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Low Energy Beam Pumping in a TARA Reactor Design

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Abstract

We explore the possibility of neutral beam pumping in a tandem mirror thermal barrier region. In order to reduce the necessary neutral beam pumping power, we investigate substituting low for high energy neutral beams. Using a two-step square well model and the variational expression for the trapping current, in a simplified one dimension calculation, we illustrate the competition between the trapping process and the drag effect for the deeply trapped particles. This competition can lead to a stable equilibrium with reduced pumping power.

1. Introduction

It has been shown that the performance of a tandem mirror can be importantly enhanced by the use of a "thermal barrier."[1] In this situation a potential depression is interposed between the central cell and plug to thermally isolate the respective electron species. Maintenance of the potential depression depends critically on the abiltiy to purge the barrier of thermal ions that tend to trap there, and which would otherwise cause a decrease of the depth of the potential depression. The purging of trapped ions has been termed barrier pumping.

One scheme that has been proposed involves the use of energetic neutral beams, injected at such an angle that upon charge exchange (with an ion trapped in the thermal barrier) the resulting ion will be in the loss cone of the thermal barrier mirror cell. The neutral beam produced ions must have sufficient energy to overcome the total thermal barrier potential and some fraction of the pump beam current must be injected at the total thermal potential, the so-called high energy pump beams. In this work we show that the pump beams can be effective with an energy that is significantly reduced from the barrier potential. This result will impact positively on the power balance due to the reduced energy expended per particle pumped as well as the enhanced charge-exchange cross section of the lower energy neutral beams.

In this study we consider a tandem mirror reactor based on the TARA type end plug[2] with central cell parameters similar to MARS. The central cell is thus bounded by axisymmetric mirror plugs which contain the thermal barrier and ion plugging potential. Beyond the plugs a short open transition maps the flux tube into ECRH heated low field quadrupole anchors.

A controlled radial diffusion may be applied in the central cell so that ions (D, T, and alphas) and possibly electrons primarily leave radially. Pumping of the thermal barrier may then be obtained by expelling ions and alphas into the central cell. Good neutral beam access into the thermal barrier region of the plugs encourages the use of neutral beam pumping. This option is desirable because the physics is conceptually simple and no difficult extrapolation of technology is required.

A preliminary estimate[3] shows that the necessary pumping power is about 150 MW which will have a negative impact on the recirculating power and therefore the value of the reactor Q (Q is defined to be the ratio of the fusion power to the injected power). Analyzing the necessary pumping power indicates that the high energy beam pumping takes two thirds of the total power consumption although it pumps only 5 percent of the total trapping current. This implies that high energy beam pumping is much less efficient than low energy pumping. Thus if we can substitute low for the high energy pumping beams due to the higher efficiency of low energy beam pumping we can decrease the total input power.

In order to analyze this trade-off, we must answer the following three questions: (1) Is there an equilibrium when we substitute low for the high energy beam pumping and if so how much power is required to maintain this equilibrium? (2) Do we have a reasonable model to analyze this equilibrium? (3) Is this equilibrium stable, i.e., does the accumulation of the deeply trapped particles eventually destroy the equilibrium?

Logically, we have to answer the second question first. Although Fokker-Planck codes would in principle solve the problem the available codes do not readily lend themselves to this calculation. Furthermore these codes do not illuminate the physics and are large and very consumptive of computer time. Recently, Futch and Logan[4] have shown that an equilibrium exists when only the local low energy beam pumping is used; and that this equilibrium may become unstable when the pumping factor g_b is about 2 ~ 3 (corresponding to barrier mirror ratio of 4.5, and ratio of barrier potential to temperature of greater than 1). Here, g_b is the ratio of total density to passing particle density at the bottom of barrier. This Memorandum encourages us to analyze this problem using the variational method which has succeeded in the case of a well-pumped thermal barrier[5,6]. Before we apply this variational method, we will set up a two-step square well model for the local pumping case.

2. Model

In Fig. 1, two step square well model is used for describing the thermal barrier region of TARA reactor. The neutral beam is injected near the mirror peak at a plateau where the magnetic field (B_1) and electric potential (ϕ_1) are higher than those at the bottom of the barrier (B_b, ϕ_b). In terms of velocity space, there is a boundary layer which is between the passing particle region and the unpumped region (Fig. 2a). In this boundary layer the pumping rate $\nu_L \neq 0$. Assuming equilibrium with a constant pumping rate ν_L, we may write the trapping current J_t as

$$J_t = \nu_L n_{tL} \quad (1)$$

where n_{tL} is the trapped particle density in the low energy beam pumping region. Using the variational principle[5,6], we may write another expression for the trapping current

$$J_t = \sqrt{2 \alpha_L g_b}\, \nu_L n_p \quad (2)$$

Here n_p is the passing particle density, g_b is the familiar pumping factor, α_L is the corresponding variational parameter

$$\alpha_L = \frac{2}{3}\frac{\nu_p}{\nu_L}\frac{1}{H^2}\left(\frac{1}{\pi R_b}\right)^{3/2} \qquad (3)$$

where ν_p is the collisional frequency, $\nu_p = \frac{\Gamma n_p}{v_t^3}$, v_t is the thermal velocity, $v_t = \sqrt{\frac{2T_p}{m}}$, T_p is the temperature of the passing particles. m is the mass of the particle. $\Gamma = \frac{4\pi e^4}{m^2} \ln\Lambda$, $\ln\Lambda$ is the Coulomb logarithm. In our case $\ln\Lambda \approx 34.9 - \ln\sqrt{\frac{3n_p}{T_p(\phi_b+T_p)}}$.
In the expression for $\ln\Lambda$, n_p is in unit of cm^{-3}; T_p and ϕ_b are in unit of keV. e is the charge of the particle. R_b is the barrier mirror ratio (the ratio of the peak magnetic field B_{mb} to the bottom magnetic field B_b in Fig. 1). H is the ratio of the passing particle density to the central cell particle density,

$$H = \mathrm{erfc}\left(\sqrt{\frac{\phi_b}{T_p}}\right)\exp\left[\frac{\phi_b}{T_p}\right] - \sqrt{\frac{R_b - 1}{R_b}}\,\mathrm{erfc}\left(\sqrt{\frac{R_b}{R_b - 1}\frac{\phi_b}{T_p}}\right)\exp\left[\frac{R_b}{R_b - 1}\frac{\phi_b}{T_p}\right] \qquad (4)$$

