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by Collective Thomson Scattering**

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Localized Magnetic Field Pitch Angle Measurements by Collective Thomson Scattering

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Abstract

A novel new method is proposed to measure the internal magnetic field pitch angle in tokamaks by measuring the frequency of the lower hybrid resonance in the collective Thomson scattered spectrum. The resonance frequency has a dependence of 1.2 and 2.0 GHz/degree pitch angle for ITER and C-MOD like plasma parameters, respectively, using a submillimeter-wave diagnostic source. Sensitivity to other plasma parameters such as electron density, temperature, energetic ion fraction, and magnetic field is shown to be weak.

Introduction

The spatially resolved measurement of the internal magnetic field pitch angle in a tokamak can be used to determine the current density and safety factor profiles, important for plasma equilibrium and stability studies. Because of the importance of these plasma parameters pitch angle diagnostics have been the focus of much development activity. Most notably motional Stark effect¹ and lithium pellet² techniques have been recently demonstrated. In this paper we propose a new technique based on collective Thomson scattering which can potentially be more sensitive with better plasma penetration capabilities.

There are a number of different ways that Thomson scattering can be applied to the measurement of local magnetic field pitch angle. All involve determining the orientation of the scattering wave vector $\mathbf{k} = \mathbf{k}_0 - \mathbf{k}_s$ relative to the total magnetic field $\mathbf{B} = \mathbf{B}_t + \mathbf{B}_p$, where \mathbf{k}_0 and \mathbf{k}_s are the incident and scattered wave vectors and \mathbf{B}_t and \mathbf{B}_p are the toroidal and poloidal magnetic field components. In the method with the longest history^{3,4} the objective is to find the angular orientation of \mathbf{k} where the electron cyclotron modulation of the scattered electron feature is a maximum. This technique suffers from poor photon statistics and requires imaging detection. Next it was shown that in collective Thomson scattering one could use the spectral narrowing of the scattered ion feature to determine the direction of \mathbf{B} ⁵. This method can have much better signal to noise performance and does not require imaging detection, but the

spectral fit for pitch angle is not always unambiguous, requiring an independent measurement for T_i . Here we will show that it is potentially possible to measure the angular orientation of \mathbf{k} relative to \mathbf{B} with much greater sensitivity than previously possible and without imaging or dependence on T_i by detecting the frequency of the lower hybrid resonance in the collective Thomson scattered spectrum.

Analytical Basis

There has been considerable interest recently in improving the modeling of collective Thomson scattered signals. This has been motivated by millimeter-wave gyrotron scattering systems being implemented on JET⁶ and TFTR⁷ for energetic ion and alpha-particle diagnostics. Vahala, Vahala, and Sigmar⁸ were first to point out the presence of the lower hybrid resonance in the energetic ion feature of the collective Thomson scattered spectrum. However, the longitudinal electrostatic plasma model they used was too much of an approximation to correctly predict the lower hybrid resonance frequency. Aamodt⁹ was first to develop a fully electromagnetic model which included transverse screening terms. He also pointed out the importance of fluctuations in terms other than the electron density, such as field fluctuations, that under certain conditions could contribute significantly to the scattered signal. By choosing a sufficiently high diagnostic frequency, $\omega_0 \gg \omega_{ce}, \omega_{pe}$, as we will be doing here, the electron density fluctuations should be the only important term in the scattered spectrum. Also the geometrical form factor, a multiplying term in the scattered spectrum would be a constant equal to one¹⁰. Using Chiu's formalism for the fully electromagnetic spectral density function^{11,12}, the shape of the scattered spectrum, where $\omega = \omega_0 - \omega_S$, is given by

$$S(\mathbf{k}, \omega) = S_e(\mathbf{k}, \omega) + \sum_i S_i(\mathbf{k}, \omega) \quad (1)$$

where

$$S_e(\mathbf{k}, \omega) = \left(\frac{\mathbf{k}}{\omega} + \frac{\mathbf{k}}{\omega} \cdot \bar{\chi}_e \cdot \bar{\mathbf{K}}^{-1} \right) \left(\frac{\mathbf{k}}{\omega} + \frac{\mathbf{k}}{\omega} \cdot \bar{\chi}_e \cdot \bar{\mathbf{K}}^{-1} \right)^* : \left(\frac{\bar{\chi}_e - \bar{\chi}_e^\dagger}{2i} \right) \frac{2T_e \omega}{m_e \omega_{pe}^2}$$

