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Abstract

The fourth order dispersion relation for D(^{3}He) obtained from a straightforward small $k_\perp \rho_D$ expansion does not properly reproduce the exact kinetic dispersion relation near ion-ion hybrid resonance. A method has been developed to obtain a correct approximate fourth order dispersion relation.

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In certain reduced descriptions of ICRF heating it is necessary to expand the dispersion relation in powers of $k_\perp \rho_i$ and then truncate the series.[1,2,3,4] However, for D(^{3}He) plasmas a standard expansion of the dispersion relation in $k_\perp \rho_D$ does not reproduce the properties of the exact dispersion relation. A spurious resonance in the ion-Bernstein wave (IBW) appears between the the position of the fast Alfvén wave (FAW) cutoff and the minority fundamental harmonic, and an associated spurious mode with very large $n_\perp$ appears on the low field side of this resonance. The asymptotic behavior of the IBW on the high field side is incorrect as well. Figure 1 gives a plot of the standard fourth-order in $k_\perp \rho_D$ dispersion relation, $\mathrm{Re}\, n_\perp^2$ vs.x, ($n_\perp = ck_\perp/\omega$), showing the spurious resonance on the low field side. It also shows the IBW root flattening out on the high field side. Figure 2 shows a plot of the exact dispersion relation, i.e. with the Bessel functions not expanded with respect to $k_\perp \rho_D$. We see only the FAW and the IBW. There is no resonance, but instead the curve has a 'dogleg' for negative $\mathrm{Re}\, n_\perp^2$. At the 'dogleg', $k_\perp \rho_D$ is much greater than 1, so that the first order standard expansion breaks down. The IBW on the high field side also does not flatten out. In the standard expansion the dispersion relation is given by:

$$a n_\perp^4 + b n_\perp^2 + c = 0, \tag{1}$$

where

$$a \simeq -K_{11}^p,$$

$$b \simeq -(K_\perp^0 - n_\parallel^2),$$

$$c \simeq (K_\perp^0 - n_\parallel^2)^2 - K_{12}^{02},$$

and

$$K_{11}^p \simeq \frac{54}{35}\beta - \frac{\eta\beta}{4\alpha_{He}} Z(\frac{x}{\alpha_{He}}),$$

$$K_\perp^0 \simeq -\frac{9}{7}\frac{c^2}{c_A^2} + \frac{3\eta}{4\alpha_{He}} Z(\frac{x}{\alpha_{He}})\frac{c^2}{c_A^2},$$

$$K_{12}^0 \simeq -\frac{12}{7}\frac{c^2}{c_A^2} + \frac{3\eta}{4\alpha_{He}} Z(\frac{x}{\alpha_{He}})\frac{c^2}{c_A^2},$$

$\alpha_{He} = N_\parallel \sqrt{\beta_{He}}$. Generally $a << b, c$ so the roots of (1) are

$$n_\perp^2 \simeq -b/a, -c/b. \tag{2}$$

The first root corresponds to the IBW and the second to the FAW. Consider the IBW root:

$$n_\perp^2 \simeq \frac{(K_\perp^0 - n_\parallel^2)}{-K_{11}^p}, \tag{3}$$

or, with $N_{\perp,\parallel} = n_{\perp,\parallel} c_A/c$,

$$N_\perp^2 \simeq \frac{\frac{9}{7} + N_\parallel^2 - \frac{3\eta}{4\alpha_{He}} Z(\frac{x}{\alpha_{He}})}{\frac{54}{35}\beta_D - \frac{\beta_D \eta}{4\alpha_{He}} Z(\frac{x}{\alpha_{He}})}. \tag{4}$$

We investigate the above expression near the mode conversion position given by the ion-ion resonance from cold theory, $x_{mc} \simeq -7/12\eta$. At $x = x_{mc}$

$$\left|\frac{x}{\alpha_{He}}\right| = \frac{7\eta}{N_\parallel \sqrt{\beta_{He}}}. \tag{5}$$

