# Incorporating Unobservable Heterogeneity in Discrete Choice Model: Mode Choice Model for Shopping Trips

by

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Submitted to the Department of Civil and Environmental Engineering

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#### **Abstract**

In this thesis, we propose a methodology for incorporating attitudinal data in a choice model to capture unobservable heterogeneity across the population. The key features of this approach are, 1) the concept of latent attitudes, and the assumption that 2) the respondent's answers to psychometric attitudinal questions relating to the importance of attributes are manifestations of these attitudes and that 3) those attitudinal data bring sufficient information to capture unobservable heterogeneity across the population in the context of choice behavior. Each individual is probabilistically assigned to a finite number of segments according to his/her own value of latent attitudinal variable(s) as well as to threshold parameter(s) common to the population. Segment-specific parameters are estimated simultaneously. An empirical case study on shopping trip mode choice demonstrates the effectiveness of the methodology.

Thesis Supervisor: Moshe Ben-Akiva

Title: Professor of Civil and Environmental Engineering

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## Chapter 1

#### Introduction

Consumers are often treated as "optimizing black boxes" in econometric models. Inputs might be attributes of alternatives, socioeconomic characteristics, market information, historical experience, and market constraints. Figure 1-1 gives a path diagram for the decision-making process. This path diagram builds on earlier conceptualizations of McFadden (1986), Ben-Akiva and Boccara (1987), Morikawa (1989, 1996), and Gopinath (1995). Terms in ovals are theoretical or latent variables, while those in boxes are observed directly or measured by suitable experiments. Perceptions and attitudes are influenced by attributes of alternatives and by decision-maker characteristics. Attitudes affect latent segments, and latent segments and perceptions together determine preference, and preferences are the major source of the market behavior.

#### 1.1 Heterogeneity across Population

Since the late 1970s, many researchers have widely employed the multinomial logit model to study consumer choice behavior (e.g. Green et al. 1977, Guadagni and Little 1983, Ben-Akiva and Lerman 1985, Krishnamurthi and Raj 1988). While the application of the multinomial logit has been widespread, research on proper control for heterogeneity has been limited. However, unmeasured individual/household-specific factors may influence an individual's/household's choice behavior. Even with the

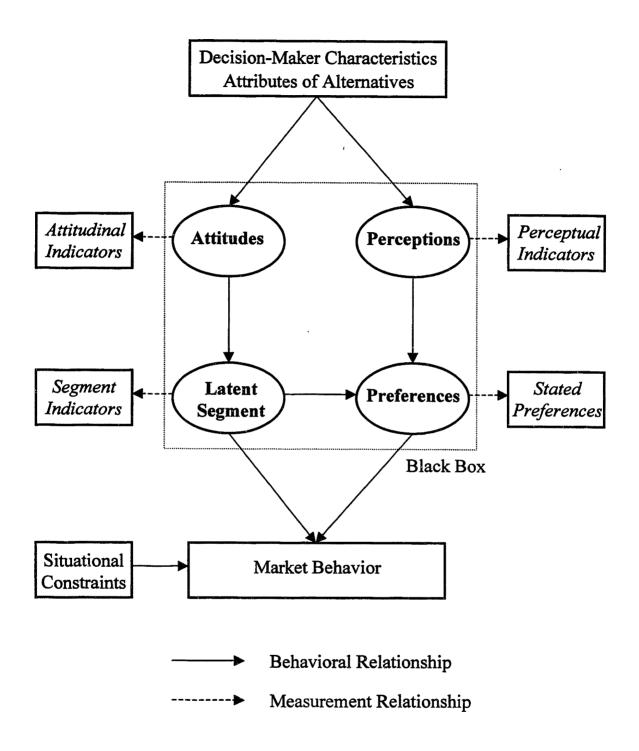


Figure 1-1: Path Diagram for Consumer Behavior

specification of demographic variables, individuals/households may differ in their responses to marketing mix variables. Failure to account for such heterogeneity results in biased and inconsistent parameter estimates.

#### 1.2 Market Segmentation

Consumers differ in one or more respects. They may differ in their socio-econometric characteristics and each individual has different attitudes and different preferences. In the context of preference heterogeneity in the market, there are three different patterns (Figure 1-2 presents an illustration of each pattern.):

#### Homogeneous Preference

All the consumers in the market have roughly the same preference. In other words, there are no natural segments in the market. However, this situation seldom happens in reality.

#### Diffused Preference

At the other extreme case, consumer preferences may be scattered through out the space. In such a case, instead of segmenting the market, estimating the parameters at the individual/household level might be desirable but is rarely, if ever, practical.

#### Clustered Preference

The market might reveal distinct preference clusters, called *natural market segments*. In this case, it is necessary to make a model capturing heterogeneity. Most practical cases might fall in this pattern.

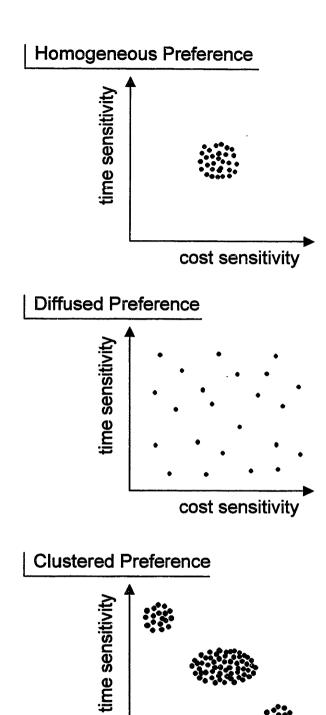


Figure 1-2: Basic Market-Preference Patterns

cost sensitivity

#### 1.3 Literature Review

#### 1.3.1 Importance of Capturing Heterogeneity

The theoretical development of discrete choice models is based on utility-maximizing behavior at the individual or household level. Therefore, ideally, the parameters of the choice model should be estimated at the individual/household level. Even in panel data, however, the number of observations per individual/household is often insufficient for consistent and efficient estimation of individual/household-specific parameters. Further, from the standpoint of marketing decision making, the parameter estimates are meaningful only at the aggregate or market level. Accordingly, researchers have resorted to pooling the data across individuals/households and estimating a set of aggregate-level parameters. A pertinent question is: What is the effect of heterogeneity across individuals/households on the estimated parameters of a logit model of mode choice? In general, failure to control for such heterogeneity is likely to yield biased and inconsistent parameter estimates, and more importantly, biased and inconsistent estimates of choice probabilities (Hsiao 1986).

#### 1.3.2 Segmentation according to Past Choice Behavior

Researchers have recently addressed the preceding issue in one of two ways. The first approach is incorporating heterogeneity in preferences in the context of a choice model using observed past choice behavior. Guadagni and Little (1983) used two variables referred to as "brand loyalty" and "size loyalty", which are exponentially weighted averages of past brand and brand-size choices, to account for heterogeneity across households. Their approach tracks changes in households tastes over time using a weighted average of past choice behavior in which recent choice is weighted more heavily. Thus, this loyalty variable captures not only much of the cross-sectional heterogeneity but also a good part the purchase-to-purchase dynamics. Krishnamurthi and Raj (1988) used a household-specific variable based on the share of purchases of a particular brand in relation to all brands. Kamakura and Russell (1989) develop a

model to identify a finite number of segments allowing parameters to remain constant within each segment but differ across segments. A latent class approach is used to estimate endogenously the size and logit-model parameters of each segment. Accordingly, the approach proposed by Kamakura and Russel to analyze brand choice of households consists of a finite mixture of logit models. They apply this approach to study the competition between national brands and private labels in one product category. Allenby and Rossi (1990) used a somewhat different operationalization, wherein heterogeneity is captured via a fixed term that is measured as the difference between the predicted choice probabilities and the probabilities estimated by a relative frequency approach. A justification for the use of such variables is that they attempt to capture the differences in brand preferences across households. Fader and Lattin (1993) separate the heterogeneity and non-stationarity components of brand loyalty. Chintagunta (1994) has developed a latent class model that segment specific brand intercepts are constrained to lie within a subspace of few dimensions, with the inferred brand and segment locations in that space constituting a product-market map. Bucklin, Gupta and Siddarth (1998) consider response segmentation through a latent class model that simultaneously incorporates all three purchase behaviors: choice, incidence, and quantity.

A question that arises in using observed past brand/mode choice histories to capture heterogeneity across households/individuals is whether it would affect the estimates of the unknown parameters such as price and travel time. Gönül and Srinivasan (1993) claim that the purchase history measure includes unobserved choice behavior and, as a result, could be correlated with the random component resulting in biased and inconsistent estimates. In other words, the effect of past brand choices on current choice will be overstated if the past choices are not adjusted for the possible effects of price and other promotional variables. Consequently, the effect of some of the other variables included in the model would be understated.

#### 1.3.3 Random-Coefficients Specification

The second approach dealing with unobserved heterogeneity is to use a randomcoefficients specification in which the parameters of the household-level logit model of brand choice are treated as realizations of random variables representing the preferences of households and their responses to marketing activities. These random variables are assumed to follow a continuous probability distribution. The studies by Chintagunta, Jain, and Vilcassim (1991), Gönül and Srinivasan (1993), and Jain, Vilcassim, and Chintagunta (1994) are in this vein. Chintagunta, Jain, and Vilcassim (1991) suggest that in attempting to capture heterogeneity, imposing a single probability distribution across all brands would not be appropriate. Their model has a choice specific random component on the intercept term and their random distributions are estimated non-parametrically by the Hecman and Singer (1984) procedure. They estimate a finite number of support points and associated probabilities for the distribution of intrinsic preferences across households. Gönül and Srinivasan (1993) use Universal Product Code scanner data on disposable diapers and present the multinomial logit model integrating random variations in intrinsic brand utilities (intercepts), random variations in response to marketing variables, and loyalty. Jain, Vilcassim, and Chintagunta (1994) approximate the unknown underlying distribution of unobserved heterogeneity by a discrete distribution. An advantage of using a random-coefficients specification to account for heterogeneity is the parsimony in the number of parameters to be estimated relative to estimating household-specific parameters (e.g., Rossi and Allenby 1993). Gopinath (1995) develops the latent class choice model (LCCM) for taste heterogeneity with specific reference to the classes characterized along cost sensitivity and time sensitivity dimensions. Since the membership of individuals in groups is unobserved, the groups are characterized by latent classes. In his model, the underlying choice process is hypothesized to vary across a finite set of groups of individuals in the population, and to be homogeneous within each such group.

#### 1.4 Objective of Research

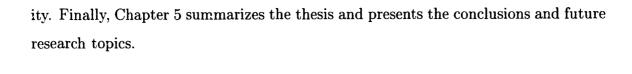
The major objectives of this research are to 1) capture the unobservable heterogeneity across the population, and 2) incorporate the captured heterogeneity in discrete choice model. Unobservable heterogeneity is captured by using structural equations with latent variables, which is well known as the *LISREL* model. The motivation of this research is based on the idea that the captured latent variables could bring "good" information to extract the difference of sensitivities across the population, and the captured heterogeneity enables to improve the explanatory power of discrete choice model. We accounted for unobserved heterogeneity across individuals in a logit model of mode choice by assuming the existence of a finite number of segments. Each segment consists of a set of individuals having identical overall mode preference and response to mode attribute variables.

It has been argued that consumer's choice behavior is affected by latent factors such as "comfort" as well as manifest ones such as "price". In some cases, to capture heterogeneity across the population, using latent variables might be more reasonable than using observable variables. We consider shopping trip mode choice as an empirical case study. Comparing mode choice for shopping mall trips with mode choice for commuting, shopping mall trip data might reflect a wider variety of decision-protocols, taste variations, and latent factors such as attitudes and perceptions which affect the decision making process, because of fewer constraints in shopping trips. In such a case, a model using only observed variables sometimes fails to control for heterogeneity, which is a typical problem encountered in estimating models from observed data. Obtaining an acceptable model specification is often very difficult because the actual behavior is influenced by related attributes, which are usually latent, while the available data are limited. Therefore, we use latent variables, which might provide useful information on heterogeneity across the population, and might enable us to obtain unbiased parameters in the resulting choice model. To deal with the latent variables, we apply structural equations with latent variable techniques. Structural equations with latent variables have some familiar specifications such as the multiple indicator multiple cause (MIMIC) model and the higher-order factor analysis (HOFA) model, which will be reviewed in the next chapter. In our case study, the HOFA model is combined with the segment likelihood membership model to capture unobservable heterogeneity. The reason we focus on the HOFA model is that assuming the existence of more general and abstract factors behind the "first-order" factor is more reasonable and realistic, and it results in the improvement of the explanatory power of the model. The MIMIC model, on the other hand, uses observed exogenous variables and those variables are assumed as a perfect measure of latent exogenous variables. This assumption could be unreasonable in some cases.

