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PASSIVE AND ACTIVE CIRCUITS FOR VERTICAL PLASMA STABILIZATION

R. J. Thome, R. D. Pillsbury, Jr., D. B. Montgomery,
P. A. Politzer, S. M. Wolfe, W. R. Mann,
W. G. Langton and P. Pribyl

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Abstract

The highly elongated plasmas associated with divertor operation in the tokamaks are subject to a strong axisymmetric instability. A combination of passive loops and active feedback coils are needed for control of vertical plasma position. Development of design techniques for this coil system are underway using two techniques. First, simplified models are used which lead to generalized diagrams for evaluation of the relative effectiveness of different locations for active or passive elements. These plots and the associated methodology are generally applicable to tokamak design. At the second level, a more detailed computer model, incorporating the specific machine configuration and constraints, is used to study the optimal active coil and passive element characteristics.

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1.0 INTRODUCTION

The determination of the characteristics required for the passive stabilization elements and active coils for vertical position control of the elongated plasma in a tokamak has been studied by using a simplified model of the plasma. The latter is represented as a rigid circuit element which can dynamically interact with the other circuits. In this way, the effort is similar to other recent studies [1], [2], [3], [4]. However, the computer code which has been developed uses more general feedback control laws involving lead, measurement lag, control delay, and gain and is capable of studying any number of circuits of specified geometry. It has been used to generate the preliminary requirements for ALCATOR DCT which are discussed at the end of this paper.

In the course of the code development, it was found that considerable insight could be gained by formulating a generalized single circuit problem involving any number of coils of arbitrary geometry in a common active/passive circuit. This allowed the criteria for stabilization to be found as a set of inequalities among dimensionless system parameters which specifically relate the passive and active circuit characteristics and feedback time constants. The criteria are discussed below and may be used in the construction of stability maps which allow stable ranges for feedback time constants to be determined graphically.

A further simplification of the generalized single circuit to a single circuit composed of two circular loops symmetrically located relative to the $z = 0$ plane, allows the stability criteria to be reduced to a form suitable for evaluation using contour of effectiveness methods. The latter have been used for independent evaluation of passive and active loop locations [5], but have been extended here to include the effects of the feedback and circuit time constants.

2.0 MODEL AND GOVERNING EQUATIONS

The basic model adopted for this report is illustrated in Fig. 1. The model consists of a plasma that is assumed to be a rigid filament with current, I_p, that is assumed to be constant throughout this analysis.

The model also includes N_{coils} coils that may have arbitrary geometries (e.g. - solenoidal, saddle, etc.) and may be placed anywhere in space. Certain of the coils may be identified as active. These coils are assumed to have an externally applied voltage, that is governed by a control law. The remaininng coils are assumed to be passive (i.e. no external voltage sources). The coils can be assembled into a number of separate circuits. The voltage equation for each can be written using cartesian index notation as:

$$\frac{\partial}{\partial t}[M_{ij}I_j] + \frac{\partial}{\partial t}[M_{ip}]I_p + R_{ij}I_j = V_i \qquad i,j = 1,2,\ldots,N_{circ} \tag{1}$$

where M_{ij} is the mutual inductance between circuit i and circuit j, M_{ip} is the mutual inductance between circuit i and the plasma. I_j is the current in circuit j. R_{ij} is the element of the resistance matrix which is diagonal - i.e. $R_{ij} = 0$ for $i \neq j$. V_i is the applied voltage which is zero for passive circuits. The repeated indices imply summation. (1) includes the coupling to the plasma (but with I_p independent of time). Expanding the time derivative yields:

$$[M_{ij}]\mathring{I}_j + I_p\frac{\partial M_{ip}}{\partial z}\mathring{z} + [R_{ij}]I_j = V_i \tag{2}$$

For a small vertical displacement, z, of the plasma:

$$\frac{\partial M_{ip}}{\partial z} \approx -2\pi r_o\left(\frac{B_r}{I}\right)_i = -2\pi r_o B_{ri} \tag{3}$$

where B_{ri} represents the radial field at the plasma produced by a unit current in circuit i.

The applied voltage on the active circuits is assumed to have the following form:

$$V_i + (t_{b_i} + t_{c_i})\mathring{V}_i + t_{b_i}t_{c_i}\ddot{V}_i = -\alpha_i(z + t_{a_i}\mathring{z}) \tag{4a}$$

where z and $\mathring{z}$ are the plasma displacement and velocity, respectively. The feedback time parameters t_a, t_b, and t_c are either machine or user specified. Such a control law would give the following transfer function:

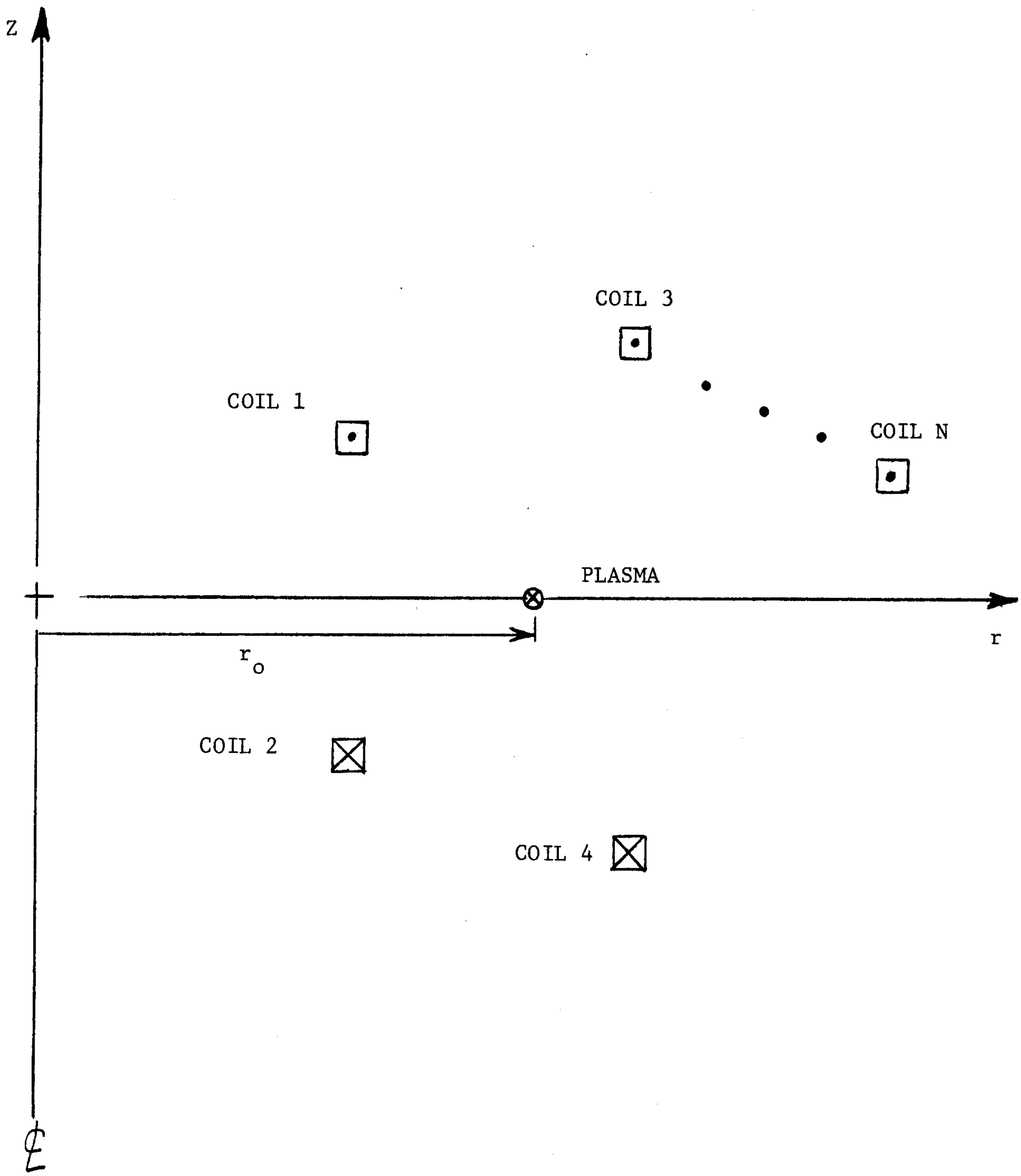

Fig. 1 Model for Plasma Stabilization

$$\frac{V_i}{z} = -\alpha_i \frac{(1+st_{a_i})}{(1+st_{b_i})} \quad \frac{1}{(1+st_{c_i})} \tag{4b}$$

The times t_a, t_b, and t_c can be identified as the lead, lag, and effective delay times. The effective delay is a machine and control circuit dependent quantity that accounts for the measurement delay and threshold delay. The remaining two times are user or controller defined: t_b is a lag, which can be thought of as a filter of input signal noise; t_a is used to cancel the effect of the measurement/machine delay t_c.

At this stage, the vertical force balance equation for the plasma is introduced. Two distinct models of the plasma were used. In the first model, the plasma mass was concentrated at the filament representing the plasma. The second model neglected the plasma mass, and, hence, the inertia term in the force balance equation. The model with mass is described first, followed by the model without mass.

2.1 Plasma Mass Included

The vertical force balance equation for the plasma is given by:

$$m\ddot{z} = 2\pi r_o I_p (B_{rk} I_k - B_{r_{ext}}) \tag{5}$$

where $B_{r_{ext}}$ is the steady state radial field component at the plasma produced by the other PF coils, and m is the mass of the plasma. $B_{r_{ext}} = 0$ at $z = 0$, which serves to define the $z = 0$ plane (especially for non-symmetric configurations). Summation is implied by the repeated index. For small displacements:

$$B_{r_{ext}} \approx \frac{\partial B_{r_{ext}}}{\partial z} z \tag{6}$$

In terms of the field index η_f, the external field is:

$$B_{r_{ext}} = -\frac{\eta_f B_z z}{r_o} \tag{7}$$

where B_z is the axial component of field at the plasma produced by the PF coil system. B_z and η_f are negative for elongated plasmas, and, hence, $B_{r_{ext}}$ is negative $z > 0$. This term is a destabilizing term. That is, if the plasma is displaced vertically, the external field produces a force on the plasma that is in the same direction as the displacement. The radial field produced at the plasma by the active and passive circuits must attempt to offset this destablizing effect. The term B_{rk} represents the radial field produced at the plasma by a unit current in the kth active or passive circuit.

