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**Complete Macroscopic Stabilization
of Tokamak Plasmas**

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Abstract

A numerical investigation of ideal magnetohydrodynamic stability in finite aspect ratio tokamaks with zero edge current density is carried out. It is shown that, assuming a conventionally D-shaped cross section and a safety factor on axis greater than 2.1, both $n = \infty$ and $n = 1$ modes are stable for any beta, even in the absence of any surrounding conductors.

One of the fundamental goals of the tokamak research program is to maximize the ratio β of plasma kinetic pressure to magnetic pressure, that can be limited by the onset of magnetohydrodynamic (MHD) instabilities. A significant advance in this regard was the discovery of the second stability region for high toroidal wavenumber (n) internal modes [1-6]. It was subsequently shown that it is possible to devise means of connecting the high- β second stability region to the conventional low- β first stability regime [7-11]. One such way of access to second stability [7-10] requires a high safety factor, with its magnetic axis value q_0 typically greater than 2. This necessitates an external control of the current density profile in order to prevent its ohmic peaking, a possibility that has already been demonstrated experimentally by means of noninductive current drive [12,13]. If q_0 can indeed be kept above 2, then a continuous trajectory, completely stable against ideal MHD internal modes into the second stability regime has been theoretically predicted in a conventionally shaped, finite aspect ratio tokamak [14].

The most serious obstacle to second stability scenarios is the low- n external mode instability. Low- n modes normally are found to become unstable [15-17] when the ratio

$$\beta_T \equiv 40\pi\beta a B_v / I_t \quad (1)$$

exceeds a critical value of the order of 3, and do not show the second stability phenomenon. In our definition of β_T , a is the plasma minor radius, B_v is the vacuum magnetic field at the plasma center R_c , I_t is the total toroidal current, and a system of units in which aB/I is dimensionless and equal to $(aB/\mu_0 I)_{MKS}$ is adopted. A conducting wall around the plasma only slows the growth of these instabilities down to the characteristic resistive diffusion time, and may not be a satisfactory answer to the external instability problem. It should be noted however, that the majority of theoretical studies and virtually all the experiments carried out so far have q_0 near unity. Also most theoretical investigations (Ref. 14 included) of MHD beta limits have used flux conserving tokamak sequences. For these the current density does not vanish at the plasma boundary and, as beta increases, an increasingly large edge current develops while the magnetic shear remains constant, which results in a strong drive for the low- n external instability. On the other hand the

pioneering Versator II lower-hybrid current drive experiment [13] at high β_p and high q_0 showed no evidence of low- n external instability.

In this work it is shown that, assuming smooth and monotonic pressure and current density profiles that vanish at the plasma edge and $q_0 \geq 2.1$ in a moderately D-shaped tokamak of conventional aspect ratio ($\epsilon = a/R_c = 1/3$), the $n = 1$ external instability can be suppressed even in the absence of any conducting wall. A continuous path into the second stability regime that stays thoroughly stable against all ideal MHD modes is shown to exist. This result is consistent with a recent theoretical analysis of MHD stability at high q [18]. Thus the same means (current profile control at high q_0) that allow a stable access to the second stability region for high- n internal modes can also stabilize the low- n external modes. This opens the possibility of seriously contemplating high beta tokamak scenarios that are completely free of macroscopic instabilities.

The theoretical results of Ref. 18 are based upon the invariance of the ideal MHD equilibrium and stability equations under the scaling $\beta_p \rightarrow \beta_p$, $\beta \rightarrow \lambda^2 \beta$, $q \rightarrow q/\lambda$, in the limit of large q . They can be summarized as follows. First, as $q \rightarrow \infty$ and up to corrections of order $1/q$, all n -modes have the same stability properties. Second, if no access to second stability for $n = \infty$ modes is possible even at large q , then the first stability beta limit must be expressed in terms of invariant parameters under the above scaling and the Troyon formula must be generalized to

$$\beta_T \leq C_R(q_0)/q_0, \quad (2)$$

where $C_R(q_0)$ becomes independent of q_0 and n as $q_0 \rightarrow \infty$. If, on the other hand, a stable access to second stability for $n = \infty$ modes exists at sufficiently high q_0 , then all n -modes must also become stable or at worst have growth rates that decay like $1/q_0$ as $q_0 \rightarrow \infty$, for any β . By restricting ourselves to smooth pressure and current density profiles that vanish at the plasma edge, the external kink instability drive is minimized at high q and the low- n external modes become more internal-like. Thus a complete stabilization of low- n modes can be expected provided the $n = \infty$ ballooning modes do exhibit a stable access to second stability.

