PFC/JA-92-29-REV

Charged-Particle Stopping Powers in Inertial Confinement Fusion Plasmas

C. K. Li and R. D. Petrasso

Submitted for publication in: *Physical Review Letters*

November 1992

Revised February 1993

Accepted for publication in *Physical Review Letters*

Plasma Fusion Center

Massachusetts Institute of Technology

Cambridge, MA 02139

This work was supported in part by U.S. DOE Grant No. DE-FG02-91ER54109

Charged-Particle Stopping Powers in Inertial Confinement Fusion Plasmas

Chi-Kang Li and Richard D. Petrasso

Plasma Fusion Center

Massachusetts Institute of Technology

Cambridge, MA 02139

Abstract

The effects of large-angle scattering , important for plasmas for which the Coulomb logarithm is of order 1, have been properly treated in calculating the range (R) and the ρR (the fuel-areal density) of inertial confinement fusion (ICF) plasmas. This new calculation, which also includes the important effects of plasma ion stopping, collective plasma oscillations, and quantum effects, leads to an accurate estimate, not just an upper limit, of ρR. For example, 3.5 MeV α's from D-T fusion reactions are found to directly deposit $\simeq 47\%$ of their energy into 20 keV deuterons and tritons. Consequently the α range (R) and ρR are reduced by about 60% from the case of pure electron stopping.

PACS Nos. 52.55.Pi, 52.40.Mj, 52.50.Gj, 52.25.Tx

The stopping of charged-particles (α's, ^{3}H, ^{3}He, hot electrons . . .) in compressed pellet plasmas is a fundamental problem with close parallels to early work of Rutherford and Bohr who studied α stopping in solid materials[1]. In the context of inertially confined fusion plasmas (ICF), it involves the deposition of energy from charged-particles, especially the α's, in the fuel material during the initial cold and compressed state and then during the evolution to full ignition and burn[2-4]. The tremendous range of pellet plasma conditions [$n_e \lesssim 10^{27}$ cm^{-3}, and $0.1 \lesssim T_e$ $(T_i) \lesssim 40$ keV] is directly reflected in the range of the Coulomb logarithm - - $1 \lesssim ln\Lambda_b \lesssim 12$ - - a parameter fundamental to many plasma properties[5-12], including charged-particle stopping[2-4,13,14]. In the context of solutions to the plasma Fokker-Planck equation, $ln\Lambda_b$ has a precise significance: it is a measure of the importance of small-angle collisions to large-angle scattering. Previously practical results based on the plasma Fokker-Planck equation have been well approximated only for $ln\Lambda_b \gtrsim 10$[5-12] because terms of $1/ln\Lambda_b$ are truncated in the collision operator. Although this issue has remained unsolved until now, Fraley *et al.*[14] and later Mehlhorn[13] clearly recognized its importance in their studies of α energy deposition in ICF. These workers noted that when ion stopping is significant for α's, large-angle scattering is also likely to be important. Fraley *et al.* did calculate ion stopping, but because they were unable to estimate the effect of the large-angle scattering within the framework of the Fokker-Planck approximation, and because they neglected collective plasma effects, they concluded that their estimate of range (R) and ρR (fuel areal-density) was in fact only an upper limit. Mehlhorn attempted to treat large-angle scattering[13], but discarded his results in favor of the standard small-angle formulation when the large-angle results proved inconsistant.

This particular problem is overcome by our recent generalization of the Fokker-Planck equation, which properly treats the effects of large-angle scattering as well as small-angle collisions[15]. In subsequent discussions of charged-particle stopping, we utilize one of the general results of that analysis:

$$\frac{dE_t^{t/f}}{dx} = -\frac{(Z_t e)^2}{v_t^2}\,\omega_{pf}^2\,G(x^{t/f})\,ln\Lambda_b\,, \tag{1}$$

where $dE_t^{t/f}/dx$ is the stopping power of a test particle (sub or superscript t) in a field of background charges (sub or superscript f) and

$$G(x^{t/f}) = \mu(x^{t/f}) - \frac{m_f}{m_t}\{\frac{d\mu(x^{t/f})}{dx^{t/f}} - \frac{1}{ln\Lambda_b}[\mu(x^{t/f}) + \frac{d\mu(x^{t/f})}{dx^{t/f}}]\}\,. \tag{2}$$

The contribution of large-angle scattering is solely manifested by $1/ln\Lambda_b$ terms of Eq. 2. In particular, if $ln\Lambda_b \gtrsim 10$ and we ignore this correction, then Eqs. 1 and 2 reduce to Trubnikov's expression[8]. In the above equations, $Z_t e$ is the test charge; v_t (v_f) is the test (field) particle velocity with $x^{t/f} = v_t^2/v_f^2$; m_t (m_f) is test (field) particle mass; $\omega_{pf} = (4\pi n_f e_f^2/m_f)^{1/2}$, the field plasma frequency. $\mu(x^{t/f}) = 2\int_0^{x^{t/f}} e^{-\xi}\sqrt{\xi}\;d\xi/\sqrt{\pi}$ is the Maxwell integral; $ln\Lambda_b = ln(\lambda_D/p_{min})$, where, for the non-degenerate regime, λ_D is the Debye length and $p_{min} = \sqrt{p_\perp^2 + (\hbar/2m_r u)^2}$; $p_\perp = e_t e_f/m_r u^2$ is the classical impact parameter for 90° scattering, with m_r the reduced mass and u the relative velocity. However, in the low temperature, high density regime, electron (not ion) quantum degeneracy effects must be considered in calculating λ_D and p_{min} (see Figs. 1a).

