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Neoclassical Plateau Regime Transport in a Tokamak Pedestal

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In subsonic tokamak pedestals the radial scale of plasma profiles can be comparable to the ion poloidal Larmor radius, thereby making the radial electrostatic field so strong that the $E \times B$ drift has to be retained in the ion kinetic equation in the same order as the parallel streaming. The modifications of neoclassical plateau regime transport – such as the ion heat flux, and the poloidal ion and impurity flows – are evaluated in the presence of a strong radial electric field. The altered poloidal ion flow can lead to a significant increase in the bootstrap current in the pedestal where the spatial profile variation is strong because of the enhanced coefficient of the ion temperature gradient term near the electric field minimum. Unlike the banana regime, orbit squeezing does not affect the plateau regime results.

I. INTRODUCTION

It is desirable that future fusion devices operate in an enhanced confinement mode [1] – including an H-mode pedestal [2]. One of the most important open questions of fusion plasma theory is the physics of pedestal and internal transport barriers. Although it is commonly assumed that turbulent transport dominates in tokamak plasmas, some neoclassical effects [3] are also significant in the pedestal. Long wavelength shear (non-zonal) flows within the flux surfaces play an important role in the suppression of turbulence in transport barriers [4], and the relation between the parallel ion flow and the global radial field, is expected to be neoclassical with a negligible turbulent contribution [5]. Furthermore, the bootstrap current [3] is modified and it plays crucial role in reactor relevant operation of fusion devices.

Experimental results show that in subsonic tokamak pedestals the radial scale of plasma profiles can be as small as the ion poloidal Larmor radius [6] of the main ion species. Accordingly, the radial electrostatic field is so strong ($\sim$ 100 kV/m, up to 300 kV/m on Alcator C-mod) that the contribution of the $\mathbf{E} \times \mathbf{B}$ drift to the poloidal ion motion is comparable to that of the parallel streaming. Therefore, these two components of the poloidal motion have to be retained in the ion kinetic equation in the same order.

The conventional way of solving the drift kinetic equation by expressing the cross field magnetic drifts in terms of a parallel gradient has to be modified, since this method cannot be applied to the $\mathbf{E} \times \mathbf{B}$ drifts.

The kinetic description of plasmas has recently been extended for pedestals with strong electric fields and short scale lengths by making use of the fact that the canonical angular momentum is a constant of the motion, and using it as the radial coordinate instead of the poloidal flux[7]. By adopting this approach herein modifications to neoclassical plateau regime transport [8, 9] are evaluated in pedestal regions. These results are relevant in existing fusion devices such as Alcator C-mod [10]. The calculation allows for $\mathcal{O}(1)$ values of the normalized electric field $U = v_{\mathbf{E}\times\mathbf{B}} B/(v_i B_{\rm pol})$, where $v_{\mathbf{E}\times\mathbf{B}}$ is the $\mathbf{E} \times \mathbf{B}$ velocity, v_i is the ion thermal speed, and B and $B_{\rm pol}$ are the magnitudes of the total and poloidal magnetic fields respectively, with $B_{\rm pol} \ll B$.

We find that the ion heat diffusivity is reduced for large values of U, as the resonance causing plateau regime transport is shifted towards the tail of the distribution, but it is enhanced if $U \approx 1$. Moreover, the poloidal ion and impurity flows are modified. The altered poloidal ion flow can lead to an increase in the bootstrap current in the pedestal where the radial profile variation is strong because of the enhanced coefficient of the ion temperature gradient term near the electric field minimum. Unlike the banana regime [11], orbit squeezing does not affect the plateau regime results.

In the next section, the perturbed ion distribution function is derived and the ion heat flux is calculated together with the parallel ion flow and the poloidal impurity rotation. The bootstrap current is evaluated in Sec. III. Finally the results are summarized in Sec. IV.

II. ION TRANSPORT AND PARALLEL ION FLOW

The magnetic field is represented as $\mathbf{B} = I\nabla\zeta + \nabla\zeta \times \nabla\psi$, where ζ is the toroidal angle and $2\pi\Psi$ is the poloidal flux. The radial electric field E_r is assumed to be of the magnitude $\sim B_p v_i/c$, so the $\mathbf{E} \times \mathbf{B}$ drift is kept in the ion kinetic equation in the same order as the parallel streaming since we assume $B_p \ll B$. Throughout the calculation we assume a quadratic electric potential well

$$\Phi(\Psi) = \Phi(\Psi_*) + (\Psi - \Psi_*)\Phi'(\Psi_*) + \frac{1}{2}(\Psi - \Psi_*)^2\Phi''(\Psi_*), \tag{1}$$

where the preceding convenient representation makes use of

$$\Psi_* = \Psi - \frac{Mc}{Ze} R\mathbf{v} \cdot \hat{\zeta} = \Psi - \frac{Iv_\parallel}{\Omega} + \frac{\mathbf{v} \times \mathbf{b} \cdot \nabla\Psi}{\Omega}, \tag{2}$$

where M is the mass and Ze is the charge of the particle, R is the major radius, $\Omega = ZeB/Mc$ is the cyclotron frequency and $\mathbf{b} = \mathbf{B}/B$. Within a constant multiplier the canonical angular momentum is Ψ_*, and it is a constant of the motion due to axisymmetry.

The ratio of the last two terms on the right side of Eq. (2) is $B/B_p \gg 1$, so we take $\Psi_* \approx \Psi - Iv_\parallel/\Omega$.

It is convenient to introduce $u = cI\Phi'/B$, the poloidal projection of the $\mathbf{E}\times\mathbf{B}$ drift velocity that competes with the poloidal component of the parallel streaming and $u_* = cI\Phi'_*/B$, where $\Phi'_* = \Phi'(\Psi_*)$. We assume B is slowly varying with Ψ, so that $B(\Psi_*,\theta) \approx B(\Psi,\theta)$. The orbit squeezing factor $S = 1 + cI^2\Phi''_*/(B\Omega)$ is considered to be constant except for its B dependence. Using the preceding notation the poloidal motion of the particles is given by

$$\dot{\theta} = \left(v_\parallel \mathbf{b} + \mathbf{v}_{\mathbf{E}\times\mathbf{B}}\right)\cdot\nabla\theta = (v_\parallel + u)\mathbf{b}\cdot\nabla\theta \approx \left(Sv_\parallel + u_*\right)/(qR), \tag{3}$$

where θ is the poloidal angle, and q is the safety factor. To find the final form of Eq (3) we use equation (1) to obtain

$$u_*(\Psi_*) = \frac{cI}{B}\left[\Phi'(\Psi) + (\Psi_* - \Psi)\Phi''(\Psi)\right] = u(\Psi) - \frac{cI^2 v_\parallel \Phi''(\Psi)}{B\Omega} = u + (1-S)v_\parallel. \tag{4}$$

