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ABSTRACT

The effect of waveguide boundary conditions on the $k_{||}$ spectrum resulting from the backscatter of lower hybrid waves near a waveguide array is calculated. It is found that the resulting $k_{||}$ spectrum is peaked at multiples of π/b, where b is the waveguide width. This calculation may partially explain recent lower hybrid heating results on Alcator A.

I. Introduction

There is currently extensive interest in using microwave radiation near the lower hybrid frequency to heat tokamak plasmas. This method of auxiliary heating has been tried on ATC,[1,2] Wega,[3] Petula,[4] Doublet IIA,[5] JFT2,[6] and recently on Alcator A.[7] The Alcator A experiment was carried out at a frequency $f = 2.45$ GHz using a two waveguide array. Ion heating was observed within narrow density bands which corresponded to the appearance of the lower hybrid mode conversion layer at the plasma center. At this value of central density ($n_e = n_{LH}$) $k_\perp = |\vec{k} \times \vec{B}|/|\vec{B}|$ is expected to become large and energetic ion heating can occur.

Nevertheless, the Alcator A experiment obtained anomalous results. Linear waveguide-plasma coupling theory[8-10] predicted a wave power spectrum characterized by $n_z \sim 3$ ($n_z = k_{||}c/\omega$, $k_{||} = k_z = |\vec{k}\cdot\vec{B}|/|\vec{B}|$) whereas the ion heating densities corresponded to $n_z \sim 5$. In addition, electron heating occurred at densities below n_{LH} which would require $\omega/k_{||}V_{Te} \sim 3$, or $n_z \sim 5$ for $T_e = 1$keV. Furthermore, the ion heating and wave amplitudes measured in the plasma interior were independent of the relative phase of the two waveguides, in clear disagreement with linear theory. It was suggested that these results could be explained by the parametric decay near the plasma surface of lower hybrid waves into other larger n_z lower hybrid waves.

Figure 1 schematically illustrates this parametric process in an inhomogeneous plasma. Here we consider decay waves which propagate along the pump wave resonance cone.[11,12] There is an interaction region driven by the pump wave in front of the $\omega = \omega_{pe}$ layer in which the plasma noise source gets amplified nonlinearly by a factor $A(k_{||})$ for decay into lower hybrid waves and ion acoustic waves. After amplification the noise source is reflected by the waveguides, which distorts $S(k_{||})$ into $S'(k_{||})$; this reflected power spectrum is then reamplified. If $S'(k_{||}) \cdot A(k_{||})$ is peaked at large values of $k_{||}$, the anomalous heating observed in Alcator A could occur when this decay wave propagates into the plasma interior. In addition, should this decay occur at the plasma edge where the pump wave is strong regardless of waveguide phasing, phase independent heating could be obtained.

This paper will not explore the details of the decay wave process, but will examine the transition of $S(k_{||})$ into $S'(k_{||})$. The results to be obtained will not be limited to parametric decay processes, but will also apply to any strong backscattering process occurring near a waveguide mouth. In such a backscattering process some fraction of the lower hybrid wave power radiated by the waveguides is backscattered into the waveguide array. We are concerned as to how the reflection of this backscattered wave from the waveguide array will modify its $k_{||}$ spectrum. It will be shown that even for broad $k_{||}$ noise sources and broad $k_{||}$ parametric growth rates or backscattering, the waveguide boundary conditions will force the reflected wave $k_{||}$ spectrum to peak near multiples of π/b, b being the waveguide width. In addition, little reflected wave power near $n_z = 1$ can be generated due to this process. These conclusions are qualitatively consistent with the Alcator A lower hybrid heating results, which observed central ion heating for densities corresponding to n_{LH} for $n_z \sim 5 = (\pi/b)(c/\omega)$.

In formulating this theory we will consider the reflection of lower hybrid waves from a waveguide array as a separate linear problem. We will cause some power spectrum $P(k_{||})$ to impinge onto the array from the plasma in the absence of any direct power flow or excitation inside the waveguides that propagates towards the plasma-waveguide interface. We will then calculate the resulting power spectrum reflected from the array. The total or net power spectrum launched by the array will be the superposition of the original lower hybrid wave spectrum launched by the waveguides and lower hybrid wave spectrum reflected from the waveguides after being backscattered from the plasma. However, if the power spectrum reflected off the array has a peak at some value of $k_{||}$ where the incident power spectrum is negligible, the net power radiated by the array will invariably also have a peak at that value of $k_{||}$.

In Section II the theory of lower hybrid wave reflection from a waveguide-plasma interface will be formulated. In Section III this theory is applied to the geometry of the Alcator A lower hybrid heating experiment and characteristic $k_{||}$ spectra are calculated. In Section IV these results are summarized and the basic conclusions emphasized.

II. Theory

Here we formulate the theory of reflection of lower hybrid waves from a waveguide-plasma interface. This treatment parallels that of Brambilla,[8] except that here the source is within the plasma, while previous treatments directly excite each waveguide. The geometry is that of Fig. 1, with N waveguides at positions $z = z_1, z_2 \ldots z_N$, each having width b. The fields in the waveguide mouths at the waveguide-plasma interface $(x = 0)$ are

$$E_z(z) = \sum_{p=1}^{N} \sum_{n=0}^{\infty} \theta_p(z)\, \beta_{np} \cos\left(\frac{n\pi(z - z_p)}{b}\right) e^{-ik_n x} \tag{1}$$

$$B_y(z) = \sum_{p=1}^{N} \sum_{n=0}^{\infty} \theta_p(z)\, \frac{\omega}{ck_n}\, \beta_{np} \cos\left(\frac{n\pi(z - z_p)}{b}\right) e^{-ik_n x}$$

where

$$\theta_p(z) = \begin{cases} 1 & z_p < z < z_p + b \\ 0 & \text{elsewhere} \end{cases}$$

$$k_n = \begin{cases} \omega/c & n = 0 \\ i[(n^2\pi^2/b^2) - (\omega^2/c^2)]^{\frac{1}{2}} & n \neq 0 \end{cases}$$

$\theta_p(t)$ specifies a zero field outside waveguide p; for $n = 0$ β_{np} is the amplitude of a waveguide mode propagating away from the plasma, whereas for $n \neq 0$ this mode is evanescent. We have let the waveguides be infinite in the y direction, as done by Brambilla,[8] and we only allow modes in the waveguide that propagate or evanesce away from the waveguide mouth at $x=0$, as the only power source we allow is the lower hybrid wave within the plasma. The final solution for the lower hybrid wave spectrum would be the superposition of this result and the wave launched by a waveguide array in the absence of any backscatter. In the plasma near $x=0$ we have

$$E_z(z) = \int_{-\infty}^{\infty} [I(k_z)e^{ik_x x} - Y(k_z)e^{-ik_x x}]e^{ik_z z}\, dk_z$$

$$B_y(z) = -\int_{-\infty}^{\infty} \frac{\omega}{c} [I(k_z)e^{ik_x x} + Y(k_z)e^{-ik_x x}]e^{ik_z z} \frac{dk_z}{k_x} \tag{2}$$

$$\text{where} \qquad k_x = \begin{cases} [(\omega^2/c^2) - k_z{}^2]^{\frac{1}{2}} & \frac{\omega}{c} > |k_z| \\ i[k_z{}^2 - (\omega^2/c^2)]^{\frac{1}{2}} & |k_z| > \frac{\omega}{c} \end{cases} .$$

We have chosen k_x so as to treat the singularities at $k_z = \pm(\omega/c)$ similarly to Brambilla.[8] $I(k_z)$ is the Fourier amplitude of the wave propagating away from the waveguide-plasma interface, while $Y(k_z)$ is that of the inward propagating wave. $I(k_z)$ and $Y(k_z)$ are determined by the plasma dynamics and the exact nature of the backscatter source within the plasma. Equation (1) and (2) can be solved for the β_{np}'s by matching $E_z(z)$ at the entire $x=0$ interface and $B_y(z)$ at $x=0$ inside the waveguide mouths.

We can first equate the $E_z(z)$ of Eq. (1) to that of Eq. (2) at $x=0$. If we multiply both equations by $\exp(-ik_z z)$ and integrate from $z=-\infty$ to $z=+\infty$ we obtain

$$I(k_z) - Y(k_z) = \sum_{p=1}^{N} \sum_{n=0}^{\infty} \beta_{np} F_{np}(k_z) \equiv Q(k_z) \tag{3}$$

where

$$F_{np}(k_z) = \frac{ik_z}{2\pi} \frac{\exp(-ik_z z_p)}{n^2\pi^2/b^2 - k_z^2} [1 - (-1)^n e^{-ik_z b}]$$

In order to solve this problem, we must further equate the B_y of Eqs. (1) and (2) inside each waveguide mouth. Multiplying Eqs. (1) and (2) by $\cos[m\pi(z - z_q)/b]$ and integrating from $z = z_q$ to $z = z_q + b$ we obtain

$$\frac{\omega}{ck_m} \beta_{mq} \frac{b}{2}(1 + \delta_{m,0})$$
$$= - \int_{z_q}^{z_q + b} \int_{-\infty}^{\infty} \frac{\omega}{c}[I(k_z) + Y(k_z)]\frac{e^{ik_z z}}{k_x} \cos\frac{m\pi(z - z_q)}{b} dk_z dz \tag{4}$$

