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Kinetic Effects in the Pedestal of a Tokamak

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We consider the effects of finite **E**x**B** on the residual and neoclassical ion heat transport and ion flow in the pedestal. For a tokamak pedestal with a density scale of the poloidal ion gyroradius the finite **E**x**B** drift modfies the ion orbits, moving the trapped-passing boundary towards the tail of the ion distribution function.

1 Introduction

In many tokamak plasmas the width of the pedestal is observed to be on the order of a poloidal ion gyroradius. Consequently, finite drift orbit effects take on a special significance. We employ a special version of gyrokinetics that replaces the standard radial gyrokinetic variable by canonical angular momentum [1] to investigate kinetic phenomena in an axisymmetric tokamak pedestal in the banana regime. An entropy production argument for a pedestal in which the density drops substantially on the scale of the ion poloidal gyroradius, finds that in the banana regime the ion temperature is not allowed to vary substantially on such a short scale if the poloidal magnetic field is small compared to the toroidal field [1]. When the toroidal ion flow in the pedestal is subsonic this requires the **E**x**B** and ion diamagnetic flows to cancel to lowest order, making the flow of the electrons sonic there. These results mean that in a subsonic banana regime pedestal of poloidal ion gyroradius width, the ions are electrostatically confined and the electrons are confined magnetically [1]. Using this observation we then calculate the collisionless zonal flow residual in the pedestal to demonstrate that it is enhanced over its core value [2,3]. We find that in the pedestal, finite orbit effects due to **E**x**B** result in a lower level of anomalous transport. Unlike the core, there is a finite **E**x**B** reduction in the trapped particle population. Our result allows us to suggest a model for the pedestal formation. We next proceed by retaining collisions in the pedestal to find that finite **E**x**B** drift orbit effects reduce the neoclassical banana regime ion heat conductivity below its core value, and substantially modify the poloidal ion flow. Both the thermal diffusivity reduction and the enhancement in the residual are caused by the trapped ion region being moved towards the tail of the ion distribution function once the **E**x**B** drift begins to compete with the poloidal projection of parallel streaming. These calculations ignore **E**x**B** shear in the pedestal for simplicity, but work in progress will address this more realistic situation.

2 Pedestal background

In the alternate gyrokinetic technique that has been developed to analyze the tokamak pedestal canonical angular momentum $\psi_* \equiv \psi - (Mc/e)R^2\vec{v}\cdot\nabla\zeta = \psi + \Omega_i^{-1}\vec{v}\times\vec{n}\cdot\nabla\psi - (Iv_{||}/\Omega_i)$ is taken as the gyrokinetic radial variable [1] rather than the radial guiding center location $\Psi \equiv \psi + \Omega_i^{-1}\vec{v}\times\vec{n}\cdot\nabla\psi$ of typical gyrokinetic treatments. This approach allows strong radial plasma gradients to be treated when considering zonal flow, neoclassical effects, and the turbulent behavior. The nonlinear gyrokinetic equation retains large poloidal $\vec{E}\times\vec{B}$ drift and orbit squeezing effects on zonal flow and neoclassical ion transport in the pedestal, while allowing the toroidally rotating Maxwellian solution of the isothermal tokamak limit to be exactly recovered [4]. Indeed, a physically acceptable solution for the lowest order ion distribution function in the banana regime in a tokamak pedestal is nearly this same isothermal Maxwellian solution provided that the ion temperature variation radial scale is much greater than poloidal ion gyroradius ρ_{pi} [1] as is briefly discussed next. Consequently, the background radial ion temperature profile there cannot be as narrow as that of plasma density or electron temperature when they vary on the scale of a poloidal ion gyroradius.

