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Effect of Electron Beam Temperature on the Gain of a Collective Free Electron Laser

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Abstract

At sufficiently low beam currents, electron beam temperature effects cause the gain of collective (Raman) regime free electron lasers to be lower than the predictions of cold beam theory. This gain degradation has been measured as a function of the beam current, the wiggler magnetic field, and the interaction frequency. The measurements are used to estimate the electron beam temperature, and the estimated temperature is close to the temperature predicted by numeric simulations.

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Introduction

Elevated electron beam temperatures decrease the gain of collective (Raman) regime free electron lasers (FELs) [1,2,3]. The small signal gain scales as the beam temperature spread, $\Delta\gamma_{\|}/\gamma_{\|}$, divided by the beam plasma frequency $\omega_p/\gamma^{1/2}\gamma_{\|}$, and since the plasma frequency is proportional to the square root of the beam current I, temperature effects are more noticeable when the beam current is low. Here γ is the relativistic mass increase, $1/\gamma_{\|}^2 = 1 - \beta_{\|}^2$ is the Lorentz axial contraction factor, $\beta_{\|} = v_{\|}/c$ is the normalized axial velocity, and $\omega_p = (ne^2/m\epsilon_0)^{1/2}$ is the nonrelativistic plasma frequency.

These temperature effects are observed when a mildly relativistic ($V \approx$ 160KeV), helical wiggler ($k_w = 2\pi/l$, $l = 3.3$cm) FEL is operated at abnormally low beam currents ($I < 0.3$A). The properties of this FEL have been studied extensively at beam currents where temperature effects are unimportant ($I \approx 5$A). In this "high current" regime previous measurements[4,5] indicate that cold beam collective FEL theory provides an excellent description of the FEL characterisitics. Using the cold beam results as a basis for comparison, we derive the electron beam temperature necessary to produce the observed low current gain degradation. These estimated temperatures are in agreement with numeric estimates of the electron beam temperature.

Experimental Apparatus

The FEL is illustrated in Fig. 1. A thermionically emitting Pierce gun removed from a SLAC klystron (model 343) is energized by the Physics International Pulserad 615MR high voltage facility. The ensuing aperture limited, 0.25cm radius electron beam is guided magnetically into a copper plated, 2.54cm ID stainless steel evacuated drift tube which also acts as the cylindrical waveguide. Beam integrity is maintained by a uniform axial guide field $B_{\|}$, and the beam current is measured by a Rogowski coil. Experimental measurements and electron gun computer simulations reported elsewhere [4] indicate that the beam energy spread $\Delta\gamma_{\|}/\gamma_{\|}$ is less than 0.5%. this, together with the relatively high current density and modest beam voltage, assures operation in the collective (Raman) regime. The 50 period circularly polarized wiggler has a period l of 3.3cm, and is generated by bifilar conductors wound directly on the drift tube. The first six periods of the wiggler are resistively loaded to produce a smooth increase in the magnetic field amplitude at the wiggler entrance [6].

The FEL is operated as an amplifier. A microwave E-plane bend launcher placed upstream from the wiggler superimposes the input microwave signal of frequency f onto the electron beam and converts the TE_{01} waveguide mode of the rectangular input waveguide to a linearly polarized, TE_{11} mode of the circular guide in the FEL. A TE_{11} circular guide to TE_{10} rectangular guide transformer at the output end of the wiggler couples one polarization out of the system in the form of a linearly polarized wave traveling in a rectangular guide. The other polarization is attenuated by a resistive load. The system is driven by a 10W traveling wave tube.

Experimental Measurements and Analysis

For a particular frequency ω, the gain of the FEL (including launching losses is

$$G = P_{\text{out}}/P_{\text{in}} = \left[\cosh(\Gamma z) + 1\right]^2/4. \tag{1}$$

where Γ is the growth rate. The FEL gain in the cold beam collective (Raman) regime has been extensively examined with our apparatus and is found to be in excellent agreement with theoretical predictions [4,5]. A typical comparison is shown in Fig. 2, where we plot the measured and predicted normalized growth rate $\Gamma/I^{1/4}$ as a function of the electron beam current I at a frequency of 11.25GHz, a wiggler field strength $B_w = 170$G, and an axial field strength $B_{||} = 1580$G. The measured growth rate Γ is found by measuring the gain G and inverting Eq. 1. In the cold beam collective regime, the growth rate Γ is proportional to the square root of the plasma frequency, and thus to the fourth root of the electron beam current. As is seen from Fig. 2, this relationship is closely followed for beam current between 6.0 and 0.3A, but below 0.3A, $\Gamma/I^{1/4}$ decreases sharply.