We adopt the treatment of Ref. [7] and use Ψ_* as radial coordinate, rather than the poloidal flux function Ψ, thereby allowing for the handling of strong gradients in the plasma profiles and the electrostatic potential. A convenient energy variable is then defined as

$$\mathcal{E} = \frac{v^2}{2} + \frac{Ze}{M}\left[\Phi(\Psi) - \Phi(\Psi_*)\right] = E - \frac{Ze}{M}\Phi(\Psi_*) = S\frac{v_\parallel^2}{2} + \mu B + v_\parallel u_*, \tag{5}$$

where $E = v^2/2 + Ze\Phi/M$ is the total energy and $\mu = v_\perp^2/2B$ is the magnetic moment. Note that $\mathcal{E}$ is conserved by the Vlasov operator $d_t \equiv \partial_t + \mathbf{v}\cdot\nabla + (\Omega\mathbf{v}\times\mathbf{b} - Ze\nabla\Phi/M)\cdot\nabla_v$ since $d_t\mathcal{E} = 0$, because E and Ψ_* are constants of the motion.

In Ref. [7] it is shown that the lowest order solution of the electrostatic gyrokinetic equation must be a Maxwellian even in the pedestal where the density and potential can vary on the scale of the poloidal ion Larmor radius. Proceeding to higher order we write the gyro-averaged distribution function as

$$\bar{f} = f_*(\Psi_*,\mathcal{E}) + h(\Psi_*,\mathcal{E},\mu,\theta,t), \tag{6}$$

where f_* is a stationary near Maxwellian that is only a function of the constants of the motion $\mathcal{E}$ and Ψ_*. Upon Taylor expanding the slowly varying functions η and T_i as in Ref. [7] we obtain

$$\begin{aligned} f_* &= \eta(\Psi_*)\left(\frac{M}{2\pi T_i(\Psi_*)}\right)^{3/2} e^{-\frac{mE}{T_i(\Psi_*)}} \\ &\approx f_{Mi}\left\{1 - \frac{Iv_\parallel}{\Omega_i}\left[\frac{\partial \ln p_i}{\partial\Psi} + \frac{Ze}{T_i}\frac{\partial\Phi}{\partial\Psi} + \left(\frac{mv^2}{2T_i} - \frac{5}{2}\right)\frac{\partial \ln T_i}{\partial\Psi}\right] + \ldots\right\}, \end{aligned} \tag{7}$$

where

$$\eta(\Psi_*) = \eta(\Psi) + (\Psi_* - \Psi)\frac{\partial\eta(\Psi)}{\partial\Psi} + \ldots, \tag{8}$$

$$\eta(\Psi) = n_i(\Psi) \exp\left[\frac{Ze\Phi(\Psi)}{T_i(\Psi)}\right], \tag{9}$$

and the stationary Maxwellian on a flux surface is

$$f_{Mi} = n_i(\Psi)\left(\frac{M}{2\pi T_i(\Psi)}\right)^{3/2} e^{-\frac{Mv^2}{2T_i(\Psi)}}. \tag{10}$$

We consider subsonic flows, so that in the pedestal $\partial_\Psi \ln n_i \approx -(Ze/T_i)\partial_\Psi \Phi$. The governing equation for a time independent perturbed ion distribution, in accordance with Ref. [7], is

$$\dot{\theta}\frac{\partial h_{1i}}{\partial \theta} - C^l_{ii}\left\{h_{1i} - \frac{Iv_\parallel f_{Mi}}{\Omega_i}\left(\frac{Mv^2}{2T_i} - \frac{5}{2}\right)\frac{\partial \ln T_i}{\partial \Psi}\right\} = 0, \tag{11}$$

where C^l_{ii} is the linearized ion-ion collision operator, which is momentum conserving, and the θ derivative is taken keeping $\mathcal{E}$, Ψ_* and μ fixed. The kinetic equation Eq. (11) can be rewritten as

$$\left(Sv_\parallel + u_*\right)\mathbf{b}\cdot\nabla\left[H_i + \frac{Iv_\parallel f_{Mi}}{\Omega_i}\left(\frac{Mv^2}{2T_i} - \frac{5}{2}\right)\frac{\partial \ln T_i}{\partial \Psi}\right] - C^l_{ii}\left\{H_i\right\} = 0, \tag{12}$$

where we have introduced

$$H_i = h_{1i} - \frac{Iv_\parallel f_{Mi}}{\Omega_i}\left(\frac{Mv^2}{2T_i} - \frac{5}{2}\right)\frac{\partial \ln T_i}{\partial \Psi}. \tag{13}$$

In the plateau regime, the form of the collision operator cannot affect the transport when the kinetic equation is written in the form of (12). Therefore we can use a simple Krook operator to model the collisions. However, the replacement $C^l_{ii}\left\{H_i\right\} \to -\nu H_i$ destroys the momentum conserving property of the operator. This defect is remedied by adding a homogeneous solution to H_i, and then determining its free coefficient by making use of the fact that $C^l_{ii}\{v_\parallel f_M\} = 0$ [3, 9, 12]. This addition modifies the ion flow, thus it should be done so that the resulting flow is divergence free. Accordingly, we adopt the following replacement

$$H_i \to H_i + \frac{MBkv_\parallel f_{Mi}}{T_i}, \tag{14}$$

where the unknown k is to be determined by requiring that the solution gives no radial particle flux.

