COUPLING TO THE FAST WAVE AT LOWER HYBRID FREQUENCIES*

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ABSTRACT.

Launching of the fast wave in the lower hybrid frequency range is described. This wave is excited at the plasma edge by RF electric fields perpendicular to those required for the lower-hybrid wave. In high temperature plasmas, where the lower hybrid wave may not penetrate because of Landau damping or other effects near the edge, the fast wave might provide an alternative for heating and/or current generation in the central portion of the plasma. In addition, for high density plasmas this has the advantage that lower frequencies than those required for the lower hybrid excitation can be used. Thus waveguides of convenient dimensions for maximum power transmission and ease of fabrication can be employed. Coupling from a waveguide array into an inhomogeneous plasma is analysed. Power reflection in the waveguides is found as a function of array design and density gradient at the edge. This reflection is fairly large (> 20%). Propagation into the plasma is then considered and the field structure and dispersion of the fast waves are found as a function of distance of penetration. Unlike the lower hybrid waves, fast waves do not form resonance cones and energy is dispersed over a large volume.

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1. INTRODUCTION

Much attention has been focused in past years on the excitation of lower hybrid waves for plasma heating and/or current generation. These waves can be excited, for instance, by a waveguide array which imposes at the edge of the plasma an electric field parallel to the confining magnetic field[1-3]. However, in the same frequency regime ($\Omega_i << \omega << \Omega_e$) there is another possibility for RF plasma heating and/or current generation. This is through the waves which propagate on the smaller k-branch ("fast" branch) of the cold plasma dispersion relation and are excited by a field perpendicular to the magnetic field. We consider in this paper coupling and propagation of these "fast" waves which, in the plasma interior, have the dispersion characteristics of Whistler or high frequency Alfven waves.

Both the coupling and the propagation of the lower hybrid ("slow") waves have been studied extensively. Depending on density, magnetic field strength and wave number k_z parallel to the magnetic field, these waves can convert linearly to a warm plasma mode ("thermal turnaround") or to the cold fast wave[4]. Both processes are to be avoided in the outer layers of the plasma, as they reflect the incident power. To avoid conversion to the fast wave, the parallel wave number must be sufficiently large; this is expressed by an accessibility condition, $n_z > n_{za}$ ($n_z \equiv ck_z/\omega$)[5,6]. On the other hand, to avoid thermal mode conversion before reaching the center, the lower hybrid waves must be excited at frequencies above lower hybrid resonance at the center. At these frequencies n_{za}, which increases with frequency, is above 1, say in the range $1.2 < n_{za} < 2$. In reactor-grade plasmas, with temperatures near ignition and beyond ($T_e \simeq 10$-15KeV), even $n_z \simeq 3$-4 will be strongly electron Landau damped at the edge. In such cases, the range of lower hybrid waves which are neither wave-converted nor Landau damped at the edge will be narrow.

For instance, consider a deuterium plasma with toroidal magnetic field $B_0 = 5$ T, center density $n_0 = 10^{14}$ cm^{-3} and temperature in the outer layers $T_e \simeq 2$ KeV. The spatial damping of lower hybrid waves is (Eqs.(1-3) are summarized in ref.(4)) :

$$k_{xi} = \pi^{1/2} \zeta^3 \, exp(-\zeta^2) \, k_{xr} \qquad \zeta = \frac{c}{\sqrt{2}\, n_z \, v_{Te}} \tag{1}$$

where k_{xr} is the perpendicular wave number, obtained from warm plasma theory. With T_e= 2 KeV, we have $\zeta \simeq 11/n_z$. To keep z-integrated damping at the edge small, ζ must be fairly large. Roughly, we need $\zeta > 4$ or, for this example, $n_z < 3$. To avoid early thermal mode conversion of the spectrum in $n_z < 3$ we must choose a sufficiently high operating frequency. For each n_z in this range we need:

$$\omega^2 \geq \omega_{LH}^2 \left(\frac{a}{2}\right) \left(1 + 2\sqrt{3}\, n_z \frac{v_{Te}}{c}\right) \tag{2}$$

where $\omega_{LH}(\frac{a}{2})$ is the lower hybrid resonance frequency midway into the plasma. The maximum lower bound occurs for n_z= 3 and the minimum allowed frequency is then $\omega = 125\omega_{LH} \simeq 2\pi$ 85 GHz. At this frequency, to avoid fast wave conversion we need:

$$n_z > n_{za} = \left(1 - \frac{\omega^2}{\Omega_e \Omega_i}\right)^{-1/2} \simeq 1.2 \tag{3}$$

where $\Omega_{e,i}$ are the cyclotron frequencies. Thus, the accessible spectrum is fairly narrow: $1.2 < n_z < 3$.

Furthermore, for high-magnetic field, "compact" reactors operating at high densities, the lower hybrid frequency reaches into the high microwave regime. At these frequencies waveguides are of small cross-section. With the high power levels required for RF heating, intricate arrays containing many waveguides will be necessary. For instance, with center density $n_0 = 10^{15}$ cm^{-3} and B_0= 10 T, we find f≥ 6 GHz, for which the waveguide width is 1÷2 cm. Large arrays of such narrow waveguides, carrying large amounts of RF power, may present difficult technical problems.

As noted before[4,6], this suggests that the fast wave be investigated as an alternative for plasma heating. As it does not suffer thermal turnaround, lower operating frequencies can be used. As n_{za} is closer to 1 at these frequencies, this broadens the accessible spectrum. Secondly, because waveguides are larger, simpler arrays with fewer elements can be used. Finally, because the feasibility of lower hybrid heating at high temperatures and densities may be marginal, fast wave heating deserves a more detailed look, despite (as will be seen) greater technical difficulties in achieving coupling. When in the future all effects are accounted for (toroidal, nonlinear, etc), the fast wave scheme may appear equally viable.

Though some authors have studied propagation of the fast wave in connection with coupling to the slow wave[2,7], no detailed study of coupling to the fast wave has been made. The absence of a resonance also rules out thermal mode conversion as an energy transfer mechanism in the plasma center. However, other processes, also studied in connection with the slow wave, could perform the transfer: parametrics, stochastic heating or even Landau damping in the interior. In the plasma interior, the dispersion relation of the fast wave and its electron Landau damping are given by:

$$n_{xr} = \frac{\omega_{pe}^2}{\omega\Omega_e}\,(n_z^2 - K_\perp)^{-1/2} \tag{4a}$$

$$k_{xi} = k_{xr} \cdot \frac{\pi^{1/2}\zeta^3 e^{-\zeta^2}}{1 + \dfrac{\Omega_e^2\,(n_z^2 - K_\perp)^2}{\omega_{pe}^2\, n_z^2}} \tag{4b}$$

where k_x, k_z are the wave numbers perpendicular and parallel to the magnetic field and n_x, n_z their indices of refraction (ω_{pe}, Ω_e and $K_\perp$ are, respectively, the plasma frequency, cyclotron frequency and dielectric tensor element, eq.(11)). These equations are obtained, respectively, from the cold-plasma dispersion relation and from a perturbation of the Vlasov dispersion relation. They assume $\omega_{pe}^2 << \Omega_e^2$, a condition generally valid in high magnetic fields ($B_0 \simeq 10$ T), and at lower densities, near the plasma edge. The damping term is similar to that of the slow wave, it has the same exponential character, but the coefficient is smaller (as k_{xr} is smaller for the fast wave and the denominator is large). From the equation for n_x, we can show that for $n_z^2 >> |K_\perp| \simeq 1$ the dispersion relation is that of a Whistler wave:

$$\omega = \frac{\omega_{pe}^2}{\Omega_e} c^2 \cdot k_x k_z \tag{5}$$

For a narrow range near $n_z \simeq 1$, the dispersion relation is that of a "high frequency Alfven" wave:

$$\omega = k_x\, c_A \qquad c_A = \frac{B_0}{(n_0 \mu_0 m_i)^{1/2}} \tag{6}$$

For intermediate values of n_z, the wave is of a "mixed" character.