The final set of equations for the analysis with mass included is given by:

$$[M_{ij}]\mathring{I}_j - 2\pi r_o I_p B_{r_i}\mathring{z} + [R_{ij}]I_j = V_i \tag{8}$$

the active circuit(s) voltage/feedback relation:

$$V_i + (t_{b_i} + t_{c_i})\mathring{V}_i + t_{b_i}t_{c_i}\ddot{V}_i = -\alpha_i(z + t_{a_i}\mathring{z}) \tag{9}$$

and the vertical force balance equation:

$$m\ddot{z} = 2\pi r_o I_p\left(B_{rk}I_k + \frac{\eta_f B_z}{r_o}z\right) \tag{10}$$

These equations may be cast into dimensionless form by introducing the following parameters:

$$\mathcal{I}_i = I_i/I_p \quad ; \quad \xi = z/r_o$$

$$\mathcal{M}_{ij} = M_{ij}/L_o \quad ; \quad \mathcal{B}_j = B_{rj}I_p/B_z$$

$$\mathcal{R}_{ij} = R_{ij}/R_o \quad ; \quad \mathcal{V}_j = V_j/V_o$$

$$\mathcal{A}_j = \alpha_j r_o/V_o \quad ; \quad \tau = t/\tau_o$$

$$\tau_a = t_a/\tau_o \quad ; \quad \tau_b = t_b/\tau_o$$

$$\tau_c = t_c/\tau_o$$

where

$$L_o = -\frac{2\pi r_o^2 B_z}{I_p}$$

$$\tau_o = \sqrt{\frac{m}{2\pi I_p(-B_z)}}$$

$$R_o = L_o/\tau_o$$

$$V_o = R_o I_p$$

The final dimensionless equations are:

$$[\mathcal{M}_{ij}]\mathring{I}_j + \mathcal{B}_i\mathring{\xi} + [\mathcal{R}_{ij}]I_j = \mathcal{V}_i \tag{11}$$

$$\mathcal{V}_i + (\tau_{b_i} + \tau_{c_i})\mathring{\mathcal{V}}_i + \tau_{b_i}\tau_{c_i}\ddot{\mathcal{V}}_i = -\mathcal{A}_i\Big(\xi + \tau_{a_i}\mathring{\xi}\Big) \tag{12}$$

$$\ddot{\xi} = \mathcal{B}_k I_k - \eta_f \xi \tag{13}$$

This set represents the equations governing the active stabilization of a plasma that is modeled as a single rigid filament with constant current, I_p and mass, m. The second derivative terms may be replaced by a new variable - for example :

$$s = \mathring{\mathcal{V}}$$

The set, then, reduces to a set of coupled, first order, linear, ordinary differential equations. This set may be solved using any time integration algorithm. However, since all coefficients are constant, an eigenexpansion technique has the advantage of indicating stability by inspection of the eigenvalues. These eigenvalues are, in general, complex. The system is considered to be stable if the real parts for all eigenvalues are less than zero (i.e. - an exponential decay rather than growth).

The number of equations and unknowns (and, hence, eigenvalues) in this set varies with the assumptions for the control law. If τ_b and τ_c are zero, the voltage law is substituted directly into (11), and there are $N = N_{circ} + 2$ unknowns - where N_{circ} is the number of circuits, and, hence, the number of unknown currents. The $+2$ comes from an unknown for both the displacement ξ and velocity $\mathring{\xi}$. If $\tau_b = 0$ and $\tau_c \neq 0$, then the number of unknowns becomes $N = N_{circ} + 2 + N_{\tau_c}$ where N_{τ_c} is the number of active circuits with $\tau_c \neq 0$. Finally, if both τ_b and τ_c are not zero, the number of unknowns becomes:

$$N = N_{circ} + 2 + N_{\tau_c} + N_{\tau_b}$$

The eigenexpansion technique leads to a solution of the above set of equations that has the form:

$$I_i = \sum_{j=1}^{N} c_{ij} e^{\lambda_j \tau} \qquad i = 1, 2, \ldots N_{circ}$$

$$\mathcal{V}_i = \sum_{j=1}^{N} c_{ij} e^{\lambda_j \tau} \qquad i = N_{circ} + 1, N_{circ} + 2, \ldots N_{circ} + N_{active}$$

$$\xi = \sum_{j=1}^{N} c_{ij} e^{\lambda_j \tau} \qquad i = N_{circ} + N_{active} + 1$$

where N_{active} is the number of active circuits defined by a nonzero τ_b and/or τ_c, and N is the total number of unknowns.

The c_{ij}'s are normalized eigenvectors associated with the eigenvalues, λ_j. The normalization factors are determined from the initial conditions. It is not necessary to specify the initial conditions in order to determine the eigenvalues and, in turn, whether or not the system is stable. The initial conditions need only be specified after a stable solution has been found, and the displacement, current, voltage, power, etc. versus time behavior of the system is desired. The instantaneous power required for the active circuit is given by:

$$P = Re\{V\}Re\{I\}$$

The application of this analysis of the model with mass is discussed in Section 3.0. The modifications to these equations for the model without mass are given below.

2.2 Plasma Mass Neglected

If the mass of the plasma is neglected - which is equivalent to ignoring the inertia term - then the force balance equation can be written as:

$$z = \frac{r_o}{\eta_f B_z} B_{rk} I_k \tag{14}$$

where the repeated index implies summation. (14) can be substituted directly into the circuit equations and voltage law. The equations may be normalized using the dimensionless parameters given in the preceding section, with the exception of the normalizing time τ_o, which will remain undefined at this point. The following set of linear, coupled, first order, ordinary, differential equations then govern the behavior of the model:

$$\left[\mathcal{M}_{ij} + \frac{\mathcal{B}_i \mathcal{B}_j}{\eta_f}\right]\mathring{\mathcal{I}}_j + [\mathcal{R}_{ij}]\mathcal{I}_j = \mathcal{V}_i \tag{15}$$

$$\mathcal{V}_i + (\tau_{b_i} + \tau_{c_i})\mathring{\mathcal{V}}_i + \tau_{b_i}\tau_{c_i}\ddot{\mathcal{V}}_i = -\frac{\mathcal{A}_i}{\eta_f}\left(\mathcal{B}_k \mathcal{I}_k + \tau_{a_i}\mathcal{B}_k \mathring{\mathcal{I}}_k\right) \tag{16}$$

It will be seen in a later section that for a single circuit with massless plasma, a natural choice for the presently undefined normalizing time is $\tau_o = \mathcal{M}/\mathcal{R}$.

3.0 SINGLE CIRCUIT ANALYSIS

A general purpose computer program was written to solve the sets of equations defined in the preceding section for specific coil, circuit, and machine geometries. The program sets up and solves the generalized eigenvalue problem for any number of circuits. However, insight into requirements for the active and passive control coils can be gained by solving the single circuit cases discussed in this section.

The two models discussed in the preceding section have been used in a simple analysis of a single circuit that has both active and passive characteristics. The stability of the plasma as a function of selected dimensionless parameters associated with the coil/plasma model can then be investigated analytically. This section presents the results of this investigation - first for the model with mass, and then for the model without mass. The results are then compared.

3.1 Plasma with mass - single circuit analysis

If the passive and active system consists of a single circuit with windings distributed in any arbitrary arrangement above and below the $z = 0$ plane, the governing set of differential equations given by eqns (10)-(12) reduce to:

$$[\mathcal{M}]\mathring{I} + \mathcal{B}\mathring{\xi} + [\mathcal{R}]I = \mathcal{V} \tag{17}$$

$$\mathcal{V} + (\tau_b + \tau_c)\mathring{\mathcal{V}} + \tau_b\tau_c\mathring{s} = -\mathcal{A}\Big(\xi + \tau_a\mathring{\xi}\Big) \tag{18}$$

$$\mathring{y} = \mathcal{B}I - \eta_f\xi \tag{19}$$

$$s = \mathring{\mathcal{V}} \tag{20}$$

$$y = \mathring{\xi} \tag{21}$$

where the coil related subscripts have been dropped for convenience.

The governing equations given by (17)-(21) can be cast in the form of the generalized eigenvalue problem. The characteristic equation from which the eigenvalues can be found is a quintic. If it further assumed that the two control time constants (τ_b and τ_c) in the voltage relation are zero, the characteristic equation reduces to a cubic of the form:

$$A\lambda^3 + B\lambda^2 + C\lambda + D = 0 \qquad ; \qquad \tau_b = \tau_c = 0 \tag{22}$$

where the coefficients are given by:

$$A = 1$$

$$B = \frac{\mathcal{R}}{\mathcal{M}}$$

$$C = \eta_f + \frac{\mathcal{A}\tau_a\mathcal{B}}{\mathcal{M}} + \frac{\mathcal{B}^2}{\mathcal{M}}$$

$$D = \frac{\mathcal{R}}{\mathcal{M}}\left(\eta_f + \frac{\mathcal{A}\mathcal{B}}{\mathcal{R}}\right)$$

Routh's stability criteria require that $A, B, C, D > 0$ (actually that they have the same sign) and that $BC > AD$. Investigating these criteria leads to the following conditions on the parameters:

$$\frac{\mathcal{R}}{\mathcal{M}} > 0 \tag{23}$$

$$\frac{\mathcal{A}\tau_a\mathcal{B}}{\mathcal{M}} + \frac{\mathcal{B}^2}{\mathcal{M}} > -\eta_f \tag{24}$$

$$\frac{\mathcal{A}\mathcal{B}}{\mathcal{R}} > -\eta_f \tag{25}$$

and

$$\frac{\mathcal{A}\tau_a\mathcal{B}}{\mathcal{M}} + \frac{\mathcal{B}^2}{\mathcal{M}} > \frac{\mathcal{A}\mathcal{B}}{\mathcal{R}} > -\eta_f \tag{26}$$

The first condition is equivalent to requiring a (normalized) coil time constant to be positive and is met by all designs. The field index, η_f, is negative for all cases of interest to this study. The second and third conditions put bounds on the normalized coil geometry and feedback gain required for stability. The fourth condition is a restatement or nesting of the second and third conditions, which are, therefore, redundant.

The major parameters are $\mathcal{B}^2/\mathcal{M}$, $\mathcal{A}\mathcal{B}/\mathcal{R}$, and $\mathcal{A}\tau_a\mathcal{B}/\mathcal{M}$. The third parameter, which is a normalized form of the $\dot{z}$ feedback, has the effect of increasing the effective normalized radial field per unit inductance. Eqn. (24) shows that if τ_a is increased, the range of stable solutions increases. It can also be seen from (25) or (26) that a purely passive system, $\mathcal{A} = 0$, does not lead to a stable solution.

For the special case where the single circuit consists of two series opposing circular cross section loops symmetrically located above and below the $z = 0$ plane, the parameter $\mathcal{B}^2/\mathcal{M}$ can be written as:

$$\frac{\mathcal{B}^2}{\mathcal{M}} = \frac{\mu_0 I_p}{(-B_z) r_o} G_{20} \tag{27}$$

where G_{20} is a function of the normalized coil locations and size only. Fig 2 shows contours of constant G_{20} in normalized space - with $(\rho, \eta) = (a/r_o, d/r_o)$ where r_o is the plasma radius, a is the coil radius, and $+d$ and $-d$ are the axial coordinates of the two coils.