This decrease is readily understood by examining the axial phase velocity $v_p = \omega/k$ of the slow space charge wave that is intrinsic to the collective

FEL, namely

$$v_p = v_{\parallel} - \frac{p_1 \omega_p / \gamma^{1/2} \gamma_{\parallel}}{(k + k_w)}. \tag{2}$$

Since k_w, γ, $\gamma_{\parallel}$, the wavenumber k and the space charge reduction factor p_1 are approximately fixed for any given set of experimental parameters, the phase velocity v_p approaches the electron beam velocity $v_{\parallel}$ when the beam current $I(\propto \omega_p^2)$ is decreased. When the phase velocity approaches the velocity of the bulk of the drifting electrons in the temperature broadened electron beam distribution, the resonant beam-wave Landau interaction strongly damps the space charge wave and abruptly lowers the FEL gain. The gain degredation has been calculated analytically[1,5] when the damping is small. The observed growth rates in Fig. 2 are accurately predicted for all beam currents when a beam temperature of $\Delta\gamma_{\parallel}/\gamma_{\parallel} = 0.0034$ is assumed, as is illustrated by the dashed line.

In Fig. 3a we keep the beam current I approximately constant (at $I \approx$ 0.2A) and vary the wiggler field B_w. When the wiggler field is low the observed growth rate increases linearly as predicted by cold beam collective theory. Above B_w=250G, however, the growth rate no longer increases linearly with the wiggler field because, as is discussed below, the wiggler itself induces a velocity spread on the electron beam. For strong wiggler fields, this wiggler induced temperature is large enough that the FEL gain saturates.

The wiggler field induced beam temperature arises from two separate causes. The first cause arises from the three dimensional nature of the wiggler field, which in cylindrical coordinates (r, ϕ, z) is,

$$\begin{aligned} \mathbf{B}_w &= 2B_w\Big(\hat{\mathbf{e}}_r I_1'(k_w r)\cos(\phi - k_w z) - \hat{\mathbf{e}}_\phi \frac{I_1(k_w r)}{k_w r}\sin(\phi - k_w z) \\ &\qquad + \hat{\mathbf{e}}_z I_1(k_w r)\sin(\phi - k_w z)\Big) \end{aligned} \tag{3}$$

where I_1 is the modified Bessel function [8]. Inspection of Eq. 3 shows that away from the $r = 0$ axis, the radial wiggler field becomes stronger and an axial field component is also present. Recent calculations and computer simulations [9] have shown that despite the complicated three dimensional nature of the magnetic field, electrons launched into a wiggler with a gradually rising (adiabatic) entrance field travel at nearly constant axial velocity

$\beta_\parallel$, and acquire a transverse velocity given by

$$\beta_\perp = \frac{2\Omega_w \beta_\parallel I_0(k_w y_g) I_1(\lambda)/\lambda}{k_w \beta_\parallel c - \Omega_\parallel - 2\Omega_w I_0(k_w y_g) I_1(\lambda)}, \tag{4}$$

where $\Omega_w = eB_w/\gamma mc$ is the relativistic cyclotron frequency associated with the wiggler magnetic field, $\Omega_\parallel = eB_\parallel/\gamma mc$ is the relativistic cyclotron frequency of the axial field, y_g is the distance of the electron guiding center from the wiggler axis, and $\lambda = \beta_\perp/\beta_\parallel = \pm k_w r$ is the normalized size of the orbit, such that $\lambda = -k_w r$ when $\Omega_\parallel > k_w \beta_\parallel c$, and $\lambda = +k_w r$ when $\Omega_\parallel < k_w \beta_\parallel c$. When Eq. 4 is combined with the energy conservation relation,

$$1/\gamma^2 = 1 - \beta_\parallel^2 - \beta_\perp^2 \tag{5}$$

electrons far from the wiggler axis are found to move more slowly than electron close to the wiggler axis. This electron velocity shear across the electron beam gives rise to an effective beam temperature.

In practice, the nearly constant axial electron velocities predicted by Eq. 4 are not easily attained because the electron beam is not launched with the exactly correct initial conditions. However, an electron launched into a wiggler with a slowly increasing field at the wiggler entrance will enter a stable trajectory [10] and the electron's axial velocity will oscillate around the value predicted by Eq. 4. By properly tapering the wiggler field, this trajectory "quiver" can be minimized, [11] but it cannot be entirely removed. The quiver constitutes a second source of wiggler induced temperature. A third source of beam temperature, the space charge depression across the beam, is not important in this experiment.

