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**SATURATION OF SRS
BY SPATIOTEMPORAL CHAOS
IN COUPLED LANGMUIR DECAY**

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Saturation of SRS by Spatiotemporal Chaos in Coupled Langmuir Decay

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The nonlinear evolution and saturation of stimulated Raman scattering (SRS) in a homogeneous plasma has been studied recently by numerically integrating an appropriate set of one-dimensional model equations that represent the nonlinear coupling of a laser wave field to electron plasma (EPW) and ion-acoustic waves (IAW) [1, 2, 3, 4]. Many experiments have observed SRS in laser-plasma interactions [5], but no theory or numerical simulation accounts for all of these observations. Here we present the results of a new model for understanding the saturated state of SRS based upon the rapid evolution of spatiotemporal chaos (STC) in the Langmuir decay instability (LDI) [6, 7] which is driven by, and coupled to, SRS. We restrict ourselves to interactions at low densities (below quarter-critical) where collapse is not present, and assume that multiple LDI cascades are not dominant, as would be the case for an inhomogeneous plasma. We compare our results with recent experiments [8].

The LDI can be treated approximately as an independent parametric process from SRS provided the growth rate of LDI $\{\gamma^{LDI} \approx (v_{03}/4v_{Te})(\omega_4\omega_5)^{1/2}\}$ exceeds that of SRS $\{\gamma^{SRS} \approx (k_3v_{01}/4)(\omega_p^2/\omega_2\omega_3)^{1/2}\}$. The integer subscript of the (k_j, ω_j) waves are indexed in order of decreasing frequency (i.e. $j = 1$ for the laser to $j = 5$ for the {decay} DIAW). Thus, $v_{01} = eE_1/m_e\omega_1$, where e is the electron charge, m_e is the electron mass and E_1 is the laser electric field amplitude; $v_{03} = eE_3/m_e\omega_3$ is the electron oscillating velocity in the electric field of the EPW; $v_{Te} = (\kappa T_e/m_e)^{1/2}$ is the electron thermal velocity; and ω_4 and ω_5 denote the angular frequencies of the (decay) DEPW and DIAW, respectively. By assuming that $\omega_3 \approx \omega_{pe}$, $\omega_4 \approx \omega_{pe}$, $\omega_5 \approx 2c_s k_3$, where ω_{pe} is the electron plasma frequency, c_s is the sound speed and k_3 is the wavenumber of the EPW, we find that $\gamma^{LDI} > \gamma_{max}^{SRS} \approx (v_{01}/4c)\omega_1$ implies:

$$\frac{E_3}{E_1} > \frac{v_{Te}}{c} \sqrt{\frac{c}{2c_s}} \sqrt{\frac{k_1}{k_3}} \left(\frac{\bar{n}}{1 - \bar{n}} \right)^{1/4}, \quad (1)$$

where (k_3/k_1) is given in [1], $\bar{n} = n/n_{cr}$, n is the electron density, and n_{cr} is the critical electron density. For the parameters corresponding to the computer simulation of [2]: $n/n_{cr} = 0.2$, electron temperature $T_e = 2\text{keV}$, $m_i/Z_i m_e = 3672$ (m_i the ion mass and Z_i the ion charge number), $T_i/Z_i T_e = 0.1$ (T_i is the ion temperature), $I_o \lambda_o^2 = 10^{14}$ (I_o is the intensity of the laser in W/cm^2 and λ_o is its wavelength in μm), we find that $E_3/E_1 > 0.833$. Upon comparison with the simulation (Fig. 1 in [2]), we find that this is indeed the threshold condition for the onset of the LDI. Beyond this threshold value, the growth rate of the LDI is seen [2] to be much larger than that for SRS.

