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Introduction

There are two events that are particularly ubiquitous in space plasmas. One is the existence of short scale-length (of the order of a few ion Larmor radii) density gradients and the other is transverse (to the geomagnetic field) energization of ionospheric ions which, eventually, make their way into the outer reaches of the terrestrial magnetosphere. Rocket observations have shown the existence of density depleted structures in the auroral ionosphere which are associated with enhanced electric fields [1, 2]. These structures are a few tens of meters in width transverse to the geomagnetic field and are believed to be of the order of 100 km along the geomagnetic field. Associated with these enhanced fields, transversely accelerated ions with energies in the range of tens of eV are observed [1, 2]. These energies should be compared to the ambient thermal energies of approximately 0.3 eV.

Resonant Electrostatic Fields in Density Gradients

If we assume a cylindrical density depression with its radial density variation being across the geomagnetic field, then, in the cold plasma approximation, the spatial evolution of the electrostatic potential is given by:

$$\frac{d^2\phi}{dr^2} + \left\{ \frac{1}{r} + \frac{1}{K_\perp} \frac{dK_\perp}{dr} \right\} \frac{d\phi}{dr} + \left\{ \frac{m}{r} \frac{1}{K_\perp} \frac{dK_\times}{dr} - \frac{m^2}{r^2} - k_z^2 \frac{K_\parallel}{K_\perp} \right\} \phi = 0 \qquad (1)$$

where we have assumed that the azimuthal, longitudinal, and temporal variations of the fields are of the form $\exp(im\theta + ik_z z - i\omega t)$,

$$K_\perp = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} - \sum_i \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} \qquad (2)$$

$$K_\times = -\frac{\omega}{\omega_{ce}} \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} + \sum_i \frac{\omega_{ci}}{\omega} \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} \qquad (3)$$

$$K_\parallel = 1 - \frac{\omega_{pe}^2}{\omega^2} - \sum_i \frac{\omega_{pi}^2}{\omega^2} \qquad (4)$$

where ω_{pe} and ω_{pi} are the electron and ion plasma frequencies, respectively, and ω_{ce} and ω_{ci} are the electron and ion cyclotron frequencies, respectively. The plasma and cyclotron frequencies are assumed to have a radial dependence due to the radial variation of the plasma density and the geomagnetic field. Generally the radial scalelength of the density depression is much smaller than the radial scalelength of the geomagnetic field variation, so that the geomagnetic field can be considered constant over the transverse dimension of the density depression. From (1), it is clear that the differential equation for ϕ has a regular singularity at $K_\perp = 0$ corresponding to either the lower hybrid resonance frequency ω_{LH} or the upper hybrid resonance frequency ω_{UH}. In the vicinity of either of these resonances, ϕ has a logarithmic singularity. For space plasmas of interest, we have considered a linear electron density profile:

$$n = \begin{cases} n_0, & \text{for } r \le r_1 \\ n_0 + (n_1 - n_0)\left(\dfrac{r - r_1}{r_2 - r_1}\right), & \text{for } r_1 < r < r_2 \\ n_1, & \text{for } r \ge r_2 \end{cases}$$

where n_0 and n_1 are constants. In the regions $r < r_1$ and $r > r_2$, the solution to (1) can be readily expressed in terms of ordinary Bessel functions or modified Bessel functions, depending on whether $K_\parallel / K_\perp$ is less than or greater than zero, respectively. The solutions have to further satisfy that ϕ remains finite as $r \to 0$ and as $r \to \infty$. The complete solution to (1) over the entire density gradient is obtained by requiring that ϕ match onto these Bessel function solutions at $r = r_1$ and $r = r_2$. This then becomes an eigenvalue problem. For fixed frequency ω the eigenvalue is k_z while for fixed k_z the eigenvalue is ω.

For illustrative purposes, let us assume the following parameters which correspond to the plasma conditions in the auroral ionosphere around an altitude of 1000 km: $n_0 = 3.4 \times 10^9$ m^{-3}, $n_1 = 4.3 \times 10^9$ m^{-3}, singly charged oxygen O^+ and hydrogen H^+ ions with the density ratio of 9:1, respectively, and $B_0 = 0.36$ Gauss. For a fixed ω corresponding to $K_\perp(r_0) = 0$ for $r_1 < r_0 < r_2$ we solve, numerically, (1) for the eigenvalue k_z. It is worth noting that k_z will depend on ω and the width of the gradient region $\Delta r = r_2 - r_1$ and not on the specific values of r_2 and r_1. We find, numerically, that for a given ω and Δr there are two distinct k_z's which satisfy the boundary conditions. For $\Delta r = 1$ m and a frequency of $f_{LH} = \omega / 2\pi = 4.55$ kHz (corresponding to the lower hybrid resonance being halfway up the density gradient), the two eigenvalues are $k_{z1} = (0.022 + 0.006i)$ m^{-1} and $k_{z2} = (0.011 - 0.008i)$ m^{-1}. If we choose $\Delta r = 10$ m and the same frequency, then the two eigenvalues are $k_{z1} = (0.029 + 0.3i)$ m^{-1} and $k_{z2} = (0.023 - 0.02i)$ m^{-1}. Thus, the lower hybrid fields propagate along the geomagnetic field for sharp density gradients and are essentially evanescent for more gradual density gradients. It is worth noting that even when the imaginary part of k_z is small compared to the real part, the fields are not "propagating" over long distances before damping out spatially. The spatial extents are typically of the order of 100 m. These distances are relatively short compared to what is inferred from observations. Hence such fields would have to be maintained by a source of free energy (e.g., an independent streaming instability) inside the density depleted region.