In what follows we first consider coupling to the fast waves from a waveguide array and find power reflection as a function of array design and density gradient at the edge. In this we parallel the treatment of Brambilla[1] and Krapchev and Bers[3] for a two-dimensional slab geometry. We then calculate the field structure inside the plasma by evaluating the integral of Fourier components which have penetrated. We shall not consider the problem of energy transfer to the medium.

2. FORMULATION

We consider a simple slab model for coupling and propagation (see fig.(1)). The coupling structure is like the "grill" array of waveguides sketched in fig.(1)[8]. Because we want to excite fast waves, with E in the fundamental mode perpendicular to B_0, the waveguides are placed with their long edge parallel to the magnetic field. In our model the array is taken as infinite in y, an idealization of a large array of stacked waveguides. Magnetic field is along z (and assumed constant), the density gradient is in x. We assume no vacuum layer, the plasma edge is flush with the waveguide mouthes.

We shall evaluate the reflection coefficient for an array of arbitrary extent in z. As a first step, we obtain the "input admittance" of the plasma, the admittance (H_z/E_y) for an infinite, single k_z excitation at the edge. We isolate these k_z components by considering a Fourier transformation:

$$E_y(x,z) = \int_c E_y(x,n_z)\, e^{in_z z}\, dn_z \tag{7}$$

Here, $n_z \equiv k_z/k_0$ and similarly x and z are normalized b multiplication by k_0 $(k_0=\omega/c)$. The contour C (fig.(2)) is chosen so that $E_y(x,z)$ satisfies causality: it passes below any singularities in the right half plane and above singularities in the left hand plane. As will be seen, this choice insures that only outgoing waves propagate at $z \to \infty$. The basic equation for wave propagation in a cold plasma is:

$$\nabla \times \nabla \times E = K\, E \tag{8}$$

where $K = K(\omega, x)$ is the dielectric tensor. Eq.(8) contains both slow and fast waves. Near the edge, where $\omega_{pe}^2(x) << \Omega_e^2$, the waves are uncoupled by assuming $E_y \simeq 0$ for the slow and $E_z \simeq 0$ for the fast waves[2]. For the fast wave, the transformed equation for $E_y(x,n_z)$ is then:

$$\frac{d^2E_y}{dx^2} + n_x^2(x,n_z)E_y = 0 \tag{9}$$

where n_x is the "local" normalized k_x:

$$n_x^2 = \frac{K_x^2}{n_z^2 - K_\perp} - (n_z^2 - K_\perp) \tag{10}$$

$K_\perp$ and K_x are the usual elements of the cold plasma dielectric tensor. In the plasma center, where $K_x >> 1$ and $K_\perp = O(1)$, eq.(10) is the dispersion relation of a Whistler wave. Assuming a linear density profile, and constant B_0, we write:

$$K_x = \beta x \qquad K_\perp = 1 - \gamma x \tag{11}$$

where ($\Omega_i << \omega << \Omega_e$):

$$\beta \equiv \frac{d}{dx}\left(\frac{\omega_{pe}^2}{\omega\Omega_e}\right)_{x=0} \qquad \gamma \equiv \frac{d}{dx}\left(\frac{\omega_{pi}^2}{\omega^2}\right)_{x=0} \tag{12}$$

If the scale length near the edge is of order a:

$$\beta \simeq \frac{\omega_{pi}(a)}{\omega}\,\frac{\omega_{pi}(a)}{\Omega_i}\,\frac{1}{k_0 a} \tag{13a}$$

$$\gamma = \frac{\Omega_i}{\omega}\beta \tag{13b}$$

For instance , consider $n_0(a) = 10^{14}$ cm^{-3}, $B_0 = 5$T and a=10 cm. Choosing the operating frequency well below lower hybrid, $f = 0.6 f_{LH}(a)$, we find $\beta = 10$, $\gamma = 0.6$ and $f \simeq 1.4$ GHz. The normalized x in eq.(9) varies from 0 to 2.8 over the width of the density gradient. The accessibility condition is $n_z > n_{za} = 1.1$. Finally, $\omega_{pe}^2(a)/\Omega_e^2 = 0.4 < 1$, so that eq.(9) is roughly valid even in the center.

We solve (9) in a region which extends from the wall (x=0) to a point where $n_x^2 >> 1$ (where we can connect to the WKB solution). This region is: $0 \le x <$ a few$(1/\beta)$. Eq.(9) is greatly simplified when $|n_z^2 - 1| >> \gamma/\beta = \Omega_i/\omega$: in this domain we neglect γx in the region of integration (so that $K_\perp \simeq 1$) and obtain:

$$\frac{d^2E_y}{dx^2} - \left(\frac{\beta^2 x^2}{1-n_z^2} - (1-n_z^2)\right)E_y = 0 \tag{14}$$

As $\Omega_i << \omega$, eq.(14) is valid for all but a very narrow part of the n_z spectrum. For instance, with the parameters given above, we have $\Omega_i/\omega = 1/25$, and eq.(14) ceases to be valid in the range $|n_z - 1| < 0.02$. In what follows, we assume eq.(14) to be valid for the entire n_z spectrum, neglecting the effects of the small region near $n_z=1$. Eq.(14) has a solution in terms of parabolic cylinder functions. We write:

$$E_y(x,n_z)=E_y(0,n_z)\,\frac{U(a,\xi)}{U(a,0)} \tag{15}$$

where:

$$\xi=\frac{(2\beta)^{1/2}}{(1-n_z^2)^{1/4}}\,x \qquad\qquad a=-\frac{(1-n_z^2)((1-n_z^2)^{1/4})^2}{2\beta} \tag{16}$$

where U is the parabolic cylinder function[9]. The branch cuts of the function $(1-n_z^2)^{1/4}$ are shown in fig.(2). They are chosen so that, when C is close to the real axis, $(1-n_z^2)^{1/4}\simeq|1-n_z^2|^{1/4}$ when $|n_z|<1$ on C and $(1-n_z^2)^{1/4}\simeq e^{i\pi/4}\,|1-n_z^2|^{1/4}$ when $|n_z|>1$. The solution of eq.(14) satisfies two boundary conditions: first a condition at $x=0$, $E_y(x\to 0,n_z)=E_y(0,n_z)$ where $E_y(0,n_z)$ is the spectrum of the excitation; secondly a radiation condition for large x: only evanescent or outgoing waves are allowed.