Taking the LDI as an independent parametric process driven by a *growing* (due to SRS) EPW, nonlinearly coupled to a damped DEPW and a damped DIAW, we model it by the

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following nonconservative form of the three wave interaction [6, 7]:

$$(\partial_t + v_3 \partial_x - \gamma_3) a_3 = -K a_4 a_5 + D \partial_x^2 a_3, \quad (2)$$

$$(\partial_t + v_4 \partial_x + \gamma_4) a_4 = K^* a_3 a_5^*, \quad (3)$$

$$(\partial_t + v_5 \partial_x + \gamma_5) a_5 = K^* a_3 a_4^*, \quad (4)$$

where the a 's are the slowly varying complex wave-packet amplitudes ($|a|^2$ is the wave action density [9]), v 's are the group velocities, γ_3 is the growth rate for the EPW, γ_4 and γ_5 are the damping rates, K is the complex coupling coefficient, and D is the diffusion coefficient. This diffusion coefficient has the effect of limiting the growth at short wavelengths [otherwise (2)–(4) do not have a stationary state] and plays the role of a model Landau damping. D is evaluated by expanding the (Landau damping) imaginary part of the frequency for the EPW in the vicinity of k_3 ; the second order term in this expansion gives D . In (2)–(4), $\gamma_3 = \gamma^{SRS}$ is the SRS growth rate, γ_4 is the Landau damping rate of the DEPW, and γ_5 is the Landau damping rate of the DIAW; they are evaluated at their local (average) values of k . Assuming $v_5 \approx 0$, and $v_4 \approx -v_3$, transforming to the frame moving with the group velocity of the EPW, rescaling the length, time and amplitudes by $(\gamma_5/v_3)x \rightarrow x$, $\gamma_5 t \rightarrow t$, $(K/\gamma_5)a_n \rightarrow a_\alpha$, ($n = 3, 4, 5 \rightarrow \alpha = i, j, k$ respectively), and rescaling the damping and diffusion coefficients by $\gamma_3/\gamma_5 \rightarrow \gamma_i$, $\gamma_4/\gamma_5 \rightarrow \gamma_j$, $(\gamma_5/v_3^2)D \rightarrow D$, (2)–(4) become

$$\partial_t a_i - D \partial_x^2 a_i - \gamma_i a_i = -a_k a_j, \quad (5)$$

$$\partial_t a_j - 2\partial_x a_j + \gamma_j a_j = a_i a_k^*, \quad (6)$$

$$\partial_t a_k - \partial_x a_k + a_k = a_i a_j^*. \quad (7)$$

We have numerically integrated (5)–(7) in periodic boundary conditions for the following plasma parameters: $I_o = 5 \times 10^{14}$ W/cm², $\lambda_o = 0.35$ μ m, $T_e = 1$ keV, $T_i = 0.4$ keV, $Z_i = 3.5$, $m_i = 6.5$ amu, and $n/n_{cr} = 0.075$. These parameters correspond to a set of experiments reported in [8]. For these parameters we find that $\gamma_i = 7.62$, $\gamma_j = 3.25$, and $D = 0.011$. The results are shown in Figs. 1 and 2. Figure 1 shows the autocorrelation function $S(x, t) = \langle a_i^*(x', t') a_i(x - x', t - t') \rangle$, where $\langle \cdot \rangle$ indicates an average over all positions x' and time t' . This averaging is equivalent to an ensemble average over initial conditions. $S(x, t)$ decays rapidly over a short correlation length and after a short correlation time. This decorrelation is due to STC and has been discussed extensively in [6, 7]. The wave number spectrum of the EPW autocorrelation function at $t = 0$, $S(q, t = 0) = \langle |a_i(q, t = 0)|^2 \rangle$, where q is the wave number, is shown in Fig. 2. The cutoff in the spectrum reflects the finite correlation length. Figure 2 exhibits the spectral broadening of the EPW seen in the simulations of [2]. The results are independent of the periodic length of the system as long as it is larger than the correlation length.

Weak growth and diffusion in Eq. (5) corresponds to a regime of near integrability where perturbative analysis around the underlying integrable solutions of the conservative three wave interaction is possible [6, 7]. These calculations show that $S(x, t)$ has a correlation time (temporal width) that scales as $1/\gamma_i$ and a correlation length (spatial width) that scales as $\sqrt{D/\gamma_i}$. The average maximum amplitude of the EPW will saturate at a value of approximately $q_0 = \sqrt{\gamma_i/D}$. This leads to an estimate for the amplitude of the correlation function of $S(0, 0) = q_0^2/4$. For the parameters corresponding to our simulation this gives $S(0, 0) \approx 170$ which is close to the value observed in Fig. 1.

