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Evaluation of turbo-Brayton cycle for cooling current leads: Integrated current lead/heat exchanger

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Abstract

We investigate the optimization of turbo-Brayton cycles for cooling current leads. Simple models of single stage current lead, two-stage current lead and two stage current lead coupled with a double stage turbo-Brayton cycle have been used to provide understanding about the issues and the tradeoffs. In addition, we discuss the possibility of using the heat exchanger in the turbo-Brayton system as the current lead.

I. Introduction

Current leads to cryogenic environment introduce substantial heat loads. When the leads are designed to carry substantial current, and the cryostat is small (as in the case of distribution systems), the cryogenic load is dominated by the current leads. Thus, there has been substantial effort in optimizing the current leads.

The cross section of the current lead is the results of a tradeoff between thermal loads, which increase with current lead cross section, and dissipated power, which decreases inversely proportional with current lead cross section. McFee [McFee] demonstrated a method for optimizing the current leads when the cryogenic environment was at 4 K. He showed that the use of two stages was beneficial in decreasing the cryogenic load, with an intermediate stage around liquid nitrogen temperature. A substantial fraction of the power dissipated in the upper, hotter regions of the current lead are intercepted at this intermediate temperature.

We have extended McFee's calculation to the case when the cryogenic temperature is around liquid nitrogen. In this case, there are advantages of using an intermediate stage at temperatures around 150 K. In addition to decrease electrical power consumption to drive the refrigerators, multiple stages improves the thermal stability of the current leads, decreasing the change for burn-out.

In this report we investigate an alternative approach, where the leads are integrated with the refrigerator. We have chosen to use a turbo-Brayton (TB) cycle, operating with neon. At the operating temperature, neon behaves as an ideal gas.

Several cases are investigated. The first case, investigated in section II, considered a single stage current lead with a single stage TB refrigerator, each optimized separately. The second case, covered in section III, considers a two stage current lead with a single stage TB cycle. Finally, the case where the current lead is combined with the heat exchanger of a TB cycle is considered. The latter configuration has been suggested by Maguire [Maguire].

The different cases are shown in Figure 1. In all cases it is assumed that before returning to the heat exchanger, after the expander the working gas cools the superconducting cable. The current leads are shown in orange color. There is an expander at the low temperature end that reduces the temperature of the gas, while at the top there is a compressor followed by a cooler to decrease the temperature of the working fluid before feeding it back into the heat exchanger. It is assumed that the temperature downstream from the cooler is 300 K, and that both the compressor and the expander see a pressure ratio of 3. The pressure drop of the gas along the heat exchanger or the cable is ignored.

The geometry of the heat exchanger is proprietary and will not be discussed here, as the details of the heat exchanger are not relevant for the calculations. The only relevant parameter is the cross section area, which is assumed to be 8.6 10^{-4} m^2 for each leg of the heat exchanger, and the length, which is assumed to he 0.87 m.