Since the coil-pair inductance is used, a cross-sectional area or dimension is required. It is assumed that the coil has a round cross section of radius r_w. Fig 2. is for a ratio of $r_w/r_o = 0.005$. Figs. 3, 4, and 5 present contours of constant G_{20} for this ratio of 0.01, 0.02 and 0.03, respectively. Once the major machine parameters of plasma current, plasma radius, field index, and steady state background field (B_z) are specified, (27) and these plots may be used to determine coil locations that ensure plasma stability under the assumptions described above. For example, for the Alcator DCT, ($I_p = 1MA$, $r_o = 2m$, $B_z = -0.15T$, and $\eta_f = -2$), therefore,

$$\frac{\mathcal{B}^2}{\mathcal{M}} = 4.19 G_{20}$$

If $\tau_a = 0$, it is required that

$$G_{20} > 0.477 \tag{28}$$

for conditions (24) and (26) to be satisfied. For $\tau_a > 0$ this requirement can be relaxed according to (26).

The additional information on the feedback gain must still be investigated. The normalized parameter of interest is $\mathcal{A}\mathcal{B}/\mathcal{R}$, which can be written (for the special case of two circular loops) as:

$$\frac{\mathcal{A}\mathcal{B}}{\mathcal{R}} = \alpha \left(\frac{\mu_0 \sigma}{(-B_z)} \right) \left(\frac{r_w}{r_o} \right)^2 G_{21} \tag{29}$$

