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Plasma Heating by Fast Magnetosonic Waves in Tokamaks†

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Abstract

The fundamental theory of plasma heating by the fast magnetosonic wave in toroidal plasma configurations is reviewed and extended. The particular wave damping processes considered include cyclotron damping at the fundamental ion cyclotron frequency and its harmonics, and electron Landau damping and transit time magnetic pumping (TTMP). The latter processes heat electrons and may also be exploited to drive toroidal plasma currents. The wave absorption and damping decrements are obtained by using Stix's approach, namely by computing the dissipated power, $Re(\bar{J} \cdot \bar{E})$ in terms of the hot plasma dielectric constant (where $\bar{J}$ is the wave induced current). This approach is compared with power absorption calculations from quasi-linear theory, and exact agreement is found for a Maxwellian distribution of particles. Wave absorption in the presence of a small group of energetic particles is also examined for all three types of damping processes. The limitations of theory owing to mode conversion phenomena are indicated. Finally, a brief discussion of recent experimental results is given, verifying the reality of Landau damping of magnetosonic waves by electrons.

I. Introduction

The absorption of the fast magnetosonic wave by electrons in high temperature tokamak plasmas is of great practical importance because of the feasibility of heating to ignition-like temperatures,[(1)] as well as driving the toroidal plasma current.[(2)] An attractive steady state reactor concept could be developed based on fast wave heating and current drive if the bootstrap-current fraction were sufficiently high.[(3)] From a practical point of view, of considerable importance is the single pass absorption of fast waves by electrons in high temperature plasmas. A single pass absorption of 10% or more is thought to be desirable to ensure unidirectional wave propagation and absorption. The earliest correct calculation of fast wave absorption by electrons was performed by Stix in 1975,[(1)] who considered only the low frequency limit, $v_{te} \gtrsim \omega/k_{\|}$ where $v_{te} = (2T_e/m_e)^{1/2}$, $\omega/2\pi$ is the wave frequency, and $k_{\|} = \vec{k} \cdot \vec{B}/|\vec{B}|$ is the parallel component of the wave-vector. In such a limit the single pass absorption is rather weak, and the current drive efficiency is also low. As a consequence, one may have to deal with eigen-mode excitation in the toroidal plasma. The low frequency regime ($\omega < \omega_{ci}$) has the advantage of avoiding "parasitic" ion absorption, including that by alpha particles. On the other hand, one may be faced with partial mode conversion into shear Alfven waves[(4)] where $n_{\|}^2 = S$ ($n_{\|} = ck_{\|}/\omega$ and S is the perpendicular component of the dielectric constant[(1)]). More recent calculations emphasized the regime $\omega/k_{\|} \gtrsim v_{te}$, $\omega > \omega_{ci}$, so that the theoretically predicted stronger absorption by electrons could be tested in present-day tokamak plasmas.[(5-8)] Here we shall review the calculations of fast wave absorption on electrons for arbitrary parallel phase velocities.

The dissipation of wave power by electrons is manifested through Landau-type wave-particle resonances, $\omega \simeq k_{\|}v_{e\|}$, and the strongest interaction results when the wave resonates with the bulk-electrons, namely $v_{e\|} \simeq v_{te}$. It will be shown below that Stix's approach can be followed through even for the case of resonance with bulk electrons if one considers the use of the plasma dispersion function (or Z-function). Similarly to Stix we find[(8)] that three terms contribute to the damping of this wave: the electron Landau damping being proportional to ImK_{zz} (where ImK is the imaginary part of the hot plasma dielectric tensor), the transit time magnetic pumping (or "TTMP") being proportional to ImK_{yy}, and the cross term, being proportional to ImK_{yz}. Quasi-linear estimates have shown that for $\omega \lesssim 2k_{\|}v_{te}$, in the presence of a fast magnetosonic wave the electron distribution remains nearly a Maxwellian.[(9)] We shall find that in this case for low frequencies the Landau damping term dominates while the other two terms cancel. However, at finite frequencies ($\omega > \omega_{ci}$) an additional term survives from the cross-term and increases wave damping. We shall also find that significant absorption will result only at finite values of the electron beta, typically $\beta_e \gtrsim 0.01$.

The present calculations were motivated by testing experimentally this kind of heating and current drive concept in the General Atomics DIII-D tokamak.[10] The initial calculations by the author using the Stix-formalism[5,6] was followed by S.C. Chiu et al.[7] who calculated the damping decrement from the determinant of the hot plasma dielectric tensor. These calculations are more direct, however, they do not display the physical importance of the various absorption and wave polarization terms. Therefore, here we shall follow the physically more transparent derivation of taking the ratio of the absorbed power to the power flux which yields the spatial damping rate. For electromagnetic waves, to a good approximation, the power flux is simply given by the Poynting flux. We note that for $\omega > 2k_{\parallel}v_{te}$, the absorption is generally too weak to be of practical interest.

We shall also generalize these results to the case of two-component plasma, for example when a small fraction of energetic electron component is present. Such a situation may be produced in "synergistic" experiments where in addition to the magnetosonic wave, a lower-hybrid wave may also be present in the plasma. The question of direct fast wave absorption by the energetic electron tail has been raised as an important issue in connection with recent experiments on JET where an enhanced current drive effect was noted.[11]

We shall also consider the case of competing wave absorption process by ions. This is detremental to current drive or direct heating by electrons, and must be carefully considered. We shall make simple estimates of ion absorption by harmonic ion cyclotron damping, minority absorption and absorption by energetic ions (due to neutral beam injection or alpha particles). Finally, a brief summary of recent experimental results will be presented, supporting evidence of direct absorption of fast waves by electrons in tokamak plasmas. It should be noted that the absorption of fast waves by ions has been tested experimentally over the past decade and a half, and these results are summarized in a companion lecture by J. Hosea.

II. Absorption of Magnetosonic Waves by Electrons

The dispersion relationship of fast waves in the ion cyclotron frequency range may be written in the following form:[1,12]

$$n_{\perp}^2 = \frac{(n_{\parallel}^2 - R)(n_{\parallel}^2 - L)}{S - n_{\parallel}^2} \tag{1}$$

where in the cold plasma limit the following approximate expressions hold:

$$
\begin{aligned}
R =& 1+\sum_i \frac{\omega_{pi}^2}{\omega_{ci}(\omega+\omega_{ci})} , \\
L =& 1-\sum_i \frac{\omega_{pi}^2}{\omega_{ci}(\omega-\omega_{ci})} , \\
S =& 1-\sum_i \frac{\omega_{pi}^2}{\omega^2-\omega_{ci}^2} ,
\end{aligned}
$$

where $S = (R+L)/2$. Here $\omega_{pi} = (4\pi n_i e^2 Z_i^2/m_i)^{1/2}$ is the angular ion plasma frequency and $\omega_{ci} = eZ_iB/m_ic$ is the angular ion cyclotron frequency. Equation (1) predicts a rather complex behavior for fast wave propagation, especially in the case of multi-ion species plasmas. In addition to Refs.(1,12) an excellent summary of such phenomena can be found in Ref.(13). Here we simply want to point out a few of the salient features of Eq.(1).

The region of $n_\parallel^2 = R$ corresponds to the right hand cut-off layer ($n_\perp^2 = 0$), and the fast wave is evanescent at densities lower than this critical density, n_R. The right hand cut-off layer always exists in the plasma, regardless of the relative value of ω/ω_{ci}. Consequently, the wave has to "tunnel through" an evanscent layer in the plasma periphery and the reflected rf power must be prevented from getting back into the rf source (usually a high power tetrode) by an external tuning (matching) network. The $n_\parallel^2 = L$ layer is also a cut-off layer. In a single ion species plasma, this layer occurs only for $\omega < \omega_{ci}$, and typically it occurs at densities n_L such that $n_L < n_S < n_R$, where n_S is the critical density at the resonance layer where $n_\parallel^2 = S$ and $n_\perp^2 \rightarrow \infty$. At the resonance layer finite temperature effects must be included and mode conversion into the kinetic shear Alfven wave will take place. The inhomogeneous magnetic field of a tokamak will only quantitatively change this picture. The presence of the cut-off-resonance-cut-off "triplet" complicates the prediction of rf power flow into the fast magnetosonic wave. If $\omega > \omega_{ci}$ everywhere in the plasma, such a complication does not arise and one only need to be concerned with the right hand cut-off layer. If a minority ion species (or a second majority ion species) is present in the plasma, the mode conversion layer ($n_\parallel^2 = S$) will be affected by the second ion species, and the cyclotron frequency of the lighter ion species (ω_{cm}) will dominate. For example, if $\omega \simeq \omega_{cm}$ near the center of the plasma column, the $n_\parallel^2 = L$ and $n_\parallel^2 = S$ layers will be located on the high field side of the minority species cyclotron resonance layer. The $n_\parallel^2 = R$ layer will remain near the plasma periphery, maintaining the presence of the evanescent layer.