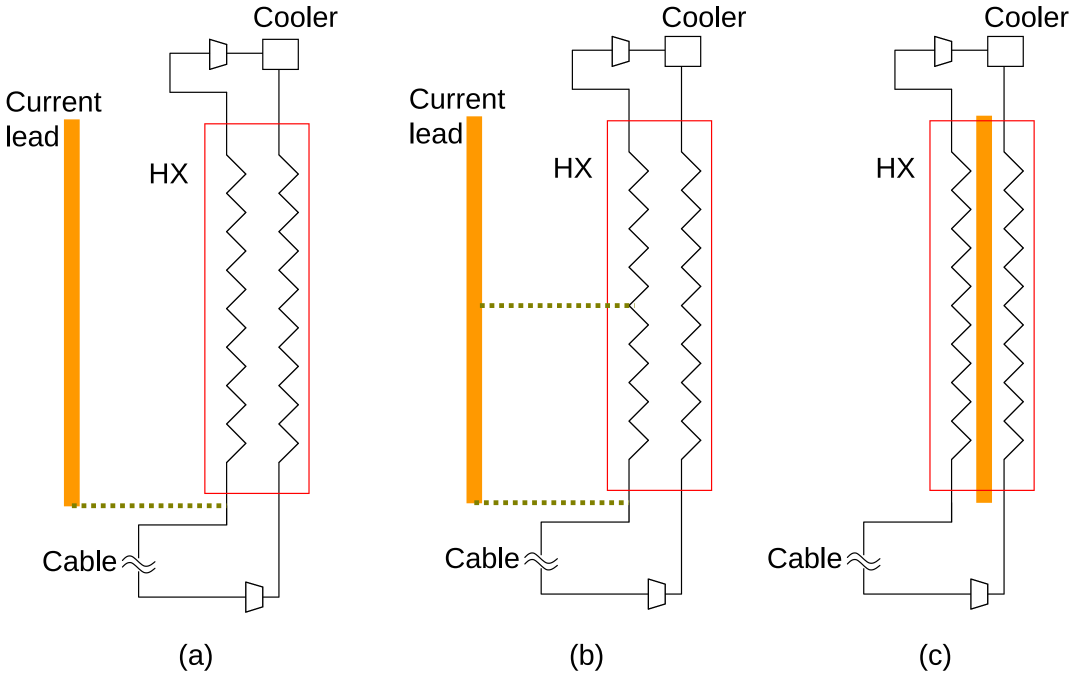

Figure 1. Cases analyzed in this report.

A simple model has been built to analyze the overall the performance of the system. The heat exchanger is divided in multiple sections of equal lengths. It is assumed that the temperature profile is linear from room temperature to the cryogenic temperature. Thus the distance between the discrete temperature points is constant.

It is assumed that the compression and expansion are ideal processes, with $\gamma = 5/3$, resulting in $(\gamma - 1)/\gamma = 0.398$.

It is assumed that the minimum temperature difference between the cold flow and the hot flow is 2 K, occurring at the warm end of the heat exchanger.

The flow rates and the lower temperature are adjusted in order to maintain a temperature of 72 K downstream from the low temperature expander. It is assumed that the current leads carrying 10000 A.

II. Single stage current lead

In this section a single stage current lead is used, with a single stage TB cooler.

The calculations performed by Bromberg [Bromberg], for copper current leads, optimized a current lead (from room temperature to 72 K) by having $IL/A = 3.64$ MA/m. The corresponding cryogenic heat load at 72 K from the Joule dissipation in the current lead is 0.042 W/A when the lead is energized to full current, or 420 W for the case under investigation (there is no thermal conduction from room temperature, as the temperature gradient at the high temperature end is 0 for an optimized current lead). Similarly, the cryogenic load at the low temperature is 270 W in the case of no current to the lower temperature, due to thermal conduction from the room temperature environment.

The results of the calculations are shown in Figure 2 (full current) and 3 (no current). It should be stressed that the current lead has been optimized for 10,000 A, and thus it operates suboptimally at lower currents [Bromberg]. The temperature along the heat exchanger is shown. It is assumed that the heat from the current lead is added to the gas downstream from the expander, shown as a step in the "up" curve in Figures 2 and 3 (up refers to the return flow from low temperature to room temperature, while "down" refers to the flow from room temperature to the cryogenic temperature). Note that under the assumptions, there is a constant separation between the down and up curves in the heat exchanger, as no additional heat is generated/removed along the path of the heat exchanger.

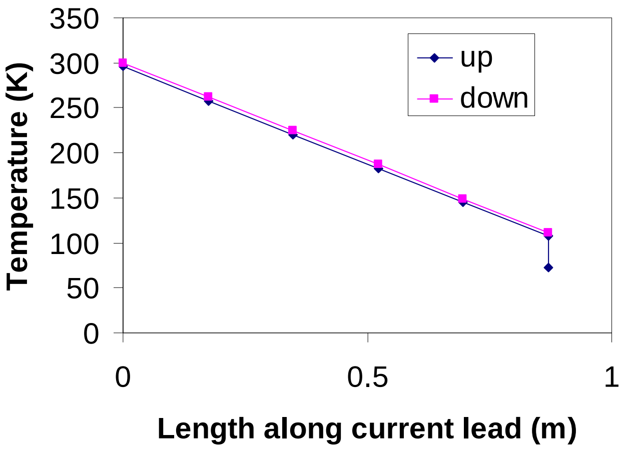

Figure 2. Temperature along the heat exchanger for the case of single current lead, single expander with full current (10000 A).

