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The Invalidity of a Mach Probe Model

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The Invalidity of a Mach Probe Model

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Abstract

Despite its recent application to the interpretation of directional Langmuir probe measurements, the model of Hudis and Lidsky[1] for ion collection in a flowing plasma is no more than an ad hoc mathematical ansatz. A reliable theoretical interpretation of directional Langmuir probe measurements in unmagnetized plasmas is still lacking.

The Mach probe tries to deduce plasma flow velocity from observations of the upstream to downstream asymmetry of ion collection. Much recent theory and experiment has led to an understanding[2] of its use in magnetized plasmas, that is, where the ion Larmor radius is smaller than the probe size. There is no such established model for unmagnetized probe measurements. Nevertheless many recent authors, and specifically three recent papers in the Physics of Plasmas[3, 4, 5] have used the convenient but unjustified formulas from Hudis and Lidsky[1]. The purpose of this comment is to emphasize again that the model of Hudis and Lidsky is unfounded in physics and cannot be expected to give a reliable calibration of Mach probes other than by coincidence.

In outline, their model assumes the ion flow at the sheath is given by the usual Bohm criterion (that the flow velocity should equal the sound speed c_s), and the density there is given by $n_s = \exp e\phi_s/T_e$, the Boltzmann factor for whatever the sheath potential, ϕ_s is. Undoubtedly, the potential drop on the down-stream side of the probe is deeper than on the upstream side in a plasma flow, because it will be "harder" to accelerate the ions to the sound speed in a direction opposite to their external drift than it is in the same direction. The question, though, is how much deeper? This is the heart of the Mach probe calibration problem.

It would be nice if there were a convincing simple argument that gave the potential ϕ_s based on energy conservation. The simple, standard, stationary plasma, low T_i value, $e\phi_s/T_e = -1/2$, is based on just such a plausible argument[6]. The ions acquire a velocity equal to the potential drop and their velocity at infinity can be ignored. Unfortunately, this argument *cannot* be simply modified to account for a drift velocity v_d at infinity.

If ion temperature at infinity were still ignored, and it were permissible to regard the problem as simplified to one cartesian dimension (which is what Hudis and Lidsky do), then unfortunately the collection current on the downstream side would be zero. This difficulty

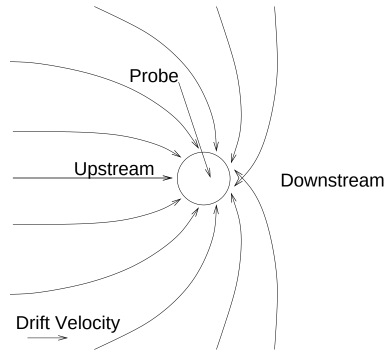

Figure 1: Schematic illustration of possible ion orbits showing how collection to the downstream side of the probe is essentially multidimensional.

arises because at infinity on the downstream side all the ions are drifting away from the probe, and can never be collected. This is part of an inherent problem with the one cartesian dimension, sourceless, collisionless, model. It has no valid solutions for the quasi-neutral presheath[6, 7]. In the real situation, even for very small ion temperatures, a finite drift at infinity will not lead to zero current on the downstream side of the probe, because ions will be drawn in from the directions perpendicular to the external flow, in a two-dimensional pattern. They will be accelerated transversely by the presheath potential, and then curved inward by the electric field at the downstream side, to hit that side, as schematically illustrated by figure 1.

To the present author's knowledge, no complete analysis of this situation has been documented, although a coarse-mesh numerical solution was published long ago[8] without including the data necessary for probe interpretation. But one obvious result is that these ions cannot possibly satisfy the implicit assumption that ion energy parallel to the probe surface can be ignored. A major fraction of the presheath potential energy that they acquire goes into this transverse velocity component. So no model based purely on one-dimensional energy conservation arguments can hope to relate the ion velocity perpendicular to the sheath surface to the local value of potential. The bare one-dimensional model is incapable of properly describing this situation.

Hudis and Lidsky nevertheless assume inconsistently that there *is* such a simple energy conservation relationship in a flowing plasma. They suppose that the potential drop on the upstream side must be reduced by the amount of the incoming ion drift energy. It is on the downstream side where the problem most obviously lies. It would be obviously unphysical to suppose the potential drop is just increased by the amount of the outgoing drift kinetic energy. So instead they invoke an ad hoc additional incoming velocity at infinity, which must be presumed to exceed the drift velocity. They call this velocity v_t and suppose that it is the ion thermal velocity. In order not to bias the problem, this arbitrary velocity must be added to both upstream and downstream sides. Notice, though that even in the absence of an external drift, the bald assumption of an inflow velocity at infinity of any magnitude other than c_s is inconsistent with continuity in one cartesian dimension (that is, in a planar slab model).

The assumed distant inflow velocity, provided it is bigger than the drift, $v_t > v_d$, persuades Hudis and Lidsky to suppose that the sheath edge potential can be deduced from energy conservation and the sheath edge Bohm criterion. Thus they take the potential ϕ_s to be that drop necessary to give ion velocity equal to c_s at the sheath edge, and deduce the sheath edge density from the Boltzmann relation obtaining

$$n_s = n_\infty e^{1/2} \exp(v_\infty^2/c_s^2), \tag{1}$$

where n_∞ is the density far from the probe, and

$$v_\infty = v_t \pm v_d, \tag{2}$$

with the upper and lower (plus and minus) signs corresponding to up and down stream respectively. This is the totality of their model.

