PSFC/JA-11-38

Neoclassical theory of pedestal flows and comparison with Alcator C-Mod measurements

Grigory Kagan, Kenneth D. Marr, Istvan Pusztai, Matt Landreman, Peter J. Catto and Bruce Lipschultz

December 2012

Plasma Science and Fusion Center
Massachusetts Institute of Technology
Cambridge MA 02139 USA

This work was supported by the U.S. Department of Energy, Grant No. DE-FG02-91ER-54109 .

To be submitted for publication in the Contributions to Plasma Physics (2012)

Neoclassical theory of pedestal flows and comparison with Alcator C-Mod measurements

Grigory Kagan, Kenneth D. Marr, Istvan Pusztai,
Matt Landreman, Peter J. Catto and Bruce Lipschultz

(Dated: December 17, 2011)

Abstract

Neoclassical implications of the strong radial electric field, inherently present in an H-mode tokamak pedestal, are considered. The main ion poloidal flow in the pedestal is predicted to be reduced in magnitude, or even reversed, compared with its core counterpart. The resulting change in the neoclassical formula for the impurity flow is shown to result in improved agreement with boron measurements in the Alcator C-Mod pedestal. In addition, due to the ion flow being modified, the bootstrap current is expected to be enhanced in the pedestal over conventional predictions.

I. INTRODUCTION

Conventional neoclassical theories [1] are essentially built upon the original ion orbit evaluation by Galeev and Sagdeev [2], which assumes that scale-length of background quantities such as the plasma density or electrostatic potential are much larger than the orbit width. For the main ion species, this assumption is found to break down in many pedestal experiments. The pedestal width is often comparable to the poloidal ion gyroradius and the variation of electrostatic energy over an ion orbit is then comparable to the ion's kinetic energy. As a result, the pedestal flow of main ions can be substantially different from its core counterpart.

The short background scale of the pedestal does not have a similar impact on individual motion of impurity ions and electrons. The former usually have a high charge number and collide before drifting a distance comparable to their poloidal gyroradius. The latter's poloidal gyroradius is much less than that of the ions and therefore the pedestal width. However, both electron and impurity ion species experience friction with background ions and therefore their flows are modified as well. The associated discrepancy for the poloidal impurity flow has been measured to be substantial for the banana regime high confinement (H) mode pedestal of Alcator C-Mod [3].

We evaluate main ion orbits accounting for the radial electric field inherently present in a subsonic H-mode tokamak pedestal. We are then able to carry out the kinetic calculation to find pedestal modifications to the conventional neoclassical prediction for the banana regime main ion flow [4]. This result, in turn, allows us to deduce the pedestal expressions for the poloidal impurity flow and the bootstrap current. We then proceed by comparing the revised formula for the impurity flow with the boron flow measured in banana regime C-Mod pedestals to find that agreement between the theory and experiment is noticeably improved upon accounting for the electric field effect on the main ion orbits [5]. This comparison verifies the role of the electric field in modifying the main ion flow and supports our conclusion that the bootstrap current is enhanced in a banana regime pedestal [6].

II. PEDESTAL BASICS

The key pedestal feature can be recovered with the help of the pressure balance equation for the main ion species [7]. That is, considering the radial component of this equation and employing standard expressions for the tokamak magnetic field $\vec{B} = I\nabla\zeta + \nabla\zeta \times \nabla\psi$ and the net ion flow $\vec{V_i} = \omega_i R^2 \nabla\zeta + K(\psi)\vec{B}$, where $K(\psi)$ is a free function and $2\pi\psi$ is the poloidal flux, one obtains the leading order relation

$$\omega_i = -c\frac{d\phi}{d\psi} - \frac{c}{en}\frac{dp_i}{d\psi} \approx -c\frac{d\phi}{d\psi} - \frac{cT_i}{Zen}\frac{dn}{d\psi}, \tag{1}$$

where ω_i denotes a toroidal angular velocity of background ions, ϕ is the electrostatic potential, n is the plasma density and $p_i = nT_i/Z$. Also, ζ stands for the toroidal angle and I is defined by the toroidal magnetic field B_t through $I \equiv B_t R$, where R is the major radius of the tokamak. The last term on the right side of Eq. (1) represents the ion pressure gradient with the ion temperature gradient dropped. It therefore relies on the fact that in a banana regime H-mode pedestal the main ion temperature scale is larger than the poloidal ion gyroradius scale-length of the density. This feature has been deduced mathematically in Ref. [7] by assuming that the entropy flux from the pedestal region is negligible. In more collisional pedestals or in the presence of a substantial entropy flux this equation can be challenged.