If $\omega > \omega_{cm}$ everywhere in the plasma column, and if we consider regions of fast wave propagation well away from any cut-off or resonance layer, the fast wave dispersion can be approximated fairly well by the following simple relationship:

$$\omega \simeq k_\perp v_A (1 + c^2 k_\parallel^2/\omega_{pi}^2)^{1/2} , \tag{2}$$

where $v_A \simeq c\omega_{ci}/\omega_{pi}$ is the Alfven speed, and where usually $n_\parallel^2 \ll n_\perp^2$ so that $\omega \simeq k_\perp v_A$. In the discussions below, we shall use only Eq.(2) for the real part of the dispersion relationship when we calculate power absorption.

As shown by Stix, the absorbed power in the plasma can be determined by calculating the dissipated wave power, $Re(\vec{J} \cdot \vec{E})$ which can be written in the form

$$P_{abs} = \frac{-i\omega}{16\pi} \sum_{modes} \vec{E}^* \cdot (\overleftrightarrow{K} - \overleftrightarrow{I}) \cdot \vec{E} + c.c. \tag{3}$$

where the summation is over different modes, $\overleftrightarrow{K}$ is the hot plasma dielectric tensor, $\overleftrightarrow{I}$ is the unit diadic, and *c.c.* represents the complex conjugate. Thus, the contribution will come from the anti-Hermitian part of $\overleftrightarrow{K}$, and the Hermitian part will cancel. Now we will be interested in Landau-type of resonance of electrons, namely $\omega \simeq k_\parallel v_{e\parallel}$. Examining the hot plasma dielectric tensor,[12] we find that the following terms may contribute:

Landau Damping:

$$K_{zz} = 1 + \frac{1}{k_\parallel^2 \lambda_{De}^2}\Big[1 + \zeta_e Z(\zeta_e)\Big]; \tag{4a}$$

Transit Time Magnetic Pumping:

$$K_{yy} = 1 + 2k_\perp^2 r_{ce}^2 \frac{\omega_{pe}^2}{\omega^2} \zeta_e Z(\zeta_e); \tag{4b}$$

Cross-Terms:

$$K_{yz} = -K_{zy} = -i\frac{\omega_{pe}^2}{\omega\omega_{ce}} \frac{k_\perp}{k_\parallel} [1 + \zeta_e Z(\zeta_e)]; \tag{4c}$$

Here the cyclotron (harmonic) resonance terms have been neglected. In Eq.(3) we defined $\lambda_{De} = v_{te}/\sqrt{2}\omega_{pe}$, $r_{ce} = v_{te}/\sqrt{2}\omega_{ce}$, $v_{te}^2 = 2T_e/m_e$, $\omega_{pe}^2 = 4\pi n_e e^2/m_e$, $\omega_{ce} = eB/m_e c$, $\zeta_e = \omega/k_\parallel v_{te}$ and $Z(\zeta_e)$ is the Fried-Conte plasma dispersion function. We note that while Landau damping is the result of the force on a charge due to the parallel wave electric field, $(eE_\parallel)$, transit time magnetic pumping results from the force associated with the magnetic moment and the wave magnetic field,[12] $(-\nabla_\parallel(\mu B))$.

The spatial damping decrement is given by the ratio of the absorbed power, P_{abs} and the Poynting flux, $S_\perp$

$$2k_{\perp Im} = P_{abs}/S_\perp \tag{5a}$$

where we take $S \simeq S_\perp \sim cn_x|E_y|^2/8\pi$, we assumed $k_\parallel^2 \ll k_\perp^2$, and $k_\perp = k_x$. Evaluating Eq.(3), the contributions from 4(a-c) are given by

$$P_{abs} = \frac{\omega}{4\pi^{1/2}} \frac{\omega_{pe}^2}{\omega^2} \left[k_\perp^2 r_{ce}^2 |E_y|^2 - \frac{\omega}{\omega_{ce}} \frac{k_\perp}{k_\parallel} |E_z||E_y| + \frac{\omega^2}{k_\parallel^2 v_{te}^2} |E_z|^2 \right] \zeta_e e^{-\zeta_e^2} . \tag{5b}$$

III. Wave Polarization

To proceed, we need to evaluate E_z in terms of E_y, and then substitute for E_z in Eq.(5). This can be carried out with the help of the dielectric matrix equation,[12]

$$\begin{pmatrix} K_{xx} - n_z^2 & K_{xy} & K_{xz} + n_x n_z \\ K_{yx} & K_{yy} - n^2 & K_{yz} \\ K_{zx} + n_x n_z & K_{zy} & K_{zz} - n_x^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0 \tag{6}$$

which results in three equations relating E_x, E_y, E_z. Here $n^2 = n_z^2 + n_x^2$, and $n_z = n_\parallel = ck_\parallel/\omega$, $n_x = n_\perp = ck_\perp/\omega$ are the parallel and perpendicular components of the index of refraction. Eliminating E_x in favor of E_y and E_z, the following result can be deduced:

$$\frac{E_y}{E_z} = \frac{n_x^2 n_z^2 - (K_{xx} - n_z^2)(K_{zz} - n_x^2)}{-n_x n_z K_{xy} + (K_{xx} - n_z^2) K_{zy}} . \tag{7}$$

We now consider the relative magnitudes of various terms in Eq.(7) for $\omega_{ci} \sim \omega < \omega_{pi}$, i.e. the ion-cyclotron frequency range. The other important approximation is that $n_x^2 < |K_{zz}|$. This usually implies that $\omega^2 < \omega_{LH}^2 \sim \omega_{pi}^2$, the lower-hybrid (ion-plasma) frequency. This follows from the scaling $n_x^2 \sim \omega_{pi}^2/\omega^2$, $K_{zz} \sim (\omega_{pe}^2/\omega^2)$ or $\sim 1/(k_z^2 \lambda_{De}^2)$. Thus in the numerator, $n_x^2 n_z^2$ is negligible for $n_x \sim c/v_A \sim \omega_{pi}/\omega_{ci}$, $n_z = ck_z/\omega \sim c/v_{te}$, and $|K_{xx}| \sim \omega_{pi}^2/\omega_{ci}^2$. The next simplification occurs if we neglect the first term in the denominator, namely $n_x n_z K_{xy}$. As will be shown later, this corresponds to the low frequency, hot plasma limit, namely

$$\frac{\omega^2}{\omega_{pi}^2} \ll \frac{T_e}{m_e c^2} \tag{8}$$

in which case $E_y/E_z \simeq -K_{zz}/K_{zy}$, or as shown by Stix,[1]

$$\frac{E_z}{E_y} = -\frac{ik_\perp k_\| v_{te}^2}{2\omega\omega_{ce}} \ . \tag{9}$$

This result is valid for arbitrary values of $\omega/k_\| v_{te}$, (as long as the unity term in K_{zz} can be ignored). This results from the fact that $[1+\zeta_e Z(\zeta_e)]$ cancels in the ratio of K_{zz}/K_{zy}. Note that for $\omega \sim k_\| v_{te}$, $|E_z/E_y| \sim k_\perp r_{ce}/2 \sim 10^{-3}$ and therefore the electron absorption will be relatively weak. Using Eq.(9) in (5b) results in a cancellation of the first and second terms (i.e., TTMP and the cross term cancel) and only the third term, namely Landau damping survives. As noted by Stix,[1] its magnitude is 1/2 that of TTMP. Thus, the damping of the fast magnetosonic wave in a Maxwellian plasma at low frequencies and high temperatures is entirely due to Landau damping for arbitrary values of the wave phase velocity.

IV. Damping Rate

Dividing the last term of Eq.(5) with S and using Eq.(9), we obtain for the spatial damping rate

$$2k_{\perp Im} = k_{\perp Re}\left(\frac{\pi^{1/2}}{2}\right)\beta_e\zeta_e\exp(-\zeta_e^2) \tag{10}$$

where $\beta_e = 8\pi n_e T_e/B^2$ is the electron beta. Replacing $k_{\perp Re} \sim \omega/v_A$, we get

$$2k_{\perp Im} = \frac{\pi}{2}^{1/2}\frac{\omega}{\omega_{ci}}\frac{\omega_{pi}}{c}\beta_e\zeta_e\exp(-\zeta_e^2) \tag{11}$$

so that for $\zeta_e \sim 1$, single pass absorption is proportional to ω, $n^{3/2}$, T_e and B^{-3}. We also note that the maximum absorption occurs for $\zeta_e \sim 0.7$. For example, for present day machines, $T_{eo} \simeq 6.0$ keV, $B_o \simeq 2.0$ T, $n_o \sim 5 \times 10^{19}\text{m}^{-3}$, $\beta_e \sim 0.03$, $f = 76$ MHz in D plasma, $\zeta_e \sim 0.7$, $\Delta x \sim a/2 \sim 0.50$ m, $2k_\perp\Delta x \simeq 0.62$, and the single pass absorption is $[1-\exp(-2k_{\perp Im}\Delta x)] \simeq 0.47$. This may be a typical achievable value in the DIII-D tokamak. The required parallel wavelength at the antenna would be $\lambda_\| \simeq 60$ cm which is very reasonable (toroidal wave propagation effects would reduce this to $\lambda_\| \simeq 44$ cm near the center of the plasma where $\zeta_e \simeq 0.7$).

