Balancing Container Inventories for Ocean Carriers

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Abstract

Over the last twenty years the transportation industry has undergone a dramatic shift into container operations. The advantages of this mode of transportation are numerous, especially for the ocean carriers. The use of containers adds a high degree of versatility to their ships and increases the utilization of the vessels by means of a remarkable decrease in the loading and unloading operations time. However, the introduction of the containers adds, as well, a considerable investment cost to an industry that was already very capital intensive. The pressure of the high cost investment in equipment in addition to a remarkable competition in the sector forces every player in the industry to try to obtain the maximum efficiency in the utilization of its assets.

Global trade is not in general balanced, and so the demand for containers at the different ports of the world varies greatly. As a result of this unbalanced situation, empty containers must be reallocated from mainly importing areas to those at which the overall outflow of freight is larger than the inflow. Managing the container inventory and the container reallocation, subject to the particular requirements of the industry and the present and future demand, is known as the Container Allocation Problem. The purpose of this thesis is the development of a model for this problem so as to maximize the profit to be obtained from the management of a shipping line container inventory.

The container allocation problem is modeled by the use of a large-scale, multi-stage stochastic network formulation that incorporates the uncertainty factor in the demand side of the problem. This network formulation captures the space-time dynamics of the reallocation process while using an objective function that minimizes the cost of the container operations in the long run. A continuous rolling horizon to limit the number of nodes in the network is used in the modeling of this system so as to make this problem tractable.

Finally, a solution algorithm for this problem is proposed. The algorithm decomposes the initial non-linear network formulation into an iteration of successive linear approximations that can be solved via a classical linear programming method.

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CHAPTER ONE

1.1 INTRODUCTION:

With the increasing globalization of world economy, container-shipping lines have seen a steady growth in volume proportional to the growth in global trade. Virtually all ocean cargo, apart from bulk commodities such as oil, gas or grain, has switched to this containerized mode of transportation. The benefits of the modularization of the general cargo into separate containers include very fast handling and the extreme versatility of the vessels, since one ship can deal with any number of different types of cargo. However, there is a counterpart for all of these benefits. World trade flows do not produce a balanced freight flow network. As a result, container inventories of the shipping lines end up in locations of very little use.

Starting in the nineteen seventies, containerized transportation practices have increased to be almost the only mode of transportation in shipping lines for general cargo. Shipping companies, following this trend, have acquired large inventories of containers to provide transportation services from initial shippers to final consignees. These large inventories represent a very important part of the total capital investment of shipping companies, and also represent a very high operation cost. The operational management of such a container inventory will be the scope of this thesis.
An important part of container inventory management is the reallocation of empty containers; that is how to reposition the containers to meet present and future demand. Relocation of this excess inventory, empty containers, from places with lower outbound than inbound trade volumes and into high container demand locations is one of the major problems faced by the liner operators.

Historical data shows that a container travels a significant distance while empty in an average turnaround cycle. This comes from two basic problems that containerized shipping lines are constantly facing. The first problem is that global trade is fundamentally unbalanced. It is rare to find a port that has the same volume of goods flowing in and out. But that is not the only reason why containers have to be reallocated. Even if the flow of goods were perfectly balanced in the long run, temporary fluctuations in trade would enforce the reallocation of empty containers. Locations where the outflow of merchandise exceeds the inflow need a constant supply of containers to be able to match the demand. Locations where inflow of goods is more important than export usually generate a surplus of containers, the supply of containers is larger than the real demand. As a result, in the same way that there is an imbalance in global trade, the liners have an imbalance in the arrangement of their container inventories.

The objective of this thesis is to develop an operational model for container allocation, so that the container inventory management in a shipping company is as efficient as possible. This is generally called a container allocation model. How to determine the distribution of empty containers to meet customer demands at all different locations or ports at the right time will be
the basis for an efficient operating model. This paper will address the dynamic container allocation problem and propose a new model for the shipping industry, which is based on an approach that has already been used very successfully for the dynamic vehicle allocation problem in the long-haul trucking industry. (Sheffi and Powell). The goal will be to assist the liner operators in the efficient reallocation of their container inventory and the appropriate pricing for each distinct dispatch.

1.2 SHIPPING INDUSTRY OVERVIEW

The first step to take prior the explanation of the container reallocation problem itself will be a description of the container shipping industry. Any shipping company business consists on carrying goods from shippers to consignees through its transportation network. This network includes a number of ports and inland terminals, a fleet of ships and the container inventory.

Since the first introduction of the general cargo container, the ocean shipping industry has seen almost all its general cargo shift into this mode of transportation. Containerizing the cargo has solved or at least improved most of inefficiencies of the slow handling of general cargo in ports and transshipment areas. It has facilitated the connection between different modes of transportation (i.e.: trucks or train), reducing both loading and unloading times. As a result of this speeding up, the utilization of the ship inventory has been improved by shortening the turnaround time of vessels in ports. In addition, cargo loses and freight damage has been significantly reduced.
Notwithstanding the benefits of containerization, embedding the cargo in an “envelope” or a “cover” has created some other problems that are still to be solved. Such problems include, for example, the large investments in the container inventory and a high maintenance cost that occurs because the containers are now exposed to damage during trips. Most importantly containerization represents a new coordination problem, namely, how to best distribute the container inventory subject to the demand for containers and the shipping line’s transportation capacity.

To make the problem more challenging for the liner operators, a container is actually a simple envelope, with almost no differentiation from one to the next, and with very little technology involved. As such, containers are a classic example of a commodity. Competition thus is vigorous, the service provided is usually very similar for all the liner companies. Prices are as a result low and margins very slim. Understanding the real cost of every operation and choosing the right repositioning policies can make the difference between failure and success for a containership liner company.

In the case of a container shipping line, goods are handled in containers that usually come in two different sizes and approximately fifteen different types. These two sizes are usually either twenty or forty feet in length. Although there are some other sizes, these two are the most widely used. Containers are available in a variety of heights and can have other special characteristics that differentiate them, such controlled atmosphere. For a complete list see reference [8].
Containers are assembled for transportation purposes into container ships. A modern container vessel can carry up to almost 6000 TEU's (Twenty foot Equivalent Unit) at one time. The size and speed of these vessels has continuously improved since their first introduction in the early 1970's. This growth has been parallel to the demand for this type of service.

The focus of this thesis is the management of the container inventory of a shipping company. The container inventory of a shipping liner in general consists on two different categories of containers, namely leased or owned. The ratio of owned to leased containers depends mainly on the shipping company policy. We can find among the containers shipping lines almost every option ranging from all leased to all owned. The leased containers are usually on long-term leases, typically three or five year leases. For the purpose of this thesis these containers will be treated as one common group, all owned by the shipping company.

Due to general imbalances in trade as well as statistical fluctuation in this trade, the shipping companies are forced to carry empty containers from surplus to deficit regions on a short-term basis to meet the general demand. Decisions about the filling of immediate demand, whether done with owned or short-term leased containers, and especially decisions regarding reallocation of empty containers, will determine the ability of the company to meet future demand. This is a very delicate problem in the management of container inventories that we will define as Dynamic Container Allocation (DCA). The decision is dynamic because the decisions taken today will affect the future evolution of the container distribution. Today's decisions will directly affect the future supply of containers in the network depots and therefore the decisions to take in those coming periods.
As a conclusion of this section a final definition of the container allocation problem: *Decisions or set of decisions regarding the management and replenishment of a container inventory so as to "minimize" the total cost of the inventory operations in the long run while maintaining quality service levels.*
2. CHAPTER TWO:

2.1. THE CONTAINER ALLOCATION PROCESS:

In a typical ocean shipment, a shipper requests a container from a shipping line. The shipping company then provides the empty container to the shipper from its closest depot. This container is generally picked up by the shipper who takes it into its own facility to fill it with cargo. Once the container is filled, the shipper returns it to the same depot where it was picked up. Then, the shipping line sends it to its final destination, loading it directly into a ship in the case of a port or via train and later into a vessel in the case of an inland terminal.

