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# Calculation of a Hyperbolic Corrugated Horn Converting the $TEM_{00}$ Mode to the $HE_{11}$ Mode

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# Calculation of a Hyperbolic Corrugated Horn Converting the $TEM_{00}$ Mode to the $HE_{11}$ Mode

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**Abstract**

Corrugated waveguide transmission lines are in use to transmit high power mm-wave radiation from gyrotrons to the plasma for electron cyclotron plasma heating in tokamaks such as ITER. The coupling efficiency of the gyrotron output radiation formed as a quasi-Gaussian beam to the waveguide mode is a critical issue. A hyperbolic corrugated horn serves as a converter of the $TEM_{00}$ Gaussian mode to the $HE_{11}$ mode of a corrugated waveguide. We report the design of a hyperbolic horn for application in the ITER transmission line at 170 GHz. The theoretical conversion efficiency of the horn is higher than 0.995.

## I. Introduction

Oversized circular corrugated waveguides are used for low loss transmission of high power mm-waves in electron cyclotron heating (ECH) of fusion plasmas [1]–[3]. A complex of 24 corrugated waveguide transmission lines (TL) is under development for ECH in ITER. Each line will be rated to transmit up to 2 MW of continuous wave (CW) power, generated by gyrotrons, to the tokamak in the mm-wave range (at the frequency of 170 GHz) [4]–[7]. The gyrotron is a high power vacuum electron device producing coherent radiation from an electron beam gyrating in a dc magnetic field. High power mm-waves are generated in the cylindrical cavity of a gyrotron in a very high order cavity mode such as the $TE_{31,8,1}$ mode [8] and then converted into a Gaussian-like quasioptical beam by an internal mode converter. The gyrotron internal mode converter consists of a helical launcher antenna and three or four quasioptical mirrors (Fig. 1) with the final mirror directing the linearly-polarized output beam through a diamond window. A mirror optics unit (MOU) [9] that contains two quasioptical mirrors (Fig. 1) is connected to the gyrotron window and converts the gyrotron output beam to a Gaussian beam ($TEM_{00}$ mode) with the optimal parameters (the beam waist radius and phase front curvature radius) for coupling into the corrugated waveguide. For transmission at 170 GHz at ITER, a corrugated waveguide with a diameter of 63.5 mm has been specified.

The MOU couples the Gaussian beam to the aperture of the corrugated waveguide. In Sec. II, we show that the efficiency of aperture coupling is 0.98, and that the loss is mostly due to Gaussian beam truncation by the aperture. The coupling loss can be reduced by using a matching horn (Fig. 1). A smooth-wall matching horn of a parabolic shape was proposed in Ref. [10], while a corrugated horn of parabolic shape was designed in Ref. [11], [12]. Optimized smooth-wall and corrugated horns are calculated in Ref. [13].

The corrugated horn described in this paper differs from the corrugated horn launchers for two-mirror antennas [14], which aim to reduce sidelobes and increase the effective area of the antenna. In contrast, we design a horn that is highly overmoded and quasioptical. It converts the $HE_{11}$ mode in an overmoded waveguide to the $TEM_{00}$ Gaussian beam with an almost flat phase front. Mode converting corrugated horns have been previously described in Ref. [15]. Those horns were calculated using the coupled mode equations [16]. The design consists of two horns that are optically connected by a matching mirror, together forming a low diffraction loss miter bend. This work differs in the application to matching from free space to a corrugated waveguide, rather than matching within a waveguide at a miter bend.

In this paper, we design a corrugated horn of hyperbolic shape to match the Gaussian output beam of the MOU to the corrugated waveguide TL. As we show in Sec. VII, the hyperbolic horn design can be simplified. It can be designed analytically because the modes of the hyperbolic horn can be found in separated variables. Also, the hyperbolic horn can be designed for use with the existing MOU. The MOU is designed to match the gyrotron output Gaussian beam to the open end of a corrugated waveguide. The matching hyperbolic horn can be designed without changing this MOU, that is, the matching horn would use the same mirrors and mirror positions as in the design of the MOU for aperture coupling. The horn could be connected to the open end of the corrugated waveguide, which would only require that the open end of the waveguide be moved back a certain distance (Fig. 1).