Now we consider the more general result in Eq.(7), namely retain the first term in the denominator (i.e. do not assume Eq.(8)). Taking the inverse of Eq.(7), we obtain

$$\frac{E_z}{E_y} = -i\frac{k_\perp k_\| v_{te}^2}{2\omega\omega_{ce}} + \frac{K_{xy} n_x n_z}{(K_{xx} - n_z^2) K_{zz}} \ . \tag{12}$$

Now we find that as before, in Eq.(5) the TTMP term cancels with the cross term for the first term of Eq.(12). However, the second term of Eq.(12) survives with the cross-term and adds to the Landau damping term, increasing its effectiveness at higher frequencies. The net damping decrement is obtained by combining Eqs.(5) and (12), and upon dividing by the Poynting flux we obtain

$$2k_{\perp Im} = k_{\perp Re}(\frac{\pi}{2})^{1/2}\beta_e\zeta_e e^{-\zeta_e^2}[1 + \frac{1}{\alpha^2}] \tag{13}$$

where the surviving cross-term gives

$$\alpha = \frac{T_e}{m_i c^2}(\frac{\omega^2 - \omega_{ci}^2}{\omega_{pi}^2})(S - n_\|^2)|K_{zz}| \tag{14}$$

where

$$S = 1 - \sum_j \omega_{pj}^2/(\omega^2 - \omega_{cj}^2)$$

is the cold plasma limit of K_{xx}. Note that in Eq.(14) the absolute value of K_{zz} is to be taken which requires special attention if $\zeta_e \simeq 0(1)$. For $|S| \gg n_\|^2$, and $|S| \gg 1$, Eq.(14) can be written in the following form:

$$\frac{1}{\alpha^2} = \left(\frac{m_e c^2}{T_e}\right)^2 \frac{\omega^4}{4\omega_{pi}^4}\frac{1}{\zeta_e^4|1 + \zeta_e Z(\zeta)|^2} \ . \tag{15}$$

In the cold plasma limit ($\zeta_e^2 \gg 1$), Eq.(15) reduces to

$$\frac{1}{\alpha^2} \simeq \left(\frac{m_e c^2}{T_e}\right)^2 \frac{\omega^4}{\omega_{pi}^4} \ , \tag{16a}$$

whereas in the hot plasma limit ($\zeta_e^2 \ll 1$), Eq.(15) becomes

$$\frac{1}{\alpha^2} \simeq \left(\frac{m_e c^2}{T_e}\right)^2 \frac{\omega^4}{4\omega_{pi}^4}\frac{1}{\zeta_e^4(1 - 0.86\zeta_e^2)} \simeq \left(\frac{\omega^4}{\omega_{pi}^4}\right) n_\|^4. \tag{16b}$$

The result Eq.(16a) has been noted previously by Moreau et al.[9] For example, for $\zeta_e \simeq 0.7$ and the previously listed plasma parameters $1/\alpha^2 \simeq 0.31$. However, lowering the electron temperature from $T_e \simeq 6.0$ keV to $T_e \simeq 3.0$ keV increases $1/\alpha^2$ to unity. At higher phase velocities ($\zeta_e \gtrsim 1$), $1/\alpha^2$ is less significant for $T_e \gtrsim 3.0$ keV. In Fig. 1 we give a numerically evaluated plot of Eq.(15). Note the dramatic increase of $1/\alpha^2$ for $\zeta_e \leq 1$, reflecting the ζ_e^{-4} dependence of $1/\alpha^2$ in this limit.

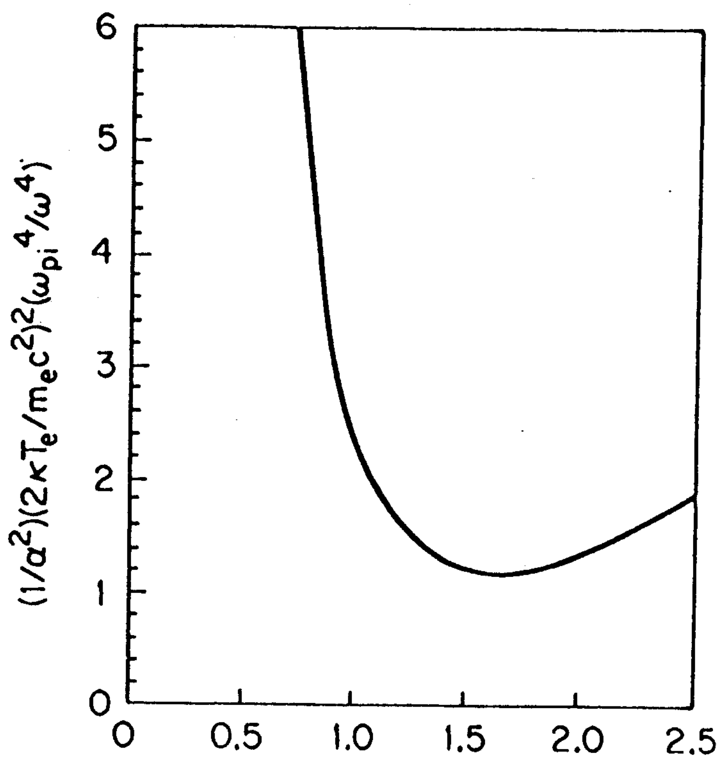

Fig. 1 The normalized value of $1/\alpha^2$ as a function of ζ_e.

V. Two Component Plasma with a High Energy Electron Tail

In recent experiments in JET a synergism between high phase velocity fast waves and a pre-formed electron tail, driven by lower-hybrid waves, has been discovered.[11] Similar phenomena might be expected to occur if fast waves were launched into a run-away dominated discharge. Here of interest is a range of phase velocities such that

$$v_t \ll \omega/k_\parallel \simeq v_{h\parallel} \tag{17}$$

where $v_{h\parallel} = (2T_{h\parallel}/m_e)^{1/2}$ is the effective mean velocity of the hot electrons, while v_t is that of the cold electrons. Such high phase velocity waves can be launched by a "monopole" type phasing of the fast wave antenna current straps (i.e., 2 or more current straps fed in phase from the transmitters). For example, in a typical lower-hybrid current driven discharge, $T_{h\parallel} \simeq 100 - 500$ keV, $T_{h\perp} \sim 70$ keV, $T_e \sim 1 - 5$ keV, and $0.005 \lesssim n_h/n_e \lesssim 0.01$, where n_h is the density of hot electrons.[14] Here we shall assume that the hot component is also characterized by a Maxwellian:

$$f_e(\vec{v}) = \frac{n_h}{\pi^{3/2} v_{h\perp}^2 v_{h\parallel}} e^{-\left(\frac{v_\parallel^2}{v_{h\parallel}^2} + \frac{v_\perp^2}{v_{h\perp}^2}\right)} \tag{18}$$

We can repeat the previous calculations for an anisotropic plasma. The polarization will be mainly determined by the cold plasma, whereas the absorption will be on the hot plasma component. In the limit where the effective $1/\alpha^2$ is negligible, the result of the calculations is as follows:[(8)]

$$2k_{\perp Im} = k_{\perp Re}\left(\frac{\pi^{1/2}}{2}\right)\beta_{h\perp}\frac{T_{h\perp}}{T_{h\parallel}}\zeta_h e^{-\zeta_h^2}\left[2\left(1 - \frac{T_\perp}{T_{h\perp}}\right) + \frac{T_\perp^2}{T_{h\perp}^2}\right] \tag{19}$$

where the first term in the bracket corresponds to TTMP, the second term is the cross-term and the third term is the Landau damping term. Here $T_\perp$ designates the temperature of the cold plasma component. We have designated the beta of the hot perpendicular component by

$$\beta_{h\perp} = \frac{8\pi n_{eh} T_{h\perp}}{B^2} \tag{20}$$

and $\zeta_h = \omega/k_\parallel v_{h\parallel}$. Since in the present case $T_\perp \ll T_{h\perp}$, we see that the cross term and the Landau damping terms are negligible as compared to TTMP (the first term in the bracket). Furthermore, if $T_{h\perp} \ll T_{h\parallel}$, as is often the case, for $\zeta_{h\parallel} \simeq 1$, $\beta_{h\perp} = \beta_{bulk}$, the absorption is reduced as compared to the bulk absorption case when $\zeta_{bulk} \simeq 1$. Substituting in typical numbers from the JET experiments we find that Eq.(19) predicts very weak single pass absorption, of the order of one percent. Therefore, we expect a rather weak effect on the overall current drive efficiency, in disagreement with experimental results. These experiments were complicated by the fact that a minority component and ion cyclotron resonance layer were also present. This would introduce an ion Bernstein wave (IBW) mode-conversion layer and diffusion of fast electrons by these waves would have to be invoked to explain the results.[(15)]