The vanishing of the entropy production on a core flux surface, $\langle \int d^3v \ell n f_{0i} C_{1ii}\{f_{0i}\}\rangle_\psi = 0$, requires the lowest order axisymmetric ion distribution function f_{0i} to be a local Maxwellian, with f_{0i} independent of poloidal angle in the banana regime. However, drift departures from pedestal flux surfaces can become comparable to the poloidal ion gyroradius scale length ($\rho_{pi}\nabla \ell n n \sim 1$ with n the ion density). As a result, the entropy production argument has to be modified to account for the non-locality of finite poloidal ion gyroradius effects that requires the equilibrium to be established over the entire pedestal. Using the new gyrokinetic variables, vanishing entropy production in the pedestal requires [1]:

$$\int_{\Delta V} d^3r \int d^3v \ell n f_{0i} C_{1ii}\{f_{0i}\} = 0, \qquad (1)$$

where ΔV is the volume of the pedestal (between the top of the pedestal where $\rho_{pi}\nabla \ell n n << 1$ and the separatrix). Therefore, f_{0i} must be a drifting Maxwellian independent of poloidal angle in the banana regime. To make the Vlasov operator vanish and for f_{0i} to remain Maxwellian f_{0i} cannot depend on magnetic moment μ so that $f_{0i} = f_{0i}(\psi_*,E)$, where $E = v^2/2 + e\Phi/M$ is the total energy and independent of poloidal angle the magnetic moment. For a nearly rigid toroidal rotation frequency ω_i it is possible to construct such a Maxwellian if the ion temperature variation is slow compared to the poloidal ion gyroradius ($\rho_{pi}\nabla \ell n T_i << 1$, $\rho_{pi}\nabla \ell n \omega_i << 1$) [1,4]:

$$f_{0i}(\psi_*,E) = n(M/2\pi T_i)^{3/2}\exp[-M(\vec{v}-\omega_i R^2\nabla\zeta)^2/2T_i] = \eta(M/2\pi T_i)^{3/2}\exp(-ME/T_i - e\omega_i\psi_*/cT_i),$$

where $\eta = n\exp[(e\Phi/T_i)+(e\omega_i\psi/cT_i)-(M\omega_i^2R^2/2T_i)]$ must also be nearly constant ($\rho_{pi}\nabla \ell n\eta << 1$). Thus, the background ion temperature profile must have a scale length much larger than a density pedestal with a scale length L ~ ρ_{pi}. If T_i varied with ψ or ψ_* on the ρ_{pi} scale either the Vlasov operator or the collision operator would not vanish to lowest order. In principle, it is might be possible for the pedestal to be narrower than ρ_{pi} if the inverse aspect ratio ε is very small, however, in practice $\varepsilon^{1/2} \sim 1$ at the pedestal so the $\varepsilon << 1$ assumption used in subsequent sections is only useful for obtaining analytic results that must be extended by simulations.

Lowest order momentum balance gives $\omega_i = -c[d\Phi/d\psi + (en)^{-1}d(nT_i)/d\psi]$ with $\Phi(\psi)$ the axisymmetric electrostatic potential. For a density scale length of ρ_{pi} we notice that $cR(en)^{-1}d(nT_i)/d\psi \sim v_i$ = ion thermal speed. Consequently, in a subsonic pedestal it must be that to lowest order $en d\Phi/d\psi \approx -d(nT_i)/d\psi \approx -T_i dn/d\psi$. Thus the ions are essentially electrostatically confined [1] as observed in the banana regime H mode pedestal of DIII-D [5] and in the slightly more collisional Alcator C-Mod H mode pedestal [6]. Total pressure balance then gives that the electrons are magnetically confined with a mean flow $\vec{V}_e$ comparable to the ion thermal speed ($\vec{V}_e \sim v_i$) and a large bootstrap current in the pedestal.

3 Gyrokinetic treatment of the pedestal residual

Zonal flow behavior in the pedestal is substantially different from that in the core because of differences in the polarization of the plasma [2]. Plasma polarization was shown by Rosenbluth and Hinton [7] to be the key factor affecting the linear stage of zonal flow dynamics since the plasma "shields" any turbulence generated zonal flow potential, resulting in only a fraction of it, the residual, surviving. Classical polarization is due to the dipole moment associated with ion gyrocenters and becomes relevant on a time scale greater than the cyclotron period. Neoclassical polarization is due to the magnetic drift shift in the center of the banana or passing orbits of the ions in a tokamak and occurs on time scales greater than the bounce period. The residual is then defined for a step function change in the perturbed density as the ratio of the final perturbed potential $\delta\Phi(t\rightarrow\infty)$ to its initial value $\delta\Phi(t=0)$:

$$\frac{\delta\Phi(t\rightarrow\infty)}{\delta\Phi(t=0)} = \frac{\varepsilon_{pol-cl}}{\varepsilon_{pol-cl}+\varepsilon_{pol-nc}} = \frac{1}{1+1.6(q^2/\varepsilon^{1/2})Y}\ , \qquad (2)$$

where ε_{pol-cl} and ε_{pol-nc} are the classical and neoclassical plasma polarizations relating the perturbed polarization densities δn_{pol} and potential $\delta\Phi$ [assumed proportional to $\exp(-i\int\kappa d\psi)$] in $\varepsilon_{pol}\kappa^2\delta\Phi = -4\pi e\delta n_{pol}$. The factor Y = 1 for the familiar Rosenbluth and Hinton case [7], with q the safety factor and ε is the inverse aspect ratio.

The strong poloidal $\vec{E}\times\vec{B}$ drift and its associated finite orbit effects that arise in the pedestal modify the collisionless zonal flow residual of Rosenbluth and Hinton [7] because the influence of the banana regime axisymmetric radial electric field, approximately satisfying $ed\Phi_0/d\psi = -(T_i/n)dn/d\psi$, on the particle trajectories must be retained. This difference causes the residual associated with the small amplitude, shorter wavelength, axisymmetric zonal flow potential $\delta\Phi$ to differ substantially from that of Rosenbluth and Hinton as briefly discussed in the remainder of this section.

We are interested in the case where poloidal $\vec{E}\times\vec{B}$ drift and parallel streaming compete so the the ion poloidal drift frequency is

$$\dot{\theta} = [v_{||} + cIB^{-1}\Phi'(\psi)]\vec{n}\cdot\nabla\theta\ . \qquad (3)$$

In the core of a tokamak the second term on the right side of (8) is negligible, whereas in the pedestal these terms are comparable, and Φ' modifies the poloidal motion of particles. Notice that the $\vec{E}\times\vec{B}$ drift, $|\vec{v}_E| \sim v_i\rho_i/\rho_{pi}$, remains much less than the ion thermal speed $v_i = (2T_i/M)^{1/2}$ as required by our gyrokinetic ordering because $\vec{v}_E$ is nearly parallel to the poloidal plane while $v_{||}\vec{n}$ has only a small poloidal component $v_{||}\rho_i/\rho_{pi}$ since we assume $B_p \ll B$.

In the conventional core case, a particle is trapped if its $v_{||}$ is small enough, but for a particle to be trapped in the pedestal its $v_{||}$ must be close to $-cIB^{-1}\Phi'(\psi)$. Consequently, as a trapped particle in the pedestal undergoes its banana motion projected onto the poloidal cross-section, its parallel velocity oscillates around the value of $-cIB^{-1}\Phi'(\psi)$ (rather than zero). As trapped and barely passing particles play a key role in neoclassical phenomena such as radial ion heat flux or neoclassical polarization, the evaluation of these effects has to be revisited in the pedestal as they will differ from the core.

We can verify these differences analytically in the large aspect ratio limit ($\varepsilon \ll 1$) where the effect of a strong electric field with shear on particle orbits can be treated by considering a quadratic potential well in ψ. It is important to realize that it is the ε << 1 assumption allows us to analytically treat the trapped and passing orbit widths as small compared to the poloidal ion gyroradius scale of the pedestal. Expanding about ψ_*, we obtain

$$\Phi(\psi) = \Phi_* + (\psi - \psi_*)]\Phi'_* + (1/2)(\psi - \psi_*)^2\Phi''_* \ , \tag{4}$$

with $\Phi_* \equiv \Phi(\psi_*)$, $\Phi'_* \equiv \Phi'(\psi_*)$, and $\Phi''_* \equiv \Phi''(\psi_*)$. If we assume the radial variation of B and I are weak, and define u = cIΦ'/B, equation (3) can be written as