In addition to stopping by binary collisions, both small- and large-angle, stopping occurs due to plasma oscillations[16,17]. As this contribution is only important when $x^{t/f} \gg 1$, a generalized stopping formula is

$$\frac{dE_t^{t/f}}{dx} \simeq -\frac{(Z_t e)^2}{v_t^2}\omega_{pf}^2[G(x^{t/f})ln\Lambda_b + \theta(x^{t/f})ln(1.123\sqrt{x^{t/f}})] \ , \tag{3}$$

Collective effects are represented by the second term $[ln\Lambda_c^{t/f} \equiv ln(1.123\sqrt{x^{t/f}})]$ where $\theta(x^{t/f})$ is a step function whose value is identically 0 (1) for $x^{t/f} \leq 1$ (>1). Note that for all charged fusion products (α's, ^{3}H, ^{3}He ...) interacting with *field electrons*, $x^{t/f}$ is usually much less than 1, indicating collective contributions can be ignored (see $ln\Lambda_c^{\alpha/e}$ in Fig. 1a). In contrast, for charged fusion products interacting with *field ions*, $x^{t/f}$ is usually much larger than one, and therefore collective effects are significant (dashed line, Fig. 1b).

In order to illustrate the results of the generalized stopping power (Eq. 3), we consider 4 cases: α's, ^{3}H, ^{3}He, and hot electrons each interacting with field ions and field electrons. For 3.5 MeV α's in a $10^{26}/cm^3$ D-T plasma, Figs 1a and 1b show the corresponding Coulomb logarithms for α - electron ($ln\Lambda^{\alpha/e}$) and α - ion ($ln\Lambda^{\alpha/i}$) interactions. In the case of α - electron interactions, Eqs. 1 and 2 nearly reduce to Trubnikov's results[8] because the mass ratio of field-to-test particles, m_e/m_α, is of order 10^{-4}. However, when the field-to-test mass ratio (m_f/m_t) is of order 1 or 10^3 - - as it is for α's, ^{3}H and ^{3}He interacting with field ions, or for test electrons interacting with field ions - - Eqs. 1, 2 and 3 must be used instead of Trubnikov's. Table 1 shows the relative importance of ion and electron stopping for α's that thermalize from 3.5 MeV. Note that ion stopping becomes significant for $T_e \sim T_i \gtrsim 5$ keV. In more detail, Fig. 2 plots the ion stopping fraction, $(dE^{\alpha/i}/dx)/(dE^{\alpha/i}/dx + dE^{\alpha/e}/dx)$, for relevant α energies ($\leq$ 3.5 MeV) and plasma temperatures. Fig. 3 shows the corresponding ρR for α's (calculated from the 3.5 MeV birth energy to background thermal temperature). For example, at 20 keV inclusion of ion stopping (binary plus collective) reduces the ρR of pure electron stopping by about 60%. Also for the α's, Fig. 4 shows the density dependence of ρR. Effects of electron degeneracy can be clearly seen for density $\gtrsim 10^{27}/\text{cm}^3$ and temperature $\lesssim 5$ keV. Degeneracy effects enter in both the calculation of $ln\Lambda$ and the parameter $x^{t/f}$. (In the degenerate regime, our calculations are only semi-quantitative.) In the non-degenerate regime of Fig. 4, Fraley *et al.*'s[14] results, which ignored (large-angle) scattering and collective effects, are about 20% larger. (They did not treat the degenerate regime.)

The development of novel ρR diagnostics are currently based upon the 1.01 MeV ^{3}H and 0.82 MeV ^{3}He[4,18] that result from D-D fusion. Because of the relevance of this diagnostic to present experiments, we show in Figs. 5a and 5b ρR with and without the effect of ion stopping. As is evident, even for fairly low plasma temperatures the effects of ion stopping are extremely important.

In contrast to charged-fusion products interacting with background electrons and ions, for which the scattering is small either because $m_e/m_\alpha \sim 10^{-4}$ or $ln\Lambda_b^{\alpha/i} \sim 10$, scattering must be included in treating hot electrons interacting with cold electrons and ions. Such a situation arises in considering hot corona electrons interacting with the cold core[19]. The dashed line in Fig. 6 shows ρR due only to small-angle binary collisions, which is the conventional calculation. The solid line includes as well large-angle scattering off electrons and ions plus the collective effects of the background electrons. As can seen, these contributions are important.

In summary, we have calculated the stopping powers and ρR of charged-fusion products and hot electrons interacting with plasmas relevant to inertial confinement fusion. For the first time the effects of scattering, which limited previous calculations to upper limits[13,14], have been properly treated. In addition, the important effects of ion stopping, electron quantum properties, and collective plasma oscillations have also been included. Ion stopping is found to be important for all charged-fusion products. For hot electrons interacting with cold dense plasmas, the contributions of scattering and collective oscillations are significant.