Next, utilizing the experimental observation that the characteristic radial density scale in the H-mode pedestal is on the order of the poloidal ion gyroradius, the last term on the right side of Eq. (1) is estimated to be comparable to v_i/R, where $v_i \equiv (2T_i/M_i)^{1/2}$ with T_i and M_i being the main ion temperature and mass, respectively. Thus, for the term on the left side of Eq. (1) to contribute the net ion velocity $\omega_i R$ must be sonic. For a subsonic pedestal, as is the case in many experiments, it must then be that the two terms on the right side of Eq. (1) cancel each other to leading order; i.e. the main ions are *electrostatically* confined.

With this result in hand it is straightforward to estimate the contribution of the $E \times B$ drift to the poloidal motion of an ion to find that it is comparable to that of the parallel velocity. In other words, the poloidal angular velocity of a pedestal ion is given by

$$\dot{\theta} = [v_{||} + cI\phi'/B]\vec{n} \cdot \nabla\theta, \tag{2}$$

where $\vec{n} \equiv \vec{B}/B$ and $\phi = \phi(\psi)$. In contrast to the conventional core case, the two terms in the square brackets on the right side of Eq. (2) are comparable. Importantly, as long as $B_p << B$, this feature does not require the $E \times B$ drift to be on order of the ion thermal speed v_i. That is, the $E \times B$ drift is much less than v_i still and able to compete with $v_{||}$ due to geometrical factors only, as illustrated in Figure 1 of Ref. [8]. Recalling that in the conventional neoclassical theory ion orbits are evaluated from $\dot{\theta} = v_{||}\vec{n} \cdot \nabla\theta$, we therefore conclude that this conventional theory is inapplicable in the pedestal.

III. ION ORBITS IN THE PRESENCE OF A STRONG RADIAL ELECTRIC FIELD

As deduced in the preceding section, to develop a neoclassical theory of a subsonic pedestal one must calculate ion drift orbits accounting for a strong radial electric field, inherently present in this tokamak region. To simplify the problem, we consider the case of concentric flux surfaces and assume the inverse aspect ratio ϵ to be small. Then, the magnetic field as a function of θ can be written in a relatively simple form

$$B(\theta) = B_0(1+\epsilon)/(1+\epsilon\cos\theta), \tag{3}$$

where $B_0 \equiv B(\theta = 0)$ and $\theta = 0$ is taken to be at the outer equatorial plane. Equally important, in the large aspect ratio case the ion can drift radially only by $\sqrt{\epsilon}\rho_{pi}$ that is much less than the characteristic scale of the electrostatic potential, ρ_{pi}. Therefore, variation of ϕ over the orbit is small and its Taylor expansion can be employed

$$\phi(\psi) \equiv \phi_* + (\psi - \psi_*)\phi'_* + (1/2)(\psi - \psi_*)^2\phi''_*, \tag{4}$$

where

$$\psi_* \equiv \psi + (M_i c/Ze)R^2\vec{v} \cdot \nabla\zeta \approx \psi - Iv_{||}/\Omega_i, \tag{5}$$

where Ω_i is the main ion cyclotron frequency. According to Eq. (5), ψ_* is proportional to the toroidal component of the canonical angular momentum and therefore conserved over a particle orbit. Due to Eq. (4) the problem with a realistic potential becomes essentially equivalent to that with a quadratic potential. Accordingly, in what follows we proceed as if ϕ is indeed quadratic in ψ.