We must determine $I(k_z)$ and $Y(k_z)$ to proceed further. For $|n_z| < 1$, the lower hybrid waves are evanescent at densities greater than $n_e = n_c$ $(4\pi n_c e^2/m_e = \omega^2)$ and carry no power. Assuming a linear density profile we can relate $I(k_z)$ to $Y(k_z)$ for $|n_z| < 1$

$$Y = I\frac{1 - Z}{1 + Z} \tag{5}$$

where

$$Z = i \frac{J_{1/3}(\frac{2}{3}\xi_o^{3/2}) + J_{-1/3}(\frac{2}{3}\xi_o^{3/2})}{J_{2/3}(\frac{2}{3}\xi_o^{3/2}) - J_{-2/3}(\frac{2}{3}\xi_o^{3/2})}$$

$$\omega_{pe}^2 = \omega^2 \frac{x}{L} \qquad \xi_o = (\frac{\omega L}{c})^{2/3} |1 - n_z^2|^{1/3}$$

and $J_{1/3}(x)$ is the ordinary Bessel function. For $|n_z| > 1$ the lower hybrid waves are evanescent for $\omega_{pe} > \omega$ and propagate for $\omega_{pe} > \omega$. Here, as with Brambilla[8] we will ignore the mode conversion of lower hybrid waves into whistler waves for $n_z < (1 - \omega^2/\omega_{ce}\omega_{ci})^{-\frac{1}{2}} \sim 2$. The equation describing a Fourier component of the lower hybrid wave is for $|n_z| > 1$

$$\frac{\partial^2 E_z}{\partial \xi^2} + \xi E_z = 0 \tag{6}$$

where $\xi = \xi_o(\frac{x}{L} - 1)$. The solution to Eq. (6) for $x > L$ is

$$E_z = \xi^{\frac{1}{2}}[A(n_z)J_{1/3}(\tfrac{2}{3}\ \xi^{3/2}) + B(n_z)J_{-1/3}(\tfrac{2}{3}\ \xi^{3/2})] \tag{7}$$

For $\xi >> 1$ the WKB solution of Eq. (6) is

$$\begin{aligned} E_z \simeq\ & [\rho(n_z)e^{-i\frac{2}{3}\xi^{3/2}} + \gamma(n_z)e^{i\frac{2}{3}\xi^{3/2}}] \\ & * (1 - e^{-2\pi i/3})\ \xi^{-\frac{1}{4}} \end{aligned} \tag{8}$$

Since the lower hybrid waves are backward propagating, $\rho(n_z)$ represents waves travelling away from the waveguides whereas $\gamma(n_z)$ is the source impinging on the array. In normal waveguide plasma coupling theory $\gamma(n_z) = 0$ and each waveguide is internally excited; here $\gamma(n_z)$ is a given function detailing the backscattered lower hybrid wave impinging on the array and each waveguide is only excited by this external lower hybrid wave. At some large ξ we can match Eq. (8) with the asymptotic expansion of Eq. (7) to obtain $A(\rho,\gamma)$ and $B(\rho,\gamma)$. The connection formula between $x < L$ and $x > L$ then fixes

$$E_z(x<L) = (-\xi)^{\frac{1}{2}}[-A(\rho,\gamma)I_{1/3}(\tfrac{2}{3}(-\xi)^{3/2}) + B(\rho,\gamma)I_{-1/3}(\tfrac{2}{3}(-\xi)^{3/2}]$$

$$B_y(x<L) = \frac{i(-\xi)(c/\omega L)^{1/3}}{(n_z^{\ 2}-1)^{2/3}} \times [-A(\rho,\gamma)I_{-2/3}(\tfrac{2}{3}(-\xi)^{3/2}) + B(\rho,\gamma)I_{2/3}(\tfrac{2}{3}(-\xi)^{3/2}] \qquad (9)$$

where $I_{1/3}(x)$ is the modified Bessel function of the first kind. Equation (9) thereby gives $I(k_z)$ and $Y(k_z)$ in terms of ρ and γ, by letting $x = 0$. $I(k_z) - Y(k_z)$ was already obtained in Eq. (3). We can now use Eq. (3) to obtain $\rho(n_z)$, which is the Fourier amplitude of the lower hybrid wave spectrum reflected off the array and is what we seek. After much algebra we obtain for $|n_z| > 1$

$$i\,\rho(k_z) = \frac{Q(k_z)e^{\pi i/4}}{\beta[I_{1/3}(X)e^{\pi i/3} + I_{-1/3}(X)]} - \gamma(k_z)\,\frac{I_{1/3}(X) + e^{\pi i/3}I_{-1/3}(X)}{I_{-1/3}(X) + e^{\pi i/3}I_{1/3}(X)} \qquad (10)$$

where $X = (2/3)\xi_o^{3/2}$ and $\beta = (\omega L/c)^{1/3}(n_z^{\ 2}-1)^{1/6}$. We have now determined $I(k_z) + Y(k_z)$ in Eq. (4) as

$$I(k_z) + Y(k_z) = \frac{Q(k_z)}{Z(k_z)} + S(k_z) \qquad (11)$$

where $Z(k_z) = [I_{1/3}(X)e^{\pi i/3} + I_{-1/3}(X)]/[I_{-2/3}(X)e^{\pi i/3} + I_{2/3}(X)]$

and $$S(k_z) = \gamma(k_z)\beta e^{-\pi i/4}\Big\{I_{-2/3}(X) + e^{\pi i/3}I_{2/3}(X) - \frac{[I_{-2/3}(X)e^{\pi i/3} + I_{2/3}(X)]}{I_{-1/3}(X) + I_{1/3}(X)e^{\pi i/3}}[I_{1/3}(X) + e^{\pi i/3}I_{-1/3}(X)]\Big\}$$

This $Z(k_z)$ is identical to that of Brambilla and describes the plasma impedance. We can solve Eq. (4) to obtain the system of equations

$$\beta_{mq} + \sum_{n=0}^{\infty}\sum_{p=1}^{N} \beta_{np}K_{mq,np} = S_{mq} \tag{12}$$

where

$$K_{mq,np} = \frac{4\pi k_m}{b(1+\delta_{m,o})} \int_{-\infty}^{\infty} \frac{F_{np}(k_z)F^*_{mq}(k_z)dk_z}{Z(k_z)k_x}$$

and

$$S_{mq} = -\frac{4\pi k_m}{b(1+\delta_{m,0})} \int_{1}^{\infty} \frac{dn_z}{i(n_z^2 - 1)^{\frac{1}{2}}} [S(n_z)F^*_{mq}(n_z) + S(-n_z)F_{mq}(n_z)]$$

Equation (12) can be truncated for a given number of modes and waveguides and solved. It is similar to previous formulations, except that the source term S_{mq} is new and reflects the excitation of the system by plasma waves. The solution of Eq. (12) yields the β_{np}'s which allow us to calculate $\rho(n_z)$ and therefore the power spectrum reflected by the array into the plasma. The superposition of this spectrum and the originally launched wave constitutes the total or net power spectrum that actually could heat the plasma.

The power spectrum resulting from this formulation can be calculated as $S_x = -(c/8\pi)\,\mathrm{Re}(E_z B_y^*)$. For $|n_z| > 1$ we have $S_x = \int dk_z P\ (k_z)$ and

$$P(k_z) = \frac{c}{2}\,\frac{\mathrm{Im}[I(k_z)Y^*(k_z)]}{k_x} \tag{13}$$

This becomes

$$P(k_z) = C\,\frac{|\gamma|^2 - |\rho|^2}{(n_z{}^2 - 1)^{1/6}} \tag{14}$$

$$* \ [I_{1/3}(X)I_{2/3}(X) - I_{-2/3}(X)I_{-1/3}(X)]$$

where C is a constant independent of k_z. Noting that $I_{-1/3}(x)I_{-2/3}(x) - I_{+2/3}(x)I_{+1/3}(x) = 2[\sin(\pi/3)]/[\pi x]$, we can identify the $|\gamma|^2$ term as the power spectrum incident on the array. The $|\rho|^2$ term is what we have solved for and is the final power spectrum radiated by the waveguides after reflection. In the remainder of this paper we shall define $P(k_z)$ as only the $|\rho|^2$ term which is the component of the power spectrum propagating into the plasma.

$\gamma(k_z)$ would be set by the details of the noise source or backscattering processes. However, we can specify $\gamma(k_z)$ so that the power impinging on the waveguide is constant for $|n_z| > 1$; this obtains

$$\gamma_o(k_z) = C \frac{(n_z{}^2 - 1)^{1/12}}{[I_{1/3}(X) I_{2/3}(X) - I_{-2/3}(X) I_{-1/3}(X)]^{\frac{1}{2}}} \tag{15}$$

We can add additional factors to $\gamma(k_z)$ that would narrow its k_z spectrum. In addition, phase variations can be present. However, we should note that if near $x = L$, $\gamma(n_z >> 1)$ is given, at some $x >> L$, $\gamma(n_z)\exp(i\frac{2}{3}\xi^{3/2}) \simeq \gamma(n_z)\exp(ik_z\Delta z)$, where Δz is just the shift in z of the resonance cone and follows the group velocity trajectory. Thus, any well behaved k_z spectrum generated or backscattered at some $x > L$ will be well behaved and not rapidly phase varying at $x = L$ if it propagates along the resonance cone. We will therefore consider in our calculations the spectrum

$$\gamma(k_z) = \gamma_o(k_z) e^{-ik_z\Delta z_o} [e^{-(k_z - k_o)^2/\Delta k^2} + e^{-(k_z + k_o)^2/\Delta k^2}] \tag{16}$$

where Δz_o is the shift of the wave packet impinging on the array from the lower hybrid resonance cone, and therefore from the waveguide mouth. While other forms of $\gamma(k_z)$ are certainly possible, the $\gamma(k_z)$ of Eq. (16) is general enough to explore the basic properties of this formulation. Using this $\gamma(k_z)$ we can calculate the conditions imposed on $P(k_z)$ by the waveguide array boundary conditions.