We gratefully acknowledge useful conversations with Drs. M. D. Cable, E. M. Epperlein, H. Griem, S. P. Hatchett, S. Skupsky, T. Mehlhorn, and G. B. Zimmerman. Supported in part by U. S. DOE Contract No. DE-FG0291ER54109 and LLNL Contract No. B160456.

References

[1] N. Bohr, *Philos. Mag.*, **25**, 10 (1913).

[2] J. D. Lindl, R. L. McCrory, and E. M. Campbell, *Phys. Today*, **45**(9), 32 (1992); G. B. Zimmerman, UCRL-JC-105616 report, Nov. 1990.

[3] S. Skupsky, *Phys. Rev.*, **16**(2), 727 (1977); C. Deutsch, *Ann. Phys. Fr.*, **11**, 1 (1986).

[4] M. D. Cable and S. P. Hatchett, *J. Appl. Phys.*, **62**(6), 2233 (1987); private communication, 1993.

[5] L. D. Landau, ZhETF *J. Exptl. Theoret. Phys. USSR*, **7**, 203 (1937).

[6] R. S Cohen, L. Spitzer and P. McR. Routly, *Phys. Rev.*, **80**(2), 230 (1950).

[7] M. N. Rosenbluth, W. M. MacDonald, and D. L. Judd, *Phys. Rev.*, **107**(1), 1 (1957).

[8] B. Trubnikov, *Review of Plasma Physics 1*, Consultants Bureau, New York, 1965.

[9] S. I. Braginskii, *Review of Plasma Physics 1*, Consultants Bureau, New York, 1965.

[10] D. J. Sigmar and G. Joyce, *Nucl. Fusion*, **11**, 447 (1971).

[11] D. C. Montgomery and D. A. Tidman, *Plasma Kinetic Theory*, McGraw-Hill Book Company, New York, 1964; I. P. Shkarosky, T. W. Johnston, and M. P. Backynski, *The Particle Kinetics of Plasmas*, Addison-Wesley, Mass., 1966.

[12] D. L. Book, *NRL Plasma Formulary*, (1990).

[13] T. A. Mehlhorn, *J. Appl. Phys.*, **52**(11), 6522 (1981).

[14] G. S. Fraley, E. J. Linnebur, R. J. Mason, and R. L. Morse, *The Physics of Fluids*, **17**(2), 474 (1974).

[15] C. K. Li and R. D. Petrasso, submitted to *Phys. Rev. Lett.*, Nov. 1992.

[16] *I. E. Tamm: Selected Papers*, p128, B. M. Bolotovskii and V. Ya. Frenkel (Editors), Springer-Verlag, New York, 1991.

[17] J. D. Jackson, *Classical Electrodynamics*, Chap. 13, John Wiley & Sons, New York, 1975.

[18] H. Azechi, N. Miyanaga, R. O. Stapt, K. Itoga, H. Nakaishi, M. Yamanaka, H. Shiraga, R. Tsuji, S. Ido, K. Nishhara, Y. Izawa, T. Yamanaka, and C. Yamanaka, *J. Appl. Phys.*, **49**(10), 555 (1986).

[19] J. J. Duderstadt and G. A. Moses, *Inertial Confinement Fusion*, John Wiley and Sons, New York, 1981.

[20] Y. T. Lee and R. M. More, *The Physics of Fluids*, **27**(5),1273 (1984).

Table 1. The relative importance of 3.5 MeV α stopping by deuterons and tritons compared to that by electrons (n_e=$10^{26}/cm^3$, $T_e \simeq T_i$).

T_e (keV)	D-T ion stopping (% of total)	electron stopping (% of total)
1.0	$\simeq 6$	$\simeq 94$
5.0	$\simeq 19$	$\simeq 81$
10.0	$\simeq 32$	$\simeq 68$
20.0	$\simeq 47$	$\simeq 53$
40.0	$\simeq 64$	$\simeq 36$

Fig. 1a. The Coulomb logarithms for α-electron interactions, for 3.5-MeV α's originating from D-T reactions ($n_e = 10^{26}/cm^3$). The quantum calculation (solid line) is used in our text and subsequent figures. The classical (Spitzer) calculation (long-dashed straight line) is given for reference. The effect of collective plasma oscillations, for this particular case, is unimportant. Stopping power and ρR are calculated only for $T_e \gtrsim 1$ keV. (For $ln\Lambda_b^{\alpha/e} < 2$, strongly-coupled effects become an issue[15,20].) Fig. 1b. The Coulomb logarithm for α-ion (deuteron and triton) interactions ($ln\Lambda_b^{\alpha/i}$) and α-ion collective interactions ($ln\Lambda_c^{\alpha/i}$). In contrast to the α-electron interaction (Fig. 1a), quantum effects are unimportant. However, collective effects are significant since $v_\alpha \gg v_i$ [v_i, the background ion (D or T) velocity].

Fig. 2. The relative fraction of α - field ion (D-T) stopping as a function of the α kinetic energy (E_α) and plasma temperature ($T_e \sim T_i$). For $E_\alpha \lesssim 1.5$ MeV and $T_e \gtrsim 15$ keV, ion stopping is dominant. A 3.5 MeV α in a 40 keV plasma deposits its energy along the dotted trajectory. It initially deposits $\sim$ 35% of its energy to ions, but, by the end of its range, $\gtrsim$ 95% is going into the ions. By integrating over the trajectory, the total ion stopping is $\simeq$ 64% (see Table 1). This effect significantly reduces the α range (R) and ρR by about 73% (see Fig. 3).