Vessels traveling from port to port carry usually a mixture of loaded and unloaded containers. When loaded containers arrive to their destination port they are sent to their final consignees. If the final destination is situated inland, the containers are typically sent via train or truck to and inland depot where they are forwarded via truck to the end consignee. The container is then emptied by the consignee who returns the empty back to the shipping line once this operation is finished. This container is then ready to carry another shipment, therefore, restarting the cycle.
As a result, during the cycle the containers describe, each container out of the inventory will find itself at one of the following stages:

1. Stored in a port or depot
2. Traveling in a vessel, train or truck
3. Being loaded at the shipper’s site.
4. Being unloaded at the consignee’s location, prior to the return to the shipping line depot.
5. Waiting in a transshipment point for its next stage of transportation.

The objective of the container shipping lines is to achieve the most efficient utilization of their inventory in order to minimize its total managing cost. This must be done while maintaining a sufficient level of service to the clients and without compromising on time deliveries.

In order to achieve their goal, shipping lines must make the following decisions:

- *Decide the number of containers to reallocate into each port to satisfy future demand.*

- *Decide which freights or services to provide.* In general due to a surplus capacity, especially regarding vessels, shipping liners tend to respond to any demand or order placed by a shipper. With an overabundance of vessels a loss of freight might very well bear more adverse consequences than just the lost revenue of that particular load. One unsatisfied shipment could, in fact, cause the client to choose another transportation provider for future loads.

- *Decide the number of additional units to be leased to satisfy booked demand.*
Up to now, the general practice consists of maintaining a constant level of units at each depot based on historic demand and container supply levels. This policy has had, as a result, a greater number of units than are necessary in a daily operation, driving the utilization of the equipment to very low levels compared to its potential maximum.

The strategy of maintaining a constant level of containers has been, and still is, most probably reinforced by two causes. The first cause is the simplicity of implementation and latter control of the system. This control of the constant level of inventory system is, in general, exerted by local dispatchers. This factor leads to considerable inertia that prevents the implementation of new ideas, especially if they involve centralized decisions. The second cause is applying local instead of global performance measures. Measures such as local minimization cost policies would tend to build large inventories of containers since short-term leasing costs are typically large. The same result would occur under service level measures or percentages of owned containers supplied to customers. This bounded strategy, reinforced by obsolete policies and local measures will only drive the utilization of the container inventory to sub-optimal solutions. Holding a larger number of containers than needed at each depot is the end result of this situation with total costs far higher than those which could be achieved by a global and more coordinated allocation scheme.
2.2 DEFINITIONS AND VARIABLES INVOLVED IN THE PROCESS:

2.2.1 Entities involved in the container allocation problem:

- **Shippers**: The container allocation problem is highly dependent upon the nature of the shippers. They can be divided according to their behavior in two groups:

  1. Occasional clients: These firms usually book their freight service a very short time in advance, normally within three or four days of the departure of the vessel. These customers introduce uncertainty into the demand for empty containers and are very difficult to predict.

  2. Regular shippers: This firms have a stable history of business with the shipping line and usually book freight in advance and with a clear pattern for their shipments. They make up a large proportion of the total loads, and it is in general easier to get a good estimate of their future demand.

- **Consignees** also play an important role in this problem, since, once they have received the cargo, they hold the container for a period of time prior to return it to the shipping company. This period of time is not fixed, since each consignee may employ different times to unload the freight and return the container. Typically, shipping lines charge some fee after the first week or eight days to ensure that the containers are not standing underutilized in some customer yard and to discourage consignees for using the containers as temporary storage for their cargo. Because returned containers dictate the supply of empty containers, consignees introduce a new source of uncertainty to the future availability of containers to meet demand.
• **Freight forwarders**: Most of the ocean cargo is currently booked with the intervention of freight forwarders. This trend might change in the future, since freight forwarders are no more than intermediaries, and the use of the Internet to book shipments directly with shipping lines could displace the freight forwarders’ traditional role. At this point the information freight forwarders provide is very important in the forecasting of future demand. However, freight forwarders usually tend to overpredict demand to make sure they never run out of containers. This practice clearly reduces the utilization of containers.

• **Shipping agents** are sales and operations representatives of the shipping line for a certain region. They are very actively involved in the management of the container inventory, because they generally manage all the commercial and operational functions within their region.

• **Terminal operators**: Shipping lines use the services of the terminal operators who carry out all the handling and storage operations in depots and ports. Terminal operators are the source of a large part of the cost of container inventory management. Ship loading costs, unloading costs, and storage fees, that will be further described in section 2.2.3, are established by the terminal operators.

• **Container Lessors**: Regardless of the size and distribution of its own container inventory, a shipping line often needs to arrange short term leases to be able to fulfill all the immediate demand. These short-term leases cover in general either a one-way trip or as much as a round trip. A short-term lease covers a period on the order of fifteen to twenty days as opposed to the long-term leases mentioned in section 1.3 where shipping lines lease the containers for periods on the order of three to five years.
Short-term lease costs consist of 3 different charges: Pick-up fees, drop-off fees, and a daily cost. These costs will be discussed in the next section; they are in general very large compared to the rest of the costs involved in the management of the container inventory.

- **Inland transportation operators**: These are mainly trucking and rail companies. They are used by the shipping line to provide transportation out of its transportation network.

- **Container repair shops**: Containerization has brought improvements through decreasing cargo damage. However, now the containers themselves suffer the damage due to the handling of the freight. As a result containers need periodic overhauls and repairs carried out by the container repair shops.

Although these two last entities are directly related to the container inventory operation and the shipping carrier needs to assign a budget for these purposes, these functions are beyond the scope of this thesis.

**2.3. REVENUE AND COST STRUCTURE:**

**2.3.1. Revenue Structure:**

As opposed as less-than-container loads where rates and fees to customers are charged depending on a variety of characteristics, such as type of commodity, consolidation level, volume or weight, most of the full loaded container charges have very little variation and depend mainly on two factors. The first factor is the origin and destination pair of the shipment, and the
second factor is the transportation service provided to the customer. These services are divided into four general categories, which are listed below:

- Pier-to-Pier (P/P)
- Pier-to-House (P/H)
- House-to-Pier (H/P)
- House-to-House (H/H)

There is as well a slight variation in price due to the type of commodity carried, but in general this variation is not significant, probably due to tremendous competition in the market and the high level of available capacity.

2.3.2. Cost Structure:

The cost structure is by far more complex and interesting from the point of view of this analysis. On this side of the profit equation the total cost associated with the management of the container inventory can be split into several parts:

- **Loading Cost**: Charged by terminal operators to the shipping carrier, it is the cost to have the container loaded into a vessel. It varies from port to port, and it is the same, in general, whether the container is loaded or empty. It covers the handling operation from storage or shipping areas into the vessel.
• **Unloading Cost:** All the prior handling costs associated with loading also apply, except that in this case the handling operation takes place from the ship to the storage or departure areas.