In the large mirror ratio ($R_b \gg 1$) and large potential ($\phi_b/T_p \gg 1$) case,

$$H \approx \frac{1}{R_b}\sqrt{\frac{T_p}{\pi(\phi_b + T_p)}} \qquad (5)$$

There is an important difference between the local pumping and uniform pumping cases: for local pumping the expression for g_b obtains the general form

$$g_b = 1 + \frac{n_{tL}}{n_p}\left(1 + \frac{n_{tH}}{n_{tL}}\right) \tag{6}$$

Here, n_{tH} is the trapped particle density in the unpumped region. When we uniformly pump thermal Barrier, $n_{tH} = 0$ and $n_{tL} = n_t$. Then equation (1), (2) and (6) give the previously obtained uniform pumping result,

$$\alpha = \frac{(g_b - 1)^2}{2g_b} . \tag{7}$$

In the local pumping case, however, $n_{tH} \neq 0$ and we obtain

$$\alpha_L = \frac{(g_b - 1)^2}{2g_b} \; \frac{1}{\left(1 + \frac{n_{tH}}{n_{tL}}\right)^2} \tag{8}$$

Comparing equation (8) with (7), we have

$$\alpha_L = \alpha \; \frac{1}{\left(1 + \frac{n_{tH}}{n_{tL}}\right)^2} \tag{9}$$

From equation (3), we know that the α_L is inversely proportional to pumping rate ν_L. Therefore, for a given set of parameters (n_p, T_p, R_b, ϕ_b, g_b) the local pumping needs a pumping rate

$$\nu_L = \nu \left(1 + \frac{n_{tH}}{n_{tL}}\right)^2 \tag{10}$$

Here ν is the necessary uniform pumping rate for the same set of parameters.

The physics in equation (10) is clear. Local pumping always requires a higher pumping rate than uniform pumping. For the same g_b, if $n_{tH} \neq 0$, of course, the pumping rate in the boundary layer (Fig. 2a) must be stronger than before ($n_{tH} = 0$ case).

Evaluation of the pumping rate requires an estimate for the ratio (n_{tH}/n_{tL}).

3. The Ratio n_{tH}/n_{tL}

We require an estimate of the ratio n_{tH}/n_{tL} to evaluate the trade-off between increased pumping of a restricted region of velocity space and uniform pumping.

The exact calculation should invoke a two-dimensional Fokker-Planck code. We will use a one dimension model to have an estimate of the maximum power requirement. Consistent with the two sphere model[6], we may simply estimate the density at one of the two tips of the separatrixes in velocity space. The balance equation is as follows:

$$-\nabla \cdot \vec{I} - \nu_L f = 0. \tag{11}$$

Here $\vec{I}$ is the Coulomb collision current in velocity space, f is the distribution function. Under the assumption of

$$f = h\, f_m \tag{12}$$

we have

$$\nabla \cdot \overleftrightarrow{D} f_m \cdot \nabla h - \nu_L\, f_m\, h = 0 \tag{13}$$

Here, f_m is a center-shifted maxwellian distribution function[5], and $\overleftrightarrow{D}$ is the diffusion tenser. In the one dimension model, the equation may be written as

$$\frac{1}{v^2}\ \frac{d}{dv}\ (v^2\, f_m\, D_\parallel\ \frac{dh}{dv}) - \nu_L\, f_m\, h = 0 \tag{14}$$

$$D_\parallel \approx \frac{2\pi e^4 \ln\Lambda}{m^2}\ \frac{n_b}{2}\ \frac{1}{v_T}\ \frac{4}{3\sqrt{\pi}} \tag{15}$$

$D_\parallel$ is the parallel component of the diffusion tensor[7]. We evaluate $D_\parallel$

using a low velocity approximation, since we do not expect very large relative velocity in the low energy beam pumping scheme. n_b is the total density at the bottom of the barrier. Thus the approximate equation in the pumped region is

$$f_m D_\parallel \frac{d^2h}{dv^2} + \frac{1}{v^2} \left(\frac{d}{dv} \left(v^2 f_m D_\parallel \right) \right) \left(\frac{dh}{dv} \right) - \nu_L f_m h = 0 \qquad (16)$$

It should be noted that this is an equation in the co-moving frame. When $v = v_1$ (see Fig. 2b) in the co-moving frame, it corresponds to the $v_\parallel = (v_{\phi b} - v_1)$ in the stationary frame at the bottom of the barrier. In the co-moving frame, $v = v_1$ is the boundary between unpumped and pumped region.

In the unpumped region, the solution is simply

$$h = h(v_1) = \text{constant}, \qquad (17)$$

since there is no flow and ∇h must be zero.

In the pumped region $\nu_L \neq 0$, the exact solution is complicated and and a Taylor expansion may be used to obtain a rough estimate

$$h(v) = h(v_1) + \left.\frac{dh}{dv}\right|_{v = v_1} (v - v_1) + \frac{1}{2!} \left.\frac{d^2h}{dv^2}\right|_{v = v_1} (v - v_1)^2 + \ldots. \qquad (18)$$

The continuity condition for $\frac{dh}{dv}$ makes

$$\left.\frac{dh}{dv}\right|_{v = v_1} = 0 \qquad (19)$$

Hence in the pumped region we can estimate

$$h(v) = h(v_1) + \frac{1}{2} \left.\frac{d^2h}{dv^2}\right|_{v = v_1} (v - v_1)^2 \qquad (20)$$

Using equation (16), we have

$$h(v) = h(v_1) + \frac{1}{2}\frac{\nu_L}{D_\parallel} h(v_1)(v - v_1)^2 \tag{21}$$

Therefore, the ratio of the density

$$\frac{n_{tH}}{n_{tL}} = \frac{\int_{v_1}^{\infty} v^2 dv\ f}{\int_0^{v_1} v^2 dv\ f}$$

$$= \frac{\int_{v_1}^{\infty} v^2 dv\ \ h(v_1) f_m}{\int_0^{v_1} v^2 dv\ \ h(v_1)\left[1 + \frac{1}{2}\frac{\nu_L}{D_\parallel}(v - v_1)^2\right] f_m} \tag{22}$$

Using $f_m(v) = C_o e^{-v^2/v_T^2}$, we have

$$\frac{n_{tH}}{n_{tL}} = \frac{C_1}{1 + \frac{\nu_L}{g_b} C_2} \tag{23}$$

where

$$C_1 = \frac{3}{2}\left(\frac{T_p}{\phi_1}\right)^{3/2} \left\{\Gamma\left(\frac{3}{2}, \frac{\phi_1}{T_p}\right) - \Gamma\left(\frac{3}{2}, \frac{\phi_b}{T_p}\right)\right\} \tag{24}$$

$$C_2 = \frac{3}{20}\sqrt{\pi}\ \frac{1}{\nu_c H}\ \frac{\phi_1}{T_p} \tag{25}$$

Here $\phi_1 \equiv \frac{1}{2} m v_1^2$,

$$\nu_c \equiv \frac{\Gamma n_c}{v_T^3} \tag{26}$$

and n_c is the central cell density. $\Gamma(\frac{3}{2}, \frac{\phi_1}{T_p})$ is the incomplete Gamma function, which may be expressed analytically as

$$\Gamma\left(\frac{3}{2},x\right) = \frac{1}{2}\sqrt{\pi}\left[1 - \Phi(\sqrt{x})\right] + \sqrt{x}\, e^{-x} \tag{27}$$