Fig. 3. ρR for 3.5 MeV α interacting in a $10^{26}/cm^3$ D-T plasma. The dashed line represents pure electron stopping (scattering is negligible). The solid line results from

the cumulative effects of electron binary, ion binary (small-angle plus scattering), and ion collective oscillations.

Fig. 4. ρR curves for 3.5 MeV α interacting with D-T plasmas of various densities. Quantum degeneracy is important for $n_e \gtrsim 10^{27}/cm^3$ and $T_e \lesssim 5$ keV.

Fig. 5a (5b). ρR for 1.01 MeV ^3H (0.82 MeV ^3He) interacting in a $6\times10^{24}/cm^3$ D plasma. The dashed line represents pure electron stopping (scattering is negligible). The solid line results from the cumulative effects of electron binary, ion binary (small-angle plus scattering), and ion collective oscillations.

Fig. 6. ρR for hot corona electrons interacting with a cold (core) D plasma ($n_e \sim 10^{23}/cm^3$ and $T_e \sim 50$ eV). The dashed curve shows the effects of pure small-angle binary collisions. The solid line results from the cumulative effects of electron small-angle collisions and large-angle scattering, electron collective oscillations, and large-angle ion scattering.

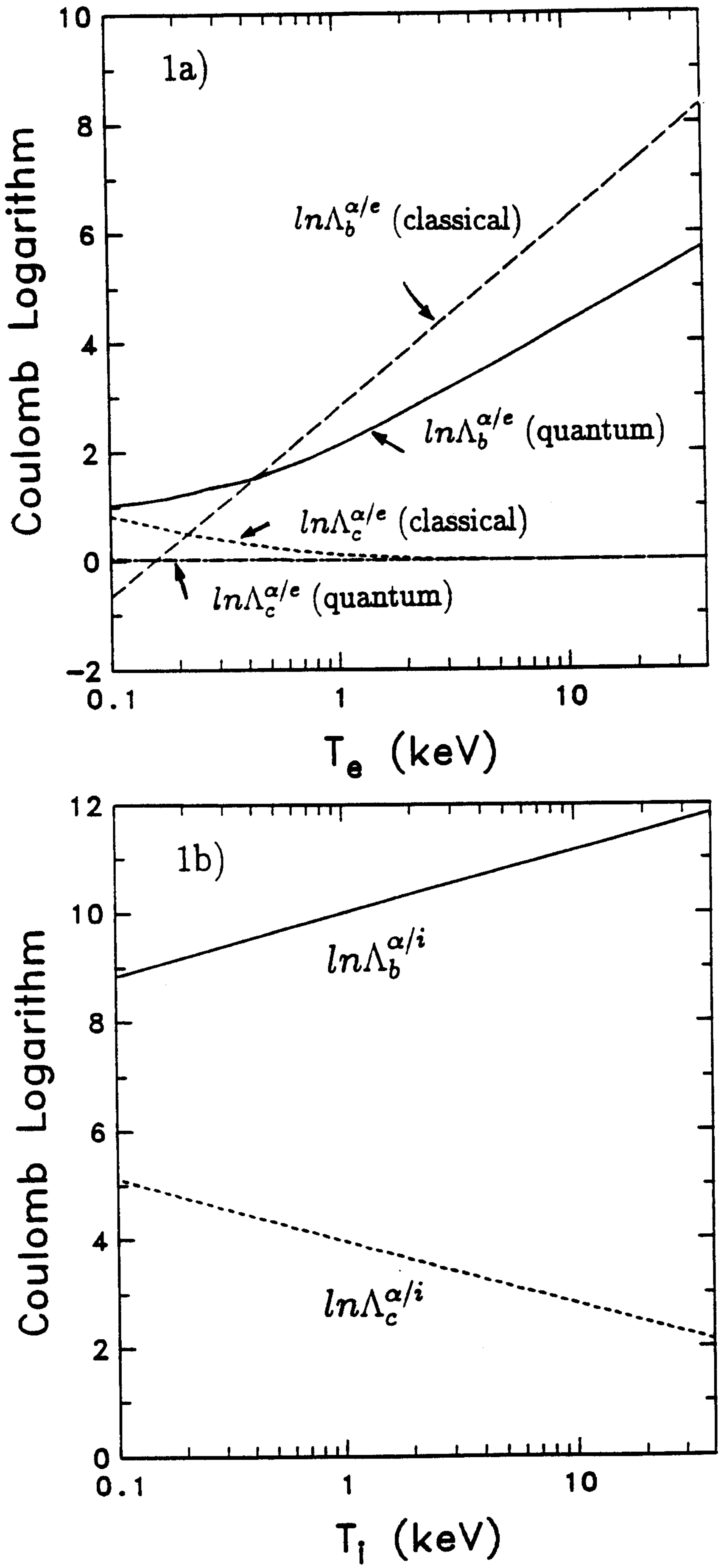

1a)
Coulomb Logarithm
$ln\Lambda_b^{\alpha/e}$ (classical)
$ln\Lambda_b^{\alpha/e}$ (quantum)
$ln\Lambda_c^{\alpha/e}$ (classical)
$ln\Lambda_c^{\alpha/e}$ (quantum)
T_e (keV)
1b)
$ln\Lambda_b^{\alpha/i}$
$ln\Lambda_c^{\alpha/i}$
T_i (keV)

