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Abstract: Bounds on the ergodic classical and private capacities for optical communication through turbulence are presented, showing that high photon and spectral efficiencies can be achieved simultaneously in the near-field power transfer regime.

Introduction

Photon-starved optical communication links place a premium on achieving high photon information efficiency (PIE), i.e., many bits/detected photon, which can be realized with pulse-position modulation (PPM) and direct detection [1]. Future developments in fiber-optic communication are increasingly focused on achieving high spectral efficiency (SE), viz., many bits/sec-Hz, which can be realized with high-order quadrature-amplitude modulation and coherent detection Should *both* high photon efficiency *and* high spectral efficiency be desired, however, the preceding approaches fail. Recourse to the ultimate quantum-mechanical (Holevo-limit) capacity of the bosonic (optical communication) channel does *not* eliminate the conflicting demands of high PIE and high SE. The single spatial-mode Holevo capacity of the vacuum-propagation channel [3] is a function of the average number of transmitted photons, $\bar{N}_T$, and the channel transmissivity, η. The left panel of Fig. 1 shows the Holevo-limit PIE versus SE for that channel along with the Shannon-limit behaviors for heterodyne and homodyne detection, plus that of on-off keying (OOK) with direct detection. None of these approaches realizes both high PIE and high SE.

High PIE and high SE *can* be realized together over the vacuum channel with M spatial-mode operation, whose Holevo-limit results, when all modes have the same transmissivity, are shown in the right panel of Fig. 1. Thus far, no explicit approach to realizing this capacity is known. As seen in this panel, heterodyne and homodyne detection have hard limits on their PIE_M values, whereas the Shannon capacity of direct detection with on-off keying mimics the 189 spatial-mode Holevo capacity but requires 4500 equal-transmissivity spatial modes to do so [4].

Well-known vacuum-propagation characteristics imply that high photon efficiency with high spectral efficiency can only be obtained in the near-field power transfer regime. For an L-m-long line-of-sight vacuum-propagation link at wavelength λ between coaxial square transmitter and receiver pupils with side lengths d_T and d_R, respectively, near-field power transfer prevails when the Fresnel-number product, $D_f = (d_T d_R/\lambda L)^2$ satisfies $D_f \gg 1$. Then, there are approximately D_f orthonormal spatial modes in the transmitter pupil—for each of two orthogonal polarization states—that transfer almost all of their power *and* retain their orthogonality after propagation into the receiver pupil. It follows that PIE = 10 bits/photon and SE = 5 bits/sec-Hz will only be practical for very modest path lengths: for $d_T = d_R = d$ operation at $\lambda = 1.55\,\mu\text{m}$, $D_f = 189$ (the Holevo limit) requires $d = 14.6\,\text{cm}$ when $L = 1\,\text{km}$, and for OOK direct detection $D_f = 4500$ requires $d = 32.3\,\text{cm}$ for $L = 1\,\text{km}$ operation at $\lambda = 1.55\,\mu\text{m}$. Application scenarios for these short path lengths will almost certainly involve terrestrial paths, which preclude high data-rate operation in low-visibility weather conditions. Putting aside the modest extinction loss associated with atmospheric absorption and scattering at a well-chosen laser wavelength, it is then the parts per million refractive-index fluctuations associated with turbulent mixing of air parcels with $\sim 1\,\text{K}$ temperature differences that distinguish clear-weather atmospheric propagation from propagation through vacuum. This paper presents bounds on the ultimate PIE versus SE behavior that can be realized using multiple spatial modes in the presence of atmospheric turbulence.