$\Phi(\sqrt{x})$ is the error function: $\Phi(\sqrt{x}) = \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{x}} e^{-t^2} dt$. Combining equation (23) with equations (6), (1) and (2), we have the solution for g_b

$$g_b = \frac{\nu_L}{C} x^2 \tag{28}$$

$$C = \frac{4}{3} \frac{\nu_c}{H} \left[\frac{1}{\pi R_b}\right]^{3/2}$$

Here, x is defined as the ratio of trapped particle density in the low energy beam pumping region, n_{tL}, to the passing particle density, and it is the solution of the following equation

$$\frac{\nu_L}{C} x^2 = \frac{1}{2}\left\{-\left[C_2\nu_L - C_1 x - (1 + x)\right] + \sqrt{\left[C_2\nu_L - C_1 x - (1 + x)\right]^2 + 4\,(1 + x)\, C_2\nu_L}\right\} \tag{29}$$

Once we have the solution for x we can calculate g_b using equation (28), and the total density in the barrier, i.e.

$$\begin{aligned} n_b &= g_b\, n_p \\ &= g_b\, n_c\, H \quad . \end{aligned} \tag{30}$$

Therefore we may calculate the variation of the total density when the potential ϕ_b is perturbed. In the same time, the density ratio $(\frac{n_{tH}}{n_{tL}})$ can be evaluated using equation (23).

4. The Stability Analysis

In Section 2 we found an equilibrium for the local pumping case when the pumping rate is higher than the uniform pumping case by a factor of the density ratio square (equation (10)). In Section 3 we made an estimate of the density ratio (n_{tH}/n_{tL}). We now consider the stability of this equilibrium, i.e. whether a deviation from equilibrium tends to grow or decay. We first examine a schematic plot of the electron and ion density as a function of potential, ϕ_b, shown in Fig. 3. Equilibrium occurs at the quasi-neutral potential(ϕ_b^*) when $n_i = n_e$ and stability depends on the relative slope $\partial n/\partial \Phi$ for ions and electrons. A fluctuation of potential, $\Delta\phi_b$, causes a deviation of ion density from the equilibrium, Δn_i. Neutrality however requires a corresponding deviation of electron density, Δn_e, which requires a further deviation in potential, $(\Delta\phi_b)_1$. Thus when

$$\left|\frac{\Delta n_e}{(\Delta\phi_b)_1}\right| < \left|\frac{\Delta n_i}{\Delta\phi_b}\right| \tag{31}$$

electrical neutrality requires

$$\left|(\Delta\phi_b)_1\right| > \left|\Delta\phi_b\right| . \tag{32}$$

Consequently, the fluctuation in potential will increase and this equilibrium is observed to be unstable. In the other limit, when

$$\left|\frac{dn_e}{d\phi_b}\right| > \left|\frac{dn_i}{d\phi_b}\right| \tag{33}$$

a similar analysis leads to the conclusion

$$|(\Delta\phi_b)_1| < |\Delta\phi_b| \tag{34}$$

Consequently, a fluctuation in potential will damp and this equilibrium is stable (Fig. 4).

We can in fact use Eq. (30) to calculate the curve n_{bi} as a function of ϕ_b for a fixed central cell density, n_c. For Maxwell-Boltzman electrons the electron density n_{be} changes with ϕ_b exponentially. We note that the parameter ϕ_1, comes into determining $dn_i/d\phi_b$ through equations (24) and (25). ϕ_1 may itself be changing with ϕ_b, since the electrical potential at the $B = B_1$ point is not fixed. However, for a given magnetic field profile, the ratio of ϕ_1/ϕ_b may be assumed to be a constant. Therefore we choose fixed ϕ_1/ϕ_b in the stability criteria. Fig. 5 illustrates the case where $\phi_1/\phi_b \sim 0.444$, the local pumping rate $\nu_L = 42\ \text{sec}^{-1}$, the central cell density $n_c = 3.50 \times 10^{14}\ \text{cm}^{-3}$ Line 1 represents the $n_{bi}(\phi_b)$ curve and line 2 $n_{be}(\phi_b)$. Their intercept gives the equilibrium result; $\phi_b/T_p \sim 3.8$ and since $\left|\frac{dn_e}{d\phi_b}\right| > \left|\frac{dn_i}{d\phi_b}\right|$, this is a stable equilibrium. Line 3 in Fig. 5 shows $n_i(\phi_b)$ for the corresponding uniform pumping case. We can see that the uniform pumping is more stable than the local pumping case. When we draw these curves, the hot electron component (typically for a reactor $T_{eh} = 820$ kev, $n_{eh} = 4 \times 10^{13}\ \text{cm}^{-3}$) and the sloshing ion component ($n_{is} = 1 \times 10^{13}\ \text{cm}^{-3}$) have been included. These energetic components may affect the equilibrium parameter but not the stability, since they are too energetic to respond to a small potential fluctuation. Fig. 6 shows a case where the local pumping is very close to uniform pumping, i.e. $\phi_l/\phi_b = 0.9$. As we expect, line 1 approaches line 3 and their stability features are almost the same. Fig. 7 shows a case where the local pumping is more concentrated in small region, i.e. $\phi_l/\phi_b = 0.333$. This is a marginal case, and line 2 is tangent to line 1. Although there is an equilibrium, any deviation toward left would lead to further deviation. If we reduce the pumped region further, there will not be an equilibrium. In this marginal case, if we increase the pumping rate to 45 sec^{-1}, the equilibrium becomes stable again (Fig. 8). However, if we reduce the pumped region too much, i.e. $\phi_l/\phi_b = 0.25$, then the equilibrium is unstable even if the pumping rate ν_L is as high as 80 sec^{-1} (Fig. 9).

Through this analysis, we can see this sensitivity to the ratio (ϕ_l/ϕ_b). In fact it is easy to see this effect in the velocity space (Fig. 2b); this ratio will determine the size of the pumped region. If the size of this region is too small to reduce the height of function $h(v_l)$, it is difficult to stop the accumulation of the trapped particle in the unpumped region. Then the equilibrium is impossible or unstable.

Although the foregoing analysis is based on a simple one dimension model for the ratio of (n_{tH}/n_{tL}), the result is useful since it is based on a conservative analysis. The one dimension model chosen to estimate (n_{tH}/n_{tL}) integrates along the direction of minimum n_{tL} and this will tend to overestimate this ratio.