Fig. 1

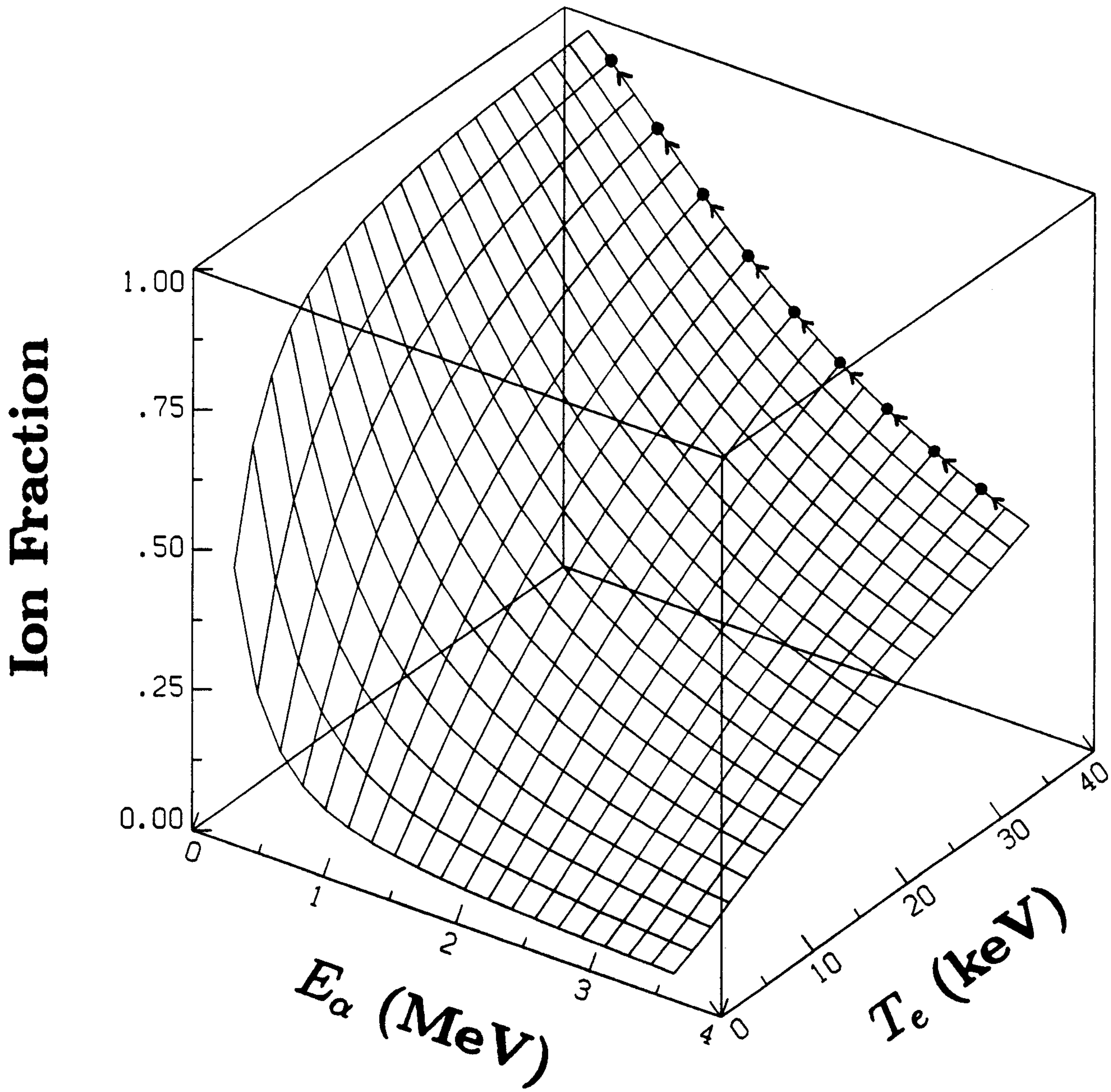

Ion Fraction
1.00
.75
.50
.25
0.00
0
1
2
3
4
E_α (MeV)
0
10
20
30
40
T_e (keV)