Let us now introduce the finite frequency correction ($1/\alpha^2$ term) into the wave polarization while maintaining absorption on the hot tail (i.e., $\zeta_{h\parallel} \simeq 1$, $\zeta_e \gg 1$, where $\zeta_e = \omega/k_\parallel v_{t\parallel}$ designates the bulk plasma). Then it is straightforward to show that the polarization will be determined by the bulk plasma. Carrying out the algebra, Eq.(19) will be modified by inclusion of two additional terms:

$$2k_{\perp Im} = k_{\perp Re}\left(\frac{\pi^{1/2}}{2}\right)\beta_{h\perp}\frac{T_{h\perp}}{T_{h\parallel}}\zeta_h e^{-\zeta_h^2}\Big[2(1 - \frac{T_\perp}{T_{h\perp}}\Big) + \frac{T_\perp^2}{T_{h\perp}^2}$$

$$+\frac{2}{\alpha}\frac{T_\perp}{T_{\perp h}}\left(1-\frac{T_\perp}{T_{\parallel h}}\right)+\frac{1}{\alpha^2}\frac{T_\perp^2}{T_{\perp h}^2}\Big] \quad (21)$$

where we assumed that $K_{zzRe}/|K_{zz}| \simeq 1$, and in particular, $|K_{zzRe}| \simeq \omega_{pe}^2/\omega^2$ due to the "fluid" approximation on the bulk plasma. Note that for $T = T_h$, Eq.(21) will reduce to Eq.(13) as expected. Furthermore, for $\alpha \to \infty$, Eq.(21) reduces to Eq.(19). In the ICRF regime, for $T_{\perp h} \gg T_\perp$, the first term (TTMP) will remain dominant. However at higher frequencies ($\omega_{ci} \ll \omega < \omega_{LH}$) the last term could dominate. We note that at $\omega \sim \omega_{LH}$, Eq.(21) is not strictly valid since some of the approximations may not hold (in particular, as discussed earlier the approximation $n_x^2 \ll K_{zz}$ may not hold when $\omega \sim \omega_{LH}$). We note that a result similar to Eq.(21) has been derived recently by Moroz et al.[16]

VI. Absorption of Magnetosonic Waves on Ion Cyclotron Harmonics

One of the competing mechanisms for the absorption of fast magnetosonic waves is absorption on ions near the fundamental, or the harmonics of the ion cyclotron frequency.[7] This may occur on bulk ions, or fast ions due to simultaneous neutral beam injection, or even on alpha particles in a reactor grade plasma. From the current drive point of view this must be regarded as "parasitic" absorption since it removes effective power from electrons which drive the current. The ion absorption can occur by means of direct ion absorption, or by means of mode conversion into an ion Bernstein wave (IBW). The latter process may dominate if $n_\parallel \simeq 0$. Here we shall assume that $n_\parallel$ is finite (and in particular, $\omega/k_\parallel v_{te} \sim 1$ or $n_\parallel \simeq v_{te}/c$), and that the fast wave power density is not high enough to distort the initial Maxwellian distribution of ions. When this condition is violated the situation becomes considerably more complex.[1]

Ion cyclotron harmonic absorption in the limit of near-perpendicular propagation may be obtained from the general result Eq.(3). After a considerable amount of algebra, for $b_i \ll 1$ and $\omega \simeq \ell\omega_{ci}$, Eq.(3) reduces to

$$P = \frac{\omega_{pi}^2}{16\pi^{1/2}}\frac{b_i^{(\ell-1)}}{k_\parallel v_{ti}}\frac{\ell}{(\ell-1)!2^{(\ell-1)}}|E_+|^2 \exp\left[-\frac{(\omega-\ell\omega_{ci})^2}{k_\parallel^2 v_{ti}^2}\right] \quad (22)$$

where we retained hot plasma terms with E_x, E_y, K_{xx}, K_{yy}, K_{xy} and K_{yx}. Here $E_+ = E_x + iE_y$ is the left hand polarized component of the wave electric field, and $b_i = k_\perp^2 r_{ci}^2$ is the finite ion Larmor radius factor ($r_{ci} = v_{ti}/2^{1/2}\omega_{ci}$). This formula is valid for $b_i \ll 1$, which is usually satisfied for not too high harmonics since $k_\perp \sim \omega/v_A$ so that $k_\perp^2 r_{ci}^2 \sim \ell^2\beta_i/2$.

In the simplest case, wave polarization is obtained from the cold plasma dielectric tensor Eq.(6) where the following dielectric constant terms are used:[12]

$$K_{xx} = K_{yy} = S = 1 - \sum_j \frac{\omega_{pj}^2}{\omega^2 - \Omega_j^2} \simeq -\frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} \tag{23}$$

$$K_{xy} = -iD = -i\sum_j \frac{\omega_{pj}^2 \Omega_j}{(\omega^2 - \Omega_j^2)\omega} \simeq -\frac{i\omega_{pi}^2}{\omega^2 - \omega_{ci}^2}\left(\frac{\omega}{\omega_{ci}}\right) \tag{24}$$

where the last equality in each line is valid in the limit $\omega \sim \ell\omega_{ci} \ll \omega_{pi}$, and $1 \ll \omega_{pi}^2/\omega_{ci}^2$. We also ignore K_{zz} and E_z. Then the polarization is obtained by combining the first two rows of Eq.(6):

$$\left|\frac{E_+}{E_y}\right|^2 \simeq \frac{(\frac{\omega}{\omega_{ci}} - 1)^2(\frac{\omega}{\omega_{ci}} - \cos^2\theta)^2}{|\frac{\omega}{\omega_{ci}}(1 + \cos^2\theta)|^2} \Rightarrow (\ell - 1)^2 \tag{25}$$

where $\ell = \omega/\omega_{ci}$, $\cos\theta \simeq k_\parallel/k$, and the last limit is valid for $\cos\theta \to 0$. Equation (25) predicts the well known result that at $\omega = \omega_{ci}$, $E_+ \to 0$ since the ions shield out the left-hand polarized component of the electric field (i.e., the magnetosonic wave becomes purely right-hand polarized for perpendicular propagation). As is well known,[1] if strong ion absorption is desired, this problem may be remedied by injecting a minority ion species into the plasma, typically a few percent of hydrogen or helium-3 ions into a deuterium majority plasma. Thus ion cyclotron absorption becomes effective again at $\omega = \omega_{cm}$, where the subscript m designates the minority species, since $E_+(\omega \neq \omega_{cM}) \neq 0$ (where M designates the usually heavier ion species). For $\omega = 2\omega_{CD} = \omega_{CH}$, $|E_+/E_y|^2 = 1$, while for a He-3 minority resonance in a deuterium plasma $\omega = (4/3)\omega_{CD}$, and $|E_+/E_y|^2 = 1/9$. As a consequence, in a deuterium plasma He-3 minority absorption is weaker than that due to H minority.

We can obtain the damping by integrating the power absorbed across a cyclotron harmonic resonance layer in a radially inhomogeneous magnetic field, and divide the absorbed power by the Poynting flux, $S_\perp = (c/8\pi)n_\perp|E_y|^2$. The dominant factor in the integral comes from the exponential factor,

$$\frac{\omega - \ell\omega_{ci}(x)}{k_\parallel v_{ti}} \simeq \frac{x}{2^{\frac{1}{2}}\Delta} \tag{26}$$

where we wrote $\omega_{ci}(x) \simeq \omega_{ci}(1 + x/R)$, $\omega \simeq \ell\omega_{ci}$, R is the tokamak major radius and $\Delta \equiv k_\parallel v_{ti} R/2^{1/2}\omega$. Without loss of generality, the limits of integration may be extended to infinity and we have

$$2\int_{-\infty}^{\infty} k_{Im}(x)dx \propto \frac{1}{2^{\frac{1}{2}}\Delta}\int_{-\infty}^{\infty} \exp(-x^2/2\Delta^2) = \pi^{1/2} \ ,$$

so that the spatial damping decrement $2\int k_{Im}(x)dx \equiv 2\eta$ becomes

$$2\eta = \frac{\pi}{2}\frac{\omega_{pi}}{c}\frac{R\beta_i^{(\ell-1)}\ell^{2(\ell-1)}}{(\ell-1)!2^{2(\ell-1)}}\frac{|E_+|^2}{|E_y|^2} \ . \tag{27a}$$

Here we assumed $\omega \simeq k_\perp v_A$ (neglecting $k_\|$) so that $k_\perp \simeq \ell\omega_{pi}/c$, and $\beta_i = 8\pi n_i T_i/B^2$ is the ion beta. This result can be evaluated easily for polarization in the cold plasma limit, in particular, combining Eqs.(25) and (27) gives

$$2\eta = \frac{\pi}{2}\frac{\omega_{pi}}{c}R(\frac{\ell^2\beta_i}{4})^{(\ell-1)}\frac{(\ell-1)}{(\ell-2)!} \tag{27b}$$

where we rearranged some of the numerical factors. The transmission coefficient is given by

$$T = e^{-2\eta} \ , \tag{28a}$$

and absorption is given by

$$A = 1 - T = 1 - e^{-2\eta} \ . \tag{28b}$$

In particular, we have the following results for ℓ =1 to 4:

Table I

ℓ	2η
1	0
2	$R\pi\omega_{pi}\beta_i/2c$
3	$R\pi\omega_{pi}\beta^2(81/16c)$
4	$R\pi\omega_{pi}\beta^3(48/c)$

We recall once more that for the low harmonic numbers which are of practical interest we assumed $\ell^2\beta_i/2 \ll 1$.

