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Statistical Assessment of Fatigue Life for TF Coil Case

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Abstract

This memo is an intermediate report for the sub-task: "Fatigue assessment of the TF coil case," in the task: "MD10 - Magnet description of the work program." A probabilistic assessment to the fatigue life of TF case has been performed by using statistical and uncertainty analysis including: small sample statistics to analyze the effect of Paris parameters; Monte Carlo simulation to evaluate the effect of 3D crack parameters; uncertainty estimation for the effect of stress, Walker's exponent and load ratio. These results are then incorporated to obtain the final fatigue life at given odds.

Assume that ITER consists of 100 significant components with a total confident limit of 95%, then the confident limits for each component should be 99.95% (i.e., odds of 2,000 to 1). For such odds, the fatigue life is estimated to be 45,844 cycles for Case 1 with high stress peak, and 36,754 cycles for Case 2 with large stress range in DDR design. It is also found that the 3D crack parameters including the initial crack depth, the aspect ratio and the eccentricity have the most significant effect on the fatigue life. Fracture toughness shows the least effect. The uncertainties due to the Paris parameters, 3D crack parameters, fracture toughness and the Walker's exponent can be reduced if more test data are available by further experiments and literature survey.

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1. Introduction and Problem Statement

This memo is an intermediate report for the sub-task: "Fatigue assessment of the TF coil case," in the task: "MD10 - Magnet description of the work program."[1] A preliminary probabilistic assessment to the fatigue life of TF case has been performed by using statistical and uncertainty analysis,[2,3,4] for two typical cases in the TF case: case 1 with high stress peak, and case 2 with large stress range. The stress parameters for those two cases are outlined in Table 1.[5] Case 1 is analyzed in the text as an example, and the results of case 2 are attached in the Appendix as reference.

Table 1 Stress Pulses for TF Case (DDR)

Case	First Peak	Second Peak
1	340 MPa, R=0, x 1	526 MPa, R=0.54, x20
2	200 MPa, R=0, x1	412 MPa, R=0.19, x20

The fatigue life is estimated by a numerical code developed at MIT[2] based on the following assumptions: (a) Linear elastic fracture mechanics (LEFM) is applied; Paris equation is used for the life estimation. (b) A part is fractured as the stress intensity factor at crack tip reaches the fracture toughness. (c) Mean stress effect is accounted by the Walker exponent (d) Crack growth model: a crack grows in an elliptical shape with major axis "a" and minor axis "b"; The crack size is governed by the growth rates at two points: a point at the major axis "a" and a point at the minor axis "b"; The axis lengths (a and b) as a function of the cycles are obtained by the numerical integration at these two points respectively. (e) The local residual stress at the crack tip is neglected during life estimation.

The technique of statistical and uncertainty analysis includes: (a) Small sample statistics to analyze the effect of Paris parameters; (b) Monte Carlo simulation to estimate the effect of 3D crack parameters; (c) Analytical evaluation for the effect of stress, Walker's exponent and load ratio. These results are then incorporated to obtain the final fatigue life at given odds.

2. Basic Equations

2.1 Fatigue life

Referring to Ref. 6, an analytical expression for the fatigue life of a 3D crack is derived as a function of the Paris parameters, initial crack size, critical crack size, load ratio and mean stress. This expression will be used frequently in the following uncertainty estimation.

The fatigue crack growth rate with constant stress amplitude is expressed by the Paris equation, which is valid only for small-scale yielding at the crack tip (linear-elastic fracture mechanics),

$$\frac{da}{dN} = c(\Delta K)^n , \qquad (2.1)$$

where stress intensity factor range •K = K_{max} - K_{min}, c and n are Paris parameters, a and N are the half crack length and fatigue cycle respectively.

Increasing the mean stress ($(s_{max} + s_{min})/2$) for an applied stress range ($s_{max} - s_{min}$) generally shortens fatigue life. The effect of mean stress is often expressed by an effective stress intensity factor range:[7]

$$\Delta K_{eff} = K_{max}(1-R)^m = \Delta K(1-R)^{m-1}, \qquad (2.2)$$

where R is the stress ratio (s_{min} / s_{max}) and m is the Walker exponent.

Combining the above two equations gives:

$$\frac{da}{dN} = c(K_{max})^n (1-R)^{mn}, \qquad (2.3)$$

where: $K_{max} = Y s_{max} \sqrt{pa}$.

Integration of Eq. 2.3 gives the fatigue life (the number of cycles to failure):

$$N_f = s_{max}{}^{-n}(1-R)^{-mn} x, \qquad (2.4)$$

where : $x = \frac{1}{c}\int_{a_i}^{a_f} \frac{da}{Y^n (pa)^{n/2}}$.

The expression for x can also be obtained in terms of crack depth b, instead of the semi-crack length a, based on the known crack aspect ratio of b/a. Note that the Y expression must be modified accordingly:

$$x = \frac{1}{c}\int_{b_i}^{b_f} \frac{db}{Y^n (pb)^{n/2}},$$

where b_i and b_f are initial and final crack depths respectively.

Taking logarithms of both sides of Eq. 2.4 gives

$$\ell n(N_f) = \ell n(x) - mn \cdot \ell n(1-R) - n \cdot \ell n(s_{max}), \qquad (2.5)$$

For a thick plate such as the TF case, the crack growth rate is very large in its final stage of approaching final fracture so that some variation of the final crack depth only has a little effect on total life or the value of x. Therefore, $\ell n(N_f)$ can be treated approximately as a linear function of $\ell n(s_{max})$.