After the replacements the kinetic equation becomes

$$\begin{aligned}&\left(Sv_\parallel + u_*\right)\mathbf{b}\cdot\nabla H_i + \nu H_i \\ &\quad = -\left(Sv_\parallel + u_*\right)\mathbf{b}\cdot\nabla\left\{\frac{Iv_\parallel f_{Mi}}{\Omega_i}\left(\frac{Mv^2}{2T_i} - \frac{5}{2}\right)\frac{\partial \ln T_i}{\partial \Psi} + \frac{MBkv_\parallel f_{Mi}}{T_i}\right\}.\end{aligned} \tag{15}$$

The spatial derivatives can be performed by recalling the B dependence of u_* and S to find

$$\left(Sv_\parallel + u_*\right)\mathbf{b}\cdot\nabla|_{\mathcal{E},\mu,\Psi_*} v_\parallel = \left[-\mu B + v_\parallel u_* + 2(S-1)\frac{v_\parallel^2}{2}\right]\mathbf{b}\cdot\nabla \ln B, \tag{16}$$

which then follows from $\mathbf{b}\cdot\nabla|_{\mathcal{E},\mu,\Psi_*}\mathcal{E}=0$. Employing Eq. (16) we find the relations

$$\left(Sv_\parallel + u_*\right)\mathbf{b}\cdot\nabla|_{\mathcal{E},\mu,\Psi_*}\left(\frac{v_\parallel}{\Omega_i}\right) = \frac{1}{2\Omega_i}\left[2v_\parallel^2 + v_\perp\right]\mathbf{b}\cdot\nabla\ln R, \tag{17}$$

$$\left(Sv_\parallel + u_*\right)\mathbf{b}\cdot\nabla|_{\mathcal{E},\mu,\Psi_*}\left(v_\parallel B\right) = \frac{B}{2}\left[v_\perp^2 - (4S-2)\,v_\parallel^2 - 4v_\parallel u_*\right]\mathbf{b}\cdot\nabla\ln R. \tag{18}$$

The plateau regime only exists for large aspect ratio ($\epsilon \ll 1$, where $\epsilon = r/R_0$ with the minor radius r) and requires $\epsilon^{1/2} \ll \nu_i qR/v_i \ll 1$. As a result, we can approximate $\mathbf{b}\cdot\nabla\ln R$ by $-\epsilon\sin\theta/(qR)$ to obtain

$$\left(Sx_\parallel + \frac{u_*}{v_i}\right)\frac{\partial H_i}{\partial\theta} + \frac{\nu qR}{v_i}H_i = Q_i\sin\theta, \tag{19}$$

where $x = v/v_i = (x_\perp^2 + x_\parallel^2)^{1/2}$ is the velocity normalized to the ion thermal speed $v_i = (2T_i/M)^{1/2}$, and

$$Q_i = \epsilon v_i f_{Mi}\left\{\frac{2x_\parallel^2 + x_\perp^2}{2\Omega_i}I\left(x^2 - \frac{5}{2}\right)\frac{\partial\ln T_i}{\partial\Psi} + \frac{MBk}{2T_i}\left[x_\perp^2 - (4S-2)\,x_\parallel^2 - 4x_\parallel\frac{u_*}{v_i}\right]\right\}. \tag{20}$$

Due to the simple model for collisions the kinetic equation Eq. (19) can easily be solved to find

$$H = Q_i\frac{\frac{\nu qR}{v_i}\sin\theta - \left(Sx_\parallel + \frac{u_*}{v_i}\right)\cos\theta}{\left(Sx_\parallel + \frac{u_*}{v_i}\right)^2 + \left(\frac{\nu qR}{v_i}\right)^2} \approx Q_i\left[\pi\delta\left(Sx_\parallel + \frac{u_*}{v_i}\right)\sin\theta - \frac{\cos\theta}{Sx_\parallel + \frac{u_*}{v_i}}\right], \tag{21}$$

since most passing ions are nearly collisionless. The trapped and barely passing ions are collisional, thus collisions only enter to resolve the singularity at $Sx_\parallel + u_*/v_i = 0$.

The full gyro-averaged perturbed distribution $\bar{f}_{1i} = \langle f_i - f_{Mi}\rangle_\varphi$ is given by

$$\bar{f}_{1i} = h_i - \frac{Iv_\parallel}{\Omega}\frac{\partial f_{Mi}}{\partial\Psi} = H_i + \frac{MBkv_\parallel f_{Mi}}{T_i} - \frac{Iv_\parallel f_{Mi}}{\Omega_i}\left(\frac{\partial\ln p_i}{\partial\Psi} + \frac{Ze}{T_i}\frac{\partial\Phi}{\partial\Psi}\right). \tag{22}$$

Note that from all the terms in $\bar{f}_{1i}$ only the $\propto\sin\theta$ part of H_i has a finite contribution to the cross-field transport fluxes, and it does not depend on the radial electric field, in accordance with the requirement of intrinsic ambipolarity.

In order to determine the unknown k, we now make the radial ion particle transport vanish

$$0 = \langle\mathbf{\Gamma}_i\cdot\nabla\Psi\rangle = \left\langle\int d^3v\bar{f}_{1i}\mathbf{v}_d\cdot\nabla\Psi\right\rangle \approx -\left\langle\frac{I\epsilon}{2\Omega qR}\int d^3v(2v_\parallel^2 + v_\perp^2)H_i\sin\theta\right\rangle, \tag{23}$$

where $\mathbf{v}_d$ is the magnetic drift velocity. The velocity integral is to be performed holding Ψ constant, thus $H_i(\Psi_*)$ needs to be transformed back to flux surfaces. The orbit squeezing factor S is not affected by the transformation, since Φ'' is considered to be constant, while from (4) we see that $u_*(\Psi_*)$ is replaced by $u(\Psi) + (1-S)v_\parallel$. Accordingly,

$$\delta\left(Sx_\parallel + \frac{u_*}{v_i}\right) \to \delta\left(x_\parallel + U\right) \quad \text{and} \quad x_\perp^2 - (4S-2)\,x_\parallel^2 - 4x_\parallel\frac{u_*}{v_i} \to x_\perp^2 - 2x_\parallel^2 - 4x_\parallel U, \tag{24}$$

where we introduced $U = u/v_i$. Equation (24) shows that the resulting transport is insensitive to the orbit squeezing. Substituting Eqs. (21) and (20) into Eq. (24) the integrals can be evaluated yielding

$$\langle \mathbf{\Gamma}_i \cdot \nabla\Psi \rangle \approx -\sqrt{\frac{\pi}{2}} \frac{I^2\epsilon^2 n_i}{\Omega_i^2 q R_0} \left(\frac{T_i}{M}\right)^{3/2} \times e^{-U^2} \left\{ \left(\frac{1}{2} - U^4 + 2U^6\right) \frac{\partial \ln T_i}{\partial \Psi} + \left[1 + 2\left(U^2 + U^4\right)\right] \frac{Zek\langle B^2\rangle}{IT_i c} \right\}, \tag{25}$$

where n_i is the ion density. The ambipolarity condition requires that

$$k = -\frac{J(U^2)}{2} \frac{\partial \ln T_i}{\partial \Psi} \frac{IT_i c}{Ze\langle B^2\rangle}, \tag{26}$$

with

$$J(U^2) = \left[\frac{1 - 2U^4 + 4U^6}{1 + 2\left(U^2 + U^4\right)}\right], \tag{27}$$

which is consistent with the usual ($U = 0$) plateau result. As illustrated in Fig. 1, $J(U)$ has minimum of ≈ 0.39 at $|U| \approx 0.76$, and $J \to 2U^2 - 3 + \mathcal{O}(U^{-2})$ as $|U|$ goes to infinity.