In principle, our approach is an extension of the work of Gopinath (1995). Gopinath doesn't specify explicit indicators for the classes in LCCM. Rather, only the choice indicator was utilized as an indirect indicator of the latent class. Our approach, on the other hand, uses psychometric data as direct indicators of latent segments. Those indicators specify latent attitudinal variables that are included in the segment likelihood membership model, not in the choice model. Further, latent attitudinal variables are captured by the HOFA model which includes both latent endogenous and exogenous variables. The latent exogenous variable is used in the segment likelihood membership model.

#### 1.5 Outline of Thesis

This thesis is composed of five chapters. Chapter 2 reviews structural equations with latent variables, focusing on the higher-order factor analysis model. The multiple indicator multiple cause model is also presented in this chapter. Chapter 3 proposes a methodology of incorporating unobservable heterogeneity in demand modeling, in which the mode choice model includes latent variables to capture the heterogeneity across the population. In Chapter 4, an empirical analysis of the model is presented using shopping trip survey data collected in a shopping mall. This survey includes psychometric data, obtained through perceptual and attitudinal questions. Those psychometric data are used as observable indicators of the latent heterogene-



## Chapter 2

# Structural Equations with Latent Variables

#### 2.1 Introduction

In this section, we review structural equations with latent variables and their sub-models: multiple indicator multiple cause (MIMIC) model and higher-order factor analysis (HOFA) model. The purpose of this chapter is to introduce the HOFA model and clarify the difference between the MIMIC and the HOFA model.

The structural equation model is used to specify the phenomenon under study in terms of cause and effect variables and various causal effects. Each equation in the model represents a causal link rather than a mere empirical association, and the structural parameters do not, in general, coincide with coefficients of regressions among observed variables. The structural parameters represent relatively unmixed, invariant and autonomous features of the mechanism that generates the observable variables (Jöreskog 1982). Background material and advanced topics are covered by Everitt (1984) and Bollen (1989). In this chapter, starting with the review of structural equation models, we will discuss two sub-models, the MIMIC model and the HOFA model, and then the rules of identification.

# 2.2 Structural Equation Model and Measurement Equation Model

Structural equations with latent variables consist of two parts: structural equation model and measurement equation model. The structural equation model specifies the causal relationships among the latent variables and are used to describe the causal effects and the amount of unexpected variance. The measurement equation model specifies how the latent variables or hypothetical constructs are measured in terms of the observed variables and is used to describe the measurement properties (validity and reliabilities) of the observed variables. The general matrix representation of the structural equations for the latent variable model is given by:

Structural Equation Model

$$\eta^* = B_{\#}\eta^* + \Gamma\xi^* + \zeta, \tag{2.1}$$

Measurement Equation Model

$$z = \Lambda \eta^* + K \xi^* + \epsilon, \tag{2.2}$$

where

 $\eta^*$ :  $m \times 1$  vector of latent endogenous variables,

 $\xi^*$ :  $q \times 1$  vector of latent exogenous variables,

 $B_{\#}$ :  $m \times m$  coefficient matrix for latent endogenous variables, where all diagonal elements are zero,

 $\Gamma: m \times q$  coefficient vector for the latent exogenous variables,

 $\zeta : m \times 1$  vector of latent random components,

 $z : \bar{p} \times 1$  vector of observable multivariate data,

 $\Lambda: \bar{p} \times m$  coefficient matrix for the latent endogenous variables,

 $K: \bar{p} \times q$  coefficient matrix for the latent exogenous variables,

 $\epsilon$ :  $\bar{p} \times 1$  vector of random components.

In general, the models involve some standard assumptions:

Assumption 1

$$E(\xi^*) = O,$$

$$E(\eta^*) = O,$$

$$E(\zeta) = O,$$

$$E(\epsilon) = O.$$

Assumption 2

Covariance-variance matrix of exogenous variables  $\Sigma_c$ , where  $c = (\epsilon' \zeta' \xi^{*'})'$ , is restricted as

$$\Sigma_c = \left[ egin{array}{ccc} \Theta & O & O \ O & \Psi & O \ O & O & \Xi \end{array} 
ight].$$

Assumption 3

 $(1 - B_{\#})$  is non-singular.

#### 2.3 Basic Structure of Covariance Matrix

By using the Assumption 3, we can rewrite Equation 2.1 as

$$\eta^* = B\Gamma \xi^* + B\zeta, \tag{2.3}$$

where  $B = (1 - B_{\#})^{-1}$ .

Substituting Equation 2.3 into Equation 2.2, the observable variables are described as linear function of latent exogenous variables.

$$z = (\Lambda B\Gamma + K)\xi^* + \Lambda B\zeta + \epsilon. \tag{2.4}$$

Since the variance-covariance matrix of observable variables is described as

$$Cov(z) = \Sigma_z = E(zz'), \tag{2.5}$$

with Assumption 2, the variance-covariance matrix  $\Sigma_z$  becomes

$$\Sigma_z = (\Lambda B \Gamma + K) \Xi (\Lambda B \Gamma + K)' + \Lambda B \Psi B' \Lambda' + \Theta. \tag{2.6}$$

This  $\Sigma_z$  shows the basic structure of the variance-covariance matrix of structural equations model. The variance-covariance matrix  $\Sigma_z$  is a function of the elements of  $\Lambda, K, B, \Gamma, \Xi, \Psi$ , and  $\Theta$ . In applications some of these elements are fixed and equal to assigned values. For the remaining non-fixed elements of the seven parameter matrices one or more subsets may have identical but unknown values. Thus, the elements in  $\Lambda, K, B, \Gamma, \Xi, \Psi$ , and  $\Theta$  are categorized into three types:

- 1. fixed parameters that have been assigned given values,
- 2. constrained parameters that are unknown but equal to one or more other parameters, and
- 3. free parameters that are unknown and not constrained to be equal to any other parameter.

A structural equation model is fully defined by the specification of the structure of the following seven matrices:

$$\Lambda, K, B, \Gamma, \Xi, \Psi, \Theta, \tag{2.7}$$

where the first four matrices are the parameter matrices and the last three matrices are the variance-covariance matrices of  $\xi^*$ ,  $\zeta$ , and  $\epsilon$ , respectively. Also, all the matrices may contain fixed, free, or constrained elements.

#### 2.4 Estimation

Let  $\tau$  be the vector of all free and constrained parameters in all seven parameter matrices. In estimation of the structural equations, maximum likelihood (ML), least squares (LS), or generalized least squares (GLS) is usually used. The goal of estimation is to choose values for the unknown parameters that lead to an implied covariance matrix,  $\Sigma_z(\hat{\tau})$ , as close to sample covariance matrix,  $S_z$  as possible. The unknown parameters are estimated by minimizing the one of the following functions.

$$F_{SAS\ LS}[\Sigma_z(\tau)] = \frac{1}{2} tr(S_z - \Sigma_z(\tau))^2, \qquad (2.8)$$

$$F_{SAS\ GLS}[\Sigma_z(\tau)] = \frac{1}{2} tr((S_z - \Sigma_z(\tau))S_z^{-1})^2, \qquad (2.9)$$

$$F_{SAS\ MLE}[\Sigma_z(\tau)] = tr(\Sigma_z(\tau)^{-1}S_z) - \ln|\Sigma_z(\tau)^{-1}S_z| - p,$$
 (2.10)

$$F_{LISREL}[\Sigma_z(\tau)] = \ln |\Sigma_z(\tau)| + tr(\Sigma_z(\tau)^{-1}S_z) - \ln |S_z| - p.$$
 (2.11)

where  $F_{SAS}[\cdot]$  are the functions used in SAS's CALIS procedures and  $F_{LISREL}[\cdot]$  is the function used in LISREL. Two stage least squares (2SLS) estimation method presented by Bollen (1996) is estimable with standard statistical software.

Adding some restrictions to Equations 2.1 and 2.2, one can handle all kinds of reasonable sub-models in a simple manner (e.g. Bollen 1989), such as the multiple indicator multiple cause (MIMIC) model and the higher-order factor analysis (HOFA) model.

# 2.5 Multiple Indicator Multiple Cause (MIMIC) Model

The MIMIC model assumes the existence of certain latent variables which are directly affected by two or more observable dependent variables and which are indicated directly by one or more observable independent variables. In other words, the MIMIC model is a three step causal chain system model, in which the variables in the middle (second) step are described as latent variables (Figure 2-1). In the MIMIC model,

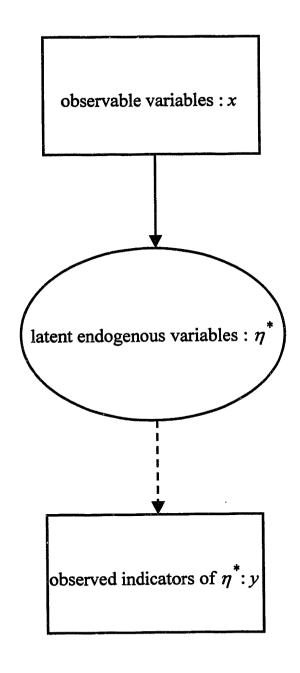


Figure 2-1: Path Diagram for MIMIC Model

Equation 2.2 is constrained as:

$$z = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} O \\ \Lambda_y \end{bmatrix} \eta^* + \begin{bmatrix} I \\ O \end{bmatrix} \xi^* + \begin{bmatrix} O \\ \epsilon \end{bmatrix}, \tag{2.12}$$

and the structural equation of the MIMIC model is given by

$$\eta^* = B_{\#}\eta^* + \Gamma x + \zeta, \tag{2.13}$$

where

 $x: q \times 1$  vector of observed indicators of  $\xi_n^*$ ,

 $y : p \times 1$  vector of observed indicators of  $\eta_n^*$ ,

 $\eta^*$ :  $m \times 1$  vector of latent endogenous variables,

 $\xi^*$ :  $q \times 1$  vector of latent exogenous variables,

 $B_{\#}$ :  $m \times m$  coefficient matrix for latent endogenous variables,

 $B = (1 - B_{\#})^{-1},$ 

 $\Gamma : m \times n$  coefficient vector for the latent exogenous variables,

 $\Lambda_y: p \times m$  coefficient matrix for the latent endogenous variables,

 $\zeta$ :  $m \times 1$  vector of random components,

 $\epsilon~:~p\times 1$  vector of random components,

 $I : q \times q \text{ identity matrix},$ 

O: zero matrix.

Note that in the MIMIC model, x is assumed as a perfect measure of  $\xi^*$  and that only one latent variable,  $\eta^*$ , is present in the model.

Since each element of variance-covariance matrix  $\Sigma_z$  is given by

$$\Sigma_{\nu} = \Lambda_{\nu} (B \Gamma \Xi \Gamma' B' + B') \Lambda_{\nu}' + \Theta, \tag{2.14}$$

$$\Sigma_{yx} = \Lambda_y B \Gamma \Xi, \tag{2.15}$$

$$\Sigma_{xy} = \Xi \Gamma' B' \Lambda'_{y}, \tag{2.16}$$

<sup>&</sup>lt;sup>1</sup>Details are shown in Appendix A

$$\Sigma_x = \Xi, \tag{2.17}$$

variance-covariance matrix of MIMIC model is

$$\Sigma_{z}^{mimic} = \begin{bmatrix} \Sigma_{y} & \Sigma_{yx} \\ \Sigma_{xy} & \Sigma_{x} \end{bmatrix} = \begin{bmatrix} \Lambda_{y}(B\Gamma\Xi\Gamma'B' + B')\Lambda'_{y} + \Theta & \Lambda_{y}B\Gamma\Xi \\ \Xi\Gamma'B'\Lambda'_{y} & \Xi \end{bmatrix}, \quad (2.18)$$

where

 $\Psi = E(\zeta\zeta'),$ 

 $\Theta = E(\epsilon \epsilon').$ 

#### 2.6 Higher-Order Factor Analysis (HOFA) Model

Factor analysis is used based on the analyst's recognition that relatively few underlying latent variables may underlie a large number of indicators. Moreover, the assumption that the latent variables more closely correspond to the concepts of psychometric theory than do the indicators is another motivation. In some cases, assuming the existence of more general and abstract (second-, third-, or higher-) factors behind the "first-order" factors is more reasonable and realistic, and it might improve the explanatory power of the model. Such a model is a higher-order factor analysis (HOFA) model. Here, we review the second-order factor analysis (SOFA) model. Figure 2-2 represents a path diagram for the SOFA model.

