PSFC/JA-10-1

Enhancement of the Bootstrap Current in a Tokamak Pedestal

Grigory Kagan and P.J. Catto

February 2010

Plasma Science and Fusion Center
Massachusetts Institute of Technology
167 Albany Street
Cambridge, MA 02139 USA

This work was supported by the U.S. Department of Energy, Grant No. DE-FG02-91ER-54109 . Reproduction, translation, publication, use and disposal, in whole or in part, by or for the United States government is permitted.
Submitted to Phys. Letters 2010

Enhancement of the Bootstrap Current in a Tokamak Pedestal

Grigory Kagan[1,2] and Peter J. Catto[1]

[1]*Massachusetts Institute of Technology, Plasma Science and Fusion Center, Cambridge, Massachusetts 02139*

[2]*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*

The strong radial electric field in a subsonic tokamak pedestal modifies the neoclassical ion parallel flow velocity, as well as the radial ion heat flux. Existing experimental evidence of the resulting alteration in the poloidal flow of a trace impurity is discussed. We then demonstrate that the modified parallel ion flow can substantially enhance the pedestal bootstrap current when the background ions are in the banana regime. Only the coefficient of the ion temperature gradient drive term is affected. The revised expression for the pedestal bootstrap current is presented. The prescription for inserting the modification into any existing banana regime bootstrap current expression is given.

The bootstrap current [1,2] due to the diamagnetism associated with trapped and barely passing particles is a key feature of advanced tokamak operating regimes since it can dramatically reduce the need to drive current [3]. Moreover, the stability of tokamaks is sensitive to the details of the bootstrap as well as driven current profiles, especially for the density and temperature pedestal just inside the last closed flux surface [4-7].

Recent impurity flow measurements in the pedestal of Alcator C-Mod indicate that the poloidal flow of the background banana regime ions can be in the direction opposite to the one predicted by conventional neoclassical theory [8,9]. Any sign change in the poloidal ion flow has important implications for the bootstrap current since it is sensitive to the parallel background ion flow due to momentum exchange between electrons and ions.

However, experimental studies of the pedestal bootstrap current [5-7] measure currents large enough to impact edge stability and claim qualitative agreement with the Sauter, Angioni and Lin-Liu model [10] based on the conventional neoclassical bootstrap expression [11-13] derived by assuming the poloidal ion gyroradius is small compared to the shortest pedestal scale length. Here we reconcile these seemingly contradictory experimental results for the bootstrap current and ion flow by first demonstrating that the change in the sign of the background poloidal ion flow is due to the strong radial electric in the pedestal [14] enhancing the pedestal bootstrap current. We then show that no contradiction arises because the new formula for the bootstrap current continues to be of the conventional form and therefore of the general form of Sauter, Angioni, and Lin-Liu [10] provided the ion temperature gradient coefficient is allowed to depend on the radial electric field in the pedestal.

The bootstrap current is normally regarded as the most important prediction of neoclassical theory, and is enhanced by strong density and pressure gradients because of its diamagnetic nature. Conventional tokamaks have $B/B_p \sim qR/a \gg 1$, where B is the total magnetic field, B_p is the poloidal magnetic field, q is the safety factor, and R/a is the aspect ratio. As a result, the diamagnetism associated with the trapped and barely passsing particles is larger than that associated with their gyromotion since the magnetic drift departure from a flux surface is roughly a poloidal gyroradius $\rho_{pj} = \rho_j B/B_p$ rather than a gyroradius $\rho_j = \left(2T_j M_j\right)^{1/2} c/|Z_j|eB$, where the subscript j denotes the species of temperature T_j, mass M_j and charge Z_j, and e is the magnitude of the charge of an electron and c the speed of light.

During high confinement operation of Alcator C-Mod the pedestal is found to have radial density and electron temperature variations on the scale of the poloidal ion gyroradius ρ_{pi} [15]. However, the flows in C-Mod are subsonic so the only way to satisfy radial ion pressure balance is for the ions to be nearly electrostatically confined with a somewhat weaker background ion temperature variation than the density [16]. This weaker ion temperature variation also enhances the bootstrap current and is required to minimize entropy production in the pedestal; however, it is not the primary effect of interest for the discussion that follows.