In Fig. 4 we show the axial velocity of electrons entering a wiggler with a smooth, adiabatic entrance, as a function of the axial distance into the wiggler. The trajectories are computed numerically for electrons with zero initial radial velocity. The dashed line shows the axial velocity of an electron launched on the wiggler axis ($r = 0$). The solid line is the velocity of an electron launched at an initial radial position of 0.254cm. The difference in the average axial velocities of the two electrons is the result of the radial shear of the wiggler field.

In Fig. 3b we show the numerically computed beam temperature for our electron beam as a function of the wiggler field B_w. The effective temperature $\Delta\gamma_\parallel/\gamma_\parallel$ is found by determining the standard deviation of the electron beam's axial velocity in a volume that extends across the beam radius and along the wiggler length. The solid line is the total beam temperature including both of the previously mentioned effects. The dashed line includes only the effect of the radial shear, and is calculated from Eq. 4. Note that the total temperature including the wiggler quiver temperature is usually two to three times larger than the radial shear temperature alone.

The points in Fig. 3b give the estimated temperatures that are required in order to match the computed and observed growth rates. For much of the data the gain is sharply reduced from the cold beam gain, and the analytic, small damping formulas[1,5] are no longer appropriate. To more accurately determine the gain in this warm beam case, it is necessary to evaluate the longitudinal plasma susceptibility, which is a lengthy undertaking. Instead we use a heuristic formula for the gain which is derived by Jerby and Gover[3]. The formula states that the ratio of warm to cold beam gain at frequency ω is given by

$$\frac{\log G_{\text{warm}}}{\log G_{\text{cold}}} = \frac{1}{1 + (\overline{\theta}_t / \overline{\theta}_t^{ac})^2} \tag{6}$$

where $G_{\text{cold}} = \cosh(\Gamma_{\text{cold}} z)$ is the gain predicted by standard cold beam theory, G_{warm} is the warm beam gain, $\overline{\theta}_t = (\omega/\gamma_{||}^2, \beta_{||}^3 c) z (\Delta\gamma_{||}/\gamma_{||})$ is the normalized beam temperature, and $\overline{\theta}_t^{ac}$ is the normalized acceptance temperature such that when $\overline{\theta}_t = \overline{\theta}_t^{ac}$, $G_{\text{warm}} = G_{\text{cold}}^{1/2}$. The normalized acceptance temperature is a function of beam current and the interaction strength, and is plotted in Ref. 3. Over their range of common validity, the small temperature analytic formulas[1,5] and the heuristic formula (Eq. 6) predict temperatures that agree to $\sim 25\%$.

As is seen from Fig. 3b, below ~ 250G the beam temperatures obtained from the gain measurements are approximately constant, while above 250G the temperature increases with B_w and follows the temperature found by numeric simulations. The measurements imply that the total beam temperature is the sum of an "intrinsic" beam temperature probably generated in the electron gun and the beam transport, for which $\Delta\gamma_{||}/\gamma_{||} < 0.002$, and a temperature induced by the wiggler field. The data also indicates that both the initial condition derived quiver velocity temperature and the wiggler radial shear temperature degrade the FEL interaction. Because the quiver motion is more or less coherent, its effect is not completely equivalent to the usual concept of temperature. Therefore, it is not a priori obvious that the quiver motion should in fact degrade the FEL interaction, as is indicated by our measurements. In contrast, in a previous experiment [12] which measured beam temperatures by laser scattering, only the wiggler shear temperature was included in the analysis of the data.

The error bars in Fig. 3b are calculated from the estimated calibration and noise errors, and when the temperature is high the error is quite small. However, when the temperature is low, small calibration errors cause a large error in the estimated temperature. Inaccuracies in the theory used to calculate the cold beam gain or the acceptance temperature cause additional errors in the estimated temperature. For example, both the analytic formulas and the heuristic formulas are derived for Maxwellian velocity distributions and other distributions would cause the results to be somewhat modified.