For tokamak plasmas such as Alcator C-mod this is typically much greater than unity. Hence we expand the Z functions in the asymptotic regime yielding

$$N_\perp^2 \simeq \frac{\frac{9}{7} + N_\parallel^2 + \frac{3\eta}{4x}}{\frac{54}{35}\beta_D + \frac{\eta\beta_D}{4x}}. \tag{6}$$

We see that there is a new resonance, but it is purely an artifact of the standard expansion. In fact the resonance exists for all orders of the Taylor expansion of the Bessel functions because $k_\perp \rho_D$ becomes greater than unity very quickly away from the coupling region. To correct for this we expand (6) to first order in x around the mode conversion point x_{mc}, giving

$$N_\perp^2 \simeq \frac{\frac{9}{7} + N_\parallel^2 + \frac{3\eta}{4x}}{-\frac{13}{20}\frac{\eta\beta_D}{x}}. \tag{7}$$

Now we can restore the Z function via $-\frac{1}{x} \Rightarrow \frac{1}{\alpha_{He}} Z(\frac{x}{\alpha_{He}})$, to get a new form for K_{11}^p:

$$K_{11}^p = -\frac{13}{20}\frac{\beta_D \eta}{\alpha_{He}} Z(\frac{x}{\alpha_{He}}).$$

The exact dispersion relation is modeled very well with this new element in (1) as seen in figure 3. The IBW now has the correct high field behavior and the spurious resonance is gone. The Z function restored in the denominator has the correct real part; it's imaginary part is very small near $x = x_{mc}$. Near $x = 0$ one finds numerically (from the exact and approximate dispersion relations) that $|\mathrm{Im}\, n_\perp| \gg |\mathrm{Re}\, n_\perp|$. Hence the discrepency between the exact (fig.3) and the approximate (fig.4) values of $\mathrm{Re}\, n_\perp^2$ near $x = 0$ is unimportant. The imaginary part of $K_\perp^0$ is larger than that of K_{11}^p by a factor of $1/\beta_D$ so the Z function restoration does not affect the conditon $a \ll b$ in (2), and thus will not alter the FAW. This corrected fourth order equation will reproduce the actual dispersion relation and not significantly alter the damping. These new dispersion elements were used to calculate the scattering coefficients for $D(^3He)$ heating in Alcator C-mod using the Fuchs-Bers formalism.[5]