The preceding calculation of J is based on the observation that if we artificially set $k = 0$ in Eq. (25) the resulting ion particle flux would be much higher than the electron particle flux (given in Appendix A for completeness). For $U = 0$ these fluxes are separated by the square root of the electron to ion mass ratio. However, for higher values of U the $\exp(-U^2)$ factor appearing in the expression for the ion particle flux (25) reduces it to the level of neoclassical electron transport. Therefore, our ambipolarity assumption (23) must be modified to include the electrons. As this does not happen until around $U = 3.5$ it need not concern us here.

Having calculated the full H_i distribution, we are in the position to evaluate the radial ion heat flux

$$\langle \mathbf{q}_i \cdot \nabla\Psi \rangle = \left\langle \int d^3v \frac{Mv^2}{2} \bar{f}_{1i} \mathbf{v}_d \cdot \nabla\Psi \right\rangle \approx -\left\langle \frac{I\epsilon}{2\Omega q R} \int d^3v \frac{Mv^2}{2} (2v_\parallel^2 + v_\perp^2) H_i \sin\theta \right\rangle. \tag{28}$$

We find

$$\langle \mathbf{q}_i \cdot \nabla\Psi \rangle \approx -3\sqrt{\frac{\pi}{2}} \frac{I^2\epsilon^2 p_i}{\Omega_i^2 q R_0} \left(\frac{T_i}{M}\right)^{3/2} \frac{\partial \ln T_i}{\partial \Psi} L(U^2), \tag{29}$$

with

$$L(U^2) = e^{-U^2} \frac{1 + 4\left\{U^2 + 2U^4 + \left[(4U^6 + U^8)/3\right]\right\}}{1 + 2(U^2 + U^4)}. \tag{30}$$

The preceding reduces to the standard plateau result [3, 9, 13] in the $U \to 0$ limit. The function $L(U)$ is plotted in Fig. 2 to show that plateau ion heat flux is almost 50% higher when the parallel projection of the poloidal component of the ion $\mathbf{E} \times \mathbf{B}$ drift velocity is close to the ion thermal speed [$L(|U| \approx 0.91) \approx 1.46$], but it rapidly drops off for higher values of U, as the resonance causing the plateau transport is shifted towards the tail of the ion distribution.

To calculate the electron transport and the bootstrap current the ion parallel flow needs to be evaluated. Neglecting the small local contribution from H_i we obtain

$$n_i V_{\|i} = \int d^3v v_\| \bar{f}_{1i} \approx -\frac{Ip_i}{M\Omega_i}\left[\frac{\partial \ln p_i}{\partial \Psi} + \frac{Ze}{T_i}\frac{\partial \phi}{\partial \Psi} + \frac{J(U^2)}{2}\frac{B^2}{\langle B^2\rangle}\frac{\partial \ln T_i}{\partial \Psi}\right]. \tag{31}$$

To relate the poloidal flow of a collisional trace impurity to the poloidal flow of a background ion in the plateau regime we note that the flux surface average of B times their parallel flows must be related by $\langle BV_{\|i}\rangle = \langle BV_{\|Z}\rangle$ [14, 15]. Using radial pressure balance for the ions and impurities along with the preceding result for $V_{\|i}$ gives the impurity poloidal flow to be

$$V_{Z,\theta} = \frac{cIT_iB_\theta}{eZ_i\langle B^2\rangle}\left[\frac{T_zZ_i}{T_iZ_Z}\frac{\partial \ln p_Z}{\partial \Psi} - \frac{\partial \ln p_i}{\partial \Psi} - \frac{J(U^2)}{2}\frac{\partial \ln T_i}{\partial \Psi}\right]. \tag{32}$$

The preceding expression can be used in C-Mod when the background ions are in the plateau regime [10].

III. BOOTSTRAP CURRENT

Even if the radial electric field is high enough to modify the ion transport and flows, i.e. $U = \mathcal{O}(1)$, electron orbits are practically unaffected by the strong radial electric field due to their large thermal speed. However, because electron-ion collisions depend on the ion mean flow, the electron distribution indirectly experiences this friction and is thereby influenced by the presence of the electric field. To evaluate this ion flow effect we next consider the electron problem.

We start with the electron kinetic equation [3]

$$v_\|\mathbf{b}\cdot\nabla h_{1e} + \frac{e}{T_e}f_{Me}v_\|E_I = C_e^{(1)}\left\{\bar{f}_{1e}\right\}, \tag{33}$$

where the spatial derivatives are performed holding E and μ fixed, the parallel induced electric field is denoted by $E_I = \mathbf{b}\cdot\nabla(\mathbf{E}+\nabla\Phi)$,

$$\bar{f}_{1e} = h_{1e} - \frac{Iv_\|}{\Omega_e}\frac{\partial f_{Me}}{\partial \Psi}, \tag{34}$$

and

$$C_e^{(1)}\left\{\bar{f}_{1e}\right\} = C_{ee}^{(1)}\left\{\bar{f}_{1e}\right\} + \mathcal{L}_{ei}\left\{\bar{f}_{1e} - \frac{m}{T_e}V_{\|i}v_\|f_{Me}\right\} \tag{35}$$

is the full linearized electron collision operator, with $C_{ee}^{(1)}$ the linearized electron-electron collision operator and $\mathcal{L}_{ei}$ the Lorentz operator (the operators are given in Appendix B). Since $C_{ee}^{(1)}\left\{f_{Me}v_\|\right\} = 0$, the $V_{\|i}v_\|f_{Me}$ term under $\mathcal{L}_{ei}^{(1)}$ can be added to $\bar{f}_{1e}$ under $C_{ee}^{(1)}$. Introducing f_S, the solution of the Spitzer problem