III. Calculation of $P(n_z)$ Spectra

In calculating the $P(n_z)$ resulting from the reflection of lower hybrid waves from an array, we will use parameters similar to those of the Alcator A experiment. In this experiment a two waveguide array was used, each waveguide being 1.275 cm wide and separated by a 1 mm wide septum. The frequency was 2.45 GHz and we will take $L = 0.01$ cm or $\nabla n = 7.4 \times 10^{12} cm^{-4}$. In these calculations we will truncate $\gamma(k_z)$ at $|n_z| = 50$, at which point the waves would be strongly electron Landau damped at the plasma edge.

Figure (2a) shows the familiar power spectrum launched by a two waveguide system with oppositely phased waveguides. A large fraction of the power is between $1 < n_z < 2$, which would form surface waves and not penetrate into the plasma interior. The remainder of the power is characterized by $n_z \sim 3$ with very little power at $n_z > 5$. Figure (2b) shows the reflected power spectrum for the same waveguide array under similar conditions. Here, as in succeeding figures, we will plot the power spectrum reflected by the array and not the superposition of this reflected power spectrum and the originally launched lower hybrid power spectrum. We have set $\Delta k = 0$ and $\Delta z_o = 0$ in Eq. (16) which describes the backscattered wave impinging on the array. A strong peak at $n_z \sim 6$ occurs in the reflected wave power spectrum and there is very little power at $n_z < 3$. This is because the $m = 1$ modes in the waveguides are strongly excited by the source radiation; the $m = 0$ modes are weakly excited due to the lack of source power near $n_z = 0$ where the $m = 0$ mode is peaked. Figure (2b) also shows the reflected power spectrum for $L = 0.001$ cm and illustrates that the basic features of this $P(n_z)$ are only weakly dependent on ∇n.

In Fig. (2) and succeeding figures there is an integrable singularity at $n_z = 1$. Near $n_z = 1$ $P(n_z)$ varies like

$$P(n_z) = \frac{C_o}{(n_z^2 - 1)^{2/3}} \quad . \tag{17a}$$

It can easily be seen that $\int_1^{1+\Delta} P(n_z)dn_z$ will then be

$$\int_1^{1+\Delta} P(n_z)dn_z \simeq 3P_o\,\Delta \tag{17b}$$

where $P_o = P(n_z = 1 + \Delta)$ and $\Delta << 1$. From Eq. (17b) the total power for $1 < n_z < 1.1$ can be estimated, and is completely negligible for most of the power spectra resulting from reflection off the waveguide array. However, for the Brambilla[8] spectrum of Fig. (2a) it is of the order of 10% of the total power.

In Fig. (2b) only the first three modes were included in the calculation. Fig. (3a) shows the same spectrum as that of Fig. (2b) except over a larger n_z band. Higher order peaks at larger n_z are present. Figure (3b) shows $P(n_z)$ for the same conditions as those of Fig. (3a), except that now four modes are included in each waveguide. The lowest order peak is only slightly changed, whereas the spectrum for $n_z > 10$ is strongly affected. This illustrates that including more modes of order n will affect $P(n_z)$ for $n_z \gtrsim (n\pi)/b$, but for $n_z << (n\pi)/b$ $P(n_z)$ will be slightly altered. Of course, here we have allowed $\gamma(n_z)$ to extend up to $n_z = 50$; truncating $\gamma(n_z)$ at lower n_z will also truncate $P(n_z)$, as will be shown below. In the remaining calculations we will concern ourselves with $n_z < 10$ and need only include three waveguide modes.

Figure (4) is the graph of $P(n_z)$ for a single waveguide having $b = 2.6$ cm under the same conditions as those of Fig. (3). Here there is no peak at $k_z = \pi/b$ but there is one at $k_z = 2\pi/b$. This is due to the fact that our $\gamma(k_z)$ spectrum is chosen so that for $\Delta z_o = 0$ the excitation has even symmetry about $z = 0$, thereby excluding the oddly symmetric $m = 1$ mode in the waveguide. There is little difference between the $P(n_z)$ of the single and double waveguide arrays having the same total width. However, for a $\gamma(k_z)$ that is not evenly symmetric the $m = 1$ mode could be excited, causing a peak in $P(n_z)$ near $k_z = \pi/b$. This is illustrated by Fig. (4b), which is generated for the same conditions as those of Fig. (4a), except that now $\Delta z_o = 1$ cm, destroying the even symmetry of $\gamma(k_z)$. The $m = 1$ mode is strongly excited and results in a peak at $n_z = 2.4$ or $k_z = \pi/b$, as shown in Fig. (4b).

What these previous figures illustrate is that even for $\gamma(k_z)$ that form relatively flat in k_z incoming power spectra, the power spectra reflected from the array will have peaks at $k_z = \pi/b$. Of course, the net power radiated by the array will be the superposition of this reflected power spectrum with the original $P(k_z)$ launched by the waveguide array. However, a two waveguide array would ordinarily launch negligible power at $k_z = \pi/b$; any reflected power spectrum having a peak at $k_z = \pi/b$ would certainly result in a net radiated power spectrum that would have substantial power at $k_z = \pi/b$. Several successive reflections and backscatters from the plasma could further enhance this peak.

Figure (5) shows $P(n_z)$ resulting from reflection from a conducting wall; i.e., $\Sigma\beta_{np}F_{np}(k_z) = 0$. Here we have let $\Delta z_o = 0$ and $\gamma(k_z)$ is a constant between $1 < n_z < 50$. As expected, the resulting spectrum is exactly flat and reflects the incident $\gamma(k_z)$ shape. Figure (5) also shows the $P(n_z)$ resulting from setting $\Delta z_o = -5$cm and $\Sigma\beta_{np}F_{np}(k_z) \neq 0$. This finite Δz_o effectively shifts the incident wave packet away from the waveguide mouth. Again, $P(n_z)$ is almost flat, except for small oscillations resulting from the interaction of the array with fringe fields. Figure (5) then verifies that we obtain the correct result from this formulation when the incident radiation impinges on a conducting wall.

In the previous results, we have truncated $\gamma(k_z)$ at $n_z = 50$. However, we can narrow $\gamma(k_z)$ by letting Δk in Eq. (16) be finite. If the source radiation has a z dependence $\sim \exp(-z^2/b^2)$, its Fourier transform would have a form similar to that of Eq. (16) with $\Delta k = 2/b$. For $b = 1.275$ cm, this corresponds to $\Delta n_z = \Delta k\ c/\omega = 3.06$. Figure (6) shows the results of setting $\Delta n_z = 3.06$ and $n_{zo} = k_o c/\omega = 5$. $P(n_z)$ is then also peaked at $n_z = 5$, and has little spectral power for $n_z > 10$. Modes having $m\pi/b$ at values of k_z where $\gamma(k_z) \simeq 0$ are weakly excited; in these cases the reflected spectral power is negligible where $\gamma(k_z) = 0$.

Figure (7) illustrates the effect of shifting the origin of the wave packet of Fig. (6). In Fig. (7) $\gamma(k_z)$ is centered at $n_{zo} = 3$ with $\Delta n_z = 3.06$ and $\Delta z_o = 5$ cm. As expected, the resulting $P(n_z)$ is also centered about $n_{zo} = 3$. Figure (7) also shows

$P(n_z)$ for $\Delta z_o = 0$. Now $P(n_z)$ is centered near $n_{zo} = 4.5$, and reflects the strong excitation of the $m = 1$ mode. As before, the resulting emitted n_z spectrum is strongly peaked near $k_{||} = \pi/b$ when the packet is positioned at the center of the array.

As a final point, we can examine how a spectrum that is compressed near $n_z = 1$ is altered by the reflection process. In Fig. (8) a $\gamma(k_z)$ having $n_{zo} = 1.5$ and $\Delta n_z = 2$ impinges on a double waveguide array; this incident power spectrum has negligible power at $n_z = 5$. Upon reflection the resulting $P(n_z)$ still has a substantial amount of power for $n_z < 3$, but the power in the peak at $n_z = 5$ is roughly equal to that below $n_z = 3$. What this illustrates is that even if a power spectrum compressed near $n_z = 1$ is backscattered into the waveguide mouths, a substantial portion of the reflected power that re-enters the plasma will be characterized by $k_{||} = \pi/b$. If this reflected power spectrum is superimposed onto the Brambilla type two waveguide spectrum of Fig. (2a), the resulting net power will have a substantial amount of power at $k_{||} = \pi/b$. Several successive reflections and backscatters will further enhance this upshifted in $k_{||}$ power spectrum.

IV. Conclusions

In summary, we have calculated the effect of the waveguide array boundary conditions on lower hybrid radiation that scatters off it. In general the resulting power spectrum that reflects off the array will peak at values of k_z that are multiples of π/b; the amount of power near $n_z = 1$ will usually be small and under certain conditions the $m = 1$ mode feature will dominate the spectrum. Under this condition the net power radiated by the array will also have a peak near $k_{\|} = \pi/b$. It also was shown that by shifting the position of the impinging wave packet away from the array, we obtain the usual result of reflection off a plane wall.