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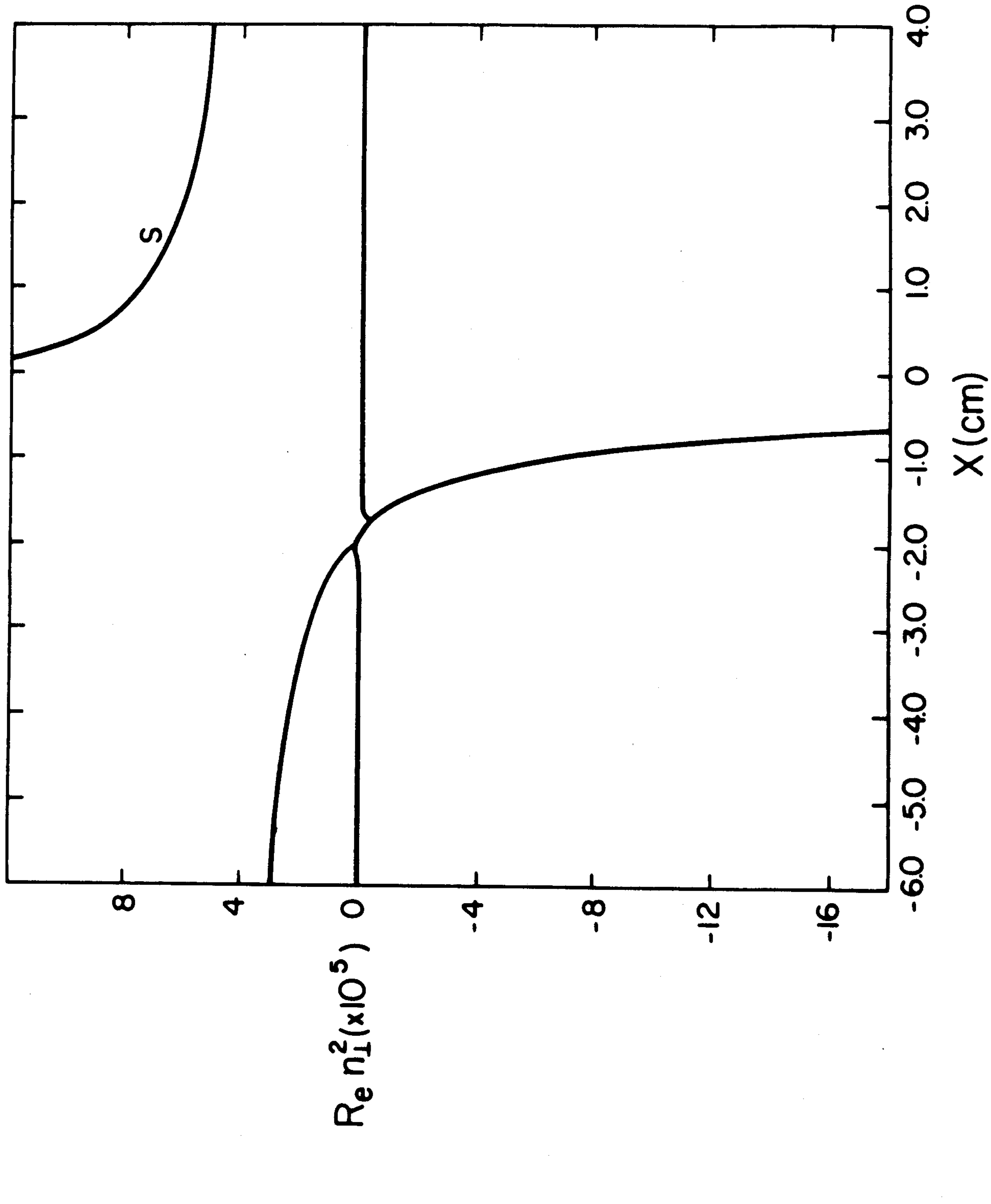
FIGURES

1. The standard fourth order in $k_\perp \rho_D$ dispersion relation , Re $n_\perp^2$ vs. x for Alcator C-mod parameters: $B_0 = 9$T, $n_e = 5 \times 10^{14}$cm^3, $T = 2$KeV, $R_0 = 64$cm, $\eta = 0.05$. The spurious mode is labeled by s. The position $x = 0$ is where $\omega = \omega_{3He}$.(Note: The FAW branch has $n_\perp^2 \simeq 2.3 \times 10^3$; on the chosen veritical scale it appears very close to zero.
2. The exact dispersion relation Re $n_\perp^2$ vs. x for the same parameters as Fig. 1.
3. The corrected fourth order (in $k_\perp \rho_D$) dispersion relation, Re $n_\perp^2$ vs. x for the same parameters as Fig. 1.



$R_e\, n_\perp^2 (\times 10^5)$
8
4
0
-4
-8
-12
-16
S
-6.0
-5.0
-4.0
-3.0
-2.0
-1.0
0
1.0
2.0
3.0
4.0
X (cm)