The more important effect is the strong radial electric field needed to keep the ion flow subsonic in the pedestal since the poloidal $\vec{E} \times \vec{B}$ drift can compete with the small poloidal projection of the parallel ion streaming, thereby modifying conventional neoclassical results in the banana regime [14]. The ion flow is altered because the passing ion constraint on the ion-ion collision operator [11,12] must be imposed along the $\vec{E} \times \vec{B}$ modified ion trajectory by holding canonical angular momentum fixed, rather than the poloidal flux function. This difference alters the nonlocal part of the ion distribution function and leads to a poloidal ion flow sensitive to the radial electric field when this passing collisional constraint is evaluated retaining the strong poloidal radial electric field variation along the trajectory. Orbit squeezing [17] does not play a role in modifying the nonlocal portion of the ion distribution function, but does, of course, change the localized contribution and, thereby, radial transport.

The preceding discussion indicates that ion behavior in the pedestal can be expected to be rather different than in the core. Of particular interest for the bootstrap current calculation is the change in the parallel ion flow on a flux surface caused by the strong radial electric field inherent in a subsonic pedestal because of the need to maintain radial pressure balance. Experiments find that in many tokamaks the pedestal width can be of order of the poloidal ion gyroradius. Thus, for a pedestal ion, variation of the electrostatic energy across a neoclassical orbit is comparable to that of the kinetic energy, causing this orbit to be substantially different from those in the core. Of course, electron orbits are essentially unchanged since $\rho_{pe} \ll \rho_{pi}$. However, even though electrons do not feel the pedestal electric field directly, it affects them indirectly through their friction with the modified parallel velocity of the bulk ions. Consequently, the coefficient preceding the ion temperature gradient term in the conventional formula and the Sauter, Angioni, and Lin-Liu form [10] is importantly modified in the pedestal.

To evaluate the neoclassical ion flow in the pedestal the full Maxwellian Rosenbluth potential form of the like particle collision operator must be employed [18] along with a term that insures momentum conservation for ion-ion collisions. This more complete operator [19,20] captures both energy and pitch angle scattering ion transitions across the electric field modified trapped-passing boundary that is no longer at constant pitch angle. This operator is used to evaluate the passing collisional constraint in the pedestal to determine the nonlocal and localized neoclassical corrections to the leading order ion distribution function, which is assumed to be a stationary Maxwellian f_{i0}. The Maxwellian remains stationary because the $\vec{E} \times \vec{B}$ drift cancels the ion diamagnetic drift in the pedestal to lowest order, and f_{i0} permits the strong density variation required for near electrostatic ion confinement through its dependence on total energy.

There are two changes that occur when evaluating the nonlocal portion of the constraint equation. The first is that the parallel velocity must be shifted by the poloidal $\vec{E} \times \vec{B}$ drift $u \equiv cI\phi'(\psi)/B$ since the deeply trapped particles are at $v_{\|} + u \approx 0$ rather than at $v_{\|} \approx 0$. Here ϕ is the electrostatic potential with $\vec{E} = -\phi'\nabla\psi = -(\partial\phi/\partial\psi)\nabla\psi$ and ψ the poloidal flux function. The second is in the factors that must be introduced to insure momentum conservation in ion-ion collisions change due to the finite electric field modication of the orbits. As a result of these changes, the perturbed ion distribution function becomes

$$f_{i1} = -\frac{Iv_{||}f_{i0}}{\Omega_i}\left[\frac{1}{p_i}\frac{dp_i}{d\psi} + \frac{Ze}{T_i}\frac{d\phi}{d\psi} + \left(\frac{Mv^2}{2T_i} - \frac{5}{2}\right)\frac{1}{T_i}\frac{dT_i}{d\psi}\right] + g \tag{1}$$

where $g = h_\sigma + g - h_\sigma$ vanishes for the trapped particles (but not the passing), $g - h_\sigma$ is the small local term giving an order $\varepsilon^{1/2}$ correction to the ion flow (that we can neglect), and

$$h_\sigma = \frac{I(v_{||} + u)}{\Omega_i}\left[\frac{M(v^2 + u^2)}{2T_i} - \sigma\right]\frac{f_{i0}}{T_i}\frac{dT_i}{d\psi} \ , \tag{2}$$

with $I = RB_t$ with B_t the toroidal magnetic field, $\Omega_i = ZeB/Mc$, $\vec{B} = I\nabla\zeta + \nabla\zeta \times \nabla\psi$, and n_i, T_i, $p_i = n_i T_i$, Z and M the background ion density, temperature, pressure, charge number and mass. Notice that in addition to $v_{||} \to v_{||} + u$ and $v^2 \to v^2 + u^2$ in h_σ, the factor σ determined by demanding like particle momentum conservation when evaluating $g - h_\sigma$ is modified to become