Structural Equation Model

$$\eta^* = \Gamma \xi^* + \zeta, \tag{2.19}$$

Measurement Equation Model

$$z = \Lambda \eta^* + \epsilon. \tag{2.20}$$

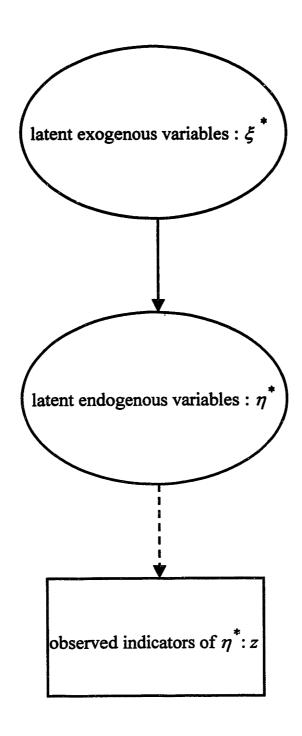


Figure 2-2: Path Diagram for HOFA Model

The variance-covariance matrix of the SOFA model is <sup>2</sup>

$$\Sigma_{z}^{sofa} = \Lambda \Gamma \Xi \Gamma' \Lambda' + \Lambda \Psi \Lambda' + \Theta, \qquad (2.21)$$

where

 $\eta^*$ :  $m \times 1$  vector of latent endogenous variables,

 $\xi^*$ : 1 × 1 vector of latent exogenous variables,

 $z : p \times 1$  vector of observed indicators of  $\eta^*$ ,

 $\Gamma: m \times 1$  coefficient vector for the latent exogenous variables,

 $\Lambda: p \times m$  coefficient matrix for the latent endogenous variables,

 $\zeta : m \times 1$  vector of random components,

 $\epsilon$ :  $p \times 1$  vector of random components,

 $\Xi = E(\xi^* \xi^{*\prime}),$ 

 $\Psi = E(\zeta \zeta'),$ 

 $\Theta = E(\epsilon \epsilon').$ 

In the HOFA model, Equation 2.19 linking  $\eta^*$  to  $\xi^*$  is part of the measurement model, yet it also is an equation linking latent variables. Note that if you add the assumption that  $\xi^* = x$ , the HOFA model will be identical to the MIMIC model.

#### 2.7 Identification

One of the issues an analyst should pay attention before specifying models and estimating unknown parameters is *identifiability* of the model. The parameters in  $\tau$  are globally identified only if no vector  $\tau_1$  and  $\tau_2$  exist such that  $\Sigma(\tau_1) = \Sigma(\tau_2)$  unless  $\tau_1 = \tau_2$ . If all unknown parameters in  $\tau$  are identified, then the model is identified. Rules exist that aid in the identification of the general model. We review some of these next. Unfortunately, none of these is a necessary and sufficient condition for needel identification.

<sup>&</sup>lt;sup>2</sup>Details are shown in Appendix B

#### 2.7.1 *t*-Rule

The t-rule for identification is that the number of non-redundant elements in the covariance matrix of the observed variables must be greater than or equal to the number of unknown parameters in  $\tau$ :

$$t = (\frac{1}{2})(p)(p+1),$$
 (2.22)

where p is the number of observed variables and t is the number of free parameters in  $\tau$ . The t-rule is a necessary but not sufficient condition for identification.

#### 2.7.2 MIMIC Rule

The identification rule for MIMIC models is

$$p \ge 2, \ q \ge 1,\tag{2.23}$$

where p is the number of y's, and q is the number of x's. The MIMIC rule is a sufficient condition for identification but not a necessary one.

#### **2.7.3 HOFA** Rule

The identification rule for HOFA models is

$$p \ge 2, \ m \ge q, \tag{2.24}$$

where p is the number of indicators, q is the number of  $\xi^*$ 's, and m is the number of  $\eta^*$ 's. The HOFA rule is also a sufficient condition for identification but not a necessary one.

#### 2.8 Summary

In this chapter, we reviewed the structural equations with latent variables, which enable us to deal with unobservable variables. The difference of the MIMIC model and the HOFA model are clarified. By definition, latent variables are unobservable, so that we should pay attention to the identifiability of the model and method of the model evaluation. This topic is briefly reviewed in this chapter.

In the next chapter, we present a methodology for incorporating psychometric data in discrete choice models to capture unobservable heterogeneity across the population. In principle, this methodology builds on the theme reviewed in this chapter.

## Chapter 3

# Framework of Segment-Specific Choice Model and Segment Likelihood Membership Model with Attitudinal Data

#### 3.1 Introduction

Our approach is based on the assumption that each individual can be placed into a small number of segments according to the differences of attitude, and the differences of attitude causes the differences of preference. The model includes two sub-models:

1) segment-specific choice model, and 2) segment likelihood membership model. We will discuss those models in turn.

#### 3.2 Segment-Specific Choice Model

In deriving the model, we start with the usual assumptions of random utility theory: when facing a mode choice decision, each individual assigns random utilities to each mode considered and then selects the one with the highest derived utility. This utility is decomposed into a deterministic component and a random component. Therefore,

the random utility assigned to alternative i by individual n who belongs to segment s is

$$U_{ni|\alpha_s}^* = \alpha_s X_{ni} + \nu_{ni}, \tag{3.1}$$

where

 $\alpha_s$ : unknown parameter vector for segment s,

 $X_{ni}$ : attributes of alternative i and decision maker n's characteristics,

 $\nu_{ni}$ : random component.

Assuming that  $\nu_{ni}$  derive from independent, identical Gumbel distributions, individual n's conditional probability of choosing i is given by the multinomial logit model,

$$P_{ni|\alpha_s} = \frac{\exp(\mu \alpha_s X_{ni})}{\sum_{i \in C_n} \exp(\mu \alpha_s X_{ni})},$$
(3.2)

where

 $\mu$ : scale parameter of Gumbel distribution,

 $C_n$ : choice set of individual n.

Therefore, the log-likelihood function for individual n choosing alternative i is given by

$$L_{n|\alpha_s} = \prod_{i=1}^{I} (P_{ni|\alpha_s})^{\kappa_{ni}}, \tag{3.3}$$

where  $\kappa_{ni} = 1$ , if individual n chooses alternative i, and  $\kappa_{ni} = 0$ , otherwise.

#### 3.3 Accounting for Heterogeneity

The formulation in Equation 3.2 assumes that  $\alpha_s$  varies across the segments. Hence,  $\alpha_s$  can be assumed to be a realization of the random variable (vector)  $\alpha$  that has a

distribution  $A(\alpha)$ . Therefore, the unconditional likelihood function for individual n is given by

$$L_n = \int_{\alpha} \left[ \prod_{i=1}^{I} (P_{ni|\alpha_s})^{\kappa_{ni}} \right] dA(\alpha). \tag{3.4}$$

We approximate  $A(\alpha)$  by a discrete distribution with a finite number of supports S and their associated probabilities  $Q(\alpha_s)$ , s = 1, 2, ..., S, such that  $\sum_{s=1}^{S} Q(\alpha_s) = 1$ . Therefore, individual n's likelihood is given by

$$L_n = \sum_{s=1}^{S} \prod_{i=1}^{I} (P_{ni|\alpha_s})^{\kappa_{n_i}} Q(\alpha_s).$$
 (3.5)

The locations  $\alpha_s$  and probabilities  $Q(\alpha_s)$  for all  $s=1,2,\ldots,S$  are then estimated from the sample data by maximizing the sample likelihood function.

$$L = \prod_{n=1}^{N} L_n. \tag{3.6}$$

#### 3.4 Segment Likelihood Membership Model

We assume the existence of s = 1, 2, ..., S homogeneous segments in preference with relative sizes. We further assume that likelihood of finding individual n in segment s is characterized by criterion function  $G^*$ . The criterion function  $G_n^*$  for individual n can be written as<sup>1</sup>

$$G_n^* = DF_n^* + \delta_n, (3.7)$$

where

 $D: 1 \times (q+m)$  coefficient vector,

 $F^*: (q+m) \times 1$  vector of latent attitudinal variables,

q: number of latent exogenous variables,

<sup>&</sup>lt;sup>1</sup>Criterion functions by Gopinath (1995) use observable variables, such as *income*, age, and gender, instead of  $F^*$ .

m: number of latent endogenous variables,

 $\delta_n\,$  : random components following the distribution  $N(0,\sigma_\delta^2).$ 

The value of each element of D can be zero or one. The specification of the criterion function is aided by prior behavioral hypotheses that the relevant individual attitudinal latent variables affect each dimension. For example, an individual's sensitivity to travel time might be hypothesized to be affected by the individual's latent time consciousness. By assuming the existence of threshold values  $(\theta)$ , the latent segment levels in particular dimension are associated with the underlying criterion functions.

$$s = k, if \theta_{k-1} < G_n^* \le \theta_k, k = 1, \dots, S,$$
 (3.8)

where  $\theta_k$  is threshold parameter ( $\theta_0 = -\infty$  and  $\theta_S = +\infty$ ). Further, assuming a parametric probability density function for  $\delta_n$ , the probability that individual n falls in the segment s is

$$Q_{n}(\alpha_{s} \mid F_{n}^{*}; D, \theta) = Q_{n}(\theta_{s-1} < DF_{n}^{*} + \delta_{n} \leq \theta_{s})$$

$$= Q_{n}(\theta_{s-1} - DF_{n}^{*} < \delta_{n} \leq \theta_{s} - DF_{n}^{*}). \tag{3.9}$$

Since the criterion function  $G^*$  is latent, it is necessary to set the scale of the function. Otherwise, the unknown parameters cannot be identified because:

$$Q_n(\theta_{s-1} - DF_n^* < \delta_n \le \theta_{s-1} - DF_n^*) \tag{3.10}$$

$$= Q_n(c \cdot \theta_{s-1} - c \cdot DF_n^* < c \cdot \delta_n \le c \cdot \theta_{s-1} - c \cdot DF_n^*)$$

$$= Q_n(\theta_{s-1}^{\tilde{}} - \tilde{DF_n^*} < \tilde{\delta_n} \le \theta_{s-1}^{\tilde{}} - \tilde{DF_n^*}), \tag{3.11}$$

where c is an arbitrary positive scalar, and  $\tilde{\theta}$ ,  $D\tilde{F}_n^*$ , and  $\tilde{\delta}_n$  are scaled form of the corresponding parameters in Equation 3.9. This equation shows that the probability of individual n falling in latent segment s does not change by scaling values of  $\theta$  and  $DF_n^*$ . Therefore, it is necessary to fix the scale of the criterion function  $G^*$  to identify the parameters. To make the criterion function  $G^*$  identifiable, the following

assumption is added.

$$\delta_n \sim N(0, 1). \tag{3.12}$$

Hence, Equation 3.9 becomes

$$Q_n(\alpha_s \mid F_n^*; D, \theta) = \Phi(\theta_s - DF_n^*) - \Phi(\theta_{s-1} - DF_n^*), \tag{3.13}$$

where  $\Phi(\cdot)$  denotes the cumulative distribution function of the standard normal.

## 3.5 Structural Equations with Latent Variables

Unobservable (latent) variable  $F_n^*$  can be represented by the structural equations, as we reviewed in Chapter 2. Here, we assume the existence of latent endogenous variables  $(\eta_n^*)$  and latent exogenous variable  $(\xi_n^*)$ . Thus,  $F_n^* = [\xi_n^{*'} \eta_n^{*'}]'$  is given by

$$\eta_n^* = \Gamma \xi_n^* + \zeta_n, \tag{3.14}$$

$$y_n = \Lambda_y \eta_n^* + \epsilon_n, \tag{3.15}$$

where

 $\eta_n^*$ :  $m \times 1$  vector of latent endogenous variables,

 $\xi_n^*: q \times 1$  vector of latent exogenous variables where  $\xi_n^* \sim MVN(0, \Xi)$ ,

 $y_n: p \times 1$  vector of observed attitudinal indicators of  $\eta_n^*$ ,

 $\Gamma : m \times q$  coefficient vector for the latent exogenous variables,

 $\Lambda : p \times m$  coefficient matrix for the latent endogenous variables,

 $\zeta_n : m \times 1$  vector of random components where  $\zeta_n \sim MVN(0, \Psi)$ ,

 $\epsilon_n : p \times 1$  vector of random components where  $\epsilon_n \sim MVN(0, \Theta)$ .