5. The Power Requirement for Pumping

We now estimate the required pump power. From Fig. 5 we observe that the necessary energy for the low energy beam is about 40 kev (ϕ_b is chosen to be 90 kev[3]). The necessary power is therefore

$$P = V \phi_1 J_t$$
$$= V \phi_1 \nu_L n_c H x. \qquad (35)$$

In Eq. (35) x is determined by equation (28). From Fig. 5 we find

$\frac{n_b}{n_c} = 0.15.$ Hence

$$g_b = \frac{n_b}{n_p} = \frac{n_b}{n_c H} = \frac{0.15}{H} \qquad (36)$$

and

$$x = \sqrt{\frac{C g_b}{\nu_L}} = \sqrt{\frac{0.15\ C}{\nu_L H}} \qquad (37)$$

Therefore, we find

$$P = V \phi_1 n_c \sqrt{\frac{4}{3} \nu_L \nu_c \left(\frac{1}{\pi R_b}\right)^{3/2} 0.15} \qquad (38)$$

with V the volume of the barrier (about 2.51 M^3 [3]). The necessary power is thus about 17 MW. Considering the efficiency for the low energy beam pumping, we multiply the power by a factor of 4. Even though this power is greater than the low energy pumping power in the uniform pumping case (57 MW)[3], it is much less than the total pumping power in the uniform pumping case (150 MW) (including power to a high energy pump beam). These results agree qualitatively with Fokker-Planck code calculations performed by R.S. Devoto[8].

6. Summary

We have shown that local pumping using a low energy beam is more economical than uniform pumping utilizing high energy neutral beams. A simple model has been used to illuminate the physics. In a frame moving with one group of the passing particles, the deeply trapped particles experience a drag effect exerted by the shallowly trapped particles and this group of passing particles. In certain cases, this drag effect may be enough to purge these deeply trapped particles. Then substituting low for the high energy beam pumping is possible.

A number of effects which have not been included in this work and which may prove to be important are listed below:

(i). The spacial variaiton of the magnetic field may be important. Presently a self-consistent calculation using a bounce average Fokker-Planck code is not available. For uniform pumping cases the bounce average effect has been seen to produce a change of up to a factor of 2 in the pump requirements.

(ii). The D and T mass effect. Our variational method is only applied to single species cases (e.g. D and D reactor). When D and T are the collisional particles this variational formula can not be used directly. If we may use the effective mass $m = 2.5\ m_p$ (m_p is the mass of proton) we will introduce an error into our result.

(iii). The electron effect. Previously, the electron effect was neglected because the pitch angle scattering was thought to be the dominant trapping mechanism. Our variational calculation shows, however, that the drag term plays an important role in the trapping process[5][6]; therefore, one may wonder whether the electron drag should be included in the trapping mechanics.

Acknowledgements

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Captions

Fig. 1 Two Step Square Well Model for the local pumping

Fig. 2a The Boundary Layer for the Local Pumping Case in Velocity Space

Fig. 2b One Dimension Model for the Evaluation of Ratio $\left(\frac{n_{tH}}{n_{tL}}\right)$

Fig. 3 The Curve of Total Density versus the Potential at the Bottom of Barrier (Unstable Equilibrium)

Fig. 4 The Curve of Total Density versus the Potential at the Bottom of Barrier (Stable Equilibrium)

Fig. 5-9 $(n_b/n_c) \sim (\phi_b/T_p)$ curves

line 1 is for the ion density for the local pumping case
line 2 is for the electron
line 3 is for the ion density for the uniform pumping case