A comparison of Eqs.(13) and (27) give the single-pass electron to ion cyclotron harmonic absorption ratio, $r = 2ak_{\perp Im}^{avg}/2\eta_{ion}$, or

$$r = (\frac{2}{\pi})^{1/2}(\frac{a}{R})\frac{\ell(\ell-2)!}{(\ell-1)}\frac{\beta_e < \theta >}{(\ell^2\beta_i/4)^{(\ell-1)}} , \tag{29}$$

where a is the minor radius, and we assumed that the effective "average" absorption distance is the hot core region of the plasma column, or $< \Delta r >_{eff} \simeq d/2 = a$, with d being the plasma diameter. In this region we take

$$< \theta > = < \zeta_e \exp(-\zeta_e^2)(1+\alpha^{-2}) >_{avg} ,$$

where an optimized value of $< \theta > \simeq 0.5$ is assumed (i.e., $\zeta_e \lesssim 1.0$). Thus, for $a/R \simeq 1/3$, $\ell = 2$ we get $r \simeq (\beta_e/4\beta_i)$ so that absorption on ions dominates for $\beta_i \simeq \beta_e$. However, for $\ell \geq 3$, $\beta_e \simeq \beta_i \lesssim 0.10$, electron absorption dominates over ion cyclotron harmonic absorption. Thus, for effective fast wave current drive we should use $\omega \gtrsim 4\omega_{ci}$ or $\omega \lesssim \omega_{ci}$. Note that additional ion absorption mechanisms due to minority species (H or He-3), neutral beam particles or alpha particles need to be considered.

VII. Mode Conversion Regime

We can further improve on this theory by including the hot plasma contribution to the polarization calculation. In particular, by retaining the hot plasma contribution to K_{xx} and K_{xy}, we have

$$\frac{E_x}{E_y} = -\frac{K_{xy}}{K_{xx}} = \frac{-i\ell A - iBZ_{Im}}{A + BZ_{Im}} \tag{30}$$

where

$$A = -\omega_{pi}^2/(\omega^2 - {\omega_{ci}}^2)$$

$$B = \frac{b_i^{\ell-1}\omega_{pi}^2\ell^2}{2k_{\|}v_{ti}\omega 2^{(\ell-1)}\ell!}$$

and

$$Z_{Im} = \sqrt{\pi}\exp(-(\omega - \ell\omega_{ci})^2/k_{\|}^2 v_{ti}^2).$$

Thus, we can evaluate $|E_+/E_y|^2$ and obtain the following result for the polarization:

$$\frac{|E_+|^2}{|E_y|^2} = \frac{(\ell-1)^2}{1+\sigma_\ell^2} \tag{31a}$$

where we assumed $\ell \gtrsim 2$, and

$$\sigma_\ell^2 \simeq \pi^{1/2}\left(\frac{\ell^2\beta_i}{4}\right)^{\ell-1}\frac{(\ell^2-1)}{2\ell!}\zeta_{oi}F\ , \tag{31b}$$

where $\zeta_{oi} = \omega/k_{\|}v_{ti}$ and $F \simeq \exp(-2\zeta_{-\ell}^2)$. Thus, to correct for polarization effects, including hot plasma corrections, we simply divide Eq. 27(b) by $(1+\sigma_\ell^2)$, where for simplicity we take $F \Rightarrow 1$ (more accurately, we should repeat the intergration over x but the final result remains the same as taking $F \Rightarrow 1$).

Note that σ_ℓ corresponds to the ratio of δ, the separation between the cyclotron resonance layer and the mode conversion layer, to $\Delta \propto k_{\|}v_{ti}R/\omega$, the width of the cyclotron resonance layer. The "cold plasma" result is obtained in the limit $\sigma_\ell^2 \ll 1$. For example, for $\ell = 2, \delta \simeq \beta_i R/2$ and $\delta/\Delta \simeq \beta_i\zeta_{oi} \sim \sigma_2$. For $\sigma^2 \ll 1$ cyclotron harmonic absorption dominates, and if $2\eta > 1$, we have effective absorption on ions. In the opposite limit, $\sigma_\ell > 1$, we have effective "depolarization," and we end up in the mode-conversion regime (from the fast wave to ion Bernstein wave or IBW). It has been shown[18] that in the mode conversion regime for low field side launch the transmission (T), reflection (R) and mode conversion (M) coefficients are given by the Budden factors

$$T = \exp(-2\eta) \tag{32a}$$

$$R = (1 - T)^2 \tag{32b}$$

$$M = T(1 - T) \tag{32c}$$

so that $R + T + M = 1$.

The results for high-field side launch are

$$T = e^{-2\eta} \tag{33a}$$

$$R = 0 \tag{33b}$$

$$M = 1 - T \tag{33c}$$

so that effective mode conversion takes place since waves arrive at this layer first. The fate of the mode converted IBW is somewhat complicated. On the "midplane" it may simply convect out of the plasma, while off the midplane it may be absorbed by electrons.

VIII. Minority Absorption Regime

It is straightforward to include absorption by minority ion species in the previous theory. The absorbed power is

$$P_m = \frac{\omega_{pm}^2}{16\pi^{1/2}} \frac{|E_+|^2}{k_\parallel v_{tm}} \exp\Big[-\frac{(\omega - \omega_{cm}^2)}{k_\parallel^2 v_{tm}^2}\Big] \tag{34}$$

where m is the minority species. Integrating across the resonance layer as before, the damping decrement is given by

$$2\eta = \frac{\pi}{2}\frac{\omega_{PM}}{c}\frac{n_m}{n_M}\frac{Z_m}{Z_M}R|E_+|^2/|E_y|^2 \tag{35}$$

where R is the major radius, M designates the majority species, ω_{PM} is the majority angular ion plasma frequency, Z is the ion charge, and n_m/n_M is the minority to majority density ratio. We can again carry out the hot plasma polarization calculations and obtain

$$\frac{|E_1^+|^2}{|E_y|^2} = \frac{1}{1+\sigma_1^2} \tag{36}$$

where

$$\sigma_1^2 = \frac{\pi}{4}\left(\frac{n_m}{n_M}\frac{M}{m}\frac{Z_m^2}{Z_M^2}\right)^2\left(1-\frac{\omega_{cM}^2}{\omega^2}\right)^2\left(\frac{\omega}{k_\parallel v_{ti}}\right)^2 \tag{37}$$

again separates the ion absorption regime ($\sigma_1^2 \ll 1$) to the mode-conversion regime ($\sigma_1^2 > 1$). Here M/m is the majority to minority ion mass ratio, and ω_{cM} is the majority ion cyclotron frequency (here $\delta \sim Rn_m/n_M$ and $\Delta \sim k_\parallel v_{tm}R/\omega$ and the ratio gives σ_1). We note that for H^+ minority, D^+ majority ions $k_\perp \sim 2\omega_{pM}/c$, and

$$2\eta = \frac{\pi}{2}\frac{\omega_{pM}}{c}\frac{n_m}{n_M}R\ . \tag{38}$$

Thus, the role of β_i (second harmonic deuterium absorption) has been replaced by n_m/n_M, the fraction of minority ions. If $\beta_M < n_m/n_M$, minority absorption dominates. Note that minority absorption is very effective even in a relatively cold plasma when β_M may be low.

