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Spin Stability of Asymmetrically Charged Plasma Dust

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Abstract

An isolated spherical insulating particulate suspended by an electric field in a flowing plasma exhibits an instability that causes it to spin. The instability, which spontaneously breaks the symmetry about the flow direction, arises when the electric dipole moment, caused by asymmetric charging, points opposite to the external electric field. The theoretical instability criterion derived is that the Debye length should exceed approximately the probe radius divided by the Mach number, when the ion flux asymmetry has its intuitive direction and size. The saturated state is then that the particle rotates at a fraction of the ion plasma frequency. However, for the conditions of many experiments, intuition is incorrect, and the direction of charge asymmetry is such as to yield stability.

1 Introduction

Dust in plasmas often resides in regions such as the edge of electrode sheaths where the drag of ions flowing at approximately the sound speed towards the electrode is balanced by a normal electric field repelling the negatively charged particle[1]. Recently it has been reported that under some circumstances the dust particles spin [2, 3, 4]. Particulate spin has also been hypothesised as a mechanism for the production of cosmic radiation[5]. Previously considered mechanisms have mostly focussed on particle asymmetries [4] or the possibility of there being shear or rotation in the background ion flow [3], or mechanisms arising from radiation [2]. The present paper shows that there is a natural mechanism that should cause even perfectly spherical particles in a perfectly irrotational, magnetic-field-free flow to spin. The stability criterion and the final spin state are obtained.

When the particle is made of an insulating material, it can support potential differences around its surface. In the limit of zero conductivity, and ignoring all charging effects other than electron or ion collection, the surface charge density accumulates in such a way as to bring the local electric current density to zero. This occurs by the surface potential

adjusting to repel just the right fraction of the electron collection current. To an excellent approximation if the electron distribution is thermal, the electron flux density is given [6] by

$$\Gamma_e = n_\infty \sqrt{T_e/2\pi m_e} \exp(e\phi/T_e) , \quad (1)$$

where T_e , m_e , and n_∞ are the electron temperature, mass, and density far from the particle, and ϕ is the surface potential relative to infinity. (We take the ion charge to be unity for notational convenience; the generalization to charge Ze is trivial.)

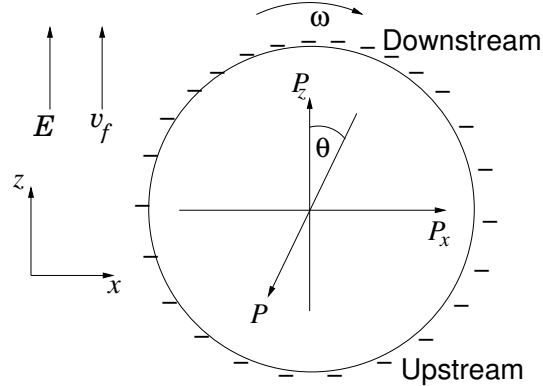


Figure 1: Schematic illustration of of the charged sphere and dipole moments.

The ion flux to the spherical surface is asymmetric in a flowing plasma. The intuitive direction of asymmetry, which does not always occur [7], is that there should be greater ion flux density to the upstream side. In that case, as illustrated in Figure 1, the upstream side will acquire a less negative potential, and a less negative surface charge than the downstream side, causing an electric dipole moment, P that is opposite the direction of the flow, which we take to be the z -direction. Since the particle is on average negatively charged, and stationary within a plasma flowing past and giving rise to an ion drag force, it requires a compensating electric field E pointing in the direction of the flow. An electric dipole pointing opposite to an external electric field is unstable with respect to rotation. This is the underlying mechanism discussed here.