## II. Coupling to Corrugated Waveguide

For low loss transmission, the desired operating mode of corrugated waveguide is the $HE_{11}$ mode [1]. It is a linearly polarized mode in the waveguide which has a corrugation depth of a quarter-wavelength. The $HE_{11}$ mode is a Gaussian-like mode since it closely matches the free space Gaussian beam, $TEM_{00}$. The coupling efficiency, $\eta_0$, of the $TEM_{00}$ mode, with electric field radial amplitude distribution

$$A_G = \exp(-r^2/W^2) \tag{1}$$

and a flat phase front, to the $HE_{11}$ mode [1], with electric field radial amplitude distribution

$$A_0 = J_0(\nu_{11} r/a) \tag{2}$$

(where $a$ is the waveguide radius and $\nu_{11}$=2.405 is the zero of Bessel function $J_0$), is the following:

$$\eta_0 = \frac{(\int_0^a A_G A_0 r dr)^2}{\int_0^a A_0^2 r dr \int_0^\infty A_G^2 r dr} \tag{3}$$

The coupling efficiency $\eta_0$ is a function of the Gaussian beam width $W$, where the optimum width, $W$=0.64$a$, gives $\eta_0$=0.980 [1]. The 2% loss in coupling to the $HE_{11}$ mode comes from two effects: the truncation of the Gaussian beam by the waveguide aperture and a mismatch of the amplitudes in Eqs. (1) and (2). The coupling loss in practice may be larger than 2% due to imperfections in the Gaussian beam, tilts or offsets of the beam, or ellipticity in the shape of the beam when the widths along each transverse axis are not equal [17].

For 1 MW CW operation, even the 2 % loss in coupling to the $HE_{11}$ mode is significant in terms of its effect on gyrotron operation and due to the strict requirement on total losses in the TL between the gyrotron and the plasma [7]. It is therefore worth utilizing an element to improve mode conversion between the $TEM_{00}$ mode and the $HE_{11}$ mode. We descibe a hyperbolic horn that can significantly reduce this 2% loss.

## III. Hyperbolic Horn

A corrugated horn (with the same quarter-wavelength corrugation depth as the corrugated waveguide) that smoothly expands as a function $r = \overline{r}(z)$ of the axial coordinate $z$ (Fig. 2) can be used for conversion of the $TEM_{00}$ mode generated by the MOU to the $HE_{11}$ mode of the TL [11], [12]. Conversely, the $HE_{11}$ mode radiated from the TL by use of this horn is converted to the $TEM_{00}$ Gaussian beam in the opposite direction.

In this paper, we consider a hyperbolic horn:

$$\overline{r}(z) = a\sqrt{1 + (z/z_0)^2}. \tag{4}$$

Other horn expansion functions $\overline{r}(z)$ can be calculated and optimized numerically. In Ref. [18], a design for the conversion of the TEM mode of a planar waveguide to a Gaussian beam used a function $\overline{r}(z)$ expressed as a square root of a fourth order polynomial as opposed to the second order polynomial (Eq. (4)). In general, for conversion of a waveguide mode to a free space beam, the horn has to expand slightly faster than the beam expands while radiating from the open end of waveguide [18]. In Ref. [10], [11], a parabolic function $\overline{r}(z) = a[1 + (z/z_0)^2]$ was used. With the hyperbolic horn, the expansion of the horn walls is very similar to the expansion of the Gaussian beam [19], therefore, the beam does not completely decouple from the horn wall. As our simulations show, the efficiency of conversion in a hyperbolic horn can be sufficiently high with a major advantage being that the design of a hyperbolic horn is simplified because the boundary problem for the wave equation can be solved in separated variables.