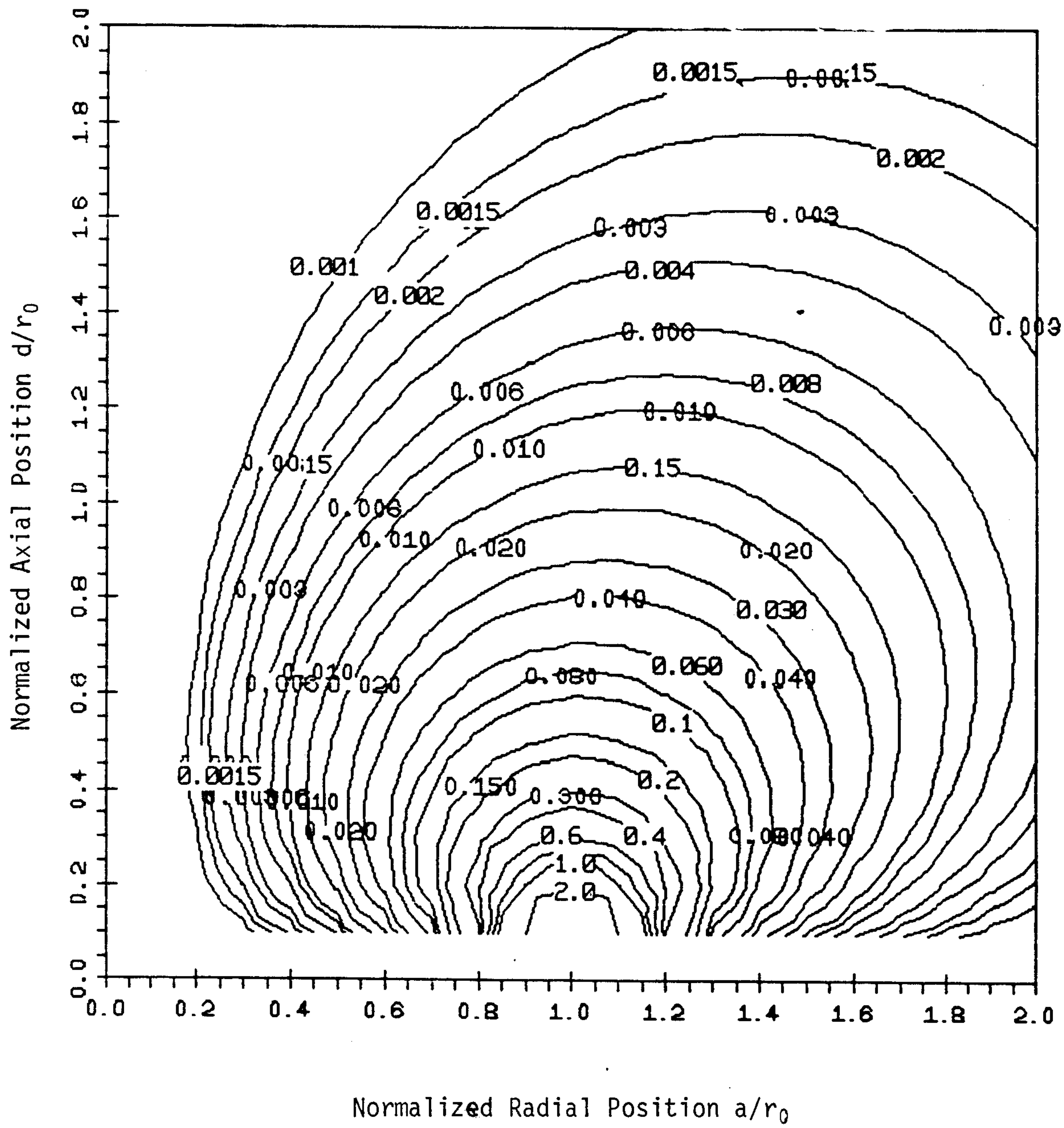

Fig. 2 Contours of Constant G_{20} in Normalized Space for $r_w/r_o = 0.005$

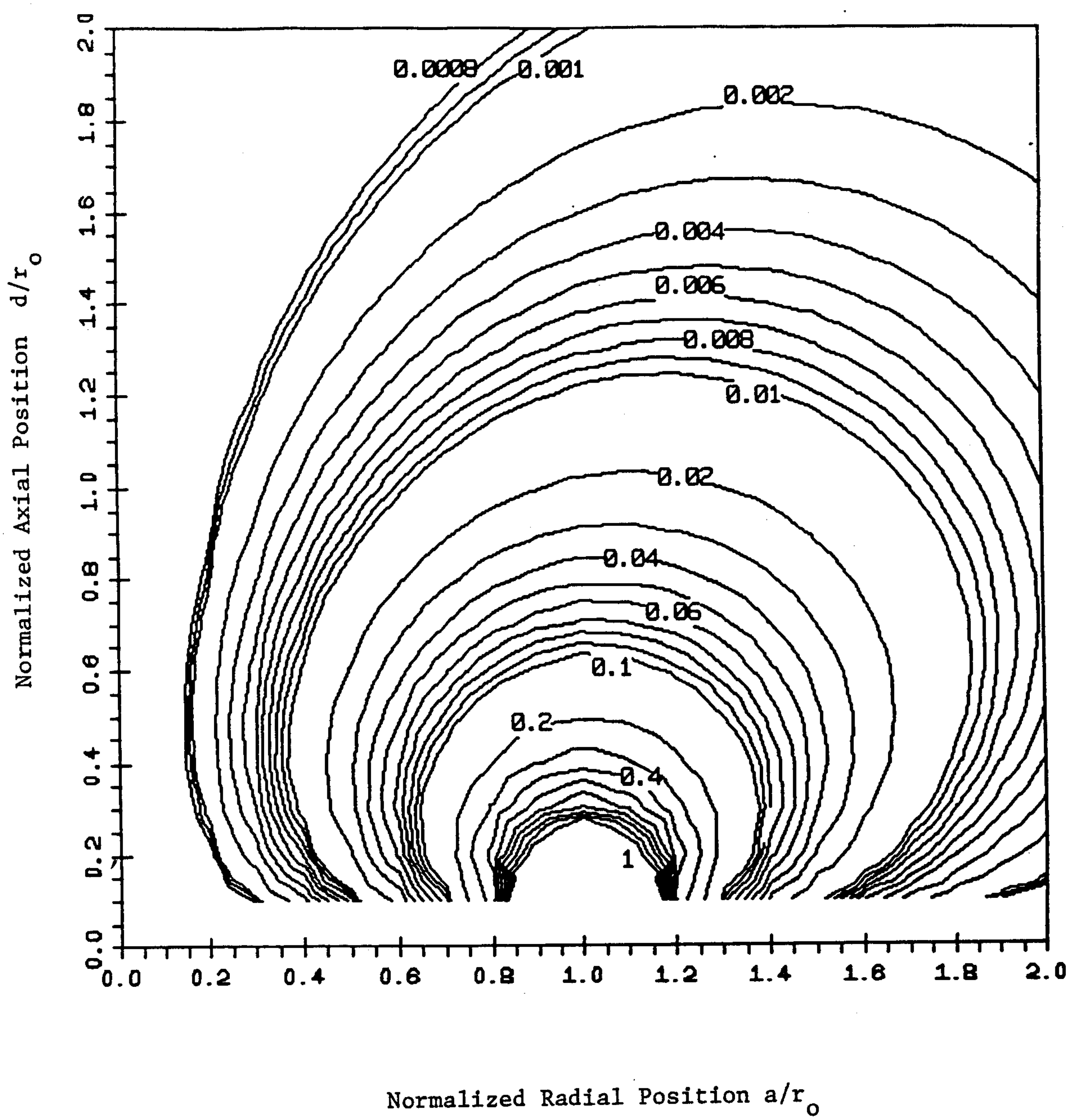

Fig. 3 Contours of Constant G_{20} in Normalized Space for $r_w/r_o = 0.01$

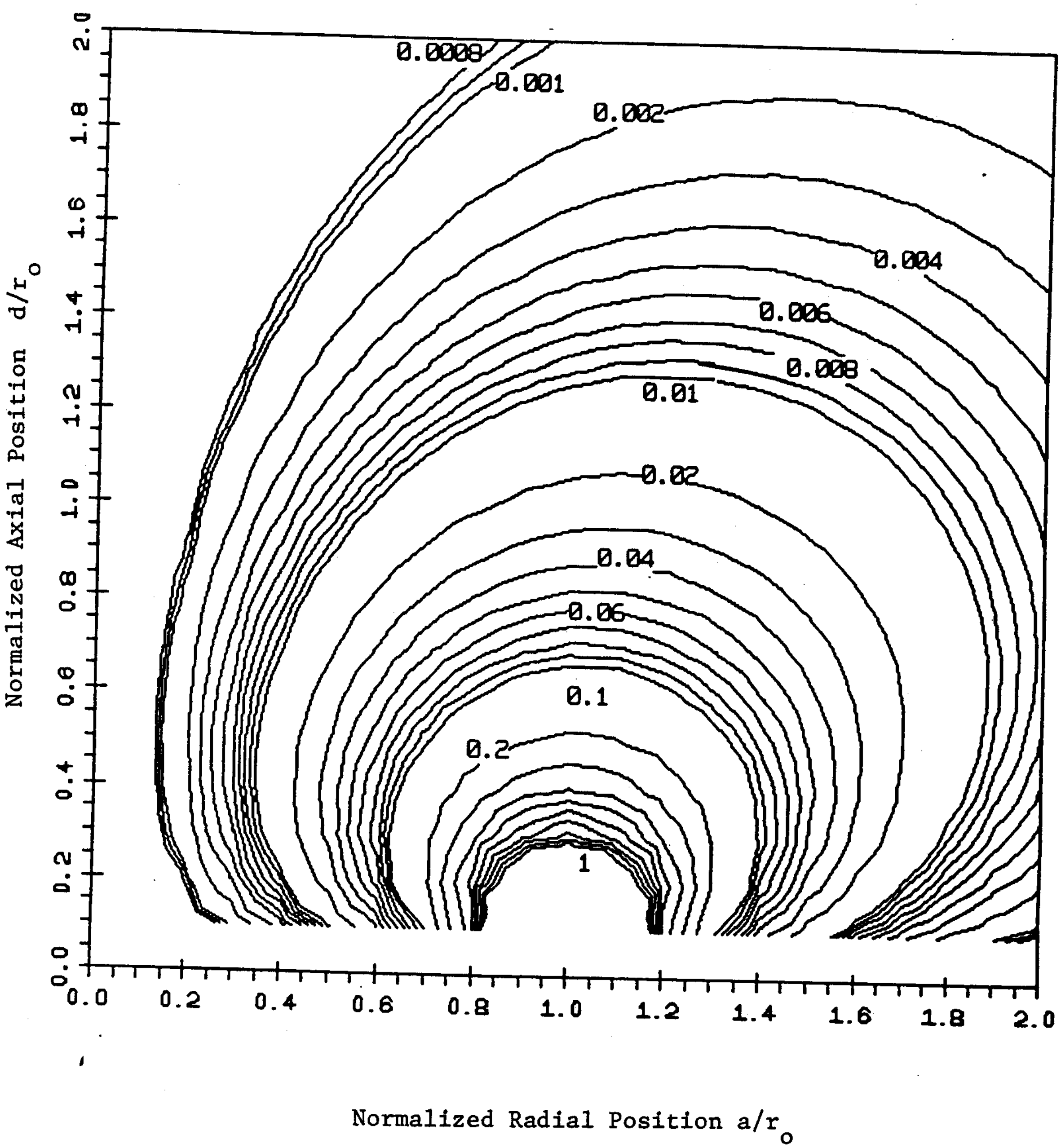

Fig. 4 Contours of Constant G_{20} in Normalized Space for $r_w/r_o = 0.02$

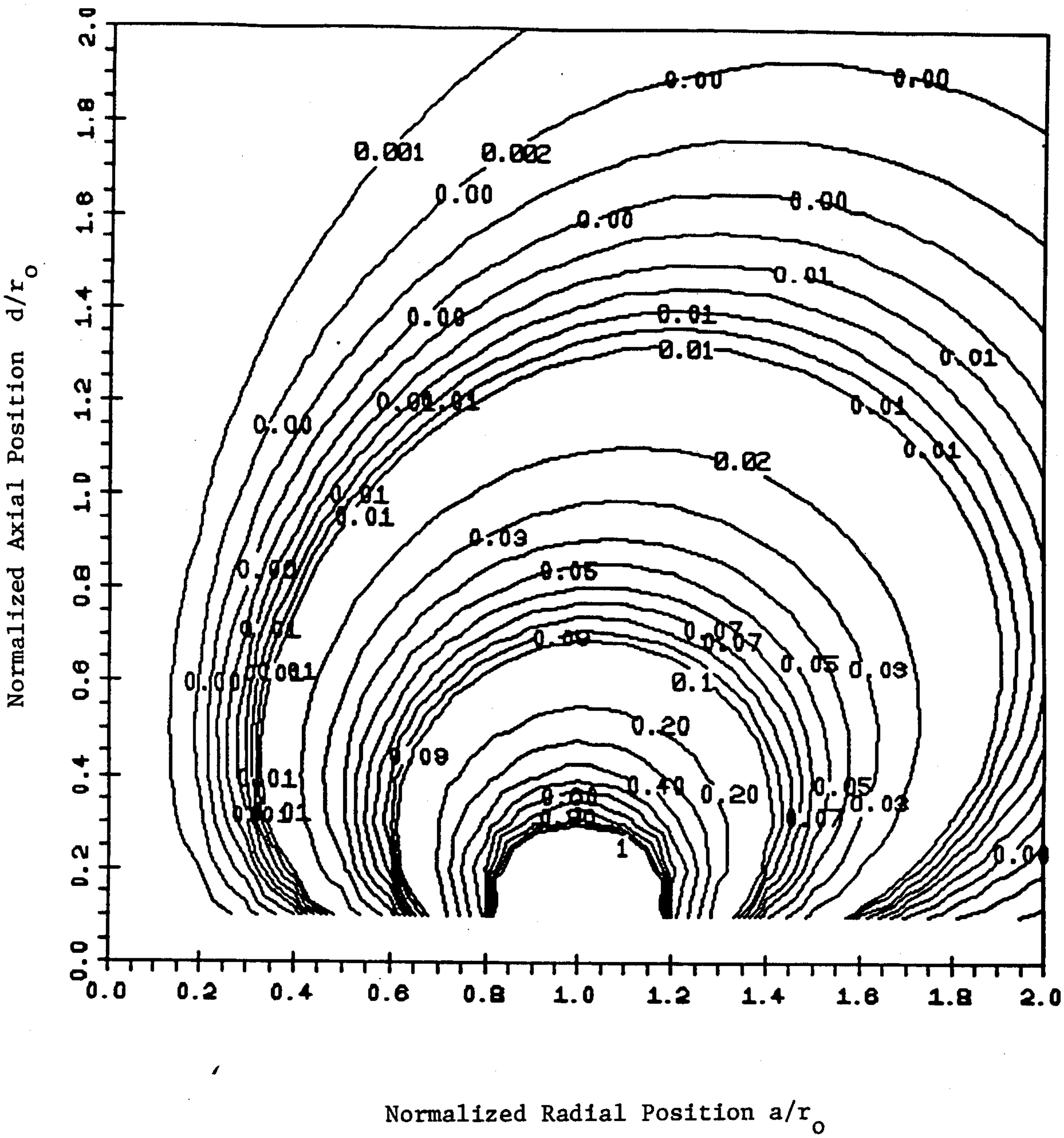

Fig. 5 Contours of Constant G_{20} in Normalized Space for $r_w/r_o = 0.03$

where α is the feedback gain and σ is the electrical conductivity of the coil material. The parameter G_{21} is a function of the coil-pair normalized position, (ρ, η), only. Contours of constant G_{21} may be generated and plotted in normalized space in the same manner as the G_{20} contours. Once a machine is specified and the coil material and size are chosen, these contours can be used along with the G_{20} contours to choose a coil (pair) location that will yield a stablized plasma. For example, the Alcator DCT machine with copper coils , will require by (26), (27), and (29):

$$4.19G_{20} > \alpha\left(\frac{r_w}{r_o}\right)^2 485.9G_{21} > 1 \tag{30}$$

or, equivalently,

$$.00862\left(\frac{G_{20}}{\left(\frac{r_w}{r_o}\right)^2 G_{21}}\right) > \alpha > \frac{.00206}{\left(\frac{r_w}{r_o}\right)^2 G_{21}} \tag{31}$$

Figs. 6 and 7 show contours of constant G_{21} and $1/G_{21}$ (a potentially more useful form than G_{21}). Once the coil cross section radius to plasma radius ratio is chosen, the limits on the feedback gain can be determined from the Figs. 2-5 and 6-7, and (31). For example, for $r_w/r_o = 0.01$, and $(\rho, \eta) = (1.2, 0.3)$, $G_{20} = 0.4$ - using Fig. 3. This does not satisfy (24), but the example will continue for illustration. Fig. 7 shows that $1/G_{21} = 7.0$, and, hence (31) implies

$$241.4 > \alpha > 288.2$$

Therefore, this choice of coil location, and size will not allow stabilization for any feedback gain.

If instead, a coil location of (1.2, .2) is chosen, $G_{20} = 0.5$ (Fig. 3), which satisfies (24), and $1/G_{21} = 6.0$ (Fig. 7). (31) then becomes:

$$258.7 > \alpha > 246.9$$

This specifies the range on feedback gains for $\tau_a = 0$ that will stabilize the plasma. For $\tau_a > 0$, the effect indicated in (26) would be to broaden the upper limit, since, the effect is to increase the radial field and decrease the value of G_{20} required.

Real machine designs will have regions in which coils cannot be placed due to constraints imposed by other subsystems. A machine overlay on Figs. 3 to 7 would facilitate the selection of possible coil locations for plasma stabilizing coils and allow their relative effectiveness to be compared rapidly.