2.2 Math structure

Suppose that the nominal life N can be modeled as a product of a statistical mean N_{st} multiplied by factors F_i (i = 2, 3, 4, ...) giving the effects of the test variables (e.g., Paris parameters, 3D crack parameters, fracture toughness, stress, load ratio and Walker's exponent):[3,4]

$$N = N_{st} F_2 F_3 F_4 \cdots \tag{2.6}$$

or write in an abbreviation:

$$N = N_{st} \prod_{i \ge 2} F_i \ . \tag{2.7}$$

Eqs. 2.6 or 2.7 can be linearized by taking logarithms of the factors as:

$$\ell n(N) = \ell n(N_{st}) + \sum_{i \ge 2} \ell n(F_i) \ . \tag{2.8}$$

The mean of $\ell n(N)$ can be assessed as

$$\overline{\ell n(N)} = \overline{\ell n(N_{st})} + \overline{\sum \ell n(F_i)} \ . \tag{2.9}$$

The total uncertainty of $\ell n(N)$ is the square root of the sum of square for each variable uncertainties at given odds, according to the uncertainty propagation theory,[8] as:

$$u_{tot} = \sqrt{u_{st}^2 + \sum_{i \ge 2} u_{ri}^2} \tag{2.10}$$

The log of nominal value at given odds after uncertainties is the mean minus the total uncertainty:

$$\ell n(N) = \overline{\ell n(N)} - u_{tot} \ . \tag{2.11}$$

The nominal life is then obtained from:

$$N = \exp(\ell n(N)) \ . \tag{2.12}$$

2.3 Analysis of uncertainties[8]

Assume that F is a function of various variables x_1, x_2, ...

$$F = F(x_1, x_2, x_3, \cdots) \ . \tag{2.13}$$

The center of the uncertainty interval of the results, F_0 , is a function of the center of the uncertainty intervals of the variables (or typical ones):

$$F_0 = F(x_{01}, x_{02}, x_{03}, \cdots) \ . \tag{2.14}$$

The effect of the uncertainty Δx_i from variable x_i on the uncertainty in the result is evaluated by:

$$\Delta F_i = F(x_{0i} \pm \Delta x_i, x_{0(j \neq i)}) - F_{0i} \ . \tag{2.15}$$

The uncertainty of $\ell n(F)$ is approximately the relative uncertainty of F:

$$u_{ri} = \ell n(F_0 + \Delta F_i) - \ell n(F_0) = \ell n\left(1 + \frac{\Delta F_i}{F_0}\right) \approx \frac{\Delta F_i}{F_0} \ . \tag{2.16}$$

3. Small Sample Statistics [9]

Assume that variable x (e.g., $\ell n(N)$) has a set of n data: x_1, x_2, x_3, ..., x_n. The central tendency is usually measured by mean of these n values as:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \ . \tag{3.1}$$

The data dispersion is measured by the standard deviation as:

$$s_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2} \ . \tag{3.2}$$

It is found that the data distribution is approximately normal if the data number is greater than 30, and it becomes a perfect normal distribution if the data number is infinite. Unfortunately, most of the engineering problems have data number less than 30, that is, a small sample problem (e.g., the present case). One approach to solve such a small sample problem is to use student's t distribution, which is a modification of the normal distribution. The flatness of a student's t distribution is a function of the data number n (or, degree of freedom, it is equal to n-1 for a single variable). Less data number leads to flatter curve than the normal distribution. The confident limits for the mean in the student's t distribution are:

$$x_c^m = \bar{x} \pm t_{c,f} s_x \left(\frac{1}{n}\right)^{1/2} , \tag{3.3}$$

where $t_{c,f}$ is a critical value, and is a function of the confident limits and degree of freedom. Note that as n becomes very large, the confident limits approach the mean value. Prediction limits for one more observation are

$$x_c^p = \bar{x} \pm t_{c,f} s_x \left(1 + \frac{1}{n}\right)^{1/2} \quad . \tag{3.4}$$

3.1 Paris parameters

Table 3.1 shows the fatigue lives for 17 sets of the Paris parameters in 316-type wrought stainless steels at 4K.[10-13] These data include those with annealing or aging conditions, and variation of chemical compositions. The fracture toughness K_{1c} is an average over 6 sets of data.[14]

Table 3.1 Fatigue Life at 4K for TF Case (DDR) with Different c and n

No.	c (m/cycle) $\times 10^{-12}$	n	Ref.	Life N (cycle)	$\ell n(N)$
1	12.3	2.5	10	525805	13.17269
2	0.756	3.26	10	998129	13.81364
3	20	2.35	11	503182	13.12871
4	0.617	3.44	11	749958	13.52777
5	13.2	2.56	11	411264	12.92699
6	13.2	2.56	11	411264	12.92699
7	0.21	3.8	12	844495	13.64649
8	0.789	3.26	12	956387	13.77092
9	0.12	3.96	12	971946	13.78706
10	0.012	4.55	12	2142467	14.57747
11	0.09	3.96	12	1295603	14.07449
12	0.35	3.69	12	677595	13.4263
13	0.25	3.65	12	1054813	13.86887
14	0.5	3.57	12	652790	13.38901
15	0.14	3.81	12	1233796	14.02561
16	0.26	3.75	12	778260	13.56482
17	4.81	2.91	13	414414	12.93462

Case 1 - Stress cycle: 340 MPa x 1, R=0; 526 MPa x 20, R=0.54
Initial crack depth = 0.4 mm
Eccentricity = 25 mm
Aspect ratio = 0.5
Critical stress intensity factor = 128 $MPa\sqrt{m}$
Walker exponent = 0.67

The statistic mean and standard deviation for $\ell n(N)$ are listed in Table 3.2.