These results are in qualitative agreement with the observations of the Alcator A lower hybrid heating experiment,[7] which inferred a k_z spectrum peaked at π/b. Thus, a parametric decay occurring in front of the array could cause the observed effects, even if the decay process itself was broad in k_z. However, Fig. (8) illustrates that even backscatter from a process that doesn't alter k_z can eventually lead to a strongly peaked power spectrum at $k_z = \pi/b$. Backscattering from long wavelength drift wave turbulence[13-15] near the waveguide mouth could then cause an effective upshift in k_z if the backscattering is strong enough and if it is close enough to the waveguide mouth.

It should be noted that other effects, such as variations in k_z as the wave propagates into the plasma interior, may also have contributed to the anomalous results of the Alcator A experiment.

In this paper we have not delved into the exact nature of the parametric or turbulent backscattering process. Obviously, if it is weak it can have little effect on the net power radiated by the waveguide array. In addition, we have shown that if the backscattered waves are offset from the waveguide mouth, they will be substantially unchanged in their $k_{\|}$ spectrum upon reflection.

This latter condition requires that the backscattering region be close to the waveguide mouth, so that deviations of the back-scattered wave from the originally launched lower hybrid wave are smaller than the waveguide array width. However, it should be noted that this formulation can be extended to cover the case of a backscattered wave striking any opening or corruga-tion in the vacuum vessel, and predicts that any such obstruction will leave its imprint on the Fourier spectrum of the reflected waves.

Acknowledgments

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Figure Captions

Fig. 1 Schematic of backscatter interaction discussed in text. $S(k_{||})$ is a noise source, $A(k_{||})$ is the parametric amplification factor, and $S'(k_{||})A(k_{||})$ is the spectral power after reflection from the waveguide array. The z_n are the waveguide positions and b is the waveguide width. $\omega = \omega_{pe}$ at $x = L$.

Fig. 2 $P(n_z)$ power spectrum for two waveguide array having $b = 1.275$ cm, septum 1 mm thick and $f = 2.45$ GHz. a) Brambilla spectrum; $L = 0.01$ cm b) Reflected power spectrum. 1- $L = 0.01$ cm and the impinging power spectrum is constant for $1 < n_z < 50$. 2 - Same as 1, except that $L = 0.001$ cm. In (b) $\Delta z_o = 0$, three modes are included in each waveguide and the normalization is arbitrary.

Fig. 3 a) $P(n_z)$ spectrum for the same conditions as those of Fig. (2a), but over a wider n_z band. b) Same as (a), except 4 modes are included. Below $n_z = 10$ both (a) and (b) are similar.

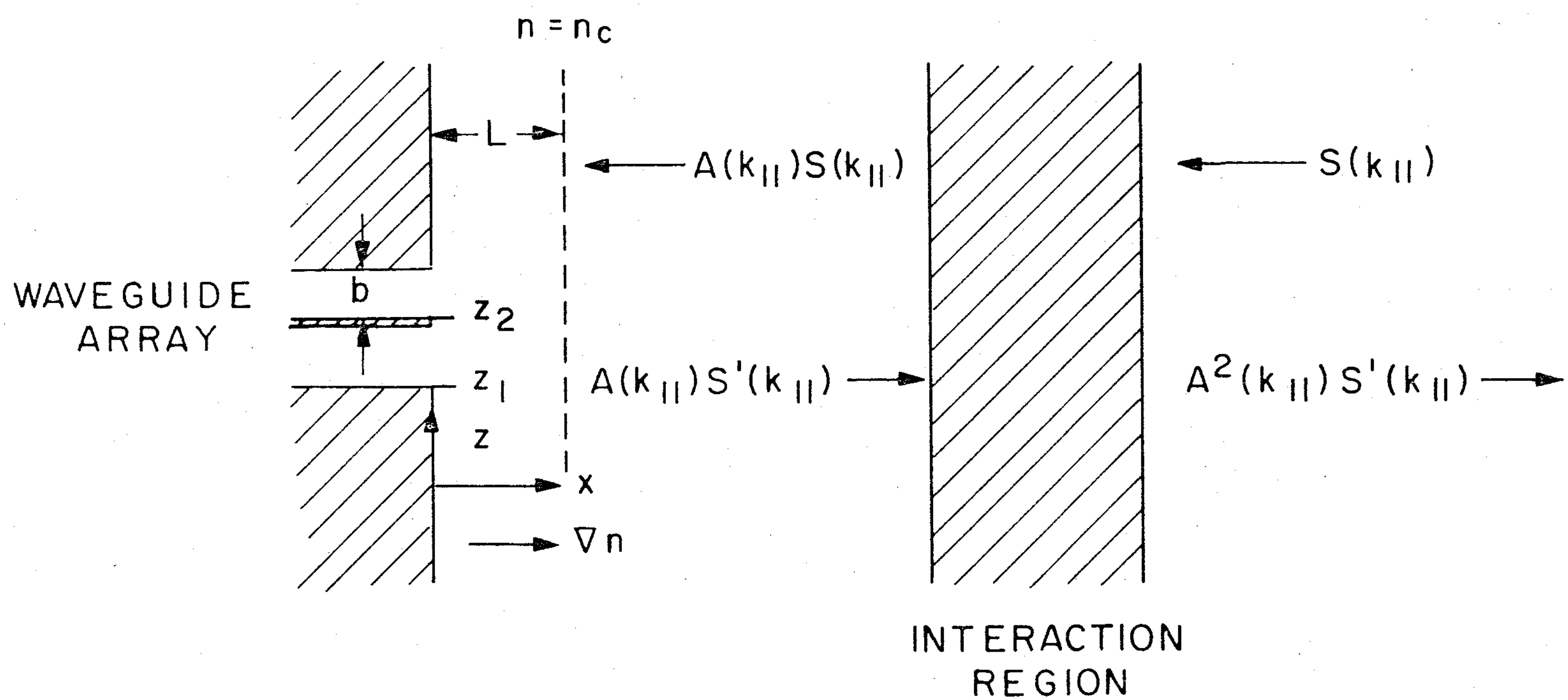
Fig. 4 $P(n_z)$ spectrum for a single waveguide having $b = 2.6$ cm, $f = 2.45$ GHz, $L = 0.01$ cm, and an impinging power spectrum that is constant for $1 < n_z < 50$. a) $\Delta z_o = 0$; the $m = 1$ mode is not excited. b) $\Delta z_o = 1$ cm; the loss of even symmetry in $\gamma(k_z)$ allows the $m = 1$ mode to be excited.

Fig. 5 1- Spectrum resulting from setting $\Sigma\beta_{np}F_{np}(k_z) = 0$; $L = 0.01$ cm, $f = 2.45$GHz, and the impinging power spectrum is constant for $1 < n_z < 50$. 2- Same as 1, except a double waveguide array is employed with $b = 1.275$ cm, $\Delta z_o = 5$ cm, and $\Sigma\beta_{np} F_{np}(k_z) \neq 0$. The normalizations are arbitra

Fig. 6 $P(n_z)$ spectrum for double waveguide array having $b = 1.275$ cm, $f = 2.45$ GHz and $L = 0.01$ cm. $\gamma(k_z)$ is set by letting $\Delta n_z = 3.06$, $n_{zo} = 5$ and $\Delta z_o = 0$. As expected, the reflected $P(n_z)$ is peaked at $n_z = 5$.

Fig. 7 $P(n_z)$ for double waveguide array having $b = 1.275$ cm, $f = 2.45$ GHz, and $L = 0.01$ cm. $\gamma(k_z)$ is set by letting $\Delta n_z = 3.06$ and $n_{zo} = 3$. 1- $\Delta z_o = 5$ cm. $P(n_z)$ is also peaked at $n_z = 3$. 2- $\Delta z_o = 0$. $P(n_z)$ is now peaked near $n_z = 4.5$. The normalizations are arbitrary.