In Fig. 5 we show the growth rate as a function of frequency f at a low and approximately constant beam current of $I \approx 0.15$A. The solid line is the prediction of cold beam theory, and the dashed line is the prediction of the warm beam theory (Eq. 6) with an assumed constant temperature $\Delta\gamma_{\|}/\gamma_{\|} = 0.0067$. Although this temperature is adjusted to give the best fit between experiment and theory, it is close to the predictions of Fig. 3. Since the the cold beam growth rate, the normalized temperature $\overline{\theta}_t$ and the normalized acceptance temperature $\overline{\theta}_t^{ac}$ all depend on the interaction frequency, these measurements are a good test of the theory.

Conclusions

We observe a large decrease in gain when temperature effects become important in a collective (Raman) FEL. These effects have been measured as a function of the beam current, the wiggler field strength, and the interaction frequency f. The results can be used to estimate the electron beam temperature, and these estimates are in good agreement with numeric simulations. We find that the heuristic temperature model of Jerby and Gover [3] is useful in analyzing our experiment.

Our results are complimentary to a previous experiment in which laser scattering was used to determine the temperature of an electron beam in a wiggler. [12] However, because of the complexity of the scattering experiment, the system was not operated simultaneously as a free electron laser. In contrast, our noninterfering technique permits measurements of the beam temperature while the FEL is in operation.

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Figures

Fig. 1 Schematic of the FEL

Fig. 2 Normalized growth rate $\Gamma/I^{1/4}$ as a function of the beam current I. The points are the experimental measurements. The straight line is the prediction of cold beam theory, and the dashed line includes the effect of temperature.

Fig. 3 The effect of the wiggler field B_w on the electron beam temperature. a) Growth rate vs. wiggler field. The points are the experimental measurements, and the line is the prediction of cold beam theory. b) Temperature vs. wiggler field. The dots give the temperature estimated from the gain reduction. The solid line is the beam temperature found by numeric simulations, including both the quiver and the radial shear components. The dashed line is the temperature from the radial shear alone. For this data, $B_{\parallel} = 1512$G, $I = 0.15 - 0.20$A, and $f = 10.63$GHz.

Fig. 4 The axial velocity of electrons with energy $\gamma = 1.297$ launched into a $B_w = 300$G wiggler field, and a $B_{\parallel} = 1512$G axial field. The wiggler entrance begins at $z = 0$cm and the wiggler has reached its full strength at $z = 20$cm. The dashed curve is for an electron initially at $r = 0$cm and the solid curve is for an electron initially at $r = 0.254$cm. The solid arrow indicates the radial shear velocity spread, and the dashed arrow indicates the quiver velocity spread for the $r = 0.254$cm case.

Fig. 5 Growth rate as a function of frequency at $I = 0.15$A, $B_{\parallel} = 1512$G, and $B_w = 305$G. The solid line is the prediction of cold beam theory, and the dashed line includes the effect of a constant beam temperature of $\Delta\gamma_{\parallel}/\gamma_{\parallel} = 0.0067$.