## IV. Parabolic Equation and Lens Transformation

The linearly polarized field in an axially symmetrical horn is expressed within the paraxial approximation as $A(r, z) \exp(-ikz)$ where $k$ is the wavenumber and $A$ is a complex slowly varying amplitude that satisfies the parabolic equation

$$-2ik\frac{\partial A}{\partial z} + \frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r} = 0. \tag{5}$$

At the horn boundary, where $r = \overline{r}(z)$ is an envelope of the corrugated surface, the boundary condition $A$=0 is imposed.

To simplify the analysis, we use the coordinate and field transformation that allows one to convert the boundary problem with curvilinear boundaries to the boundary problem with rectilinear boundaries. The lens transformation, sometimes called Talanov's transformation, has been widely used in self-focusing theory [20] and waveguide quasioptics [18]. This transformation of coodinates from $(r, z)$ to $(\rho, \tau)$ is defined by:

$$r = \frac{\rho}{\sigma(\tau)}a \tag{6}$$

$$\frac{dz}{d\tau} = \frac{ka^2}{\sigma^2(\tau)} \tag{7}$$

such that

$$\overline{r}(z) = \frac{a}{\sigma(\tau)} \tag{8}$$

and

$$\sigma'(\tau) = -ka\frac{d\overline{r}}{dz} \tag{9}$$

allows one to transform the problem with the boundary $r = \overline{r}(z)$ (Fig. 2) to the problem with the linear boundary $\rho = 1$ (Fig. 3).

Using the transformation of field amplitude

$$A = \sigma(\tau)\widetilde{A}(\rho,\tau)\exp\left(i\frac{1}{2}\rho^2\frac{\sigma'}{\sigma}\right) \tag{10}$$

we convert the parabolic equation of Eq. (5) to the following parabolic equation

$$-2i\frac{\partial\widetilde{A}}{\partial\tau} + \frac{\partial^2\widetilde{A}}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\widetilde{A}}{\partial\rho} - \chi(\tau)\rho^2\widetilde{A} = 0 \tag{11}$$

where

$$\chi(\tau) = -\frac{\sigma''(\tau)}{\sigma(\tau)} = k^2\overline{r}^3(z)\frac{d^2\overline{r}}{dz^2} \tag{12}$$

Eq. (11) is the parabolic equation for a lens-like medium with the dielectric constant

$$\epsilon(\rho,\tau) = 1 - \chi(\tau)\rho^2 \tag{13}$$

The lens-like medium is focusing when $\chi(\tau) > 0$.

For the hyperbolic horn Eq. (4)

$$\chi = \alpha^2 = (ka^2/z_0)^2, \tag{14}$$

therefore the dielectric constant $\epsilon$ does not depend on $\tau$. This allows us to solve the problem using separated variables.

We show further in Sec. V and VI that the solutions of Eqs. (11) and (12) can be found to describe the modes of a hyperbolic horn and the beams in free space.

## V. Modes of a Hyperbolic Horn

The solution of the parabolic equation Eq. (11) with the boundary condition $\widetilde{A}(\rho = 1) = 0$ is the following

$$\widetilde{A}(\rho,\tau) = \frac{1}{\rho}M\left(\frac{1}{4}\frac{\xi^2}{\alpha}, 0, \alpha\rho^2\right)\exp\left(i\frac{1}{2}\xi^2\tau\right) \tag{15}$$

where $M$ is the Whittaker function, also known as the function of paraboloid of revolution [21]. The constants $\xi = \xi_n$, where $n$=1,2,.., are zeros of the Whittaker function.

$$M\left(\frac{1}{4}\frac{\xi_n^2}{\alpha}, 0, \alpha\right) = 0 \tag{16}$$

They determine the modes of the waveguide filled with the lens-like medium (Fig. 3).

