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Efficiency Scaling Law for the Two-Stream Amplifier

Chiping Chen

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MIT Plasma Fusion Center
Cambridge, Massachusetts 02139 USA

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Chiping Chen
Plasma Fusion Center
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

ABSTRACT

The mechanism of conversion from the kinetic energy of electron beams to electromagnetic wave energy is investigated in the two-stream amplifier. It is shown that the optimal efficiency of the amplifier scales according to $\Delta V/V$ for relativistic electron beams and $(\Delta V/V)^2$ for nonrelativistic electron beams, where ΔV is the axial velocity difference in the two beams and V is the mean axial beam velocity. This scaling law not only explains intrinsically low efficiencies ($\lesssim 5\%$) observed in numerous nonrelativistic two-stream amplifier experiments but also predicts that the optimal efficiency of a relativistic two-stream amplifier is typically greater than that of a nonrelativistic two-stream amplifier by one order of magnitude.

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I. INTRODUCTION

Since research began in the area of relativistic electronics three decades ago, a variety of coherent radiation devices powered by relativistic electron beams has been studied extensively [1, 2]. Vigorous research on such devices has been motivated by the need for high-power microwave and millimeter wave sources in wide applications ranging from the development of high-power, high-resolution radar to the development of advanced radio-frequency (rf) accelerators. Examples of such devices are the free-electron laser (FEL), the cyclotron resonance maser, the relativistic traveling wave tube, the relativistic magnetron, and the relativistic klystron. In essence, all of these relativistic coherent radiation devices were derived from their nonrelativistic counterparts except the cyclotron resonance maser.

However, the relativistic two-stream amplifier (RTSA) [3-5] has not been explored until recently, because earlier two-stream amplifier experiments in which nonrelativistic electron beams were employed yielded low energy conversion efficiencies (5% at best) [6-9], and also because the poor performance of nonrelativistic two-stream amplifiers was not understood [6]. As in nonrelativistic two-stream amplifiers, it has been recognized recently [3-5] that large-amplitude space-charge waves can be excited via a stimulated amplification process in a RTSA employing two co-propagating relativistic electron beams with different axial velocities. Because it is not required to insert either a passive cavity or a slow-wave structure between the rf input and rf output sections, the RTSA avoids the problem of self-oscillations which often occurs in high-gain operation of relativistic traveling wave tube (TWT) amplifiers. Furthermore, with a traveling-wave output structure, the RTSA is anticipated to overcome the problem of rf breakdown which often occurs in high-power operation of relativistic klystrons. Therefore, the RTSA offers an attractive alternative as a high-gain, high-power microwave source, provided it can operate efficiently.

The purpose of this paper is show that the efficiency of the two-stream amplifier can be improved dramatically by increasing the voltages of the electron beams. In particular, use is made of kinetic power theorem [10] to analyze the conversion of the kinetic energy of electron beams into electromagnetic wave energy in the two-stream amplifier. It is shown that the optimal efficiency of the amplifier scales according to $\Delta V/V$ for relativistic electron beams and $(\Delta V/V)^2$ for nonrelativistic electron beams, where ΔV is the axial velocity difference in the two beams and V is the mean axial beam velocity. This scaling law not only explains intrinsically low efficiencies observed in numerous nonrelativistic two-stream amplifier experiments but also predicts that the optimal efficiency of a relativistic two-stream amplifier is typically greater than that of a nonrelativistic two-stream amplifier by one order of magnitude.

II. POWER RELATION

In this section, we make use of kinetic power theorem [10] to derive an analytical expression for the amount of rf power extractable from a two-stream amplifier. Figure 1 illustrates a two-stream amplifier involving two annular electron beams co-propagating through a drift tube of constant radius b placed between the input rf cavity and the output rf cavity. To model the two-stream interaction, we assume that (1) the annular electron beams are infinitely thin, (2) the strength of an applied axial magnetic field is infinite, (3) the drift tube is made up of perfect conductors, (4) there is no background plasma, and (5) the field perturbations are axisymmetric transverse-magnetic (TM) modes with $\partial/\partial\theta = 0$.