2 Stability Analysis

This spherical particle is not a dipole of fixed magnitude, however. Its surface charge is determined dynamically. If it starts to spin, its surface charge will no longer be in current-balance, so the dipole moments will evolve with time. If we suppose the y -axis to be chosen along the axis of rotation, then the important dipole components are in the z - and x -directions (P_z, P_x), and can be adequately described by the following coupled equations.

$$\frac{dP_x}{dt} + \frac{P_x}{\tau_c} = \omega P_z \quad (2)$$

$$\frac{dP_z}{dt} + \frac{P_z - P_o}{\tau_c} = -\omega P_x, \quad (3)$$

where ω is the instantaneous angular spin rate, P_0 is the natural dipole moment for a non-spinning sphere, negative in value when the upstream side is more positive, and τ_c is the characteristic charging time of the dipolar component of the charge on the sphere. That the charge density may have other spherical harmonics than just the first (which corresponds to a pure dipole) is not important for the rotational dynamics.

If the particle experiences a drag torque from ion or atom impacts that gives rise to a spin damping time-constant τ_d , then its equation of motion is

$$\frac{d\omega}{dt} + \frac{\omega}{\tau_d} = -\frac{EP_x}{I} \quad (4)$$

where I is the moment of inertia.

The stability of this system may be determined by noting that the steady state ($\omega =$ constant, $d/dt = 0$) solution of eqs (2,3) is

$$P_z = P_0/(1 + \omega^2\tau_c^2), \quad P_x = P_0\omega\tau_c/(1 + \omega^2\tau_c^2) \quad (5)$$

showing that the linear (low spin velocity) approximation for P_x is $P_x \approx P_0\omega\tau_c$. Substituting into eq(4) we obtain

$$\frac{d\omega}{dt} = \left[\frac{\tau_c}{\tau_r^2} - \frac{1}{\tau_d} \right] \omega, \quad (6)$$

where the rotational time constant is defined by $\tau_r^2 \equiv -I/EP_0$. Equation (6) shows immediately that the state $\omega = 0$ is unstable and spontaneous spin up will occur if

$$\tau_c > \tau_r^2/\tau_d. \quad (7)$$

The spin velocity will saturate when the equation of motion eq (4) is satisfied with $d\omega/dt = 0$. Substituting for P_x from eqs (5), immediately yields the final state:

$$\omega = \frac{1}{\tau_c} \sqrt{\frac{\tau_d\tau_c}{\tau_r^2} - 1}. \quad (8)$$

Thus the spinning state will occur if the product of the surface charging time-constant and the drag time-constant is larger than the square of the rotational time-constant. To relate this to the plasma parameters we now obtain expressions for the values of these time-constants.

3 Time-constants and Plasma Parameters

In general the ion flux density is a weaker function of surface potential than the electron flux density. In the limit of small Debye length, when the sheath is thin, it becomes independent

of surface potential[6]. We therefore ignore any ion flux density variation in calculating the *charging time constant*, τ_c .

We write the time derivative of the surface potential as being the current density divided by the capacitance per unit area:

$$\begin{aligned} \frac{d\phi}{dt} &= -e(\Gamma_e - \Gamma_i)/C_d \\ &= \frac{-en_\infty}{C_d} \left(\frac{T_e}{2\pi m_e}\right)^{1/2} \left[\exp\left(\frac{e(\phi - \phi_f)}{T_e}\right) - 1 \right] \exp\left(\frac{e\phi_f}{T_e}\right), \end{aligned} \quad (9)$$

where ϕ_f is the (constant) equilibrium floating potential for this surface element. The capacitance per unit area, $C_d = \sigma/\phi$, to be precise, is the capacitance attributable to a dipolar (first spherical harmonic) distribution of surface charge density σ over the sphere. For a sphere of radius a in vacuum, this capacitance may be shown to be $3\epsilon_0/a$, while for a sphere shielded by a short Debye length plasma ($\lambda_D \ll a$) it is ϵ_0/λ_D . Therefore a reasonable approximation is to take $C_d = \epsilon_0(1/\lambda_D + 3/a) \equiv \epsilon_0/l_c$.