Much is known about optical propagation and communication through atmospheric turbulence, but little of it is germane to determining the degree to which this channel permits the simultaneous achievement of high photon efficiency and high spectral efficiency. It is known that the atmospheric channel has a near-field power transfer regime that is similar to that of vacuum propagation [5]. Relatively little has been done, prior to our work, to assess the communication performance obtainable in this regime. This is due to the difficulty of determining exact statistics of the turbulent
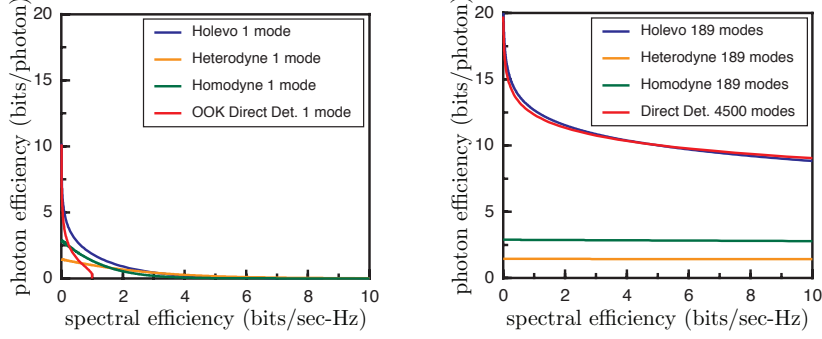
Fig. 1. Photon efficiency versus spectral efficiency for single spatial-mode (left panel) and multiple spatial-mode (right panel) vacuum-propagation links.

channel's near-field modal transmissivities. We have developed near-field capacity bounds [6] from near-field power transfer statistics [7] that depend solely on the turbulent channel's well-known mutual coherence function [8].

Transmitter and Receiver Assumptions

We assume coaxial square pupils at the transmitter ($\mathscr{A}_T$) and the receiver ($\mathscr{A}_R$) with common side length $d = 17.6\,cm$ operating at $\lambda = 1.55\,\mu$m wavelength over an $L = 1\,km$ path so that the free-space Fresnel number product is $D_f = 400$. The transmitters we consider will employ one of the vacuum-propagation mode sets: the $M = 225$ focused-beams (FB) modes,

$$\Phi_{n_x,n_y}^{(0),\text{FB}}(\rho) = \frac{\exp[-ik(x^2+y^2)/2L+i2\pi(n_x x+n_y y)/d]}{d}, \quad \text{for } -7 \leq n_x, n_y \leq 7, \tag{1}$$

for $\rho \in \mathscr{A}_T$, where $\rho = (x,y)$ and $k = 2\pi/\lambda$; the $M = 231$ Hermite-Gaussian (HG) modes,

$$\Phi_{n_x,n_y}^{(0),\text{HG}}(\rho) = \frac{\sqrt{2\pi}(D_f)^{1/4}}{d\sqrt{\pi\,n_x!\,n_y!\,2^{n_x+n_y}}} H_{n_x}\left(\frac{\sqrt{2\pi}(D_f)^{1/4}}{d}x\right) H_{n_y}\left(\frac{\sqrt{2\pi}(D_f)^{1/4}}{d}y\right) \exp\left[-\left(\frac{\pi\sqrt{D_f}}{d^2}+i\frac{k}{2L}\right)(x^2+y^2)\right],$$
$$\tag{2}$$

for $\rho \in \mathscr{A}_T$ and nonnegative integer n_x, n_y satisfying $n_x + n_y \leq 20$, where $H_n(\cdot)$ is the nth Hermite polynomial; and the $M = 231$ Laguerre-Gaussian (LG) modes,

$$\Phi_{n_p,n_\ell}^{(0),\text{LG}}(\rho) = \sqrt{\frac{n_p!}{\pi(|n_\ell|+n_p)!}} \left[\frac{\sqrt{2\pi}(D_f)^{1/4}}{d}\right]^{|n_\ell|+1} r^{|n_\ell|} L_{n_p}^{|n_\ell|}\left(\frac{2\pi\sqrt{D_f}}{d^2}r^2\right) \exp\left[-\left(\frac{\pi\sqrt{D_f}}{d^2}+i\frac{k}{2L}\right)r^2+in_\ell\theta\right], \tag{3}$$

for $\rho \in \mathscr{A}_T$ and integer n_ℓ and nonnegative integer n_p satisfying $2n_p + |n_\ell| \leq 20$, where $L_{n_p}^{|n_\ell|}(\cdot)$ is the generalized Laguerre polynomial, and $\rho = (r,\theta)$ in polar coordinates. Two receiver architectures will be considered: an adaptive receiver which uses perfect (full-wave) adaptive optics to track the spatial patterns produced, in $\mathscr{A}_R$, by transmission of these mode sets from $\mathscr{A}_T$, and a non-adaptive receiver which extracts the M mode patterns that would be produced in $\mathscr{A}_R$ under vacuum propagation.