Under these assumptions, the equilibrium motion of the electrons is one-dimensional, and is represented by two cold, thin annular electron beams with axial velocities $V_\alpha \vec{e}_z$, currents I_α ($I_\alpha > 0$), and radii a_α ($a_\alpha < b$), where the indices $\alpha = 1, 2$ designate the inner and outer beams, respectively. The equilibrium electron and current densities can be expressed as

$$n_0(r) = \sum_{\alpha=1,2} n_{0\alpha}(r) = \sum_{\alpha=1,2} \frac{I_\alpha}{2\pi e a_\alpha V_\alpha} \delta(r - a_\alpha) \tag{1}$$

and

$$\vec{J}_0(r) = \sum_{\alpha=1,2} \vec{J}_{0\alpha}(r) = -\sum_{\alpha=1,2} e n_{0\alpha}(r) V_\alpha \vec{e}_z, \tag{2}$$

respectively. Here, $r = (x^2 + y^2)^{1/2}$ is the radial coordinate, $-e$ is the electron charge, and $\delta(x)$ is the Dirac delta function.

The linearized cold-fluid equations for the two-stream interaction are:

$$\left(\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\delta E_z = 4\pi \sum_{\alpha=1,2}\left(\frac{\partial \delta\rho_\alpha}{\partial z} + \frac{1}{c^2}\frac{\partial \delta J_\alpha}{\partial t}\right), \tag{3}$$

$$\frac{\partial \delta\rho_\alpha}{\partial t} + \frac{\partial \delta J_\alpha}{\partial z} = 0, \tag{4}$$

$$\left(\frac{\partial}{\partial t} + V_\alpha \frac{\partial}{\partial z}\right)\delta V_\alpha = -\frac{e}{\gamma_\alpha^3 m}\delta E_z|_{r=a_\alpha}. \tag{5}$$

In Eqs. (3)-(5), c is the speed of light *in vacuo*, $\delta E_z = \delta E_z(r,z,t)$ is the axial electric field perturbations for axisymmetric TM modes, $\delta V_\alpha = \delta V_\alpha(z,t)$ is the axial velocity perturbations for beam α, m is the electron rest mass, $\beta_\alpha = V_\alpha/c$ and $\gamma_\alpha = (1-\beta_\alpha^2)^{-1/2}$ are the normalized axial velocity and relativistic mass factor of an unperturbed fluid element in beam α, respectively. The charge and axial current density perturbations are defined by

$$\delta\rho_\alpha(r,z,t) = -e\delta n_\alpha(r,z,t), \tag{6}$$

$$\delta J_\alpha(r,z,t) = -e n_{0\alpha}(r)\delta V_\alpha(z,t) - e\delta n_\alpha(r,z,t)V_\alpha, \tag{7}$$

where $\delta n_\alpha(r)$ is the electron density perturbations in beam α.

Expressing the perturbations in terms of an eigenmode of the form

$$\delta\psi(r,z,t) = \psi(r,z,t) - \psi_0(r) = \delta\psi(r)e^{i(k_z z - \omega t)} \tag{8}$$

with $\psi_0(r)$ denoting an unperturbed fluid or field variable and (k_z, ω) being the axial wave number and frequency, it is readily shown that the eigenvalue equation for the space-charge waves on the two electron beams can be expressed as

$$\left\{\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \left(\frac{\omega^2}{c^2} - k_z^2\right)\left[1 - \sum_{\alpha=1,2} \frac{c^2 \epsilon_\alpha}{a_\alpha \ln(b/a_\alpha)} \frac{\delta(r-a_\alpha)}{(\omega - k_z V_\alpha)^2}\right]\right\}\delta E_z(r) = 0. \tag{9}$$

In Eq. (9), the electric field amplitude $\delta E_z(r)$ must be finite at $r = 0$, and vanish at $r = b$ except at two small windows where couplings take place between the space-charge waves in the drift tube and the electromagnetic waves in the input and output rf cavities. The dimensionless coupling constants ϵ_α are defined by

$$\epsilon_\alpha = \frac{2}{\gamma_\alpha^3 \beta_\alpha}\left(\frac{I_\alpha}{I_A}\right)\ln\left(\frac{b}{a_\alpha}\right), \tag{10}$$

where $I_A = mc^3/e \cong 17$ kA.