Waves with $|n_z|>1$ are cutoff near the plasma edge, but can "tunnel through" to higher densities where they are propagating. For instance, for large $|n_z|$ and large x we find:

$$E_y(x,n_z)\simeq exp\left(i\,\frac{\beta x^2}{2n_z}\right)x^{-1/2}\,\frac{n_z^{1/4}}{(2\beta)^{1/4}}\,2^{1/4}|a|^{1/4}e^{-|a|\pi/2} \tag{17}$$

where the exponential "tunnelling factor" $e^{-|a|\pi/2}=\exp(-\pi(n_z^2-1)^{3/2}/4\beta)$ accounts for the large cutoff layer. Waves with $|n_z|<1$ propagate near the edge but are cutoff further on. For these waves we have for large x:

$$E_y(x,n_z)\simeq exp\left(-\frac{\beta x^2}{2(1-n_z^2)^{1/2}}\right)x^{|a|-\frac{1}{2}}\,2^{|a|}\,\beta^{\frac{1}{2}(|a|-\frac{1}{2})}\,(1-n_z^2)^{\frac{1}{8}-\frac{1}{4}|a|}\,\Gamma\left(\frac{3}{4}-\frac{1}{2}|a|\right) \tag{18}$$

where $|a|\simeq 1$. The first term is evanescent: for $\beta\simeq 1$ the waves decay rapidly for $x>1$. The gamma function may have singularities for real n_z. These singularities correspond to multiple reflections between the edge and the cutoff. A finite number of resonances occur when $|n_z|<1$ and:

$$n_z^2=1-(3\beta)^{2/3},\ 1-(5\beta)^{2/3},\ \ldots\ 1-((2p+1)\beta)^{2/3},\ \ldots \qquad p=0,\,1,\,2,\ldots \tag{19}$$

A necessary condition for resonance is that the cutoff layer (for $n_z<1$) be large enough: $\beta<1/3$.

The input admittance Y_{in} is evaluated from Faraday's law:

$$H_z(0,n_z) = -i\,\frac{dE_y(x,n_z)}{dx}\Big|_{x=0} \tag{20}$$

and from eq.(10) we get:

$$Y_{in}(n_z) \equiv \frac{H_z(0,n_z)}{E_y(0,n_z)} = i\,\frac{2\beta^{1/2}}{(1-n_z^2)^{1/4}}\,\frac{\Gamma(\frac{3}{4}+\frac{a}{2})}{\Gamma(\frac{1}{4}+\frac{a}{2})} \tag{21}$$

where a is complex.

To evaluate coupling we use Brambilla's method[1]. The waveguide fields are:

$$E_y(x,z) = \sum_m E_m(z)\,(A_m e^{in_m x} + B_m e^{-in_m x}) \tag{22}$$

$$H_z(x,z) = \sum_m Y_m^W\,E_m(z)\,(A_m e^{in_m x} - B_m e^{-in_m x}) \tag{23}$$

where $Y_m^W = n_m$ is the admittance of a given mode in a given waveguide and $n_m = (1 - (m\,\pi/b)^2)^{1/2}$ its wavenumber (b is the waveguide width). The modes have a sinusoidal dependence on z: $E_m(z)$ $\sin(m\pi z/b)$. A_m and B_m are the amplitudes of incident modes (known) and reflected modes (to be determined). All modes are of the TE_{m0} type, with $E_z=0$. Eq.(18) relates $H_z(x=0)$ to $E_y(x=0)$. Another relation is found with the plasma admittance:

$$H_z(0,z) = \int_c Y_{in}(n_z)\,E_y(0,n_z)e^{in_z z}\,dn_z \tag{24}$$

Fourier transforming (22), substituting it in the right hand side of (24), putting (23) on the left hand side and using mode orthogonality we obtain for each s:

$$Y_s^W\,(A_s - B_s) = \sum_m Y_{sm}^P\,(A_m + B_m) \qquad s = 1, 2, 3, \ldots \tag{25}$$

where:

$$Y_{sm}^P = \int_c E_s^*(n_z) Y_{in}(n_z) E_m(n_z)\,dn_z \tag{26}$$

We assumed the normalization:

$$\int_{-\infty}^{\infty} E_s(z)E_m^*(z)\,\frac{dz}{2\pi} = \delta_{sm} \tag{27}$$

We shall simplify (25) by retaining only fundamental modes in the expansion.

Y_{sm}^{P} can be evaluated by collapsing C onto the real axis:

$$Y_{sm}^{P} = PP\int_0^1 Y_{in}(n_z)\, 2\, Re[E_s^*(n_z)E_m(n_z)]\, dn_z$$

$$+ \int_1^\infty Y_{in}(n_z)\, 2\, Re[E_s^*(n_z)E_m(n_z)]\, dn_z$$

$$+ \sum_{p\geq 0}\beta \frac{16}{3\pi} \frac{\Gamma(\frac{3}{2}+p)}{n_{zp}(4p+3)^{1/2}} Re[E_s^*(n_{zp})E_m(n_{zp})] \tag{28}$$

The first integral above is always a reactive term: it accounts for reflection by the cutoff layer of $|n_z|<1$ waves. We have from eq.(21):

$$|n_z|<1 \qquad Y_{in}(n_z)= i\,\frac{2\beta^{1/2}}{(1-n_z^2)^{1/4}} \frac{\Gamma(\frac{3}{4}-\frac{1}{2}|a|)}{\Gamma(\frac{1}{4}-\frac{1}{2}|a|)} \qquad |a|= \frac{|1-n_z^2|^{3/2}}{2\beta} \tag{29}$$

For β large (steep gradient) and n_z moderate ($n_z<\beta^{1/3}$) we get:

$$Y_{in}(n_z)\simeq i\,\frac{0.68\beta^{1/2}}{(1-n_z^2)^{1/4}} \tag{30}$$

The second integral in eq.(28) is a mixed term containing both a reactive part (impedance mismatch) and a real part (radiation into the plasma). We have:

$$|n_z|>1 \qquad Y_{in}(n_z)=e^{i\frac{\pi}{4}}\, \frac{2\beta^{1/2}}{(n_z^2-1)^{1/4}} \frac{\Gamma(\frac{3}{4}+i\frac{|a|}{2})}{\Gamma(\frac{1}{4}+i\frac{|a|}{2})} \qquad |a|= \frac{|n_z^2-1|^{3/2}}{2\beta} \tag{31}$$

We saw in eq.(17) that for waves with $|n_z|>1$, $|a|$ is a "tunneling" factor: when $|a|>>1$ there is complete reflection of incident power. The admittance reduces to its free-space value:

$$Y_{in}(n_z)\simeq i(n_z^2-1)^{1/2} \tag{32}$$

Thus evanescence imposes a second accessibility condition on n_z: n_z must not be too large, $|n_z|<\beta^{1/3}$. For small $|a|$ (β large or $|n_z|\simeq 1$) we have:

$$Y_{in}(n_z)\simeq e^{i\frac{\pi}{4}}\, \frac{0.68\beta^{1/2}}{(n_z^2-1)^{1/4}} \tag{33}$$

The last term in Eq.(28) comes from the poles of the admittance, which lie on the real axis in $|n_z|<1$ (Eq.(19)). This contribution is purely resistive: it accounts for power loss to surface waves, and can be comparable to the two other terms. Note that surface waves are excited only for small β, $\beta<1/3$.

3. NUMERICAL RESULTS FOR COUPLING

Eq.(28) suggests the kind of excitation which optimizes coupling: to minimize the reactive part of the admittance there must be little energy in $|n_z|<1$ and $|n_z|>>\beta^{1/3}$. The first condition is a stringent one: remember that for a vacuum filled waveguide operating above cutoff, we have, at fixed frequency, $b>\pi$. This keeps most of the spectrum in $|n_z|<1$. To broaden the spectrum, we must artificially decrease the cutoff width: ridge waveguides or waveguides filled with dielectric are possibilities[1]. We illustrate what follows with such waveguides filled with dielectric ϵ_r. To further improve the spectrum, we narrow it about some value of $n_z>1$ by using a phased array.