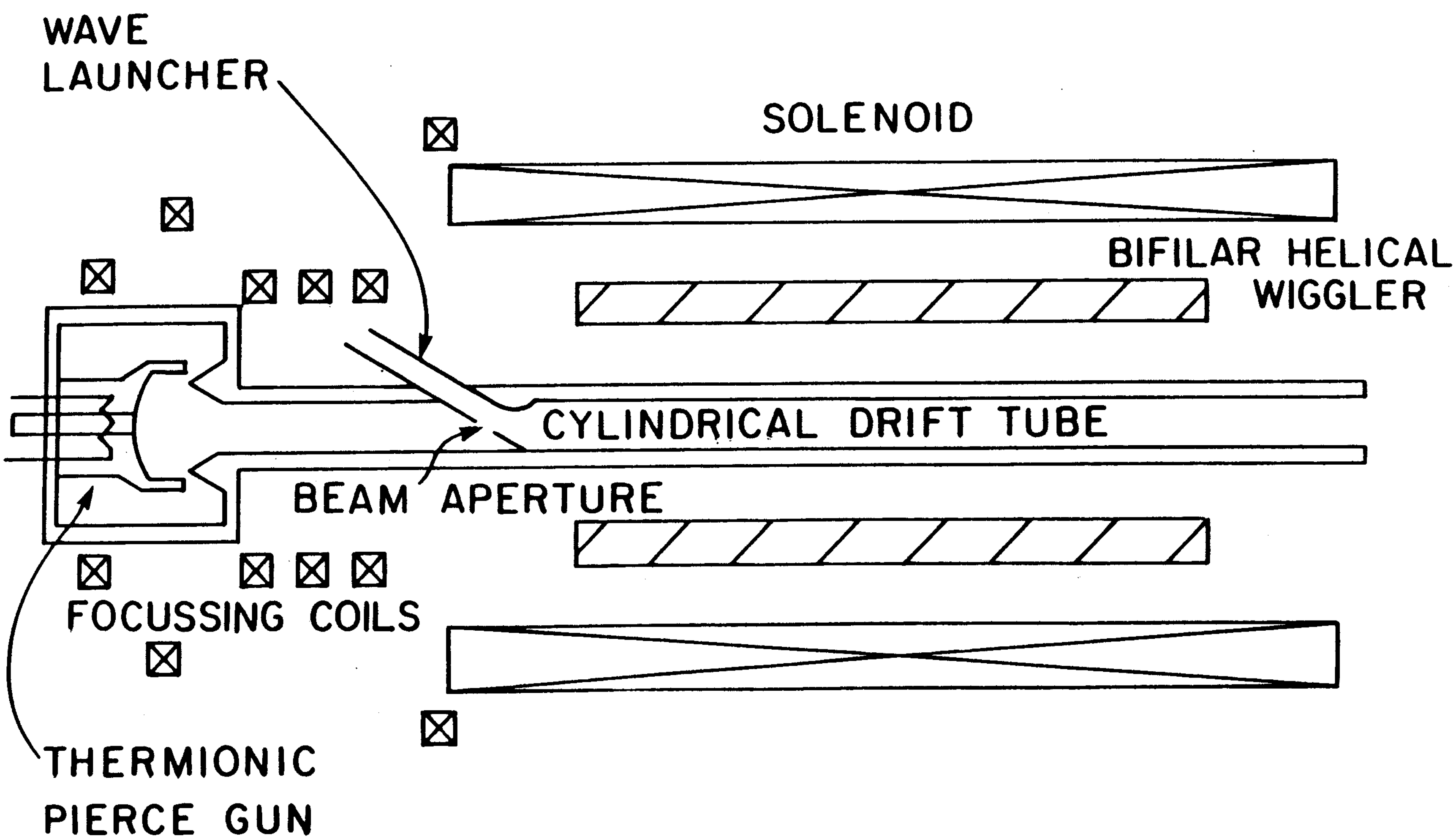

Fig. 1
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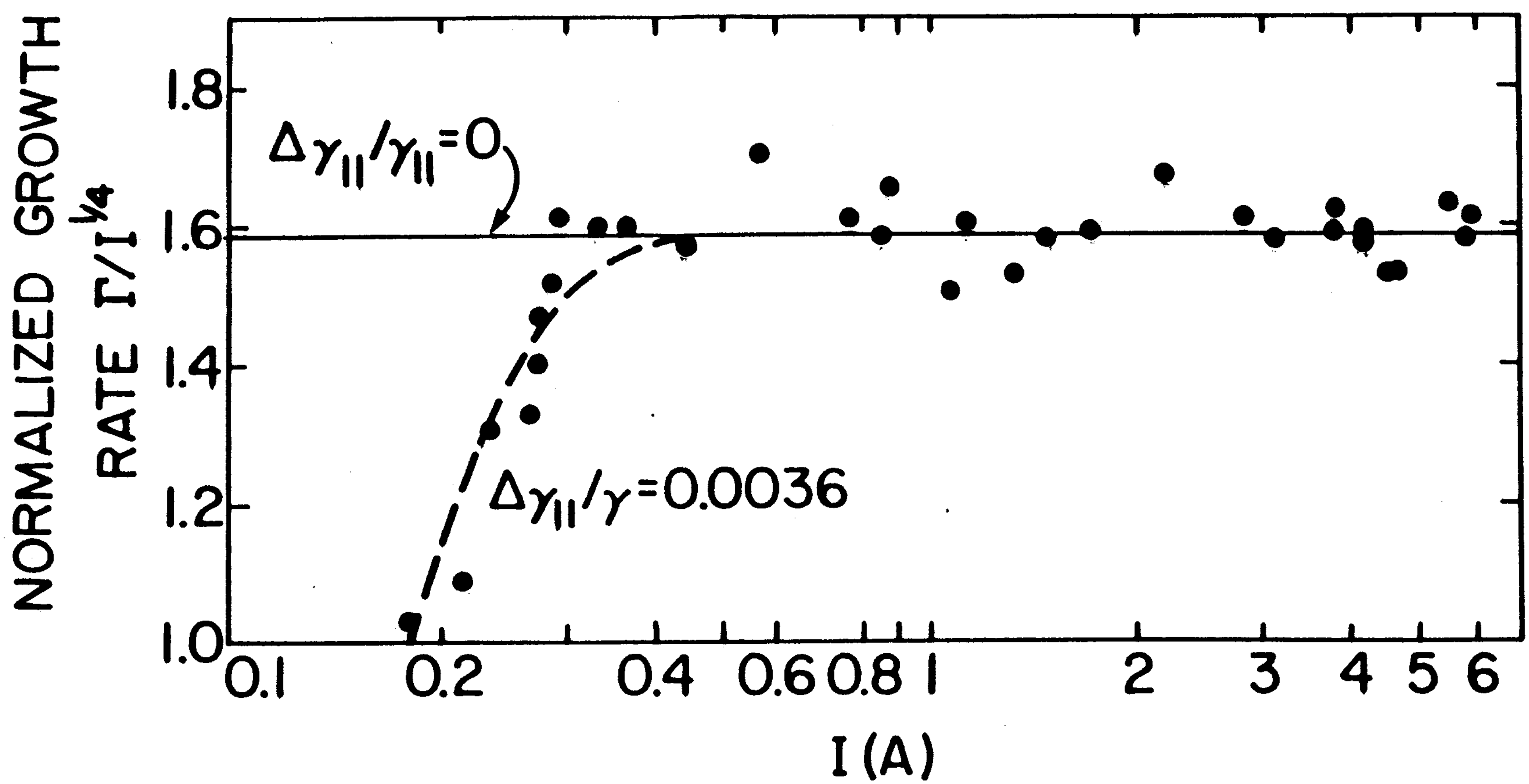