• **Storage cost:** The storage costs are the charges that depots charge in exchange for the place to store the container. It varies with the region, and in general is very closely related to the availability or value of land. For that reason depots in Japan are far more expensive than those in places like Australia or the US. There are in general two types of storage contracts with depots. The first type is the renting of a specific area where the shipping carrier can store its own containers. The second contract type, which will be used in this thesis, consists of a per day and per unit storage fee.

• **Short term leasing costs:** As was mentioned, the short term leasing costs consist of three components. These cost have a very large impact on the total cost associated with a given containerized shipment. In general shipping companies will only incur this short-term lease cost when they are unable to cover all demand at a port with their own containers. Some authors, as for example Cheung and Chen [10], see these short-term leasing cost as the main driver of the reallocation process.

The first two components that form the short-term leasing cost are a pick up fee and a drop off fee. These fees depend on the area or region involved in the traffic. In general, regions with high outbound volume of trade have high pick up and low drop off fees, while regions where the inbound traffic prevails over the outbound will have low pick up and high drop of fees. For the purpose of this study these costs are going to be treated as fixed over time for each port or depot, yet in reality these fees are contracted on a per unit basis. This is a
reasonable simplification since the variations in these fees for a given port and time frame are in general very small.

The third part of the short-term leases cost is a rental fee per day plus and insurance charge to cover for possible damage to the container.

• Transportation cost: For the purpose of this study, the cost to move a container from port to port will be ignored. The cost associated with a vessel transporting goods from port to port remains the same regardless the number and contents of containers aboard. This cost, therefore, should be allocated to the vessel herself not the particular containers.

Hence, this cost should be allocated to the ship or the particular route that she covers. This thesis will not question the routes that a specific shipping company decides to cover. Should the shipping company desire to allocate part of the transportation cost into particular containers, it would be a sunk cost, not subject to any minimization or optimization. Therefore, this cost should not be part of the container deployment decision.

However, if transportation is done by means of a third company, outside of the shipping line, this cost must be considered. This is typically associated with trucking or rail lines services when shipping lines are unable to provide the services themselves.
2.4 INFORMATION AND ASSUMPTIONS FOR THE MODELS

Decisions about reallocation of empty containers, short-term leases to cover demand and whether to provide service to a potential customer are made based on the following information. These data will be considered known for all the models described later in this thesis:

1. Ship and train schedules. Note that these timetables are in general fixed and offered on a regular basis.

2. Future demand, part of which will be already booked and part of which would be forecasted based on historical data.

3. Inventory of empty containers at each depot. While the actual data is certain and known, the future supply remains uncertain. The shipping company does not know where or when the consignees of the freight will return the container. This rate of return must be treated as a random variable.

4. Costs. For the purpose of this study, all the cost related to the inventory operations are considered as known in advance.

5. Available capacities on each of the routes of the service network.

In addition this information, in order to build the models that simulate this problem, some assumptions must be made. Most of them can be easily relaxed or very accurately fit the container shipping industry. The following list enumerates and explains these assumptions in further detail:
• *All the containers will be considered as the same size.* This reduces the problem from a multi-commodity into a single commodity problem. However, since the market has basically two types of containers, 20 footer and 40 footer, I am going to address this problem by considering one forty as equivalent to two twenties.

• *The supply of leased containers is immediate and there are no restrictions to the number of containers that can be leased.* This assumption does represent reality fairly well since the number of containers available for leasing purposes is about half the total number of containers in the world. Even in the unlikely case that the shipping line could not obtain a container from a leasing company, is a common practice to borrow it from another shipping line under the charge of a premium.

• *Rolling horizon.* All the models to be proposed use a rolling horizon to account for the downstream effects of the current allocation decisions. The length of the planning horizon will be a compromise between a short period that would not capture downstream effects and a long horizon that would make the problem numerically intractable. For these reasons, the minimum time would not be shorter than the longest trip pair involved in the network, but the horizon should not be longer than a complete schedule cycle.

• *Only first period decisions actually implemented:* Each time the model is run, its purpose is to determine only the next set of decisions which must be made. The decisions for the subsequent periods would be delayed so the model has the maximum amount of information prior to make the next recommendation.

• *All the information for each period known at the same time.* Information coming after that time will be introduced into the model the following day.
• \textit{No two ships with the same port origin/destination pair will be leaving the departure port during the same day or period.} This assumption is mainly to eliminate further notational complications in the model, but it does not impose any significant limitation to the model.

2.4 USING DYNAMIC NETWORKS TO MODEL THE ALLOCATION PROCESS.

The idea of using dynamic networks to model the dynamic allocation problem is straightforward. Container movements are modeled using a space-time representation network as shown figure 2.1. As is shown in this figure, the two "axis" of this network represent a space and time framework. In such a network model, each port at a given day is represented as a node. This particular figure represents a set of four ports or depots along a five-day period. Arcs, in this network, represent traffic routes between any two depots at different times. Note that not all the ports are connected to each other. Only ports with a scheduled traffic have connections between each other. Therefore, the arc that connects port two in period 1 with port 1 in period 3 represents a vessel to depart port 2 on day one with arrival to port 1 on day 3.
Figure 2.1: Transportation Network, 4 ports and 5 periods of time.

The nature of these arcs is defined by the size of the respective vessel. The ship capacity will dictate the number of containers that she is able to carry, and so the arc capacity in the model is limited to take into account these restrictions. In general the flow of full and empty containers can be modeled as separated arcs. However, to avoid confusion in the figure both flows are joined together in one single arc.

In addition from the arcs connecting two different depots, figure 2.1 also has arcs connecting the same port in two consecutive days. These arcs have no limiting arc capacity, as many containers as required can “move” from day \( i \) to \( i+1 \) by staying in the same port. There are two costs associated with this type of arc which will be captured by the model. The first one is the direct storage cost at the port. The second and more subtle cost is the opportunity cost of having the
container standing in one port and not in any other where the demand might be greater or where a larger profit potential exists.

The combination of nodes and the arcs that connect them is exactly the service network of a given shipping company. With this representation it is clear that decisions made on a given day will affect the distribution of the container inventory in the future.

2.5. LITERATURE REVIEW

In this section two models for container allocation will be reviewed. The common scheme for both of these models is that they only model the movements of empty containers. The loaded containers simply leave the network model and are, when these models are implemented, treated by a separate simulation system. The situation or status of loaded containers is used to predict the supplies of empties once they become available at the different depots. For these empty container models the number of containers returned by the consignees is assumed to be known in the deterministic model, or forecasted, in the case of the stochastic model.

These only empty container models have another source of uncertainty. This comes from the fact that these simulations only take account of the number of empties carried in a vessel, so the remaining space in the ship, which is also dependent on the number of full containers onboard, must be treated as another random variable. To overcome this problem, this space is handled as a stochastic parameter predicted on the basis of past history.
2.5.1 Deterministic model, single commodity and only empty containers.

There is no uncertainty in this type of model. All the input variables of the problem are assumed to be known with certainty for both the present and the future periods. Although the model in this section involves only one type of container, it can be easily extended to a multi-commodity system where all the different types of containers present in the inventory of the shipping line are treated explicitly.

The following is a network representation for this model:

![Network Diagram](image)

Figure 2.2: Deterministic Network. Only empty containers.
There are arrows escaping from each port with no destination which represent the full containers that the model does not follow. They leave this network and appear some time in the future as an exogenous supply of containers at each depot.

There is an assumption that has been used to simplify the objective function in these models. All the demand will be fulfilled either by owned or leased containers. This assumption can be easily relaxed, yet we have seen in section 2.3 that it matches very well with the current shipping companies' practices.