Table 3.2 Statistic results of $\ell n(N)$

Mean	Standard deviation
13.563	0.458

Student's t distribution is applied to the small sample statistics for $\ell n(N)$ data, listed in Table 3.1. The critical value of "t " is a function of the confident limits and degree of freedom. Given freedom of 16 based on 17 sets of independent data, the various t values as well as the confident limits are listed in Table 3.3.[15]

Table 3.3 "t" Values and Confident Limits for $\ell n(N)$

Degree of Freedom = 16

Odds	t value	Confident limit for mean of $\ell n(N)$	Confident limit for one more prediction of $\ell n(N)$
100 :1	2.584	13.275	12.345
2,000:1	4.015	13.116	11.670
10,000:1	4.791	13.030	11.304

For the two-sided confidence limits of 67% in normal distribution, we have:

$$\ell n(N) = 13.563 \pm 0.458 \quad (2 \text{ to } 1) \ .$$

Uncertainties at three different odds (e.g., confident limits of 99%, 99.95% and 99.99%) for mean and one more prediction based on student's t distribution are summarized in Table 3.4. These values differ from the standard deviation by factors 2.58, 4.02 and 4.79 rather than 2.3, 3.3 and 3.7 in normal distribution because the smaller the sample the fatter the tail of the t distribution.[8,9].

Table 3.4 Uncertainty of $\ell n(N)$ due to Uncertainty of c and n

Odds against failure	Uncertainty for mean of population	Uncertainty for one more prediction
2-sided 2:1		0.458
1-sided 100:1	0.288	1.218
1-sided 2,000:1	0.447	1.893
1-sided 10,000:1	0.533	2.259

3.2 Fracture toughness

Table 3.5 shows $\ell n(N)$ for 6 fracture toughness values of 316LN after aging at 4K. The mean value of $\ell n(N)$ (see Table 3.6) is slightly larger than the last one with an increase of $\overline{\ell n(F_2)} = 0.153$. The standard deviation is 0.0212. It indicates that the fracture toughness only have minor effect on life because of the acceleration of crack growth near the end of life.

Table 3.5 Fatigue Live at 4K for Various Fracture Toughness [14]

No.	Fracture toughness ($MPa\sqrt{m}$)	Life N (Cycle)	$\ell n(N)$
1	67	874135	13.68099
2	80	894700	13.70424
3	90	902852	13.71331
4	167	919575	13.73167
5	160	918002	13.72995
6	202	924448	13.73695

Case 1 - Stress cycle: 340 MPa x 1, R=0; 526 MPa x 20, R=0.54

c = 6.65×10^{-13} (m/cycle), n = 3.34

Initial crack depth = 0.4 mm

Eccentricity = 25 mm

Aspect ratio = 0.5

Walker exponent = 0.67

Table 3.6 Statistic results of $\ell n(N)$

Mean	Standard deviation
13.716	0.0212

Student's t distribution is applied. The t values and confident limits for three different odds are listed in Table 3.7, as well as the prediction limits in Table 3.8. We obtain $\ell n(F_2) = 0.153 \pm 0.0212$ (2 to 1).

Table 3.7 "t" Values and Confident Limits for $\ell n(N)$

Degree of Freedom = 5

Odds	t value	Confident limits for mean of $\ell n(N)$	Confident limits for one more prediction of $\ell n(N)$
100 :1	3.366	13.687	13.639
2000:1	6.869	13.657	13.559
10000:1	9.678	13.632	13.494

Table 3.8 Uncertainty of $\ell n(N)$ due to Uncertainty of Fracture Toughness

Odds against failure	Uncertainty for mean of population	Uncertainty for one more prediction
2-sided 2:1		0.0212
1-sided 100:1	0.029	0.077
1-sided 2,000:1	0.059	0.157
1-sided 10,000:1	0.084	0.222

$\overline{\ell n(F_2)} = 0.153$

4. Monte Carlo Simulation[16]

4.1 Crack size distribution

The distribution of defects in the TF case is probably the most significant issue for any probabilistic assessment of the fatigue life. Unfortunately, there is no such a data base for the TF case. In the study, we use the published data from the nuclear and piping industry.

Among many models for the crack size distribution,[17,18] the Marshall model[19] is perhaps the most typical one. It is expressed as an exponential form:

$$p(b) = \frac{1}{m}\exp\left(-\frac{b}{m}\right) , \tag{4.1}$$

where b is the crack depth and m is the mean depth. The cumulative distribution of the crack depth is then obtained from the integration of $p(b)$ from 0 to b:

$$F(<b) = 1 - \exp\left(-\frac{b}{m}\right) . \tag{4.2}$$

Ref. 20 defines the maximum allowable crack depth for the TF case is 3.65 mm. In terms of statistics, we assume that the odds of a crack size greater than 3.65 mm is 1:10,000. It then givens $m = 0.4mm$. A crack size distribution from random sampling of 1,000 points based on the Marshall equation is shown in Fig. 1.