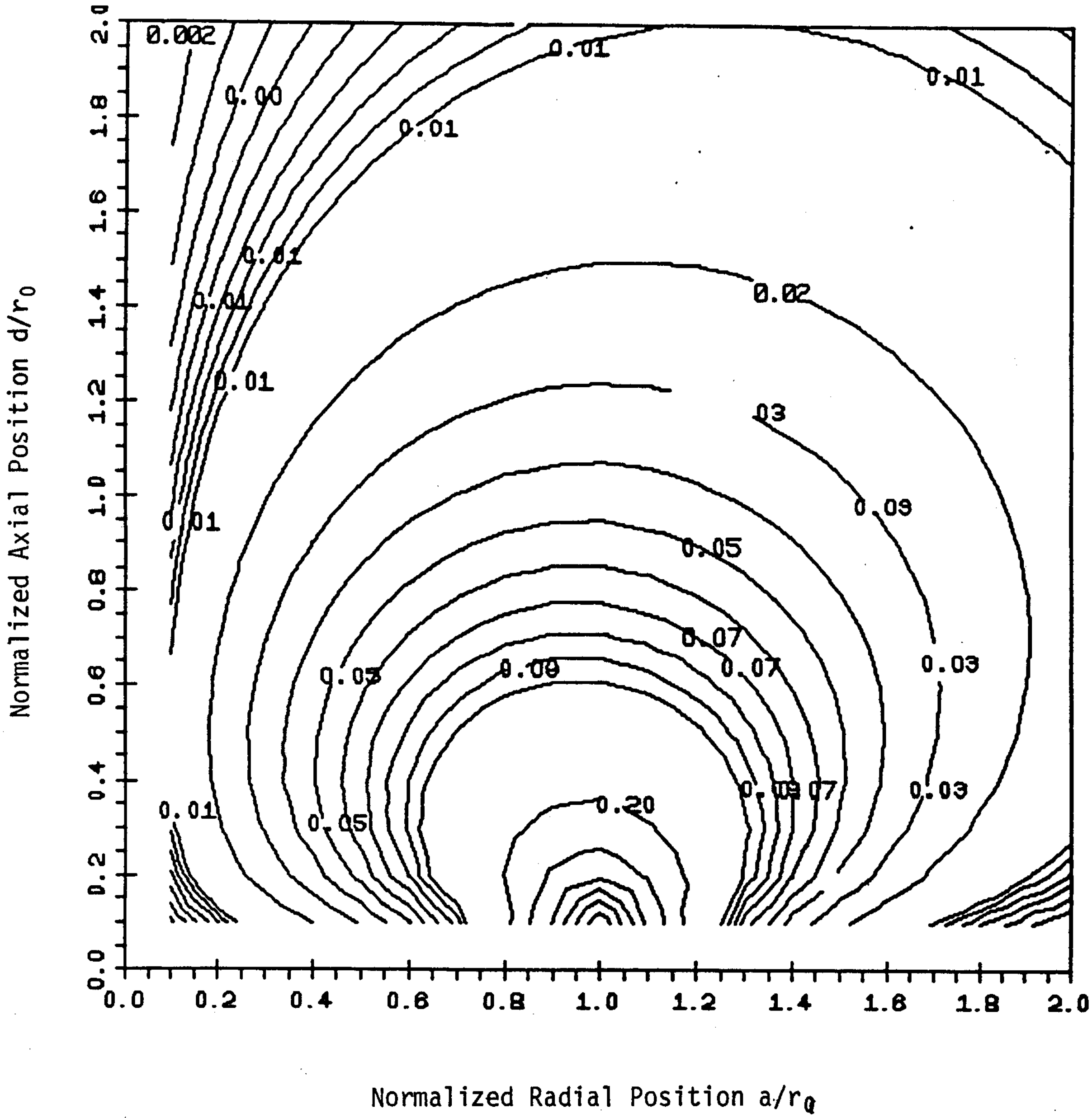

Fig. 6 Contours of Constant G_{21} in Normalized Space

Multiple circuit analyses introduce additional parameters and become too unwieldly for presentation in this form. However, general analysis programs that set up and solve the matrix eigenvalue/eigenvector problems given machine, and coil characteristics have been implemented.

The next section discusses the effect of neglecting the plasma mass, and, hence, the inertia term in the force balance equation.

3.2 Plasma without mass - single circuit analysis

If the mass of the plasma is neglected, the force balance equation for a general single circuit is:

$$z = \frac{\tau_o}{\eta_f B_z} BI$$

where B is the radial field at the plasma per unit current in the stabilizing circuit carrying a total current I. This can be substituted directly into the circuit equations and voltage law. Normalization using the dimensionless parameters given in section 2.1, except for τ_o, yields the following set of linear, coupled, first order, ordinary, differential equations:

$$\left[\mathcal{M} + \frac{\mathcal{B}^2}{\eta_f}\right]\mathring{\mathcal{I}} + [\mathcal{R}]\mathcal{I} = \mathcal{V} \tag{32}$$

$$\mathcal{V} + (\tau_b + \tau_c)\mathring{\mathcal{V}} + \tau_b\tau_c\ddot{\mathcal{V}} = -\frac{\mathcal{A}}{\eta_f}\Big(\mathcal{B}\mathcal{I} + \tau_a\mathcal{B}\mathring{\mathcal{I}}\Big) \tag{33}$$

where the coil subscript has been dropped for convenience. The dimensionless time constant τ_o remains undefined at this point in the massless analysis.

The eigenvalue formulation for this case leads to a cubic equation where the coefficients are given by:

$$A = -\tau_b\tau_c\left(1 + \frac{\mathcal{B}^2}{\eta_f\mathcal{M}}\right)$$

$$B = -\tau_b\tau_c\left(\frac{\mathcal{R}}{\mathcal{M}}\right) - (\tau_b + \tau_c)\left(1 + \frac{\mathcal{B}^2}{\eta_f\mathcal{M}}\right)$$

$$C = -(\tau_b + \tau_c)\frac{\mathcal{R}}{\mathcal{M}} - \frac{\mathcal{A}\tau_a\mathcal{B}}{\eta_f\mathcal{M}} - \left(1 + \frac{\mathcal{B}^2}{\eta_f\mathcal{M}}\right)$$

$$D = -\frac{\mathcal{R}}{\mathcal{M}} - \frac{\mathcal{A}\mathcal{B}}{\eta_f \mathcal{M}}$$

The characteristic equation has been divided by the normalized coil inductance, $\mathcal{M}$ to arrive at these coefficients.

The term $\mathcal{R}/\mathcal{M}$ is $\tau_o R/M$, and, since, τ_o has not been defined it can be chosen such that $\mathcal{R}/\mathcal{M} = 1$. Routh's stability criteria require that $A, B, C, D > 0$ and that $BC > AD$. Investigating these criteria leads to the following conditions on the parameters:

$$\frac{\mathcal{B}^2}{\mathcal{M}} > -\eta_f \tag{34}$$

$$\frac{\mathcal{B}^2}{\mathcal{M}} > -\eta_f\left(1 + \frac{\tau_b \tau_c}{\tau_b + \tau_c}\right) \tag{35}$$

$$\frac{\mathcal{A}\tau_a \mathcal{B}}{\mathcal{M}} + \frac{\mathcal{B}^2}{\mathcal{M}} > -\eta_f(1 + \tau_b + \tau_c) \tag{36}$$

$$\frac{\mathcal{A}\mathcal{B}}{\mathcal{M}} > -\eta_f \tag{37}$$

The first criterion, (34), is identical to (24) for the model with mass and for $\tau_a = 0$. Eqn. (34), however, is a more stringent requirement than that for the system with mass, since the feedback term on $\mathring{z}$ does not aid in meeting the criterion. The conditions (35) and (36) impose additional requirements on $\mathcal{B}^2/\mathcal{M}$. For non-zero τ_b and τ_c, a larger $\mathcal{B}^2/\mathcal{M}$ is required than if these controller time constants were zero.

If the following notation is introduced:

$$X = -\frac{\mathcal{B}^2}{\eta_f \mathcal{M}}$$

$$f_o = \frac{\tau_b \tau_c}{(X-1)^2}$$

$$g_o = \frac{\tau_b + \tau_c}{(X-1)}$$

$$\tau_{a_o} = \frac{\tau_a}{(X-1)}$$

$$Y = \frac{\mathcal{A}\mathcal{B}}{\eta_f \mathcal{M}}$$

the stability criteria become:

$$X > 1 \tag{38}$$

$$g_o > f_o \tag{39}$$

$$g_o < 1 - Y\tau_{a_o} \tag{40}$$

$$Y < -1 \tag{41}$$

and the last stability criterion can be written as:

$$f_o < \frac{g_o[g_o + (Y\tau_{a_o} - 1)]}{g_o + Y[\tau_{a_o} + 1]} \tag{42}$$

The major parameters are X, Y, τ_{a_o}, f_o, and g_o, and are for a single generalized circuit. The first condition is equivalent to requiring a (normalized) radial field per unit inductance to be greater than the absolute value of the field index and can be satisfied by a passive system (i.e. - one close enough to the plasma and in an effective location).

Note that the parameter $X = -\mathcal{B}^2/(\eta_f \mathcal{M})$ can be determined from (27) for the special case of a single circuit composed of two circular loops, symmetric about the $z = 0$ plane, and with round cross sections. The G_{20} contours discussed in the preceding section remain unchanged, and are valid for models with and without mass. Similarly, (29) yields the relationship between Y and G_{21}. The G_{21} contours are also equally valid for the analysis without mass for the single circuit composed of two loops.

For the generalized single circuit, the other stability criteria (39), (40), and (42) may be presented in graphical form as a function of the parameters $X, f_o, g_o, Y, \tau_{a_o}$. Fig. 8 shows the stability region for the case $\tau_{a_o} = 0$ as a function of g_o and f_o for a Y of -1.1. The horizontal line $g_o = 1$ is condition (40). For stability the point (f_o, g_o) must lie below this line. The straight line

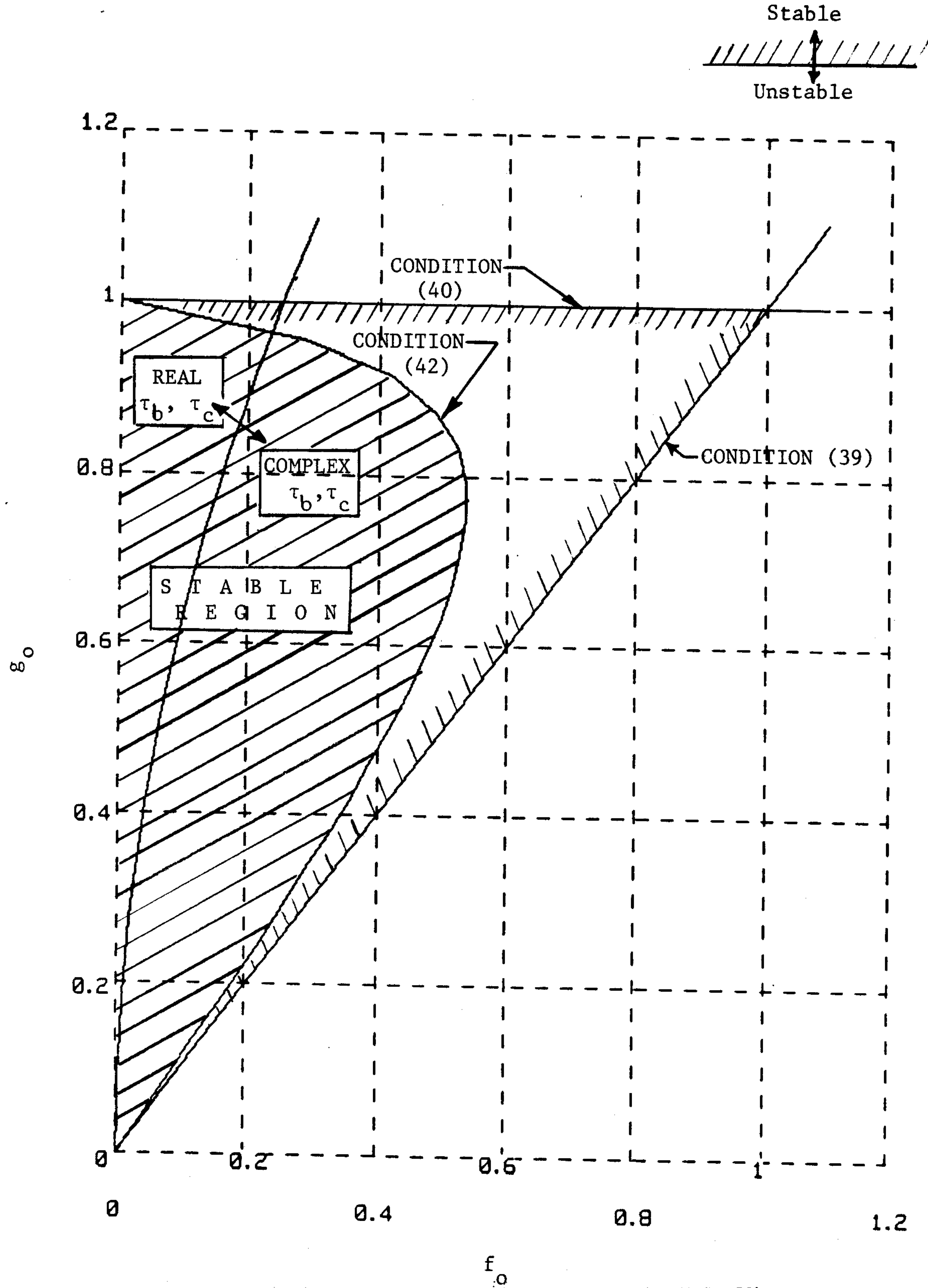

Fig. 8 Stability Region as a function of $g_o = \frac{\tau_b + \tau_c}{x - 1}$ and $f_o = \frac{\tau_b \tau_c}{(x-1)^2}$ for $\tau_{a_o} = 0$ and $Y = -1.1$

from $(0,0)$ through $(1,1)$ is condition (39) and coordinate pairs (f_o, g_o) to the right of this line lead to unstable solutions. The curved line corresponds to condition (42) and points between the f_o-axis and the curve are stable; everything outside of the curve is unstable. This condition is the most stringent of the criteria since all other conditions are satisfied by points satisfying (42), hence, (42) is the stability criteria for the generalized single circuit for a model with a massless plasma.