**Model inputs:**

- \( P \): Set of port or depots in the network at a given time period.
- \( T \): Number of periods to cover in the planning horizon
- \( \tau_{ij} \): Travel time from port \( i \) to \( j \)
- \( CL_{ij} \): Average cost of moving a leased container from port \( i \) to \( j \). It takes into account the handling and leasing costs. Leasing costs are calculated by adding pick up and drop off fees, plus the rental cost over the average transit time of shipments going from \( i \) to \( j \).
- \( CO_{ij} \): Cost of moving an owned container from \( i \) to \( j \). Note that if \( i=j \) is only the storage cost while if \( i\neq j \), \( CO_{ij} \) are only the handling costs.
- \( \xi_{ij}(t) \): Market demand for empty containers from \( i \) to \( j \) starting at time \( t \).
- \( \eta_{ij}(t) \): External supply of empty containers into port \( j \) at time \( t \).
- \( U_{ij}(t) \): Remaining capacity for containers in the arc that connects depots \( i \) and \( t \) departing from \( i \) at time \( t \).
**Decision Variables:**

- \( x_{ij}(t) \) Number of empty containers departing port \( i \) at time \( t \) with destination port \( j \).
- \( y_{ij}(t) \) Demand from port \( i \) to \( j \) at time \( t \) satisfied only with owned containers.
- \( l_{ij}(t) \) Demand from port \( i \) to \( j \) at time \( t \) satisfied only with leased containers.

**Objective function:**

\[
\min \sum_{i,j \in P} \sum_{t \in T} \left[ CO_{ij} \cdot x_{ij}(t) + CL_{ij} \cdot y_{ij}(t) - R_{ij}(\xi_{ij}(t)) \right]
\]  \hspace{1cm} (2.1)

The first term in this formula addresses the total cost of handling the empty inventory container operations. The second term covers the cost of the leased containers, and finally the third one, is the total revenue obtained from each arc.

Note that the number of leased containers can be rewritten as the total demand minus the demand covered by owned containers, in such a way the last equation turns into:

\[
\min \sum_{i,j \in P} \sum_{t \in T} \left[ CO_{ij} \cdot x_{ij}(t) + CL_{ij} \cdot (\xi_{ij}(t) - y_{ij}(t)) - R_{ij}(\xi_{ij}(t)) \right]
\]  \hspace{1cm} (2.2)

Which regrouping the terms is:

\[
\min \sum_{i,j \in P} \sum_{t \in T} \left[ CO_{ij} \cdot x_{ij}(t) - CL_{ij} \cdot y_{ij}(t) - \xi_{ij}(t)(R_{ij} - CL_{ij}) \right]
\]  \hspace{1cm} (2.3)

Now, note that the last term of this equation is revenue minus leasing cost, both fixed for the purpose of the model, times the demand. These are parameters not subject to any change since they are not part of decision variables. Thus we can safely take them out of the equation to obtain the definitive objective function for a deterministic model:

\[
\min \sum_{i,j \in P} \sum_{t \in T} \left[ CO_{ij} \cdot x_{ij}(t) - CL_{ij} \cdot y_{ij}(t) \right]
\]  \hspace{1cm} (2.4)

Subject to the following constraints
\[ \sum_{j \in P} (x_{ij}(t) + y_{ij}(t)) = \eta_i(t) \quad \forall i \in P \]  
\[ 0 \leq x_{ij}(t) \leq U_{ij}(t) \quad \forall i, j \in P \]  
\[ 0 \leq y_{ij}(t) \leq \xi_{ij}(t) \quad \forall i, j \in P \]

For \( t=1 \) while for the rest of the rolling horizon, \( t=2, \ldots, T \)

\[ \sum_{j \in P} (x_{ij}(t) + y_{ij}(t)) = \eta_i(t) + \sum_{i \in P} x_{ui}(t - \tau_u) \quad \forall i \in P \]  
\[ 0 \leq y_{ij}(t) \leq \xi_{ij}(t) \quad \forall i, j \in P \]  
\[ 0 \leq x_{ij}(t) \leq U_{ij}(t) \quad \forall i, j \in P \]

In both cases, the first constraint is basically a flow continuity equation, the flow out of any port at a given time period can never exceed the amount of containers flowing into it. The second constraint is simply a vessel capacity limitation, and finally the third one prevents the flow of loaded containers from being larger than the demand for them.

This is a classical minimum cost flow problem formulated as a linear programming minimization problem that can be efficiently solved via optimization techniques such as the Simplex algorithm.

This algorithm returns as a solution a set of optimal flows, both, \( x \) and \( y \) for \( t \) going from period 1 to \( T \). However, only period one flows would actually be implemented. The solution would be run again the next day once the new data for the next day in the planning horizon is known so to obtain the new optimal \( x \) and \( y \). \( x_{ij}(1) \) is the set of reallocation decisions to do in the current day from ports \( I \) to \( j \). \( y_{ij}(1) \) is the part of the demand that has to be served with the shipping line.
owned inventory. And finally the \( l_{ij}(1) \) which are the containers leased at each location \( I \) to cover routes \( I-j \) are calculated as:

\[
L_{ij}(1) = x_{ij}(1) - y_{ij}(1)
\]

2.11

2.5.1.b Stochastic Model. Single Commodity. Only Empties Count.

This model is a natural extension to the previous one. In the deterministic system all planning data were assumed to be fixed and known quantities. In general only the current period data is known with absolute certainty. The data for the future is not complete and it must be estimated with forecasted parameters, based on the past functioning of the network.

The quality of the solution of the deterministic model depends on the quality of the predictions used as known data in the system. If the future predictions are subject to wide variability the solution provided by the deterministic model will be very poor in the long run. If the data have a large dispersion range the realizations of these parameters in the future will most likely be far from the average. Since the deterministic model typically uses these values as inputs, the recommendations of the model will, in general, not produce a distribution of the inventory close to that one which minimizes the total cost.

To avoid this difficulty, the network optimization problem can be rewritten as a stochastic recursive problem. This stochastic recursive formulation has two stages; the first one (the current period) where all the input variables are known as in the deterministic system. The second stage covers the rest of the periods up to the end of the planning horizon.
Let then \((\Omega, T, P)\) be a probability space and \(\xi, \eta\) and \(U\) be functions of random variables defined in this probability space. Let \(\omega \in \Omega\) be an outcome. Then \(\xi(\omega), \eta(\omega)\) and \(U(\omega)\) represent particular realizations of the random variables for \(\omega\).

The next figure is a representation of four ports in a two stage stochastic problem with a planning horizon of five days. The dotted component is the second stage of the problem, where the inputs are stochastic. The full lines represent the first stage with deterministic values for which we know all the information. Note that all the arcs starting in stage 1 are deterministic values, even when running into stage 2. These arcs are to be used as an interface or connection from stage 1 to the second stage.

![Stage One](image)

**Stage One**  
**Stage Two**

Figure 2.3: Stochastic Network. Two Stages. Only empty containers.
The common inputs for this model are now,

- \( P \) set of ports in the network.
- \( T \) planning horizon, \( t=1, 2, 3, \ldots, T \)
- \( CO_{ij} \) Handling cost of moving an owned container from \( i \) to \( j \).
- \( CL_{ij} \) Cost of moving a leased container from port \( i \) to \( j \).

While inputs for stage one are,

- \( \xi_{ij}(1) \) Demand of containers departing port \( i \) on period one and with port \( j \) as the final destination.
- \( \eta_i(1) \) External supply of empty containers on period 1 at depot \( i \).
- \( U_{ij}(1) \) Remaining capacity of arc connecting depots \( i \) and \( j \) departing on the current period.