Fig. 2
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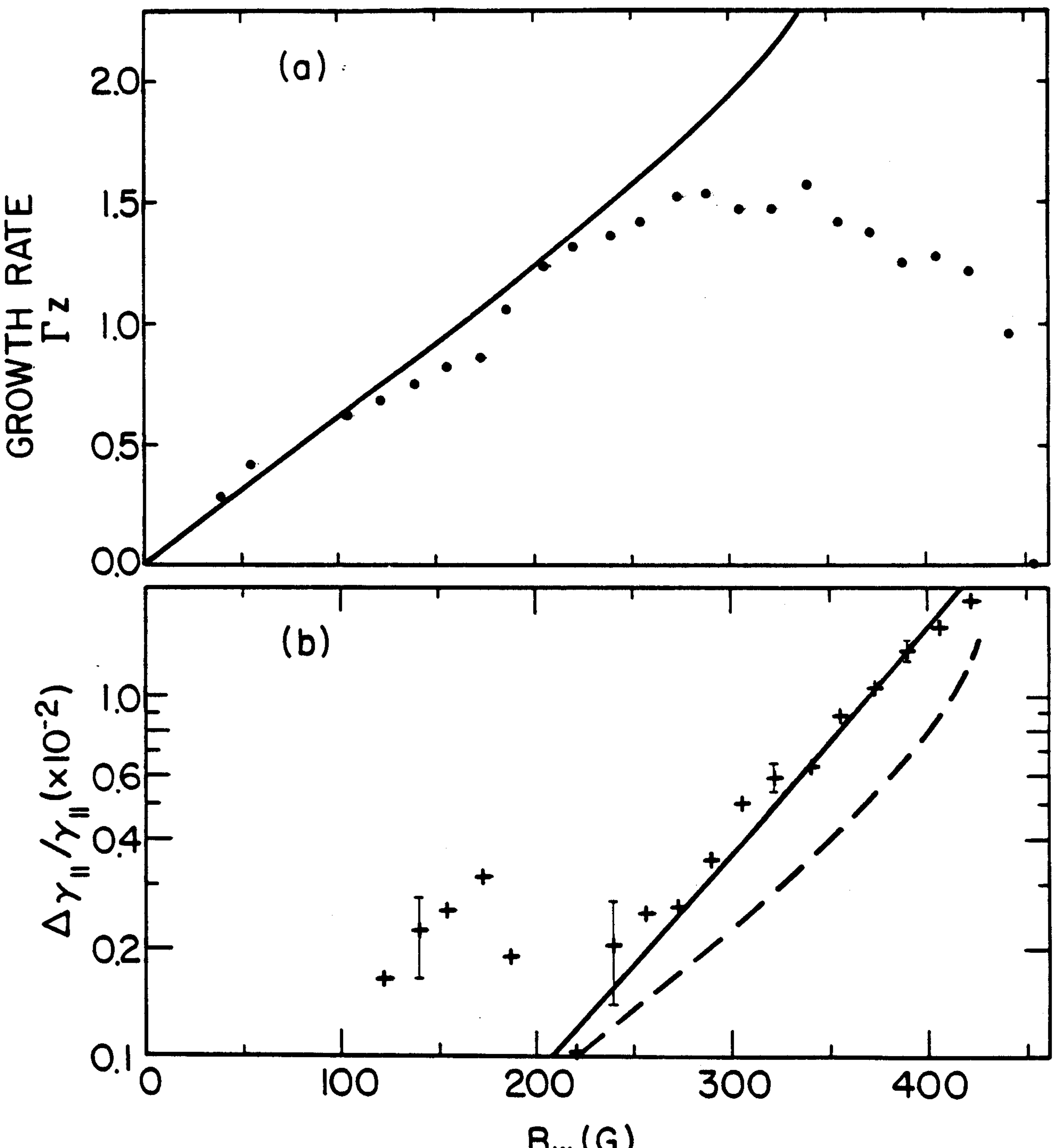