$$S_i(\mathbf{k}, \omega) = \left(\frac{\mathbf{k}}{\omega} \cdot \bar{\chi}_e \cdot \bar{\mathbf{K}}^{-1} \right) \left(\frac{\mathbf{k}}{\omega} \cdot \bar{\chi}_e \cdot \bar{\mathbf{K}}^{-1} \right)^* : \left(\frac{\bar{\chi}_i - \bar{\chi}_i^\dagger}{2i} \right) \frac{\langle v_i^2 \rangle \omega Z_i n_i}{\omega_{pe}^2 n_e}$$

$$\bar{\mathbf{K}} = (N^2 \bar{\mathbf{I}} - \mathbf{N}\mathbf{N}) - \bar{\boldsymbol{\epsilon}} \quad \text{dispersion tensor}$$

$$\bar{\boldsymbol{\epsilon}} = 1 + \sum_{\beta} \bar{\chi}_{\beta} \quad \text{dielectric tensor}$$

and $N = ck/\omega$ is the refractive index. The * signifies taking the complex conjugate and the † signifies taking the transpose and complex conjugate.

Calculated spectra using Equation 1 with magnetized electrons and unmagnetized ions are shown in Figure 1 for plasma parameters representative of a 5 Tesla D-T burning tokamak such as ITER. A submillimeter-wave diagnostic wavelength of $152 \mu\text{m}$ and scattering angle of 4° are also assumed. The only parameter varied in the series of solid curves is the \mathbf{k} to \mathbf{B} angle, ϕ , which ranges from 89.9° to 84° . The resonance peak in these spectra varies almost linearly at the rate of 1.2 GHz/degree for \mathbf{k} to \mathbf{B} angles greater than 89° . Similar calculations for plasma parameters relevant to C-MOD result in a rate of 2.0 GHz/degree. Therefore from a measurement of the frequency of this resonance peak the direction of the total magnetic field relative to the diagnostic \mathbf{k} vector can be determined with high sensitivity because frequency measurements can be made with high accuracy.

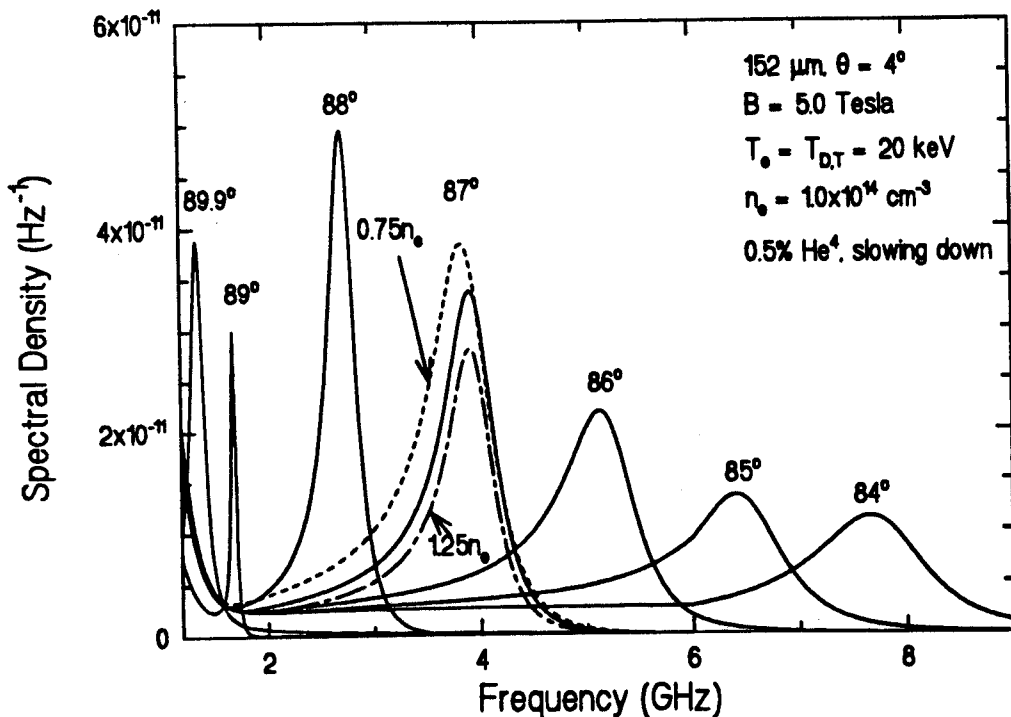


Figure 1. Calculated collective Thomson scattered spectra for ITER like tokamak plasma parameters. The lower hybrid resonance peak is a sensitive function of \mathbf{k} to \mathbf{B} angle which is varied from 89.9° to 84° .