As for stage two are,

- \( V_{ij}(t) \) State variable representing the flow going from stage 1 to stage 2 through the interface arcs.
- \( \xi_{ij}(t, \omega) \) Demand for containers at port \( i \) on period one and with port \( j \) as the final destination.
- \( \eta_i(t, \omega) \) External supply of empty containers on period \( t \) and depot \( i \).
- \( U_{ij}(t, \omega) \) Remaining capacity of arc connecting depots \( i \) and \( j \) departing on period \( t \).

Decision variables:

Again they are split in two sets,

- \( x_{ij}(1) \) Number of empty containers departing port \( i \) at time 1 with destination port \( j \).
\( y_{ij}(t) \) Period 1 demand from port \( i \) to \( j \) satisfied only with owned containers.

\( x_{ij}(t, \omega) \) Number of empty containers departing port \( i \) on period \( t \), \( t \) greater than 1, with destination port \( j \).

\( y_{ij}(t, \omega) \) Demand from port \( i \) to \( j \) satisfied only with owned containers for all the periods after the first.

Recall that for this model the assumption is that demand will be covered by the carrier, either with owned or leased containers. The deterministic objective function saw in the previous section is now extended into:

\[
\min \{ CO_{ij} x_{ij}(1) - CL_{ij}(1) + EQ(V_{ij}, \omega) \} 
\]

Subject to:

\[
\sum_{j \in P} \left( x_{ij}(1) + y_{ij}(1) \right) = \eta_{i}(1) \quad \forall i \in P
\]

\[
0 \leq x_{ij}(1) \leq U_{ij}(1) \quad \forall i, j \in P
\]

\[
0 \leq y_{ij}(1) \leq \xi_{ij}(1) \quad \forall i, j \in P
\]

\[
V_{ij}(\tau_{ij}) = x_{ij}(1)
\]

Where,

\[
Q = \sum_{i, j \in P} \sum_{t=2}^{T} \left( CO_{ij} \cdot x_{ij}(t, \omega) - CL_{ij} \cdot y_{ij}(t, \omega) \right)
\]

Subject to the following constraints,

\[
\sum_{j \in P} \left( x_{ij}(t, \omega) + y_{ij}(t, \omega) \right) = \sum_{j \in P} V_{ij}(t) + \eta_{i}(t, \omega) + \sum_{k = 1}^{P} x_{ik}(t - \tau_{ik}) \quad \forall i \in P
\]

\[
0 \leq y_{ij}(t) \leq \xi_{ij}(t) \quad \forall i, j \in P
\]

\[
0 \leq x_{ij}(t) \leq U_{ij}(t) \quad \forall i, j \in P
\]
Constraints are exactly the same as they were in the deterministic case. There is only one basic distinction, which is the connection between the two stages.

Note that equation 2.18 that deals with the flow constraint has now an extra term, the $V_{ij}$. The interface between stages is a deterministic value as opposed to the rest in of the variables in the equation. Note that this variable represents the empty containers sent on period 1 from I to j to arrive on period $\tau_{ij}$.

Note as well that for this formulation to work without inconsistencies the empty flows $x_{ij}(1,\omega)$ have to be all zeroes. If not, we would be double counting as the flow of empty containers during stage 1 has already been taken into account through the new variables $V_{ij}$.

The solution derived from this model, is the set of optimal flows, $x_{ij}(1)$ and $y_{ij}(1)$, both at current time,

$x_{ij}(1)$ Containers to reposition from port I to port j.

$y_{ij}(1)$ Number of owned containers to cover the demand from I to j.

Again, the number of leased containers needed to cover the remaining demand from I to j will be,

$$l_{ij}(1) = x_{ij}(1) - y_{ij}(1)$$ 2.21

As in the previous model shown only the decision for the current period will be implemented, delaying further decisions for the following days so that the amount of certain information is larger.
Cheung and Chen solve a very similar formulation to this one in [10] using two different methods. The first one is a stochastic quasi-gradient method or SQG. That is a typical approach for recourse transportation problems, and it is typically not very fast in converging to a solution. The reason for the slow convergence is that the non-linear expected value function is approximated by a set of linearized stochastic sub-gradients, converting the non linear problem into a series of LP’s that are solved iteratively. These sub-gradients can be quite different in each iteration and this can make the solution vary greatly from iteration to iteration. The second approach used by these authors is a stochastic hybrid approximation procedure similar to an SQG but approximating the expected recourse function with a piecewise linear model.

A natural extension to this two-stage model is a multistage problem with network recourse. In this case the random variable \(\omega\) that was used in the two-stage problem would be split into \((T-1)\) \(\omega_t\)'s to form a distinct random variable for each period after the current one and until the end of the planning horizon.

In this case the recurse function \(Q\) for period 2 would have embedded in it the rest of them. The objective function will then look like this,

\[
\begin{align*}
\min \sum_y & \quad CO_y \cdot x_y(1) - CL_y \cdot y_y(1) + \\
E \{ & \sum_y CO_y \cdot x_y(2, \omega_2) - CL_y \cdot y_y(2, \omega_2) + \ldots \ldots \} + \\
E \{ & \sum_y CO_y \cdot x_y(t, \omega_t) - CL_y \cdot y_y(t, \omega_t) + \ldots \ldots \} 
\end{align*}
\]
The constraints for this last formulation are very similar to those seen in the previous model. A solution for this case is proposed by Chu in [17] using a decomposition method that takes advantage of the tree structure of the problem.
3. CHAPTER THREE

3.1 COMPLETE MODELS (BOTH FULL AND EMPTY CONTAINERS)

This chapter outlines the development of a new model for dynamic container allocation. The main difference between the previous models and the model to be explained here is that this takes into account both, full and empty containers throughout the transportation network. This new model comes from an extension of a dynamic vehicle allocation (DVA) model developed and implemented to improve the fleet management of a truckload carrier operation.

The extension to the DVA model has been developed to take account of the differences between the trucking and shipping modes. DVA is not subject to any fixed vehicle schedule, and as different independent units, trucks can move around at their discretion. The dynamic container allocation (DCA) problem, on the other hand, is subject to the vessel voyage schedules. That is not the only distinction; the trucking scheduler has full control of the truck fleet, since the driver goes along with the truck and the company never loses track of its equipment. Shipping lines, on the other hand, forward the freight to the consignee that chooses when to return the container. Lastly, the current models for trucking companies do not deal with leased units. While that may not represent a major part of the cost in a trucking company, it does represent a large cost for a shipping line. As we have seen, leasing is a common practice in ocean transportation, and the associated cost can account for a very large part of the operational management budget.
3.2 FORMULATION OF THE MODEL

This model will also assume that all the demand is to be covered, either by leased or owned containers. As in the latter models of the previous chapter, the voyage schedules and all the costs are considered deterministic and known. Stochastic variables will be used for the rest of the parameters.

For the purpose of this formulation, two new concepts are to be introduced prior to the description of the model itself.

First, the average delay: This concept represents the average time elapsing between the arrival of a container into a depot, full or empty, and the time to reuse that specific container. It is different for each depot, since the ratio full/empty arriving into each port is historically different. The empty containers are generally ready to use the next day after the arrival, if not the same day. The full containers, as mentioned before, must be forwarded to the consignees for unloading them prior to the return of the empty container ready to be reused. This variable will be called in the model \( AV_i \) and will only depend on the specific port or depot where the containers arrive.