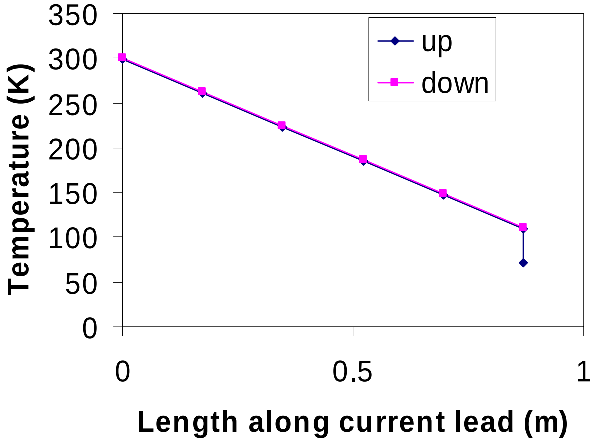

Figure 3. Temperature along the heat exchanger for the case of single current lead, single expander in the case of zero current flowing through the lead (optimized for 10000 A).

Table 1. Parameters from single stage current lead, single stage TB cooler

	One stage	
Expansion	3	3
Ip	0	0
Power at Lower stage	420	270
Power at higher stage	0	0
Area	0.0008575	0.000858
Mass flow rate	0.016	0.011
Compressor power	3338	2316
Lower temperature	107	110
Upper temperature	296	299
Dissipation	0	0
thermal conduction	214	212
IL/A	0	0

Table 1 shows details of the calculation. It is assumed that there is not current flowing through the heat exchanger, thus IL/A for the heat exchanger is 0. In this case, as well as in the other cases below, it is assumed that there is thermal conduction from the room temperature to the low temperature. Because the temperature gradient is considered top be linear, we have assumed that the heat conducted from room temperature (determined by the temperature gradient at the hot end of the heat exchanger, $- k\, A\, dT/dx|_{x=0}$) flows down to the cold environment. In reality, some of this heat is exchanged to the gas, but for the simple analysis in this report, it is equivalent as being "dumped" at the low temperature end, as the gas needs to be cooled at the bottom of the heat exchanger, irrespective or where it is heated (and since the gas is assumed ideal, the heat capacity is constant).

The mass flow rate is adjusted in both cases, however assuming constant expansion/compression ratios irrespective of the flow rate.

Also shown in Table 1 are the temperatures upstream from the expander at the low temperature. The temperatures are about 110 K.

The compressor power is shown in the table. In the case of a single stage, it is 3.3 kW for the full current case, while in the case of no current it is decreased to 2.3 kW. It should be noted that the powers are per lead.

III. Double stage current lead.

For the case of two stages, it is assumed that a fraction of the thermal load is intercepted at a higher temperature and removed by the return (cold, or in the figure, "up") leg of the heat exchanger.

The heat loads from optimized current leads is 388 W at the high end, and 170 W at the low end (in the case of full current); these values decrease to 220 W and 140 W, respectively, in the case of no current. The corresponding values of the value of *IL/A* are 2.72 MA/m for the upper stage and 2.88 MA/m for the lower stage.

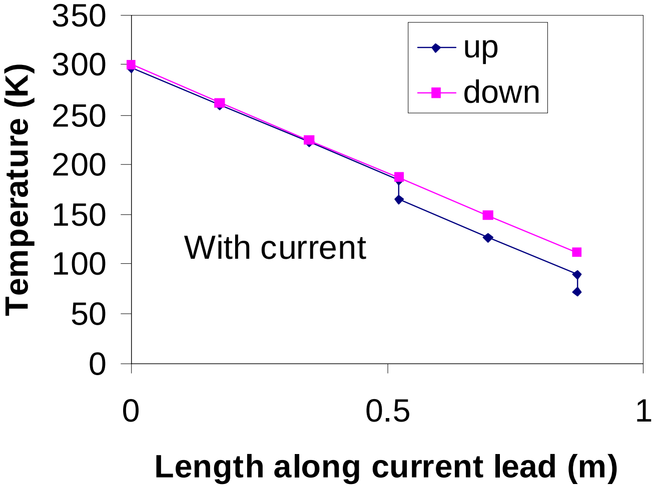

Figure 4. Temperature along the heat exchanger for the case of two-stage current lead, single expander with full current (10000 A).