Radial variations of B and I over the orbit are negligible and Eq. (2) transforms into

$$qR\dot{\theta} = v_{||} + u = S(v_{||} + u_*), \tag{6}$$

where q is the safety factor, $u \equiv cI\phi'/B$ and $u_* \equiv cI\phi'_*/SB$ with $S \equiv 1 + cI^2\phi''_*/\Omega_i B$ being the orbit squeezing factor. Then, using conservation of energy, canonical momentum and magnetic moment μ gives

$$v_{||} + u = (v_{||0} + u_0)\sqrt{1 - \kappa^2 \sin^2(\theta/2)}, \tag{7}$$

where

$$\kappa^2 \equiv 4\epsilon S \frac{(\mu B_0 + u_0)}{(v_{||0} + u_0)^2}. \tag{8}$$

In Eqs. (7) - (8) subscript "0" indicates that the quantity is to be evaluated at the outermost point of the particle trajectory, and under the square root we have neglected terms that are small in ϵ. At a banana tip, $\psi = \psi_* - Iu_*/\Omega_i$.

As seen from Eq. (7), $\kappa^2 < 1$ and $\kappa^2 > 1$ correspond to the passing and trapped regions of the velocity space, respectively. In contrast to the conventional case, where the trapped-passing boundary is a cone centered at the origin, Eq. (8) gives that in the pedestal the trapped-passing boundary is curved and shifted so that $v_{||} = 0$ is no longer the axis of symmetry. This change is solely due to the strong net electric field. As given by Eq. (8), the electric field shear, which manifests itself through the orbit squeezing, is not capable of qualitatively modifying the shape of the trapped-passing boundary. In the section to follow we discuss complications introduced into the kinetic part of calculation by the just mentioned change in the trapped-passing boundary and present a technique for handling them.

IV. PASSING CONSTRAINT FOR MAIN IONS IN THE PEDESTAL

The entropy production analysis demonstrates that in the pedestal the leading order ion distribution function is Maxwellian [7] f_M. Thus, it is possible to write the distribution function as in the conventional case [1]

$$f = f_M - \frac{Iv_{||}}{\Omega_i}\frac{\partial f_M}{\partial \psi} + g, \tag{9}$$

where the second and third terms on the right side represent the diamagnetic drive and neoclassical response, respectively. As in the conventional case, $g = 0$ for trapped particles, and the problem is to find g for passing particles. To do so, the distribution function (9) is inserted into the axisymmetric gyrokinetic equation [7] for ions and finite Larmor radius effects are neglected to obtain

$$\overline{C_{ii}\{g - h\}} = 0, \tag{10}$$

where C_{ii} is the ion-ion collision operator, overbar denotes the transit average and

$$h = \frac{Iv_{||}}{\Omega_i}\frac{M_i v^2}{2T_i^2}\frac{\partial T_i}{\partial \psi} \tag{11}$$

is the drive term. Written in this form, the passing constraint looks identical to the conventional one. However, in contrast to the core case, transit averaging in the pedestal must be performed at constant canonical angular momentum ψ_* rather than ψ, and the average must also account for the effect of the $E \times B$ drift on the poloidal ion velocity. Also, since the problem is no longer symmetric about $v_{||} = 0$ in the velocity space, the drive term as given by Eq. (11) is not convenient to employ and the pitch-angle scattering model for the collision operator does not capture all the relevant physics. The former issue is easily addressed by utilizing the number, momentum and energy conserving properties of the like particle collision operator to redefine h by

$$h = \frac{I(v_{||} + u)}{\Omega_i}\frac{M_i(v^2 + u^2)}{2T_i^2}\frac{\partial T_i}{\partial \psi}, \tag{12}$$

thereby modifying Eq. (10) accordingly. The latter issue requires much more care and is discussed next.