Fig. 2

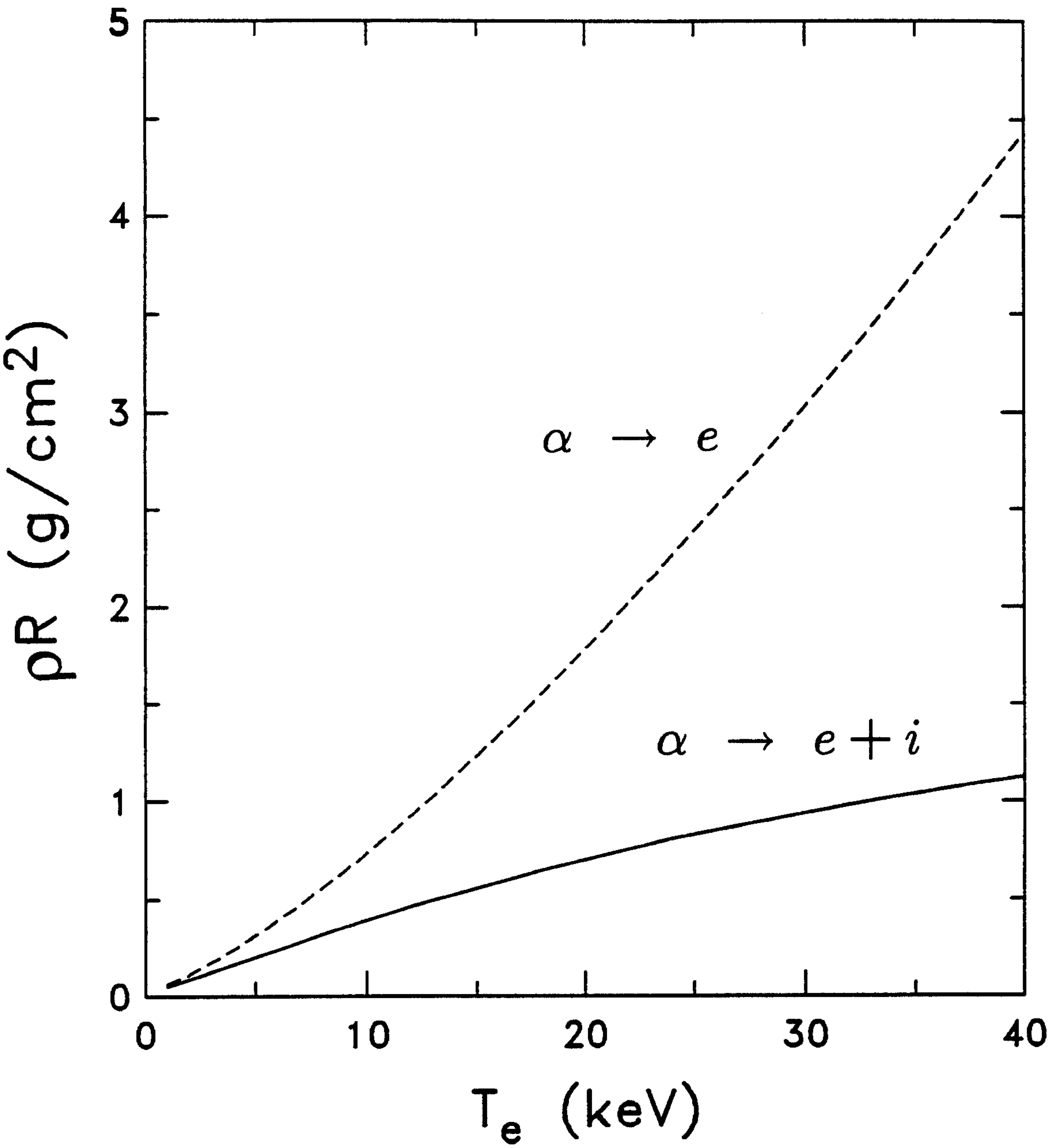