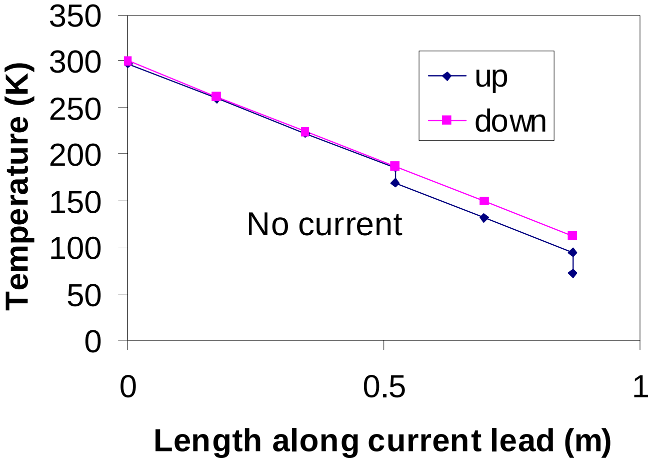

Figure 5. Temperature along the heat exchanger for the case of two-stage current lead, single expander with no current.

The results are shown in Figures 4 (full current) and 5 (no current). As in the previous case, there is thermal conduction along the lead that is dumped at the low temperature end, as well as the cryogenic load from the current leads. In addition, there is a secondary stage at intermediate temperature, shown in the "up" curve. In the case with current, it is assumed that there is no heat flowing from the intermediate stage to the low temperature stage due to convection, that is, the temperature gradient along the low stage of the current lead at the intermediate stage is 0. Figures 4 and 5 have a step associated with the change in temperature of the return leg of the heat exchanger ("up" leg) from the thermal load due to the intermediate temperature.

Table 2. Results for the cases of two stage current leads, single stage TB, optimization with current, and results without current

	Two stages	
Expansion	3	3
Ip	0	0
Power at Lower stage	170	140
Power at higher stage	388	220
Area	0.000858	0.000858
Mass flow rate	0.019	0.0135
Compressor power	3983	2837
Lower temperature	89	94
Upper temperature	297	298
Dissipation	0	0
thermal conduction	227	223
IL/A	0	0

The compressor power are shown in Table 2. The values are higher than in the case of a single stage. It should be pointed out that in the case of a single stage cryocooler, effectively all the loads (at any temperature) are being cooled by the expansion of the gas at the low temperature. The case with two stage current leads has increased thermal loads than the case with a single stage, although the bulk of the heat is being removed at the intermediate temperature. However, the TB cycle effectively removes all the heat at the lower temperature, and thus the case with two stage current lead it is less efficient than the case with a single stage. On consequence is that the lower temperature of the heat exchanger (~ 90 K), upstream from the expander, is substantially higher than the case with a single stage current lead.

IV. Heat exchanger as the current lead

In this section we discuss the option of the heat exchanger serving as the current lead itself (case (c) in Figure 1). Maguire {Maguire] has suggested the use of this approach for the manufacturing of current leads.

It is assumed that the material of the heat exchanger is copper. The heat generated by the Joule dissipation is transferred locally to the gas flow. It is assumed that the temperature of the "down" leg is linear along the current lead, as in the previous cases.

The results are shown in Figures 6 and 7. The results are shown for larger cross sectional area of the heat exchanger, as the original design of the heat exchanger resulted in excessive Joule dissipation, although the thermal conduction was small. The jump in temperature at the low temperature end of the heat exchanger is due to the thermal conduction associated with increased cross sectional area. It should be noted that the temperature change is higher in the case of the no current case than in the case with full current. The reason for the larger temperature increase in the no current case is due to the lower flow rates, resulting in increased enthalpy change for the same thermal conduction power.