In fig.(3) we consider coupling of a four-waveguide array. Only fundamental modes were included in the calculation. The incident fields are of equal amplitude and phased 180^o. The individual waveguides are each of normalized width b. The dielectric is ϵ_r=4.0, for which the cutoff b is $\pi/2$. Total power reflected $|R|^2$ is plotted against b with β as a parameter (for a given ϵ_r, these two variables determine entirely $|R|^2$). First, for small β, coupling deteriorates as $\beta\to 1/3$: we approach one of the resonances of eq.(19) and $|R|^2\to 1$. For a series of values $\beta<1/3$ (not shown on the figure) $|R|^2$ has a series of peaks where it takes the value $|R|^2=1$ irrespective of b. These peaks occur whenever a new resonance in eq.(14) is excited by $n_z=0$. Thus, when $\beta<1/3$, power is also coupled to surface waves, an undesired effect. For larger β, coupling improves: a minimum $|R|^2$ is obtained for $\beta\simeq 10.$ and $b\simeq 2.5$. Power reflected is then about 35% and the VSWR in each waveguide about 3 . For larger β, coupling deteriorates again because of the impedance mismatch between the short plasma waves and the longer waveguide waves (which have an impedance comparable to that of free space). Finally, $|R|^2=1$ at the cutoff $b=\pi/2$ and for $b\to\infty$.

In fig.(4), we consider an infinite array of waveguides phased 180^o. Again, ϵ_r=4.0. This array produces a delta-function spectrum at $n_{z\infty}=\pi/b$. We plot $|R|^2$ against $n_{z\infty}$ for some values of β.

Coupling is improved moderately over the previous case of four waveguides.

In the previous examples we chose $\varepsilon_r = 4$. For smaller ε_r ($1 \leq \varepsilon_r \leq 4$) coupling worsens for all β. This is because to get a sizeable part of the spectrum in $|n_z| > 1$ we need $b < \pi$ and for these values of b the waveguide admittance, $(\varepsilon_r - (\pi/b)^2)^{1/2}$, is small: this means larger admittance mismatch and more reflection. For large ε_r ($\varepsilon_r >> 1$) we can estimate more quantitatively the reflection: consider a large array, phased by 180^0, with a narrow spectrum centered about some n_{z0}. For this array, reflection is approximately:

$$|R|^2 = \frac{|Y_1^W - Y_{in}(n_{z0})|^2}{|Y_1^W + Y_{in}(n_{z0})|^2} \qquad Y_1^W = (\varepsilon_r - n_{z0}^2)^{1/2} \tag{34}$$

where Y_1^W is the waveguide impedance of the fundamental mode (Eq.(23)) and $Y_{in}(n_z)$ the plasma impedance (Eq.(21)). If $|n_z^2 - 1| << \beta^{2/3}$ we can use eq.(33) as an approximation to $Y_{in}(n_z)$. Also, with $\varepsilon_r >> 1$, $Y_1^W \simeq \varepsilon_r^{1/2}$. Using these approximations we find a relatively simple expression for $|R|^2$. This has a minimum when:

$$n_{z0}^2 = 1 + 0.7\frac{\beta}{\varepsilon_r} \qquad for\ \varepsilon_r >> 0.45\,\beta^{1/3} \tag{35}$$

for which:

$$|R(n_{z0})|^2 = 0.18 \tag{36}$$

Eq.(35) is consistent with the curves of fig.(4), at least for $\beta < 10.$. We obtain roughly the minimum predicted above, $|R|^2 \simeq 20\%$. When $\beta > 10.$, eq.(35) is no longer valid. n_{z0} as predicted from eq.(35) becomes large and approximation (33) does not hold. In fact, $Y_{in}(n_z)$ is more reactive than predicted by eq.(33) (there is more tunnelling through the cutoff layer), and the minimum $|R|^2$ worsens. For $\beta \rightarrow 0$ the minimum of (6) is reached very near to $n_{z0} = 1$, but worsens for all other $n_z > n_{z0}$ because the cutoff layer has become large. In fig.(3), the finite width of the excitation spectrum tends to worsen both impedance mismatch and tunneling, and the transmission is correspondingly poorer. The minimum of eq.(36), $|R|^2 = 18\%$, appears to be a minimum for all array designs.

Having obtained reflection coefficients, we can predict the power spectrum of waves excited in the plasma. Total power transmitted into the plasma is given by:

$$P_t = \int_c \frac{1}{2} Re\, E_y(n_z) H_z^*(n_z)\, dn_z$$

$$= \int_1^\infty Re[Y_{in}(n_z)\, |E(n_z)|^2]\, dn_z + \sum_{p\geq 0} \beta \frac{8}{3\pi} \frac{\Gamma(\frac{3}{2}+p)}{n_{zp}(4p+3)^{1/2}} |E(n_{zp}|^2 \tag{37}$$

where we used the symmetry of Y_{in} and $E(n_z)$ to reduce the integration to one over only positive n_z. $E(n_z)$ is obtained from eq.(22). The integral on $n_z \geq 1$ gives the power which actually penetrates into the plasma. The second term is the power coupled into surface waves (right and left-going pairs). This latter term is present only if $\beta < 1/3$, in which case power coupled into surface waves may be considerable and comparable to the power coupled into Whistler waves. For the curves of fig.(3) however, $\beta > 1/3$ and no surface waves are excited. We plot in fig.(5) the function:

$$P_t(n_z) = \frac{1}{2} Re\, E_y H_z^* = \frac{1}{2} Re\, (Y_{in}(n_z)\, |E_y(n_z)|^2) \tag{38}$$

with β=10. and b=2.5 . For comparison, incident power with no reflection is also shown. Incident power is infinite near n_z=1, but this is an integrable singularity. A narrow range of n_z close to 1 is below accessibility (n_z<1.1), and power in this range does not reach the plasma interior, but this range is narrow so that despite the singularity, the integrated power is not too large (< 25%).

We have not mentioned the effect of varying the relative amplitudes of fields in inner and outer waveguides. For the four waveguide array, this decreases somewhat the coupling but could reduce the energy in the range below accessibility, so that overall power penetrating to the plasma center would be increased[3]. However, as we are interested in regimes where n_{za} is close to 1, the energy below accessibility will be small to start off with, and so will be any improvement in overall coupling.

To summarize, an array for coupling fast waves might be designed as follows: first, given the plasma properties (maximum density, magnetic field) a range of operating frequencies is chosen. These frequencies are fairly low, to insure $n_{za} \simeq 1$ and reasonable waveguide dimensions. Secondly, this range is narrowed, both in consideration of whatever heating mechanisms are to be used and also so as to insure that the gradient parameter β is in the range 1-10 where optimum coupling takes place. The density scale length, which in part determines β might also be adjustable, through the use of plasma limiters for instance. Fair coupling ($|R|^2 < 40\%$) can then be achieved with a modest dielectric $\varepsilon_r \simeq 3$-4. ε_r is an "effective" dielectric: it could be obtained, for instance, with ridge waveguides. Finally, the fairly large reflection can in principle be tuned out, so that all of the generator power ends up in the plasma. However,on account of the large standing wave ratios there will be a lot of stored energy in the tuner and waveguides.