FIGURE 1

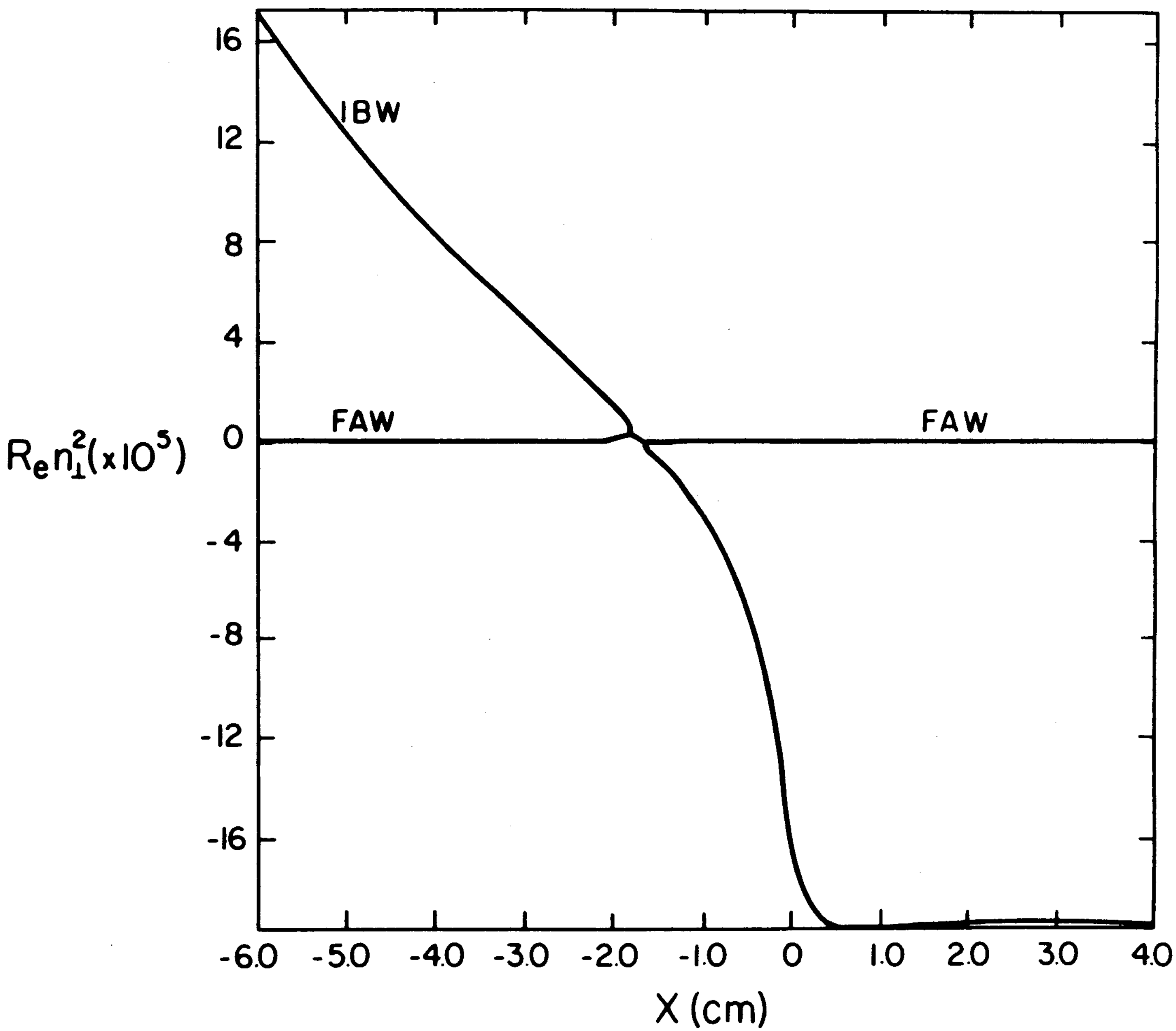

IBW
FAW
FAW
16
12
8
4
0
-4
-8
-12
-16
$R_e n_\perp^2 (\times 10^5)$
-6.0
-5.0
-4.0
-3.0
-2.0
-1.0
0
1.0
2.0
3.0
4.0
X (cm)