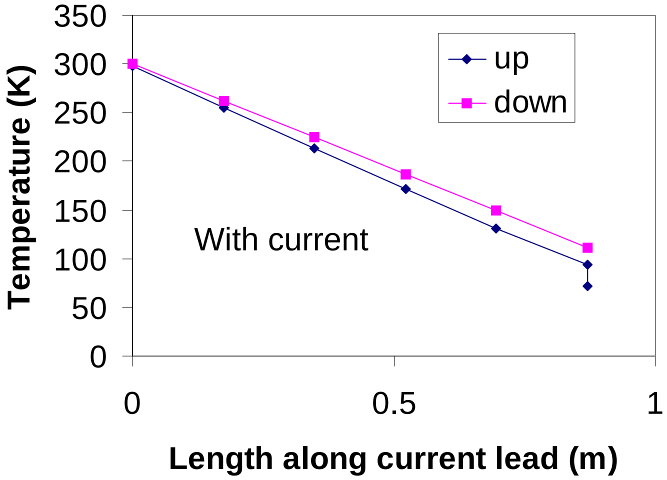

Figure 6. Temperature along the heat exchanger for the case of current lead integrated with the heat exchanger of a TB cooler, with full current (10000 A).

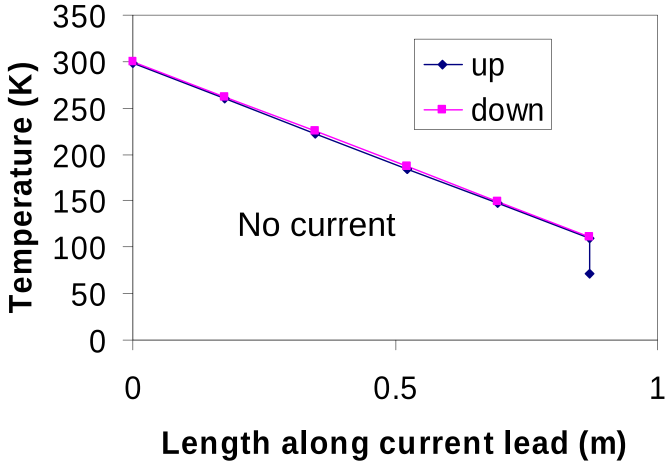

Figure 7. Temperature along the heat exchanger for the case of current lead integrated with the heat exchanger of a TB cooler, with zero current

In the case of full current, the temperature lines are not parallel. This is due to the increased temperature in the "up" leg (return leg) due to the local heat generation from the Joule heating. However, in the case of no current (and thus, no local heat generation), the temperatures profiles are again parallel lines. As in the previous cases, the temperature difference at the warm end of the heat exchanger is 2 K.

Table 3. Results for integrated heat exchanger/current lead, for the case of optimized cross section of the heat exchanger.

Expansion	3	3
Ip	10000	0
Power at Lower stage	0	0
Power at higher stage	0	0
Area	3.4E-03	3.4E-03
Mass flow rate	0.034	0.0168
Compressor power	7134	3527
Lower temperature	94	109
Upper temperature	298	298
Dissipation	608	0
thermal conduction	891	851
IL/A	2.5E+06	0

It was mentioned about that the original value for the cross sectional area of the heat exchanger was far away from the optimal. In the case where the heat exchanger is not used to conduct current, it is designed to minimize thermal conduction, and thus, minimize the cross sectional area. When used to carry current, this cross sectional area results in very large Joule dissipation. To investigate the effect, the cross sectional area has been changed (while keeping the length of the heat exchanger constant). The results are shown in Table 4.