After pressure and current profile optimization subject to our smoothness and zero

boundary value constraints, whether or not a stable access to second stability exists depends on the geometry of the plasma boundary. Generally speaking, stable access is possible if the geometry produces a sufficiently large favorable average magnetic curvature. In Ref. 18 one example was shown where this is not the case and the second stability region is not accessible, namely a large aspect ratio circular cross section. The $\beta_T \leq C_R/q_0$ form of the first stability limit was clearly demonstrated in that instance.

In this work we present a set of detailed numerical calculations at finite aspect ratio. We consider two different tokamak cross sectional shapes, both with an aspect ratio $A = 3$. The first one is a rather elongated ellipse without triangularity ($\kappa = 2.5$, $\delta = 0$) for which no access to second stability is possible due to its poor average magnetic curvature. The second one is a moderately shaped D ($\kappa = 1.6$, $\delta = 0.4$), similar to the one considered in Ref. 14 which allows stable access to second stability against $n = \infty$ modes for q_0 roughly above 2. The two flux functions used to specify the tokamak equilibria are the pressure $p(\psi)$ and the ohmic current $j_{OH}(\psi) = \langle \mathbf{j} \cdot \mathbf{B} \rangle / \langle R_0 \nabla \phi \cdot \mathbf{B} \rangle$ where $\langle \dots \rangle$ stands for the conventional flux surface average and ϕ is the toroidal angle. For both geometries we consider the same profiles:

$$p = p_0(1 - \hat{\psi})^{3/2}, \quad (3)$$

$$j_{OH} = j_0(1 - \hat{\psi}^\alpha), \quad (4)$$

where $\hat{\psi}$ is the normalized poloidal flux ($0 \leq \hat{\psi} \leq 1$). We vary p_0 and j_0 to span a $\epsilon\beta_p$ versus q_0 parameter space. While so doing, α is adjusted in such a way that the ratio of the magnetic axis MHD safety factor q_0 to the cylindrical q_* is kept constant. Here we adopt the definition

$$q_* \equiv \frac{I_{pv}}{I_t} \left(\oint \frac{d\ell_p}{2\pi R} \right)^2, \quad (5)$$

where $I_{pv} = 2\pi R_c B_v$ is the vacuum poloidal current, I_t is the total plasma toroidal current and the line integral extends over the plasma boundary circumference. Therefore, by virtue of Schwartz's inequality, q_* is less than or equal to the edge MHD safety factor q_a , the equality holding if $|\nabla\psi|$ were uniform around the plasma boundary. In our present study we have set $q_*/q_0 = 2.72$. Since j_0 scales uniformly the current density profile, only a relatively small variation in α is needed to compensate for beta effects. Through our

study, with $\epsilon\beta_p$ ranging from 0 to 2 and q_0 ranging from 1 to 4, α is found to vary between 1.1 and 1.7. If we define the beta and poloidal beta parameters as

$$\beta \equiv 2p_{av}/B_v^2, \quad (6)$$

$$\beta_p \equiv 8\pi p_{av}S/I_t^2, \quad (7)$$

where p_{av} is the volume averaged pressure and S is the area of the plasma cross section, then we have

$$\epsilon\beta_p = g(\Gamma)q_*\beta_T \quad (8)$$

where $g(\Gamma)$ depends only on the geometry of the plasma boundary. Therefore, for constant q_*/q_0 , $\epsilon\beta_p$ is directly proportional to $q_0\beta_T$, hence $\epsilon\beta_p$ and $q_0\beta_T$ scales are equivalent. A first stability region limit of the form $q_0\beta_T \leq C_R$ reflects a linear relationship between $\epsilon\beta_p$ which is a global measure of the pressure gradient, and q_*/q_0 which is a global measure of the magnetic shear, a result that can be expected to follow from local ballooning theory [19,20].