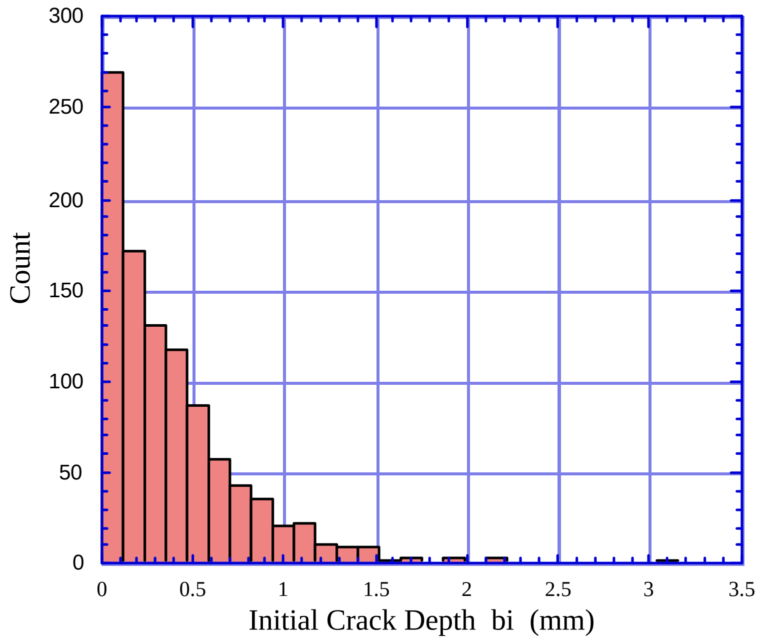

Fig. 1 Crack size distribution from random sampling based on Marshall equation

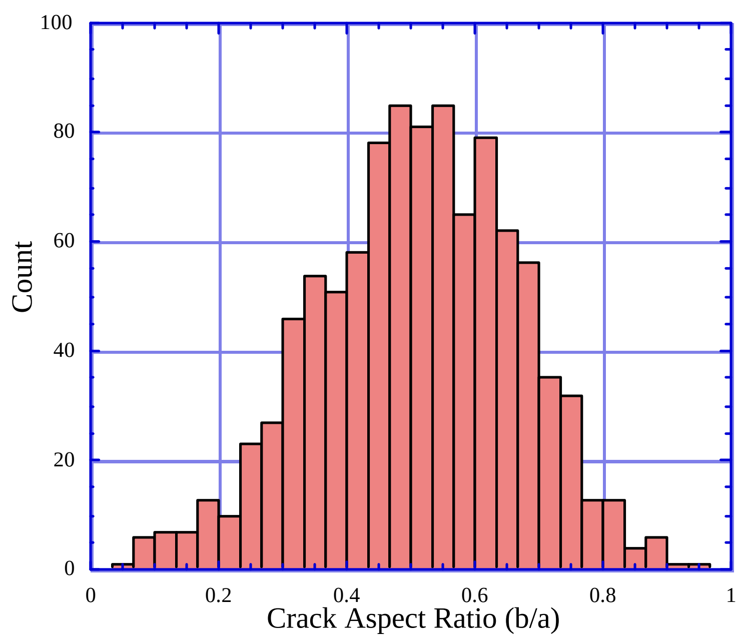

Fig. 2 A normal distribution of the aspect ratio of a 3D crack

4.2 Aspect ratio distribution

Aspect ratio is another important parameter of a 3D crack. However, very little information is available in this field. References 18 and 21 use the data from NDE test, and give a normal distribution of the aspect ratio:

$$p(r = b / a) = \frac{1}{s\sqrt{2p}} \exp\left(-\frac{(r - \bar{r})^2}{2s^2}\right) \quad , \qquad (4.3)$$

where mean $\bar{r} = 0.5$, and standard deviation $s = 0.16$. A random sampling of 1000 points based the above normal distribution is shown in Fig. 2.

4.3 Eccentricity distribution

There is no any published data available for the crack location distribution (i.e., the eccentricity of a 3D crack). Previous experience tells us that most cracks likely stay on surface or near the surface. However, a surface or near-surface crack is more easily found by a NDE test, and then removed. Therefore, we assume that the eccentricity of a 3D crack follows a uniform distribution with mean $\bar{e} = 25mm$ from the center to the surface of the TF case. Fig. 3 shows a uniform distribution drawn from 1,000 random sampling for the eccentricity of a 3D crack.