Table 4. Results for integrated heat exchanger/current lead, as a function of the cross sectional area

Expansion	3	3	3	3	3
Ip	10000	10000	10000	10000	10000
Power at Lower stage	0	0	0	0	0
Power at higher stage	0	0	0	0	0
Area	8.6E-04	1.7E-03	2.6E-03	3.4E-03	4.3E-03
Mass flow rate	0.067	0.041	0.035	0.034	0.035
Compressor power	14059	8597	7341	7134	7345
Lower temperature	78	83	89	94	97
Upper temperature	298	297	297	298	298
Dissipation	2371	1196	805	608	489
thermal conduction	239	464	680	891	1100
IL/A	1.0E+07	5.1E+06	3.4E+06	2.5E+06	2.0E+06

The original design of the heat exchanger, with an area of 8.575 10-4 m2, resulted in a thermal conduction cryogenic load of 239 W, and a Joule dissipation of 2371 W, and large compressor power (14 kW). Increasing the cross sectional area results in larger decrements of the Joule dissipation than increment of the thermal conduction load. Increasing the cross sectional area to 3.4 10^{-3} m^2 results in comparable thermal conduction and Joule dissipation, and a decrease of the compressor power of 2 compared to the original case.

The results are shown in Figure 8. *IL/A* decreases with increasing cross sectional area A of the heat exchanger/current lead

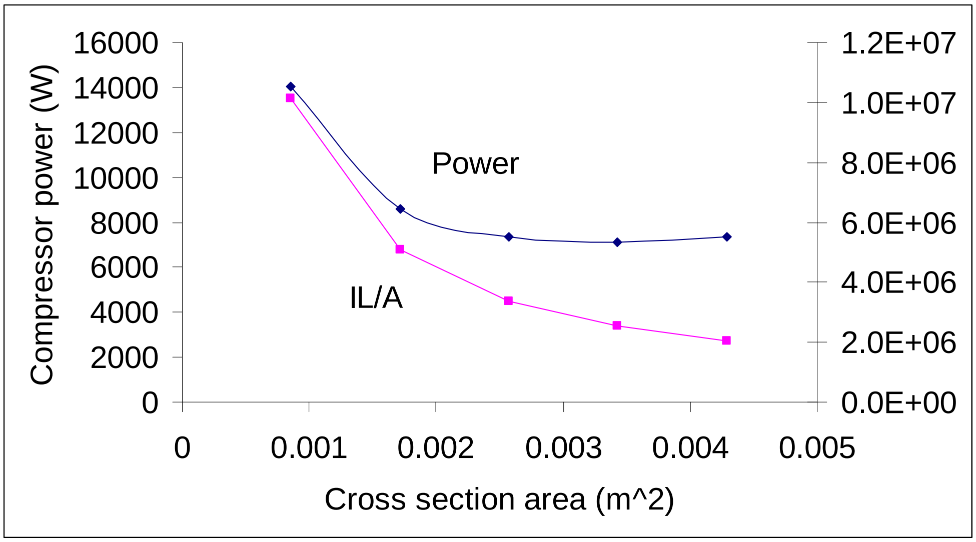

Figure 8. Optimization of the cross sectional area of the heat exchanger/current lead.

V. Double stage turbo-Brayton cooler, two stage current lead

In this section, the case with two stage cryocooler and two stage current lead is discussed.

Figure 8 shows the case analyzed in this section. It is assumed that a fraction of the heat exchanger down leg goes through an expander at intermediate temperature, where it intercepts the current lead intermediate temperature cryogenic load and a fraction of the thermal load conducted along the heat exchanger. The cross sectional area of the heat exchanger at the lower section is smaller as lower flows are being transporter through this section of the heat exchanger. It is assumed that the cross sectional area of the lower section is reduced proportionally to the fraction of the flow that is directed to the lower temperature.

The results are presented in Table 5. As the case discussed has the separate current lead, the loads at the lower stage and at the intermediate stages are the same as those in Table 3, for both the cases of full current and zero current.

There is a substantial saving in total power, with the power being about 2.6 kW, as opposed to the case with a single stage current lead and cryocooler, with a electrical power of about 3.3 kW.