IX. Absorption of Magnetosonic Waves at the Fundamental Majority Ion Cyclotron Resonance ($\ell = 1$)

Although in the cold plasma limit for perpendicular propagation we found no absorption, by introducing finite values of $k_\parallel$, $E_+ \neq 0$ if the ion temperature is sufficiently high. It is a simple matter to show that by including Doppler-shifts due to finite $k_\parallel v_{ti}$, the left hand polarized component of the fast wave is finite and Eq.(31a) would be replaced with

$$|E_+|^2/|E_y|^2 = \frac{2}{\pi}k_\parallel^2 r_{ci}^2\ . \tag{39}$$

where r_{ci} is the ion gyro-radius $(b_i^{1/2}/k_\perp)$. Combining this with the power absorption formula at $\omega = \omega_{ci}$ we obtain

$$2\eta = \frac{\omega_{pi}}{c} R k_\parallel^2 r_{CM}^2 \, . \tag{40}$$

Noting that $k_\parallel^2 r_{CM}^2 = k_\parallel^2 v_{ti}^2/2\omega^2 = n_\parallel^2 T_i/m_i c^2$ for $\omega = \omega_{CM}$, the finite absorption depends on Doppler shift. We note that majority ion cyclotron resonance absorption is weaker than harmonic ion cyclotron absorption by the ratio $(2/\pi) k_\parallel^2 r_{CM}^2/\beta_M \simeq (n_\parallel^2 \omega_{ci}^2/\omega_{pi}^2 \pi) \ll 1$. Similar results apply in comparison with minority absorption or with electron absorption. It should be noted, however, that this treatment ignores potential difficulties with mode conversion into shear Alfven waves on the high field side of the resonance.[4]

X. Harmonic Ion Cyclotron Absorption by High Energy Ions

Absorption of magnetosonic waves by hot ions may be of importance when simultaneous neutral beam injection is taking place, or when the rf power is strong enough to form an energetic ion tail by quasi-linear diffusion.[1] The ions have a slowing-down energy distribution which is best modelled by Monte-Carlo techniques in the case of neutral beam injection. As shown by simulations, a typical distribution function may be characterized in an approximate way by a Maxwellian with energies $T_{h\parallel} \sim \varepsilon_{bmax}/3$, $T_{h\perp} \sim \varepsilon_{bmax}/4$, and an ion population of a few percent of the bulk ions. If a harmonic cyclotron resonance layer is present in the plasma, significant wave power loss to the beam ions may result, while the beam ions would be accelerated to higher energies. This may be beneficial to beam penetration near the center of the plasma while detrimental near the edge (depending on the location of the cyclotron harmonic layer). In any case, it would be detrimental to driving plasma currents with the wave. It should be noted that such beam acceleration has been found in recent 3rd harmonic resonance experiments.[19]

We can easily repeat the previous calculations, with special care given to absorption by the hot ions while using the bulk plasma parameters for dispersion and polarization. The result of the calculation is

$$2\eta = \frac{\pi}{2} \frac{\omega_{pb} R}{c} \left(\frac{\ell^2 \beta_{h\perp}}{4}\right)^{(\ell-1)} \left(\frac{n_b}{n_h}\right)^{(\ell-2)} \frac{(\ell-1)}{(\ell-2)!} \, , \tag{41}$$

where sub-b designates the bulk (background) ions, sub-h designates the hot ions, $\beta_{h\perp} = 8\pi n_h T_{h\perp}/B^2$ is the hot ion beta perpendicular component, and ℓ is the harmonic number

($\ell = \omega/\omega_{ci}$). Note again, that this result is valid only for $k_\perp^2 r_{ch}^2 = (\ell^2/2)(n_b/n_h)\beta_h \ll 1$ and $\ell \geq 2$. We see that formula (41) has some interesting dependence on harmonic numbers. Comparing Eqs.(27b) and (41), we find for the ratio of cyclotron harmonic absorption on hot (beam) ions versus that on bulk ions is

$$r = \left(\frac{\beta_{h\perp}}{\beta_b}\right)^{\ell-1} \left(\frac{n_b}{n_h}\right)^{\ell-2} . \tag{42}$$

It is interesting to note that for $\ell = 2$, $r = \beta_h/\beta_b$ and absorption on hot ions may be comparable to, or less than that on the bulk ions. However, for $\ell \geq 3$, even if $\beta_h/\beta_b \lesssim 1$, absorption on the hot ions may well dominate if $n_b \gg n_h$.

XI. Quasi-Linear Theory

Another way to consider absorption of waves in the plasma is by means of quasi-linear theory. This formalism has the advantage of easy generalization to non-Maxwellian distributions, including that created by the incident rf waves themselves. Although the waves may be coherent, the particles transiting them lose phase memory as they pass around the torus hundreds of times and experience rare collisions.(1) To obtain the true distribution function of a species of charged particles, one must solve a Fokker-Planck equation, including quasi-linear diffusion and collisional drag and diffusion of the form(1)

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial t}|_{QL} - \nabla_{\vec{v}}(< \Delta\vec{v} > f) + \frac{1}{2}\nabla_{\vec{v}} \cdot [\nabla_{\vec{v}} \cdot (< \nabla\vec{v}\nabla\vec{v} > f)] \tag{43a}$$

where the 2nd and 3rd terms on the RHS correspond to collisional tems and

$$\frac{\partial f}{\partial t}|_{QL} = \frac{\partial}{\partial \vec{v}} \cdot \overleftrightarrow{D}_{QL} \cdot \frac{\partial f}{\partial \vec{v}} \tag{43b}$$

is the quasi-linear term. In general this is a difficult problem which has been solved only in a few instances. For example the case of a minority species distribution function in the presence of ICRF heating and collisional drag has been determined by Stix.(1) The experimental verification of this theory has been one of the triumphs of ICRF experiments on tokamaks during the past decade, and it will be discussed by other authors in these Proceedings. In the steady state the result is the characterization of a high energy minority tail by an effective temperature(1)

$$\frac{1}{T_{eff}} = \frac{1}{T_e(1+\zeta)}\left[1 + \frac{R_j(T_e - T_j + \zeta T_e)}{T_j(1 + R_j + \zeta)} \frac{1}{1 + (E/E_j)^{3/2}}\right] \tag{44}$$

where

$$R_j = \frac{n_j Z_j^2}{n_e}\left(\frac{v_{te}}{v_{tj}}\right),$$
$$\zeta = \frac{m\langle P\rangle v_{te}}{8\pi^{1/2} n_e n Z^2 e^4 \ell n\Lambda},$$

and $\langle P\rangle$ is the average power per unit volume deposited. Here the majority ion species is characterized by density n_j, temperature T_j, charge eZ_j, thermal speed $v_{tj} = (2T_j/m_j)^{1/2}$; electrons are characterized by density n_e, and thermal speed v_{te}; the minority species being accelerated by cyclotron resonance are designated by n, m, v, Z and $E = mv^2/2$. We note that for $\zeta \simeq 0$ the minority ion species is characterized by a temperature close to that of the majority ion temperature, whereas for $E \gg E_j(\zeta)$, $T_{eff} \simeq T_e(1+\zeta)$ and ion acceleration is entirely balanced by electron drag.

Let us now ignore collisions and discuss the quasi-linear diffusion term in the presence of rf waves. In particular, the power absorbed by charged particles can be calculated as follows:

$$P = \int d^3v \frac{mv^2}{2}\left(\frac{\partial f_o}{\partial t}\right)_{QL}. \tag{45}$$

The quasi-linear evolution of the distribution function in a magnetized plasma has been given by Kennel and Engelmann nearly three decades ago[17] and it may be written in the following form:

$$\left(\frac{\partial f}{\partial t}\right)_{QL} = \lim_{V\to\infty} \sum_{\ell} \frac{\pi Z^2 e^2}{m^2}\int \frac{d^3k}{(2\pi)^3 V}\mathcal{L} v_\perp \delta(\omega - \ell\Omega - k_\parallel v_\parallel)|\Theta_{\ell,k}|^2 v_\perp \mathcal{L} f \tag{46}$$

where

$$\mathcal{L} = \left(1 - \frac{k_\parallel v_\parallel}{\omega}\right)\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} + \frac{k_\parallel}{\omega}\frac{\partial}{\partial v_\parallel}$$
$$\Theta_{\ell,k} = \frac{1}{2}e^{+i\psi}(E_x - iE_y)_k J_{\ell+1}\left(\frac{k_\perp v_\perp}{\Omega}\right) + \frac{1}{2}e^{-i\psi}(E_x + iE_y)_k J_{\ell-1}\left(\frac{k_\perp v_\perp}{\Omega}\right) + \frac{v_\parallel}{v_\perp}E_{zk} J_\ell\left(\frac{k_\perp v_\perp}{\Omega}\right)$$

where $\vec{k} = k_\perp \cos\psi\hat{x} + k_\perp \sin\psi\hat{y} + k_\parallel\hat{z}$. Here V is the plasma volume, $\vec{E}_k$ are the Fourier amplitudes of the electric field and ψ is the angle between the $\hat{x}$ direction and $\vec{k}_\perp$, the wave vector in the plane perpendicular to $\vec{B}$, the ambient dc magnetic field. The absorption of fast waves by ion cyclotron resonance can be deduced from terms being proportional to $E_+ = E_x + iE_y$, $\ell \geq 1$. Carrying out the integration over $\vec{k}$, we readily deduce the following relevant expression for ion cyclotron resonance absorption:

$$\frac{\partial f_o(\vec{v})}{\partial t} = \frac{\pi Z^2 e^2}{8m^2|k_\parallel|}\sum_\ell \left|E_+\right|^2 \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} v_\perp^2 J_{\ell-1}^2\Big(\frac{k_\perp v_\perp}{\omega_{ci}}\Big)\delta\Big(v_\parallel - \frac{\omega - \ell\omega_{ci}}{k_\parallel}\Big)\frac{1}{v_\perp}\frac{\partial f_o}{\partial v_\perp} \qquad (47)$$

