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KINETIC THEORY OF THE PONDEROMOTIVE FORCE NEAR GYRORESONANCE*

by

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Abstract

A high frequency, spatially modulated electric field leads to an average effective force, called the ponderomotive force. We examine the case of a magnetized plasma in which an imposed radio frequency field is launched by an external source. The field polarization is perpendicular to the magnetic field (B) and is spatially modulated along B. The analysis from the Vlasov equation to second order in the electric field amplitude shows that the usual result for the ponderomotive force [1] is valid for frequencies (ω) not very close to the gyrofrequency (Ω). At $\omega = \Omega$ there is a substantial enhancement of the ponderomotive potential, which is finite, but exhibits a temperature dependence. The particles at very low velocity are responsible for this resonant effect. This phenomenon will not be observed in the case of a beam of particles at $V \neq 0$ [2].

Large amplitude, spatially modulated, high frequency (HF) waves produce a change in the plasma equilibrium. To find the new equilibrium on a time scale shorter than given by the collisional frequency one has to examine the Vlasov equation and determine the time averaged distribution function. The nonresonant part of the wave-particle interaction leads to changes in both density and temperature. These are related to an effective average force and a nonresonant diffusion. It is this force, called ponderomotive ($\overline{F}_p$), that is an often studied feature of the high frquency waves [3]. It is responsible for the low frequency density change and plays an important role in many nonlinear processes in the plasma. F_p is related to the ponderomotive potential (ϕ_p): $\overline{F}_p = -\overline{\nabla}\phi_p$. For a magnetized plasma, when the HF field is polarized perpendicularly to B, the ponderomotive potential is usually given by the expression [1]

$$\phi_p = \frac{e^2|\overline{E}_\perp|^2}{4m(\omega^2 - \Omega^2)} \tag{1}$$

where $\overline{E}_\perp$ is the HF electric field with a frequency ω and Ω is the gyrofrequency. It is clear that this expression becomes singular near gyroresonance. This nonphysical singularity is due to the method of derivation of (1), which assumes that a local expansion of the form

$$\overline{E}(\overline{x}) = \overline{E}(\overline{x}_s) + \overline{x}_\omega \cdot \overline{\nabla}\overline{E}(\overline{x}_s) + \ldots \tag{2}$$

is valid even near $\omega \simeq \Omega$. Here $\overline{x}_s$ is the slow and $\overline{x}_\omega$ the fast variable. It was pointed out correctly [2] that the separation of scales is related to the adiabaticity in the problem, $\ell|\omega - \Omega|/v \gg 1$, where ℓ is the modulational scale length and v is the particle velocity. For frequencies $\omega \simeq \Omega$ the adiabaticity is violated for any finite $v \neq 0$ and one has to take into account the spatial modulation nonperturbatively. The result is that the ponderomotive potential is not infinite at $\omega = \Omega$ but is actually small and the maximum of the ϕ_p occurs off the resonance. This is a single particle picture which holds true for any finite value of $v \neq 0$. In the experiment [2] the plasma is essentially a beam of velocity v_B with a distribution function $f \sim \delta(v - v_B)$.

In most realistic cases the plasma has a Maxwellian distribution function. This includes particles with $v \simeq 0$, which makes the adiabatic condition valid for very small differences $\omega - \Omega$. In this case one has to examine the ponderomotive effect from a kinetic point of view. It becomes clear that the concept of a force is inadequate. After determining the Vlasov equation for the time averaged f_0 we can define a global force and potential from the momentum conservation equations. The ponderomotive force in that case balances the pressure gradient. By properly integrating over the spatial modulation we determine the force both near and off resonance. Off resonance we find the old expression (1). Near resonance we obtain a temperature dependent term which is much bigger than previously suggested [2]. This is due to the resonant effect on the slow moving particles at $v = 0$, which could not be predicted by a single particle treatment [2]. This result is very significant, since it raises again the possibility for RF plugging in mirrors [4] and isotope separation [5,6].

In the model problem which we solve, the HF electric field is imposed by the external source and is of the form:

$$\vec{E} = \hat{y} E_0(z) \cos(\omega t + \psi) \quad , \quad E_0(z) \equiv E_0 \exp\left(-\frac{z^2}{2\ell^2}\right) \tag{2}$$

Here $E_0(z)$ is the amplitude with a spatial modulation of scale ℓ along the magnetic field B_0. From Faraday's law the total magnetic field becomes:

$$\vec{B} = B_0 \hat{z} + \hat{x} \frac{c}{\omega} \frac{\partial}{\partial z} E_0(z) \sin(\omega t + \psi) \tag{3}$$

The Vlasov equation for the ions is:

$$\frac{\partial f}{\partial t} + \bar{v} \frac{\partial f}{\partial \bar{x}} + \frac{e}{m}(\bar{E} + \frac{\bar{v}}{c} \times \bar{B}) \frac{\partial f}{\partial \bar{v}} = 0 \tag{4}$$