Accordingly, the kinetic power theorem can be stated as [10]

$$\mathrm{Re}\nabla\cdot\left(\frac{c}{4\pi}\delta\vec{E}\times\delta\vec{B}^* - \frac{mc^2}{e}\sum_{\alpha=1,2}\delta\gamma_\alpha\delta J_\alpha^*\vec{e}_z\right) = 0. \tag{11}$$

In Eq. (11), 'Re' designates the real part, the 'star' denotes complex conjugate, $\delta\vec{E} = \delta E_r(r,z,t)\vec{e}_r + \delta E_z(r,z,t)\vec{e}_z$ and $\delta\vec{B} = \delta B_\theta(r,z,t)\vec{e}_\theta$ are the electric and magnetic field perturbations for the axisymmetric TM mode, respectively, and $\delta\gamma_\alpha mc^2$ and δJ_α are the electron kinetic energy and axial current density perturbations for beam α, respectively. From Eqs. (4)-(7), it is readily shown that the amplitudes of the electron kinetic energy and axial current density perturbations are related by

$$\delta J_\alpha(r) \;=\; -\frac{ec\omega n_{0\alpha}(r)}{\omega - k_z V_\alpha}\left(\frac{\delta\gamma_\alpha}{\gamma_\alpha^3\beta_\alpha}\right). \tag{12}$$

In other words, the amplitude of the relative current oscillations associated with the space-charge waves can be expressed as

$$\frac{\delta I_\alpha}{I_\alpha} \;=\; -\frac{\omega}{\omega - k_z V_\alpha}\left(\frac{\delta\gamma_\alpha}{\gamma_\alpha^3\beta_\alpha^2}\right), \tag{13}$$

where $I_\alpha = 2\pi e V_\alpha \int_0^b n_{0\alpha}(r) r dr > 0$ and $\delta I_\alpha = 2\pi \int_0^b \delta J_\alpha(r) r dr$.

The amount of rf power extractable from the two-stream amplifier, i.e., the net Poynting flow from the drift tube into the input and output rf cavities, is obtained by integrating Eq. (11) over the cylindrical volume from $r = 0$ to $r = b$ and $z = 0$ to $z = L$, where $z = 0$ is an axial position to the left of the input rf cavity, and $z = L$ is an axial position to the right of the output rf cavity. The result is given by

$$P = \frac{c}{4\pi}\mathrm{Re}\int \delta\vec{E}\times\delta\vec{B}^{*}\cdot d\vec{\sigma} = \frac{mc^2}{e}\sum_{\alpha=1,2} I_\alpha\gamma_\alpha^3\beta_\alpha^2\mathrm{Re}\Big(\frac{k_z V_\alpha}{\omega}-1\Big)\Big(\Big|\frac{\delta I_\alpha}{I_\alpha}\Big|^2_{z=L}-\Big|\frac{\delta I_\alpha}{I_\alpha}\Big|^2_{z=0}\Big). \quad (14)$$

Here, use has been made of the assumption that the drift tube is a perfect conductor and the fact that the Poynting flux through the cross sections at $z = 0$ and $z = L$ is negligibly small because the operating frequency of the two-stream amplifier is required to be below the cutoff frequency of the vacuum drift tube.