Equation 3.14 and 3.15 represent a special case of structural equations with latent variables. If you assume  $\xi^* = x$  then Equation 3.14 and 3.15 would represent the MIMIC model, or if you assume q = 1, then those two equations would become the

SOFA model<sup>2</sup>. The key to our model is expressing a consumer's choice probabilities in terms of the choice probabilities corresponding to the various segments.

## 3.6 Estimation Method

Assuming that the random vectors  $\zeta$ ,  $\epsilon$ , and  $\delta$  are independent, the log-likelihood (LL) function for the model is given by

$$LL = \sum_{n} \sum_{i \in C_{n}} \kappa_{in} \cdot \ln \left\{ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \sum_{s=1}^{S} \left( \frac{\exp(\mu \alpha_{k} X_{ni})}{\sum_{j \in C_{n}} \exp(\mu \alpha_{k} X_{nj})} \cdot \left[ \Phi(\theta_{s} - DF_{n}^{*}) - \Phi(\theta_{s-1} - DF_{n}^{*}) \right] \right) \cdot \prod_{d=1}^{p} \frac{1}{\sigma_{\epsilon d}} \phi\left( \frac{y_{d} - \lambda_{d} \eta_{n}^{*}}{\sigma_{\epsilon d}} \right) \cdot \prod_{h=1}^{m} \frac{1}{\sigma_{\zeta h}} \phi\left( \frac{\eta_{nh}^{*} - \Gamma \xi_{n}^{*}}{\sigma_{\zeta h}} \right) \cdot \prod_{l=1}^{q} \frac{1}{\sigma_{\xi^{*}l}} \phi\left( \frac{\xi_{nl}^{*}}{\sigma_{\xi^{*}l}} \right) d\eta_{n}^{*} d\xi_{n}^{*} \right\},$$

$$(3.16)$$

where  $\kappa_{in}=1$  if individual n chose alternative i, 0 otherwise, and  $\sigma_{\epsilon}$ ,  $\sigma_{\zeta}$ , and  $\sigma_{\xi^*}$  are square root of diagonal elements of  $\Theta$ ,  $\Psi$ , and  $\Xi$ , respectively. Because the number of segments S is unknown, parameter estimation is carried out conditional on an assumed value for S. As you might think, Equation 3.16 is computationally demanding. Usually, in such a case, it is helpful to have good starting values of unknown parameters before we start the full information estimation. In short, since the model is a combination of the structural equation model and the multinomial logit choice model, we can easily get ad-hoc estimation values of unknown parameters by doing the estimation step wisely; those values will then be good starting values for full information estimation. Consequently, we can reduce the number of iterations and lessen the chances of non-convergence for iterative estimators. In the next subsection, we will give a brief explanation of this procedure.

## 3.6.1 Estimation of Starting Values

The procedure to obtain approximate values of unknown parameters is as follows:

<sup>&</sup>lt;sup>2</sup>Details are shown in Chapter 2

#### Notation for estimation of starting values

 $\eta_n^*: m \times 1$  vector of latent endogenous variables,

 $\xi_n^*$ :  $q \times 1$  vector of latent exogenous variables,

 $F_n^*: (q+m) \times 1$  matrix of true values of  $\eta_n^*$  and  $\xi_n^*$ .

 $D: 1 \times (q+m)$  coefficient vector,

 $\theta_s$ : threshold parameter,

 $\alpha_s$ : unknown parameter vector for segment s,

 $\mu$ : scale parameter of Gumbel distribution,

 $X_{ni}$ : attributes of alternative i and decision maker n's characteristics,

 $C_n$ : choice set of individual n.

- 1. Estimate unknown parameters  $\hat{\Lambda}, \hat{\Gamma}, \hat{\Xi}$ , and  $\hat{\Psi}$  in structural equations (Equation 3.14 and 3.15) by using structural equation modeling software packages such as SAS's CALIS procedure or LISREL.
- 2. Calculate the fitted value of  $F_n^*$  using the procedure shown in Appendix D.
- 3. Estimate the unknown parameters of the utility function, and segment membership function by using maximum likelihood estimation method. The choice probability of individual n choosing alternatives i is given by

$$P_n(i) = \sum_{s=1}^{S} \frac{\exp(\mu \alpha_s X_{ni})}{\sum_{j \in C_n} \exp(\mu \alpha_s X_{nj})} \cdot [\Phi(\theta_s - D\hat{F}_n^*) - \Phi(\theta_{s-1} - D\hat{F}_n^*)] (3.17)$$

The log-likelihood function is given by

$$LL = \sum_{n} \sum_{i \in C_n} \kappa_{in} \cdot \ln \{ \sum_{s=1}^{S} \frac{\exp(\mu \alpha_s X_{ni})}{\sum_{j \in C_n} \exp(\mu \alpha_s X_{nj})} \cdot [\Phi(\theta_s - D\hat{F}_n^*) - \Phi(\theta_{s-1} - D\hat{F}_n^*)] \}.$$
(3.18)

Note that estimated values of parameters are inconsistent because latent variable vector  $F_n^*$  is treated as non-stochastic in Equation 3.18.

## 3.7 Summary

In this chapter, a methodology, for incorporating psychometric data in discrete choice models to capture the heterogeneity across the population, is presented. The proposed approach is a combined system of linear structural equations, which identify latent variables using observable indicators, and non-linear equations, which present the discrete choice model. In the next chapter, an empirical analysis will be demonstrated.

## Chapter 4

## **Empirical Analysis**

## 4.1 Introduction

To verify the advantage of the methodology presented at the previous chapter, we conduct an empirical analysis. This empirical analysis is a clear demonstration of the effectiveness and practicality of incorporating attitudinal data in discrete choice model to capture unobservable heterogeneity across the population.

## 4.2 Description of the Data

In 1995, a Japanese contractor conducted a survey of a shopping mall located in Chiba, Japan, to get some knowledge of mode choice behaviors for shopping trips. Randomly sampled visitors' data were collected by doing face-to-face interviews. Seventy seven percent of the respondents were female. Twenty one percent were less than 30 years old, 48% between 35 and 49 years old, and 31% were more than 49 years old. The sample used in this paper comprises 357 home-based person-shopping trips, which is 80.0% of the sample. The trip modes are drive(DR), shared ride(SR), and transit(TR). The mode choice shares in the sample are as follows: DR(58.8%), SR(25.1%) and TR(16.1%). The questionnaire contained some psychometric ques-

<sup>&</sup>lt;sup>1</sup>Questions in the questionnaire are shown in Appendix F

tions, which asked respondents to rate aspects for the shopping trip in five point ratings such as 1)not important at all, 2)not very important, 3)somewhat important, 4)very important, and 5)essential.

- 1. relaxation during the trip (relx)
- 2. reliability of the arrival time (rela)
- 3. flexibility of choosing departure time (flex)
- 4. ease of traveling with heavy bags (ease)
- 5. inexpensiveness of the trip (inex)
- 6. safety during the trip (safe)
- 7. travel time (ttme)

Table 4.1 presents a descriptive summary of the importance ratings of the visitors in the survey. On average, ease of traveling with heavy bags is considered as the most important attribute.

Table 4.1: Importance Ratings for Mode Attributes							
Frequency Distribution							
Mode							
Attribute	Mean	Std. dev.	1	2	3	4	5
relx	3.90	1.11	15	38	34	150	120
rela	3.83	1.04	11	35	55	157	99
a	4.00	4.04		2.0	20	100	100
flex	4.00	1.01	11	26	38	160	122
	4.01	0.00	0	1 -	40	100	100
ease	4.21	0.96	8	15	40	126	168
·	3.74	1.15	17	38	78	113	111
inex	3.74	1.15	17	30	10	119	111
safe	3.94	1.05	10	26	68	124	129
suje	0.94	1.05	10	20	00	124	129
ttme	4.08	1.02	11	20	46	132	148
COLLEC	4.00	1.02		230		102	110
total cample size							257
total sample size							357

## 4.3 Specification of the Model

### 4.3.1 Specification of Segment Likelihood Membership Model

#### Specification of the Structural Equations with Latent Variables

Assuming existence of some latent variables, we applied the second-order factor analysis (SOFA) model and the multiple indicator multiple cause (MIMIC) model to the data set to get certain knowledge of structural relationships among the latent and observable variables. The results might be helpful when specifying the full model.

#### **SOFA** Model

Figure 4-1 represents the path diagram of the SOFA model. The single second-order factor is overall consciousness ( $\xi_1^*$ ) that directly influences three first-order factors: convenience consciousness ( $\eta_1^*$ ), time & cost consciousness ( $\eta_2^*$ ), and comfort consciousness( $\eta_3^*$ ). These first-order factors have direct effects on two or three indicators apiece.

The structural equation is specified as

$$\begin{bmatrix} \eta_{1n}^* \\ \eta_{2n}^* \\ \eta_{3n}^* \end{bmatrix} = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix} \begin{bmatrix} \xi_{1n}^* \end{bmatrix} + \begin{bmatrix} \zeta_{1n} \\ \zeta_{2n} \\ \zeta_{3n} \end{bmatrix}. \tag{4.1}$$

The specification of the measurement equation is given as

$$\begin{bmatrix} relx \\ rela \\ flex \\ inex \\ ttme \\ safe \\ ease \end{bmatrix} = \begin{bmatrix} 1.00 & 0 & 0 \\ \lambda_{12} & 0 & 0 \\ \lambda_{13} & 0 & 0 \\ 0 & 1.00 & 0 \\ 0 & \lambda_{22} & 0 \\ 0 & 0 & 1.00 \\ 0 & 0 & \lambda_{32} \end{bmatrix} \begin{bmatrix} \eta_{1n}^* \\ \eta_{2n}^* \\ \eta_{3n}^* \end{bmatrix} + \begin{bmatrix} \epsilon_{1n} \\ \epsilon_{2n} \\ \epsilon_{3n} \\ \epsilon_{4n} \\ \epsilon_{5n} \\ \epsilon_{6n} \\ \epsilon_{7n} \end{bmatrix}. \tag{4.2}$$

In the  $\Lambda$ , one indicator per construct is chosen to scale the latent first-order factors. The  $\Theta$  and  $\Psi$  are assumed to be diagonal.

$$\Theta = \begin{bmatrix}
\sigma_{\epsilon 1}^{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma_{\epsilon 2}^{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sigma_{\epsilon 3}^{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_{\epsilon 4}^{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma_{\epsilon 5}^{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma_{\epsilon 6}^{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma_{\epsilon 7}^{2}
\end{bmatrix} \tag{4.3}$$

$$\Psi = \begin{bmatrix}
\sigma_{\zeta_1}^2 & 0 & 0 \\
0 & \sigma_{\zeta_2}^2 & 0 \\
0 & 0 & \sigma_{\zeta_3}^2
\end{bmatrix}$$
(4.4)

The  $\Xi$  is assumed as

$$\Xi = 1.00. \tag{4.5}$$

This assumption is necessary to identify the SOFA model. An alternative assumption to make the SOFA model identifiable is to normalize one element of  $\Gamma$ .

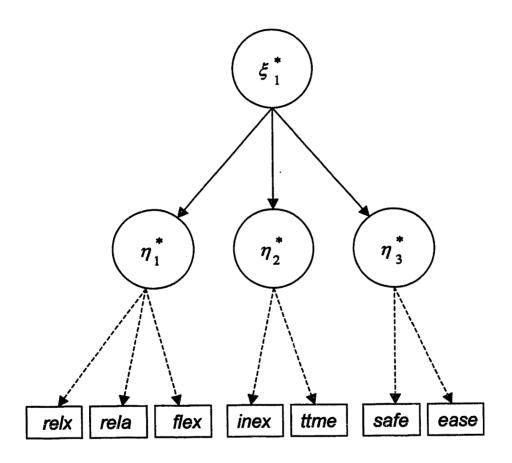


Figure 4-1: Path Diagram for SOFA Model

#### MIMIC model

By adding the assumption shown below to the SOFA model, the SOFA model will become the MIMIC model. The additional assumption is

$$\begin{bmatrix} stp \\ tpp \\ ocp \\ tspe \end{bmatrix} = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{bmatrix}, \tag{4.6}$$

where

stp: 1 if the trip is only for shopping, 0 otherwise,

tpp: 1 if the total purchase payment is over 7,000 year ( $\approx$  \$64), 0 otherwise,

ocp: 1 if full time worker, 0 otherwise,

tspe: total time spending in the shopping mall (minute(s)).