Density function of demand: The solution of this model will require a probability density function of demand for each link in the network. This distribution will describe the demand probability for each port and each time period. The simple and appropriate density function
which characterizes the possible range of demands that can occur is the Erlang Distribution, given by,

\[
f(\omega) = \frac{\lambda^\alpha \cdot \omega^{\alpha-1} \cdot e^{-\lambda \cdot \omega}}{(\alpha - 1)!}
\] 3.1

Where \( \alpha \) is positive integer and \( \lambda \geq 0 \). For a given arc, \( i, j \), starting at time \( t \), the distribution is characterized by a \( \alpha_{ij}(t) \) and \( \lambda_{ij}(t) \). Although the actual demand of containers is a discrete integer random number, the use of a continuous distribution will not affect the result.

The shape of these distributions is given by \( \alpha \) and \( \lambda \). These two parameters dictate also the mean and the variance that must be estimated by the forecasting model used to predict the future demand. Although the nature and quality of the forecasting system of the shipping line is critical for this model to function, effectively, the forecasting itself is beyond the scope of this study.

Although the Erlang distribution is used in this explanation, this model will work with any distribution since none the following calculations make any assumptions as to the nature of the density function.

The variables used in this formulation are slightly different from the previous set, due to the change in the strategy. Let then the following variables be the inputs for this model,

- \( P \) set of ports in the network.
• T planning horizon, t=1, 2, 3,....., T

• CO_{ij} Handling cost of moving an owned container from i to j.

• PL_{ij} Profit obtained from a loaded leased container contracted to carry the load from port i to j.

• PO_{ij} Profit obtained from a self owned loaded container traveling from port i to port j

• \tau_{ij} Travel time to travel from depot 1 to depot 2.

• \xi_{ij}(t,\omega) Demand distribution at time t for freight departing port i period t and with port j as the final destination.

• U_{ij}(t) Total capacity, maximum number of containers to be carried by the vessel travelling from port i to j departing port i on period t.

• R_{i}(t) Surplus or deficit of returning containers from the expected number.

• AD_{i} Average delay time to reutilize containers at port i.

And let the decision variables be the following set,

x_{ij}(t) Total number containers, both full and empty, departing port i at time t with destination port j.

This model will require the use of two auxiliary variables, namely P_{ij,\lambda}(x_{ij}(t)) and MP_{ij,\lambda}(x_{ij}(t)). P_{ij,\lambda}(x_{ij}(t)) is the total expected profit obtained from the arc (i,j) with origin on time t, given a flow of x_{ij}(t) containers from i to j. MP_{ij,\lambda}(x_{ij}(t)) is the marginal profit contributed by the link (i,j) starting at time t obtained by an extra unit of flow over x_{ij}(t)
Given the definition of the expected profit \( P_{ij,t}(x_{ij}(t)) \) we can easily write the objective function for this formulation as,

\[
\max \sum_{i \in P} \sum_{j \in P} \sum_{k \in P} P_{ij,t}(x_{ij}(t)) \tag{3.2}
\]

since the total expected contribution of the set of flows \( x_{ij}(t) \) is the summation of all separate expected contributions.

The following constraints complete the model,

\[
0 \geq x_{ij} \geq U_{ij} \quad \forall i, j \in P \tag{3.3}
\]

\[
\sum_{j \in P} x_{ij}(t) = \sum_{k \in P} x_{ki}(t - (r_{kl} + AD_{l}) + R_{i}(t)) \quad \forall i, j \in P \tag{3.4}
\]

These constraints are very similar to those seen in the previous chapters. The first one is simply a capacity limiting constraint. No vessel can carry more owned containers than its maximum capacity. Note that this model does not address the number of leased containers explicitly in the limiting capacity constraint. This fact might seem counterintuitive, but there is a reason for it. The number of leased containers is already limited by the formulation of \( P_{ij,t} \) as will be demonstrated later in the chapter.

The second constraint in this formulation is a conservation of flow constraint. It states that the number of owned containers leaving a port cannot surpass the supply in that particular port and timeperiod. Note that in this constraint the new variable \( AD_{l} \) is introduced, adding itself to the time that takes for any trip \((i,j)\) to be finished.
This practice can lead to some conservation flow problems whenever the containers are not returned in exactly the average expected time. This problem is solved by the use of the auxiliary variable \( R_i(t) \). This variable is local for each depot and reflects the realization of time 1( present) of a different number of containers that the expected by the conservation of flow equation. Is easy to establish then that,

\[
\sum_{t \in T} R_i(t) = 0 \quad \forall i \in P
\]

This equation recognizes that if the number of containers arriving on day 1 is larger or smaller than expected, then the future number to arrive at the same port must be smaller or larger than expected. In other words, the sum of the deviations from the means must cancel out.

Using this formulation all the containers owned by the shipping company are tracked and the uncertainties regarding the remaining capacities of the vessel are removed. Also, although this formulation introduces a new variable \( R \), it eliminates from the problem the necessity of a forecasted prediction of the external supply of empty containers.

The output of this model will be the optimum set of total movements \( x_{ij}(t) \), in this case full and empty containers, that minimizes the cost of the operations. The values \( x_{ij}(1) \) represent the total number of containers to be sent from \( i \) to \( j \) during the current period. Other decisions will be deferred until the model is updated and resolved in the next period.
If demand for a particular link \((i,j)\) is larger than the actual flow then the number of leased containers to fulfill that demand will be,

\[ x_{ij}(1) \quad \text{in the case where} \quad U_{ij}(1) \geq \xi_{ij}(1), \text{the demand of the arc being smaller than capacity or} \]

\[ U_{ij}(1) - x_{ij}(1) \quad \text{in the case where capacity is smaller than demand} \quad \xi_{ij}(1) \geq U_{ij}(1) \]

If, on the contrary, the demand \(\xi_{ij}(1)\) is smaller than the actual number of containers to be moved \(x_{ij}(1)\) then

\[ x_{ij}(1) - \xi_{ij}(1) \quad \text{represents the number of empty containers to be reallocated from port} \ i \ \text{to} \ j \]

in the current period.

The expected profit function, heart of this model, \(P_{ij,t}\) defined as follows,

\[
P_{ij,t}(x_{ij}(t)) = PO_{ij} \cdot x_{ij}(t) - (PO_{ij} + CO_{ij}) \int_{0}^{x_{ij}(t)} (x_{ij}(t) - \omega) f_{ij,t}(\omega) d\omega + PL_{ij} \int_{x_{ij}(t)}^{\xi_{ij}(t)} (\omega - x_{ij}(t)) f_{ij,t}(\omega) d\omega
\]

This first term of this equation simply states the possible profit to be obtained from the total flow of containers in the link. All these containers are owned and this profit is calculated regardless whether they are full or empty. The second term in this equation subtracts from this initial profit the cost and the correspondent profit of the expected number of empty containers involved in the total flow. Note that the integral in this second term adds up all the revenue contributions of all
possible number of empty containers starting at zero and up to the total flow. The integral in the second term is the expected number of empty containers given the demand probability distribution and the total number of containers moved from i to j.

For a further clarification of this point, consider the next figure. In it we can see two different demand distributions. Distribution 1 represents a port with low demand for empty containers while distribution 2 represents another port with a much higher demand. Consider a value $x$ the same total flow from both ports.