$$\sigma = \frac{\int_0^\infty dy \exp(-y)(y + Mu^2/T_i)^{3/2}[\nu_\perp y + \nu_{||} Mu^2/T_i]}{\int_0^\infty dy \exp(-y)(y + Mu^2/T_i)^{1/2}[\nu_\perp y + \nu_{||} Mu^2/T_i]} \ , \tag{3}$$

where $\nu_\perp = 3(2\pi)^{1/2}\nu_{ii}[erf(x) - \Psi(x)]/2x^3$ and $\nu_{||} = 3(2\pi)^{1/2}\nu_{ii}\Psi(x)/2x^3$ with $x = v\left(M/2T_i\right)^{1/2}$, $\nu_{ii} = 4\pi^{1/2}Z^4e^4 n_i \ell n\Lambda/3M^{1/2}T_i^{3/2}$, $erf(x) = 2\pi^{-1/2}\int_0^\infty dy \exp(-y^2)]$ and $\Psi(x) = [erf(x) - xerf'(x)]/2x^2$. Using the preceding to determine the lowest order parallel background ion velocity gives

$$V_{i||} = -\frac{cI}{B}\left(\frac{d\phi}{d\psi} + \frac{1}{Zen_i}\frac{dp_i}{d\psi}\right) + \frac{7cIBJ(U)}{6Ze\langle B^2\rangle}\frac{dT_i}{d\psi}, \tag{4}$$

Where $U \equiv u\left(M/2T_i\right)^{1/2}$, $\langle ...\rangle$ denotes a flux surface average, and

$$J(U) = (6/7)[(5/2 - \sigma) + 2(1-\sigma)U^2 + 2U^4] \ , \tag{5}$$

Adding the perpendicular ion velocity $\vec{V}_{i\perp} = (c/B^2)(\vec{B} \times \nabla\psi)[(d\phi/d\psi) + (Zen_i)^{-1}(dp_i/d\psi)]$ to the parallel ion velocity gives the poloidal ion flow to be

$$V_i^{pol} = \frac{7cIB_pJ(U)}{6Ze\langle B^2\rangle}\frac{dT_i}{d\psi}. \tag{6}$$

The parameter U accounts for the presence of the equilibrium pedestal electric field with $U \sim 1$ if the spatial scale of the potential ϕ or n_i is of order ρ_{pi}. The shaping function $J\left(U\right)$ is introduced to denote the difference between the pedestal and conventional $J = 1$, $U = 0$, and $\sigma = 4.33$ result in the core. In the pedestal, equations (3) and (5) give J to be a monotonically decreasing function of equilibrium electric field and thereby increasing bootstrap current. The function J goes negative for $U > 0.6$ to give an additive positive poloidal flow from the ion temperature gradient term. The average pedestal electric field in tokamaks such as Alcator C-Mod or DIII-D corresponds to $U \approx 0.75$ [15,21] and therefore we expect V_i^{pol} to change sign in the pedestal near the plasma edge.

Before presenting the modification to the bootstrap current due to the novel features of the ion flow outlined in the preceding paragraphs, we discuss the experimental evidence available for this effect. To this end, the impurity flow measurements recently performed at C-Mod [8,9] turn out to be important since their poloidal flow is sensitive to that flow component of the background ions and therefore measuring the former determines whether the latter is changed. Consequently, when the C-Mod study revealed impurity poloidal flows noticeably larger in banana regime pedestals than predicted by the conventional core formula, we were able to understand this seemingly contradictory result by retaining finite radial electric field modifications. We next demonstrate how this discrepancy is removed by employing our formula (5) instead of the usual $J = 1$ expression for the poloidal ion flow.

The analysis is simplified due to the high charge number and mass, and therefore high collisionality, of the boron impurities used in the experiment. These features make the impurity mean free path much less than parallel connection length qR, where R the major radius. As a result, the parallel ion and impurity flows are equal and the usual formula relating the poloidal velocities of the banana regime background ions to Pfirsch-Schluter impurities can be employed [22-25]:

$$V_z^{pol} = V_i^{pol} - \frac{cIB_p}{eB^2}\left(\frac{1}{Zn_i}\frac{dp_i}{d\psi} - \frac{1}{Z_z n_z}\frac{dp_z}{d\psi}\right) \tag{7}$$

where Z_z, n_z, and p_z are the impurity charge number, density and pressure and higher order terms in the aspect ratio expansion are omitted. The conventional formula for the poloidal ion flow ($J = 1$) makes the sum of V_i^{pol} and the diamagnetic terms tend to cancel on the right side of (7). As a result, the left side of (7) is relatively small and gives rise to the previously mentioned discrepancy between the experiment and conventional neoclassical formulas. On the other hand, accounting for the electric field makes V_i^{pol} smaller or even negative, thereby allowing the terms on the right side of (7) to add and give a larger prediction for the impurity flow in agreement with the C-Mod observations. Therefore, these impurity measurements are consistent with our first principle analysis of the background ion flow in a banana regime pedestal and we can proceed to the consideration of the pedestal bootstrap current.