We define the efficiency of the two-stream amplifier as

$$\eta = \frac{P}{\sum_{\alpha=1,2}(\gamma_\alpha - 1)mc^2(I_\alpha/e)}. \quad (15)$$

For amplifier operation, the inequality $|\delta I_\alpha(z = L)| \gg |\delta I_\alpha(z = 0)|$ holds. Therefore, it follows from Eqs. (14) and (15) that the efficiency of the two-stream amplifier can be expressed as

$$\eta = \frac{1}{\sum_{\alpha=1,2}(\gamma_\alpha - 1)I_\alpha}\sum_{\alpha=1,2} I_\alpha\gamma_\alpha^3\beta_\alpha^2\mathrm{Re}\Big(\frac{k_z V_\alpha}{\omega}-1\Big)\Big|\frac{\delta I_\alpha}{I_\alpha}\Big|^2_{z=L}, \quad (16)$$

which is one of the main results in this paper.

The axial wave number of an eigenmode of the two-stream system can be determined from the dispersion relation [3]

$$D(\omega, k_z) = 1 - \sum_{\alpha=1}^{2}\frac{\epsilon_\alpha R_\alpha(c^2k_z^2-\omega^2)}{(\omega - k_z V_\alpha)^2} + \frac{\ln(a_2/a_1)}{\ln(b/a_1)}\frac{\epsilon_1\epsilon_2 R_{12}R_2(c^2k_z^2-\omega^2)^2}{(\omega-k_zV_1)^2(\omega-k_zV_2)^2} = 0. \quad (17)$$

The wavelength-dependent geometric factors in Eq. (17) are defined by

$$R_\alpha = \frac{1}{\ln(b/a_\alpha)}\frac{I_0(pa_\alpha)}{I_0(pb)}[I_0(pb)K_0(pa_\alpha) - I_0(pa_\alpha)K_0(pb)] \quad (18)$$

and

$$R_{12} = \frac{1}{\ln(a_2/a_1)}\frac{I_0(pa_1)}{I_0(pa_2)}[I_0(pa_2)K_0(pa_1) - I_0(pa_1)K_0(pa_2)], \quad (19)$$

where $p^2 = k_z^2 - \omega^2/c^2$, and $I_0(x)$ and $K_0(x)$ are the first- and second-kind modified Bessel functions of the zeroth order, respectively.

For a given frequency, there are four physically acceptable solutions to $D(\omega, k_z) = 0$, corresponding to four space-charge waves. For an unstable two-stream system, the four waves consist of two oscillatory (stable) waves with $\text{Im}k_z = 0$, a spatially growing (unstable) wave with $\text{Im}k_z < 0$, and a spatially decaying wave with $\text{Im}k_z > 0$. The spatially growing and decaying waves result from the coupling of the slow space-charge wave on the fast beam and the fast space-charge wave on the slow beam. The unstable wave dominates the interaction process, whereas the rest of the waves play an important role in determining launching (insertion) losses. Practically, only the unstable space-charge wave is present once the current oscillations on the electron beams enter the exponential-gain regime in the amplifier. Therefore, the efficiency in Eq. (16) can be expressed as

$$\eta = \frac{1}{\sum_{\alpha=1,2}(\gamma_\alpha - 1)I_\alpha}\left[I_1\gamma_1^3\beta_1^2\left(\frac{\beta_1}{\beta_u} - 1\right)\left|\frac{\delta I_1}{I_1}\right|_{z=L}^2 + I_2\gamma_2^3\beta_2^2\left(\frac{\beta_2}{\beta_u} - 1\right)\left|\frac{\delta I_2}{I_2}\right|_{z=L}^2\right], \tag{20}$$

where $\beta_u c = \text{Re}(\omega/k_z)_u$ is the real part of the phase velocity of the unstable space-charge wave for the coupled two-stream system. Because $\beta_u c$ situates between the equilibrium velocities of the two beams, i.e., $\beta_1 < \beta_u < \beta_2$, (where $\beta_2 > \beta_1$ has been assumed), it follows from Eq. (20) that the slow space-space wave on the fast beam (i.e., beam 2) makes a *positive* contribution to the rf power, whereas the fast space-space wave on the slow beam (i.e., beam 1) makes a *negative* contribution to the rf power.