Bounds on the Ergodic Classical and Private Capacities

The two panels in Fig. 2 show bounds on the ergodic Holevo capacities' PIE versus SE tradeoffs for the FB (left) and HG or LG (right) modes. The ergodic Holevo capacity is the ultimate limit on the rate at which classical information can be reliably communicated when the transmitter and the receiver have and use knowledge of the instantaneous power-transfer characteristics of the turbulent channel. These results, and those that appear below in Fig. 3, come from general bounds derived in [6] that are evaluated using the power-transfer statistics from [7].

The two panels in Fig. 3 show bounds on the ergodic Holevo private capacities' PIE versus SE tradeoffs for the FB (left) and HG or LG (right) modes. The ergodic Holevo private capacity is the ultimate limit on the rate at Alice can reliably communicate to classical information to Bob without Eve—a passive eavesdropper who captures all the light leaked out of the Alice-to-Bob channel–getting any information.

In Figs. 2 and 3, the vacuum curves are nearly coincident with the upper bound for atmospheric propagation both with and without adaptive optics. The atmospheric propagation curves, for increasing C_n^2, are for weak, moderate, and
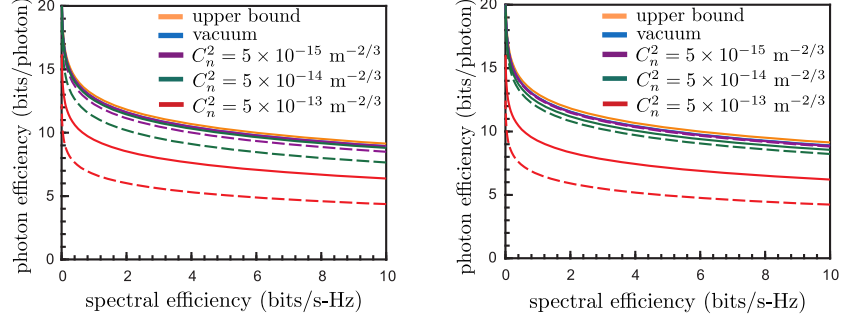
Fig. 2. Ergodic Holevo capacity bounds for PIE versus SE: $M = 225$ FB-mode system (left panel) and $M = 231$ HG-mode or LG-mode systems (right panel). A Fresnel number $D_f = 400$, transmitter and receiver apertures with sides of length $d = 17.6$cm, and wavelength $\lambda = 1.55\mu$m over an $L = 1$km propagation distance are assumed. Solid lines represent capacities of systems with perfect adaptive optics; dashed lines represent capacities of systems without adaptive optics.
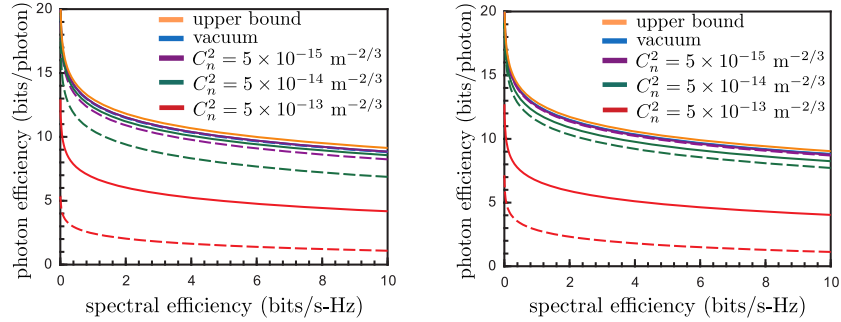

Fig. 3. Ergodic Holevo private capacity bounds for PIE versus SE: $M = 225$ FB-mode system (left panel) and $M = 231$ HG-mode or LG-mode systems (right panel). A Fresnel number $D_f = 400$, transmitter and receiver apertures with sides of length $d = 17.6$cm, and wavelength $\lambda = 1.55\mu$m over an $L = 1$km propagation distance are assumed. Solid lines represent capacities of systems with perfect adaptive optics; dashed lines represent capacities of systems without adaptive optics.

strong turbulence, with the solid (dashed) curves applying to reception with (without) adaptive optics. These results show that high PIE and high SE can be achieved simultaneously–at ultimate quantum limits—for both classical and private communication either with or without adaptive optics.

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