Due to the small electron gyroradius their orbits are insensitive to the background electric field. As a result, the electrons can only be modified by means of the altered background ion flow. Thus, we can readily adapt the usual techniques of evaluating the bootstrap current (e.g. see [11-13]) by using our electric field modified parallel ion flow result. Before doing so we remark that the electric field makes $V_{i||}$ larger than its conventional counterpart. In the conventional $J = 1$ limit the diamagnetic and neoclassical terms have opposite signs and tend to cancel each other in (4). However, upon accounting for the presence of the electric field, $J\left(U\right)$ becomes smaller or even negative (so that these terms add), thereby increasing the net parallel ion flow, as well as the resulting electron-ion friction. Hence, the bootstrap current is enhanced in the pedestal since it is in the same direction as $V_{i||}$ and depends on it through the electron friction with the ions.

Knowing (4), the pedestal bootstrap current can be rigorously evaluated in the same way as in the core. The only change is due to the modified parallel ion velocity. Therefore, only the coefficient of the ion temperature gradient term changes. Indeed, if the bootstrap current is written as the sum of pressure and temperature (instead of density and temperature) gradients, the ion temperature gradient term need only be multiplied by $J\left(U\right)$ to retain electric field effects.

For example, in the $Z \to \infty$ and arbitrary Z cases, the electric field modified bootstrap current in a quasineutral plasma ($Zn_i = n_e$), with order $\varepsilon^{1/2}$ corrections ignored, become

$$J_{||}^{bs} \approx -1.46\varepsilon^{1/2} \frac{cIB}{\left\langle B^2 \right\rangle}\left[\frac{dp}{d\psi} - 1.17J\left(U\right)\frac{n_e}{Z_i}\frac{dT_i}{d\psi}\right] \ (Z \to \infty) \quad (8)$$

and [11]

$$J_{||}^{bs} = -1.46\varepsilon^{1/2}\frac{cIB}{\langle B^2\rangle}\left[\frac{Z^2+2.21Z+0.75}{Z(Z+1.414)}\right]\times$$

$$\left[\frac{dp}{d\psi} - \frac{(2.07Z+0.88)n_e}{(Z^2+2.21Z+0.75)}\frac{dT_e}{d\psi} - 1.17J(U)\frac{n_e}{Z}\frac{dT_i}{d\psi}\right], \quad (9)$$

where n_e and T_e are the electron density and temperature, and $p = p_e + p_i = n_e\left(T_e + T_i/Z\right)$. Equations (8) and (9) illustrate the enhancement of the bootstrap current due to the finite radial electric field modification factor $J(U)$. These equations predict that the pedestal bootstrap current is larger than that given by conventional formulas since $J(U)$ becomes less than unity as U^2 increases, and then goes negative for $U > 0.6$. The modification of these results is entirely due to the finite radial electric field modification of the parallel ion flow.

The Sauter, Angioni, and Lin-Liu formula [10] can be similarly modified to retain the finite electric field effects by multiplying their αL_{34} ion temperature gradient coefficient by $J(U)$. Consequently, our finite orbit generalization is qualitatively consistent with their form. As a result, we have shown that it should not be surprising that measurements [5-7] of the bootstrap current density near the plasma edge fit the phenomenological, but theory motivated, form

$$J_{||}^{bs} = -\frac{cIBp}{\langle B^2\rangle}\left[\frac{\alpha}{n_e}\frac{dn_e}{d\psi} + \frac{\beta}{T_e}\frac{dT_e}{d\psi} + \frac{\gamma}{T_i}\frac{dT_i}{d\psi}\right], \qquad (10)$$

and thereby find reasonable qualitative agreement with the neoclassical Sauter, Angioni, and Lin-Liu model [10], where the dimensionless parameters α, β, and γ account for geometrical and collisionality effects. Our first principles approach demonstrates that in the banana regime pedestal the parameter γ is also dependent upon the equilibrium axisymmetric electric field, but still maintains the general form of equation (10).