$$C_e^{(1)}\left\{f_S\right\} = \frac{e}{T_e}E_Iv_\|f_{Me}, \tag{36}$$

the kinetic equation (33) can be rewritten as

$$v_\parallel \mathbf{b}\cdot\nabla H_e - C_e^{(1)}\left\{H_e\right\} = -v_\parallel \mathbf{b}\cdot\nabla\left(\frac{Iv_\parallel}{\Omega_e}\frac{\partial f_{Me}}{\partial\Psi} + \frac{m}{T_e}V_{\parallel i}v_\parallel f_{Me} + f_S\right)$$
$$= A_1 v_\parallel \mathbf{b}\cdot\nabla\left(\frac{v_\parallel}{\Omega_e}\right) + A_2 v_\parallel \mathbf{b}\cdot\nabla\left(v_\parallel B\right), \tag{37}$$

where

$$H_e = h_{1e} - \frac{Iv_\parallel}{\Omega_e}\frac{\partial f_{Me}}{\partial\Psi} - \frac{m}{T_e}V_{\parallel i}v_\parallel f_{Me} - f_S, \tag{38}$$

$$A_1 = -If_{Me}\left[\frac{1}{p_e}\frac{\partial p_e}{\partial\Psi} + \frac{1}{Zn_iT_e}\frac{\partial p_i}{\partial\Psi} + \left(\frac{mv^2}{2T_e} - \frac{5}{2}\right)\frac{1}{T_e}\frac{\partial T_e}{\partial\Psi}\right], \tag{39}$$

and

$$A_2 = -\frac{J(U^2)f_{Mi}}{2ZT_e}\frac{\partial T_i}{\partial\Psi}\frac{IB}{\Omega_e\langle B^2\rangle} - \frac{f_S}{v_\parallel B}. \tag{40}$$

To get A_2, we used that the Spitzer function has the form

$$f_S = -\frac{eE_Iv_\parallel}{\nu_eT_e}D(x_e)f_{Me}, \tag{41}$$

and that E_I is approximately proportional to B. In our notation $\nu_e = 4(2\pi)^{1/2}n_ee^4\ln\Lambda/(3m^{1/2}T_e^{3/2})$ is the electron-electron collision frequency, and D is a dimensionless function of $x_e = v/v_e$, which is parametrically dependent on the ion charge. The function $D(x_e)$ is calculated in the Appendix B in terms of generalized Laguerre polynomials, $L_n^{(\lambda)}$, using a variational method to find

$$D(x_e) \approx \sqrt{2}\left(a_0L_0^{(3/2)}(x_e^2) + a_1L_1^{(3/2)}(x_e^2)\right), \tag{42}$$

where the coefficients are $a_0 = (8 + 13\sqrt{2}Z)/\left[4Z(2\sqrt{2}+2Z)\right]$ and $a_1 = -3/\left[2(2+\sqrt{2}Z)\right]$.

To evaluate the bootstrap current we begin by noting that the parallel current can be written as

$$j_\parallel = \frac{B\langle j_\parallel B\rangle}{\langle B^2\rangle} - \frac{cI}{B}\frac{\partial p}{\partial\Psi}\left(1 - \frac{B^2}{\langle B^2\rangle}\right), \tag{43}$$

where the second term is the Pfirsch-Schlüter current, and the first term is defined in terms of

$$\langle j_\parallel B\rangle = e\left\langle B\int d^3v\, v_\parallel\left(Z\bar{f}_{1i} - \bar{f}_{1e}\right)\right\rangle. \tag{44}$$

Recall that $\langle j_\parallel B\rangle$ is the sum of the ohmic current $\langle j_{OH}B\rangle = -e\langle B\int d^3v\, v_\parallel f_S\rangle = e^2n_e\langle E_IB\rangle\sqrt{2}a_0/(\nu_em_e)$ and a current contribution from H_e, the bootstrap current

$$\langle j_{BS}B\rangle = -e\left\langle B\int d^3v\, v_\parallel H_e\right\rangle. \tag{45}$$

Note that the $\partial_\Psi\phi$ terms from $\bar{f}_{1e}$ and $\bar{f}_{1i}$ cancel in the integrand of Eq. (44).

The seemingly straightforward procedure for evaluating the bootstrap current would be to solve Eq. (37) for H_e and substitute it into Eq. (45). However, H_e is not accurate enough to give the correct bootstrap current Instead, it is convenient to use an adjoint method to find $\langle j_{BS}B\rangle$. The adjoint equation that must be solved is given by

$$v_\parallel \mathbf{b}\cdot\nabla G + C_e^{(1)}\left\{G\right\} = \frac{e}{T_e}E_I v_\parallel f_{Me}. \tag{46}$$

In terms of G, the bootstrap current can be calculated by the adjoint method as

$$\langle j_{BS}B\rangle = \frac{BT_e}{E_I}\left\langle \int d^3v \frac{G}{f_{Me}}\left[A_1 v_\parallel \mathbf{b}\cdot\nabla\left(\frac{v_\parallel}{\Omega_e}\right) + A_2 v_\parallel \mathbf{b}\cdot\nabla\left(v_\parallel B\right)\right]\right\rangle, \tag{47}$$

This relation is obtained by adding $\langle \int d^3v\, \text{Eq. (37)}\, G/f_{Me}\rangle$ to $\langle \int d^3v\, \text{Eq. (46)}\, H_e/f_{Me}\rangle$ and using the self-adjoint property of the linearized collision operator [16, 17]. Writing $G = f_S + g$, the unknown part g gives the only non-zero contribution to Eq. (47). It is determined by solving

$$v_\parallel \mathbf{b}\cdot\nabla g + C_e^{(1)}\left\{g\right\} = -\frac{f_S}{Bv_\parallel}v_\parallel \mathbf{b}\cdot\nabla(v_\parallel B). \tag{48}$$

Letting $C_e^{(1)}\left\{g\right\} \to -\nu g$, we find the plateau solution

$$g \approx \epsilon\frac{eE_I D(x_e)}{\nu_e T_e} f_{Me}\frac{1}{2v}(v^2 - 3v_\parallel^2)\left[\pi\delta\left(\frac{v_\parallel}{v}\right)\sin\theta + \frac{v\cos\theta}{v_\parallel}\right], \tag{49}$$

Substituting the full G into Eq. (47), evaluating the velocity integrals and performing the flux surface average, we obtain

$$\langle j_{BS}B\rangle = -\sqrt{\frac{\pi}{2}}\frac{\epsilon^2 cIp_e v_e}{\nu_e qR_0}\frac{\sqrt{2}+4Z}{Z(2+\sqrt{2}Z)}\left\{\frac{1}{p_e}\frac{\partial p}{\partial\Psi} + \frac{\sqrt{2}+13Z}{2\left(\sqrt{2}+4Z\right)}\frac{1}{T_e}\frac{\partial T_e}{\partial\Psi} + \frac{J(U^2)}{2ZT_e}\frac{\partial T_i}{\partial\Psi}\right\} \tag{50}$$

where $p = p_e + p_i$ and we ignore $\epsilon^2 v_e/(qR\nu_e) \ll 1$ corrections to the Spitzer current. In calculating the bootstrap current we have kept only two terms in the Laguerre polynomial expansion. It is shown in the Appendix B that keeping more terms or using a different expansion – as in Ref. [18] and [19] – would not give significant improvement to the preceding result.