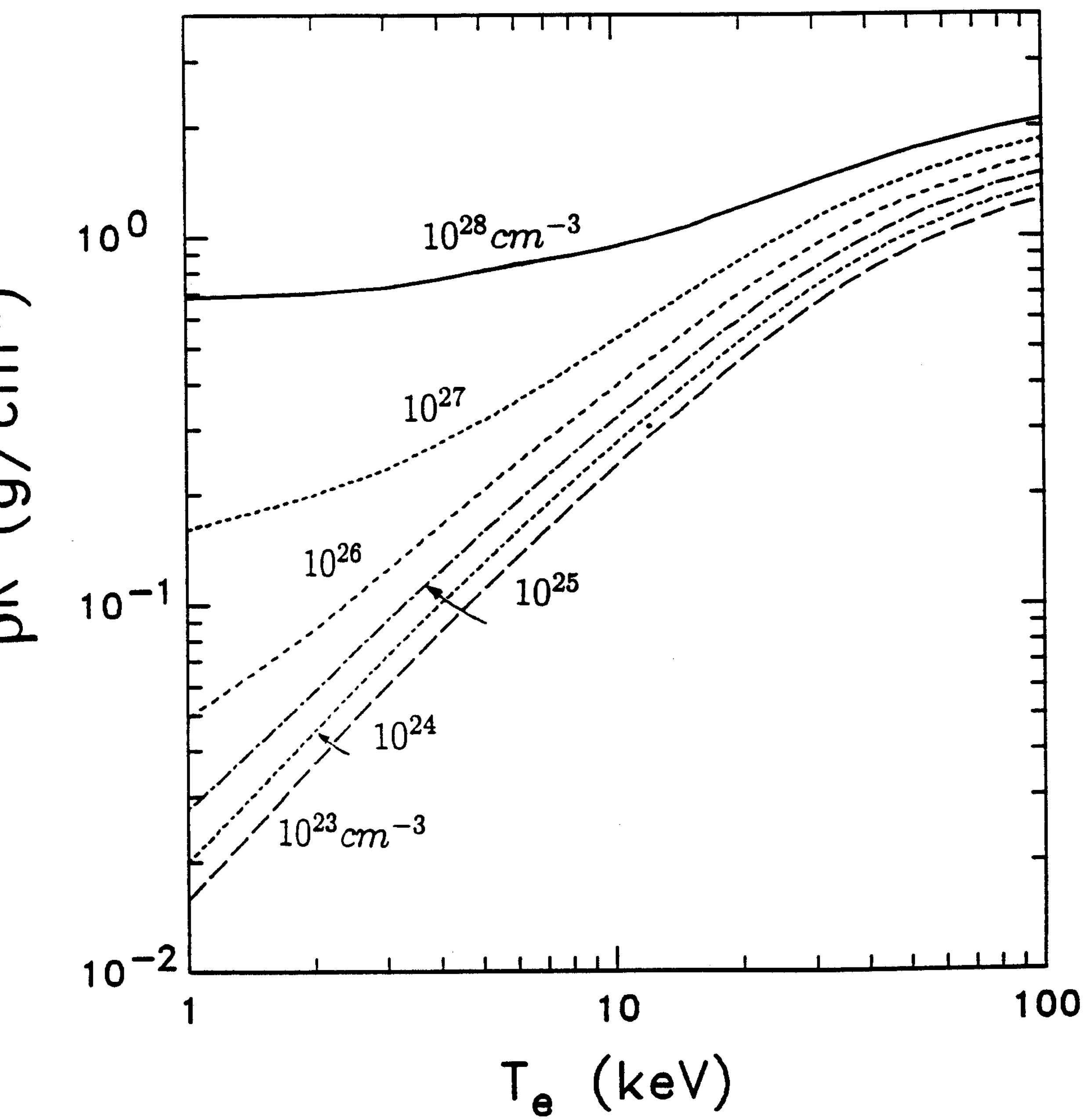
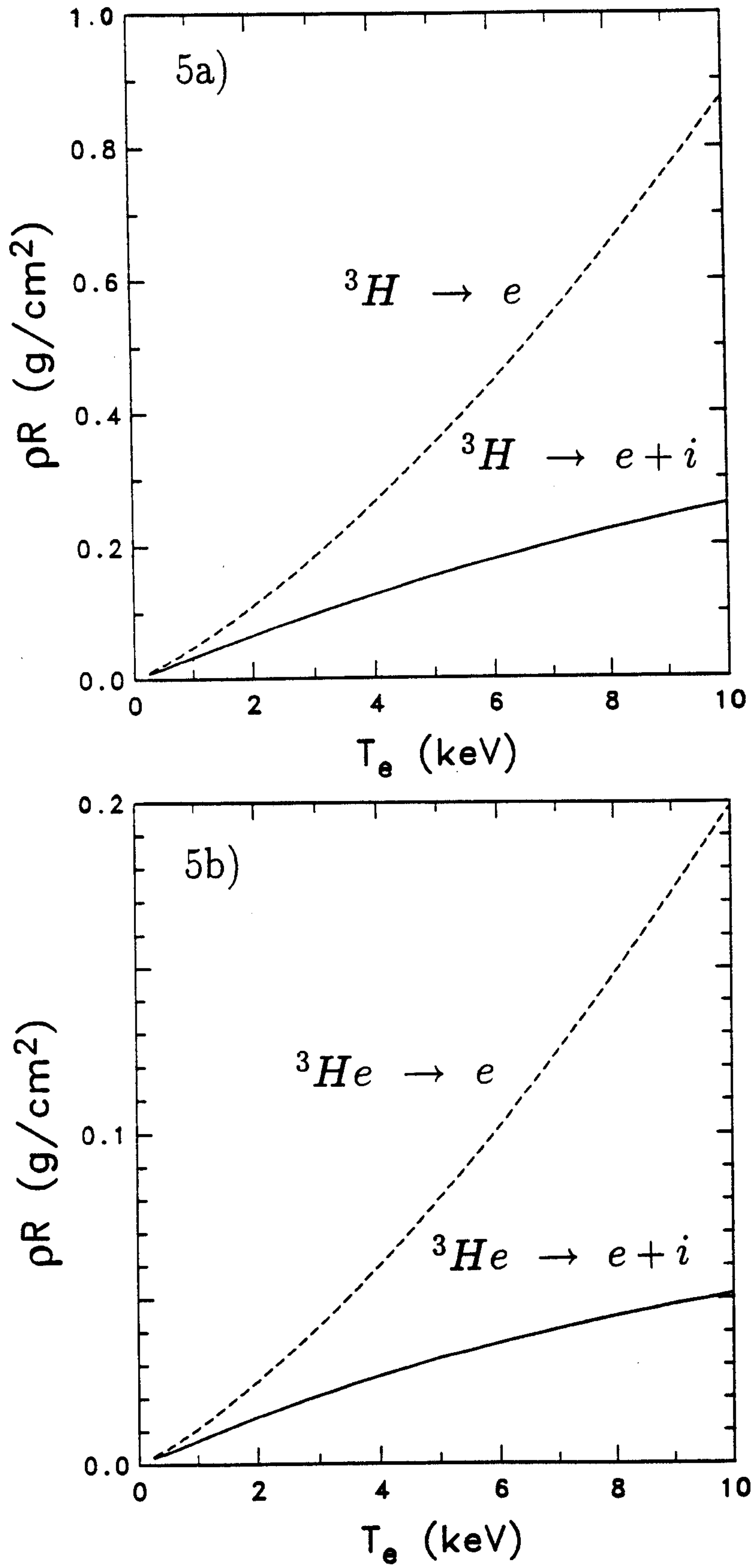