Notice that if we ignore the physical requirement of the model and use it for $v_d > v_t$ then the downstream current density *increases* with v_d when it should be decreasing, but also notice that unless $v_t \ll c_s$ the current density even for a stationary plasma would be strongly influenced by this v_t and different from the standard Bohm value. Therefore, this whole approach is obviously incorrect unless

$$v_d < (<) v_t \ll c_s, \tag{3}$$

which means the Mach number must be extremely small. [I don't mean to imply that the approach is justified if this criterion is satisfied, merely that obvious numerical absurdities occur if it is not.]

The resulting Hudis and Lidsky ratio of upstream to downstream current densities is trivially

$$\frac{j_u}{j_d} = \exp\left(\frac{[v_t + v_d]^2 - [v_t - v_d]^2}{c_s^2}\right) = \exp\left(\frac{v_d/c_s}{[c_s/4v_t]}\right) \equiv \exp\left(\frac{M}{M_c}\right). \tag{4}$$

The coefficient

$$M_c = c_s/4v_t \tag{5}$$

dividing the Mach number ($M \equiv v_d/c_s$) in the exponential in equation (4) can be regarded as the calibration factor of any Mach probe measurement. An expression identical to the right hand side of (4), $\exp M/M_c$, has been shown numerically[9, 10] and analytically[11] to govern magnetized Mach probes. Much research has been devoted to deciding just what value to use for M_c. Oversimplifying, the outcome is that a value $M_c \approx 0.5$ is approximately correct. It is plausible that a value not very much different might apply for unmagnetized probes, but no such research has established the appropriate value. For the Hudis and Lidsky model, M_c is inversely proportional to the velocity v_t, which is essentially arbitrary. Hudis and Lidsky take v_t to be the ion thermal velocity, but that is an unjustified ansatz which is almost certainly incorrect in many situations, even if the condition (3) is satisfied.

There may well be situations in which equation (5) gives quantitatively reasonable values. Indeed, when $T_e/T_i = 4$, it gives $M_c \approx 0.5$ so yielding a value consistent with the magnetized situation. If the ion temperature is large, $T_i \gg T_e$, expression (5) is reduced only modestly, to 1/4 for an isothermal-ion definition $c_s^2 = (ZT_e + T_i)/m_i$. (Some authors use $c_s^2 = ZT_e/m_i$, in which case M_c can tend to zero.) In the limit of small T_i, M_c tends to infinity like $(T_i/T_e)^{-1/2}$ — an implausible result, but one that requires very small T_i/T_e before M_c becomes ridiculous. This may explain the fact that reasonable agreement was obtained in some experiments, between independent estimates of the velocity and those based on equation (5) (corrected in some cases by additional arbitrary "calibration" factors) even when $v_d > v_t$ [3] or $T_i \sim T_e$ [4] so that even the obvious validity criterion (3) is violated. In any case, the observation of approximate consistency between the Hudis and Lidsky formula and other velocity measurements shows only that its calibration factor is in the right neigborhood. It cannot justify the adoption of a physically incorrect model, or such details as its dependence on T_i.

In summary, any flow velocity deduced from Mach probe measurements by using the formula (5), even for those situations that satisfy (3), is proportional to an arbitrary factor $1/v_t$ that is unjustified by physics.

This comment is not intended to condemn Hudis and Lidsky, whose paper is mostly about their experimental measurements. It *is* intended to criticize the continued unjustified use of their ad hoc model in modern publications, and particularly to rebut the claim that consistency of the Hudis and Lidsky formulas with some experimental measurements justifies their model.

References

[1] M. Hudis and L. M. Lidsky, J. Appl. Phys. **41**, 5001 (1970)

[2] see for example J. P. Gunn, C. Boucher, P. Devynck, I. Duran, K. Dyabilin, J. Horacek, M. Hron, J. Stokel, G. Van Oost, H. Van Goubergen and F. Zacek, Phys. Plasmas **8** 1995 (2001)

[3] L. Oksuz, M. A. Khedr and N. Hershkowitz, Phys. Plasmas **8**, 1729-1733 (2001)

[4] S. C. Hsu, T. A. Carter, G. Fiksel, H. Ji, R. M. Kulsrud and M. Yamada, Phys. Plasmas **8** 1916-1928 (2001)

[5] S. Shinohara, N. Matsuoka and S. Matsuyama, Phys. Plasmas **8** 1154-1158 (2001)

[6] I. H. Hutchinson, *Principles of Plasma Diagnostics* (Cambridge University Press, New York, 1987)

[7] K.-U. Riemann, J. Phys D **24**, 493 (1991)

[8] R. Grabowski and T. Fischer, Planet. Space Sci. **23** 287 (1975)

[9] I. H. Hutchinson, Phys. Rev. A **37** 4358 (1988)

[10] K.-S. Chung and I. H. Hutchinson, Phys. Rev. A **38** 4721 (1988)

[11] I. H. Hutchinson, Phys. Fluids **31** 2729 (1988)