Once an (f_o, g_o) pair has been chosen in the stable region, τ_c and τ_b can be determined from:

$$\tau_c^2 - g_o(X-1)\tau_c + f_o(x-1)^2 = 0 \tag{43}$$

and

$$\tau_b = g_o(X-1) - \tau_c \tag{44}$$

In general, (43) and (44) can yield complex τ_b and τ_c. The sum and product of τ_b and τ_c, however, are still real. In order to retain separate constants τ_b and τ_c (as they appear in the control law (4b)) rather than their sum and product, it is necessary to further restrict the stability region to that region which also yields real τ_b and τ_c. The additional line that separates the real and complex regions for these constants is also shown in Fig. 8. Figs 9-12 show the stability regions for selected values of Y for various values of τ_{a_o} ranging from 0.0 to 10. In summary, (42) and Figs. 9-12 are applicable to a single generalized circuit for which the normalized parameters in (34) to (37) are determined and then transformed further to those in (42).

The stability analysis for the model without mass presented in this section can also be used in the following manner, for a single, circular coil-pair. First, the G_{20} plots may be used along with (42) to determine possible locations and sizes for which $X > 1$. Then for a given X, the stability region plots are used to determine the ranges on $Y, f_o, g_o,$ and τ_{a_o} - i.e. $\alpha, \tau_a\tau_b,$ and τ_c for which the plasma is stabilized.

There are several special cases associated with the control law, (4a), and its associated transfer function given by (4b) which is repeated here for convenience.

$$\frac{V_i}{z} = -\alpha_i \frac{(1+st_{a_i})}{(1+st_{b_i})} \; \frac{1}{(1+st_{c_i})} \tag{45}$$

Some of these cases are investigated in the following sections.

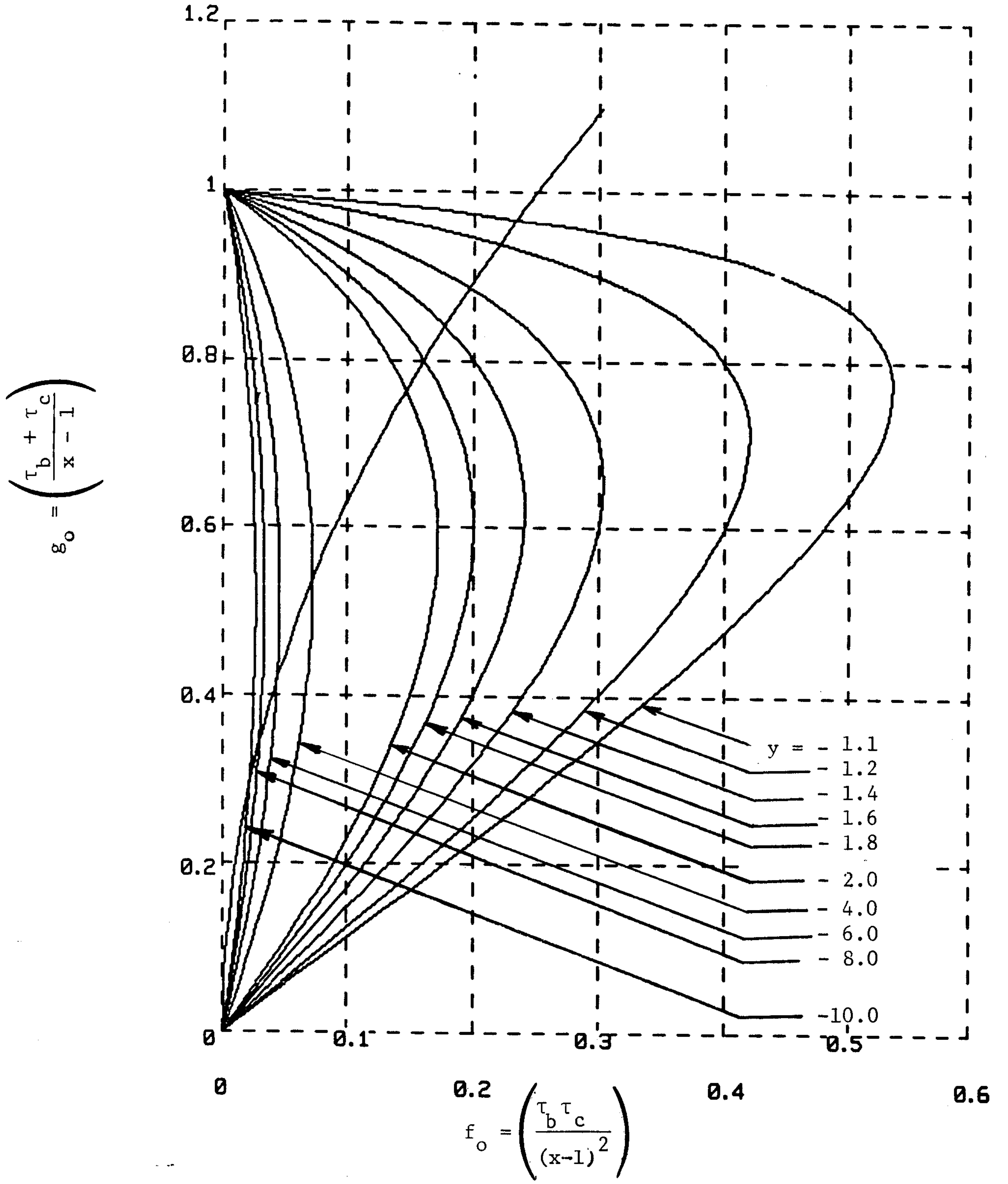

Fig. 9 - Stability regions for various values of y for $\tau_{a_o} = 0$ (no $\dot{z}$ feedback) (Stable region is inside curve defined by (42)).

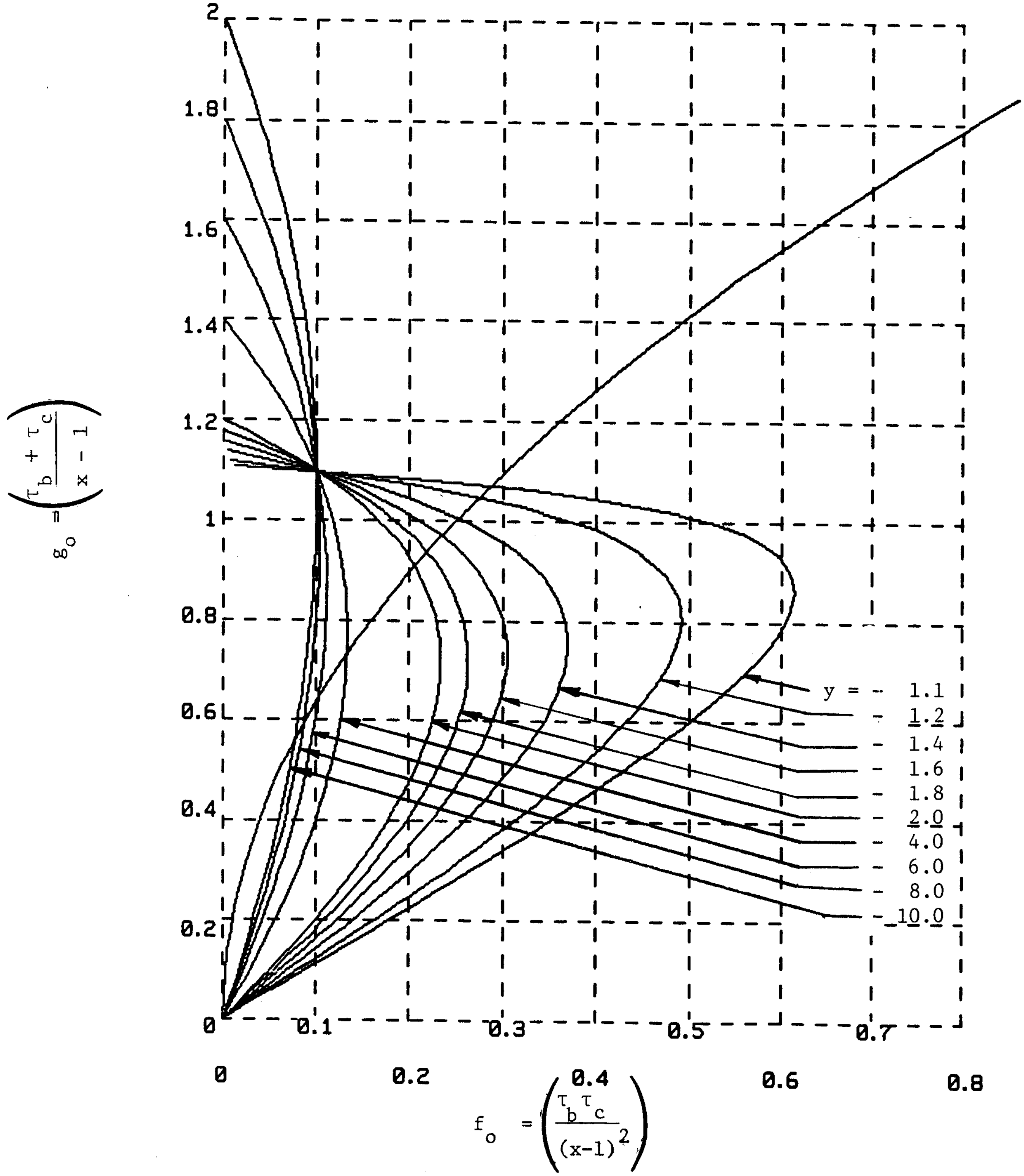

Fig. 10 - Stability regions for various values of y for $\tau_{a_o} = 0.1$ (Stable region is inside curve defined by (42)).