Fig. 3
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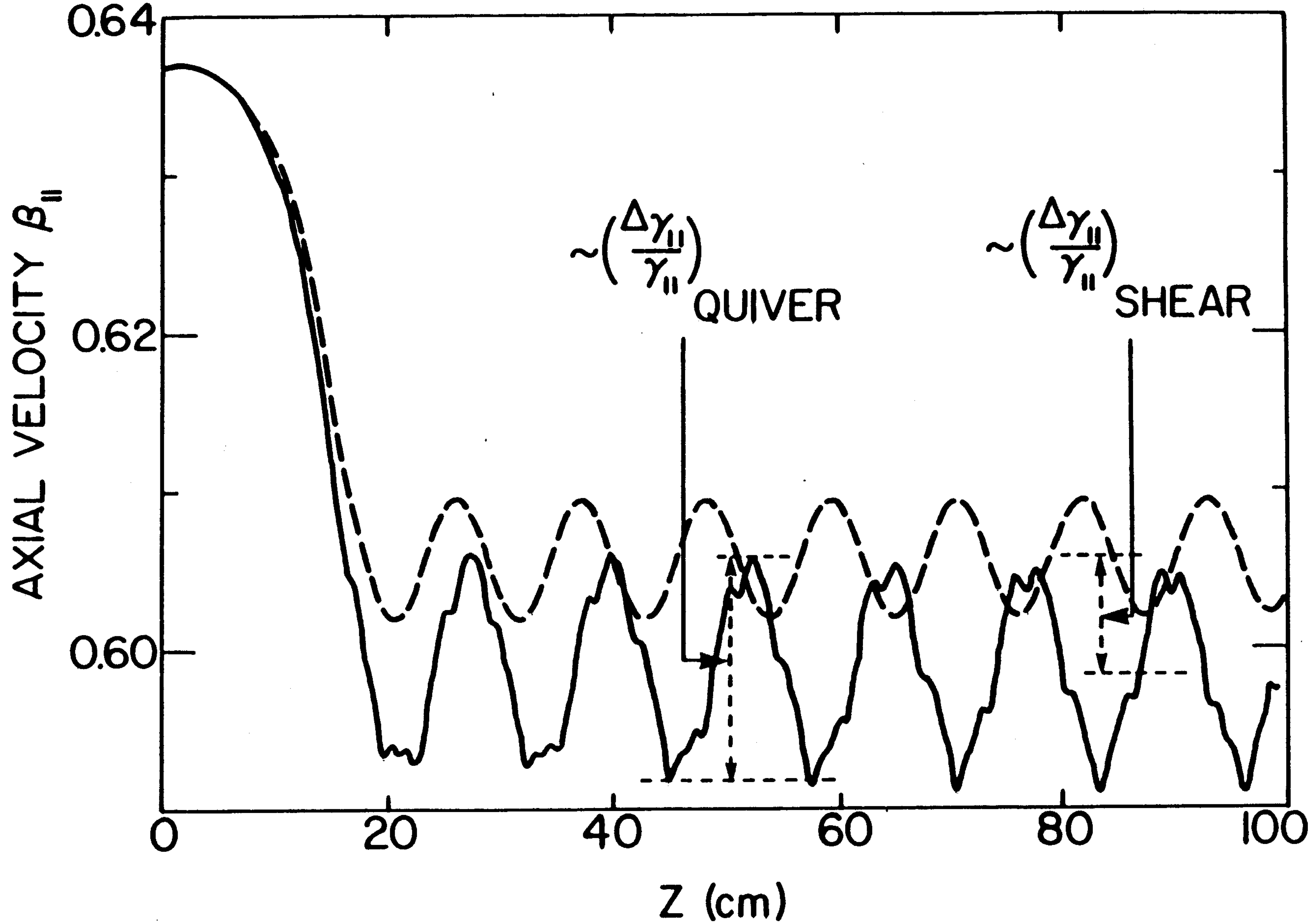