Going back to the $(r,z)$ coordinates we derive then from Eq. (15) using the lens transformation Eq. (10):

$$A_n(r,z) = \frac{a}{r}M\left(\frac{1}{4}\frac{\xi_n^2 z_0}{ka^2}, 0, \frac{ka^2r^2}{z_0\overline{r}^2(z)}\right)\times$$
$$\exp\left(-i\frac{1}{2}\frac{ka^2z}{\overline{r}^2(z)z_0^2}r^2 + i\frac{1}{2}\xi_n^2\frac{z_0}{ka^2}\arctan\frac{z}{z_0}\right) \tag{17}$$

Eq. (17) is useful as a representation of hyperbolic horn modes. The modes of the lens-like medium filled waveguide Eq. (15) are useful for designing the mode converter (Sec. VII).

## VI. Laguerre-Gaussian Beams

The lens transformation is also a method of deriving the expressions for free space beams. The parabolic equation of the lens-like medium Eq. (11) has a solution representing Laguerre-Gaussian beams if the radial coordinate is infinite ($0 < \rho < \infty$):

$$\widetilde{A}_{LG}^{(m)}(\rho,\tau) = \frac{1}{m!}L_m^{(0)}\left(\alpha\rho^2\right)\exp\left(-\frac{1}{2}\alpha\rho^2\right)\times$$
$$\exp\left(i\alpha(2m+1)\tau\right) \tag{18}$$

where $L_m^{(0)}$ ($m$=0,1,2,..) are the Laguerre polynomials. The lowest order beam is a Gaussian beam.

The Laguerre-Gaussian beams in free space can then be derived from Eq. (18) by introducing $W_0 = (2z_0/k)^{\frac{1}{2}}$:

$$A_{LG}^{(m)}(r,z) = \frac{1}{W(z)}\frac{1}{m!}L_m^{(0)}\left(\frac{2r^2}{W^2(z)}\right)\exp\left(-\frac{r^2}{W^2(z)}\right)\times$$
$$\exp\left(-i\frac{1}{2}\frac{kr^2}{R_c(z)}\right)\exp\left(i(2m+1)\arctan\frac{2z}{kW_0^2}\right) \tag{19}$$

where

$$W(z) = W_0 \sqrt{1 + \left( \frac{2z}{kW_0^2} \right)^2} \tag{20}$$

is the beam width, and

$$R_c(z) = z \left[ 1 + \left( \frac{kW_0^2}{2z} \right)^2 \right] \tag{21}$$

is the curvature radius the beam phase front.

The hyperbolic horn mode expression Eq. (17) can be reduced to the Laguerre-Gaussian beam expression Eq. (19) for the large parameter $\alpha$ when the asymptotic representation of Whittaker function is used [21].

## VII. Two-Mode Approach

In this Section we use the two-mode approach to design the hyperbolic horn. The $HE_{11}$ mode amplitude $\widetilde{A}_0 = J_0(\nu_{11}\rho)$ at the entrance $\tau = 0$ of the waveguide filled with lens-like medium (Fig. 3) can be represented as a superposition of the two waveguide modes

$$\widetilde{A}_{in} = C_1 B_1(\rho) + C_2 B_2(\rho) \tag{22}$$

where

$$B_{1,2}(\rho) = \frac{1}{\rho} M \left( \frac{1}{4} \frac{\xi_{1,2}^2}{\alpha}, 0, \alpha\rho^2 \right) \tag{23}$$

and $C_{1,2}$ are real numbers. At the output $\tau = T$, the field distribution is expressed as

$$\widetilde{A}_{out}(\rho) = C_1 \exp\left( i\frac{1}{2}\xi_1^2 T \right) B_1(\rho) + C_2 \exp\left( i\frac{1}{2}\xi_2^2 T \right) B_2(\rho) \tag{24}$$

which has to be close to the Gaussian distribution

$$\widetilde{A}_G(\rho) = \exp\left( -\frac{\rho^2}{\widetilde{W}^2} \right) \tag{25}$$