FIGURE 2

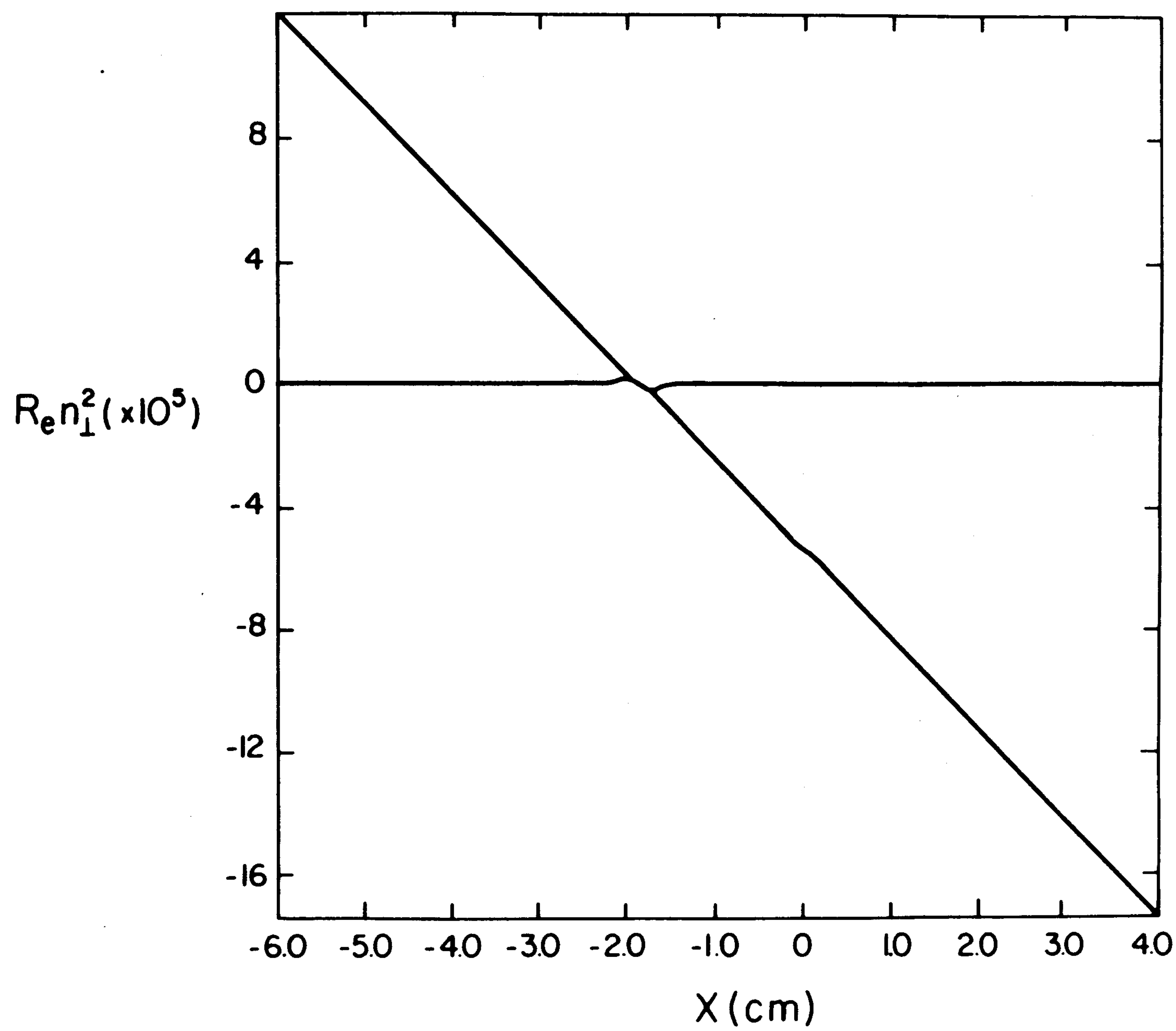


FIGURE 3