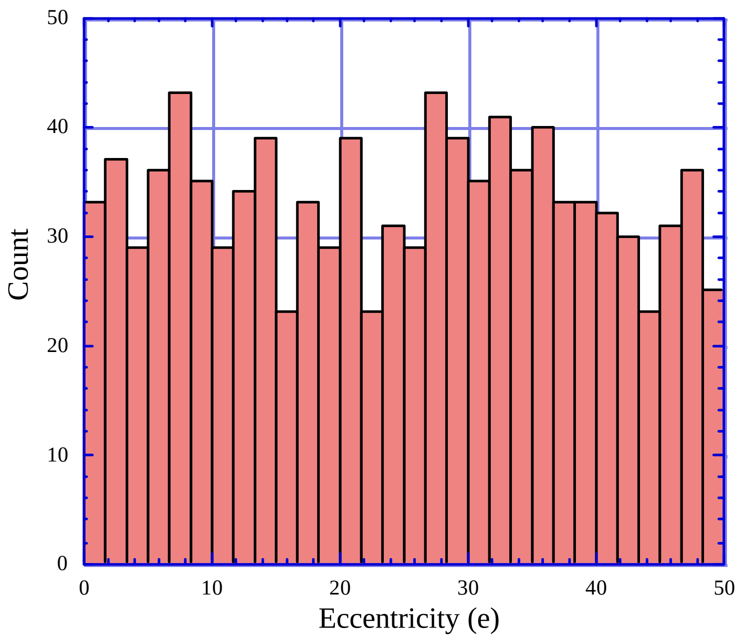

Fig. 3 A uniform distribution of the eccentricity of a 3D crack

4.4 Life distribution

A Monte Carlo simulation has been performed from the random sampling of 1,000 points for the initial crack depth "b_i", the crack aspect ratio "b/a" and the eccentricity "e", as shown in Fig. 4. The fatigue life was calculated for each random set of b_i, b/a and e, which follow the respective distribution functions. The resulted data base of the life were then analyzed by "Statgraphics."[15] It is found that the life data "N" follow approximately a log-normal distribution, as shown in Fig. 5, and the $\ell n(N)$ data follow a normal distribution with a mean and standard deviation given in Table 4.1. The confident limits for three odds (i.e., 100:1, 2000:1 and 10,000:1) are listed in Table 4.2.

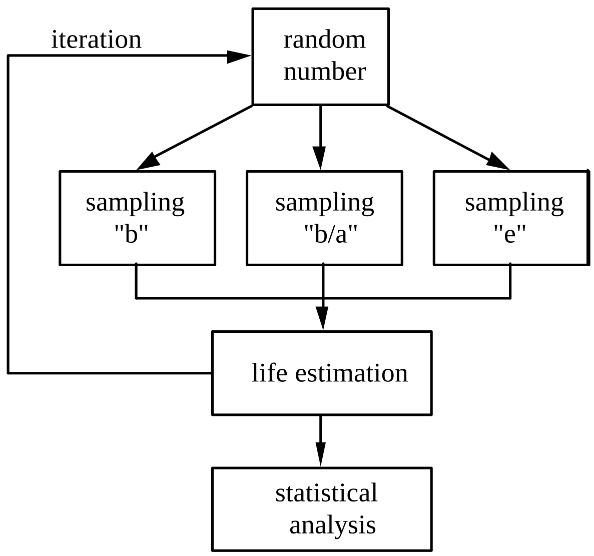

Fig. 4 Flow chat of the Monte Carlo simulation

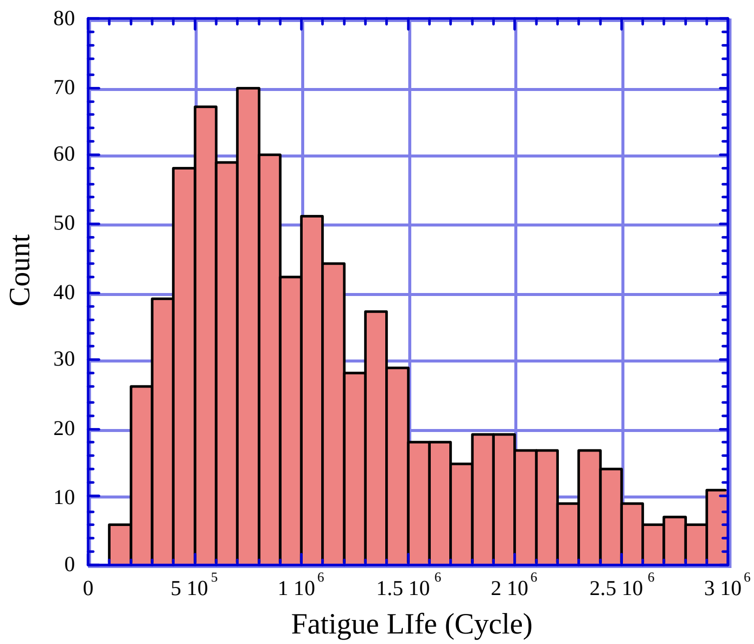

Fig. 5 A lognormal distribution for fatigue life
Case 1 - Stress cycle: 340 MPa x 1, R=0; 526 MPa x 20, R=0.54
$c = 6.65\times10^{-13}$ (m/cycle), n = 3.34, Walker exponent = 0.67

Table 4.1 Statistic results of $\ell n(N)$

Mean	Standard deviation
13.998	0.816

Note: based on 963 sets of data cut off at N=10^7 cycles

Table 4.2 Confident Limits for $\ell n(N)$

Odds	Confident limit for one more prediction of ln(N)
100 :1	12.101
2,000:1	11.314
10,000:1	10.965

For the two-sided confidence limits of 67% in normal distribution, we have:

$$\ell n(N) = 13.998 \pm 0.816 \quad (2 \text{ to } 1) \ .$$

Uncertainties at three different odds (e.g., confident limits of 99%, 99.95% and 99.99%) for one more prediction are listed in Table 4.3.