It should also be noted that the presence of energetic ions in the plasma is not necessary for this diagnostic. The 0.5% alpha-particle fraction assumed for the calculations in Figure 1 has a slowing down velocity distribution that is terminated at the 3.5 MeV alpha birth energy. There are no alpha-particles with energies greater than this. This birth energy corresponds to 6 GHz on the frequency scale in Figure 1. Consequently at the peaks at $\phi=85^\circ$ and 84° there are no energetic particles. Subsequent calculations without alpha-particles confirmed that the resonance peaks are still present with only somewhat reduced levels.

The spectral density calculations can be compared to cold plasma dispersion calculations for the lower hybrid resonance. This comparison is shown in Figure 2. The electrostatic cold plasma dispersion relation is obtained when the assumption $\tan^2\phi = -P/S$ is made, where P and S have the usual definition given by Stix^{12,13}

$$\omega^2 = \frac{1}{1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}} \left[\sum_j \omega_{pj}^2 + \frac{k_{||}^2}{k_{\perp}^2} \omega_{pe}^2 \right] \quad (2)$$

where $k_{||}$ and k_{\perp} are the components of \mathbf{k} parallel and perpendicular to the magnetic field. This equation is shown by the upper plot in Figure 2. Relaxing the electrostatic approximation produces the lower curve in Figure 2. The open circles correspond to the spectral density calculations shown in Figure 1. The spectral density calculations are valid for a hot plasma and fall between the two cold plasma approximations for plasma parameters illustrative of ITER. These cold plasma dispersion calculations for the lower hybrid resonance with electrostatic and electromagnetic approximations have also been carried out by Hughes¹⁴.

From a measurement of the ϕ angle the pitch angle $\gamma = \tan^{-1}(B_p/B_t)$ is determined as illustrated in Figure 3. The orientation of \mathbf{k} to the vacuum toroidal field is known from the installed diagnostic scattering geometry. The measured ϕ with plasma is then subtracted from the vacuum ϕ_0 to obtain γ . Only the component of \mathbf{B}_p in the \mathbf{k} \mathbf{B} plane would contribute to the measured γ .