4. FIELD STRUCTURE

Field structure is found by evaluating the Fourier integral (7) which now reads:

$$E_y(x,z) = \int_c E_y(0,n_z) \frac{U(a,\xi)}{U(a,0)} e^{in_z z} \, dn_z \tag{39}$$

This is valid as long as $K_\perp \simeq 1$, that is not too large densities (we extend eq.(39) shortly) . We evaluate (39) for fairly large x and z (as compared to the source dimensions), by using stationary phase. For z>0 we deform the contour as shown in fig.(2), picking up (if β is small enough) the residues from the poles on the real axis (the field structure for z<0 is a mirror image):

$$E_y(x,z) = \sum_p i\left[\frac{2^{1/3} 4\pi^{1/2}}{3} \, 2^{-p-1} \left(p+\frac{3}{4}\right)^{-1/3} n_{zp}^{-1} \right] \beta^{2/3} e^{-\frac{1}{4}\xi_p^2} H_{2p+1}(\xi_p/2^{1/2}) \, e^{in_{zp} z}$$

$$+ \int_1^\infty E_y(0,n_z) e^{in_z z} \frac{U(a,\xi)}{U(a,0)} \Big|_{above}^{below} \, dn_z \tag{40}$$

where:

$$\xi_p = 2^{1/2} \beta^{1/3} (4p+3)^{-1/6} \, x \tag{41}$$

and where the H_{2p+1} refer to Hermite polynomials. In eq.(40) "above" and "below" refer to ξ being evaluated above and below the branch cut. The first term in (40) is the surface-wave contribution: these waves propagate away from the source by multiple reflections between wall and cutoff layer. They decay rapidly inside the plasma, where their contribution may be neglected. As for the integral, in doing a stationary phase estimation for z>0, it is found that the contributions from above the branch cut phase mix and may be neglected. This leaves us with:

$$E_y(x,n_z) \simeq \int_1^\infty E_y(0,n_z)\, e^{in_z z} \frac{U(a,\xi)}{U(a,0)}\Big|_{below}\, dn_z \tag{42}$$

We extend eq.(42) to regions where $K_\perp \neq 1$ by using the WKB solution of eq.(3):

$$E_y(x,z) \simeq \int_1^\infty E_0(n_z)\, n_x^{-1/2}\, exp(\, i(n_z z + \int_0^x n_x(u,n_z) du)\,)\, dn_z \tag{43}$$

where, to within "slowly" varying phase terms:

$$E_0(n_z) = 2^{-1/4} \beta^{1/4} (n_z^2 - 1)^{-1/8} \frac{e^{-|a|\pi/4}}{U(a,0)}\, E_y(0,n_z) \tag{44}$$

Stationary phase integration yields:

$$E_y(x,z) \simeq E_0(n_0)\, n_x(n_0)^{-1/2}\, e^{in_0 z}\, \frac{(2\pi)^{1/2}}{\left|\frac{\partial z_R}{\partial n_z}\right|^{1/2}_{n_z = n_0}} \tag{45}$$

where the n_0, the n_z for which the phase is stationary, is determined from the ray condition:

$$z_R(x,n_0) = z \qquad z_R(x,n_z) \equiv -\int_0^x \frac{\partial n_x}{\partial n_z}(u,n_z)\, du \tag{46}$$

Note that $z = z_R(x,n_0)$ is the ray trajectory of a wave packet centered at $n_z = n_0$. Approximating $n_x \simeq K_x/(n_z^2 - K_\perp)^{1/2}$ (neglecting the cutoff) we can find an analytic expression for z_R. In particular:

$$z_R(x,n_0) \simeq \frac{\beta}{2\gamma^2} \frac{(\gamma x)^2}{n_0^2} \qquad n_0 >> 1 \tag{47a}$$

$$\simeq \frac{2\beta}{\gamma^2} (\gamma x)^{1/2} \qquad n_0 \simeq 1 \tag{47b}$$

Eq.(47) indicates how much the rays spread. The other field components are evaluated in a similar fashion, using the local polarizations in the WKB solutions. Inside the plasma, where $n_x >> 1$, E_x is the dominant component ($|E_x| >> |E_y| \simeq |E_z|$).

We show in fig.(6) the far field evaluated from (45) (shown for $z>0$ only, the fields for $z<0$ are a mirror image). In this example the source is the four-waveguide array of fig.(5), with $b=2.5$, $\beta = 10$ and phasing of 180^0. We took $B_0 = 5T$, $n_0(a) = 10^{14}$ cm^{-3}, $a = 10$ cm and $f = 0.6 f_{LH} = 1.4$ GHz, for which $\beta = 10$ and $\gamma = 0.6$. With these parameters, the total width of the array is about 35 cm. Power reflection in the waveguides, as read from fig.(3), is about 40%. The maximum amplitude in each guide, including reflection, is taken as 1: $E_y(z) = \pm\sin(\pi z/b)$. In fig.(6) we plot the field amplitudes, $|E_j(x,z)|$, as found from eq.(45). The radial distance x is fixed, x= 10 cm, so that we are looking at the fields near the center of the plasma. The amplitudes are modulated by $\exp(in_0(z)z)$, where n_0 is determined from eq.(46). For much of the field extent, $n_0 \simeq 1$.

Note that there are no resonance cones as found for slow wave penetration: the rays have spread and dispersed power. This dispersion reduces the amplitudes of E_y and E_z. For E_x, the dominant component, the change in impedance of the medium more than compensates for dispersion, and the field amplitude is quite large. We haven't mentioned the effects of confluence of slow and fast waves . The confluence layer reflects waves with $n_z<n_{za}$, where $n_{za} \simeq 1.1$ is the accessible wave number (Eq.(3)). This cuts off the part of the spectrum which contributes to much of the spreading of the fields in real space. This effect is indicated qualitatively in the figure: the dashed line indicates roughly the maximum extent of the fields when that part of the spectrum which is below accessibility is removed from the integration in eq.(43) (we simply cut off the fields in $z> z_R(n_{za})$). Everything to the right of the line would in fact be converted to slow waves, with possible multiple reflections in the outer plasma layers. These "surface waves" (different from those excited near the edge when $\beta<1/3$) carry about 25% of the transmitted power (fig.(5)).

5. CONCLUSION

Fast wave coupling by a waveguide array appears at least marginally possible.The array may need to be fairly large, to concentrate the spectrum at low n_z values for which the cutoff layer is small, and also to avoid Landau damping which in a hot plasma will damp both slow and fast waves with even moderate n_z. To avoid in addition the $|n_z|<1$ evanescence, the complication of ridge waveguides or waveguides filled with dielectric must also be considered. Also, some tuning system will be needed to compensate the fairly large reflection ($|R|^2 \simeq$ 20-40%), and large standing wave ratios will exist in the waveguides. Finally, for very gentle density gradients ($\beta<1/3$), there is the danger of resonance effects which couple energy to surface waves.

Fast waves do not form resonance cones. Rather, energy is dispersed in a large region of plasma and the amplitude of the waves is diminished in proportion. This feature is an advantage in avoiding too localized heating and may also prevent nonlinear effects such as filamentation. The other advantage was pointed out in the introduction: fast waves can penetrate with n_z very close to 1, and may avoid excessive Landau damping in very hot thermonuclear plasmas.

Our analysis of the coupling problem was greatly simplified by ignoring finite y dimension, higher order modes, etc. Also, nonlinear effects, always present at the high power levels needed to heat plasmas, may modify considerably the linear picture. Finally, with their large spatial dispersion, fast waves may fill the entire toroidal cavity. A proper treatment would then require us to find the coupling to the cavity modes and in effect solve a resonator problem.

6. ACKNOWLEDGMENTS

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7. FIGURE CAPTIONS

FIG.(1): Geometry for the Fast Wave Excitation.

FIG.(2): Contour for the Fourier Integral. Here $1/7 < \beta < 1/5$ so that ther e are two resonances in $0 < n_z < 1$.