The design is simplified when the length $T$ of lens-like medium waveguide is equal to half of the beat wavelength between the modes:

$$T = \frac{2\pi}{\xi_2^2 - \xi_1^2} \tag{26}$$

We now consider an example of hyperbolic horn design. The optimum Gaussian beam radius is $\widetilde{W}$=0.64 for coupling to the $HE_{11}$ mode at the open end of the waveguide (Sec. II). We will design a hyperbolic horn to match the $HE_{11}$ mode at the input to the Gaussian beam with $\widetilde{W}$=0.58 at the open end of the horn. The parameter $\chi = \alpha^2 = 12.5$ is determined to approximately match the function $B_1(\rho)$ (Eq. (23)) with both functions $\widetilde{A}_0(\rho)$ and $\widetilde{A}_G(\rho)$ (Eq. (25)). For this, $\alpha$=3.535 and the zeros of the Whittaker function are $\xi_1$=2.885 and $\xi_2$=5.866, therefore, $T$=0.24 (Eq. (26)).

The efficiency of conversion is determined as $\eta = \eta_{in}\eta_{out}$ where

$$\eta_{in} = \frac{\left( \int_0^1 \widetilde{A}_{in} \widetilde{A}_0 \rho d\rho \right)^2}{\int_0^1 \widetilde{A}_{in}^2 \rho d\rho \int_0^1 \widetilde{A}_0^2 \rho d\rho} \tag{27}$$

$$\eta_{out} = \frac{\left( \int_0^1 \widetilde{A}_{out} \widetilde{A}_G \rho d\rho \right)^2}{\int_0^1 \widetilde{A}_{out}^2 \rho d\rho \int_0^\infty \widetilde{A}_G^2 \rho d\rho} \tag{28}$$

The calculation result is $\eta$=0.993.

Now we determine the parameters of the hyperbolic horn for the operating frequency of 170 GHz and the corrugated waveguide radius of $a$=31.75 mm, where $ka^2$=3.6 m. The parameter $z_0 = ka^2/\alpha$=0.28$ka^2$=1.02 m. The horn length $L$ is determined using the equation for $T$

$$T = \int_0^L \frac{dz}{k\overline{r}^2(z)} = \frac{z_0}{ka^2} \arctan \frac{L}{z_0} \tag{29}$$

therefore, $L = z_0 \tan(\alpha T)$=1.16 m. The aperture radius at the horn end is 48 mm. The Gaussian beam radius at the output is $W = \widetilde{W}\overline{r}(L)$=29 mm. The output Gaussian beam curvature radius can be determined using Eq. (10): $R_c = (z_0^2 + L^2)/L$=2.0 m.

A two-mode approach was used to achieve the results in this Section. It was assumed that the horn length is equal to the beat wavelength between the two modes (Eq. (26)). A hyperbolic horn of a shorter length can be designed with a very small reduction of conversion efficiency when Eq. (26) is not satisfied. The same two-mode approach can be used to design a shorter horn. However, we will use numerical integration of Eq. (11) to include higher order modes and, therefore, calculate with higher accuracy (Sec. VIII).

## VIII. Hyperbolic Horn Design

In this section, we present the design of a hyperbolic horn to utilize as a direct addition to the existing MOU design. It is assumed that the MOU is designed to couple the gyrotron output Gaussian beam to the $HE_{11}$ mode at the open end of the waveguide (shown in Fig. 1 by a dashed line) where the MOU should form a Gaussian beam with width $W_0 = 0.64a = 20.3$ mm at the open end (Sec. II). The corrugated waveguide should thus be cut at a certain length that will be calculated and the horn should be connected to the open end as shown in Fig. 1.