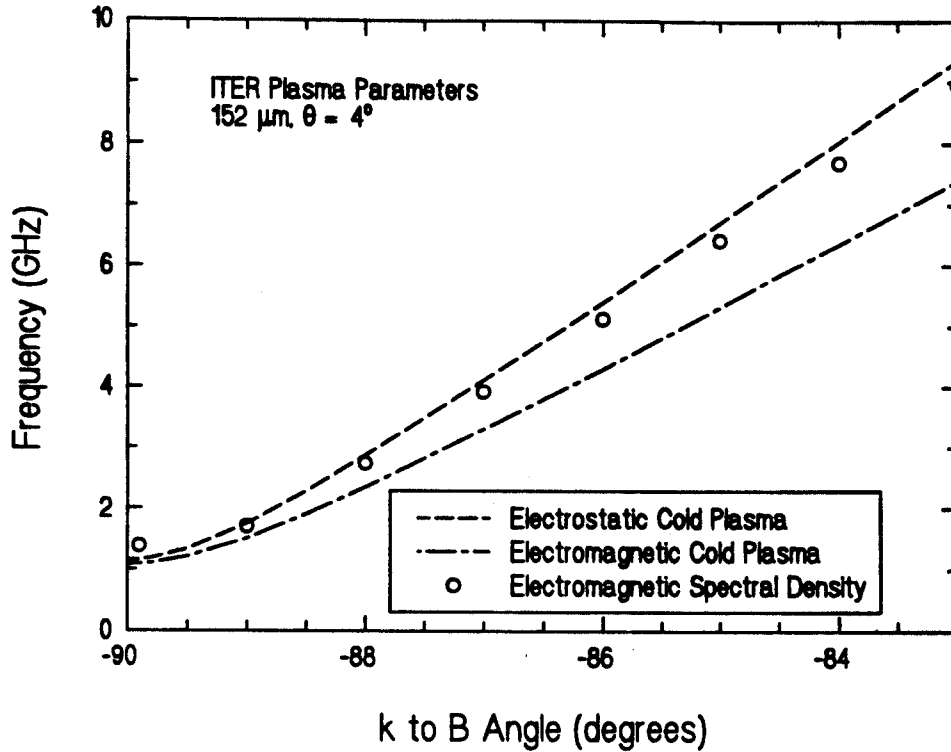


Figure 2. The lower hybrid resonance frequency is plotted as a function of k to B angle for three different theories. The open circles are from the spectral density function calculations which is considered to be an accurate model for a thermal tokamak plasma.

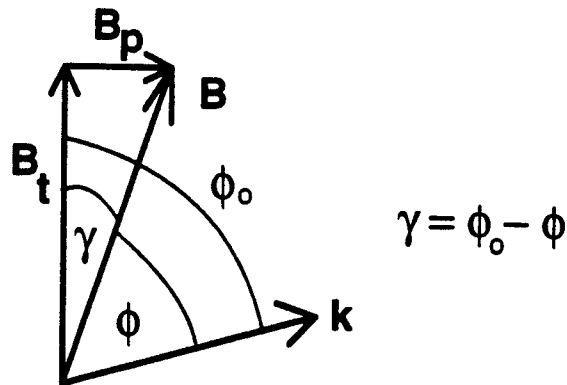


Figure 3. Relationship between the pitch angle, γ , and the measured k to B angle, ϕ .

Diagnostic Configuration

The diagnostic scattering geometry must be oriented so that the fluctuation k vector is near parallel to the poloidal magnetic field component to maximize sensitivity to the pitch angle. This necessitates a vertical scattering geometry as illustrated in

Figure 4. Consequently, this diagnostic would measure the pitch angle profile along a vertical chord which is orthogonal to particle probe diagnostics^{1,2}. The scattering angle, θ , will be relatively small in order to insure that the condition for collective Thomson scattering is met with a submillimeter-wave source. A submillimeter-wave source is preferable to a millimeter-wave source because beam refraction can effect the measurement by changing the vacuum orientation of the scattering geometry, which must be known very accurately. A submillimeter-wave source is also preferable for minimizing possible contributions to the scattered spectrum from non density fluctuation terms as described above.

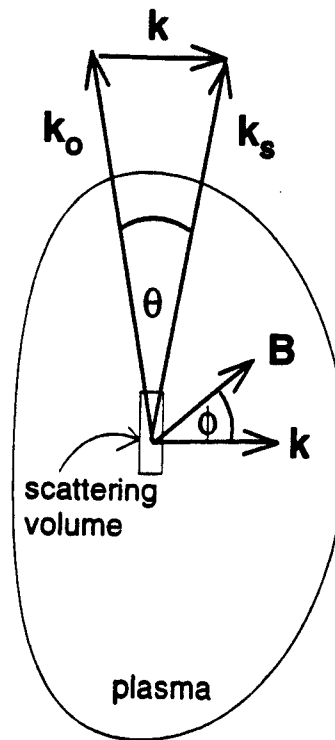


Figure 4. Diagnostic orientation must be oriented vertically for optimum pitch angle measurements.

A high power ammonia laser at $152 \mu\text{m}$ can be developed with sufficient peak and average power for this diagnostic measurement¹⁵. The scattering angle would be about 4° for ITER like plasma parameters and about 10° for a more dense, lower temperature tokamak such as C-MOD. Diagnostic port access at exactly the required angles would not be necessary because small overmoded waveguides of a suitable radiation hardened material could be used to transmit the HE_{11} mode, which launches as a Gaussian beam. A number of views could be provided for instantaneous profile measurements. Assuming diffraction limited propagation and effective launch f-

numbers of 10, a conservative estimate for spatial resolution along the laser beam would be about 8 cm for the ITER case and 3 cm for the C-MOD case. The full angle beam divergence would be approximately 0.02° . This beam divergence due to the finite beam width will slightly broaden the resonance peaks as shown in Figure 1, but would not effect their frequency position.