Fig. 8 $P(n_z)$ spectrum for a double waveguide array having $b = 1.275$ cm, $L = 0.01$ cm and $f = 2.45$ GHz. $\Delta n_z = 2$, $n_{zo} = 1.5$ and $\Delta z_o = 0$. A significant fraction of the reflected $P(n_z)$ is upshifted to $n_z = 5$.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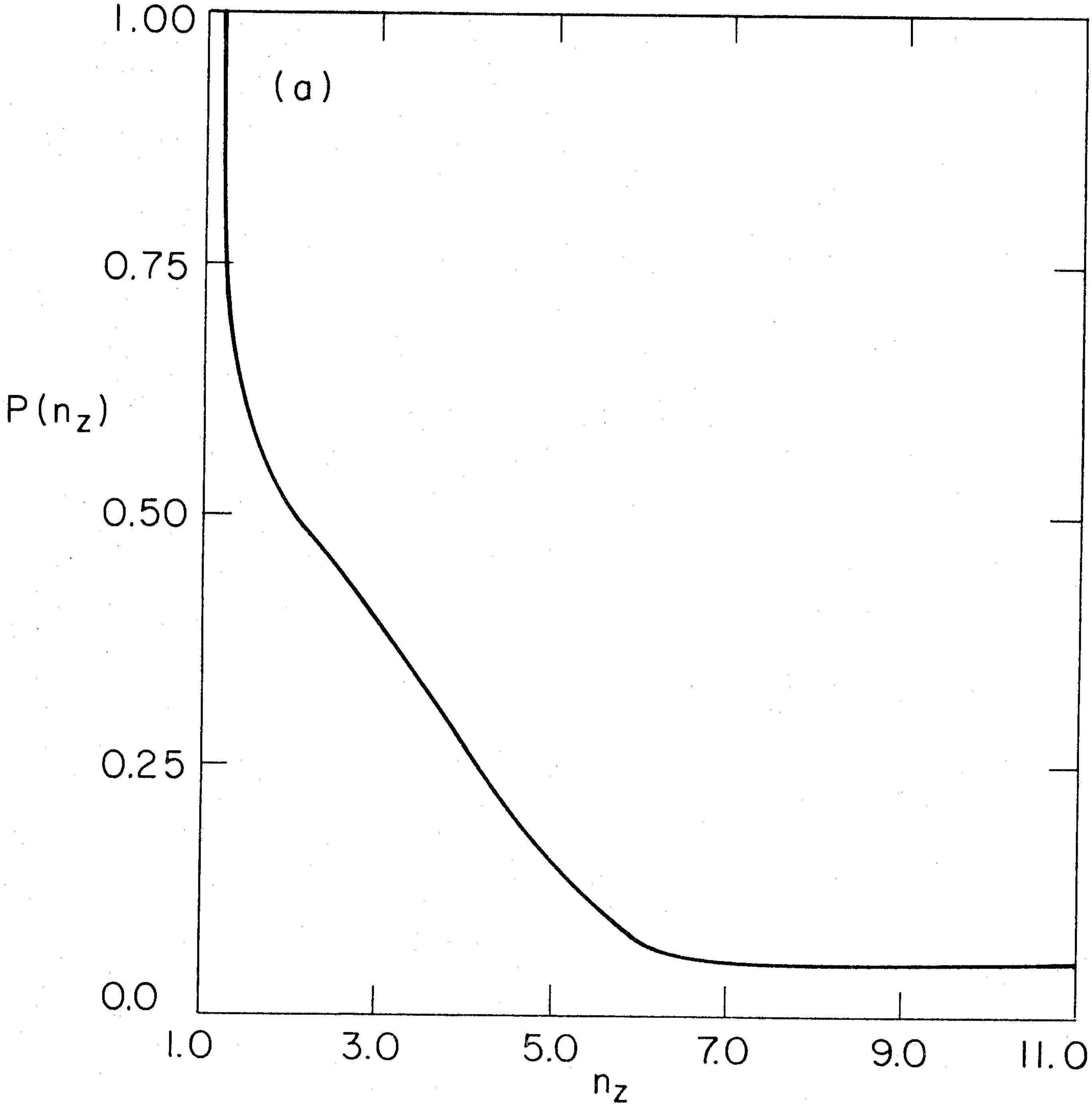
INTERACTION REGION



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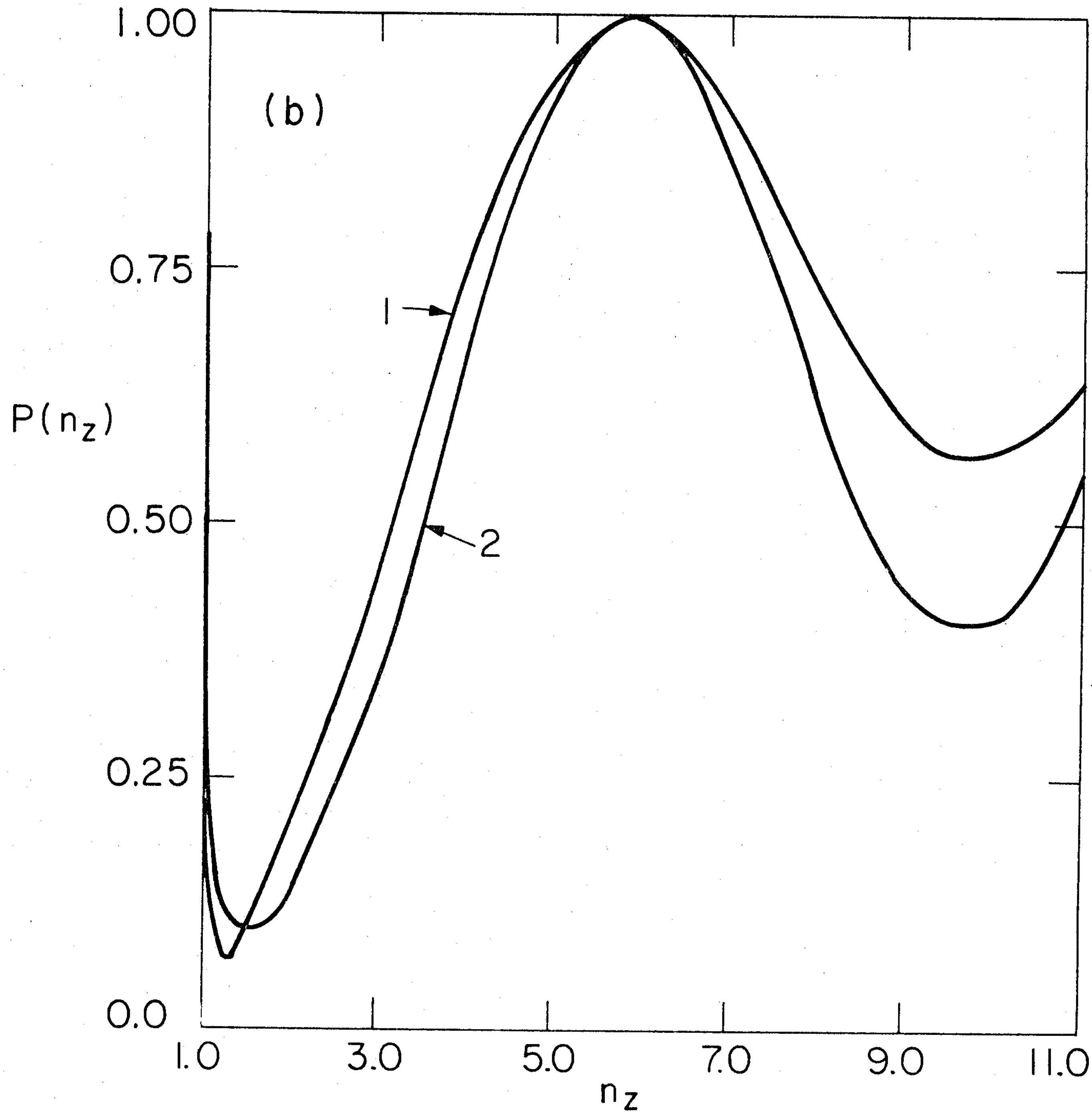
7.0
9.0
11.0
n_z



(b)
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0.75
0.50
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P(n_z)
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2
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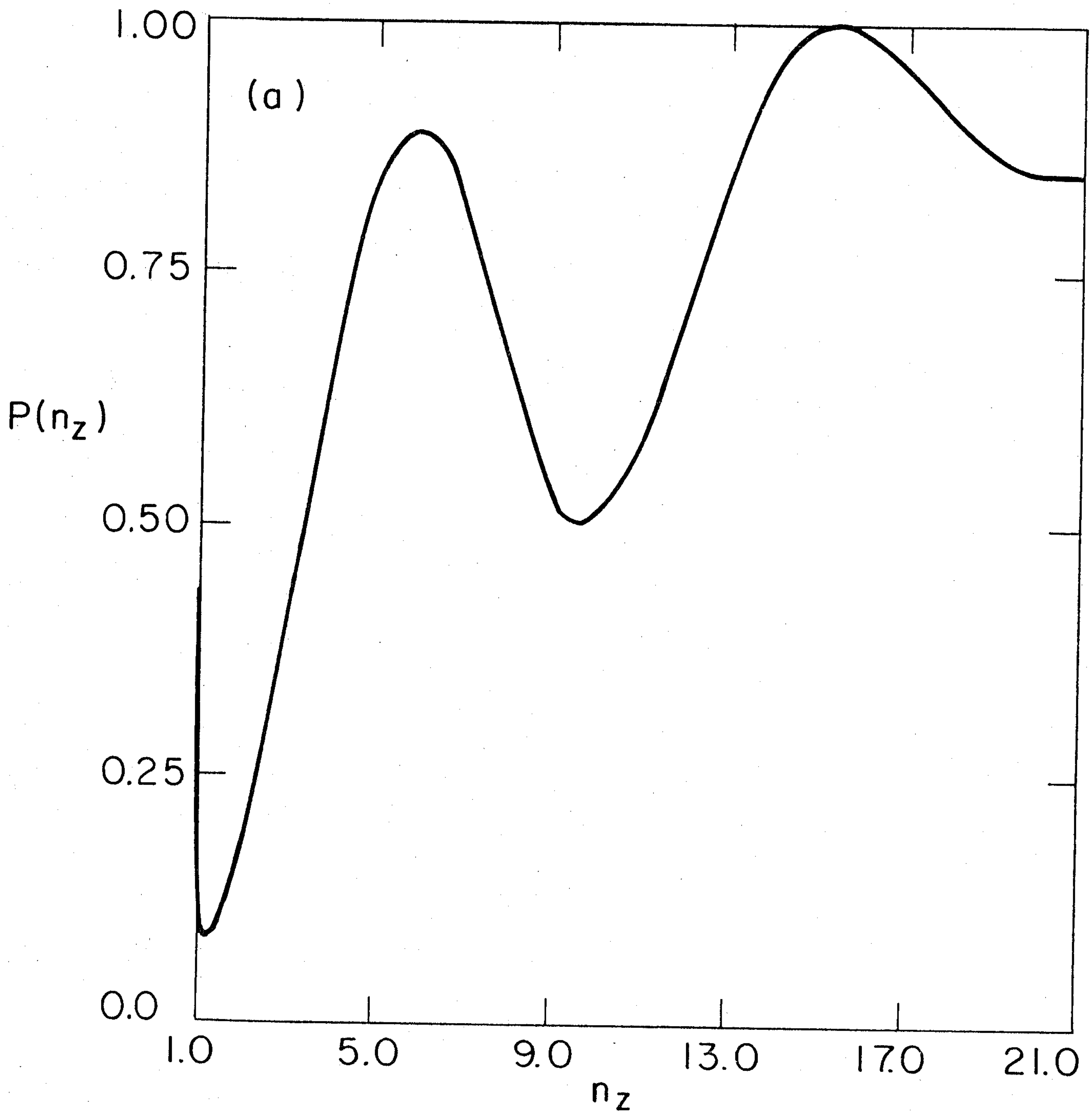
9.0
11.0
n_z