where the summation is over ion cyclotron harmonics. Using cylindrical coordinates and azimuthal symmetry, we may readily proceed with the integration in Eq.(45):

$$P = \frac{\pi^2 Z^2 e^2}{8m|k_\parallel|} f_{o\parallel}(v_{\parallel Res})|E_+|^2 \int_0^\infty f_{o\perp}\frac{\partial}{\partial v_\perp}\left[v_\perp^2 J_{\ell-1}^2\Big(\frac{k_\perp v_\perp}{\omega_{ci}}\Big)\right] dv_\perp \qquad (48)$$

where we integrated by parts twice for convenience. For a Maxwellian, Eq.(48) can be readily integrated for small arguments of the Bessel function and we obtain

$$P = \frac{\omega_{pi}^2 |E_+|^2}{16\pi^{1/2} k_\parallel v_{ti}} b_i^{(\ell-1)} \frac{\ell}{(\ell-1)!2^{(\ell-1)}} \exp\left[-\frac{(\omega - \ell\omega_{ci})^2}{k_\parallel^2 v_{ti}^2}\right] \qquad (49)$$

which is exactly the same result as Eq.(22). Thus, quasi-linear theory gives the same result as Eq.(3) for a Maxwellian distribution. We can now use Eq.(48) to integrate over a Maxwellian distribution to all orders of the finite ion Larmor radius. In particular, taking the derivative in Eq.(48) and integrating by parts, Eq.(48) can be integrated exactly and we obtain

$$P = \frac{\omega_{pi}^2 |E_+|^2 \ell}{16\pi^{1/2} k_\parallel v_{ti}}\left[I_{\ell-1}(b_i) + \frac{b_i}{\ell}\Big(I_\ell(b_i) - I_{\ell-1}(b_i)\Big)\right] e^{-b_i} \exp\left[-\frac{(\omega - \ell\omega_{ci})^2}{k_\parallel^2 v_{ti}^2}\right] \qquad (50)$$

where $I_\ell(b_i)$ is the modified Bessel function of order ℓ and argument b_i. The small Larmor radius limit of Eq.(50) results from the first term of the square bracket (expand $I_{\ell-1}(b_i)$) and it agrees with Eq.(49). It is easy to generalize Eq.(50) to include absorption on an energetic minority ion species (compare Eq.(50) with (41)). These examples show the power of using quasi-linear theory for calculating power absorption to all orders of the

Larmor radius, and for arbitrary distribution functions. In particular, larger values of b_i lead to stronger absorption, and the result is that cyclotron harmonic resonances in the plasma lead to energetic ion tail production due to quasi-linear diffusion.

We can also use Eqs.(45) and (46) to calculate power absorption due to electron Landau damping and electron transit time magnetic pumping. In this case we take the $\ell = 0$ terms and expand the Bessel functions for small arguments (i.e., small electron Larmor radius expansion) before integrating over $v_\perp$. In particular, we use $J_o(k_\perp v_\perp/\omega_{ce}) \simeq 1$, $J_{\pm 1} k_\perp v_\perp/\omega_{ce}) = \pm k_\perp v_\perp / 2\omega_{ce}$, and we obtain in Eq.(46)

$$v_\perp^2 |\Theta_o|^2 = \frac{1}{4} \frac{k_\perp^2}{\omega_{ce}^2} |E_y|^2 \left| v_\perp^2 - \frac{2i\omega_{ce}}{k_\perp} \frac{\omega}{k_\parallel} \frac{E_z}{E_y} \right|^2 . \tag{51}$$

For a Maxwellian plasma, we can use the results of Eq.(12) to express the ratio of E_z/E_y in terms of the dielectric constants. Integrating over the energy (Eq. 45) we obtain a result for the power absorbed which is identical to Eq.(5b), and the damping rates are identical to those obtained previously. We can also examine the case of high perpendicular energies and in this case the first term (TTMP) would dominate in Eq.(51). We see that in general, Eq.(51) includes TTMP (the first term) electron Landau damping (the second term) and the cross term (the product of the first and second terms). Equation (51) has been used in Fokker-Planck code calculations in connection with fast wave current drive in anisotropic plasmas which cannot be described by a Maxwellian.(20) In general, using the fast magnetosonic wave it is difficult to distort the distribution function from a Maxwellian for $\omega/k_\parallel v_{\parallel e} \lesssim 2$, which is the usual region of interest for reasonable single pass damping (i.e., several percent per pass)(9). Thus, for most cases of practical interest the results obtained earlier (Eqs. 13, 21) usually suffice. We shall now discuss some recent experimental results regarding direct electron heating by the fast magnetosonic wave, and point out possible consequences such as fast wave current drive.

XII. Recent Experiments on Direct Electron Heating by Magnetosonic Waves

Magnetosonic waves have been used for nearly two decades in tokamaks to heat ions, and indirectly electrons, by cyclotron (harmonic) resonance (commonly called "ICRF heating"). The earlier experimental results which emphasize minority heating and cyclotron harmonic heating have been summarized by P.L. Colestock,(21) and more recent references can be found in the AIP Conference Proceedings on RF heating.(22) In this section we wish to mention the very recent results obtained on the DIII-D tokamak on direct electron heating by magnetosonic waves,[23–26] and its potential use for current drive in tokamaks. In addition, we should mention the early experiments in this area on JFT-2M[27] and JET[28,] where a small fraction of the RF power (~20%) was absorbed directly by electrons. In the case of JET, the direct electron heating was limited to less than single pass absorption since a strong ion cyclotron resonance layer was present near the plasma center. In JFT-2M the single pass absorption was very weak owing to the high parallel phase velocity of the injected waves ($\omega/k_{\parallel}v_{te} \gtrsim 1.7$).

In DIII-D, a four-strap antenna was used to launch fast waves at 60 MHz.[23,25] The magnetic field is varied in the range $B_T = 1$ to 2 Tesla, so that in the deuterium plasma $f/f_{CD} = 8 - 4$ for this range of fields. The $n_{\parallel}$ spectra of the waves launched peaks at $n_{\parallel} \simeq \pm 9$ for (o, π, 0, π) phasing of adjacent antenna elements (with secondary peaks at $n_{\parallel} = \pm 2.5$) and for ($\pi/2$) phasing between adjacent current straps the spectrum peaks at $n_{\parallel} \simeq 5$ with high directionality. The spectral width is typically $\Delta n_{\parallel} \simeq \pm 2$ about the central value. The evanescent region between the $n_{\parallel}^2 = R$ cut-off layer and the antenna surface preferentially couples the lower-$n_{\parallel}$ components while toroidal effects upshift the coupled $n_{\parallel}$ spectra approximately by the inverse aspect ratio (1/2.7). The Landau absorption condition ($\omega/k_{\parallel}v_{te} \sim 1$) is satisfied for

$$T_e(keV) \simeq (250/n_{\parallel}^2)$$

or for $n_{\parallel} \simeq 9$, $T_{eo} \simeq 3$ keV, and for $n_{\parallel} \simeq 5$, $T_{eo} \simeq 10$ keV. Thus, for the heating phasing we have ideal absorption conditions, while for the current drive phasing, additional heating (for example, with ECH power) is desirable. It should be recognized, however, that for the heating case the effective $n_{\parallel}$ spectrum may be lowered by evanescence, while toroidal effects would upshift it back to near its original value. On the other hand, for current drive phasing $n_{\parallel} \simeq 5$ couples well and toroidal upshifts would result in an effective spectrum of $n_{\parallel} \sim 7$. Thus, we expect that for $T_{eo} \simeq 5$ keV the absorption of the current drive spectra should be satisfactory.