Equation (20) is a rigorous result for $|\delta I_\alpha/I_\alpha|_{z=L} \ll 1$. However, in the same spirit as in estimating the *nonlinear* efficiency of an FEL [11], Eq. (20) may be used to estimate the efficiency of the two-stream amplifier for operation at saturation where the relative current oscillation amplitudes reach $|\delta I_\alpha/I_\alpha|_s \approx 1$ for both electron beams. Analytical estimates for the saturation amplitudes of the current oscillations have been obtained in [5] and have been found to be in agreement with the results from particle-in-cell simulations [3].

As a general remark, it should be emphasized that only when *one* eigenmode with the sinusoidal dependence $e^{i(k_z z-\omega t)}$ dominates in a two-stream system are Eqs. (16) and (20) applicable. This is the case for two-stream amplifiers but not necessarily for stable two-stream systems [12].

III. EFFICIENCY SCALING LAW

To derive a simple efficiency scaling law, we now consider the long-wavelength limit in which the conditions $k_z^2 b^2 \ll \gamma_1^2$ and $k_z^2 b^2 \ll \gamma_2^2$ are satisfied. We further assume that $\beta_2 > \beta_1$, $a_1 = a_2 = a < b$, and $\epsilon_1 = \epsilon_2 = \epsilon \ll 1$, which imply that $I_2 > I_1$ and $I_1/(\gamma_1^3\beta_1) = I_2/(\gamma_2^3\beta_2)$. Let $\Delta\beta = \Delta V/c = \beta_2 - \beta_1$, $\beta = V/c = (\beta_1+\beta_2)/2$, and $\gamma = (1-\beta^2)^{-1/2}$. It can be shown [3, 4] that when $\Delta\beta = (3\epsilon)^{1/2}/\gamma$, the two-stream interaction possesses the maximum spatial growth rate and the solutions to the dispersion relation (17) are given by

$$\frac{ck_z}{\omega} = \begin{cases} \frac{1}{\beta} \pm \frac{\sqrt{5}}{2}\frac{\Delta\beta}{\beta^2}, \\ \frac{1}{\beta} \pm i\frac{1}{\sqrt{12}}\frac{\Delta\beta}{\beta^2}, \end{cases} \tag{21}$$

which yield $\beta_u = \mathrm{Re}(\omega/ck)_u = (\beta_1+\beta_2)/2$. Substituting $\beta_u = (\beta_1+\beta_2)/2$ into Eq. (20), and making use of $I_1/(\gamma_1^3\beta_1) = I_2/(\gamma_2^3\beta_2)$, we find that the optimal efficiency of the two-stream amplifier is given by

$$\eta = \frac{\Delta\beta}{2\beta[\gamma_1^3\beta_1(\gamma_1-1)+\gamma_2^3\beta_2(\gamma_2-1)]}\left(\gamma_2^6\beta_2^3\left|\frac{\delta I_2}{I_2}\right|_s^2 - \gamma_1^6\beta_1^3\left|\frac{\delta I_1}{I_1}\right|_s^2\right), \tag{22}$$

where subscript s denotes saturation at $z = L$. From Eq. (22), it is evident that the amplifier efficiency exhibits a strong dependence on the relativistic mass factors γ_1 and γ_2. It is also evident that for $|\delta I_1/I_1|_s \cong |\delta I_2/I_2|_s$, the primary electron beam (i.e., beam 2) produces more rf power than the secondary electron beam (i.e. beam 1) absorbs, so that a net amount of rf power is generated by the two electron beams.

In the *nonrelativistic* regime (i.e., $\beta_\alpha \ll 1$ and $\gamma_\alpha \cong 1$), the two terms in Eq. (22) are comparable in size. As a result, the optimal efficiency of the nonrelativistic two-stream amplifier scales according to

$$\eta = \frac{3}{2}\left(\frac{\Delta\beta}{\beta}\right)^2 \left|\frac{\delta I}{I}\right|_s^2 \cong 1.3\left(\frac{\Delta\beta}{\beta}\right)^2, \tag{23}$$

where $|\delta I/I|_s = |\delta I_2/I_2|_s \cong |\delta I_1/I_1|_s = 0.93$ according to [5]. For $\Delta\beta/\beta = 0.2$, which was typical in previous two-stream amplifier experiments [8, 9], Eq. (23) predicts that the optimal efficiency of the nonrelativistic two-stream amplifier is $\eta = 5.2\%$. This result provides the *first* explanation for intrinsically low efficiencies (≤ 5 %) observed earlier in numerous two-stream amplifier experiments with nonrelativistic electron beams [6, 8, 9].