IV. CONCLUSIONS

In subsonic tokamak pedestals having $B_{\text{pol}} \ll B$, due to the strong (~ 100 kV/m) radial electric field, the contribution of the $\mathbf{E}\times\mathbf{B}$ drift to the poloidal ion motion can be comparable to that of the parallel streaming. Thus, these contributions have to be kept in the ion kinetic equation in the same order. In the preceding sections the modifications of the neoclassical plateau regime transport are evaluated, and expressed in terms of the normalized electric field $U = v_{\mathbf{E}\times\mathbf{B}}B/(v_i B_{\text{pol}})$ which is allowed to be order unity.

Orbit squeezing effects are also taken into account but – unlike in the banana regime, – the plateau transport is found to be unaffected by them. Although the strong electric field has practically no effect on the electron orbits due to their large thermal speed, the electron dynamics is indirectly affected, since the electron-ion collisions depend on the ion mean flow. The bootstrap current is evaluated by an adjoint method that relies on the knowledge of the Spitzer function which we approximate with a truncated Laguerre polynomial series. We find that to it is unnecessary to keep more than two terms in this expansion to get reasonably accurate results. Of course, all of our results are consistent with conventional $U = 0$ plateau regime calculations.

As the electric field increases, the resonance causing the plateau transport, which would be at $v_\| \approx 0$ for $U = 0$, is now shifted towards the tail of the distribution. For strong electric field this leads to an exponential reduction of the ion heat flux for $U \gg 1$. However, for moderate values of U the ion heat diffusivity is enhanced [$L(|U| \approx 0.91) \approx 1.46$]. The shift of the resonance sets the upper limit of validity of the calculations presented at about $U = 3.5$ (we note that these higher normalized electric fields are unlikely in experiments).

The temperature gradient driven part of parallel ion flow – corresponding to the poloidal ion rotation – is multiplied by a U-dependent factor $J(|U|)$ that decreases until $J(|U| \approx 0.76) \approx 0.39$ then it starts to increase approaching an asymptote of $2U^2 - 3$. The same factor appears also in the expressions for the poloidal impurity rotation and the bootstrap current multiplying the ion temperature gradient term. It modifies these quantities for experimentally relevant values of the radial electric field. In particular, finite-U modifications might account for the discrepancy found in recent experimental comparison to neoclassical predictions [10]. However, extremely precise measurements are required for quantitative comparisons in the plateau regime because of the flatness of the $J(|U|)$ curve below $|U| \sim 1.5$.

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APPENDIX A: ELECTRON TRANSPORT

The ion flow modified by the finite poloidal Larmor radius effects appears in the drift kinetic equation (33) through the electron-ion collision operator (35). Accordingly, we expect modifications in the electron transport. Starting with the kinetic equation for H_e [Eqs. (37)-(40)], after the replacement $C_e^{(1)}(H_e) \to -\nu H_e$ (noting that the electron collision operator is not momentum conserving) we find that the equation to solve is

$$v\xi\partial_\theta H_e + \nu qRH_e = Q_e \sin\theta, \tag{A1}$$

where we introduce $\xi = v_\parallel/v$ and

$$\begin{aligned} Q_e =& \frac{\epsilon f_{Me} v^2}{T_e}\left\{\frac{I\left(1+\xi^2\right)}{2\Omega_e}\left[\frac{1}{n_e}\frac{\partial p}{\partial\Psi} + \left(\frac{mv^2}{2T_e} - \frac{5}{2}\right)\frac{\partial T_e}{\partial\Psi}\right]\right. \\ &\left. + \frac{B\left(1-3\xi^2\right)}{2}\left[\frac{J(U^2)}{2Z}\frac{IB}{\Omega_e\langle B^2\rangle}\frac{\partial T_i}{\partial\Psi} - \frac{eE_I}{\nu_e B}\sqrt{2}D(x_e)\right]\right\}, \end{aligned} \tag{A2}$$

and we use quasineutrality $n_e = Zn_i$. We obtain the plateau solution

$$H_e \approx \frac{Q_e}{v}\left(\pi\delta(\xi)\sin\theta - \frac{\cos\theta}{\xi}\right). \tag{A3}$$

The particle transport is calculated from the electron version of (23) to find

$$\begin{aligned} \langle \boldsymbol{\Gamma}_e\cdot\nabla\Psi\rangle \approx& -\left\langle\frac{I\epsilon}{2\Omega_e qR}\int d^3v v^2(1+\xi^2)H_e\sin\theta\right\rangle = -\sqrt{\frac{\pi}{2}}\frac{I^2\epsilon^2 n_e}{\Omega_e^2 qR_0}\left(\frac{T_e}{m_e}\right)^{3/2} \\ &\times\left\{\frac{1}{p_e}\frac{\partial p}{\partial\Psi} + \frac{1}{2}\frac{\partial\ln T_e}{\partial\Psi} + \frac{J(U^2)T_i}{2ZT_e}\frac{\partial\ln T_i}{\partial\Psi} - \frac{eE_I\Omega_e}{IT_e\nu_e}\sqrt{2}\left(a_0 - \frac{a_1}{2}\right)\right\}, \end{aligned} \tag{A4}$$

This expression reduces to the usual plateau result [3] in the $U\to 0$ and $E_I\to 0$ limits.