As demonstrated by conventional neoclassical calculations, it is the piece of the distribution function localized around the trapped-passing boundary that plays the key role. Practically, this is what allowed keeping only the pitch angle scattering component of the full ion-ion collision operator, while having the energy scattering component dropped. As pointed out in the preceding section, in the pedestal, the trapped-passing boundary is curved and shifted. Thus, an ion can be taken across it by scattering the energy, as well as by scattering the pitch angle, and a new model collision operator is needed. To develop this model, instead of the pitch angle and the kinetic energy employed in the conventional theory, we introduce the new variables

$$W \equiv \frac{(v_{||0}^2 + u_0^2)^2}{2S} + (\mu B_0^2 + u_0^2), \tag{13}$$

and

$$\lambda \equiv \frac{\mu B_0^2 + u_0^2}{W} \tag{14}$$

as suggested by the form of the trapping parameter κ^2 of Eq. (8). From Eqs. (8), (14) and (13) it easy to find

$$\lambda = \frac{\kappa^2}{\kappa^2 + 2\epsilon}; \tag{15}$$

i.e. λ can be expressed solely in terms of κ^2 and therefore $\nabla_v \lambda$ is perpendicular to the trapped-passing boundary. In addition to this, it can be shown that $\nabla_v \lambda \cdot \nabla_v W = 0$ to leading order in $\sqrt{\epsilon}$. Hence, we assume that once the collision operator is written in terms of these new variables, terms containing $\partial/\partial W$ can be neglected. We then start with the Rosenbluth form of like particle collision operator to obtain the following model operator

$$C\{\delta f\} = \frac{B_0(v_{||} + u)}{B} \frac{\partial}{\partial \lambda} \left[\frac{B}{B_0(v_{||} + u)} \vec{\Gamma} \cdot \nabla_v \lambda \right], \tag{16}$$

where

$$\vec{\Gamma} \cdot \nabla_v \lambda = f_M \frac{(v_{||} + u)^2}{S^2 W^2} \left[\frac{\nu_\perp}{4} v^2 + \left(\frac{\nu_{||}}{2} - \frac{\nu_\perp}{4} \right) u^2 \right] \frac{\partial}{\partial \lambda} \left(\frac{\delta f}{f_M} \right). \tag{17}$$

The model operator defined by Eqs. (16) - (17) does not manifestly conserve momentum. To restore this property we introduce an additional term inside the collision operator, namely $f_M \frac{\sigma I (v_{||} + u)}{\Omega_i T_i} \frac{\partial T_i}{\partial \psi}$, where σ is a free parameter. Adding a term of this form is valid due to the conservation properties of the linearized Fokker-Planck operator and further modifies Eq. (10). The value of σ is found after the passing constraint is solved by requiring that the operator (16) - (17), twice modified as noted, conserves momentum when operating on the first order correction to the distribution function $f - f_M$ as given by Eq. (9).

With the help of the approach outlined above, passing constraint (10) can be solved and the main ion poloidal flow can be obtained by taking appropriate moment of the resulting distribution function to find

$$V_i^{pol} = \frac{7 I B_{pol}}{6 \Omega_i M B} \frac{\partial T_i}{\partial \psi} J(U^2), \tag{18}$$

where $U \equiv u/v_i$ and

$$J(U^2) = (5/2 - \sigma + U^2)/1.17 \tag{19}$$

with

$$\sigma = \frac{\int_0^\infty dy \exp(-y)(y + 2U^2)^{3/2}(\nu_\perp y + 2\nu_{||}U^2)}{\int_0^\infty dy \exp(-y)(y + 2U^2)^{1/2}(\nu_\perp y + 2\nu_{||}U^2)}. \tag{20}$$

In Eq. (20), $\nu_\perp = 3(2\pi)^{1/2}\nu_{ii}[erf(x) - \Psi(x)]/2x^3$ and $\nu_{||} = 3(2\pi)^{1/2}\nu_{ii}\Psi(x)/2x^3$ with $x \equiv v/v_i = (y^2 + U^2)^{1/2}$, $\nu_{ii} = 4\pi^{1/2}Z^4e^4n_i ln(\Lambda)/3M_i^{1/2}T_i^{3/2}$, $\mathrm{Erf}(x) = 2\pi^{-1/2}\int_x^\infty dy \exp(-y^2)$, and $\Psi(x) = [\mathrm{Erf}(x) - x\mathrm{Erf}'(x)]/2x^2$. It should be noted that these expressions, as well as other pedestal relevant results, can also be obtained with a more general formalism presented in Ref. [9].