In the *relativistic* regime, however, the d.c. power of the secondary electron beam is negligibly small compared with that of the primary electron beam, i.e., $(\gamma_2-1)I_2 \gg (\gamma_1-1)I_1$, so that the second term is negligible in comparison with the first term in Eq. (22). Therefore, the optimal efficiency of the relativistic two-stream amplifier can be expressed as

$$\eta = \frac{\gamma_2^3\beta_2^2}{2(\gamma_2-1)}\left(\frac{\Delta\beta}{\beta}\right)\left|\frac{\delta I_2}{I_2}\right|_s^2. \tag{24}$$

Equation (24) reveals that the amplifier efficiency scales as $\eta \propto \Delta\beta/\beta = \Delta V/V$ in the relativistic regime, rather than $\eta \propto (\Delta\beta/\beta)^2 = (\Delta V/V)^2$ in the nonrelativistic regime. Furthermore, the factor $\gamma_2^3\beta_2^2/[2(\gamma_2-1)]$ in Eq. (24) is greater than unity and becomes greater than two for $\gamma_2 > 1.6$. Since $|\delta I_2/I_2|_s \cong 1$ and $\Delta\beta/\beta \cong 0.2$ are typical parameters for two-stream amplifier, the optimal efficiency of a relativistic two-stream amplifier is expected to be greater than that of a nonrelativistic two-stream amplifier by one order of magnitude.

As an example, for the choice of system parameters corresponding to: $f = 3.0$ GHz, $b = 2.54$ cm, $a_1/b = a_2/b = 0.8$, $(\gamma_1 - 1)mc^2 = 250$ keV, $I_1 = 1.3$ kA, $(\gamma_2 - 1)mc^2 = 520$ keV, $I_2 = 5.3$ kA, $\epsilon = \epsilon_1 = \epsilon_2 = 1.57 \times 10^{-2}$, and $\Delta\beta/\beta = (3\epsilon)^{1/2}/\gamma\beta = 0.16$, the estimated efficiency of the RTSA is $\eta = 0.54 \times |\delta I_2/I_2|_s^2$, indicating $\eta = 30\%$ for estimated $|\delta I_2/I_2|_s = 0.75$. In this example, the d.c. power of the primary electron beam is 2.76 GW, whereas the d.c. power of the secondary electron beam is only 0.325 GW.

IV. CONCLUSION

Use was made of kinetic power theorem to estimate the amount of rf power extractable from the two-stream amplifier. It was shown that the optimal efficiency of the amplifier scales according to $\Delta V/V$ for relativistic electron beams and $(\Delta V/V)^2$ for nonrelativistic electron beams, where ΔV is the axial velocity difference in the two beams and V is the mean beam velocity. This scaling law not only explains intrinsically low efficiencies observed in numerous nonrelativistic two-stream amplifier experiments but also predicts that the optimal efficiency of a relativistic two-stream amplifier is typically greater than that of a nonrelativistic two-stream amplifier by one order of magnitude. These results are important for future experimental studies of relativistic two-stream amplifiers.

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FIGURE CAPTION

Fig. 1 Schematic of a two-stream amplifier employing two annular electron beams with radii a_1 and a_2.