This assumption means that we treat the four observable variables (*stp*, *tpp*, *ocp*, and *tspe*) as a perfect measure of latent exogenous variables and there are no measurement errors. Hence, the structural equation of the MIMIC model is

$$\begin{bmatrix} \eta_{1n}^* \\ \eta_{2n}^* \\ \eta_{3n}^* \end{bmatrix} = \begin{bmatrix} \gamma_{11} & \gamma_{12} & 0 & 0 \\ 0 & \gamma_{22} & 0 & \gamma_{24} \\ 0 & \gamma_{32} & \gamma_{33} & 0 \end{bmatrix} \begin{bmatrix} stp \\ tpp \\ ocp \\ tspe \end{bmatrix} + \begin{bmatrix} \zeta_{1n} \\ \zeta_{2n} \\ \zeta_{3n} \end{bmatrix}. \tag{4.7}$$

Figure 4-2 shows the path diagram of the MIMIC model.

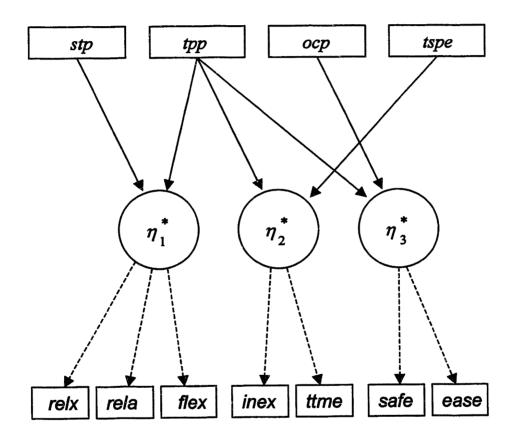


Figure 4-2: Path Diagram for MIMIC Model

#### **Estimation Results of SOFA Model and MIMIC Model**

The estimation of unknown parameters is achieved using SAS's CALIS procedure. The estimated parameters and their t values of the SOFA model are as follows:

$$\hat{\Gamma}_{sofa} = \begin{bmatrix} 0.52(10.3) & (\eta_{1n}^*) \\ 0.68(12.6) & (\eta_{2n}^*) \\ 0.81(15.6) & (\eta_{3n}^*) \end{bmatrix}$$

$$\hat{\Lambda}_{sofa} = \left[ egin{array}{ccccc} (\eta_{1n}^*) & (\eta_{2n}^*) & (\eta_{3n}^*) & & & & \\ 1.00 & 0 & 0 & (relx) & & \\ 1.13(11.4) & 0 & 0 & (rela) & & \\ 1.00(10.7) & 0 & 0 & (flex) & & \\ 0 & 1.00 & 0 & (inex) & & \\ 0 & 0 & 1.01(11.9) & 0 & (ttme) & & \\ 0 & 0 & 0.74(10.7) & (ease) & & \end{array} 
ight]$$

$$\hat{\Theta}_{sofa} =$$

	0.53(10.4)	0	0	0	0	0	0
İ	0	0.40(8.3)	0	0	0	0	0
	0	0	0.53(10.4)	0	0	0	0
	0	0	0	0.46(9.4)	0	0	0
	0	0	0	0	0.45(9.0)	0	0
	0	0	0	0	0	0.31(5.8)	0
	0	0	0	0	0	0	0.63(11.7)

$$\hat{\Psi}_{sofa} = \left[ egin{array}{cccc} 0.20(5.1) & 0 & 0 \\ 0 & 0.08(1.9) & 0 \\ 0 & 0 & 0.03(0.5) \end{array} 
ight]$$

All coefficients have reasonable sign and almost all t-statistics values are high enough to reject the null hypothesis that an estimated value is equal to zero.

The estimation results of the MIMIC model are as follows:

$$\hat{\Gamma}_{mimic} = \begin{bmatrix} (stp) & (tpp) & (ocp) & (tspe) \\ 0.06(1.5) & 0.09(2.1) & 0 & 0 & (\eta_{1n}^*) \\ 0 & 0.08(1.6) & 0 & -0.05(-1.0) & (\eta_{2n}^*) \\ 0 & -0.05(-1.4) & 0.09(1.9) & 0 & (\eta_{3n}^*) \end{bmatrix}$$

$$\hat{\Lambda}_{mimic} = \begin{bmatrix} (\eta_{1n}^*) & (\eta_{2n}^*) & (\eta_{3n}^*) \\ 1.00 & 0 & 0 & (relx) \\ 1.14(10.2) & 0 & 0 & (rela) \\ 0.95(10.1) & 0 & 0 & (flex) \\ 0 & 1.00 & 0 & (inex) \\ 0 & 0.77(1.6) & 0 & (ttme) \\ 0 & 0 & 1.00 & (safe) \\ 0 & 0 & 1.79(2.3) & (ease) \end{bmatrix}$$

 $\Theta_{mimic} =$ 

0.51(9.1)	0	0	0	0	0	0
0	0.36(6.0)	0	0	0	0	0
0	0	0.55(9.9)	0	0	0	0
0	0	0	0.27(0.6)	0	0	0
0	0	0	0	0.57(2.1)	0	0
0	0	0	0	0	0.72(5.4)	0
0	0	0	0	0	0	0.10(0.3)

$$\hat{\Psi}_{mimic} = \begin{bmatrix} 0.48(6.5) & 0 & 0 \\ 0 & 0.72(1.6) & 0 \\ 0 & 0 & 0.27(2.2) \end{bmatrix}$$

Table 4.2 presents fit indices<sup>2</sup> on each model. Most of indices indicate that the SOFA model is better than the MIMIC model from the viewpoint of fit of the data, so that we use the SOFA model to describe the latent attitudinal variables. Further, we assume overall consciousness  $(\xi_1^*)$  has essential information to capture heterogeneity across the population.

Assuming that in the population there are three latent classes at most, one or two threshold parameters,  $\theta$ , are needed in the segment likelihood membership model. Therefore, the probability that individual n has parameter vector  $\alpha_s, s = 1, \ldots, S$  is given by:

Two segments model

$$Q_{n}(\alpha_{1} \mid \xi_{n}^{*}; \theta_{1}) = \Phi[\theta_{1} - \xi_{n}^{*}], \qquad (4.8)$$

$$Q_{n}(\alpha_{2} \mid \xi_{n}^{*}; \theta_{1}) = 1 - \Phi[\theta_{1} - \xi_{n}^{*}]. \qquad (4.9)$$

$$Q_n(\alpha_2 \mid \xi_n^*; \theta_1) = 1 - \Phi[\theta_1 - \xi_n^*]. \tag{4.9}$$

<sup>&</sup>lt;sup>2</sup>Those fit indices are reviewed in Appendix C

Table 4.2: Comparison of Fit Indices

	SOFA Model	MIMIC Model
PGFI	0.51	0.55
CFI	0.97	0.60
AIC	12.1	323.1
$\hat{\Delta}$	0.96	0.59
GFI	0.97	0.83
AGFI	0.93	0.69

Three segments model

$$Q_n(\alpha_1 \mid \xi_n^*; \theta_1) = \Phi[\theta_1 - \xi_n^*], \tag{4.10}$$

$$Q_n(\alpha_2 \mid \xi_n^*; \theta_1, \theta_2) = \Phi[\theta_2 - \xi_n^*] - \Phi[\theta_1 - \xi_n^*], \tag{4.11}$$

$$Q_n(\alpha_3 \mid \xi_n^*; \theta_2) = 1 - \Phi[\theta_2 - \xi_n^*]. \tag{4.12}$$

## 4.3.2 Specification of Segment-Specific Choice Model

I arrived at the final specification based on a systematic process of eliminating variables found to be statistically insignificant in previous specifications. Each utility function is specified as:

$$U_{ni|\alpha_s}^{DR*} = \alpha_{3s}DRtt + \alpha_{5s}age + \alpha_{6s}gen + \nu_s, \tag{4.13}$$

$$U_{ni|\alpha_s}^{SR*} = \alpha_{1s} + \alpha_{3s}SRtt + \alpha_{7s}age + \alpha_{8s}gen + \nu_s, \qquad (4.14)$$

$$U_{ni|\alpha_s}^{TR*} = \alpha_{2s} + \alpha_{3s}TRtt + \alpha_{4s}cost + \alpha_{9s}np + \nu_s, \qquad (4.15)$$

where

 $\alpha_{1s}$  and  $\alpha_{2s}$  : coefficient for alternative specific constant,

DRtt, SRtt, and TRtt: total travel time for each mode,

cost: travel cost per person,

age: 1 if 30 years or older, 0 otherwise,

gen: 1 if female, 0 otherwise,

np : 1 if accompanied, 0 otherwise.

The path diagram of the full model is shown in Figure 4-3.

### 4.4 Estimation Results

Maximization of the log-likelihood function (Equation 3.16) is achieved using the GAUSS matrix programming language. To make the log-likelihood function estimable, we add the assumption that  $\zeta_{3n} = 0$ . In order to study the effectiveness of the latent segment model, we estimated four different models of mode choice. The estimated models are:

- 1. no-heterogeneity model (MNL model).
- 2. a priori segment model (2APS model).
- 3. two latent segments model with overall consciousness (2LS model).
- 4. three latent segments model with overall consciousness (3LS model).

In 2APS model, we divided the sample according to 1) trip purpose, 2) total purchase payment, 3) occupation, and 4) total time spent in the shopping mall and then model estimations were conducted for each group. As the result, the model using the total purchase payment as the criterion value showed the best fit to the data. The sample is divided into two data sets according to the individual's total purchase payment in the mall and the criterion value is 7,000 yen ( $\approx$  \$64).

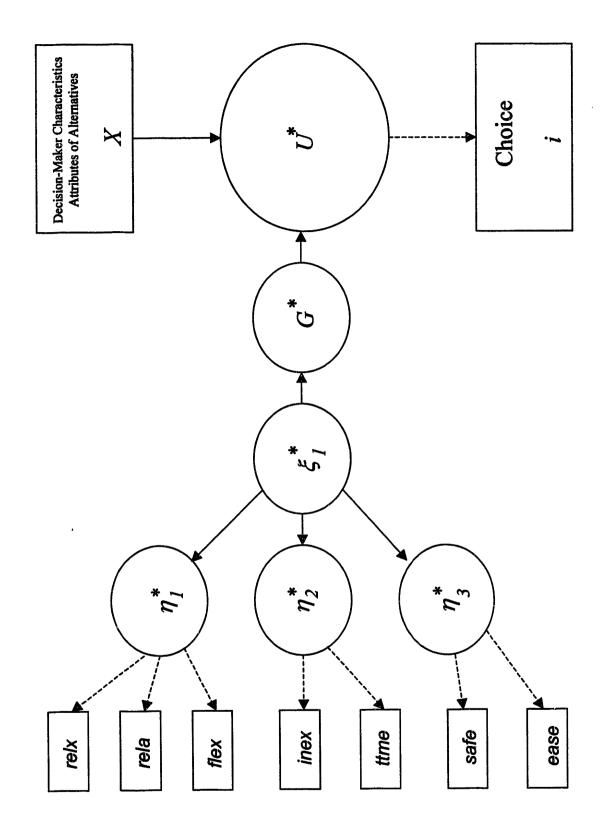


Figure 4-3: Path Diagram for Full Model

Table 4.3: Estimation Result of Multinomial Logit (MNL) Model

Parameters	Estimates	Std. err.	t-stat.
SR constant	-3.40	0.55	-6.2
TR constant	-1.59	0.64	-2.5
$travel\ time$	-0.01	0.005	-2.9
cost	0.0009	0.001	0.9
age(specific to DR)	0.14	0.39	0.4
age(specific to SR)	0.50	0.39	1.3
gen(specific to DR)	-0.94	0.45	-2.1
gen(specific to SR)	0.83	0.52	1.6
np	-0.99	0.33	-3.0
Number of observations	357		
LL of at zero	-360.98		
LL of at convergence	-249.08		

### 4.4.1 Estimated Multinomial Logit Model

This model represents the no-heterogeneity case and is the simplest of the estimated models. The estimated MNL model is presented in Table 4.3. Most of coefficients are significant; however, the coefficient for *cost* is insignificant and has a counterintuitive sign.

## 4.4.2 Estimated a Priori Segment Choice Model

The estimated a priori segment model is presented in Table 4.4. Many estimates are insignificant and the estimated parameter for *cost* in segment 1 has a positive sign, which is counterintuitive. The differences between the estimated parameters for segment 1 and segment 2 indicate that some coefficients might vary across the population.