The parabolic equation Eq. (11) was calculated numerically with the initial condition $\widetilde{A} = J_0(\nu_{11}\rho)$ at $\tau = 0$ and the boundary condition $\widetilde{A} = 0$ at $\rho = 1$. The conversion efficiency

$$\eta = \frac{\left|\int_0^1 \widetilde{A}(\rho,\tau)\widetilde{A}_G^*\rho d\rho\right|^2}{\int_0^1 |\widetilde{A}(\rho,\tau)|^2\rho d\rho \int_0^\infty |\widetilde{A}_G|^2\rho d\rho} \tag{30}$$

was calculated at each $\tau$, where the complex Gaussian amplitude is expressed as

$$\widetilde{A}_G(\rho) = \exp\left(-\frac{\rho^2}{\widetilde{W}^2} - i\frac{1}{2}\beta\rho^2\right) \tag{31}$$

The parameters $\widetilde{W}$ and $\beta$ in Eq. (31) are optimized to maximize $\eta$ (Eq. (30)) and then converted using Eq. (10) to the Gaussian beam parameters: the beam width $W$ and phase front curvature radius $R_c$ (Eq. (19)).

Figure 4 plots the conversion efficiency $\eta$ as a function of axial coordinate $z$ for hyperbolic horns of different shapes determined by the parameter $z_0$. As shown in Fig. 4, to provide $\eta > 0.995$, the horn with the parameter $z_0$ ranging from $0.28ka^2$ to $0.22ka^2$ can be used.

The beam width $W$ and phase front curvature radius $R_c$ determined at the output of the horn were converted to the waist width $W_0$ and waist axial coordinate $z_w$ using Eqs. (20,21). For the fixed parameter $z_0$, the horn length was specified to provide the beam waist width $W_0 = 0.64a$ such that the beam matches the corrugated waveguide. Figure 5 shows the horn length and Gaussian beam waist position $z_w$ measured from the beginning of the horn as functions of the parameter $z_0$. The coordinate $z_w$ should be used to determine the position of the hyperbolic horn with respect to the open end of the waveguide in the existing MOU setup (Fig. 1). As shown in Fig. 5, for $z_0 = 0.28ka^2$, the horn length is $0.22ka^2$=0.79 m, which corresponds to the output aperture of 40.3 mm. The waist position $z_w = 0.12ka^2$=0.43 m. Therefore, the waveguide should be cut by 0.36 m, and a 0.79 m-long horn should be connected to it.

The tilt of the MOU output beam with respect to the waveguide or horn axis is a critical issue [17]. Since the horn aperture and the matching Gaussian beam width are larger, the beam has to be aligned with an accuracy higher than that for the waveguide coupling. The estimate of fractional loss due to tilt is $\Delta = \pi^2(W\theta/\lambda)^2$, where $\theta$ is the tilt angle and $\lambda$ is the wavelength. The beam width $W = 0.733a$ for the designed hyperbolic horn results in $\Delta = 5.3(a\theta/\lambda)^2$. From this estimate we conclude that the tilt angle $\theta$ should not exceed 0.1 deg. to provide a horn efficiency $\eta > 0.990$.

## IX. Conclusions

A hyperbolic corrugated horn can be employed to improve the coupling of the $TEM_{00}$ Gaussian beam to the $HE_{11}$ mode of corrugated waveguide. The theory of mode conversion in such a horn was developed, and a horn was designed for the ITER 170 GHz corrugated waveguide transmission line which has a 63.5 mm diameter. A horn of length 0.79 m could replace the open end of the waveguide in the existing MOU setup, providing a coupling efficiency of >99.5%, significantly better than the coupling of a Gaussian beam to an open-ended corrugated waveguide.