Figure 3.1: Expected empty containers for two probability distributions and same total flow

Note that these two distributions have very different mean values. The given total flow $x$ exceeds by far $E(D1)$ but does not reach $E(D2)$. Therefore, for the same total outflow, in a port
with demand following distribution 1, we will find a large number of these containers to be empty. On the other hand a port with a distribution such as number 2, given a total flow of $x$, will most probably have none of the containers without freight. This is the idea captured behind the integration,

$$\int_0^{x(y(t))} (x_y(t) - \omega) f_{y,t}(\omega) d\omega$$

This integration is the shadowed areas in figure 3.1. Note that as it was expected for the same amount of total flow $x$ the shadowed area on the left (Obtained from Distribution 1) is much larger than the one on the right (Obtained with Distribution 2).

The last term of this expected profit function, function 3.6, represents the possible profit obtained by leased containers. This term is similar to the previous one. It captures the average number of containers, up to the total capacity of the arc, that a vessel will carry given a particular demand distribution and total number of owned containers moved within this arc.

Again the formulation for this third term is,

$$PL_y \int_{x_y(t)}^{u_y(t)} (\omega - x_y(t)) f_{y,t}(\omega) d\omega$$

Note that the potential profit to be obtained from this term is limited by the link maximum capacity. As the owned container flow reaches the arc maximum capacity the contribution to the final earnings from the leased containers goes to zero.

We can also see from figure 3.2 that when the total flow is larger than the expected demand the profit outcome from leased containers becomes reduced. On the other hand if the demand
distribution is shifted to the right, with greater demand as in figure 3.3, the earnings from leased containers are larger.

Figure 3.2: Expected number of leased containers for a total flow of owned containers larger than the demand distribution mean.

Figure 3.3: Expected number of leased containers for a total flow of owned containers smaller than the demand distribution mean.
As a summary of this section, the goal of this model is to maximize the total profit to be obtained from the shipping line container inventory, subject to the demands at the different ports. The function to maximize in this case is then,

$$\max \sum_{i \in P} \sum_{j \in P} \sum_{t \in T} P_{ij,t}(x_{ij}(t))$$  \hspace{1cm} 3.9

Subject to the following constraints,

$$0 \geq x_{ij} \geq U_{ij} \hspace{1cm} \forall i, j \in P$$  \hspace{1cm} 3.10

$$\sum_{j \in P} x_{ij}(t) = \sum_{k \in P} x_{ki}(t - (\tau_{ki} + AD_i)) + R_i(t) \hspace{1cm} \forall i \in P$$  \hspace{1cm} 3.11

Where the function $P_{ij,t}$ is defined as follows,

$$P_{ij,t}(x_{ij}(t)) = PO_{ij} \cdot x_{ij}(t) \cdot (PO_{ij} + CO_{ij}) \int_0^{x_{ij}(t)} (x_{ij}(t) - \omega) f_{ij,t}(\omega) d\omega +$$

$$+ PL_{ij} \int_{x_{ij}(t)}^{u_{ij}(t)} (\omega - x_{ij}(t)) f_{ij,t}(\omega) d\omega$$  \hspace{1cm} 3.12

This model uses continuous probability distributions and the flows $x_{ij}(t)$ are also considered as continuous variables. In practice the model the resulting maximizing profit flows would, in general, not be integers. Although containers are non-divisible, which might encourage us to use an integer formulation, for all the models described in this thesis a continuous formulation is used. Given that all the flows obtained as a result of these formulations are large enough, it might be preferable to round those real number solutions to integer values, even at the risk of introducing some error, rather than approaching the calculations as an integer programming problem.
4. CHAPTER FOUR

In this chapter both the solution procedure for the model and the issues related to the implementation of it in a shipping company will be addressed. However, prior to these two points the uniqueness of the solution obtained through this method will be proved.

4.1 CONDITIONS FOR UNIQUE SOLUTION

Next it will be proven that this formulation is globally concave. The advantage of a concave objective function is that it has a unique global maximum. Thus, once a solution to the optimization problem is found we are assured is not a local maximum.

The objective function for this model is,

\[ \max \sum_{i \in P} \sum_{j \in P} \sum_{t \in T} P_{ij} \left( x_{ij}(t) \right) \]  

4.1

This function is separable since every one of the expected profit functions is only dependent on its given flow \( x_{ij}(t) \). That means that to prove global concavity it is sufficient to prove that every \( P_{ij,t}(x_{ij}(t)) \) is concave with respect to its flow. The profit function is,
\[ P_{y,i}(x_y(t)) = PO_y \cdot x_y(t) - (PO_y + CO_y) \int_0^{x_y(t)} (x_y(t) - \omega) f_{y,i}(\omega) d\omega + \\
+ PL_{y,i} \int_{x_y(t)}^{u_y(t)} (\omega - x_y(t)) f_{y,i}(\omega) d\omega \quad 4.2 \]

A first derivative leads us to the marginal profit,

\[ MP_{y,i}(x_y(t)) = R_y - (R_y + CO_y) \int_0^{x_y(t)} f_{y,i}(\omega) d\omega - PL_{y,i} \int_{x_y(t)}^{u_y(t)} f_{y,i}(\omega) d\omega \quad 4.3 \]

Taking a second derivative,

\[ \frac{d^2 P_{y,i}(x_y(t))}{d(x_y(t))^2} = R_y - (R_y + CO_y) \cdot f_{y,i}(x_y(t)) + PL_{y,i} \cdot f_{y,i}(x_y(t)) = \\
= -(R_y + CO_y - PL_{y,i}) f_{y,i}(x_y(t)) \quad 4.4 \]

The distribution \( f_{y,i}(x_y(t)) \) is greater than 0 for any \( x_y(t) \geq 0 \). The term \( (R_{ij} + CO_{ij} - RL_{ij}) \) will in general be greater than 0 since the profits from an owned container are always going to be larger than those coming from a leased one. Since the second derivation of the expected profit function is always negative the problem, as stated, is concave and has a unique solution.

The only assumption made about the function \( f \) in this chapter is that for any given \( X \) the value of the function \( f(x) \) is always positive. This is true for any probability distribution. There are no further premises and the uniqueness of the optimum can be demonstrated even for a discrete probability distribution. This emphasizes the robustness of this formulation which can deal with customized stochastic demands or empirical distributions.

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4.2 SOLUTION ALGORITHM

This formulation can be solved with the use of a Frank-Wolfe algorithm. The idea is to generate a series of linear approximations attacking the problem in an iterative way. The concept is to convert the objective function into a set of sub-problems that can be solved via the Simplex method.

The linearized approximation proposed by this algorithm is simply,

$$\max \sum_{i \in P} \sum_{j \in P} \sum_{t \in T} K_{ij,t} \cdot x_{ij}(t)$$  \hspace{1cm} (4.5)

Subject to the following constraints:

$$0 \geq x_{ij} \geq U_{ij} \quad \forall i, j \in P$$  \hspace{1cm} (4.6)

$$\sum_{j \in P} x_{ij}(t) = \sum_{k \in P} x_{ki}(t - (\tau_{ki} + AD_i)) + R_i(t) \quad \forall i \in P$$  \hspace{1cm} (4.7)

Note that for this formulation the problem constraints are not modified from the original formulation of the model, which is a very convenient property of this algorithm. The $K_{ij,t}$'s are basically the slopes or gradients of the $P_{ij,t}$ functions. These slopes are the marginal profits $MP_{ij,t}$.

These slopes can be accurately and quickly obtained from formula 4.3 by numerical procedures such as the Simpson or the trapezoid methods.