Table 4.4: Estimation Result of a Priori Segment Choice (2APS) Model

Parameters	Estimates	Std. err.	t-stat.
Under \$64 segment			
SR constant	-3.41	0.65	-5.2
TR constant	-1.9	0.75	-2.6
$travel\ time$	-0.01	0.005	-2.0
cost	0.001	0.001	1.1
age(specific to DR)	-0.33	0.47	-0.7
age(specific to SR)	0.11	0.46	0.24
gen(specific to DR)	-1.13	0.54	-2.1
gen(specific to SR)	0.79	0.62	1.3
np	-1.06	0.41	-2.6
Number of observations	214		
LL at zero	-216.39		
LL at convergence	-156.42		
Over \$64 segment			
SR constant	-3.72	1.10	-3.4
TR constant	-0.57	1.50	-0.4
$travel\ time$	-0.021	0.01	-2.2
cost	-0.002	0.003	-0.7
age(specific to DR)	0.92	0.89	1.0
age(specific to SR)	1.57	0.84	1.9
gen(specific to DR)	-1.15	0.97	-1.2
gen(specific to SR)	0.32	1.08	0.3
np	-1.14	0.60	-1.9
Number of observations	143		
LL at zero	-144.59		
LL at convergence	-87.87		

Table 4.5: Estimation Result of 2LS Model, Choice Model

Parameters	Estimates	Std. err.	t-stat.
$\overline{\theta_1}$	-0.89	0.06	-15.9
Segment 1			
SR constant	-5.30	1.34	-4.0
TR constant	12.33	4.57	2.7
$travel\ time$	-0.48	0.19	-2.6
cost	-0.06	0.05	-1.1
age(specific to DR)	0.25	0.31	0.8
age(specific to SR)	0.69	0.68	1.0
gen(specific to DR)	-0.66	0.29	-2.3
gen(specific to SR)	1.09	0.78	1.4
np	-0.82	0.34	2.4
Segment 2			
SR constant	-3.96	0.61	-6.5
TR constant	10.90	3.76	2.9
$travel\ time$	-3.01	2.00	-1.5
cost	-0.08	0.05	-1.5
age(specific to DR)	0.25	0.86	0.3
age(specific to SR)	0.63	0.35	1.8
gen(specific to DR)	-0.82	0.34	-2.4
gen(specific to SR)	1.36	0.85	1.6
np	-0.80	0.35	-2.3
Number of observations	357		
LL of choice model at zero	-360.98		
LL of choice model at convergence	-235.85		

## 4.4.3 Estimated Two Latent Segments Choice Model with Overall Consciousness

The estimated two latent segments model with overall consciousness is presented in Table 4.5 and Table 4.6. In this model, most of coefficients are significant and all coefficients have expected signs. The coefficients for intercepts and alternative characteristics (travel time and cost) vary across the segments. Judging from the values of the estimated coefficients, an individual who belongs to segment 2 would have a relatively higher time and cost sensitivity, and value of time, and conscious of comforts of the modes.

Table 4.6: Estimation Result of 2LS Model, Structural Model (t-stat.)

$$\hat{\Gamma}_{2ls} = \begin{bmatrix} 0.52 & (10.3) & (\eta_{1n}^*) \\ 0.84 & (9.6) & (\eta_{2n}^*) \\ 0.91 & (13.4) & (\eta_{3n}^*) \end{bmatrix}$$

$$\hat{\Lambda}_{2ls} = egin{bmatrix} (\eta_{1n}^*) & (\eta_{2n}^*) & (\eta_{3n}^*) \ 1.00 & 0 & 0 & (relx) \ 1.00 & (9.3) & 0 & 0 & (rela) \ 0.96 & (9.9) & 0 & 0 & (flex) \ 0 & 1.00 & 0 & (inex) \ 0 & 0.98 & (11.1) & 0 & (ttme) \ 0 & 0 & 1.00 & (safe) \ 0 & 0 & 0.66 & (10.6) & (ease) \ \end{bmatrix}$$

$$\hat{\Psi}_{2ls} = \left[ \begin{array}{ccc} 0.18 \; (6.3) & 0 & 0 \\ 0 & 0.07 \; (2.0) & 0 \\ 0 & 0 & 0.00 \end{array} \right]$$

## 4.4.4 Estimated Three Latent Segments Choice Model with Overall Consciousness

The estimated three latent segments model with overall consciousness is presented in Table 4.7 and Table 4.8. In the choice model, although some t statistics are insignificant, all parameters have expected signs. All the parameters in the structural equations have positive signs as expected and sufficiently large t-statistics. It seems that segment 1 is the group which has the lowest time and cost sensitivity, while in contrast, segment 2 is the highest time and cost sensitivity group. Segment 3 has moderate sensitivity for time and cost.

#### 4.4.5 Summary of Estimated Models

Comparing the estimates across the four model specifications, we see that the coefficients of marketing variables in the no-heterogeneity model (= MNL model) are different from the estimates obtained from the other three formulations. In particular, we note that the estimates for travel time and cost variables are biased downward in magnitude in the MNL model. Table 4.9 presents the summary of the estimated models. We find a improvement in the Akaike information criterion, Bayesian information criterion<sup>3</sup>, and  $\bar{\rho}^2$  values for the two models that account for unobservable heterogeneity. In terms of the Akaike information criterion and  $\bar{\rho}^2$ , the three latent segments choice (3LS) model has the best fit among all the models. Meanwhile, looking at the Bayesian information criterion, the two latent segments choice (2LS) model fits the data the best. From the results of all the models, it might be reasonable to say that by incorporating latent attitudinal variables (in this case, overall consciousness) to segment the population, the explanatory power of the choice model is improved.

<sup>&</sup>lt;sup>3</sup>One advantage of BIC over AIC is that it takes into consideration both the number of observations and the number of parameters used in the analysis.

BIC =  $-LL + 1/2 \cdot R \cdot \log(T)$ , where LL denotes the log-likelihood, R denotes the number of parameters estimated, and T denotes the total number of observations.

Table 4.7: Estimation Result of 3LS Model, Choice Model

Parameters	Estimates	Std. err.	t-stat.
$\theta_1$	-0.87	0.05	-16.7
$ heta_2$	0.99	0.05	18.6
Segment 1	0.55	0.00	10.0
SR constant	-6.93	1.73	-4.0
TR constant	11.93	3.85	3.1
travel time	-0.25	0.13	-1.9
cost	-0.05	0.03	-1.5
age(specific to DR)	0.22	0.32	0.7
age(specific to SR)	0.68	0.40	1.7
gen(specific to DR)	-0.74	0.32	-2.3
gen(specific to SR)	1.14	0.52	2.2
np	-0.79	0.30	-2.6
Segment 2			
SR constant	-3.96	0.61	-6.5
TR constant	11.05	3.16	3.5
$travel\ time$	-3.36	1.60	-2.1
cost	-0.08	0.04	-2.0
age(specific to DR)	0.25	0.27	0.9
age(specific to SR)	0.62	0.31	2.0
gen(specific to DR)	-0.69	0.28	-2.5
gen(specific to SR)	1.46	0.61	2.4
np	-0.76	0.30	-2.5
Segment 3			
SR constant	-3.87	0.70	-5.5
TR constant	12.70	4.54	2.8
$travel\ time$	-3.11	1.24	-2.5
cost	-0.08	0.05	-1.8
age(specific to DR)	0.26	0.21	1.2
age(specific to SR)	0.60	0.40	1.5
gen(specific to DR)	-0.96	0.33	-2.9
gen(specific to SR)	1.12	0.62	1.8
np	-0.83	0.30	-2.8
Number of observations	357		
LL of choice model at zero	-360.98		
LL of choice model at convergence	-214.23		

Table 4.8: Estimation Result of 3LS Model, Structural Model (t-stat.)

$$\hat{\Gamma}_{3ls} = \begin{bmatrix} 0.53 & (9.8) & (\eta_{1n}^*) \\ 0.92 & (13.0) & (\eta_{2n}^*) \\ 0.95 & (14.4) & (\eta_{3n}^*) \end{bmatrix}$$

$$\hat{\Lambda}_{3ls} = \begin{bmatrix} (\eta_{1n}^*) & (\eta_{2n}^*) & (\eta_{3n}^*) \\ 1.00 & 0 & 0 & (relx) \\ 1.11 & (11.5) & 0 & 0 & (rela) \\ 0.93 & (9.5) & 0 & 0 & (flex) \\ 0 & 1.00 & 0 & (inex) \\ 0 & 1.06 & (11.6) & 0 & (ttme) \\ 0 & 0 & 1.00 & (safe) \\ 0 & 0 & 0.69 & (10.9) & (ease) \end{bmatrix}$$

$$\hat{\Psi}_{3ls} = \left[ \begin{array}{ccc} 0.14 \ (5.9) & 0 & 0 \\ 0 & 0.09 \ (2.3) & 0 \\ 0 & 0 & 0.00 \end{array} \right]$$

Table 4.9: Summary of Estimated Models

Indices	MNL	2APS	2LS	3LS
Number of observations	357	357	357	357
Complete LL at convergence	-249.08	-244.29	-1472.52	-1352.52
LL of choice model at convergence	-249.08	-244.29	-235.85	-214.23
# of parameters of choice model	9	18	19	29
AIC	516.15	524.58	448.54	422.71
BIC	275.52	297.19	261.11	267.58
$ar ho^2$	0.29	0.27	0.38	0.41

## 4.5 Summary

In this chapter, we have conducted an empirical analysis. The four models

- 1. no-heterogeneity model,
- 2. a priori segment model,
- 3. two latent segments model, and
- 4. three latent segments model

are estimated using shopping trip data. The data for this analysis were obtained from a survey of shopping mall visitors. In latent segment models, latent variables are captured by the SOFA structure, and one latent exogenous variable is incorporated in the choice model to account for the unobservable heterogeneity. No-heterogeneity model and a priori segment model cannot successfully estimate important parameters because those models failed to control heterogeneity across the population. The results indicate that there could be severe biases in the parameter estimates for the

effects of marketing variables if heterogeneity is not properly accounted for in the analysis. In this empirical analysis, the bias appears to be especially severe for the travel time and cost parameters, which are both important parameters in demand analysis. In contrast, parameters estimated by latent segment models have reasonable signs and statistically significant t-statistics. In the context of the fit of the data, latent segment models also show better fit than no-heterogeneity and a priori segment models. This empirical study provided a clear demonstration of the usefulness of the latent segment model.

## Chapter 5

## **Conclusions**

In this chapter, we summarize the key conclusions from the thesis, and suggest some issues to be addressed. Regarding future research topics, we mention issues of robustness of models and enhancements in data collection efforts.

## 5.1 Summary and Conclusions

In this thesis, we have provided an approach to accounting for unobservable heterogeneity across individuals in their response to marketing variables. The proposed approach used latent attitudinal variables to characterize the unobservable heterogeneity across individuals in relation to their behavior in a market. In Chapter 2, we reviewed structural equations with latent variables and presented two sub-models, the higher-order factor analysis (HOFA) model and the multiple indicator multiple cause (MIMIC) model. Our approach presented in Chapter 3, while allowing for very general patterns of heterogeneity, is also empirically tractable. In addition, the methodology makes it possible to take a very flexible approach to accounting for unobservable heterogeneity, which would be an essential consideration factor in practical situations. Maximizing log-likelihood is computationally demanding; however, a methodology to be able to reduce the number of iterations and lessen the chance of non-convergence was presented. In Chapter 4, an empirical analysis was conducted using shopping trip data. The second-order factor analysis model, which is a special

case of the HOFA model, was used to describe the latent attitudinal variables, and was combined with a discrete choice model. Simultaneous estimation was conducted to obtain unknown parameters. The empirical results indicate that there could be biases in the parameter estimates for the effects of marketing variables if heterogeneity is not accounted for in the analysis. Specifically, we find a downward bias in magnitude in the estimates of travel time and cost when the model fails to account for heterogeneity in the estimation. Proper control of heterogeneity will yield robust estimates of the model parameters. Moreover, latent segment models fit better than no-heterogeneity and a priori segment models. The effectiveness and practicality of the methodology were demonstrated through this empirical analysis.