Measurement Uncertainties

The lower hybrid resonance frequency in the collective Thomson spectra is most sensitive to the k to B angle. However, other diagnostic parameters also effect this resonance. In order to develop a feel for the uncertainties in this proposed diagnostic we systematically varied the other major diagnostic parameters to determine their influence on the resonance. Table I lists the base ITER and C-MOD parameters assumed for this study. A 0.5% fraction of alpha-particles with a slowing down velocity

TABLE I. ITER and C-MOD plasma parameters used in the pitch angle diagnostic modeling calculations.

Plasma Parameter	ITER	C-MOD
n_e	$1.0 \times 10^{14} \text{ cm}^{-3}$	$2.0 \times 10^{14} \text{ cm}^{-3}$
T_e	20 keV	4 keV
T_i	20 keV	4 keV
n_{He}/n_i	0.5% He ⁴ , slowing down	0.5% He ³ , 300 keV
B	5.0 Tesla	7.9 Tesla
θ	4°	10°
λ	152 μm	152 μm

distribution was assumed for the ITER case and a 0.5% fraction of a He³ component heated to 300 keV Maxwellian velocity distribution was assumed for the C-MOD case. As pointed out earlier energetic ions are not necessary for this diagnostic, but included here for completeness. The bulk ion composition is a 50-50 mix of D-T for ITER and all D for C-MOD.

Each of the parameters of Table I, except for the fixed diagnostic wavelength, was individually changed by +/- 25% and the spectral density function was recalculated.

The corresponding change in the observed pitch angle if the plasma parameter is not taken into account is listed in Table II. This corresponding change in the pitch angle is not symmetrical and of opposite sign. For example, a 25% increase in the electron density for ITER would make the angle ϕ appear to decrease by 0.06° and for a 25% decrease in n_e there would be no apparent change in the resonance frequency. This example is illustrated in Figure 1 by the dashed curves. The sensitivity to electron density uncertainty is higher for the colder C-MOD plasma.

TABLE II. Pitch Angle Sensitivity Analysis. The plasma parameters for ITER and C-MOD given in Table I are individually varied by $\pm 25\%$ and the corresponding change in observed pitch angle is given if the plasma parameter variation is not taken into account. Initial k to B angle is 87° .

Plasma Parameter	Pitch Angle Variance(degrees)	
	ITER	C-MOD
n_e	- 0.06, + 0.00	- 0.21, + 0.14
T_e	- 0.06, + 0.09	- 0.09, + 0.10
T_i	0	0
n_{He}/n_i	0	0
B	- 0.21, + 0.34	- 0.13, + 0.22
θ	- 0.46, + 0.57	- 0.28, + 0.32

There is no dependence of the pitch angle on accurate knowledge of the ion temperature or fraction of energetic ions. The dependence on the electron density and temperature is relatively weak. In addition, these parameters are generally measured with better than the $\pm 25\%$ uncertainty assumed in the calculation. The dependence on knowing the total magnitude of the magnetic field is somewhat stronger in the ITER case, but this parameter is generally known to much greater accuracy than the electron density and temperature. The scattering angle influences the pitch angle the most. Approximately a $\pm 0.5^\circ$ uncertainty is introduced into the pitch angle measurement in the ITER case if the scattering angle is uncertain by $\pm 25\%$. This is one reason why a short submillimeter wavelength should be used for this diagnostic, so that beam refraction uncertainties introduced into the scattering angle can be avoided.

Summary

A new approach for obtaining the local magnetic field pitch angle from collective Thomson scattering spectra has been presented. The technique is to measure the lower hybrid resonance frequency in the scattered spectrum. This frequency is highly leveraged to the pitch angle. It has been shown that the lower hybrid resonance feature has a sensitivity of over 1 GHz per degree pitch angle. Sensitivity to other plasma parameters has been shown to be relatively weak. Sensitivity to refraction and non electron density fluctuation terms in the scattered spectra can be minimized by using a short submillimeter wavelength source. With major experimental tests of collective Thomson scattering upcoming on TFTR and JET, it is expected that the existence of the lower hybrid resonance in the scattered spectra will be put on a firm experimental foundation. Those diagnostics are being implemented to measure the energetic ion density fraction and velocity distribution and will also be capable of diagnosing the bulk ion temperature. With the addition of the magnetic pitch angle to the repertoire of plasma parameters that can be diagnosed, collective Thomson scattering promises to be the single most valuable diagnostic in future tokamak diagnostic sets.

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