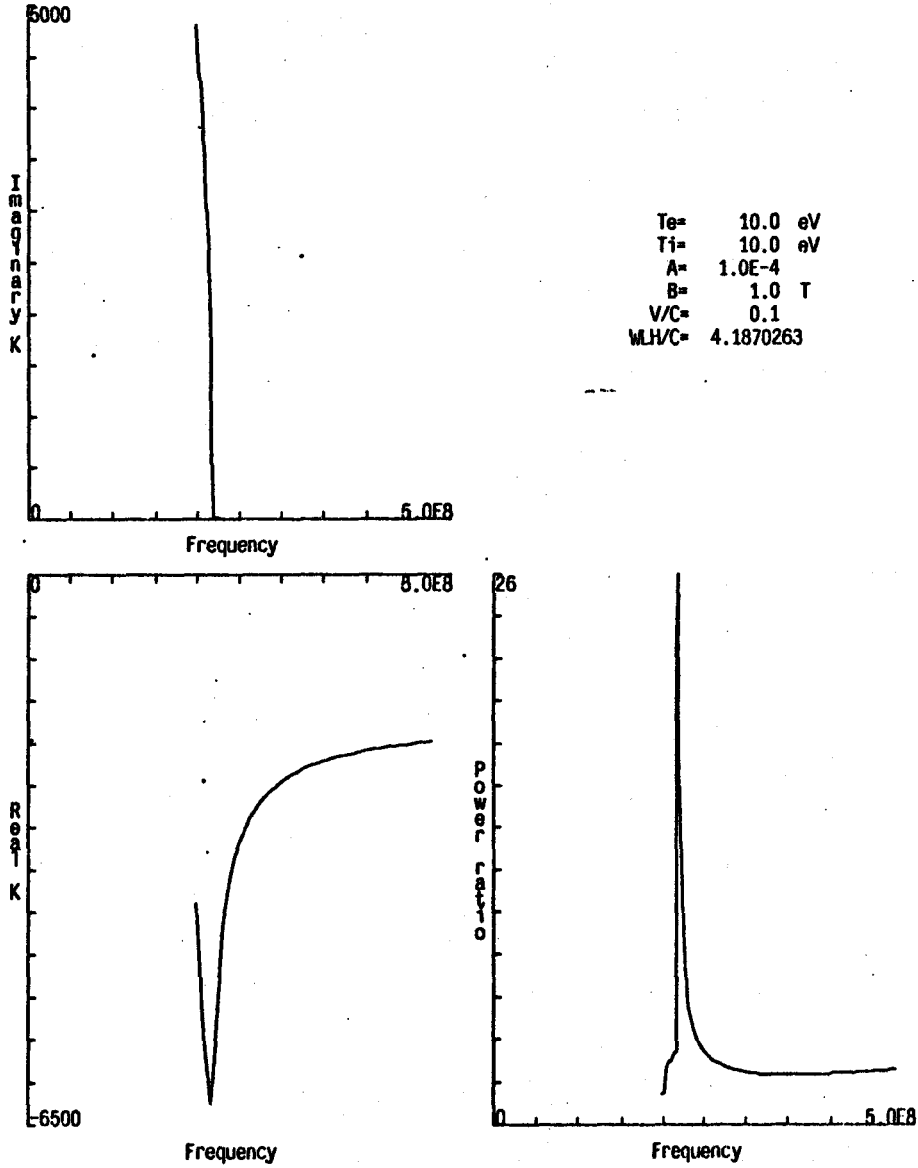


FIGURE 10

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