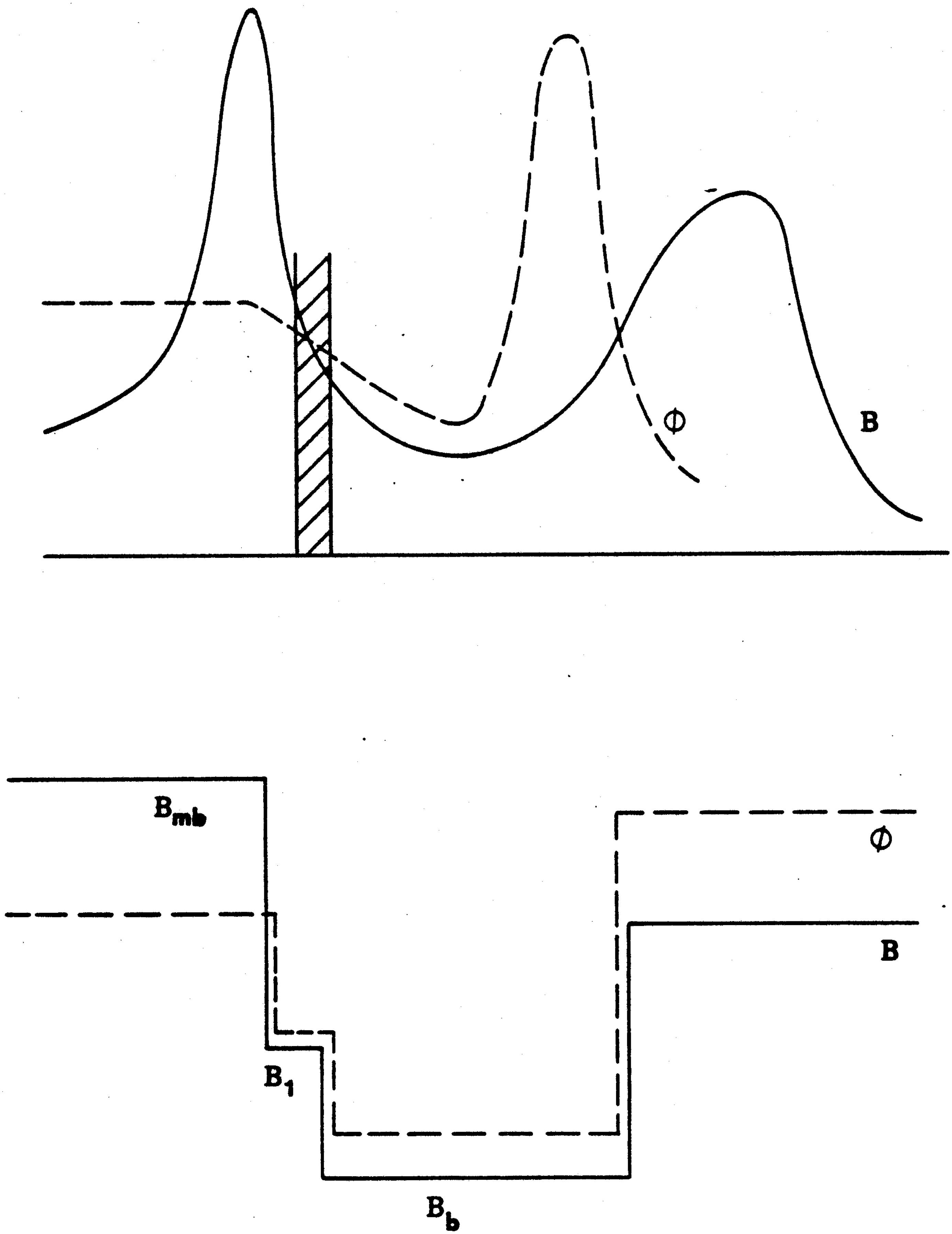
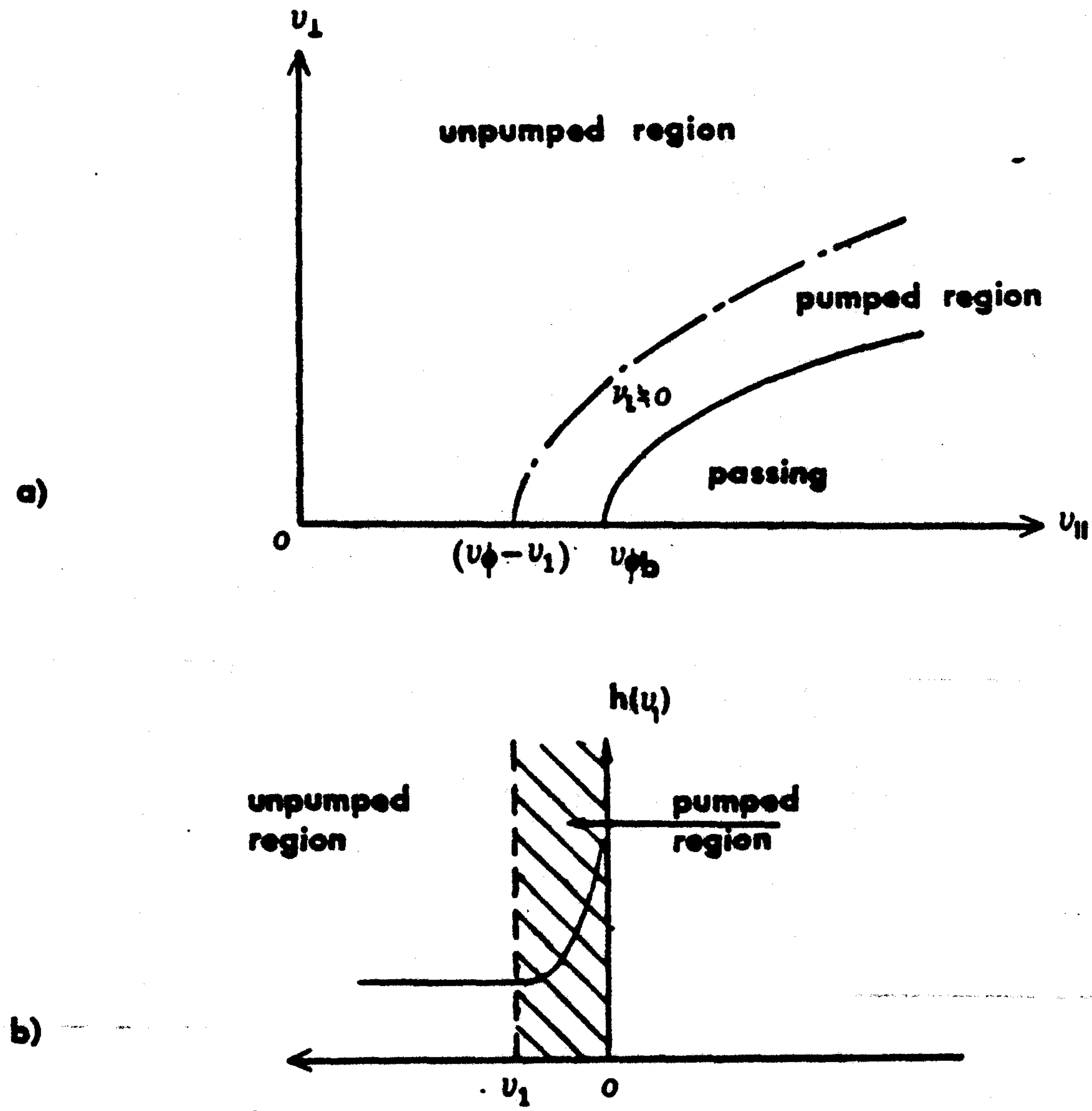