Notice that at $U = 0$, $J(U) = 1$, so Eq. (18) reproduces the conventional core result in the absence of the electric field. As U^2 grows, $J(U^2)$ decreases and becomes negative at $U \approx 1.2$. Physically, this behavior means that in a banana regime pedestal the main poloidal ion flow is in the direction opposite to its core counterpart once the radial electric field goes beyond a certain critical value. We proceed by verifying findings summarized in this section against available experimental data.

V. COMPARISON OF THE REVISED NEOCLASSICAL FORMULA WITH THE C-MOD PEDESTAL MEASUREMENTS

Unfortunately, direct measurement of the main ion poloidal flow is very challenging. What is usually measured instead is the impurity ion flow. Due to their high charge number impurities are highly collisional, making it possible to relate their poloidal flow to that of main ions through a simple expression [10]:

$$V_z^{pol} = V_i^{pol} - \frac{cIB_p}{eB^2}\left(\frac{1}{n_i}\frac{dp_i}{d\psi} - \frac{1}{Z_z n_z}\frac{dp_z}{d\psi}\right), \tag{21}$$

where Z_z, n_z and p_z stand for the impurity charge number, density and pressure, respectively. To write Eq. (21) only terms of the leading order in $\sqrt{\epsilon}$ are retained and the main ion charge number Z is set equal to unity, as it is the case in Alcator C-Mod. Recent measurements of the boron impurity flow in the C-Mod pedestal [3] provide V_z^{pol}. Also, the electric field profiles are available for the same shots, so the experimental check of Eqs. (18) - (20) can be carried out.

To do so a number of shots are considered. For each shot, the peak poloidal velocity is chosen. This peak is usually located close to where the absolute value of the electric field experiences a maximum and therefore the difference between the conventional and modified neoclassical theories is the greatest. The scatter plot of Figure 1 is then created with the theoretically predicted and experimentally observed velocities being the x and y coordinates for each point, respectively. To place the banana regime comparison into a broader context, similar plots for the Pfirsch-Schlüter (PS) and plateau regimes are presented. For the former, the electric field should not have any effect, because a background ion can only drift radially by a distance much less than the poloidal ion gyroradius before undergoing a collision. For the latter, the electric field does modify the poloidal ion flow. The altered expression has been presented in Ref. [11] and used here to create Figure 1b. Figure 1 essentially repeats Figure 2 of Ref. [5] and is reproduced here for convenience.

As expected the conventional formula is in reasonable agreement with the experiment for Pfirsch - Schlüter plasmas, as demonstrated by Figure 1a. Figure 1b shows that conventional and modified theories give different predictions in the plateau regime, but it is unclear which theory does a better job. Finally, according to Figure 1c the agreement between the neoclassical theory and experiment is clearly improved upon accounting for the electric field effect on main ion drift orbits in a banana regime pedestal.

VI. DISCUSSION

As the comparison of the preceding section demonstrates, finite electric field effects need to be accounted for when evaluating main ion poloidal flow in a banana regime pedestal. The resulting alteration in the impurity flow has been measured to be substantial in the Alcator C-Mod pedestal. Importantly, the parallel electron flow, and therefore the bootstrap current, should then be modified as well, as electrons experience friction with the main ions. Since electron orbits are not affected by the electric field due to their small gyroradius the bootstrap current can be calculated in the same way as in the conventional theory [1] with Eq. (18) for the main ion flow in place of the usual formula. As a result, the pedestal bootstrap current is found to be [6]