We consider frequencies ω in the range of the ion gyrofrequency Ω. The problem for the electrons can be considered in a similar fashion when $\omega \simeq \Omega_e$. The Vlasov equation will be treated perturbatively. The distribution function is written as $f = f_0 + f_1 + \ldots$, where f_0 is the averaged, slow time, distribution to order E_0^2 and f_1 is the fast oscillating distribution to order E_0. For f_1 we have:

$$\frac{\partial f_1}{\partial t} + v_z \frac{\partial f}{\partial z} + \Omega\left(v_y \frac{\partial}{\partial v_x} - v_x \frac{\partial}{\partial v_y}\right) f_1 = -\frac{e}{m} E_0(z) \cos(\omega t + \psi) \frac{\partial f_M}{\partial v_y} \tag{5}$$

f_M is the Maxwellian distribution. It is assumed that in the absence of a HF field the particles are Maxwellian. The solution of Eq. (5) is given by:

$$f_1 = \frac{e}{m} \int_{-\infty}^{t} dt' E_0[z + v_z(t' - t)] \cos(\omega t' + \psi)[\sin \Omega(t' - t) \frac{\partial}{\partial v_x} - \cos \Omega(t' - t) \frac{\partial}{\partial v_y}] f_M \tag{6}$$

The equation for the slow time scale distribution function to second order in the electric field amplitude is:

$$\begin{aligned} &\frac{\partial f_0}{\partial t} + v_z \frac{\partial f_0}{\partial v_z} + \Omega(v_y \frac{\partial f_0}{\partial v_x} - v_x \frac{\partial f_0}{\partial v_y}) = \\ &\quad -\frac{e}{m} \langle E_0(z) \cos(\omega t + \psi) \frac{\partial f_1}{\partial v_y} + \frac{1}{\omega} \frac{\partial E_0(z)}{\partial z} \sin(\omega t + \psi)\left(v_z \frac{\partial}{\partial v_y} - v_y \frac{\partial}{\partial v_z}\right) f_1 \rangle \end{aligned} \tag{7}$$

where the average $< \ldots >$ is taken over the phase ψ. The r.h.s. of (7) contains terms which represent an effective force and a nonresonant diffusion. This has been pointed out for the case of a unmagnetized plasma [7]. In general the r.h.s. is nonlocal in terms of $E_0(z)$, i.e. it contains derivatives of $E_0(z)$ higher than first order. Also, the ponderomotive force is velocity dependent. This nonlocal, velocity dependent character is particularly important near the resonance $\omega \simeq \Omega$ and it makes the single particle picture inadequate. Here we define the ponderomotive force as a fluid concept. It is the force which balances the pressure gradient and is derived from the "v_z" moment of Eq. (7). In our model the r.h.s. contributes to the ponderomotive force in the longitudinal direction only. This is not surprising, since the spatial modulation is along B_0. The momentum conservation equation becomes:

$$m\frac{\partial \Gamma_z}{\partial t} + \frac{\partial}{\partial z}(nT) = n_0 F_p = -n_0 \frac{\partial}{\partial z}\phi_p \tag{8}$$

Here Γ_z is the longitudinal flux, n_0 is the unperturbed (initial) density and n is the average density in the presence of a HF field. If the temperature T is considered unchanged – an assumption which is not quite correct [7] – we arrive at the classical result:

$$\delta n = n_0 - n = \phi_p / T \tag{9}$$

Here the ponderomotive density depression δn is given by the ponderomotive potential. If there is a significant enhancement of F_p and ϕ_p near gyroresonance it can be measured by the effect on the average plasma density. From (7) we obtain for the ponderomotive force:

$$\begin{aligned} F_p = &-\frac{e^2}{m\omega}\frac{\partial E_0(z)}{\partial z}\int_{-\infty}^{\infty}\frac{dv_z}{\sqrt{2\pi}v_t}\exp(-v_z^2/2v_t^2)\cdot \\ &\langle \int_{-\infty}^{t} dt' E_0[z + v_z(t'-t)]\cos(\omega t' + \phi)\sin(\omega t + \phi)\cos\Omega(t'-t)\rangle \end{aligned} \tag{10}$$

After averaging over ψ and a change of variables $\tau = t - t'$ we get:

$$F_p = -\frac{e^2}{4m\omega}\frac{\partial E_0(z)}{\partial z}\int_{-\infty}^{\infty}\frac{dv_z}{\sqrt{2\pi}v_t}\exp(-v_z^2/2v_t^2)\int_0^{\infty} d\tau E_0(z - v_z\tau)\sum_{\pm}\sin(\omega \pm \Omega)\tau\,. \tag{11}$$