Expanding the exponential in the vicinity of $\phi - \phi_f = 0$, then shows

$$\begin{aligned} \frac{1}{\tau_c} &\equiv \frac{d \ln(\phi - \phi_f)}{dt} \\ &= -\frac{\epsilon_0}{C_d} \frac{e^2 n_\infty}{\epsilon_0 T_e} \left(\frac{T_e}{2\pi m_e}\right)^{1/2} \exp\left(\frac{e\phi_f}{T_e}\right), \\ &= \frac{1}{\sqrt{2\pi}} \frac{l_c v_{te}}{\lambda_{De}^2} \exp\left(\frac{e\phi_f}{T_e}\right), \end{aligned} \quad (10)$$

The *drag time-constant*, τ_d , may be approximated by assuming that ions or atoms that collide with the particle are reemitted with angular velocity on average equal to that of the sphere surface. Therefore the rate of loss of angular momentum is equal to mass flux to the sphere, F , times $(2/3)a^2\omega$. (The surface averaging factor $2/3$ is appropriate for a uniform flux, but may be slightly altered for non-uniform.) Therefore

$$\frac{1}{\tau_d} = \frac{F2a^2/3}{I}. \quad (11)$$

The *electric dipole rotation time-constant*, τ_r , is determined by the magnitude of the electric field in which the particle resides, as well as the dipole moment.

The electric field must provide just sufficient force to balance the drag of the plasma particles (ions and neutrals mostly) upon the sphere. However, a subtlety arises in accounting for this drag. The drag force[8] is conveyed to the sphere in two main components, the first is the direct collection of ions and neutrals that hit the sphere. The momentum transfer in this form may be approximated as Fv_f , where F is the same mass flux as before, and v_f is the flow velocity, assumed equal for ions and neutrals. The second mechanism for drag arises from ions that *do not* hit the sphere. If the Debye length greatly exceeds the sphere radius, this is the larger effect, corresponding to the usual Coulomb collisional drag[9]. However,

this momentum is transferred to the charged sphere by an electric field, which arises from the variation of the plasma charge in the neighborhood of the sphere because of the effect of the charge on ion orbits. The background uniform electric field must be such that its force on the sphere balances the sum of the direct collection and the collisional electric field drag. However, the dipole sees locally an electric field that is the difference of the background and the collisional electric field. Therefore, in calculating the dipole's stability we must use an electric field that provides a force equal to only the direct momentum collection rate. Therefore, writing the monopolar capacitance per unit area of the sphere as C_m (approximately $\epsilon_o[1/\lambda_D + 1/a]$), we find that the local electric acting on the particle is required to be

$$E = -\frac{Fv_f}{\phi_f C_m 4\pi a^2}, \quad (12)$$

where $\phi_f C_m 4\pi a^2$ is the total particle charge.

The dipole moment on the sphere, when it has a surface charge density component of the form $\sigma_1 \cos \theta$ is $\sigma_1 4\pi a^3/3$. This charge component arises from a cosine potential variation $\sigma_1 = C_d \phi_1$ to accommodate the non-uniform ion flux. It has been shown [7] that one can write the ratio of the upstream to downstream flux as $\exp(KM)$, where, $M \equiv v_f/\sqrt{T_e/m_i}$ is the Mach number, and K is a calibration factor of order unity (but whose precise value depends on plasma conditions). To compensate such a flux variation requires a cosine potential variation of amplitude

$$\phi_1 = \frac{T_e KM}{e} \frac{1}{2}, \quad (13)$$

leading to an electric dipole moment

$$P_0 = -C_d \frac{T_e KM}{e} \frac{4}{2} \frac{1}{3} \pi a^3. \quad (14)$$

Substituting eqs (12) and (14) into the definition of τ_r yields

$$\frac{\tau_r^2}{\tau_d} = \left(\frac{4e|\phi_f| C_m}{T_e C_d} \right) \frac{1}{KM^2} \frac{a}{\sqrt{T_e/m_i}}. \quad (15)$$

Notice that the sphere inertia has cancelled out of this expression. It only serves to determine the overall timescale of the problem. But notice, more importantly, that the mass flux to the sphere has also cancelled out, because the local driving electric field is proportional to drag, as well as the spin damping, and both are proportional to mass flux. We denote the coefficient in parentheses by ζ .