In conclusion, we have deduced that the bootstrap current in a banana regime pedestal is larger than predicted by conventional neoclassical theory. This favorable result is due to the strong pedestal electric field that increases the poloidal ion velocity, thereby enhancing the electron - ion friction drive for the bootstrap current. Experimental support for this analytical result is provided by impurity flow observations in the Alcator C-Mod pedestal in the banana regime that can be explained by the very same finite radial electric field modification of the poloidal flow that is shown here to increase the bootstrap current [8,9]. Therefore our first principles approach is consistent with all experimental observations [5-7] and recovers a bootstrap current formula of the general Sauter, Angioni, and Lin-Liu [10] form once the coefficient preceding the ion temperature gradient term is altered in the prescribed way to account for the strong pedestal radial electric field.

Acknowledgement. We are grateful to Kenneth Marr and Bruce Lipschultz of the PSFC for helpful discussions on their impurity measurements, to Paul Bonoli of the PSFC for bringing to our attention the relevant bootstrap current experiments, and Jesus Ramos of the PSFC for conversations on the extreme sensitivity of stability to small changes in the pedestal bootstrap current. This work was performed at the MIT Plasma Science and Fusion Center and supported by the U.S. Department of Energy grant DE-FG02-91ER54109.

---

[1] A. A. Galeev, Sov. Phys. JETP 32, 752 (1971).

[2] R.J. Bickerton, J.W. Connor, and J.B. Taylor, Nat. Phys. Sci. 229, 110 (1971).

[3] S. Jardin, C. Kessel, C. Bathke, D. Ehst, T. Mau, F. Najmabadi, and T. Petrie, Fusion Eng. Des. 38, 27 (1998).

[4] P. Bonoli, R. Parker, M. Porkolab, J. Ramos, S. Wukitch, Y. Takase, S. Bernabei, J. Hosea, G. Schilling, and J. Wilson, Nucl Fusion 40, 1251 (2000).

[5] M. Wade, M. Murakami, and P. Politzer, Phys. Rev. Lett. 92, 235005 (2004).

[6] D. Thomas, A. Leonard, L. Lao, T. Osborne, H. Mueller, and D. Finkenthal, Phys. Rev. Lett. 93, 65003 (2004).

[7] D. Thomas, A. Leonard, R. Groebner, T. Osborne, T. Casper, P. Snyder, and L. Lao, Phys Plasmas 12, 056123 (2005).

[8] "Comparison of neoclassical predictions with measured flows and evaluation of a poloidal impurity density asymmetry", K. Marr, B. Lipschultz, P.J. Catto, R. McDermott, M. Reinke, A. Simakov, submitted to Plasma Phys. Control. Fusion.

[9] K. Marr, B. Lipschultz, R. McDermott, P.J. Catto, A. Simakov, I. Hutchinson, J. Hughes, M. Reinke, http://www.psfc.mit.edu/research/alcator/pubs/APS/APS2008/Marr_Poster_APS2008.pdf.

[10] O. Sauter, C. Angioni, and Y. Lin-Liu, Phys Plasmas 6, 2834 (1999).

[11] P. Helander and D.J. Sigmar, Collisional Transport in Magnetized Plasmas (Cambridge: Cambridge University Press) (2002).

[12] F. Hinton and R. Hazeltine, Reviews of Modern Physics 48, 239 (1976).

[13] S. P. Hirshman, Phys. Fluids 21, 1295 (1978).

[14] G. Kagan and P. J. Catto, submitted to Plasma Phys. Control. Fusion.

[15] R. McDermott, B. Lipschultz, J. Hughes, P. Catto, A. Hubbard, I. Hutchinson, R. Granetz, M. Greenwald, B. LaBombard, and K. Marr, Phys Plasmas 16, 056103 (2009).

[16] G. Kagan and P. J. Catto, Plasma Phys. Controlled Fusion 50, 085010 (2008).

[17] R. Hazeltine, Physics of Fluids B: Plasma Physics 1, 2031 (1989).

[18] M. N. Rosenbluth, W. M. MacDonald, and D. L. Judd, Physical Review 107, 1 (1957).

[19] I. Abel, M. Barnes, S. Cowley, W. Dorland, and A. Schekochihin, Phys Plasmas 15, 122509 (2008).

[20] P. J. Catto and D. R. Ernst, Plasma Phys. Controlled Fusion 51, 062001 (2009).

[21] J. deGrassie, Private Communication (2008).

[22] S. Hirshman and D. Sigmar, Nucl Fusion 21, 1079 (1981).

[23] Y. Kim, P. Diamond, and R. Groebner, Physics of Fluids B: Plasma Physics 3, 2050 (1991).

[24] P. Helander, Phys Plasmas 8, 4700 (2001).

[25] P. J. Catto and A. N. Simakov, Phys Plasmas 13, 052507 (2006).