The integration over velocities gives:

$$\int_{-\infty}^{\infty} dv_z \exp\left(-\frac{v_z^2}{2v_t^2} - \frac{(z - v_z\tau)^2}{2\ell^2}\right) = \sqrt{\frac{2\pi}{1/v_t^2 + \tau^2/\ell^2}}\exp\left[-\frac{z^2}{2\ell^2} + \frac{(z\tau/\ell^2)^2}{2(1/v_t^2 + \tau^2/\ell^2)}\right]. \tag{12}$$

The ponderomotive force becomes:

$$F_p = -\frac{e^2}{8m\omega v_t}\frac{\partial}{\partial z}(E_0(z))^2\int_0^{\infty} d\tau \exp\left[\frac{(z\tau/\ell^2)^2}{2(1/v_t^2 + \tau^2/\ell^2)}\right]\frac{\sum_{\pm}\sin(\omega \pm \Omega)\tau}{\sqrt{1/v_t^2 + \tau^2/\ell^2}} \tag{13}$$

The integral in (13) can not be evaluated explicitly due to the exponential factor in the integrand. This exponential factor is a slowly varying function of τ and changes its value from 1 at $\tau = 0$ to $\exp(z^2/2\ell^2)$ at $\tau \to \infty$. One can expand the result in a power series of $(z/\ell)^2$. For $z/\ell < 1$ the formula for F_p has the simple form:

$$F_p(z/e < 1) = -\frac{e^2}{8m\omega v_t}\frac{\partial}{\partial z}(E_0(z))^2 \int_0^\infty d\tau \, \frac{\sum_\pm \sin(\omega \pm \Omega)\tau}{\sqrt{1/v_t^2 + \tau^2/\ell^2}} \tag{13a}$$

This integral can be found in [8] and the result is:

$$F_p = -\frac{\pi e^2 \ell}{16m\omega v_t}\frac{\partial}{\partial z}(E_0(z))^2 \sum_\pm \left[I_0\left(\frac{|\omega \pm \Omega|\ell}{v_t}\right) - L_0\left(\frac{|\omega \pm \Omega|\ell}{v_t}\right)\right] sign(\omega \pm \Omega) \tag{14}$$

Here I_0 is the modified Bessel function of zeroth order and L_0 is the modified Struve function of zeroth order [9]. Since $\xi_+ \equiv (\omega + \Omega)\ell/v_t \gg 1$ in any realistic situation, we can use the asymptotic form for I_0 and L_0 in the term with a "+" sign. However, for the argument $\xi_- \equiv |\omega - \Omega|\ell/v_t$ there are two distinctly different regimes. In the off resonance case $\xi_- \gg 1$ and we recover the usual result [1]. Asymptotically

$$I_0(\xi_\pm) - L_0(\xi_\pm) = \frac{2}{\pi \xi_\pm} \quad , \quad \xi_\pm \gg 1 \tag{15}$$

which leads to the ponderomotive force

$$F_p = -\frac{e^2}{4m(\omega^2 - \Omega^2)}\frac{\partial}{\partial z}(E_0(z))^2 \quad , \quad \frac{|\omega - \Omega|\ell}{v_t} \gg 1\,. \tag{16}$$

This corresponds to the ponderomotive potential in Eq. (1) and is the adiabatic result when the transit time ℓ/v_t is much longer than one period of oscillation $|\omega - \Omega|^{-1}$. Near gyroresonance $\xi_- \ll 1$ and then we use the small argument limit: $I_0(\xi_-) = 1 + O(\xi_-)$, $L_0(\xi_-) = O(\xi_-)$. This gives for the ponderomotive force:

$$F_p = -\frac{\pi e^2 \ell}{16m\omega v_t}\frac{\partial}{\partial z}(E_0(z))^2 sign(\omega - \Omega) \quad , \quad \frac{|\omega - \Omega|\ell}{v_t} \ll 1\,. \tag{17}$$

The contribution from the term with a "+" sign is neglected as small ($\xi_+ \gg 1$). In the limit $\omega \to \Omega$ the ponderomotive force is finite and changes sign as we approach the gyroresonance from above or below.

The result in (17) is significant in two ways. First, it shows that the ponderomotive force is finite at $\omega \to \Omega$, but is enhanced by a factor $\ell\omega/v_t \gg 1$ over its value in an unmagnetized plasma. This is in sharp contrast to the single particle result [2]. The increase of F_p may be used for RF confinement of mirror systems and isotope separation. Secondly, the ponderomotive force at $\omega \simeq \Omega$ has a temperature dependence which scales like $T^{-1/2}$ and explicitly points to the collective nature of the effect near resonance. We hope that experiments in the near future will be able to establish the precise nature of the ponderomotive effects near the gyroresonance.

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