Similarly, the electron heat flux is

$$\begin{aligned} \langle \mathbf{q}_e\cdot\nabla\Psi\rangle \approx& -\left\langle\frac{I\epsilon}{2\Omega qR}\int d^3v\frac{m_e v^4}{2}(1+\xi^2)H_e\sin\theta\right\rangle - \frac{5T_e}{2}\langle\boldsymbol{\Gamma}_e\cdot\nabla\Psi\rangle \\ =& -\sqrt{\frac{\pi}{8}}\frac{I^2\epsilon^2 p_e}{\Omega_e^2 qR_0}\left(\frac{T_e}{m_e}\right)^{3/2}\left\{\frac{1}{p_e}\frac{\partial p}{\partial\Psi} + \frac{13}{2}\frac{\partial\ln T_e}{\partial\Psi} + \frac{J(U^2)T_i}{2ZT_e}\frac{\partial\ln T_i}{\partial\Psi}\right. \\ &\left. - \frac{eE_I\Omega_e}{IT_e\nu_e}\sqrt{2}\left(a_0 - \frac{13a_1}{2}\right)\right\}. \end{aligned} \tag{A5}$$

APPENDIX B: SPITZER FUNCTION

Considering the Ansatz given in Eq. (41) for the Spitzer problem defined by Eq. (36), the problem to solve is

$$C_e^{(1)}\left\{-\frac{v_\parallel}{\nu_e}D(x_e)f_{Me}\right\} = v_\parallel f_{Me}, \tag{B1}$$

where

$$C_e^{(1)}\{\delta f\} = C_{ee}^{(1)}\{\delta f\} + \mathcal{L}_{ei}\{\delta f\} \tag{B2}$$

is the linearized electron collision operator with $C_{ee}^{(1)}\{\delta f\} = C_{ee}\{\delta f, f_{Me}\} + C_{ee}\{f_{Me},\delta f\}$, C_{ee} is the full Fokker-Planck operator [3], and $\mathcal{L}_{ei}\{\delta f\} = (3\sqrt{2\pi}/4)\nu_e Z(T_e/m_e)^{3/2}\nabla_v\cdot[\nabla_v\nabla_v v\cdot\nabla_v(\delta f)]$ is the Lorentz operator.

An approximate solution to (A1) can be constructed variationally by maximizing the functional

$$\Lambda = \frac{\nu_e m_e}{2p_e}\left[\int d^3v\,\eta C_e^{(1)}\left\{\eta f_{Me}\right\} - 2\int d^3v\,\eta v_\parallel f_{Me}\right], \tag{B3}$$

where $\eta = -\frac{v_\parallel}{\nu_e}D(x_e)$ and we approximate the x_e dependent part of η by a truncated generalized Laguerre polynomial series expansion

$$D(x_e) \approx \sqrt{2}\left(a_0 L_0^{(3/2)}(x_e^2) + a_1 L_1^{(3/2)}(x_e^2) + a_2 L_2^{(3/2)}(x_e^2) + \ldots\right). \tag{B4}$$

By direct substitution of Eq. (B4) into Eq. (B3), using the orthogonality of the Laguerre polynomials $\int_0^\infty d\zeta e^{-\zeta}\zeta^\alpha L_j^{(\alpha)}(\zeta)L_k^{(\alpha)}(\zeta) = \delta_{jk}\Gamma(k+\alpha+1)/k!$, we obtain

$$\Lambda = -2\sum_{j,k} a_j a_k\left(\frac{M_{jk}^e}{\sqrt{2}} + ZM_{jk}^i\right) + \sqrt{2}a_0, \tag{B5}$$

where we introduce the matrix elements [13]

$$M_{jk}^e = -\frac{\sqrt{2}}{\nu_e n_e}\int d^3v x_{\parallel e} L_j^{(3/2)}(x_e^2) C_{ee}^{(1)}\left\{x_{\parallel e} L_k^{(3/2)} f_{Me}\right\} = \begin{pmatrix} 0 & 0 & 0 & \\ 0 & 1 & \frac{3}{4} & \cdots \\ 0 & \frac{3}{4} & \frac{45}{16} & \\ & \vdots & & \ddots \end{pmatrix}, \tag{B6}$$

and

$$M_{jk}^i = -\frac{1}{\nu_e n_e}\int d^3v x_{\parallel e} L_j^{(3/2)}(x_e^2)\mathcal{L}_{ei}\left\{x_{\parallel e} L_k^{(3/2)} f_{Me}\right\} = \begin{pmatrix} \frac{1}{2} & \frac{3}{4} & \frac{15}{16} & \\ \frac{3}{4} & \frac{13}{8} & \frac{69}{32} & \cdots \\ \frac{15}{16} & \frac{69}{32} & \frac{433}{128} & \\ & \vdots & & \ddots \end{pmatrix}, \tag{B7}$$

where $j,k = 0,1,2,\ldots$. Keeping the first three terms in the expansion in Eq. (B3), maximization of Λ by using the solution of the equations $\partial\Lambda/\partial a_j = 0$, gives

$$\begin{aligned} a_0 &= \left(576 + 1028\sqrt{2}Z + 434Z^2\right)/\left(8\sqrt{2}Z\gamma\right), \\ a_1 &= -3\left(30 + 11\sqrt{2}Z\right)/\left(2\gamma\right), \\ a_2 &= -3\left(4 - \sqrt{2}Z\right)/\gamma, \\ \gamma &= 72 + 61\sqrt{2}Z + 16Z^2, \end{aligned} \tag{B8}$$

which is numerically equal to the result of Ref. [18]. If we neglect the third term in (53) as well, the results are simply

$$\begin{aligned} a_0 &= \left(8 + 13\sqrt{2}Z\right)/\left[4Z\left(2\sqrt{2} + 2Z\right)\right], \\ a_1 &= -3/\left[2\left(2 + \sqrt{2}Z\right)\right]. \end{aligned} \tag{B9}$$

Denoting the approximate Spitzer function keeping n polynomials in the Laguerre polynomial expansion by $f_S^{[n]}$, we can estimate the error between $f_S^{[2]}$ and $f_S^{[3]}$ for low Z by comparing some of their moments to those of $f_S^{[4]}$. For $Z = 1$ the relative error of a_0 (i.e. the error of the ohmic current) is around 1.5 % for $f_S^{[2]}$ and 0.7 % for $f_S^{[3]}$. The error of the pressure and ion temperature gradient parts of the bootstrap current [$\sim \int dx_e x_e^5 f_S(x_e)$] is less than 0.6 % for $f_S^{[3]}$ ($Z = 1$) and happens to be even smaller for $f_S^{[2]}$.