$$qR\dot{\theta} = (v_{||} + u) = S(v_{||} + u_*) \ , \tag{5}$$

with $S = 1 + cI^2\Phi''_*/\Omega_i B$ the orbit squeezing coefficient and $u_* = cI\Phi'_*/SB$ the u shift in $v_{||}$ due to the large $\vec{E}\times\vec{B}$ drift in the pedestal for fixed ψ_*. Using conservation of total energy E and canonical angular momentum ψ_* along with the magnetic moment μ gives

$$v_{||} + u = (v_{||0} + u_0)\sqrt{1 - \kappa^2 \sin^2(\theta/2)} \tag{6}$$

where

$$\kappa^2 = 4\varepsilon S \frac{(\mu B_0 + u_0)}{(v_{||0} + u_0)^2} \tag{7}$$

with $\kappa^2 = 1$ the trapped-passing boundary and the "0" subscript denoting the value as the trajectory crosses the equatorial plane. Notice that even in the presence of a large radial electric field the behavior of the trapped and barely passing particles is as in the conventional case because we are careful to treat the distinction between ψ_* and ψ so that $v_{||} + u$ only vanishes at a banana tip. The important effect is that the trapped particle region moves onto the tail of the Maxwellian centered about v = 0 as u becomes comparable to the ion thermal speed dramatically reducing the number of trapped ions and the neoclassical polarization, and thereby enhancing the residual to make it closer to unity. To see this in detail, the preceding trajectory is employed to perform the transit averages at fixed E, ψ_* and μ. Then the residual is evaluated by performing the velocity space integrals on a fixed flux surface ψ. The final expression connects smoothly onto the core result [7], namely

$$Y = \left(1 + \frac{2iz}{k_r \rho_{pi}}\right) \frac{4\exp(-z^2)}{3\pi^{1/2}} \int_0^\infty dy (y + 2z^2)^{3/2} \exp(-y) \ , \tag{8}$$

with $z = u/v_i$ and k_r the radial wavenumber, and where we assume S = 1 for simplicity. Work in progress will properly retain **E**x**B** shear of turbulent vortices in the pedestal by extending this result of [2] to S ~ 1 to obtain the algebraic orbit squeezing modifications. The factor Y is complex because the susceptibility varies on the scale of a poloidal ion gyroradius, with the imaginary term corresponding to a shift of the entire plasma pedestal in response to the pedestal electric field.

The result for Y is rather sensitive to the electric field or z. Therefore, in the pedestal the usual core result that electric field shear controls turbulence is not strictly valid. Indeed, the residual approaches unity for $u/v_i > 1$, since Y becomes small, resulting in undamped zonal flow. Using enΦ' ~ $T_i n'$ in u ~ v_i gives a density pedestal width of order ρ_{pi} as anticipated. As in the Rosenbluth and Hinton u << v_i derivation [7] we assume $\varepsilon^{1/2} k_r \rho_{pi} \leq 1$. Moreover, the neglect of $\varepsilon^{1/2}$ corrections to the passing means that we also must assume exp $[-(u/v_i)^2] \gg \varepsilon^{1/2}$ to retain trapped and barely passing particle effects while ignoring the freely passing contribution. The more realisitic case of $\varepsilon^{1/2} \sim 1$ must be treated numerically, but the $\varepsilon^{1/2} << 1$ limit is useful to check these simulations. In the pedestal we expect the exponential decrease in the trapped population is more important than the algebraic effects associated with orbit squeezing.

The local maximum in |Y| provides a threshold at $(k_r\rho_{pi})^2 = 4z^2$ beyond which further steepening of the density profile (and through pressure balance, a further decrease in the radial electric field) leads to a larger zonal flow residual. Presumably this increase causes a reduction in turbulence, and further enhancement of the density profile to complete the feedback loop. For a well-defined threshold to exist when the radial zonal flow wavelength $2\pi/k_r$ is comparable to the pedestal width w requires $\pi\rho_{pi}/w \sim u/v_i \sim 1$. For weaker density gradients having $\pi\rho_{pi}/w \sim u/v_i << 1$ the zonal flow residual cannot be increased, no feedback occurs, and the usual Rosenbluth and Hinton result is found.