In Fig. 2 we display heating results with symmetric phasing ($\Delta\phi = \pi$) of the antenna straps. This figure shows good heating of electrons, as well as increase of the stored energy in the plasma. In Fig. 3 the calculated effective energy confinement time, normalized to ITER-P-89 scaling, is shown, as well as the calculated single pass absorption. Note that multiple pass absorption must be effective since nearly all power must be absorbed to account for the observed confinement time. There is no apparent dependence on the magnetic field, which indicates very effective multiple pass absorption. On the other hand, the exponential Landau factor is very important, as may be seen in Fig. 4. There is essentially no heating below a threshold electron temperature, while above this threshold there is rapid increase in the heating effectiveness. Finally, in Fig. 5, evidence of H-mode confinement is indicated by pure electron absorption of the fast magnetosonic wave. The threshold power is comparable to that of neutral injection or ECH power. Thus, the pure electron heating regime with the fast magnetosonic wave has been clearly verified in these seminal experiments. Very recently these experiments have been extended to $(\pi/2)$ phasing, and the existence of fast wave current drive has been demonstrated.[25,26]

XIII. Summary and Conclusions

In this treatise we summarized some aspects of plasma heating by magnetosonic waves in the ion cyclotron range of frequencies. We have considered wave propagation in the simplest possible way, namely that of slab geometry, to estimate the power absorption by electron Landau damping, transmit time magnetic pumping, and cyclotron (harmonic) damping. For simplicity, we have used the local approximation of both the real and the imaginary parts of the wave disperions relationship. It should be pointed out that toroidal wave propagation codes have recently verified these results.[30] We have shown that the simple approach outlined by Stix in 1961, namely calculating the $Re(\bar{J}\cdot\bar{E})$ contribution, gives the same result as quasi-linear theory derived by Kennel and Engelman in 1966. While we have concentrated on the relatively "new" concept of electron absorption of magnetosonic waves (advocated by Stix in his 1975 paper), a brief summary of cyclotron harmonic and minority species absorption was also presented. No attempt was made at mathematical rigor (for this see, for example, the book by Swanson[31]). In fact, the approach taken was that of an "experimentalist," who needs relatively simple absorption formulae which can be used for practical estimates. The limitation of this approach was pointed out wherever applicable, in particular, power "loss" by mode-conversion in the presence of dissipation is hard to estimate without a more rigorous approach. The conditions for efficient single pass absorption were obtained. While in present day experiments mainly minority heating is employed, for future applications the importance of absorption by electron Landau damping must be emphasized. A natural by-product of such absorption is the possibility of noninductive current drive by fast waves. In this context, cyclotron (harmonic) abosrption must be regarded as perhaps and undesirable and "parasitic" loss mechanism since it will reduce the already marginal current drive efficiency. Thus, it is clear that efficient bootstrap current generation must be part of any kind of future steady-state tokamak reactor scenario.

Finally, a very recent experiments on JFT-2M, JET, and in particular on DIII-D have clearly verified the practicality of direct electron absorption of the fast magnetosonic wave in the ICRF regime. In addition, initial results on fast wave current drive have been obtained. In the high temperature reactor regime single pass absorption of fast waves by electrons will be sufficient (of the order ~50% or greater). One of the issues still to be resolved is the competing mechanism of absorption of the fast wave by alpha particles for frequencies $\omega \gtrsim \omega_{c\alpha}$. Clearly, optimizing the choice of wave frequency will be important for good current drive efficiency.

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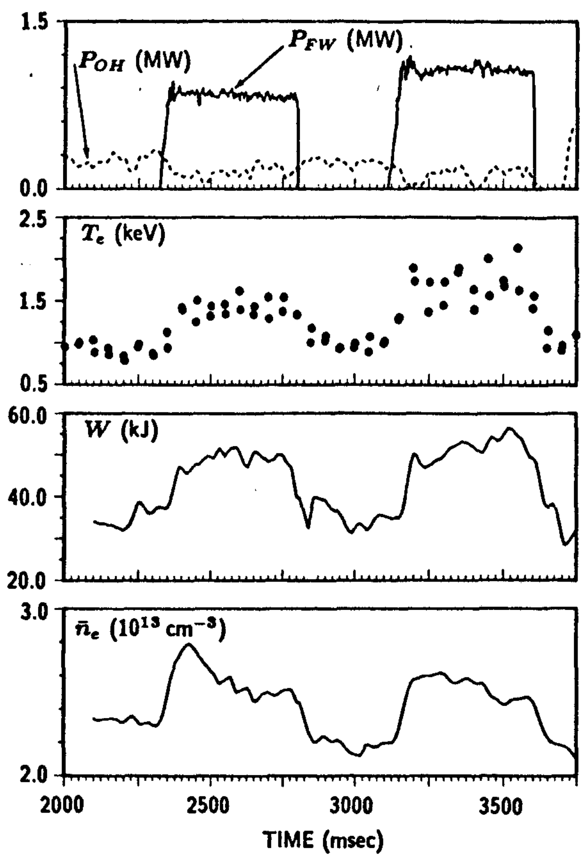

Fig. 2: Time history of (a) net fast-wave and Ohmic power, (b) central electron temperature, (c) plasma stored energy, and (d) line-averaged density. The coupled fast-wave power is 80% of the net power ($I_p = 300$ kA, $B_T = 1.6$ T, $\kappa = 1.4$, deuterium discharge with $\approx 2\%$ hydrogen fraction). (After Ref. 24)

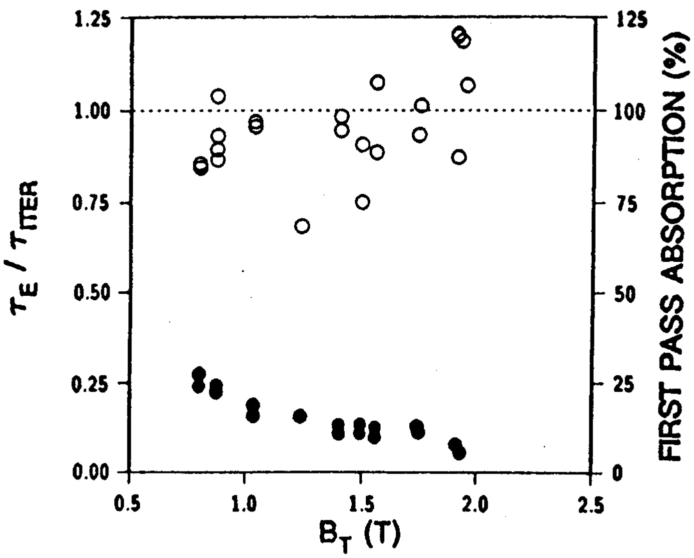

Fig. 3: Energy confinement time (open circles) normalized to the ITER-89 power-law scaling relation as a function of the toroidal magnetic field for high-aspect-ratio plasmas (R = 1.86 m, a = 0.48 m, I_p = 300 kA, $\bar{n} \approx 2.5 \times 10^{19} m^{-3}$, κ = 1.4). Also shown is the calculated first-pass absorption (solid circles). (After Ref. 24)

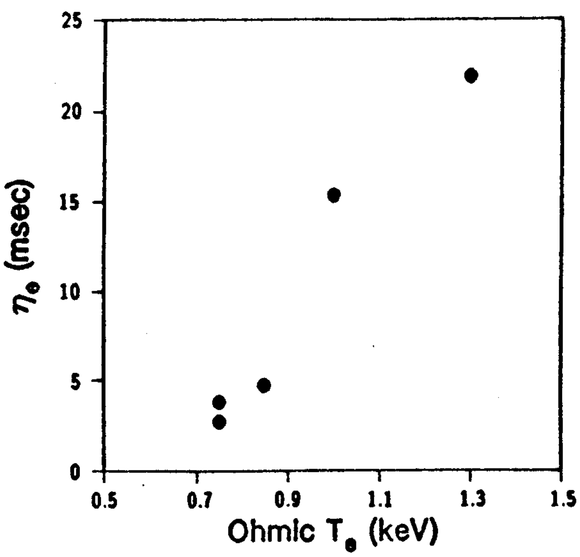

Fig. 4.: Electron heating effectiveness as a function of the target electron temperature [I_p = 510 kA, B_T = 1.4 T, $\bar{n}$ = (1.1 – 1.9) $\times 10^{19} m^{-3}$, κ = 1.9]. (After Ref. 24)

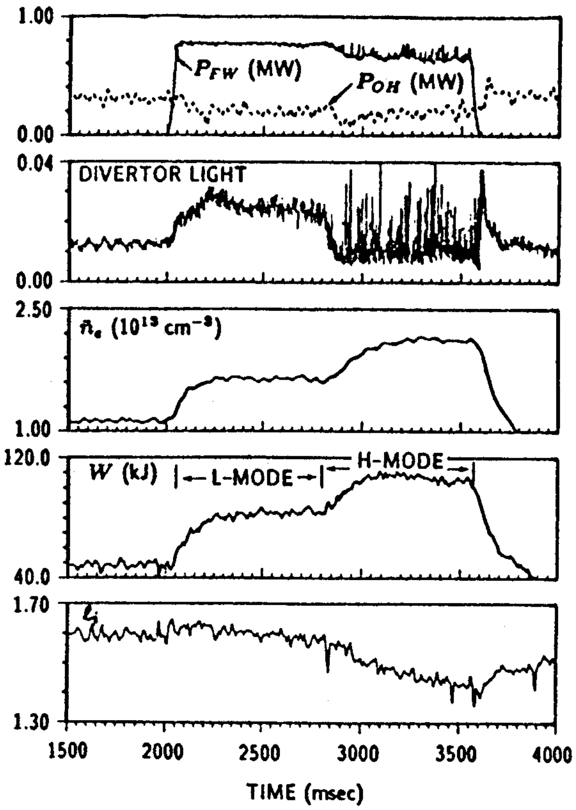

Fig. 5: Time history of (a) net fast-wave and Ohmic power, (b) divertor light, (c) line-averaged electron density, (d) plasma stored energy, and (e) normolized internal inductance. A transition form L-mode confinement to H-mode confinement occurs around 2850 msec, and the H mode continues until the fast-wave power is removed (I_p = 510 kA, B_T = 1.0 T, κ = 1.9). (After Ref. 24)

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