These results for LDI are now applied to the saturation of SRS. The SRS itself is described by a three wave interaction for the pump laser wave, the reflected wave, and the EPW. Considering that the EPW reaches the saturated STC state in a short time (of the order of a few $1/\gamma^{SRS}$), and assuming an undepleted laser pump, we need only consider the equation for the reflected wave. In the case of an inhomogeneous plasma with a linear density gradient, the phase mismatch wave number for SRS is: $\Delta k|_{SRS} = k_1 - k_2 - k_3 \equiv \kappa'x$, where x is along the density gradient. Similarly for the LDI, the phase mismatch is: $\Delta k|_{LDI} = k_3 - k_4 - k_5 \equiv \kappa'_L x$. The saturation of SRS due to the density inhomogeneity occurs after a critical time $t_{cr} \approx (2/\gamma^{SRS})\lambda(c/v_3)^{1/2}$ where $\lambda = (\gamma^{SRS})^2/(\kappa'cv_3)$ [10]. Requiring SRS to build up to appreciable amplitudes before dephasing implies that the initial SRS growth time is short compared to t_{cr} and also $\lambda \gg 1$. Since the time scale for saturation of the nonconservative LDI due to STC is of the order of $(\gamma^{SRS})^{-1}$ [6, 7], this saturation time scale is much shorter than t_{cr} , and we take the saturation of LDI to be due to STC. The consequent and ensuing saturation of SRS (on a longer time scale, of the order of t_{cr}) is then due to STC in the presence of dephasing due to inhomogeneity. This also imposes restrictions on the correlation length l_0 , for LDI. For STC we require that $L \gg l_0$. In order that inhomogeneities not be important in the evolution of the LDI we also require that $1/\sqrt{\kappa'} \gg l_0$. Assuming these limits, the evolution equation for the backscattered wave, transformed to the frame moving with the group velocity of the EPW, and rescaling the length, time and amplitudes as before, can be written as:

$$v_R \frac{\partial}{\partial x} a_2(x, \tau) = \Gamma a_1 a_i^*(x, \tau) e^{i\bar{\kappa}'x^2/2}, \quad (8)$$

where $v_R = (v_2 - v_3)/v_3$, $\tau = t - x/v_R$, Γ is the ratio of the SRS to the LDI coupling constants [9], $\Gamma = (k_3 v_{Te}/\omega_{pe})(\omega_4 \omega_5/\omega_1 \omega_2)^{1/2}/\omega_3 \omega_5 \approx (k_3 v_{Te}^2 \omega_{pe}/2cc_s \omega_2 k_1)^{1/2}$, $\bar{\kappa}' = \kappa'(v_3/\gamma_5)^2$, and we have ignored the collisional damping of the backscattered wave. Taking a_i from the saturated STC state of the LDI, (8) is a Langevin-type equation where a_2 is driven by the product of a stochastic pump a_i with the coherent laser pump a_1 . Solving (8) for the square of the amplitude and taking an ensemble average over initial conditions gives:

$$\langle |a_2(\tau)|^2 \rangle = \frac{|\Gamma|^2}{v_R^2} |a_1|^2 \int_{L/2}^{L/2} \int_{L/2}^{L/2} S(x - x', 0) e^{i\bar{\kappa}'(x^2 - x'^2)/2} dx dx'. \quad (9)$$

Assuming $S(x - x', 0) = |a_{is}|^2 \exp\{-(x - x')^2/l_0^2\}$ (see Fig. 1), and $l_0 \ll L$, (9) can be integrated to give

$$\langle |a_2(\tau)|^2 \rangle = \frac{\pi |\Gamma|^2 |a_1|^2}{v_R^2 \bar{\kappa}'} |a_{is}|^2 \left[\text{erf}\left(\beta \frac{L}{2}\right) + \text{erf}\left(\beta^* \frac{L}{2}\right) \right]. \quad (10)$$

where erf is the error function and $\beta = (\bar{\kappa}'l_0/2)/(1 - i\bar{\kappa}'l_0^2/2)^{1/2}$. From the results of our perturbation analysis of STC [6, 7], we have $l_0 \approx 2\pi/q_0$, and $|a_{is}|^2 \approx q_0^2/4$, where $q_0 = (\gamma_i/D)^{1/2}$; recall that $\gamma_i \sim \gamma^{SRS} \sim I_0^{1/2}$ where I_0 is the laser intensity. Hence (10) determines the reflection coefficient $\langle R \rangle \approx (\langle |a_2|^2 \rangle / |a_1|^2)(\omega_1/\omega_2)$, where we have taken the reflected wave group velocity equal to the incoming laser wave group velocity ($\approx c$).