FIG.(3): Power reflection $|R|^2$ for a four-waveguide array. β is the gradient parameter, eq.(7). The waveguides are filled with dielectric $\epsilon_r=4$, which reduces the cutoff width to $b = \pi/2$. Phasing is 180^0.

FIG.(4): Power reflection $|R|^2$ for an infinite array. The array produces a single $n_{z\infty}=\pi/b$. The waveguides are filled with dielectric $\epsilon_r=4$ and phasing is 180^0.

FIG.(5): Power spectrum for a four-waveguide array. We have $\beta= 10$ and b=2.5 . The dashed line (P_i) is power in the absence of reflection, the full line (P_t) power transmitted with reflection. Total power reflected is about 35%.

FIG.(6): Field structure for a four-waveguide array. Here $B_0= 5T$, $n_0(a)= 10^{14}$ cm^{-3} and f= 0.6 $f_{LH}(a) \simeq$ 1.4 GHz, b= 9 cm. We are looking at the fields in the center, x= 10 cm. The rays to the right of the ray $z= z_R(n_{za})$ have converted to slow waves.

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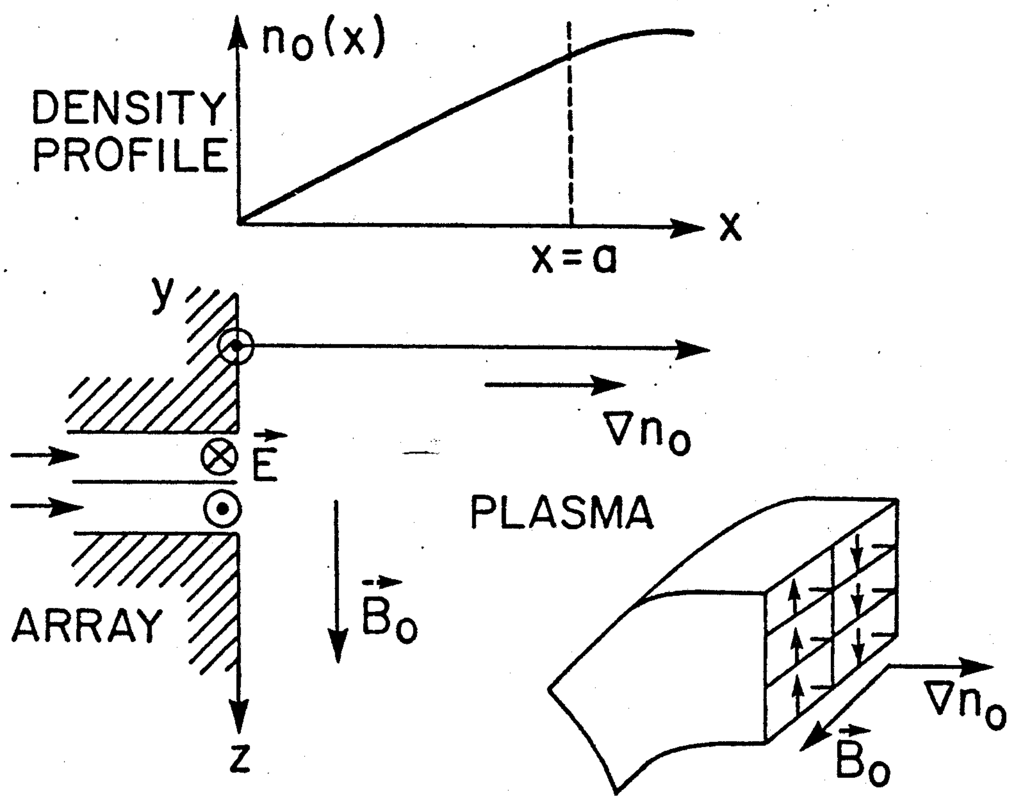


$n_0(x)$
DENSITY
PROFILE
x
x = a
y
∇n_0
$\vec{E}$
PLASMA
ARRAY
$\vec{B}_0$
z
∇n_0
$\vec{B}_0$

FIG. (1)

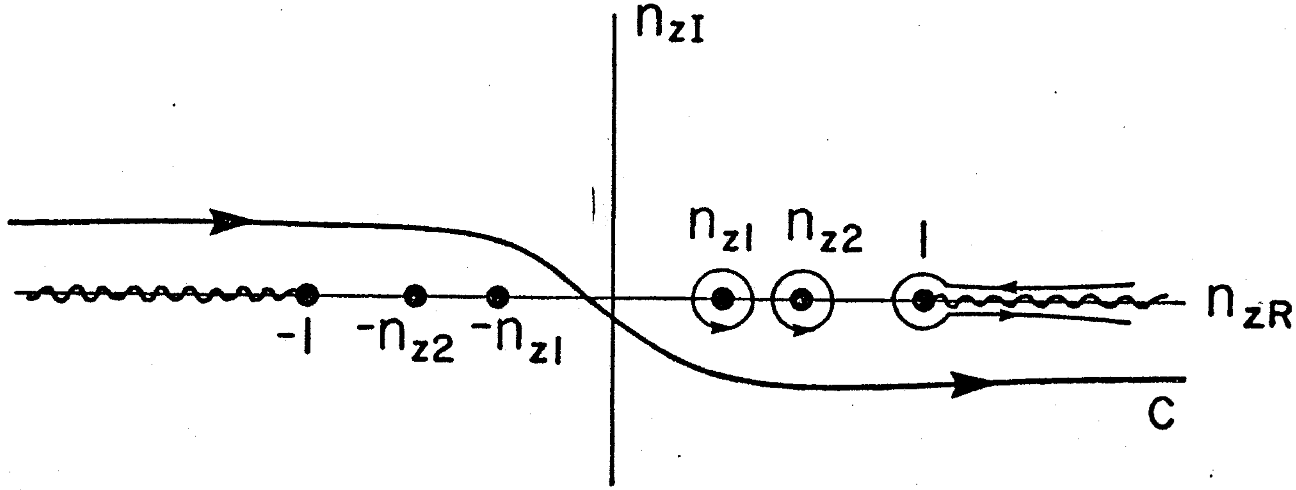


FIG.(2)