The procedure to follow will then be,
1. Set $k = 0$

2. Estimate an initial feasible solution $x_{ij}^k(t) \ \forall \ i,j \in \mathbb{N} \ \text{and} \ \forall \ t \in T$.

3. Calculate $K_{ij,t} = MP_{ij,t}(x_{ij}^k(t))$

4. Solve the LP proposed in equations 4.5, 4.6, 4.7 to obtain the solution $x_{ij}^*(t)$ for the sub-linear problem.

5. Find the size of the step maximizing the next objective function,

$$\max_{0 \leq y \leq 1} \sum_{i,j,t} x_{ij}^y(t) + \gamma^* (x_{ij}^*(t) - x_{ij}^y(t))$$

And let the $\gamma^*$ be the optimum stepsize

6. Set $x_{ij}^{k+1}(t) = x_{ij}^k(t) + \gamma^* (x_{ij}^*(t) - x_{ij}^k(t))$

7. If $|\Pi(x_{ij}^{k+1}(t)) - \Pi(x_{ij}^k(t))| < \varepsilon$ then stop, otherwise set $k = k+1$ and go to step 3.

8. Solution: $x_{ij}(t) = x_{ij}^{k+1}(t)$

The algorithm is guaranteed to converge to the solution. In addition, each iteration provides bounds to the optimum solution. However, the rate of convergence of this algorithm, especially when close to the final solution is not very fast. There are in general faster ways to provide a solution that involves more or less sophisticated procedures to obtain the gradients of the objective function. The partial solutions $x^1, x^2, x^3, \ldots$ obtained by this method tend to oscillate due to the nature of the algorithm [16]. The even numbered points $x^2, x^4, x^6, \ldots$ lie on a "line" oriented in the direction of the optimal solution, and the points $x^1, x^3, x^5, \ldots$ on another such line. Although this general tendency of the algorithm slows its convergence, it also could be exploited to make periodic approximations along the "line" generated by every other point $x^{k+2}$ and $x^k$ and accelerate the process.
4.3 DETAILS FOR THE IMPLEMENTATION OF THIS SOLUTION IN A
SHIPPING COMPANY

This section addresses several topics regarding the implementation of such a model as an aid
decision support tool for a shipping liner. In particular, it will address how to assign costs to the
different arcs or how to convert the costs that were explained in section 2 into inputs for the
model.

This model itself would have very little utility without the aid of a forecasting system to be used
as a fundamental input. The model itself must run on one particular computer, but a centralized
configuration is not required for the accumulation of the input data.

The nature of the data that shipping agents from different company depots provide to the system
permits, at least, two different levels of aggregation. On the one hand, the forecast systems could
be individual for each location. In this case the shipping agents would only enter the predicted
demand and variability from their local forecast systems into the operating model. On the other,
the model could accept specific orders or booked demand as data and process that information
from the raw data to create the forecasted demand itself.

The higher the reliability and know-how of the shipping agents along the ports, the more the
desegregate solution should be considered as best. The knowledge of these agents in the case of
a individual forecasting solution could be used to fine tune the predictions and improve the quality of the recommendations of the complete system.

On the other hand, experienced agents might be overconfident in their predictions or try to maintain a high inventory of containers and modify the forecast to their interest. Therefore, each shipping company should make the decision regarding the location and functioning of its forecasting models depending on its specific situation.

There are also some issues regarding costs and profits. As shown in section 2.2. the revenues are in general subject to only minimal variations for a given fixed trip, time and size of the container. Therefore, although the actual profit for each particular deal might be a slightly different, using an average profit for each arc and time is reasonable in order to avoid any further complications. On the cost side, when the arc takes place between a pair of different ports, the operating cost is calculated as the summation of loading and unloading costs. However, when the arc connects the same port two consecutive days, the operating cost is simply the storage cost.

The last point regarding the implementation of the is the R term in the flow constraint equation 3.4. This term resolves all possible problems concerning the arrival of containers into the port before or after their expected return time. For example, if depot I received on period 1 a container that was supposed to be received on period 3, the algorithm would have to subtract one unit from the previous value in $R_i(3)$. Note that for the R solution to work properly the container shipping line has to be able to track the origin and destination of their containers.
5. CHAPTER FIVE

5.1. FUTURE WORK

This operational model and its proposed solution suggest several threads of future development. One would be in the algorithm area. The proposed formulation, when solved using a Frank-Wolf algorithm, has been proven to provide an optimal solution. However, this is not particularly efficient when the successive iterations get close to the final solution.

Therefore accelerating the convergence of the algorithm is an area to be considered in future work. This thread of work is important in relation to the computational time that the method as established would require to provide a useful solution. The minimum time period considered for the model is one day because the carriers' operational decisions are made on a daily basis. The ratio between the actual computational time that the algorithm takes versus a day will dictate whether is important to pursue this direction of improvement.

A second area of possible improvement would consist of splitting the number of containers into full and empty once they arrive to their destination port. By doing this the model could assign the average delay in return only to full containers while the empty ones are re-usable at the same moment they arrive into the port. This improvement does not require major modifications in the
formulation or its implementation. It should improve the accuracy of the system since it will reduce the usage of the correction variable R in the flow continuity constraint.

Another area to consider that could result in improvements for this operational model is the probability functions used in the model. This issue has not been studied in depth in this thesis, but it is critical for this model. The density functions used for the model, Erlang distributions in the case of this thesis, could be improved since the model is not constrained to the use of this particular density function.

There is a final direction to consider in the development of this model. This last idea would be to extend the model from a single commodity into a multi-commodity system. This would allow explicit consideration of different container types in the analysis. Conceptually this extension is straightforward, but the implementation of this refinement can add considerable complexity. The total flow would be divided into several smaller ones, the demand forecast would have to be disaggregated, and the objective function would look far more complicated. The number of variables will grow with the different type of containers that the extended model would consider. Thus, the computational time would be lengthened.

5.2. CONCLUSIONS

The development of the container concept during the early nineteen seventies has introduced a dramatic shift in ocean cargo transportation. Most of the general cargo today is transported by means of containers.
These containers have introduced unquestionable advantages for ocean transportation. Increased cargo loading and unloading speeds, improved versatility of the ships and, finally, an enormous reduction in lost or damaged cargo are among the most remarkable.

Yet, there are also problems caused by containerization. The shipping industry, typically a very high capital asset oriented industry, is now even more so shipping companies must now make large investments in the vessels, and now must also invest in an inventory of containers. In order to obtain a substantial benefit from all these investments the shipping line must achieve high utilization of their assets, especially because competition is very aggressive and profit margins are very slim.

The efficient utilization of these assets might very well in the long run mean the difference between surviving or not in the industry. This will be even more important if the ocean transportation sector sees a deregulation similar to those which occurred in other modes of transportation such as airfreight or motor carriage. Thus container allocation models as an aid decision tool for the operation of a container inventory are important for the ocean transportation sector.

This thesis has developed a new model to solve the dynamic container allocation problem. This model can form the base for a decision support system that will minimize the operating cost associated with the container inventory and as a result drive the utilization of the equipment to desirable levels.
The main characteristic of this formulation is that it explicitly models all the container flows through the network. Previous models that only dealt with the flow of empty containers. By approaching the problem in this new manner the three sources of randomness that limit the accuracy of early models have been reduced to only one. While the earlier models treated the demand for containers, the external supply of containers into the different locations and the remaining cargo capacity on the vessels as random variables, this new formulation reduces the source of randomness to only one, namely, the customer demand for containers at each location. This reduction of the sources of random variation is very promising since the other two sources in the earlier models were essentially artificial. They were necessary in those models only due to a lack of information about full containers. By incorporating an explicit treatment of the movement of both empty and full containers, a model is constructed in which the only exogenous stochastic variable is the natural one of extrinsic customer demand.
6. REFERENCES