The above described tokamak equilibria have been tested for stability against $n = \infty$ ballooning modes and $n = 1$ external modes without any conducting wall. This numerical study was performed using the PEST package of equilibrium and stability codes [21]. Equilibrium solutions and their ballooning stability were calculated on a grid of 64 flux surfaces and 128 poloidal grid points around a flux surface, with constant poloidal arc length spacing. This grid was refined by interpolation to 200 flux surfaces and 256 poloidal grid points still equally spaced in arc length, before carrying out the $n = 1$ stability test. The convergence of stability results with respect to the poloidal mode cut-off in the $n = 1$ trial function was verified.

Figures 1 and 2 display the marginal stability curves. The $\kappa = 2.5$ elliptical shape does not allow a stable access to second stability and the curves shown in Fig. 1 represent the first stability region beta limit. For $q_0 < 1.8$ the stability limit against $n = \infty$ modes is determined by the violation of the Mercier criterion in the center of the plasma, which causes a sharp drop of the critical beta. This Mercier instability could be avoided with a flatter central pressure profile. What is of interest to us though is the behavior of the

marginal stability curves at large q_0 . There we observe how the critical $q_0\beta_T$ for $n = 1$ and $n = \infty$ approach each other and tend to a constant as $q_0 \rightarrow \infty$, in agreement with our theoretical expectations [18].

The picture changes dramatically when we consider the $\kappa = 1.6$, $\delta = 0.4$ D-shape. In this case the $n = \infty$ mode experiences a stable access to second stability for $q_0 \geq 2.1$ and the $n = 1$ mode remarkably follows suit. This behavior can be explained by our theoretical finding [18] that at sufficiently large q_0 all n -modes must approach equivalent stability properties. Any residual external kink features (necessarily of higher order in $1/q_0$) that might make the $n = 1$ mode different from the $n = \infty$ mode are suppressed by our choice of a zero edge current density profile with smooth gradient, together with our prescription of increasing beta at constant q_*/q_0 . This means that, for a fixed q_0 , β_p is increased while keeping the plasma current constant, which results in an increasing MHD safety factor q_a and an increasing edge magnetic shear that contribute to suppress the kink instability drive. In fact it is interesting to note that, on the high beta side of the stability diagram, the $n = 1$ mode is more stable than the $n = \infty$ mode. This is because, in this region of parameter space, the $n = \infty$ stability criterion is first violated in the vicinity of the plasma edge where the magnetic surfaces tend to lie in the first stability side of the local $s - \alpha$ diagram [22], since the edge pressure gradient must vanish. The fact that the edge magnetic shear increases with β_p tends to push those magnetic surfaces further into the local first stability region and makes their local access to second stability difficult. This causes some degradation of the overall $n = \infty$ stability (compared for instance to a flux conserving scenario where the magnetic shear is frozen and the second stability region is more readily reached), but is very beneficial for the $n = 1$ mode. Figure 2 indicates that there is room for some optimized scenario where, by increasing somewhat the plasma current as beta rises, the $n = 1$ and $n = \infty$ marginal curves are brought closer together to obtain a larger global second stability domain. In any case, for the profiles studied in this work, the domain $q_0 \geq 2.1$ is completely stable against $n = 1$ and $n = \infty$ modes. Given that our equilibria have finite magnetic shear and favorable average magnetic curvature throughout, we can assure that all intermediate n -modes are also stable. In fact we have tested the sequence with constant $q_0 = 2.25$ against the $n = 2$ mode and found

staying to the right of the marginal stability curves of Fig.2, completely stable trajectories can be envisioned from zero-beta up to the equilibrium beta limit. The equilibrium limit has been observed when $\epsilon\beta_p$ approaches 2, its signature being a rapid increase of the edge safety factor q_a and increasing difficulty for the equilibrium solver to reach numerical convergence. The highest beta stable equilibrium obtained at $\epsilon\beta_p = 1.97$ and $q_0 = 1.75$, has $\beta = 8.75\%$, $\beta_T = 11.50$ and the more relevant figure of merit from a theoretical standpoint $q_0\beta_T = 20.13$. The β figure is comparable to the maximum values obtained in the conventional first stability regime at low q (high current) [23], but higher values can be expected after some profile optimization. The β_T and $q_0\beta_T$ are much higher than any encountered in the first stability region. This regime of high β , very high β_p , relatively low current and complete macroscopic stability may offer a new and exciting way of tokamak operation if the required current profile control and auxiliary heating prove feasible. The fact that non-inductive current drive is an essential condition, together with the large amount of bootstrap currents expected at the characteristically high poloidal beta values, make such regime very compatible with steady state tokamak scenarios thus adding to its appeal.