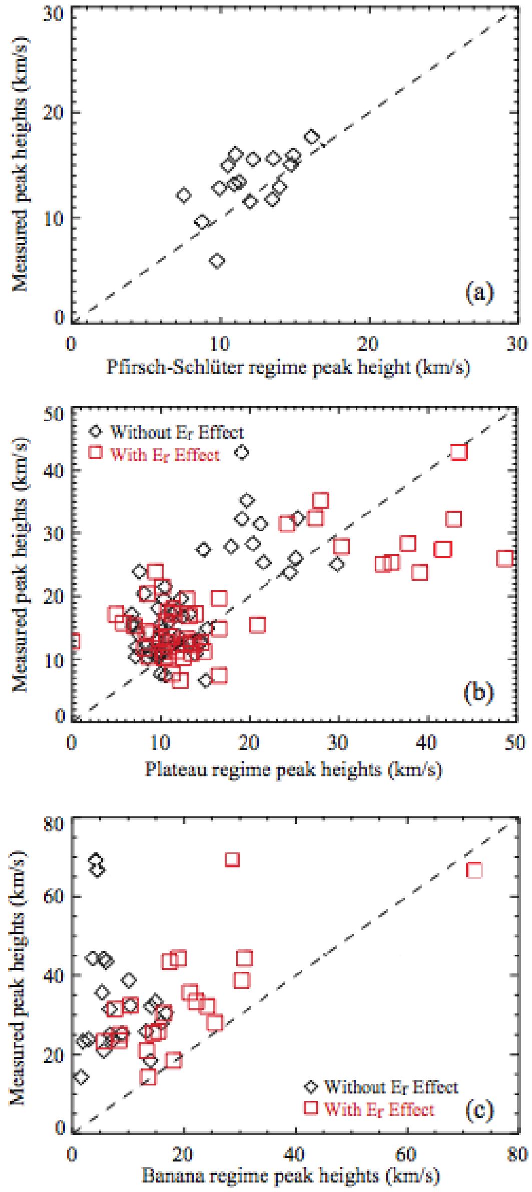

FIG. 1: Comparison of predicted and experimental peak heights found near the separatrix in the poloidal velocity profile for various collisionality plasmas: a) PS regime. b) Plateau regime. c) Banana regime. Complete agreement is indicated by the dashed line. Positive velocities indicate flow in the electron diamagnetic direction.

$$J_{bs} = -1.46\sqrt{\epsilon}\frac{cI}{B}\frac{(Z^2+2.21Z+0.75)}{Z(Z+1.414)}\left[\frac{dp}{d\psi} - \frac{(2.07Z+0.88)n_e}{(Z^2+2.21Z+0.75)}\frac{dT_e}{d\psi} - 1.17J(U^2)\frac{n_e}{Z}\frac{dT_i}{d\psi}\right], \tag{22}$$

where n_e and T_e are the electron density and temperature respectively and p denotes the total plasma pressure. In the conventional case, $J(U^2) = 1$ and the last term in the square brackets on the right side of Eq. (22) tends to lower the bootstrap current. As explained in section IV, once the pedestal electric field is taken into consideration $J(U^2)$ becomes less than unity and even goes negative provided the electric field is large enough. We therefore conclude that the modified neoclassical theory described here predicts a higher bootstrap current than the conventional one.

ACKNOWLEDGEMENTS

This work was supported by the U.S. Department of Energy Office of Fusion Energy Sciences under contracts DE-AC52-06NA25396 at the Los Alamos National Laboratory and DE-FG02-91ER-54109 at the Plasma Science and Fusion Center of the Massachusetts Institute of Technology.

[1] P. Helander and D. J. Sigmar, Collisional Transport in Magnetized Plasmas, Cambridge: Cambridge University Press (2002)

[2] A. A. Galeev and R. Z. Sagdeev, Sov. Phys. - JETP **26**, 233 (1968)

[3] K. D. Marr et al, Plasma Phys. Control. Fusion **52**, 055010 (2010).

[4] G. Kagan and P. J. Catto, Plasma Phys. Control. Fusion **52**, 055004 (2010).

[5] G. Kagan et al, Plasma Phys. Control. Fusion **53**, 025008 (2011).

[6] G. Kagan and P. J. Catto, Phys. Rev. Lett. **105**, 45002 (2010).

[7] G. Kagan and P. J. Catto, Plasma Phys. Control. Fusion **50**, 085010 (2008).

[8] G. Kagan and P. J. Catto, Phys. Plasmas **16**, 056105 (2009).

[9] P. J. Catto et al, Plasma Phys. Control. Fusion **53**, 054004 (2011).

[10] Y. Kim et al, Phys. Fluids B **3**, 2050 (1991).

[11] I. Pusztai and P. J. Catto, Plasma Phys. Control. Fusion **52**, 075016 (2010).