Table 4.3 Uncertainty of $\ell n(N)$ due to Uncertainty of 3D Parameters

Odds against failure	Uncertainty for one more prediction
2-sided 2:1	0.816
1-sided 100:1	1.897
1-sided 2,000:1	2.684
1-sided 10,000:1	3.033

$\overline{\ell n(F_3)} = 0.435$

5. Uncertainty Analysis

5.1 Uncertainty due to stress

Stress in the TF case is calculated by ANSYS based on a loading model during the operation. However, if the model neglected some minor factors, an uncertainty of the stress may arise and causes the uncertainty of the life.

For a constant amplitude fatigue process, the uncertainty of the life due to stress is obtained by taking difference at both sides of Eq. 2.5 as

$$\Delta\left[\ell n\left(N_f\right)\right] = -n \cdot \Delta\left[\ell n\left(s_{\max}\right)\right] = -n \cdot \ell n\left(1 + \frac{\Delta s_{\max}}{s_{\max}}\right) \qquad (5.1)$$

If $\Delta s_{\max} << s_{\max}$, then

$$\Delta\left[\ell n\left(N_f\right)\right] \approx -n\left(\frac{d s_{\max}}{s_{\max}}\right). \qquad (5.2)$$

However, the stress for the TF case is not constant, there are two stress peaks during operation. Assume that the life for each of stress peak s_1 and s_2 is respectively N_1 and N_2. Using Eq. 5.2 gives

$$\Delta\left[\ell n\left(N_1\right)\right] \approx -n\left(\frac{d s_1}{s_1}\right), \quad \text{and} \qquad (5.3)$$

$$\Delta\left[\ell n\left(N_2\right)\right] \approx -n\left(\frac{d s_2}{s_2}\right). \qquad (5.4)$$

Further assume that the uncertainty for log-life (or the relative uncertainty of life) in a fatigue process with two stress peaks is a geometry average over the uncertainties with two stresses acting alone:

$$\Delta\left[\ell n\left(N_f\right)\right] = \sqrt{\frac{\left[\Delta\ell n\left(N_1\right)\right]^2 + 20 \cdot \left[\Delta\ell n\left(N_2\right)\right]^2}{21}}\ . \qquad (5.5)$$

Eqs. 5.3 to 5.5 indicate that as the applied stress increases, the life decreases with a factor of n.

A realistic evaluation of the uncertainty due to stress variation would need detailed analysis for operation parameters. Pending such a study, the uncertainty limits are chosen rather arbitrarily. Assume that the relative stress varies within 5% for the two-sided confidence limits of 67% in normal distribution. Therefore, the corresponding log(life) value has a uncertainty, according to Eq. 5.5:

$$\Delta\left[\ell n\left(N_f\right)\right] \approx 0.167.$$

The critical values for a normal distribution and an extreme-value distribution with a lower limit of zero (third kind) are listed in Table 5.1. The uncertainties for the three different odds are obtained by scaling up from 0.167 with the selected scale factors in Table 5.1 using a combination of the normal and extreme-value distribution, and are summarized in Table 5.2.

Table 5.1 Critical Values for Different Odds

Odds	100:1	2,000:1	10,000:1
normal distribution	2.3	3.3	3.7
extreme-value distribution	3	4.2	5
selected scale factor	2.5	3.6	4

Table 5.2 Uncertainty of $\ell n(N)$ due to Uncertainty of Stress

Odds against failure	Uncertainty for one more prediction
1-sided 100:1	0.418
1-sided 2,000:1	0.601
1-sided 10,000:1	0.668

$\overline{\ell n(F_4)} = 0$

5.2 Uncertainty due to Walker's exponent m

Following the similar approach with the last section, the life uncertainty due to "m" for each stress peak alone is:

$$\Delta\left[\ell n(N_1)\right] = -n \cdot \ell n(1 - R_1)\Delta m \ , \tag{5.6}$$

$$\Delta\left[\ell n(N_2)\right] = -n \cdot \ell n(1 - R_2)\Delta m \ . \tag{5.7}$$

The uncertainty of life for the fatigue process with two stress peaks is then

$$\Delta\left[\ell n(N_f)\right] = \sqrt{\frac{\left[\Delta \ell n(N_1)\right]^2 + 20 \cdot \left[\Delta \ell n(N_2)\right]^2}{21}} \ . \tag{5.8}$$

Assume that the maximum possible sway is from 0.67 to 1, i.e., $\Delta m = 0.23$. It is corresponding to an odds of 10,000 to 1, at which the uncertainty is, according to Eq. 5.8:

$$\Delta\left[\ell n(N_f)\right] = 0.582 \ .$$

The uncertainties at other odds are obtained by scaling back with the same ratios in Table 5.2, and listed in Table 5.3.