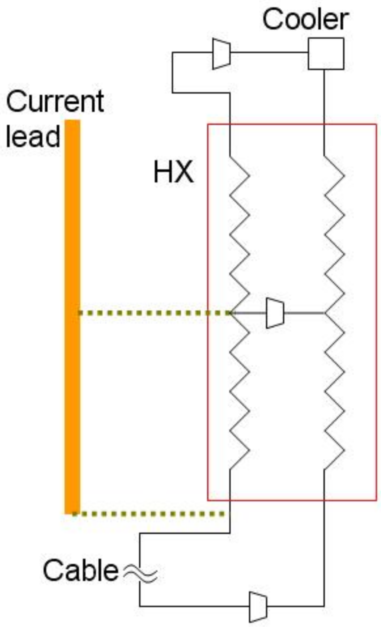

Figure 9. Two stage current lead with two stage heat exchanger, single compressor.

Table 5. Results for two-stage current lead, two stage turbo-Brayton cycle cooler

	with current	no current
Expansion	3	3
Ip	0	0
Power at Lower stage	170	140
Power at higher stage	388	220
Area of cryocooler	8.58E-04	8.58E-04
Mass flow rate	0.0125	0.0095
ratio (lower stage/total)	0.52	0.67
Compressor power, lower stage	1368	1339
Compressor power, upper stage	1263	660
mid-stage pre-expander	187	187
lower stage pre-expander	112	112
Lower temperature	104	104
Upper temperature	298	298
Dissipation	0	0
thermal conduction	216	215

VI. Discussion

In order to take full advantage of the use of TB refrigerator, an integrated optimization of the current lead and refrigerator system has to be carried out. In this report we analyzed the case of a turbo-Brayton refrigerator coupled with a single stage, a two-stage and integrated current leads. Table 6 shows a summary of the cases, for the four configurations and with and without current. For the cases investigated, the best result is obtained for the double stage cryocooler/two-stage current lead, followed by the single current lead, followed by the case with 2 leads, and finally, the case with integrated current lead/heat exchanger. We have not analyzed the case of the two-stage current lead integrated with the heat exchanger.

In addition, the overall space has not been investigated. The numbers in Table 6 are from a simple model. It is possible that increase optimization would further decrease the power consumed in the case of the double-stage cryocooler with two stage current lead. It is not expected that there is room to improve the case with single lead, as this case has already been optimized [Bromberg].

Table 6. Summary of compressor power (W) for the different configuration investigated in this report.

	With current	No current
Single lead	3338	2316
Two stage	3983	2837
HX/current lead	7134	3527
Double stage cryocooler/current lead	2631	1999

VII. Stainless steel current leads.

The work of McFee [MacFee], revisited by Bromberg [Bromberg] recently, provides a methodology and illustrative cases for analysis of current leads. It is shown that for most common metals, such as copper or aluminum, the cryogenic loads of optimized current leads is relatively independent of the choice of material. This is due to the Wiedemann–Franz-Lorenz law, which states that the ratio of the electronic contribution to the thermal conductivity (k) and the electrical conductivity (σ) of a metal is proportional to the temperature (T) [Jones]. This rule deviates in alloys. Since counter-flow heat exchangers of interest in turbo-Brayon cycles are made from Stainless steel, in this section we extend the work by McFee to 304 stainless steel.

The properties of SS304 were derived from Clark [Clark] while the thermal conductivity was obtained from Marquand [Marquand].

The steady state cryogenic characteristics of current leads depend on the ratio between the thermal and electrical conductivities. The minimum heat load is given by

$$\dot{Q}(T) = I\left[2\left(\frac{k}{\sigma}\right)_{ave}(T_H - T_L)\right]^{1/2}$$

$$\left(\frac{k}{\sigma}\right)_{ave} = \frac{1}{T_H - T_L}\int_{T_L}^{T_H}\frac{k(T)}{\sigma(T)}dT$$

clearly showing the dependence of minimum heat load and geometrical properties of the current lead on the k/σ ratio. Here $Q(T)$ is the heat load between temperatures T_H and T_L, and I is the current in the lead.