#### 5.2 Future Research

Several possible avenues for future research exist. It would be interesting to assess the robustness of the model given in this thesis using non-parametric techniques such as bootstrap, jack-knife, or cross-validation<sup>1</sup>. In general, researchers have little knowledge about the behavior or, in another word, robustness, of their models, and non-parametric techniques would appear to be an ideal means to tackle this problem. One of the advantages of non-parametric techniques is that it does not require distributional assumptions. Further, in structural equation models, very little work has been done on how well those non-parametric techniques work in practice. Chatterjee (1984) demonstrates the use of the bootstrap for estimation of the variances of the factor loading estimates. Lambert, Wildt and Durand (1991) investigate the bootstrap confidence intervals for factor loadings and suggest their usefulness as an aid in factor interpretation. Bollen and Stine (1990, 1992) discuss the application of the bootstrap methods in structural equation models. Ichikawa and Konishi (1995) conduct a Monte Carlo experiment to investigate the performance of the bootstrap

<sup>&</sup>lt;sup>1</sup>This future research direction is suggested by Associate Professor Satosi Yamasita, Institute of Statistical Mathematics.

in normal theory maximum likelihood factor analysis both when the distributional assumption is satisfied and unsatisfied. They note that more research is needed to investigate the performance of the bootstrap methods in factor analysis under various conditions.

Another area of future research interest is data collection methods. The statistically advanced and conceptually sophisticated set of tools for discrete choice analysis provided in this thesis depends heavily on the availability of high quality data such as attitudinal indicators. Such data, routinely collected in the marketing research context, are rarely collected in travel demand analysis. Therefore, more substantive research is needed in survey and questionnaire design to reflect the changing needs of travel demand analysis, and more importantly, to assess the practical significance and benefits of such modeling approaches.

## Appendix A

# Variance-Covariance Matrix of MIMIC Model

#### Notation for MIMIC model

 $x : n \times 1$  vector of observed indicators of  $\xi^*$ ,

 $y: p \times 1$  vector of observed indicators of  $\eta^*$ ,

 $\eta^*\,:\,m\,\times\,1$  vector of latent endogenous variables,

 $\xi^*$ : n × 1 vector of latent exogenous variables,

 $B_{\#}$ :  $m \times m$  coefficient matrix for latent endogenous variables,

 $B = (1 - B_{\#})^{-1}$ 

 $\Gamma : m \times n$  coefficient vector for the latent exogenous variables,

 $\Lambda_y$  :  $p \times m$  coefficient matrix for the latent endogenous variables,

 $\zeta : m \times 1$  vector of random components,

 $\epsilon$  :  $p \times 1$  vector of random components,

 $I : n \times n \text{ identity matrix},$ 

O: zero matrix.

The structural and measurement equations of MIMIC model are

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} O \\ \Lambda_y \end{bmatrix} \eta^* + \begin{bmatrix} I \\ O \end{bmatrix} \xi^* + \begin{bmatrix} O \\ \epsilon \end{bmatrix}, \tag{A.1}$$

$$\eta^* = B_\# \eta^* + \Gamma x + \zeta. \tag{A.2}$$

Thus, each element of the variance-covariance matrix is described as:

$$\Sigma_{y} = E(yy')$$

$$= E[(\Lambda_{y}\eta^{*} + \epsilon)(\Lambda_{y}\eta^{*} + \epsilon)']$$

$$= E[(\Lambda_{y}(B\Gamma\xi^{*} + B\zeta) + \epsilon)(\Lambda_{y}(B\Gamma\xi^{*} + B\zeta) + \epsilon)']$$

$$= \Lambda_{y}B(\xi^{*}\xi^{*})\Gamma'B'\Lambda'_{y} + \Lambda_{y}BE(\zeta\zeta')B'\Lambda'_{y} + E(\epsilon\epsilon')$$

$$= \Lambda_{y}(B\Gamma\Xi\Gamma'B' + B\Psi B')\Lambda'_{y} + \Theta, \qquad (A.3)$$

$$\Sigma_{yx} = E(yx')$$

$$= E[(\Lambda_{y}\eta^{*} + \epsilon)(\xi^{*})]$$

$$= E[(\Lambda_{y}(B\Gamma\xi^{*} + B\zeta) + \epsilon)(\xi^{*})]$$

$$= \Lambda_{y}B\Gamma E(\xi^{*}\xi^{*})$$

$$= \Lambda_{y}B\Gamma E(\xi^{*}\xi^{*})$$

$$= E[(\xi^{*})(\Lambda_{y}\eta^{*} + \epsilon)']$$

$$= E[(\xi^{*})(\Lambda_{y}(B\Gamma\xi^{*} + B\zeta) + \epsilon)']$$

$$= E[(\xi^{*})(\Lambda_{y}(B\Gamma\xi^{*} + B\zeta) + \epsilon)']$$

$$= E[(\xi^{*})(\Lambda_{y}(B\Gamma\xi^{*} + B\zeta) + \epsilon)']$$

$$= E(\xi^{*}\xi^{*})\Gamma'B'\Lambda'_{y}$$

$$= \Xi\Gamma'B'\Lambda'_{y}, \qquad (A.5)$$

$$\Sigma_{x} = E(\xi^{*}\xi^{*})$$

$$= \Xi. \qquad (A.6)$$

Therefore, the variance-covariance matrix of MIMIC model is

$$\Sigma_{z} = \begin{bmatrix} \Sigma_{y} & \Sigma_{yx} \\ \Sigma_{xy} & \Sigma_{x} \end{bmatrix} = \begin{bmatrix} \Lambda_{y} (B\Gamma\Xi\Gamma'B' + B\Psi B')\Lambda'_{y} + \Theta & \Lambda_{y}B\Gamma\Xi \\ \Xi\Gamma'B'\Lambda'_{y} & \Xi \end{bmatrix}, \quad (A.7)$$

where

$$\Psi = E(\zeta \zeta'),$$

$$\Theta = E(\epsilon \epsilon').$$

## Appendix B

## Variance-Covariance Matrix of SOFA Model

#### Notation for SOFA model

 $\eta^*$ :  $m \times 1$  vector of latent endogenous variables,

 $\xi^*$ : 1 × 1 vector of latent exogenous variables,

 $y~:p\times 1$  vector of observed indicators of  $\eta^*$ ,

 $\Gamma$ :  $m \times 1$  coefficient vector for the latent exogenous variables,

 $\Lambda : p \times m$  coefficient matrix for the latent endogenous variables,

 $\zeta$ :  $m \times 1$  vector of random components,

 $\epsilon$ :  $p \times 1$  vector of random components,

 $\Xi = E(\xi^* \xi^{*\prime}),$ 

 $\Psi = E(\zeta \zeta'),$ 

 $\Theta = E(\epsilon \epsilon').$ 

The second-order factor analysis is described as:

Structural equation

$$\eta^* = \Gamma \xi^* + \zeta. \tag{B.1}$$

Measurement equation

$$y = \Lambda \eta^* + \epsilon. \tag{B.2}$$

Therefore, the variance-covariance matrix of SOFA model is

$$\Sigma = E(yy')$$

$$= E[(\Lambda \eta^* + \epsilon)(\Lambda \eta^* + \epsilon)']$$

$$= E[(\Lambda(\Gamma \xi^* + \zeta) + \epsilon)(\Lambda(\Gamma \xi^* + \zeta) + \epsilon)']$$

$$= \Lambda \Gamma E(\xi^* \xi^{*'}) \Gamma' \Lambda' + \Lambda E(\zeta \zeta') \Lambda' + E(\epsilon \epsilon')$$

$$= \Lambda \Gamma \Xi \Gamma' \Lambda' + \Lambda \Psi \Lambda' + \Theta. \tag{B.3}$$

## Appendix C

## **Model Evaluation**

To help in the evaluation of a structural equation model, a number of statistical measures of fit have been proposed. At the same time, the issue of fit assessment has been the subject of both theoretical and empirical papers (e.g., Bentler and Bonnet 1980, Bentler 1990). Given the large number of alternative fit indices available, investigators may have difficulty choosing among them because the articles on this topics reach no consensus about what constitutes "good fit". Here we review some formulas for the computation of the various fit indices.

## C.1 GFI

Jöreskog and Sörbom (1981) propose a Goodness of Fit Index (GFI).

$$GFI = 1 - \frac{tr[(\Sigma_z(\hat{\tau})^{-1}S_z - I)^2]}{tr[(\Sigma_z(\hat{\tau})^{-1}S_z)^2]}.$$
 (C.1)

The GFI measures the relative amount of the variance and covariance in  $S_z$  that are predicted by  $\Sigma(\hat{\tau})$ .

#### C.2 AGFI

Adjusted GFI (AGFI) is

$$AGFI = 1 - \left[\frac{p(p+1)}{2df}\right][1 - GFI],$$
 (C.2)

where df is degrees of freedom. The AGFI adjusts for the degrees of freedom of a model relative to the number of variables.

#### C.3 PGFI

Mulaik et al. (1989) propose a Parsimonious Goodness-of-Fit Index (PGFI) defined as

$$PGFI = \left[\frac{2df}{p(p+1)}\right]GFI. \tag{C.3}$$

#### C.4 CFI

The Comparative Fit Index (CFI), (Bentler 1990) is based on the non-centrality parameter of the chi-square of the goodness-of-fit test statistic. Let a baseline model have a test statistic  $T_0$  with  $df_0$  degrees of freedom. Let the model being evaluated have associated test statistic  $T_1$ , with  $df_1$ , degrees of freedom. Let  $l_0 = T_0 - df_0$  and  $l_1 = T_1 - df_1$ . Then the CFI can be computed in sample as

$$CFI = 1 - \frac{l_i}{l_j}, (C.4)$$

where  $l_i = max(l_i, 0)$  and  $l_j = max(l_0, l_i, 0)$ .

#### C.5 Normed Fit Index $(\hat{\Delta})$

Let  $f_0$  be the value of a discrepancy function (e.g., ML, GLS) for some baseline model and f be the value of the discrepancy function for some model for interest. Then, the

normed fit index  $\hat{\Delta}$  proposed by Bentler and Bonett (1980) is defined as

$$\hat{\Delta} = \frac{f_0 - f_1}{f_0}. \tag{C.5}$$

#### C.6 AIC

Strictly applicable only for maximum likelihood estimation, the Akaike Information Criterion (AIC), (Akaike 1987) is defined as

$$AIC = -2(T - r), (C.6)$$

where r refers to the number of free parameters in the model being evaluated and T is the value of the test statistic for the model being evaluated.

### Appendix D

## Estimation of Values of Latent Variables

In most cases, the latent variables cannot be measured directly because they are "latent". Only scores on indicators for these variables can be obtained; for example, attitudes cannot be measured directly, but it is possible to collect verbal expressions of attitudinal indicators in questionnaire. Sometimes researchers wish to know the values of the latent variables for each individual observations. Getting fitted values of latent variables is also useful to get starting values for simultaneous estimation for the choice model that contains latent variables in its equation. The best we can do is to estimate these by forming some weighted function of observed variables. In estimating the values of latent variables, the observed variables are used to estimate the values of the respondents on the unmeasured variables  $(\eta^*, \xi^*)$ , given that  $\Lambda, K, B, \Gamma, \Xi, \Psi$ , and  $\Theta$  are known. Restricting ourselves to linear estimates, this means that we look for a matrix W of weights which, when multiplied with Z, gives the best estimates of  $F^*$  according to some criteria.

$$\hat{F}^* = WZ \tag{D.1}$$

where  $\hat{F}^*$  is the estimate of  $F^*$ , which is  $(m+n)\times 1^{-1}$  matrix of true values of  $\eta^*$  and  $\xi^*$ :

$$F^* = \begin{bmatrix} \eta^* \\ \xi^* \end{bmatrix}$$
 (D.2)

and W is  $(m+q) \times p^2$  matrix, and Z is  $p \times 1$  observable multivariate data. Here, we set the criterion that

$$f_{F^*} = \frac{1}{N} tr(F^* - \hat{F}^*)^2$$
 (D.3)

is minimal. We can rewrite Equation 2.2 as:

$$z = \left[ \Lambda \ K \right] \left[ \begin{array}{c} \eta^* \\ \xi^* \end{array} \right] + \epsilon \tag{D.4}$$

$$Z = [\Lambda \ K]F^* + E \tag{D.5}$$

and substituting Equation D.1 into Equation D.3 gives

$$f_{F^*} = tr(S_f + WS_zW' - 2W[\Lambda \ K]S_f)$$
 (D.6)

where  $S_f$  is variance-covariance matrix of  $F^*$ , and  $S_z$  is sample variance-covariance matrix. The first-order condition of Equation D.6 is given as

$$\frac{1}{2}\frac{\partial f_{F^*}}{\partial W} = WS_z - S_f[\Lambda \ K]' = O \tag{D.7}$$

and it is solved as<sup>3</sup>:

$$W = S_f[\Lambda \ K]'S_z^{-1}$$

<sup>1</sup>m: number of latent endogenous variables, n: number of latent exogenous variables.