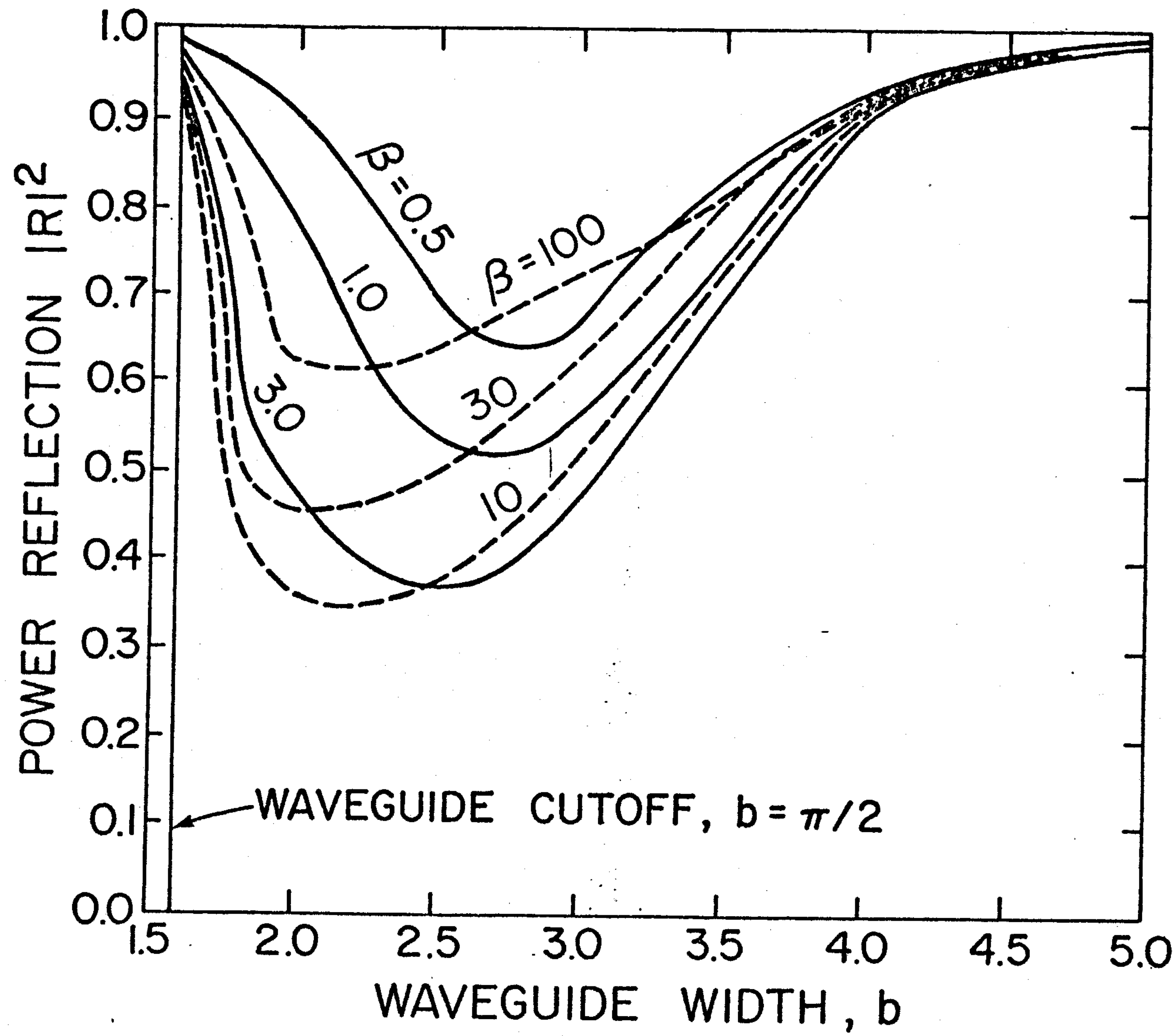

POWER REFLECTION $|R|^2$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
WAVEGUIDE WIDTH, b
WAVEGUIDE CUTOFF, $b = \pi/2$
β=0.5
1.0
3.0
β=100
30
10

FIG. (3)