Fig. 1



Co-moving frame, one dimension model

Fig. 2

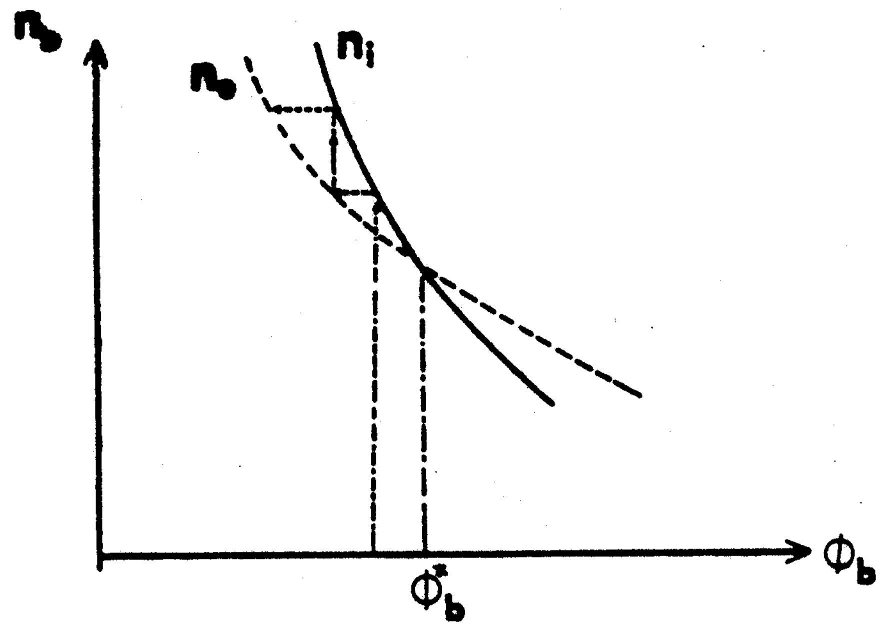


Fig. 3

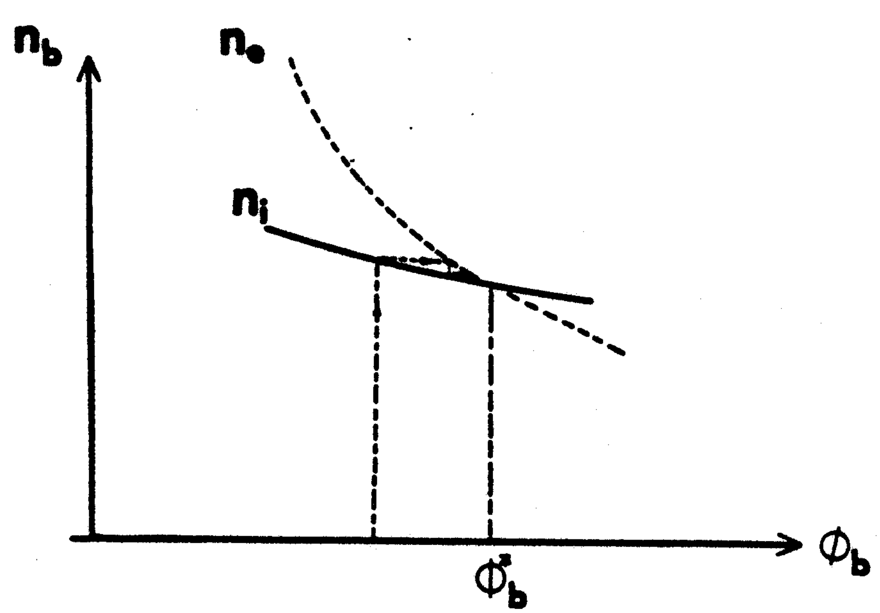
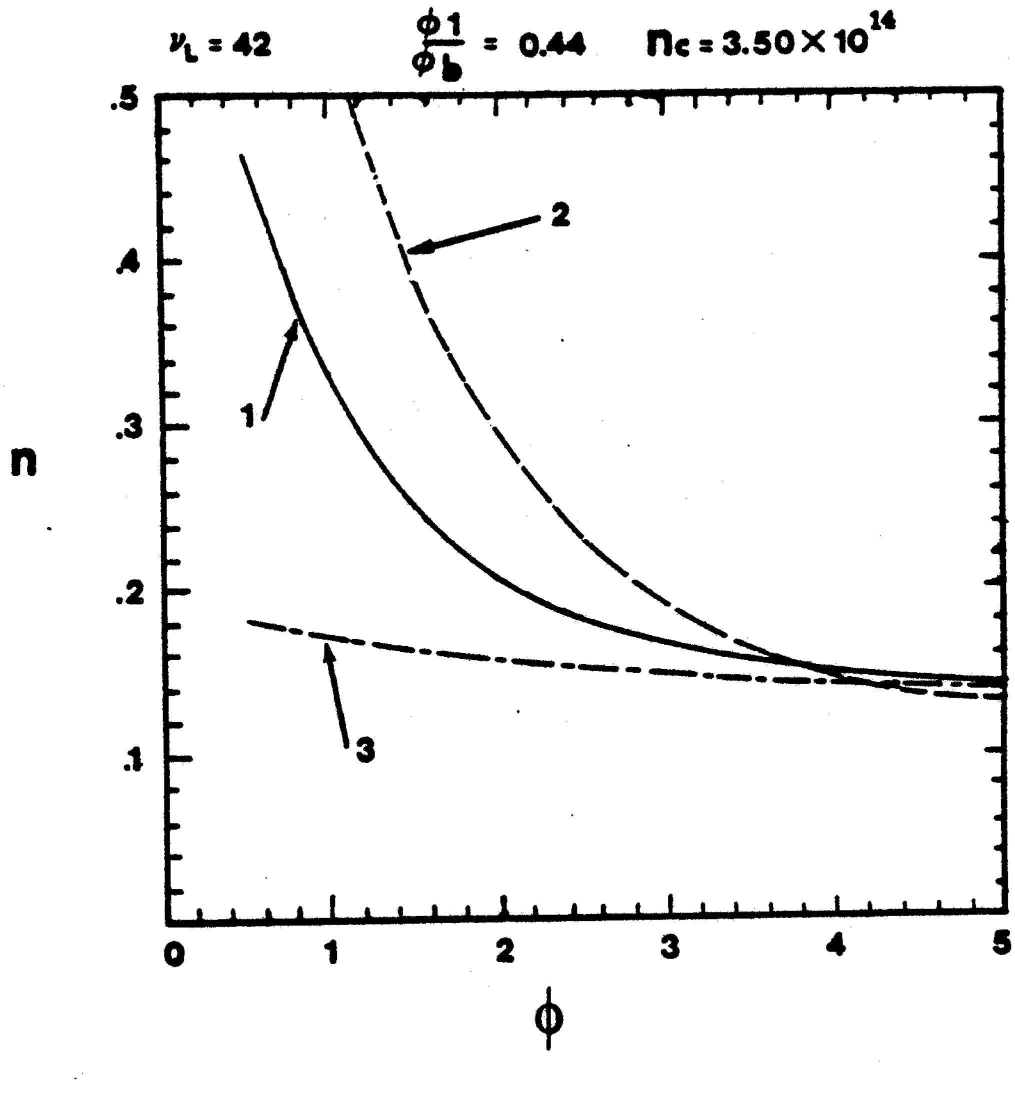


Fig. 4



ν_L = 42
$\frac{\phi 1}{\phi b}$ = 0.44
$n_c = 3.50 \times 10^{14}$
.5
.4
.3
.2
.1
n
0
1
2
3
4
5
ϕ
1
2
3