In the inhomogeneous limit when $L^2 \bar{\kappa}' \gg 1$, implying also $Ll_0 \bar{\kappa}' \gg 1$, (10) leads to

$$\langle R \rangle \sim \frac{\pi}{2} \frac{|\Gamma|^2}{v_R^2} \frac{\omega_1}{\omega_2} \frac{\gamma_i}{D} \frac{1}{\bar{\kappa}'}. \quad (11)$$

Thus we find that the inhomogeneous plasma reflection scales as the density inhomogeneity scale length, and as $I_0^{1/2}$; thus the intensity of the scattered light scales as $I_0^{3/2}$.

In the weakly inhomogeneous limit, $L^2\bar{\kappa}' \ll 1$, (10) gives

$$\langle R \rangle \sim \frac{\pi^{3/2}}{2} \frac{|\Gamma|^2}{v_R^2} \frac{\omega_1}{\omega_2} \left(\frac{\gamma_i}{D} \right)^{1/2} L \quad (12)$$

i.e., the reflection scales with plasma size and as $I_0^{1/4}$; hence the intensity of the scattered light scales as $I_0^{5/4}$. This intensity scaling is in good agreement with experimental observations of SRS for laser intensities greater than 10^{14} W/cm² [8].

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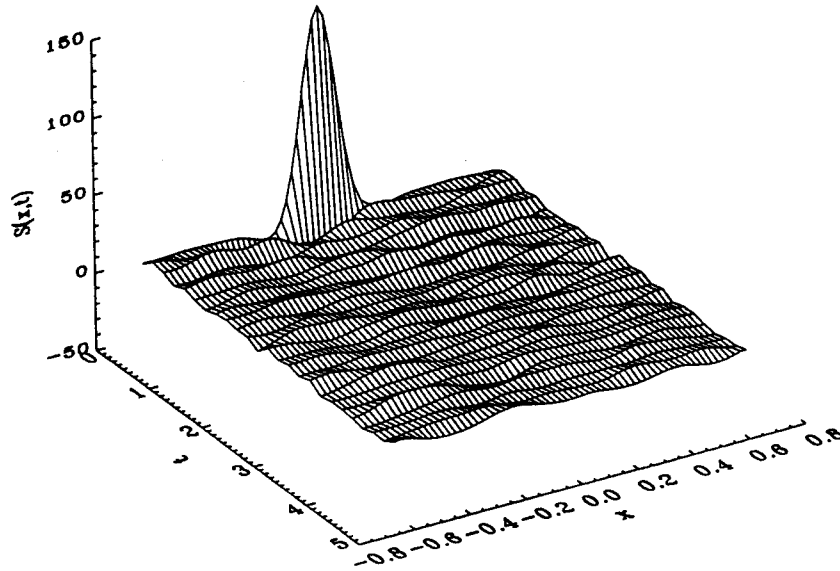


Figure 1: Correlation Function $S(x,t)$ of the EPW

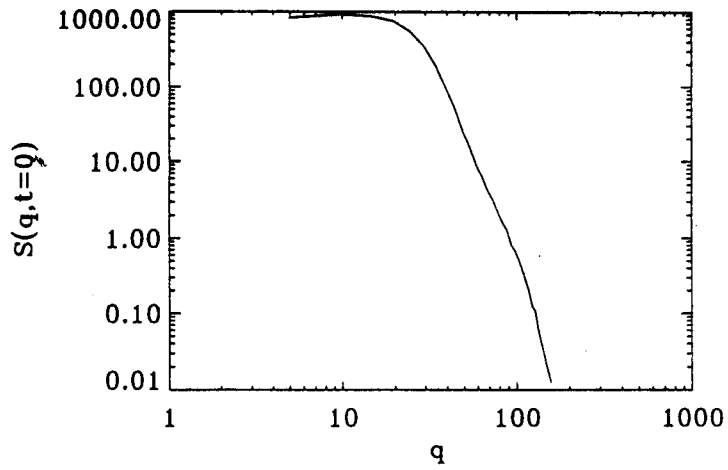


Figure 2: Wave Number Spectrum of the EPW