Interaction of Ions With Localized Fields

The interaction of ions with localized field structures is different from that with a plane wave or a set of plane waves, which we have studied in the past [3, 4]. The primary difference is that since these structures are smaller than the ion Larmor radius, the interaction of the ions with the fields occurs over only a small fraction of their orbit. Let us consider the situation in which the wave fields are independent of the coordinate along the geomagnetic field. We can then consider just the motion of the ions in a plane perpendicular to the geomagnetic field. For an ion in a uniform magnetic field ($\vec{B} = B_0\hat{z}$) interacting with electrostatic waves propagating in the radial direction across $\vec{B}$, the equations of motion are:

$$\frac{dr}{dt} = v_r, \qquad \frac{d\theta}{dt} = \frac{v_\theta}{r} \tag{5}$$

$$\frac{dv_r}{dt} = v_\theta\left(\Omega + \frac{v_\theta}{r}\right) + \frac{Q}{M}E_r, \qquad \frac{dv_\theta}{dt} = -v_r\left(\Omega + \frac{v_\theta}{r}\right) \tag{6}$$

where Ω is the angular ion cyclotron frequency, and Q and M are the charge and mass of the ion, respectively. Consider the following form of a spatially localized electric field that models the resonant electric fields in density gradients:

$$E_r = E_0 e^{-\beta\,(r-a)^2} \sum_n \sin\left(k_n r + m_n\theta - \omega_n t + \phi_n\right)$$

where a is the radius of the cavity, E_0 is the electric field amplitude, β determines the radial width of the field, and k_n, m_n, ω_n and ϕ_n are the radial wave vector, azimuthal mode number, frequency, and phase, respectively, of the n-th component. The equations of motion can be integrated numerically to determine the effect of the localized fields on the ion orbits. Our analysis shows that $\beta \gg k_n$ is required for ion energization to occur. We set $k_n = 0$ in the following results. For the ionospheric parameters discussed earlier, we consider the interaction of O^+ ions with a single wave component ($n = 1$, $\omega_1 = \omega$). We assume that $a = 10$ m and $\omega/\omega_{cO^+} = 145.74$. For an ambient temperature of 0.33 eV, the initial Larmor radii of O^+ is $\rho_{O^+} \approx 6.5$ m. The top figure in Figure 1 shows the normalized Larmor radius $\rho = \rho_{O^+}/a$ as a function of the normalized time for three values of the normalized electric field amplitude $\epsilon = QE_0/(M\Omega^2 a)$. (For $E_0 = 200$ mV/m $\epsilon = 2.6$.) It is clear that there is a threshold in the amplitude of the electric field before ion energization takes place. The energization of low energy ions is chaotic from the very beginning. This is unlike the case of ion interaction with coherent plane waves where the low energy ions are, initially, coherently energized before their motion becomes chaotic [3, 4]. The bottom figure in Fig. 1 shows the change in Larmor radius due to varying widths of the interaction region. Again there is a threshold in width, for a given amplitude, beyond which ion energization does not take place.

In the case when the ratio of the wave frequencies to the ion cyclotron frequency is a low number (corresponding to the interaction of H^+ ions with lower hybrid structures), we find that when two wave frequencies are separated by an integer multiple of the ion cyclotron frequency, the energy of the ions can increase monotonically as a function of time. The monotonic increases in energy are akin to Lévy flights [5]. An important observable signature of Lévy flights is that the ion distribution function will have long tails. Such ion distribution functions are indeed observed [2].

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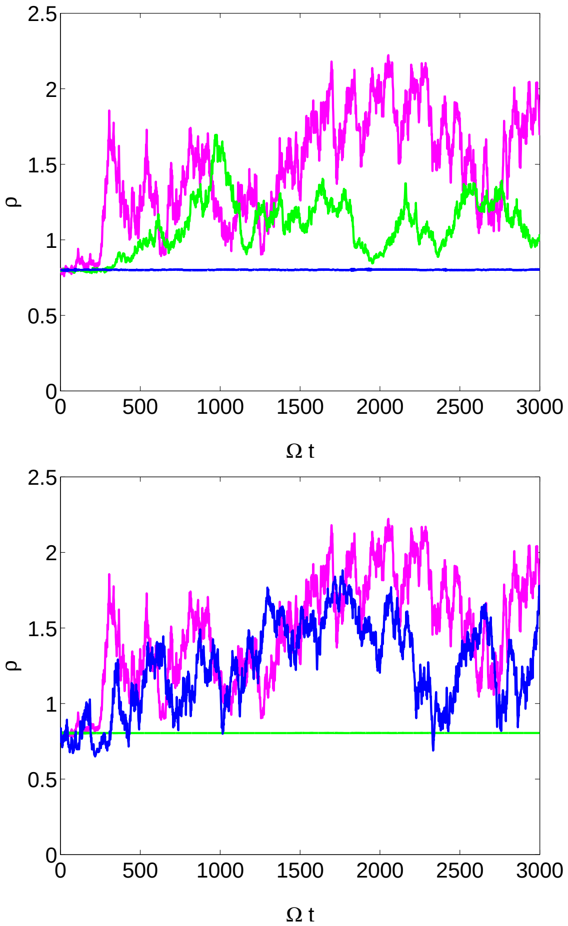

Figure 1: Normalized Larmor radius for a O^+ ion versus normalized time. Top: $\sqrt{\beta a^2} = 6.25 \times 10^3$ with $\epsilon = 1.3$ (blue), 2.6 (green), and 5.2 (red). Bottom: $\epsilon = 5.2$ with $\sqrt{\beta a^2} = 1.56 \times 10^3$ (green), 6.25×10^3 (red), and 2.5×10^4 (blue).