Fig. 4
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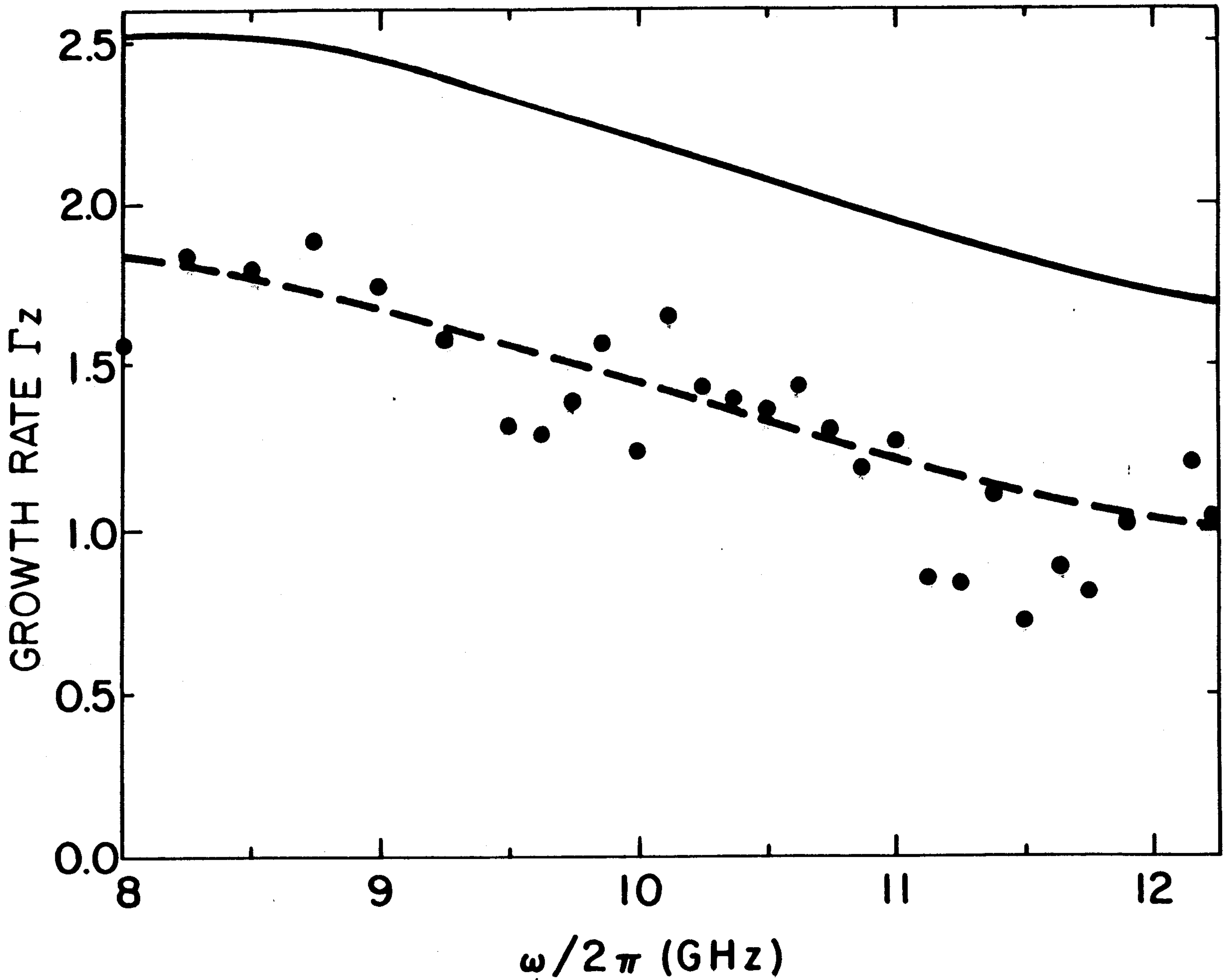

Fig. 5
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