Table 5.3 Uncertainty of $\ell n(N)$ due to Uncertainty of m

Odds against failure	Uncertainty for one more prediction
1-sided 100:1	0.364
1-sided 2,000:1	0.524
1-sided 10,000:1	0.582

$\overline{\ell n(F_5)} = 0$

5.3 Uncertainty due to load ratio R

The uncertainty of load ratio R is in fact the uncertainty of stress range during the operation. The uncertainty of life due to R can be obtained by following the same procedures in the previous sections. First consider the uncertainty of life for two stress peaks respectively as

$$\Delta\left[\ell n\left(N_1\right)\right] = -mn\cdot\Delta\left[\ell n\left(1-R_1\right)\right] = -mn\cdot\ell n\left(1-\frac{\Delta R_1}{1-R_1}\right), \tag{5.9}$$

$$\Delta\left[\ell n\left(N_2\right)\right] = -mn\cdot\Delta\left[\ell n\left(1-R_2\right)\right] = -mn\cdot\ell n\left(1-\frac{\Delta R_2}{1-R_2}\right). \tag{5.10}$$

The uncertainty of life due to combined stress peaks is then

$$\Delta\left[\ell n\left(N_f\right)\right] = \sqrt{\frac{\left[\Delta\ell n\left(N_1\right)\right]^2 + 20\cdot\left[\Delta\ell n\left(N_2\right)\right]^2}{21}}\ . \tag{5.11}$$

Assume that the maximum possible sway of R is 0.1. Then

$$\Delta\left[\ell n\left(N_f\right)\right] \approx 0.538\ .$$

The uncertainties at other odds are obtained by scaling back with the same ratios in Table 5.2, and listed in Table 5.4.

Table 5.4 Uncertainty of $\ell n(N)$ due to Uncertainty of R

Odds against failure	Uncertainty for one more prediction
1-sided 100:1	0.336
1-sided 2,000:1	0.484
1-sided 10,000:1	0.538

$\overline{\ell n(F_6)} = 0$

6. Conclusions

The uncertainties of life due to various factors are summarized in Table 6.1 for three odds (100:1, 2,000:1 and 10,000:1). The impact of each factor on the central value and their standard deviations are also evaluated, and listed in the same table. The nominal life at given odds are obtained, according to Eqs. 2.11 and 2.12, as shown in the bottom line.

Table 6.1 Fatigue Life at Given Odds from Statistics and Uncertainty Analysis (Case 1 of DDR)

No.	Variable	Central value $\bar{F}_i$	Log central value $\overline{\ell n(F_i)}$	2-side 2:1 (0.67) u_{ri}	1-side 100:1 (0.99) u_{ri}	1-side 2,000:1 (0.9995) u_{ri}	1-side 10,000:1 (0.9999) u_{ri}
F_1	Paris parameters c, n	7.768e5	13.563	0.458	1.218	1.893	2.259
F_2	Fracture toughness K_{Ic}	1.153	0.153	0.0212	0.077	0.157	0.222
F_3	3D crack parameters	1.435	0.435	0.816	1.897	2.684	3.033
F_4	Stress $s_{\max}$	1	0	0.167	0.418	0.601	0.668
F_5	Walker exponent m	1	0	0.145	0.364	0.524	0.582
F_6	Load ratio R	1	0	0.134	0.336	0.484	0.538
	$\sum \overline{\ell n(F_i)}$		14.151				
	$\sqrt{\sum u_i^2}$				2.347	3.418	3.928
	$\ell n(N)$				11.804	10.733	10.223
	N (cycle)				133,786	45,844	27,529

Assume that ITER consists of 100 significant components with a total confident limit of 95%, then the confident limits for each component should be:[22]

$$(0.95)^{0.01} \approx 0.9995 \ ,$$

which is equivalent to odds of 2,000 to 1. For such odds, the fatigue life is estimated to be 45,844 cycles according to Table 6.1.

The following Conclusions are drawn from the above analyses:

(a) For odds of 2,000 to 1, which is probably the appropriate choice for the TF case, the fatigue life is estimated to be 45,844 cycles for Case 1, and 36,754 cycles for Case 2 of DDR design.

(b) 3D crack parameters including the initial crack depth, the aspect ratio and the eccentricity have the most significant effect on the fatigue life. Fracture toughness has the least effect.

(c) The uncertainties due to the Paris parameters, 3D crack parameters, fracture toughness and the Walker's exponent can be reduced if more test data are available by further experiments and literature survey. The uncertainties due to stress and load ratio can be reduced by improving the model.

7. References

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8. Appendix:
Data of Statistical and Uncertainty Analysis for Case 2

Table A3.1 Fatigue Life at 4K for TF Case with Different c and n

No.	c (m/cycle) $\times 10^{-12}$	n	Ref.	Life N (cycle)	$\ell n(N)$
1	12.3	2.5	10	404197	12.90966
2	0.756	3.26	10	685842	13.4384
3	20	2.35	11	396364	12.89009
4	0.617	3.44	11	502763	13.12787
5	13.2	2.56	11	313157	12.65446
6	13.2	2.56	11	313157	12.65446
7	0.21	3.8	12	539571	13.19853
8	0.789	3.26	12	657103	13.3956
9	0.12	3.96	12	608114	13.31812
10	0.012	4.55	12	1241174	14.03157
11	0.09	3.96	12	810885	13.60588
12	0.35	3.69	12	439236	12.99279
13	0.25	3.65	12	687425	13.44071
14	0.5	3.57	12	430006	12.97155
15	0.14	3.81	12	787311	13.57638
16	0.26	3.75	12	500500	13.12336
17	4.81	2.91	13	299279	12.60913

Case 2 - Stress cycle: 200 MPa x 1, R=0; 412 MPa x 20, R=0.19
Initial crack depth = 0.4 mm
Eccentricity = 25 mm
Aspect ratio = 0.5
Critical stress intensity factor = 128 $MPa\sqrt{m}$
Walker exponent = 0.67