Fig. 5

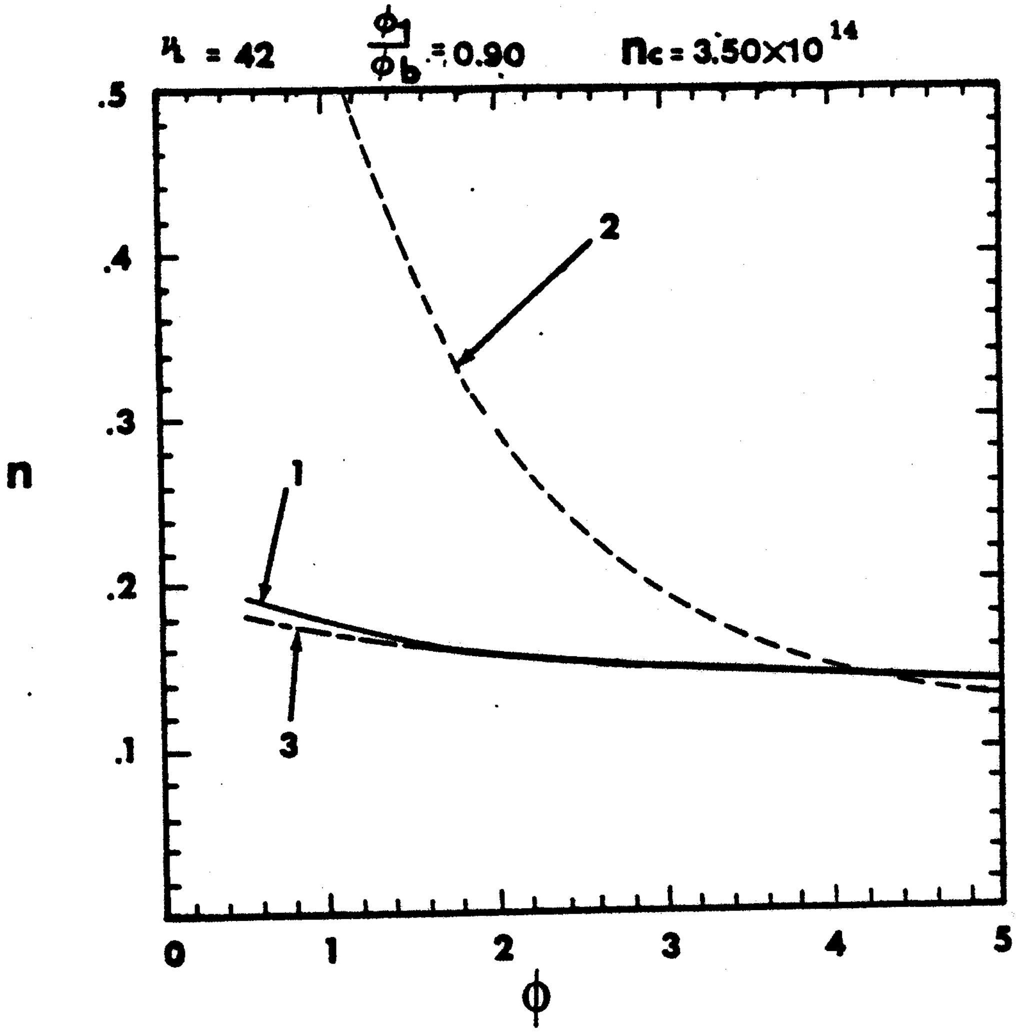
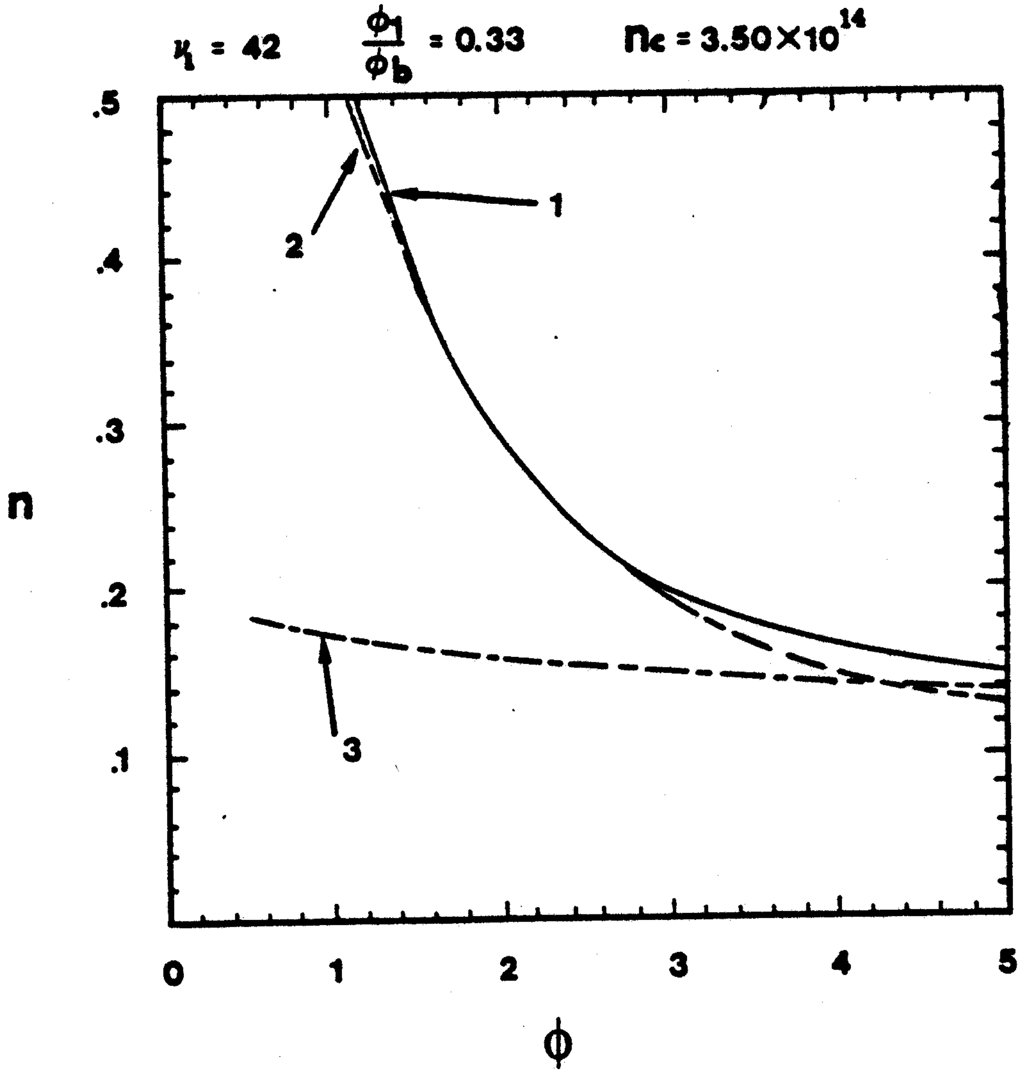


Fig. 6



$\varkappa_1$ = 42
$\frac{\phi_1}{\phi_b}$ = 0.33
$n_c = 3.50\times10^{14}$
.5
.4
.3
.2
.1
n
0
1
2
3
4
5
ϕ
1
2
3