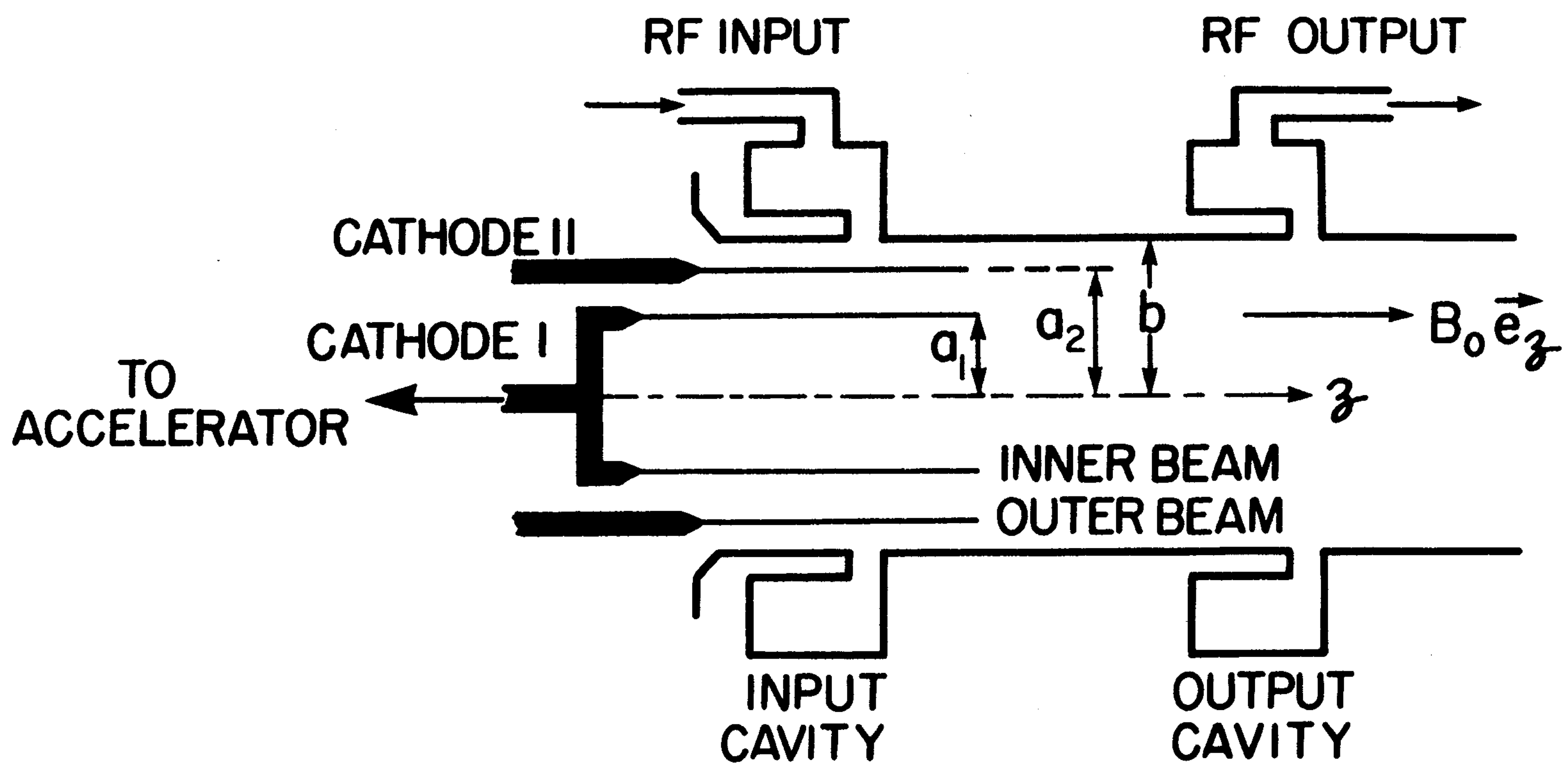
RF INPUT
RF OUTPUT
CATHODE II
CATHODE I
TO
ACCELERATOR
a_1
a_2
b
$B_o\vec{e}_z$
z
INNER BEAM
OUTER BEAM
INPUT
CAVITY
OUTPUT
CAVITY

Figure 1