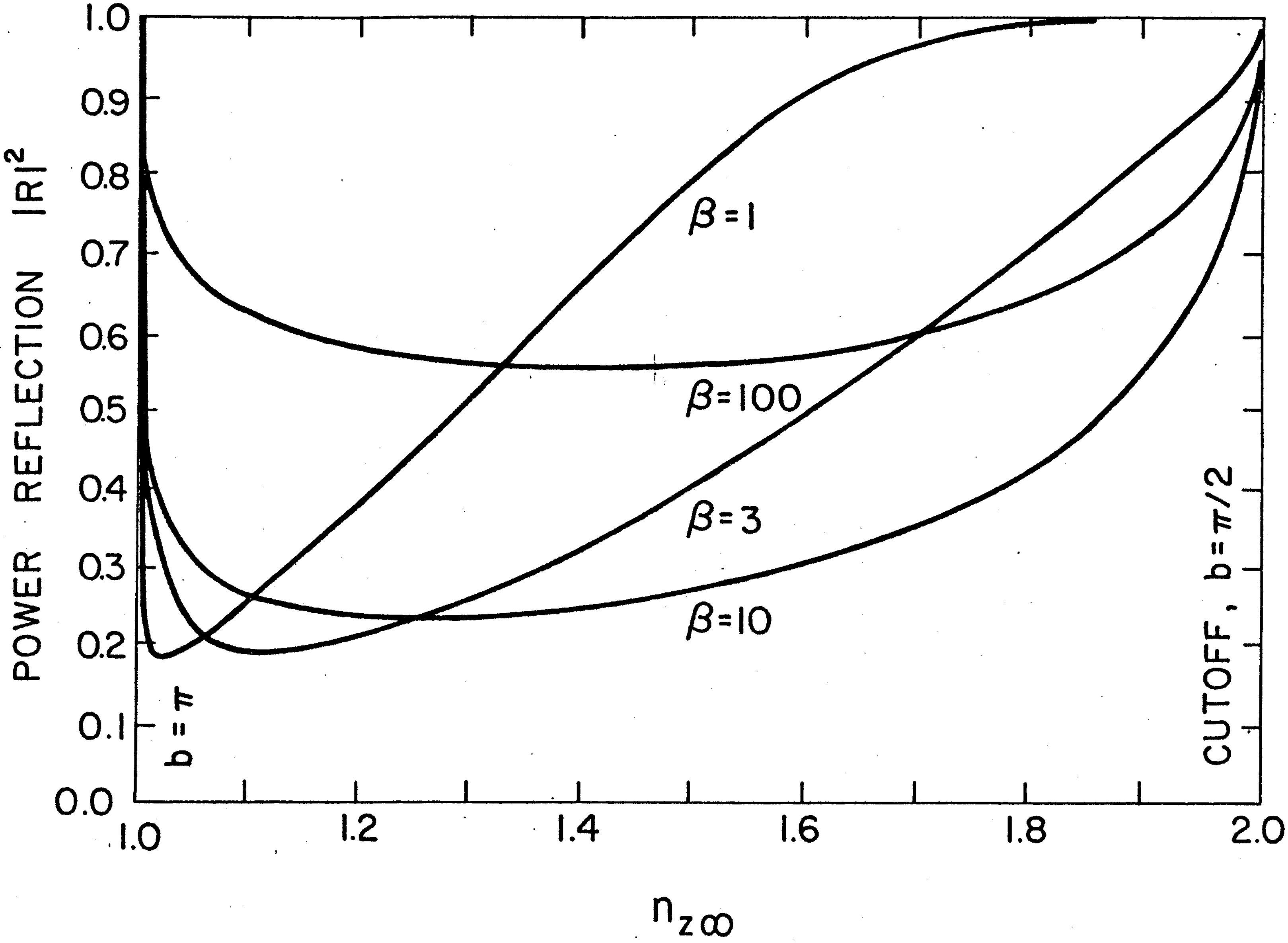
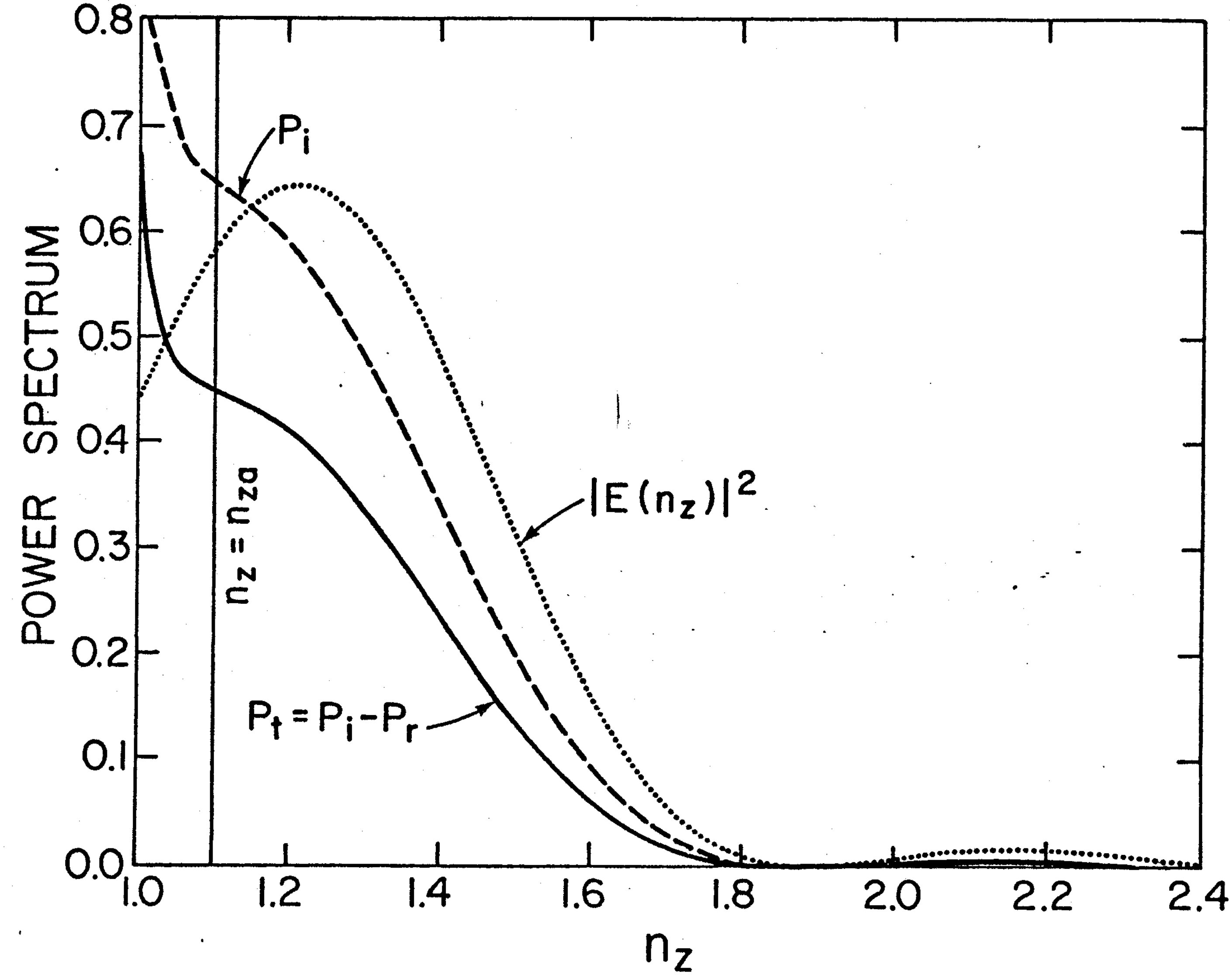
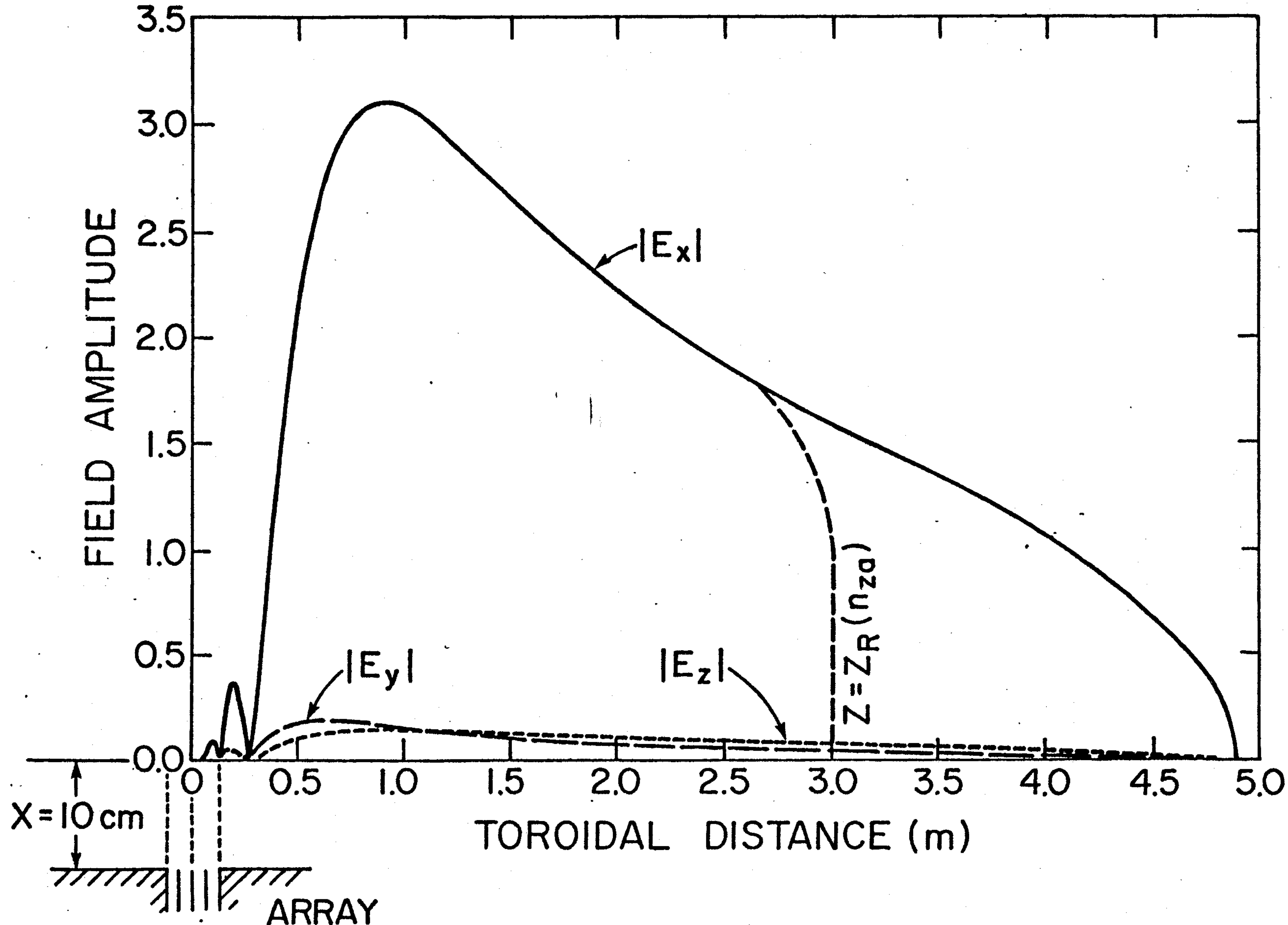

POWER REFLECTION |R|²
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1.0
1.2
1.4
1.6
1.8
2.0
β=1
β=100
β=3
β=10
b = π
CUTOFF, b=π/2
n z∞

FIG. (4)



0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
POWER SPECTRUM
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
n_z
P_i
$n_z = n_{za}$
$|E(n_z)|^2$
$P_t = P_i - P_r$

FIG. (5)



3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
FIELD AMPLITUDE
|Ex|
|Ey|
|Ez|
Z=ZR(nza)
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
TOROIDAL DISTANCE (m)
X=10cm
ARRAY

FIG. (6)