## X. Acknowledgements

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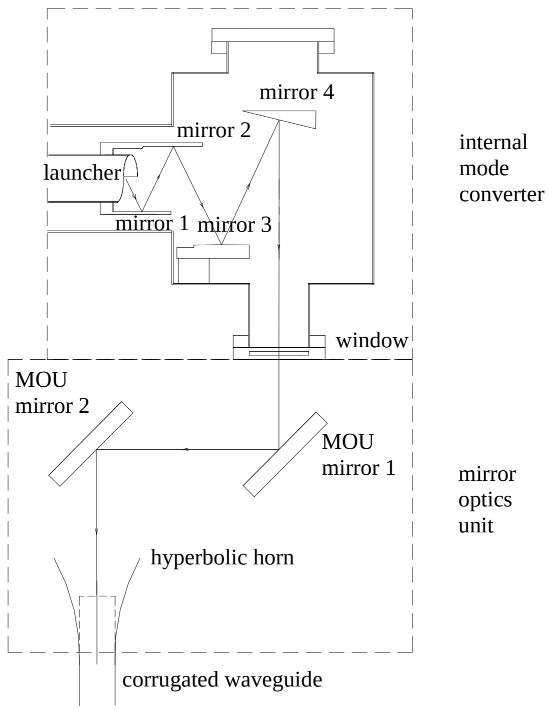


Fig. 1. Gyrotron internal mode converter and mirror optics unit (MOU) to couple the gyrotron output radiation into a corrugated waveguide. The hyperbolic horn converts the MOU output $TEM_{00}$ Gaussian beam to the $HE_{11}$ corrugated waveguide mode. The position of the corrugated waveguide in the absence of the hyperbolic horn is shown (dashed line).

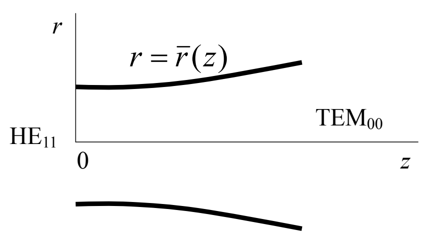


Fig. 2. Hyperbolic horn mode converter.

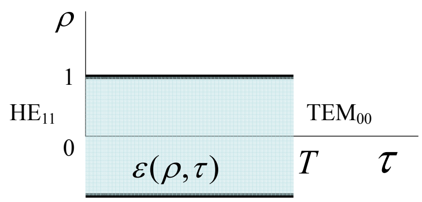


Fig. 3. Waveguide filled with lens-like medium.

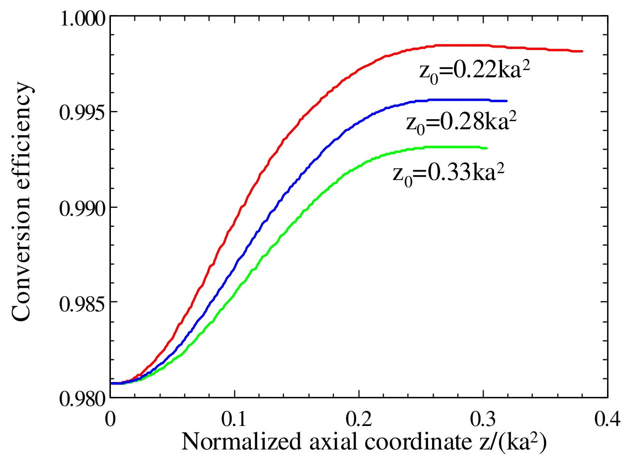


Fig. 4. Conversion efficiency as a function of axial coordinate for different horn shapes determined by the parameter $z_0$.

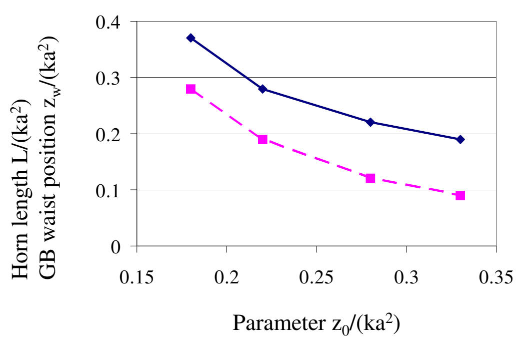


Fig. 5. Hyperbolic horn normalized length $L/ka^2$ (solid line) and Gaussian beam (GB) waist position $z_w/ka^2$ (dashed line) as functions of the horn shape parameter $z_0/ka^2$.