Fig. 3



$10^{28} cm^{-3}$
10^{27}
10^{26}
10^{25}
10^{24}
$10^{23} cm^{-3}$
10^{0}
10^{-1}
10^{-2}
1
10
100
ρR (g/cm²)
T_e (keV)

Fig. 4



5a)
ρR (g/cm²)
1.0
0.8
0.6
0.4
0.2
0.0
$^3H \rightarrow e$
$^3H \rightarrow e+i$
0
2
4
6
8
10
T_e (keV)
5b)
ρR (g/cm²)
0.2
0.1
0.0
$^3He \rightarrow e$
$^3He \rightarrow e+i$
0
2
4
6
8
10
T_e (keV)

Fig. 5

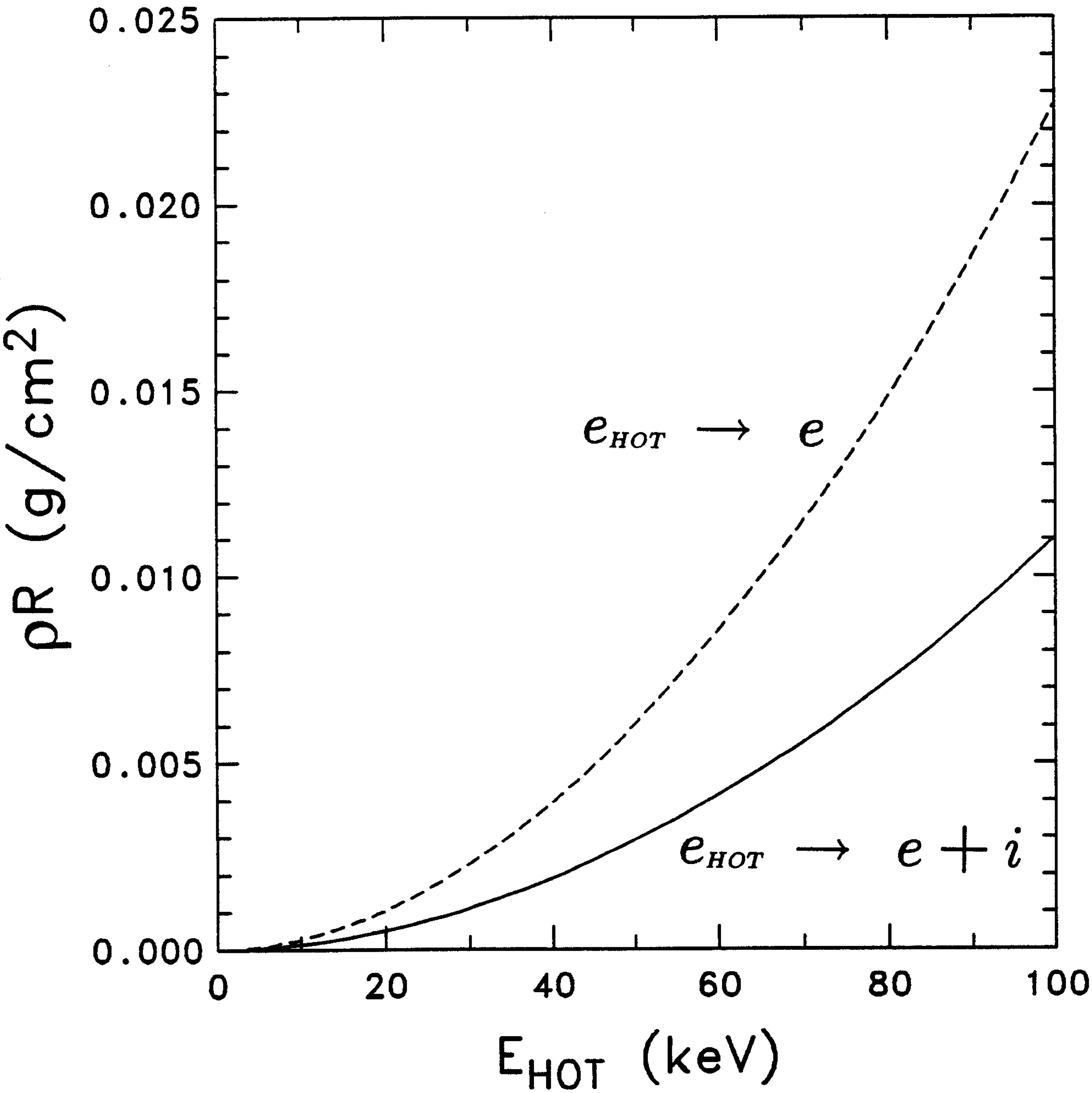

0.025
0.020
0.015
0.010
0.005
0.000
ρR (g/cm2)
0
20
40
60
80
100
EHOT (keV)
eHOT → e
eHOT → e + i

Fig. 6