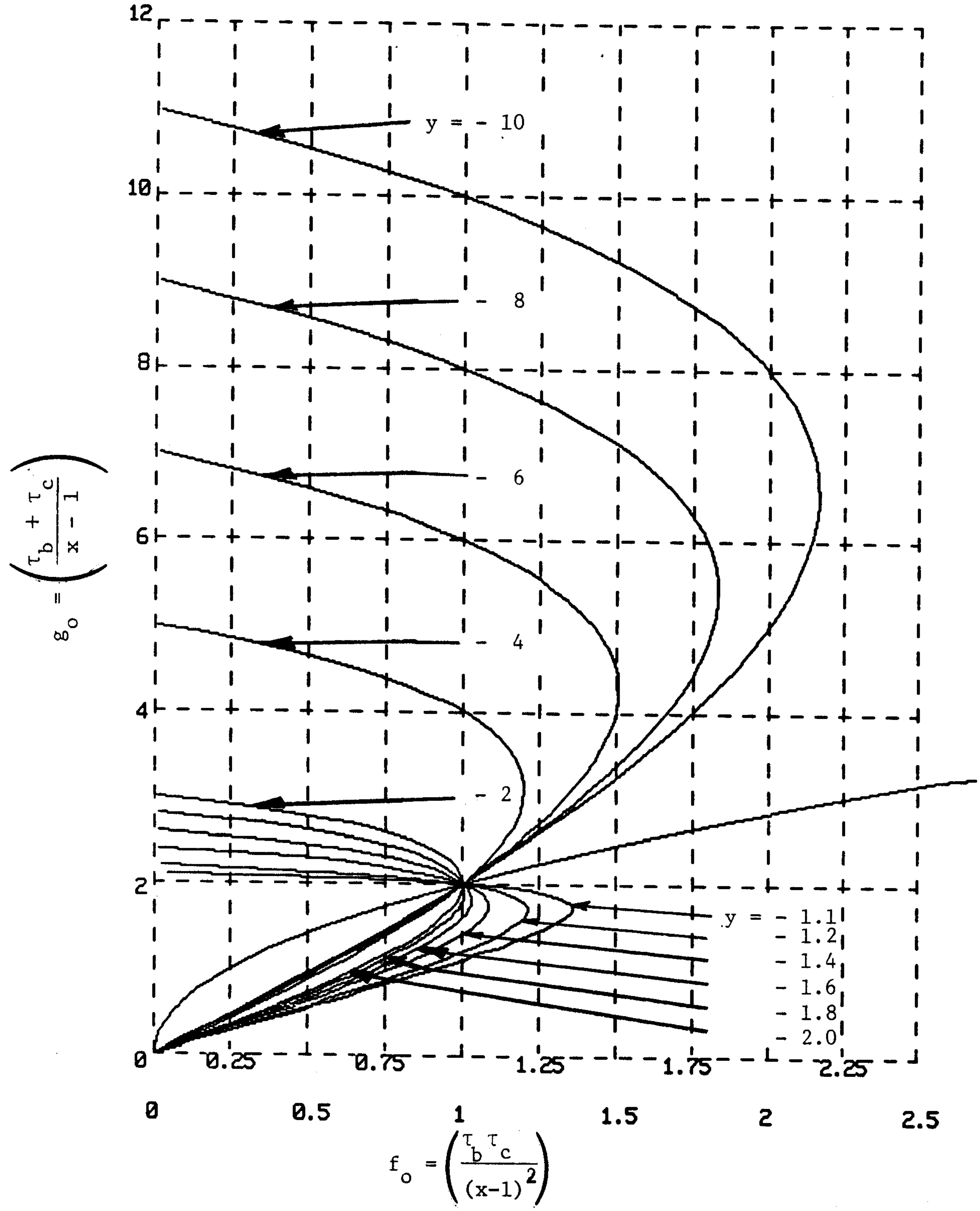

Fig. 11 - Stability region for various values of y for $\tau_{a_o} = 1.0$ (Stable region is inside curve defined by (42)).

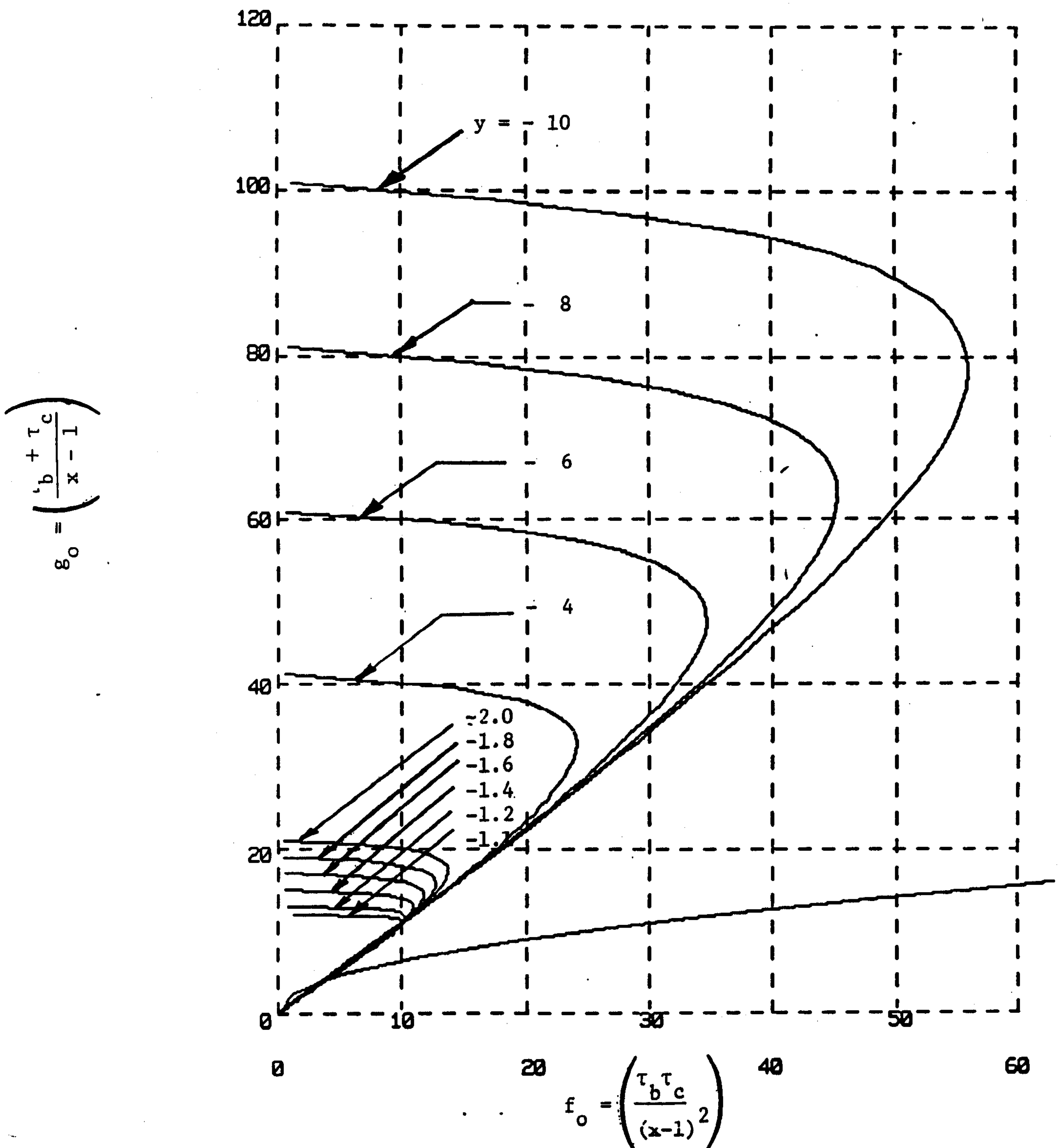

Fig. 12 - Stability regions for various values of y for $\tau_{a_o} = 10.0$ (Stable region is inside curve defined by (42)).

3.2.1 Special case: $\tau_a = \tau_c$

If the constant τ_c in the control law is assumed to be a delay time associated with the sensing and feedback, then τ_a can be used to help minimize such a delay, and the condition that $\tau_a = \tau_c$ becomes one of interest because the "lead", τ_a, exactly cancels the effect of the "lag", τ_c. Under this assumption, the stability conditions (38)-(42) become:

$$X > 1 \tag{46}$$

$$\tau_{b_o} < \frac{\tau_{a_o}}{\tau_{a_o} - 1} \qquad \tau_{a_o} > 1 \tag{47}$$

$$\tau_{b_o} < 1 - \tau_{a_o}(Y + 1) \tag{48}$$

$$Y < -1 \tag{49}$$

and

$$\tau_{b_o}^2(\tau_{a_o} - 1) + \tau_{b_o}(\tau_{a_o}^2(Y + 1) - 2\tau_{a_o} + 1) + \tau_{a_o}(1 - \tau_{a_o}(Y + 1)) > 0 \tag{50}$$

where τ_a and τ_b have been normalized by $(X - 1)$ and become τ_{a_o} and τ_{b_o} as in the previous section. Analysis of conditions (46) to (50) shows that they are all satisfied if $\tau_{b_o} < 1$ for all τ_{a_o} and Y. Mathematically, the stable region extends into the negative τ_{b_o} plane, but the implicit assumption that $\tau_{b_o} > 0$ is made.

3.2.2 Special case: $\tau_b = \tau_c = 0$

For the special case when $\tau_b = \tau_c = 0$, the governing equation becomes:

$$(Y\tau_a + (X - 1))\dot{I} + (Y + 1)I = 0 \tag{48}$$

which has an eigenvalue of:

$$\lambda = -\frac{Y + 1}{Y\tau_a + (X - 1)}$$

For an initial plasma displacement ξ_o, the normalized current in the circuit is:

$$\mathcal{I} = \frac{\eta_f \xi_o}{\mathcal{B}} e^{\lambda\tau}$$

The voltage can also be written as:

$$\mathcal{V} = -\frac{\mathcal{M} Y \eta_f \xi_o}{\mathcal{B}} (1 + \lambda\tau_a e^{\lambda\tau})$$

and the normalized instantaneous peak power can be found to be:

$$\mathcal{P}_{peak} = (\mathcal{V}\mathcal{I})_{peak} = \left(\frac{\eta_f \xi_o^2}{X}\right)\left(\frac{Y(\tau_{ao}+1)}{Y\tau_{ao}-1}\right)$$

4.0 PRELIMINARY ESTIMATES FOR ALCATOR DCT

Section 2.0 discussed the general model and indicated that a computer code had been developed to investigate the stability of systems of this type for any number of circuits of specified geometry. Table 1 outlines the procedure utilized which is based on an eigenexpansion technique. Note that stability is implied if the real part of the eigenvalues are all negative. Specification of the initial conditions is not necessary to determine stability, but is necessary to find real values of the output parameters.