Fig. 7

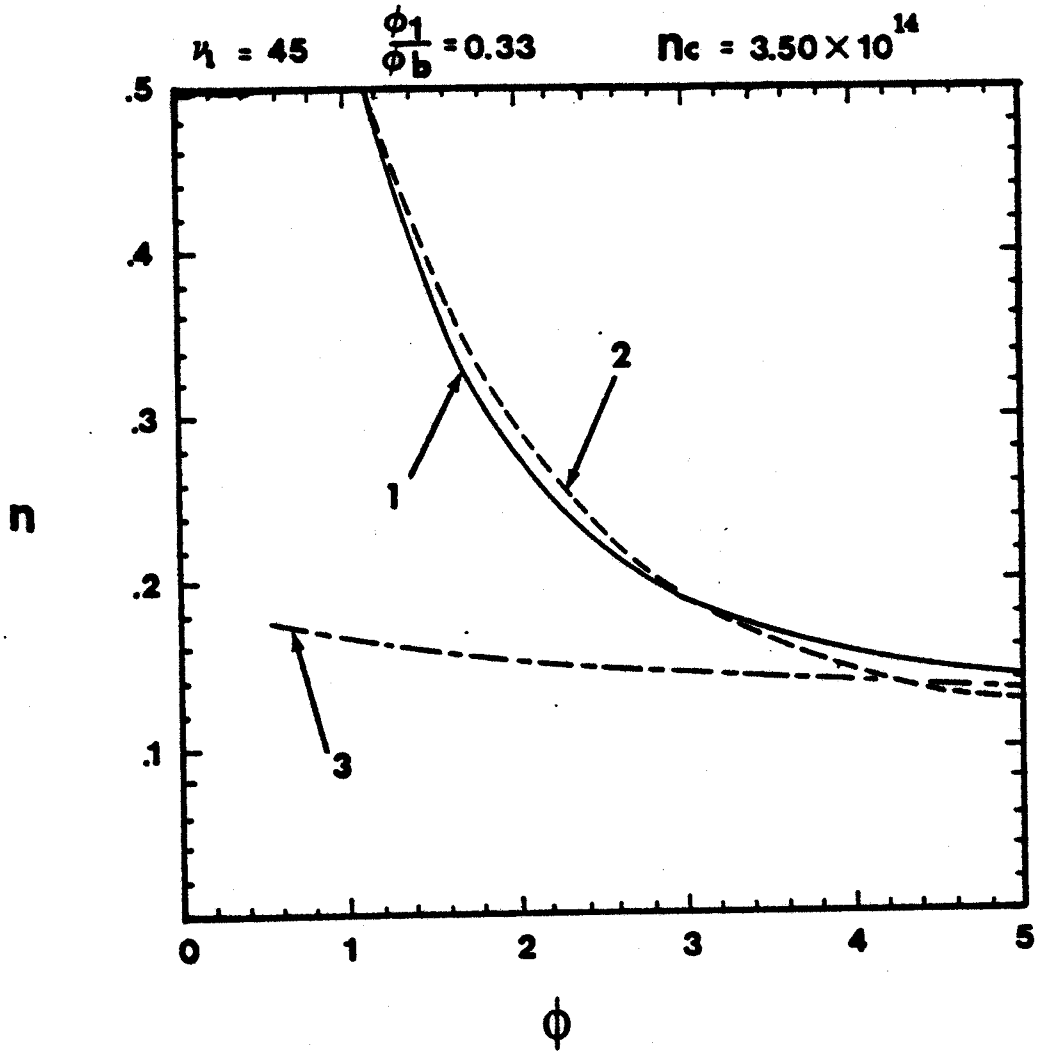


Fig. 8

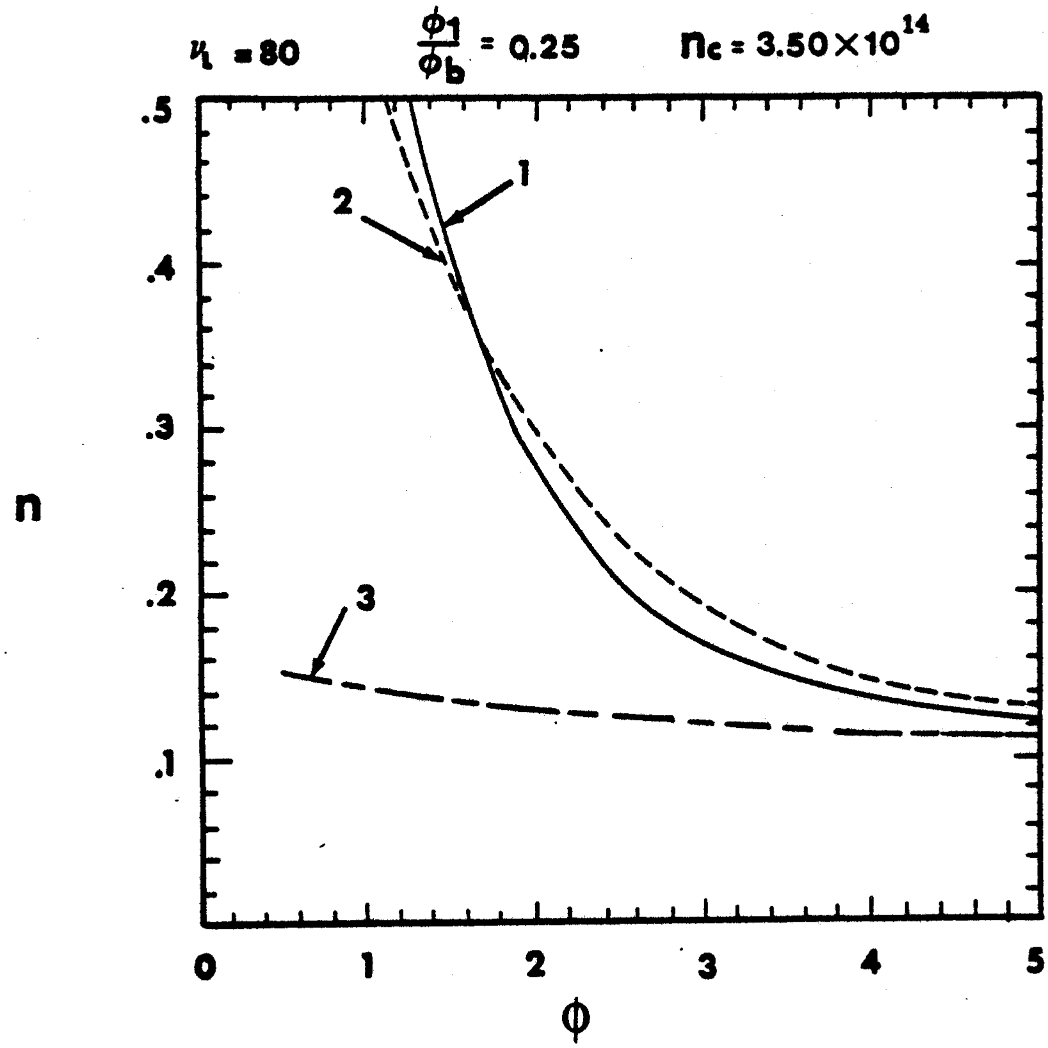


Fig. 9