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(a)
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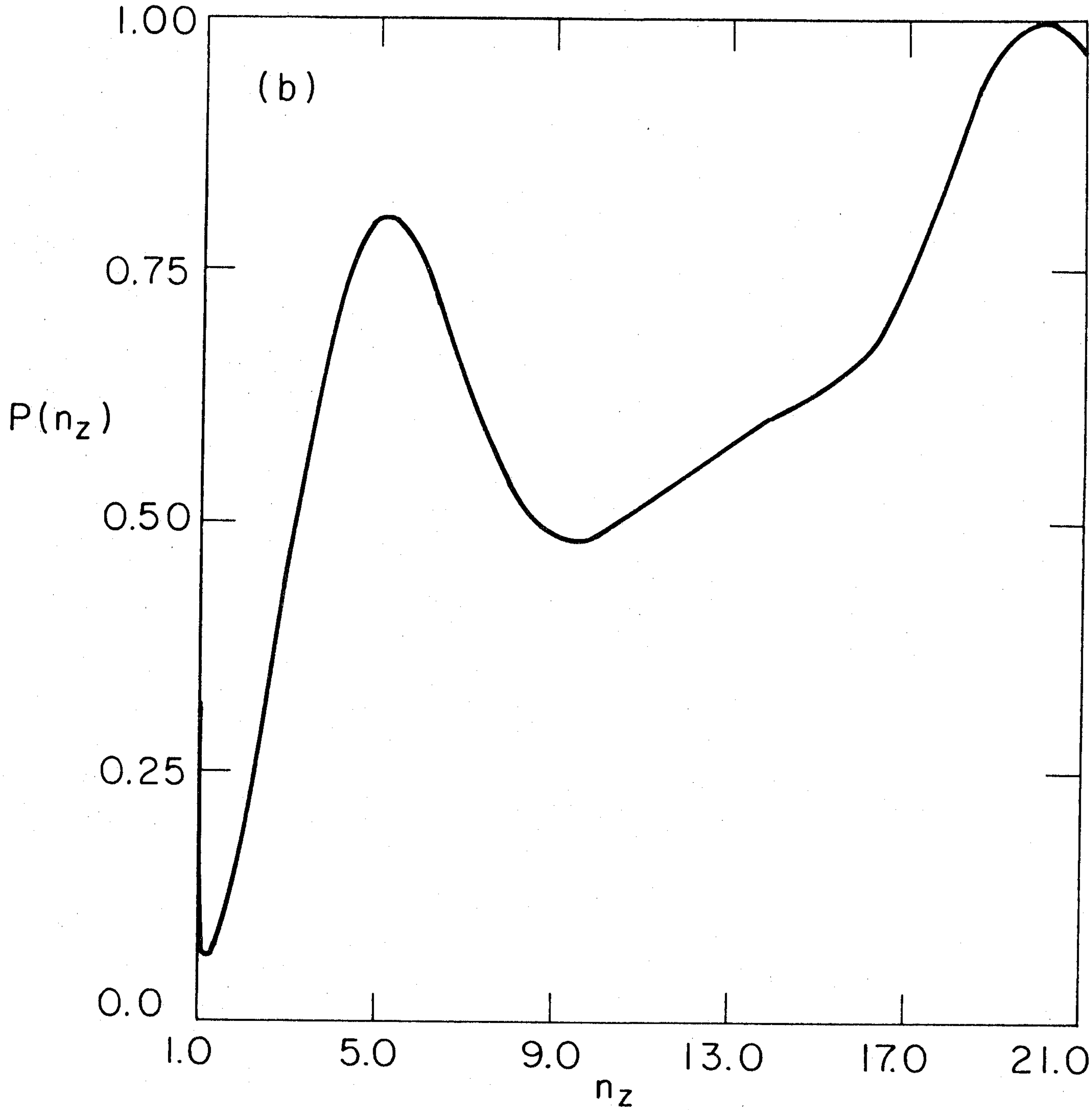
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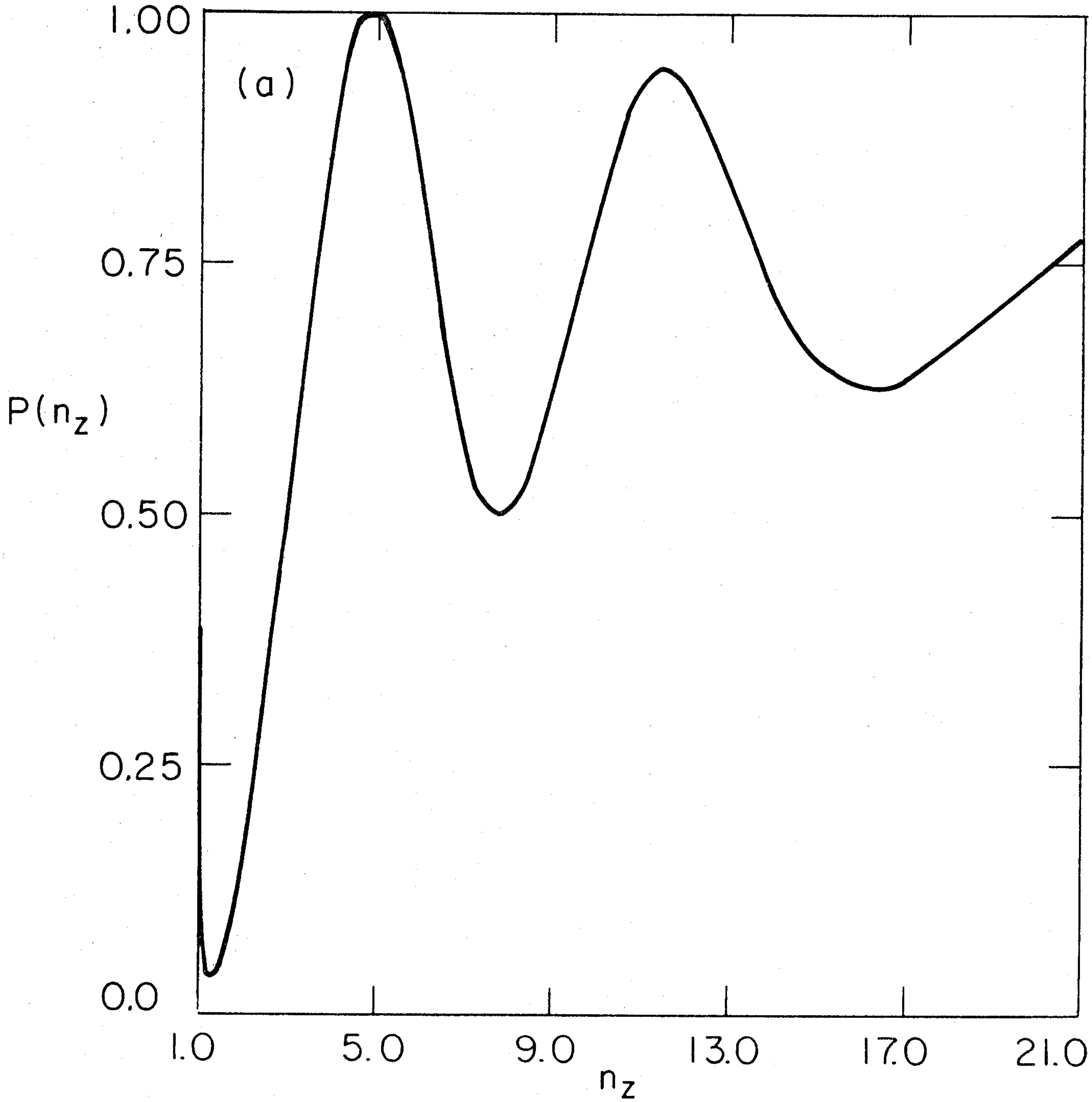