Preliminary estimates for the passive and active coil requirements to achieve vertical stability in ALCATOR DCT have been carried out with this code based on a massless plasma. Results based on utilization of passive and/or active circular elements at locations A, B, and C in Fig. 13 are given in Tables 2-5. Coils are designated in the first three columns as being either copper (CU), stainless steel (SS) or superconducting (SC). The active coil pair for each line is indicated by an asterisk (*); other coils are passive. SS is representative of the passive effect of the first wall. CU represents copper added near the first wall at the designated location to enhance passive characteristics, unless denoted with an * (e.g. - in lines 4 and 5) in which case it is an active coil set. SC is the superconducting PF coil pair likely to be most effective for this function relative to the other PF coils.

Cases were calculated in pairs corresponding to either 5 ms or 7 ms decay times for plasma position to be restored to $z = 0$. In all cases considered, less power is required for longer position decay times. In the first three cases in each table, the active coil pair is the external superconducting PF coil set located at C and the passive coils are either SS or CU located at A and B; in cases 4 and 5 the active coil pair is internal at A or B. Although an internal active coil pair requires a more complex design and assembly, the results show a substantial reduction in the required peak power for all Tables for cases 4 and 5 relative to cases 1 to 3 where the external coils are driven. A comparison of cases 2 and 3 with case 1 in each table indicates a large reduction in peak power for the external active coil pair if the passive characteristics of the first wall are enhanced with copper on either the inboard or outboard side of the first wall.

The choice of times t_a, t_b, and t_c are given in the title for each table. Power requirements can be shown to be very sensitive to these values. In particular they can be factors of two to four lower than those given for the externally driven coils if it is assumed that $t_a = t_b = t_c = 0$.

TABLE 1 COMPUTATIONAL PROCEDURE USED IN COMPUTER PROGRAM

— INPUT:	$[M_{ij}]$, $[R_{ij}]$, B_{rk};	CIRCUIT CHARACTERISTICS
	r_o, I_p, m;	PLASMA CHARACTERISTICS
	η_f, B_z;	STATIONARY FIELD CHARACTERISTICS
	t_{ai}, t_{bi}, t_{ci}, α_i;	FEEDBACK CHARACTERISTICS
— DETERMINES EIGENVALUES, λ_i;		NOTE: STABILITY REQUIRES $R_e[\lambda_i] < 0$ FOR ALL i
— DETERMINES:	$I_i(t)$	CURRENT
	$V_i(t)$	NOTE: V = 0 IF COIL IS PASSIVE
	Z(t)	DISPLACEMENT
	P(t)	POWER

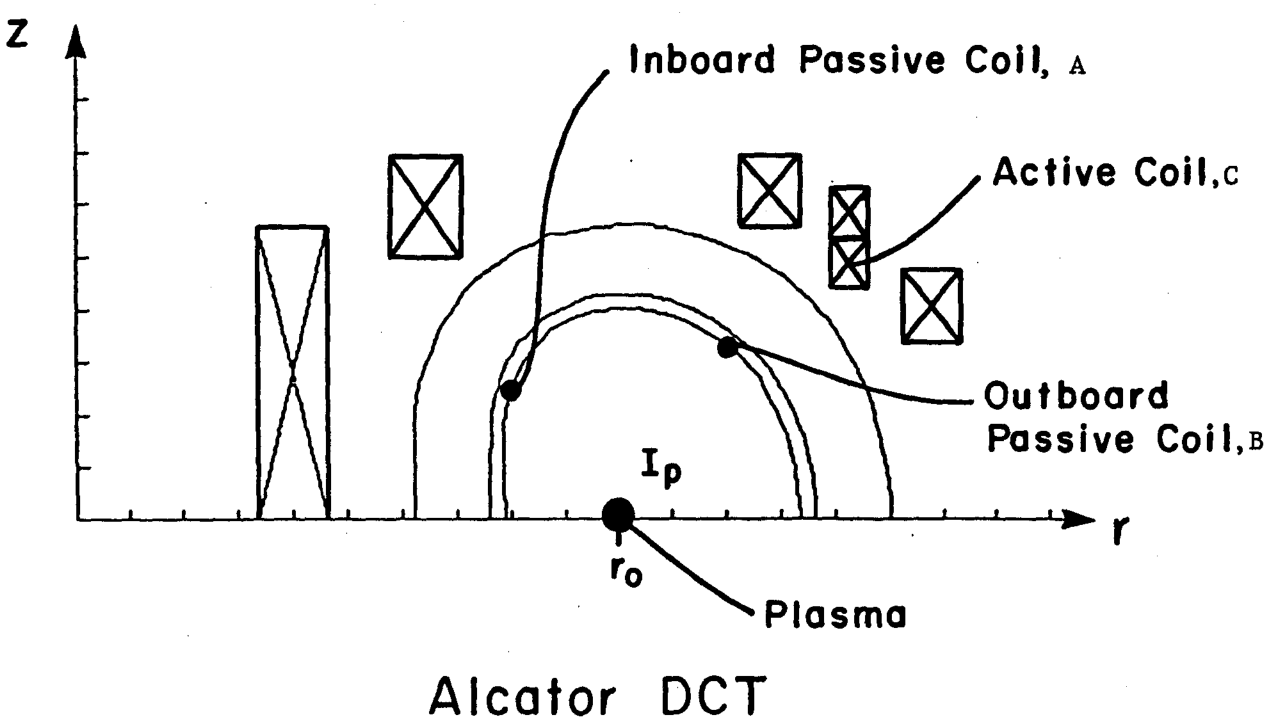

Fig. 13 Outline of ALCATOR DCT in normalized space showing coil locations for Tables 2,3,4, and 5.

TABLE 2
PEAK INSTANTANEOUS POWER REQUIREMENTS FOR ALCATOR DCT
SIX COIL MODEL - WITH $t_a = t_c = 3$ ms AND $t_b = 0$

INBOARD COIL	OUTBOARD COIL	EXTERNAL COIL	FEEDBACK GAIN, α 10^5 V/m	P_{peak} kVA	$t_{P_{max}}$ ms	τ_{decay} ms
SS	SS	SC*	1.78	463	2.8	5
			1.41	446	3.1	7
SS	CU	SC*	1.39	440	4.0	5
			1.03	401	4.7	7
CU	SS	SC*	1.19	235	3.5	5
			0.83	199	4.2	7
SS	CU*	SC	0.0107	20.9	3.0	5
			0.0079	19.4	3.6	7
CU*	SS	SC	0.0062	13.6	2.8	5
			0.0043	11.9	3.4	7

* denotes driven (active) coil

TABLE 3
PEAK INSTANTANEOUS POWER REQUIREMENTS FOR ALCATOR DCT
SIX COIL MODEL - WITH $t_a = t_c = 3$ ms AND $t_b = 0.6$ ms

INBOARD COIL	OUTBOARD COIL	EXTERNAL COIL	FEEDBACK GAIN, α 10^5 V/m	P_{peak} kVA	$t_{P_{max}}$ ms	τ_{decay} ms
SS	SS	SC*	2.15	774	3.1	5
			1.75	729	3.4	7
SS	CU	SC*	1.23	547	4.7	5
			0.98	496	5.3	7
CU	SS	SC*	1.29	313	3.9	5
			0.94	277	4.5	7
SS	CU*	SC	0.0093	24.6	3.8	5
			0.0075	22.8	4.3	7
CU*	SS	SC	0.0054	14.9	3.5	5
			0.0040	13.3	4.0	7

* denotes driven (active) coil

TABLE 4
PEAK INSTANTANEOUS POWER REQUIREMENTS FOR ALCATOR DCT
SIX COIL MODEL - WITH $t_a = t_c = 3$ ms AND $t_b = 1.2$ ms

INBOARD COIL	OUTBOARD COIL	FURTHEST COIL	FEEDBACK GAIN, α 10^5 V/m	P_{peak} kVA	$t_{P_{max}}$ ms	τ_{decay} ms
SS	SS	SC*	-	-	-	5
			-	-	-	7
SS	CU	SC*	1.30	688	5.3	5
			1.03	616	5.9	7
CU	SS	SC*	3.90	667	2.9	5
			1.66	462	4.2	7
SS	CU*	SC	0.0091	27.6	4.6	5
			0.0074	25.7	5.0	7
CU*	SS	SC	0.0049	15.4	4.4	5
			0.0039	14.0	4.8	7

* denotes driven (active) coil

- denotes no stable solution with required decay rate.

TABLE 5
PEAK INSTANTANEOUS POWER REQUIREMENTS FOR ALCATOR DCT
SIX COIL MODEL - WITH $t_a = t_c = 1$ ms AND $t_b = 0$

INBOARD COIL	OUTBOARD COIL	EXTERNAL COIL	FEEDBACK GAIN, α 10^5 V/m	P_{peak} kVA	$t_{P_{max}}$ ms	τ_{decay} ms
SS	SS	SC*	1.77	662	2.0	5
			1.41	588	2.3	7
SS	CU	SC*	1.39	588	2.8	5
			1.03	484	3.3	7
CU	SS	SC*	1.19	377	2.5	5
			0.83	283	3.0	7
SS	CU*	SC	0.0107	26.1	1.9	5
			0.0079	22.4	2.2	7
CU*	SS	SC	0.0062	17.9	1.6	5
			0.0043	14.6	2.0	7

* denotes driven (active) coil.

5.0 CONCLUSION

Preliminary estimates for ALCATOR DCT indicate that 15-25 KVA will be required for vertical stabilization if a driven coil pair is utilized near the first wall internal to the TF coils. If an external coil pair is utilized, 200-700 KVA will be required if copper passive elements in the vicinity of the first wall are used and 500-800 kVA if copper enhancement near the first wall is not employed.

In the process of studying the system requirements with a code capable of handling any number of passive and/or active coils, single circuit analyses were performed. The latter have led to the formulation of the requirements for stability using a general single circuit composed of any number of coils carrying the same current magnitude at any instant of time. These requirements were then reduced for the special case of one coil pair and related to contour plots applicable to any machine for evaluation of the relative effectiveness of different coil pair locations.

REFERENCES

1. S. J. Jardin and D. A. Larrabee, 'Feedback Stabilization of Axisymmetric Modes in Tokamaks', PPPL 1863, Jan. 82.
2. K. Ueda, S. Nishio, N. Fujisawa, M. Sugihara, S. Saito and K. Miyamoto, 'Vertical Position Control of the Elongated INTOR Plasma', JAERI-M, 82-213, Jan. 83.
3. R. Keller, 'Optimization of the Feedback Stabilization System', INTOR Workshop Phase IIA Report, EC Contribution, Oct. 82.
4. J. G. Murray and G. Bronner, 'FED Control Power', 9th Engr. Probs. of Fusion Research, Chicago, Oct. 81.
5. R. J. Thome, R. D. Pillsbury, Jr., W. R. Mann and W. G. Langton, 'Effectiveness Contour Method for Evaluating Coil Locations for Passive and Active Plasma Stabilization', MT-8, Grenoble, Sept. 83.