4 Gyrokinetic treatment of the pedestal ion heat and particle flow

We revisit the calculation of the banana regime neoclassical ion heat flux and poloidal flow by focusing on the pedestal where we need to account for the strong $\vec{E}\times\vec{B}$ drift inherent to this tokamak region. Again we specialize to the S = 1 case to focus on the strong effect of the radial electric field in modifying heat transport and flow. We also find that due to the electric field the pedestal poloidal ion flow is likely to change its direction as compared to its core counterpart. This feature may explain the discrepancy between the conventional banana regime predictions and recent experimental measurements of the impurity flow performed at Alcator C-Mod [8].

Using the orbit description of section 3 allows transit averages to be evaluated at fixed ψ_* and velocity space integrals to be performed at fixed ψ to determine the heat flux and poloidal ion flow. To do so we employ the full Rosenbluth potential form for ion-ion collisions with a background Maxwellian with a momentum conserving term inserted to determine the localized portion of the pitch angle derviative of the perturbed ion distribution function. Using this localized piece the ion heat flux is evaluated by a moment approach to find the following expression for the banana regime ion heat flux in the pedestal:

$$\langle \vec{q}\cdot\nabla\psi\rangle = -1.35 n\nu_{ii}\frac{\sqrt{\varepsilon}T_i I^2}{M\Omega_i^2}\frac{\partial T_i}{\partial\psi}G(u) \tag{9}$$

with ν_{ii} the Braginski ion-ion collision frequency and G(u) the modification due to the $\vec{E}\times\vec{B}$ drift normalized to the Helander and Sigmar result [9] so that G(0) = 1. The function G monotonically decreases and becomes exponentially small due to the exponential behavior of the localized piece of the distribution function for $u > v_i$. Consequently, the banana regime ion heat diffusivity is substantially reduced in the pedestal as compared to its core value. Our result for the banana regime ion heat flux in the pedestal differs in important ways from that obtained by Shaing [10] because we employ that ion temperature variation is slow compared to the poloidal ion gyroradius scale of the density variation and explicitly take advantage of the near cancelation of the **E**x**B** and ion diamagnetic drifts while ignoring orbit squeezing. These simplifications allow us to obtain an explicit expression valid for all u/v_i whereas Shaing is only able to do so for $u < v_i$.

Only the non-local portion of the ion distribution function matters for the evaluation of the parallel ion flow, for which no exponential reduction can occur, and we find

$$V_{\|i} = -\frac{cI}{B}\left(\frac{\partial \Phi}{\partial \psi} + \frac{1}{Zen}\frac{\partial p_i}{\partial \psi}\right) - \frac{7cI}{6ZeB_0}\frac{\partial T_i}{\partial \psi} J(u) \tag{10}$$

with J(u) the finite $\vec{E}\times\vec{B}$ modification to the result of Helander and Sigmar again so that J(0) = 1. The important feature of J is that as it decreases monotonically it changes sign at about $u/v_i = 0.6$ and becomes more negative thereafter as u/v_i increases. Consequently, the sign of the ion temperature gradient terms changes to that of the Pfirsch-Schluter regime as observed in recent C-Mod pedestal measurements of the poloidal flow [8]. This sign change enhances the pedestal bootstrap current.

5 Discussion

Some key properties of a tokamak pedestal have been determined. An entropy production proof finds that the ion temperature cannot vary on the poloidal ion gyroradius scale of a density or electron temperature pedestal. As a result, subsonic toroidal ion flow in a pedestal requires the $\vec{E}\times\vec{B}$ and ion diamagnetic drifts to cancel to lowest order and causing the electron flow to be sonic and determining the lowest order electric field once the density profile is known. In the absence of orbit squeezing, the existence of this strong radial electric field in the pedestal is found to increase the zonal flow residual found Rosenbluth and Hinton [7], thereby reducing the turbulence level in the pedestal, and intoducing the possibility for a feedback mechanism to further enhance the residual and decrease the turbulence further. In addition, we find that the banana regime ion heat conductivity in the absence of orbit squeezing is reduced by strong poloidal $\vec{E}\times\vec{B}$ drift that can also change the sign of the ion poloidal flow in a banana regime pedestal.

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