Table A3.2 Statistic results of $\ell n(N)$

Mean	Standard deviation
13.173	0.386

Table A3.3 "t" Values and Confident Limits for $\ell n(N)$

Degree of Freedom = 16

Odds	t value	Confident limit for mean of $\ell n(N)$	Confident limit for one more prediction of $\ell n(N)$
100 :1	2.584	12.931	12.147
2,000:1	4.015	12.797	11.578
10,000:1	4.791	12.724	11.270

Table A3.4 Uncertainty of $\ell n(N)$ due to Uncertainty of c and n

Odds against failure	Uncertainty for mean of population	Uncertainty for one more prediction
2-sided 2:1		0.386
1-sided 100:1	0.242	1.026
1-sided 2,000:1	0.376	1.595
1-sided 10,000:1	0.449	1.903

Table A3.5 Fatigue Live at 4K for Various Fracture Toughness

No.	Fracture toughness ($MPa\sqrt{m}$)	Life N (Cycle)	$\ell n(N)$
1	67	607141	13.31652
2	80	612868	13.3259
3	90	614911	13.32923
4	167	624463	13.34465
5	160	623938	13.34381
6	202	626148	13.34734

Case 2 - Stress cycle: 200 MPa x 1, R=0; 412 MPa x 20, R=0.19

c = 6.65×10^{-13} (m/cycle), n = 3.34

Initial crack depth = 0.4 mm

Eccentricity = 25 mm

Aspect ratio = 0.5

Walker exponent = 0.67

Table A3.6 Statistic results of $\ell n(N)$

Mean	Standard deviation
13.335	0.0125

Table A3.7 "t" Values and Confident Limits for $\ell n(N)$

Degree of Freedom = 5

Odds	t value	Confident limits for mean of $\ell n(N)$	Confident limits for one more prediction of $\ell n(N)$
100 :1	3.366	13.317	13.289
2000:1	6.869	13.3	13.242
10000:1	9.678	13.285	13.204

Table A3.8 Uncertainty of $\ell n(N)$ due to Uncertainty of Fracture Toughness

Odds against failure	Uncertainty for mean of population	Uncertainty for one more prediction
2-sided 2:1		0.0125
1-sided 100:1	0.018	0.046
1-sided 2,000:1	0.035	0.093
1-sided 10,000:1	0.05	0.131

$\overline{\ell n(F_2)} = 0.162$

Table A4.1 Statistic results of $\ell n(N)$

Mean	Standard deviation
13.648	0.853

Note: based on 978 sets of data cut off at $N=10^7$ cycles

Table A4.2 Confident Limits for $\ell n(N)$

Odds	Confident limit for one more prediction of ln(N)
100 :1	11.664
2,000:1	10.842
10,000:1	10.477

Table A4.3 Uncertainty of $\ell n(N)$ due to Uncertainty of 3D Parameters

Odds against failure	Uncertainty for one more prediction
2-sided 2:1	0.853
1-sided 100:1	1.984
1-sided 2,000:1	2.806
1-sided 10,000:1	3.171

$\overline{\ell n(F_3)} = 0.475$

Table A5.1 Critical Values for Different Odds

Odds	100:1	2,000:1	10,000:1
normal distribution	2.3	3.3	3.7
extreme-value distribution	3	4.2	5
selected scale factor	2.5	3.6	4

Table A5.2 Uncertainty of $\ell n(N)$ due to Uncertainty of Stress

Odds against failure	Uncertainty for one more prediction
1-sided 100:1	0.418
1-sided 2,000:1	0.601
1-sided 10,000:1	0.668

$\overline{\ell n(F_4)} = 0$

Table A5.3 Uncertainty of $\ell n(N)$ due to Uncertainty of m

Odds against failure	Uncertainty for one more prediction
1-sided 100:1	0.099
1-sided 2,000:1	0.142
1-sided 10,000:1	0.158

$\overline{\ell n(F_5)} = 0$

Table A5.4 Uncertainty of $\ell n(N)$ due to Uncertainty of R

Odds against failure	Uncertainty for one more prediction
1-sided 100:1	0.184
1-sided 2,000:1	0.266
1-sided 10,000:1	0.295

$\overline{\ell n(F_6)} = 0$

Table A6.1 Fatigue Life at Given Odds from Statistics and Uncertainty Analysis (Case 2)

No.	Variable	Central value $\bar{F}_i$	Log central value $\overline{\ell n(F_i)}$	2-side 2:1 (0.67) u_{ri}	1-side 100:1 (0.99) u_{ri}	1-side 2,000:1 (0.9995) u_{ri}	1-side 10,000:1 (0.9999) u_{ri}
F_1	Paris parameters c, n	5.26e5	13.173	0.386	1.026	1.595	1.903
F_2	Fracture toughness K_{Ic}	1.162	0.162	0.0125	0.046	0.093	0.131
F_3	3D crack parameters	1.475	0.475	0.853	1.984	2.806	3.171
F_4	Stress $s_{\max}$	1	0	0.167	0.418	0.601	0.668
F_5	Walker exponent m	1	0	0.04	0.099	0.142	0.158
F_6	Load ratio R	1	0	0.074	0.184	0.266	0.295
	$\sum \overline{\ell n(F_i)}$		13.81				
	$\sqrt{\sum u_i^2}$				2.282	3.298	3.775
	$\ell n(N)$				11.528	10.512	10.035
	N (cycle)				101,519	36,754	22,811