In the $Z \to \infty$ limit, electron-electron collisions can be neglected compared to electron-ion collisions, and the Spitzer problem can be solved exactly. The result has the form of Eq. (41) with $D(x_e) = 4x_e^3/(3\sqrt{\pi}Z)$. In the high Z limit the relative error of the ohmic current remains below 4.5 % for $f_S^{[2]}$, while it is less than 0.15 % for $f_S^{[3]}$. The bootstrap current for $f_S^{[3]}$ has the same asymptotic limit as the exact solution, however the form of the Spitzer function is different. The relative error for $f_S^{[2]}$ remains below 9 %. The error introduced by the uncertainty of the Coulomb logarithm (that is the ultimate error of any Fokker-Planck theory) is usually higher than the error of the two-polynomial approximation (B9), so it seems unnecessary to keep three polynomials in the variational calculation of f_s for the plateau regime.

The Laguerre polynomial expansion contain only even powers of x_e. Thus they cannot reproduce the $\propto x_e^3$ behavior expected in the high Z limit. For this reason in Ref. [19] a simple 4$^{\text{th}}$ order polynomial approximation is constructed. Although this polynomial is correct asymptotically as $Z \to \infty$, the resulting Spitzer function is not smooth at $x_e = 0$ due to the non-vanishing linear term in the polynomial, as can be seen in Fig. 3. The figure shows four approximations of the Spitzer function, and the integrand of their 5$^{\text{th}}$ moment that appears in the bootstrap current calculation. The difference between the approximations of f_S is conspicuous for low values of the normalized velocity, however the difference between the low-order (< 10) moments of f_S – which have a physical relevance – is negligibly small.

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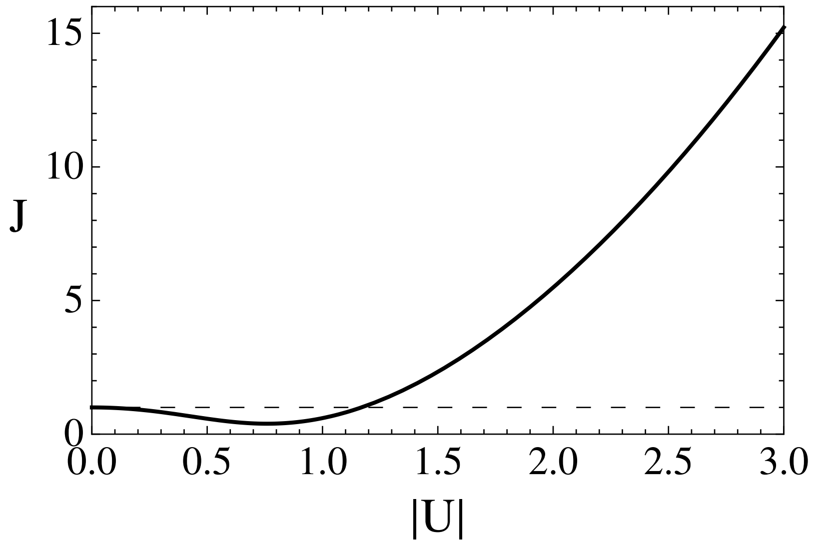

FIG. 1: The $J(|U|)$ factor multiplying the ion temperature gradient term in the ion flow.

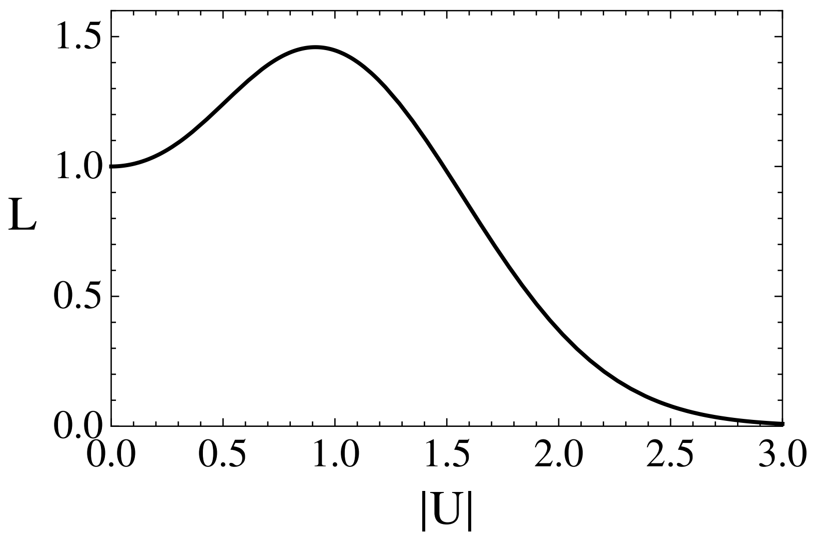

FIG. 2: The $L(|U|)$ factor multiplying the plateau ion heat diffusivity.

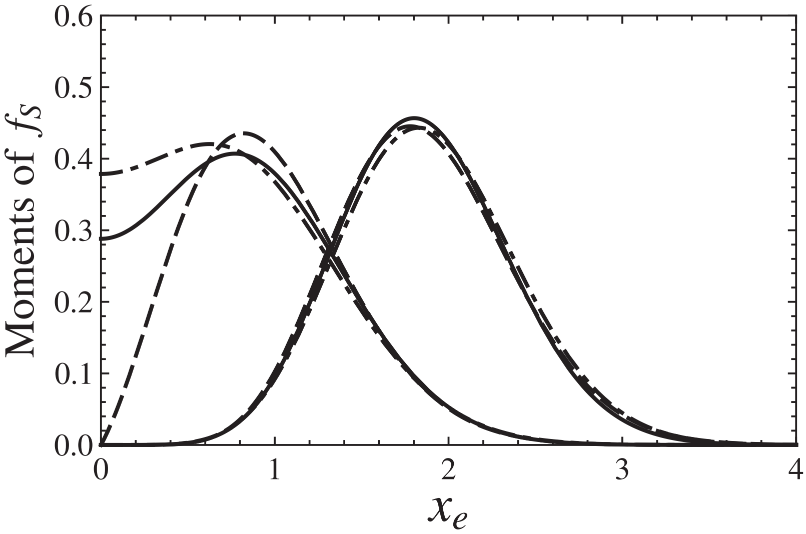

FIG. 3: Different approximations of the Spitzer function for $Z = 1$. The curves with the maximum below $x_e = 1$ represent $e^{-x_e^2} D(x_2)$, the ones centered around $x_e = 2$ are $(x_e^5/4)e^{-x_e^2} D(x_2)$. Solid – three Laguerre polynomials, Eq. (B8) and Ref. [18] (not plotted separately since they would be indistinguishable). Dash-dotted – two Laguerre polynomials, Eq. (B9). Dashed – 4^{th} order polynomial, Ref. [19].