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Figure Captions

Figure 1. Stability diagram for $n = \infty$ ballooning modes and $n = 1$ external modes without conducting wall, in a finite aspect ratio tokamak with elliptical cross section and zero edge current density.

Figure 2. Stability diagram for $n = \infty$ ballooning modes and $n = 1$ external modes without conducting wall, in a finite aspect ratio tokamak with D-shaped cross section and zero edge current density.

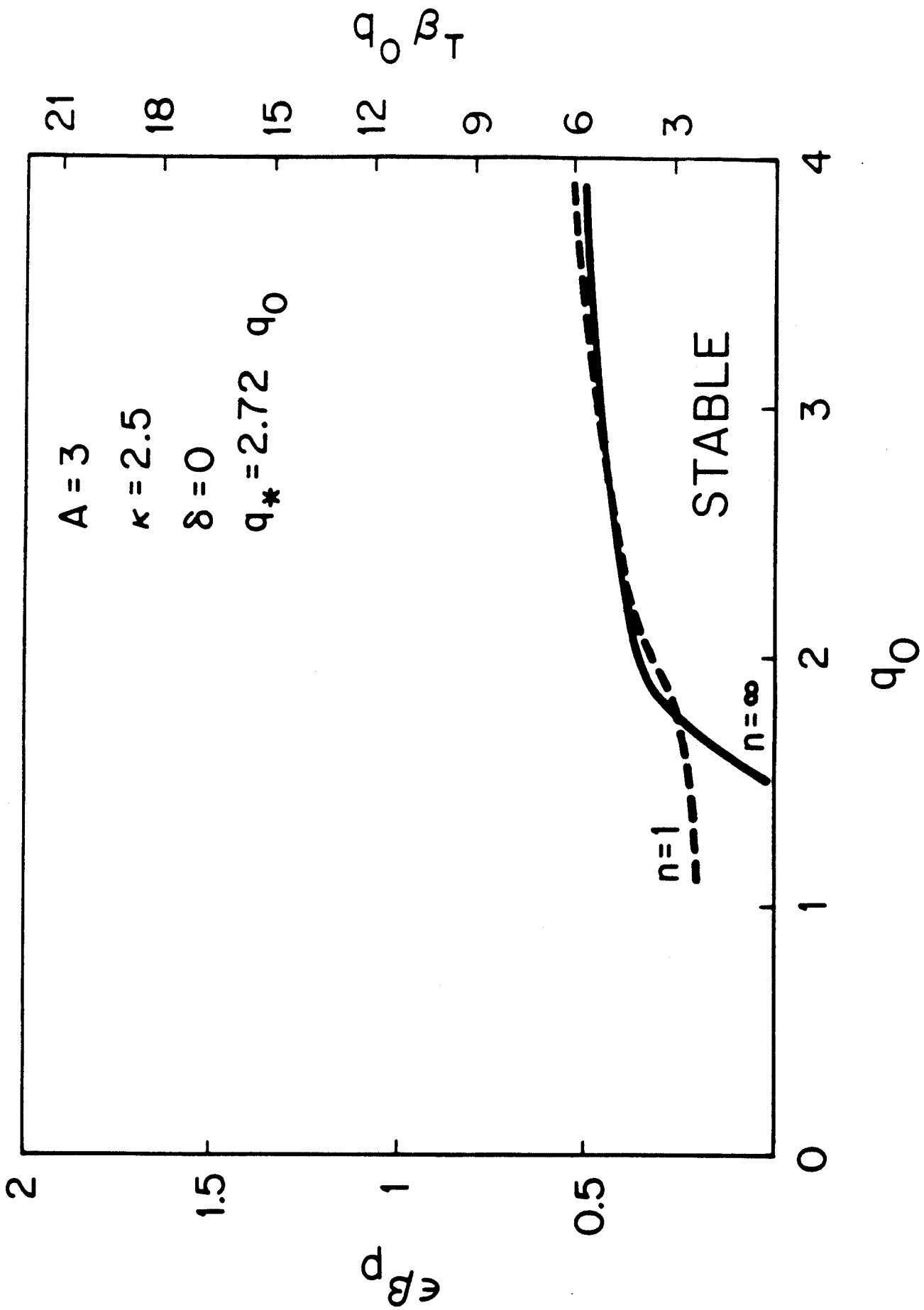


Figure 1

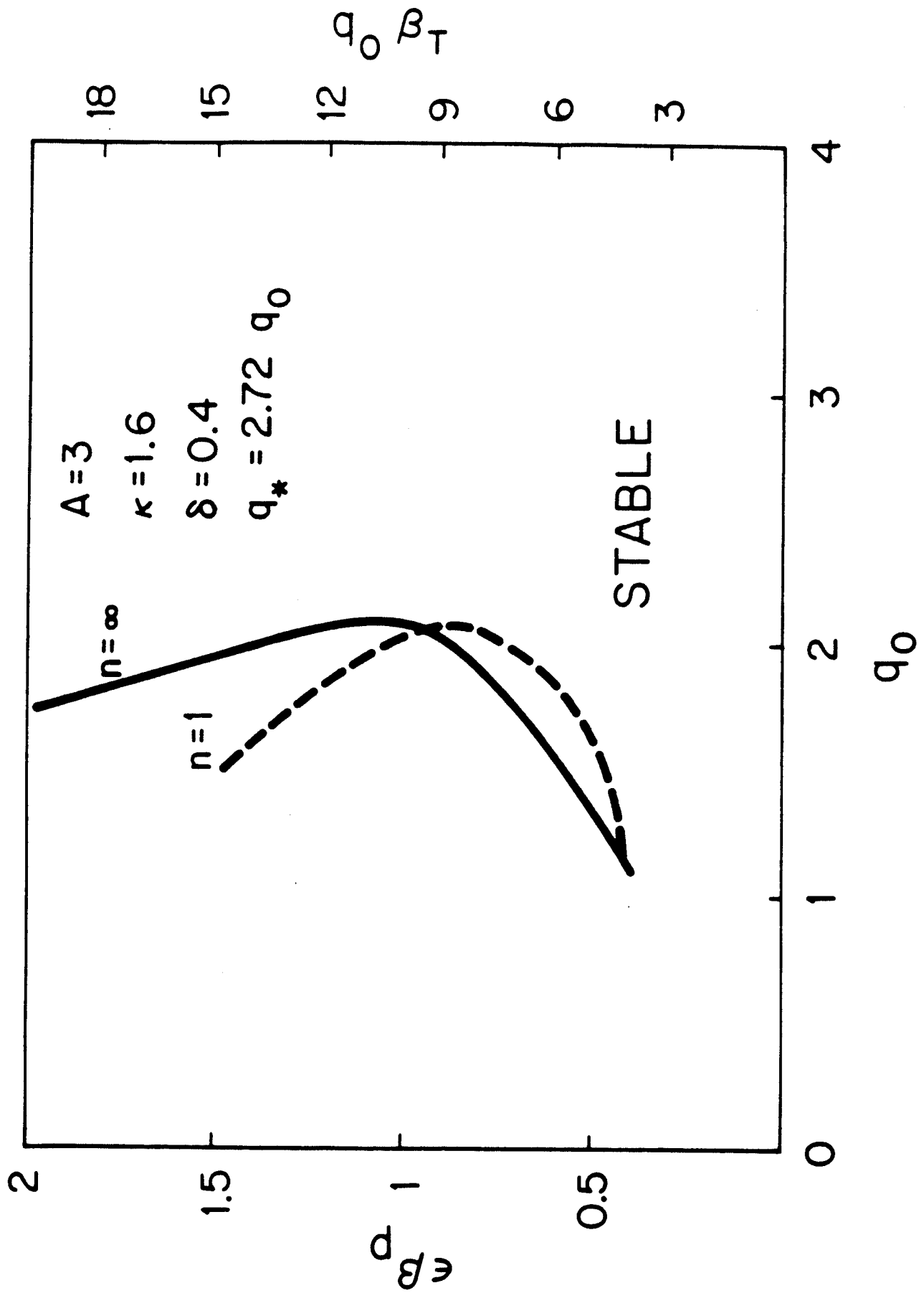


Figure 2