 $<sup>^{2}</sup>p$ : number of observed indicators of  $\eta^{*}$ , q: number of observed indicators of  $\xi^{*}$ .

<sup>&</sup>lt;sup>3</sup>How to calculate  $S_f$  is shown in Appendix E

$$= \begin{bmatrix} \Sigma_{\eta^*\eta^*} & \Sigma_{\eta^*\xi^*} \\ \Sigma_{\xi^*\eta^*} & \Sigma_{\xi^*\xi^*} \end{bmatrix} [\Lambda \ K]' S_z^{-1}$$

$$= \begin{bmatrix} B\Gamma \Xi \Gamma' B' + B\Psi B' & B\Gamma \Xi \\ \Xi \Gamma' B' & \Xi \end{bmatrix} [\Lambda \ K]' S_z^{-1}. \tag{D.8}$$

By substituting the estimated values into Equation D.8, you can get the matrix of weights W. Thus, the fitted values of  $F^*$  is given as

$$\hat{F}^* = \begin{bmatrix} \hat{\eta}^* \\ \hat{\xi}^* \end{bmatrix} = \begin{bmatrix} \hat{B}\hat{\Gamma}\hat{\Xi}\hat{\Gamma}'\hat{B}' + \hat{B}\hat{\Psi}\hat{B}' & \hat{B}\hat{\Gamma}\hat{\Xi} \\ \hat{\Xi}\hat{\Gamma}'\hat{B}' & \hat{\Xi} \end{bmatrix} [\hat{\Lambda} \quad \hat{K}]'S_z^{-1}Z. \tag{D.9}$$

Note that since  $\hat{F}^*$  are a weighted combination of Z and since  $\hat{F}^* \neq F^*$ , we can regard  $\hat{F}^*$  as an indicator of  $F^*$  that contains measurement errors. So in most cases using factor score estimates to replace latent variables and then employing classical econometric procedures on these estimates still leads to inconsistent coefficient estimators of choice model coefficients.

## Appendix E

# Variance-Covariance Matrix of Latent Variables $(S_f)$

#### Notation

```
\eta^*: m \times 1 vector of latent endogenous variables,
```

 $\xi^*$ :  $q \times 1$  vector of latent exogenous variables,

 $z : p \times 1$  vector of observed indicators of  $\eta^*$ ,

 $\Gamma : m \times 1$  coefficient vector for the latent exogenous variables,

 $\Lambda : p \times m$  coefficient matrix for the latent endogenous variables,

 $K: p \times q$  coefficient matrix for the latent exogenous variables,

 $\zeta : m \times 1$  vector of random components,

 $\epsilon$ :  $p \times 1$  vector of random components,

 $\Xi = E(\xi^* \xi^{*\prime}),$ 

 $\Psi = E(\zeta \zeta'),$ 

 $\Theta = E(\epsilon \epsilon').$ 

Structural Equation Model

$$\eta^* = B_{\#}\eta^* + \Gamma \xi^* + \zeta \tag{E.1}$$

Measurement Equation Model

$$z = \Lambda \eta^* + K \xi^* + \epsilon \tag{E.2}$$

Thus, each element of variance-covariance model is described as:

$$\Sigma_{\eta^*\eta^*} = E[(B\Lambda\xi^* + B\zeta)(B\Lambda\xi^* + B\zeta)']$$

$$= B\Lambda E(\xi^*\xi^{*'})\Lambda'B' + BE(\zeta\zeta')B'$$

$$= B\Lambda\Xi\Lambda'B' + B\Psi B', \qquad (E.3)$$

$$\Sigma_{\eta^*\xi^*} = E[(B\Lambda\xi^* + B\zeta)\xi^{*'}]$$

$$= B\Lambda E(\xi^*\xi^{*'})$$

$$= B\Lambda\Xi, \qquad (E.4)$$

$$\Sigma_{\xi^*\eta^*} = E[\xi^*(B\Lambda\xi^* + B\zeta)']$$

$$= E(\xi^*\xi^{*'})\Lambda'B'$$

$$= \Xi\Lambda'B', \qquad (E.5)$$

$$\Sigma_{\xi^*\xi^*} = E(\xi^*\xi^{*'})$$

$$= \Xi. \qquad (E.6)$$

Therefore, the variance-covariance matrix is

$$\begin{bmatrix} \Sigma_{\eta^*\eta^*} & \Sigma_{\eta^*\xi^*} \\ \Sigma_{\xi^*\eta^*} & \Sigma_{\xi^*\xi^*} \end{bmatrix} = \begin{bmatrix} B\Gamma\Xi\Gamma'B' + B\Psi B' & B\Gamma\Xi \\ \Xi\Gamma'B' & \Xi \end{bmatrix}.$$

## Appendix F

## Survey Questionnaire for Shopping Trip Data

```
    What is your sex?

            male
            female

    What is your age?

            junior age?
            junior high school student
            high school student
            college or junior college student
            employee (full time)
            self-employed
            part time job
            house wife

    other ( )
```

4.	How many people made this trip with you (yourself included)?
	1) alone
	2) 2
	3) 3
	4) 4
	5) 5
	6) 6
	7) 7
	8) 8
	9) 9
	10) 10 or more people
5.	With whom do you come?
	1) male friend(s)
	2) female friend(s)
	3) male and female friend(s)
	4) couple
	5) family
	6) alone
	7) other ( )
6.	Is shopping at this mall your main purpose of this trip?
	1) Yes
	2) No
7.	Please respond only if you answered "No" to the previous question.
	What is the main purpose for the trip?
	1) to work at a work place
	2) business
	3) school

4)	) visit				
<b>5</b> )	) recreation				
6	) other ( )				
8. Ho	w did you know	this shop	oing mall?		
1	) TV				
2	) radio				
3	) newspapers				
4	) magazines				
5	) posters				
6	) advertisement	s in train o	ears		
7	) word of mout	h			
8	) other ( )				
9. Wł	nere do you live	?			
(	) City, ( )	Ward, (	) Town, (	) Street	
10 D	a vou hava a di	ivon'a licon	202		
	o you have a di	iver s ncen	.se:		
	) Yes				
2	) No				
11. P	lease respond o	nly if you a	answered "Ye	s" to the previou	s question.
	o you have a c				_
	) Yes		·		
	l) No				
	,				
12. D	o you have a bi	cycle and/	or moped, wl	nich is usually av	ailable for you?
b	oicycle	1) Yes	2) No		
n	noped	1) Yes	2) No		

13.	Do you have a rail pass card?
	1) Yes
	2) No
	•
14.	Please respond only if you answered "Yes" to the previous question.
	What are the origin and the destination stations of your pass?
	$( )$ station $\iff$ $( )$ station
15.	Where did your trip begin?
	1) your home
	2) workplace
	3) school
	4) other ( )
16.	How did you come here?
	1) rail
	2) bus
	3) auto
	4) taxi
	5) bike/moped
	6) bicycle
	7) by foot
17	Why did you choose the mode mentioned above?
	Because
	1) The distance between home and the mall is short
	2) I thought it was the fastest mode
	3) it is easy to take
	4) it is easier to carry shopping bags
	,
	5) it is convenient

	6) I came accompanied								
	7) it is the most inexpensive								
	8) I usually choose the mode whenever I go out								
	9) other ( )								
18.	How long did it	take	you to	o get 1	to her	e?			
	( ) hour(s) (	) n	nin.						
19.	Do you usually o	choos	e the	same	$\mathbf{mode}$	as to	day's to come here?		
	1) Yes								
	2) No $\Longrightarrow$ Which	ch mo	ode do	you	choose	e usua	ally?		
	1) rail 2) bus 3)	auto	o 4) ta	xi 5)l	oike/n	noped	6) bicycle 7) by foot		
	Why did you no	ot cho	oose t	he usu	ıal mo	de to	day? ( )		
20.	Please give the a	appro	priate	level	of the	e each	mode as a means to come her	e.	
	rail	1	2	3	4	5	6		
	bus	1	2	3	4	5	6		
	auto	1	2	3	4	5	6		
	taxi	1	2	3	4	5	6		
	bike/moped	1	2	3	4	5	6		
	bicycle	1	2	3	4	5	6		
	by foot	1	2	3	4	5	6		
	availability leve	l:							
	1 = appropriate	9							
	2 = more or les	s app	oropria	ate					
	3 = neutral								
	4 = more or les	s ina	pprop	riate					
	5 = inappropria	ate							
	6 = impossible								

21.	21. Please rate the following aspects of the shopping trip.						
	A: Relaxation during the trip	1	2	3	4	5	
	B: Reliability of the arrival time	1	2	3	4	5	
	C: Flexibility of choosing departure time	1	2	3	4	5	
	D: Ease of traveling with heavy bags	1	2	3	4	5	
	E: Inexpensiveness of the trip	1	2	3	4	5	
	F: Safety during the trip	1	2	3	4	5	
	G: Travel time	1	2	3	4	5	
	aspect level:						
	1 = not important at all						
	2 = not very important						
	3 = somewhat important						
	4 = very important						
	5 = essential						
22.	Among the above seven aspects, which do yo	ou tak	e into	acco	unt m	ost in	
cho	osing your mode?						
	1) A						
	2) B						
	3) C						
	4) D						
	5) E						
	6) F						
	7) G						
23.	Please respond only if you came by auto. W	hich s	servic	es cou	ıld ma	ke you	
giv	e up using an auto?						

1) free delivery service

2) free ride service between your residential area and shopping mall							
3) charging for parking							
4) pedestrian walkway to the station with roof							
5) expanding shopping cart area availability to the station							
6) expanding shopping cart area availability to bus stops							
7) other ( )							
24. If you chose rail or bus, please give the route. If you chose auto,							
please give the route assuming you take rail or bus.							
Access							
origin 1) your home 2) other							
how? 1) by foot 2) bicycle 3) auto 4) other( )							
the name of station/bus stop you got on ( )							
the name of station/bus stop you transferrd ( )							
the name of station/bus stop you transfered ( )							
Egress							
You got off at KAMATORI station							
1) Yes							
2) No $\Longrightarrow$ Please give the station name you got off ( )							
How did you get to here from the station? ( )							
25. Please respond only if you chose auto.							
How many people were in the car?							
( )							
26. Please respond only if you took auto.							
Did you drive by yourself?							
1) Yes							
2) No							

27.	Please respond only if you took auto.
	Which highway did you take to come here?
	origin 1) your home
	2) other
	$highway ( ) \Longrightarrow highway ( ) \Longrightarrow \cdots$
	$\implies$ shopping mall
28.	Please respond only if you took auto.
	Did you follow the shopping mall's direction signs?
If so	o, please give the location of the sign(s).
29.	Where do you go after your shopping?
	1) your home
	2) visit a facility around here
	3) Chiba city
	4) KEIYO HOME CENTER, Oyumino branch
	5) KEIYO HOME CENTER, Kamatori branch
	6) Tokyo Metropolitan area
	7) other ( )
30.	What kind of mode will you take?
	1) rail
	2) bus
	3) auto
	4) taxi
	5) bike/moped
	6) bicycle
	7) by foot

31. What was your main purpose in coming here?

1) grocery / commodity
2) clothes for kids, underwear
3) stylish clothes
4) furniture, electricity
5) restaurant
6) bowling
7) culture center
8) Fantasy Island
9) other (specify) ( )
10) nothing special
32. How much money did you spend in this mall?
1) less than 1,000 yen
2) 1,000 - 2,000 yen
3) 2,000 - 3,000 yen
4) 3,000 - 4,000 yen
5) 4,000 - 5,000 yen
6) 5,000 - 6,000 yen
7) 6,000 - 7,000 yen
8) 7,000 - 8,000 yen
9) 8,000 - 9,000 yen
10)9,000 - 10,000 yen
11)10,000 - 15,000 yen
12)15,000 - 20,000 yen
13) 20,000 - 25,000 yen
14) 25,000 - 30,000 yen
15) 30,000 yen or more
33. How many hours did you spend in this mall?
( ) hour(s) ( ) min.

- 34. How many times have you come here?
  - 1) This is the first time
  - 2) 2 5 times
  - 3) 6 9 times
  - 4) 10 or more times

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