(a)
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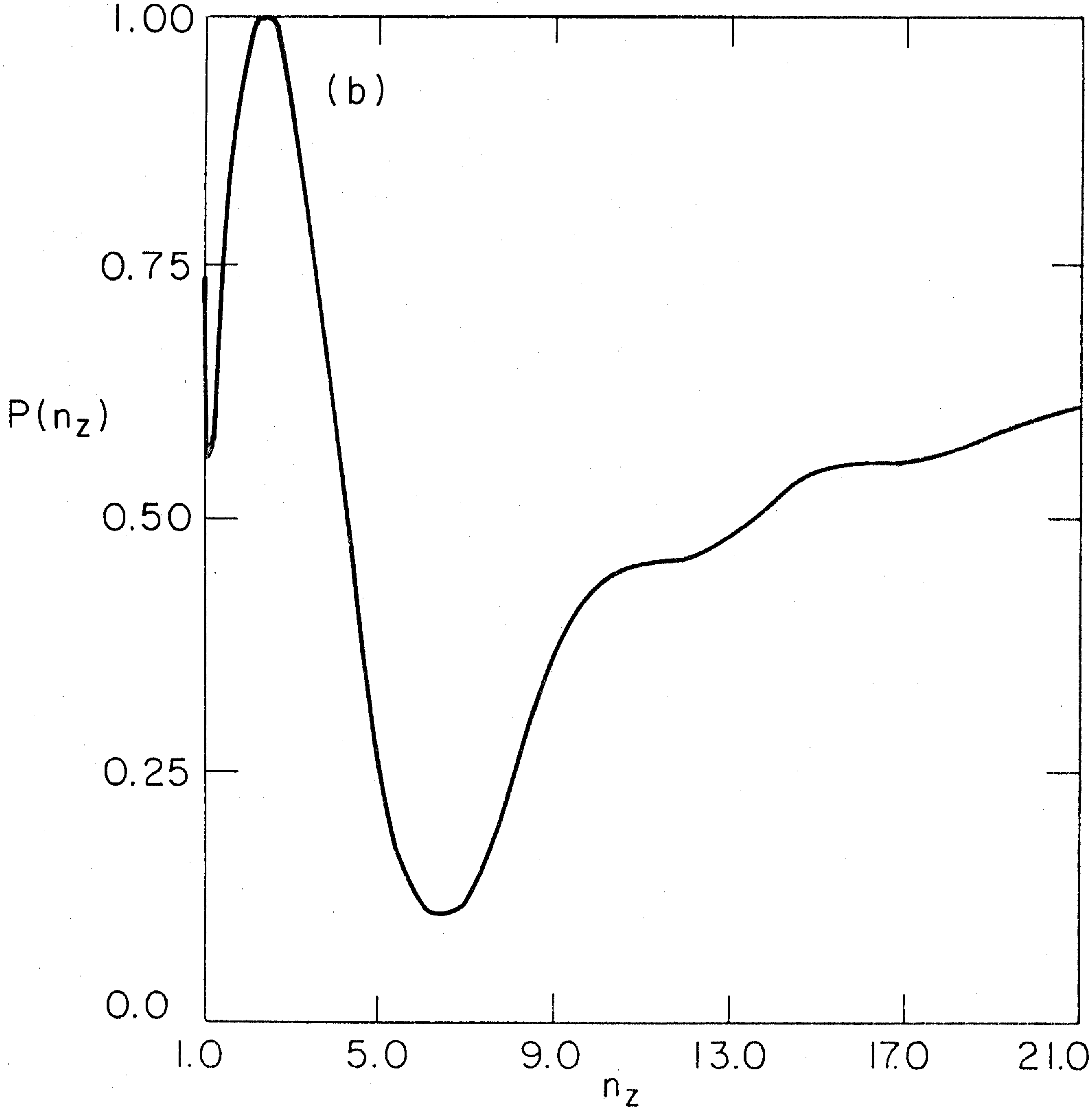
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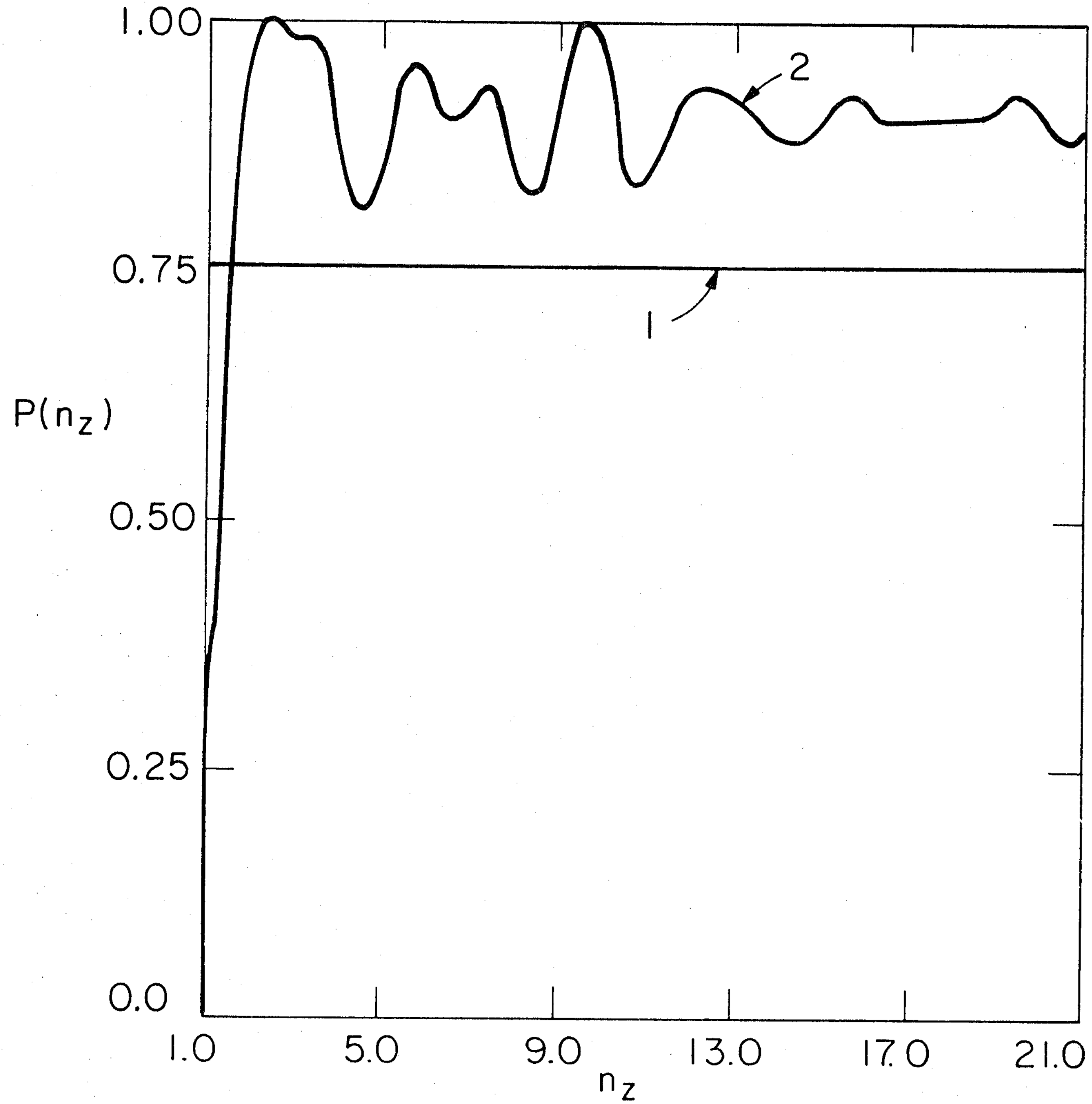
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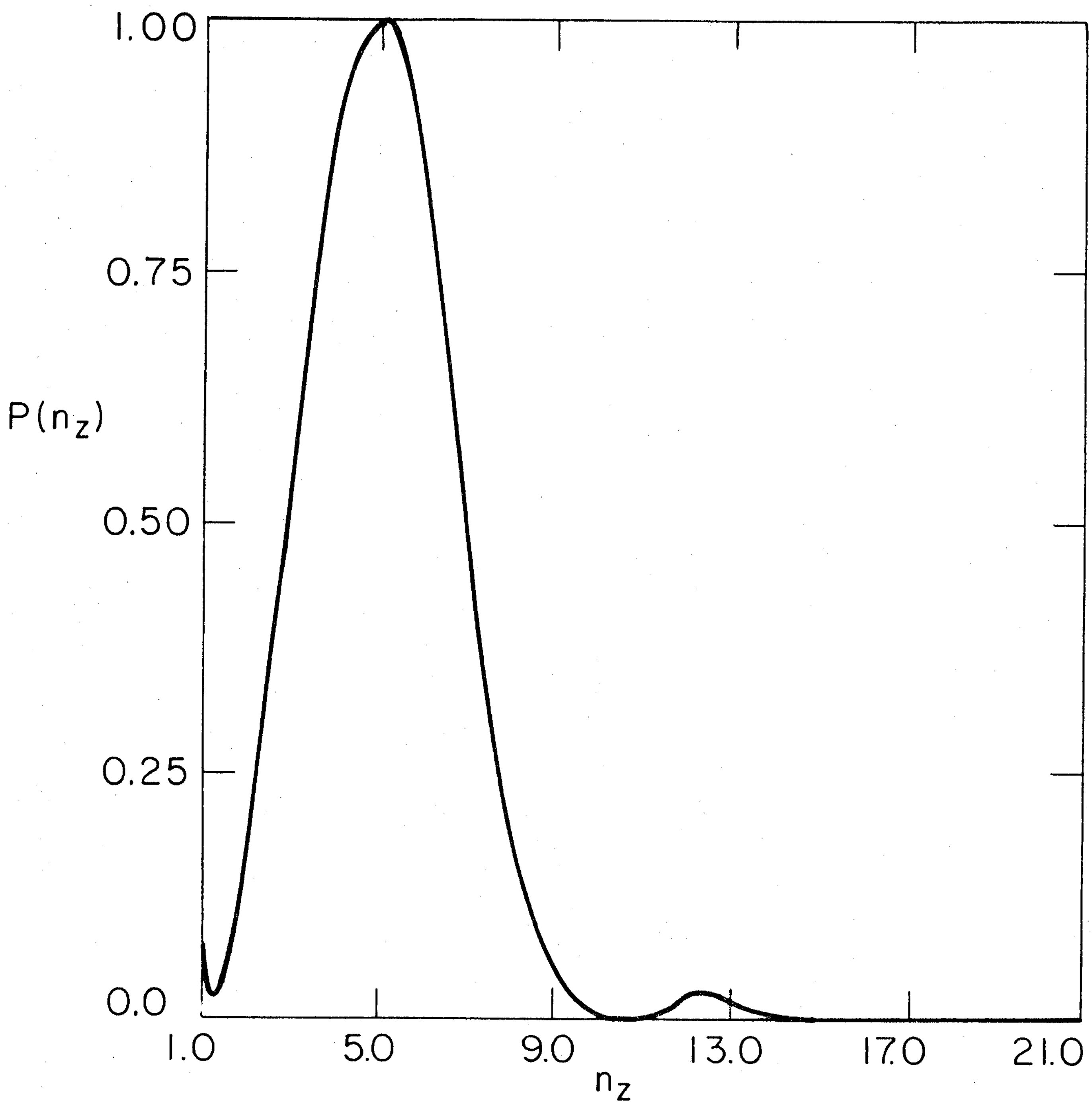