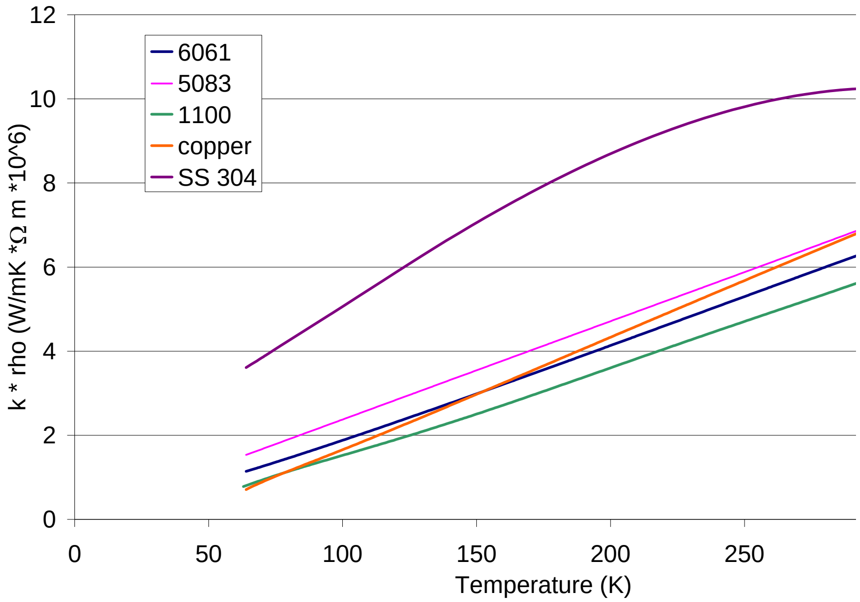

Figure 10. Ratio between thermal and electrical conductivities, k/σ, for several materials.

Figure 10 shows the ratio k/σ for copper, several aluminum alloys and SS 304. The k/σ value of SS304 is about 1.5 to 2 times the value of the other materials in Figure 10. Thus, the cryogenic losses are expected to also be higher.

Table 7 shows the cryogenic characteristics of SS 304 current leads, for the case of single stage and two stages. The high and low temperatures are assumed to be at 298 K and 65 K, respectively, and in the case of a double stage, the intermediate temperature is at 150 K. The heat loads are about 1.5-2 times those of aluminum, shown in Table 8 [Bromberg].

The larger thermal load associated with using SS304 current lead makes this approach unattractive for applications where there are concerns about the electrical power consumption.

Table 7. Stainless steel 304 optimal current lead characteristics

SS 304		one stage	Two stages
Upper stage			
Intermediate temperature	K		150
Q_dot/I	W/A	0.060	0.052
IL/A	A/m	9.09E+04	7.64E+04
Q_dot/I no Current	W/A	0.032	0.027
Lower stage			
Q_dot/I			0.030
IL/A			5.36E+04
Q_dot/I no Current			0.015

Table 8. Aluminum optimal current lead characteristics (reproduced from [Bromberg])

Aluminum		
Upper stage		
Intermediate temperature	K	150
Q_dot/I	W/A	0.035
IL/A	A/m	1.62E+06
Q_dot/I no Current	W/A	0.020
Lower stage		
Q_dot/I		0.017
IL/A		1.85E+06
Q_dot/I no Current		0.011
P_e/I	W/A	0.867

VIII. Summary

The use of a turbo-Brayton cycle for some applications where weight and volume are a concern is very attractive. In this paper we have investigated systems using a turbo-Brayton cooler for cooling a superconducting system. We have determined that the system with the smallest weight/power consumption using a single-stage turbo-Brayton cycle uses a single stage current lead. The integration of the current lead with the heat exchanger in the turbo-Brayton cooler results in about a factor of 2 increase in required electrical power.

The use of two-stage current lead with a double-stage turbo-Brayton cycle, the power consumption can be decreased by about 1/3, for comparable performance.

The integration results in a system that is very robust and capable to take overcurrents for substantial fraction of the time, safely. The single stage current lead can be burned by excessive current for a short period of time, while the two-stage is more robust, but not as robust as the integrated heat exchanger/current lead.

Integrating a two stage current lead with a two stage cryo-Brayton cycle has not been investigated. Optimization of the heat exchanger and the overall system may offer additional power savings.

Acknowledgement

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