Using these expressions, we may write the instability criterion as

$$\frac{\lambda_{De}^2}{l_c a} > \frac{1}{M^2} \frac{\zeta}{K} \sqrt{\frac{m_i}{2\pi m_e}} \exp(e\phi_f/T_e). \quad (16)$$

As is well known[6], the floating potential (here an average value over the sphere) is typically between 2 and 3 times T_e/e , assuming no electron emission. More generally, if the ion flux is written in the form

$$\Gamma_i = f n_{e\infty} \sqrt{T_e/m_i}, \quad (17)$$

then the solution for the floating potential can be written

$$\exp(e\phi_f/T_e) = f\sqrt{2\pi m_e/m_i} , \quad (18)$$

which enables us to write the instability criterion as

$$\frac{\lambda_{De}^2}{l_c a} > \frac{1}{M^2} \frac{\zeta f}{K} . \quad (19)$$

Since the Mach number, M and the coefficient K are rarely greater than 1, and the product ζf is typically of order 4 (because the enhancement of f at low T_i is removed by flow) we see that spin instability usually requires the Debye length to be somewhat greater than the sphere radius. In that limit, we can approximate $l_c \approx a/3$, $C_m/C_d \approx 1/3$, arriving at a simplified approximate criterion

$$\frac{\lambda_{De}}{a} \gtrsim \frac{1}{M\sqrt{K}} . \quad (20)$$

If K were positive, this criterion would be satisfied by most experiments, since most use particulate sizes substantially smaller than the Debye length. However, recent detailed self-consistent collisionless calculations[7] have shown that if the ion to electron temperature ratio is small, as it often is in experiments, then contrary to intuition, the value of K is *negative* for a wide range of values of λ_{De} . Negative values of K are always stable, and no particle spin then occurs. To obtain positive K , according to [7], requires either $T_i \sim T_e$ or $\lambda_{De} \gtrsim 20a$ (at $T_i = 0.1T_e$).

Employing the same approximations, the order of magnitude of the steady state spin frequency in a moderately unstable case is

$$\omega \sim \frac{1}{\tau_c} \approx \frac{a}{\lambda_{De}} f \sqrt{\frac{m_e}{m_i}} \omega_{pe} = \frac{a}{\lambda_{De}} f \omega_{pi} . \quad (21)$$

4 Conclusions

Stability analysis shows that in most situations with $\lambda_D > a$, and flow of order the sound speed, a spherical insulating grain whose drag is balanced by an external electric field is unstable to spin if ion flux is greater on the upstream side (i.e. K is positive) which is the intuitive direction of asymmetry. The grain is stable if the asymmetry is reversed, which has been shown recently elsewhere[7] often to be the case for collisionless ions at low T_i/T_e . The direction of charge asymmetry may be affected by the non-ideal environment of typical experiments, notably the effect of collisions, or non-uniformities in the background plasma. Perhaps therefore, one should regard the presence or absence of particulate spin as a diagnostic of the charging asymmetry, since that asymmetry seems likely to be the most uncertain factor in practice.

References

- [1] T. Nitter. *Plasma Sources Sci. Technol.*, 5:93, 1996.
- [2] B.T. Draine and A. Lazarian. *Ap. J.*, 508:157, 1998.
- [3] O. Ishihara and Sato N. *IEEE Trans. Plasma Sci.*, 29:179, 2001.
- [4] V.N. Tsytovich, N. Sato, and G.E. Morfill. *New J. Phys.*, 5:43, 2003.
- [5] A Ferrara and R.-J. Dettmar. *Ap. J.*, 427:155, 1994.
- [6] I.H. Hutchinson. *Principles of Plasma Diagnostics*. Cambridge University Press, Cambridge, 2nd edition, 2002.
- [7] I.H. Hutchinson. *Plasma Phys. Control. Fusion*, 45:1477, 2003.
- [8] P.K. Shukla and A.A. Mamun. *Introduction to Dusty Plasma Physics*. IOP Publishing, Bristol, 2002.
- [9] L. Spitzer. *Physics of Fully Ionized Gases*. Interscience, New York, 2nd edition, 1962.