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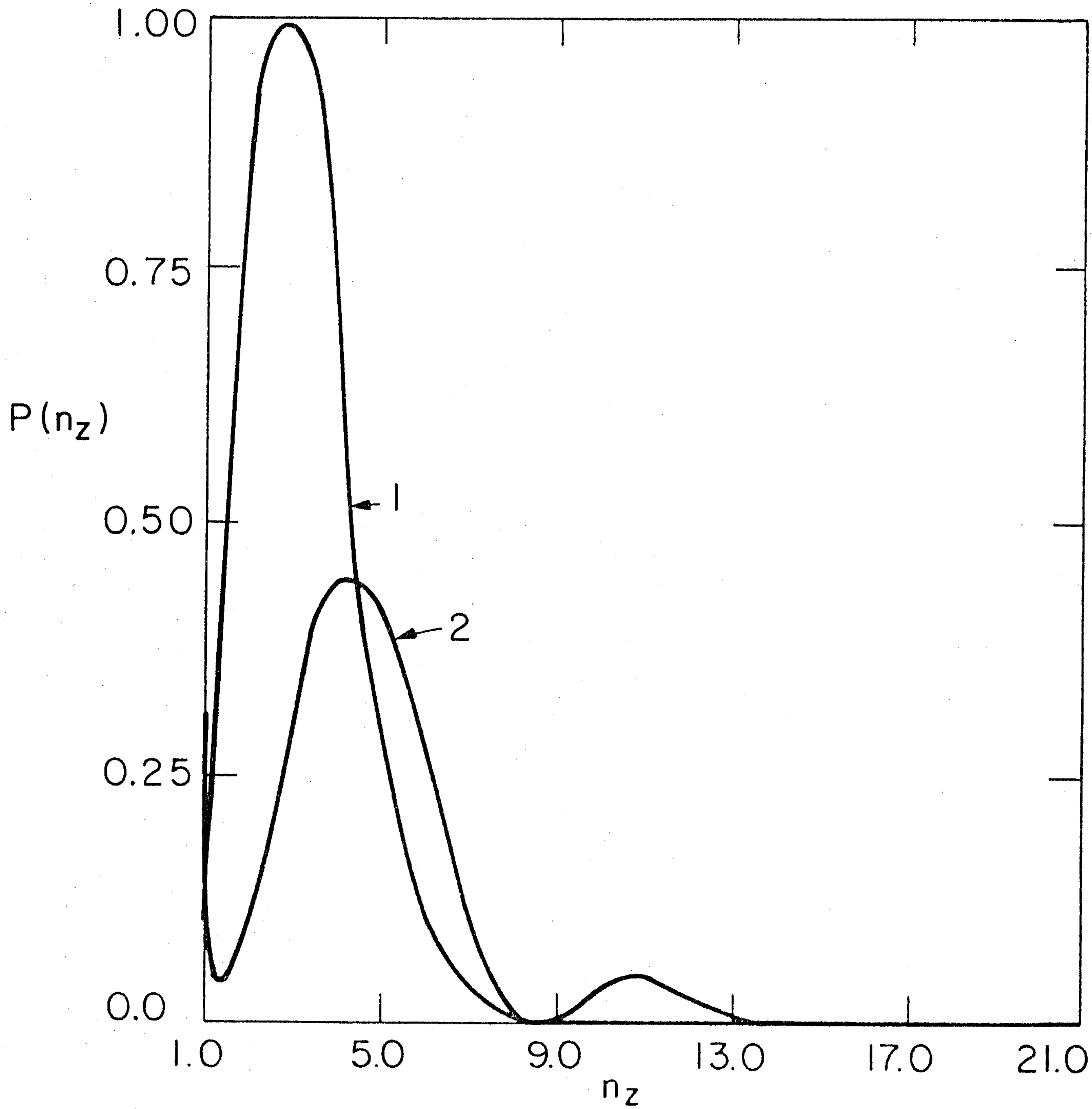
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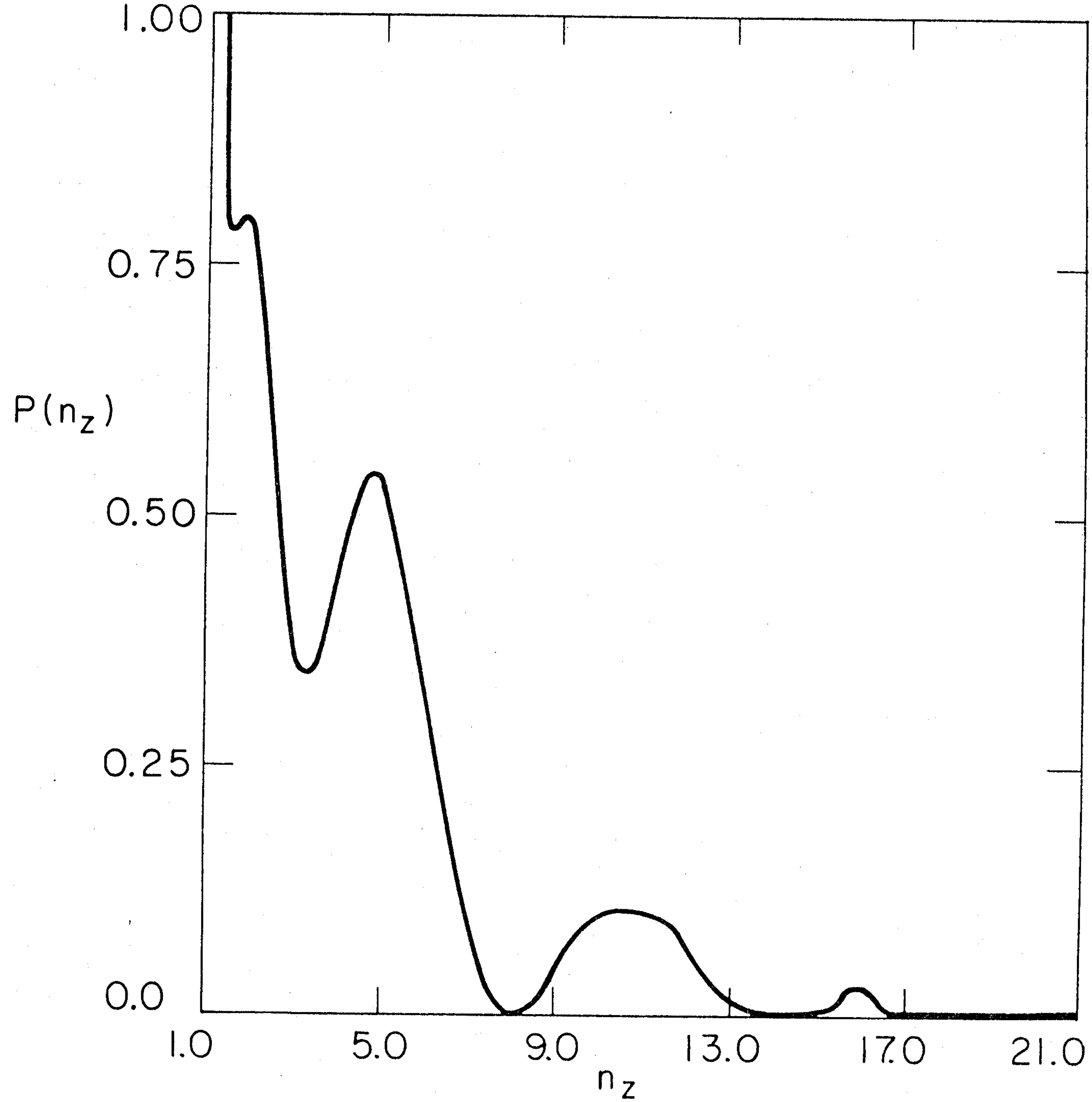
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